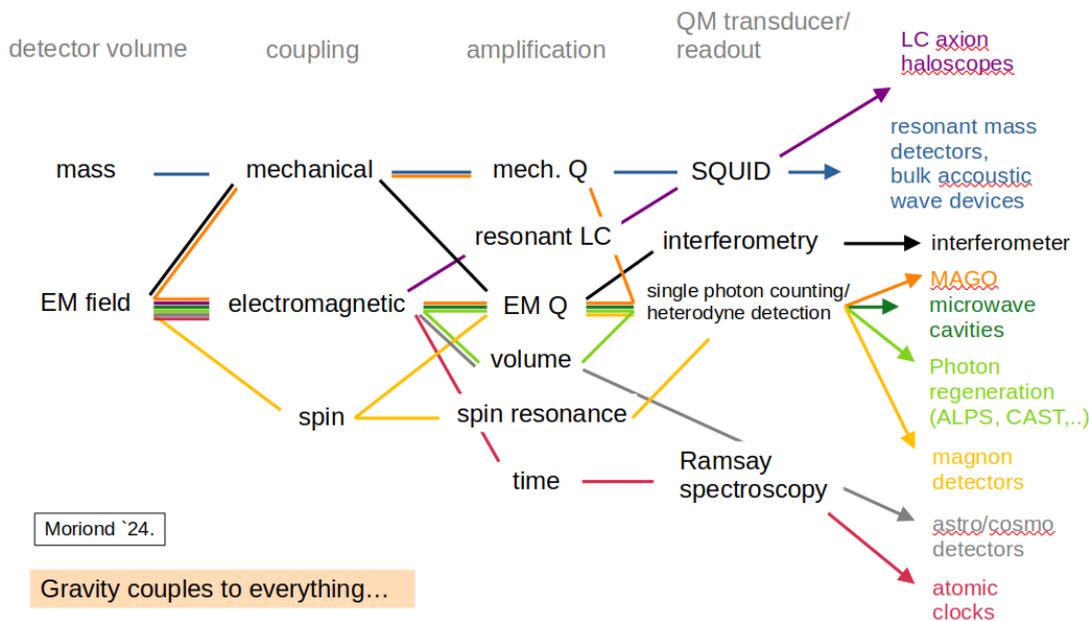




Searching for high-frequency gravitational waves

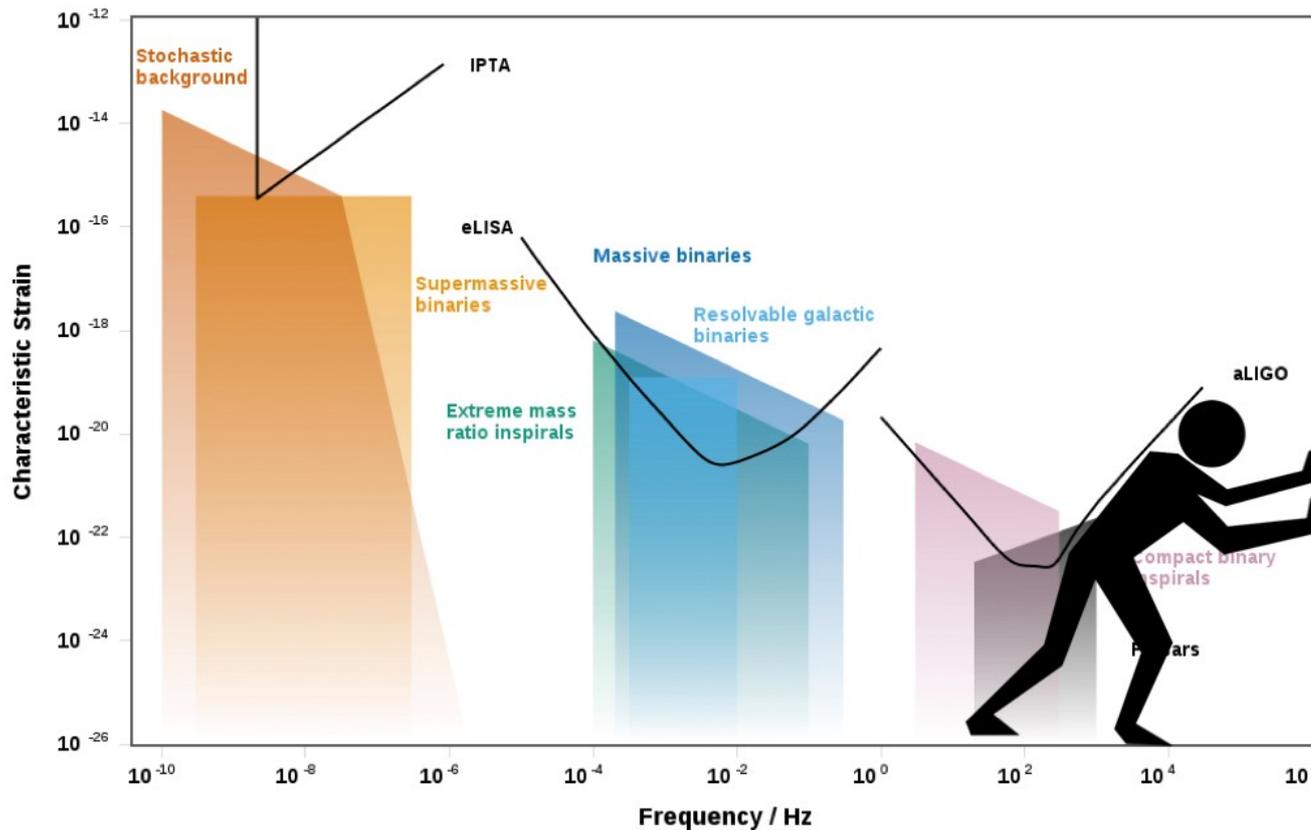
Valerie Domcke
CERN



MITP Workshop "Quantum Sensing meets UHF Gravitational Waves"
Jun 30, 2025

Based on work with Sebastian Ellis, Camilo Garcia-Cely, Joachim Kopp, Sung Mook Lee, Kalirae Pappas, Nick Rodd
+ the UHF-GW initiative
+ the ABRACADABRA collaboration

GW searches above \sim kHz ?



Why go here ?

No known astro foregrounds

SGWBs sourced at energies up to the GUT scale (hard!)

In the mean time:

Exotic astrophysical sources (PBHs, superradiance,..)

Neutron star signals up to \sim MHz ?

Outline

- 2025 Update of UHF GW Living Review
- ABRA-GW Results
- Madmax (dielectric axion haloscope) as a GW detector
- Magnetic Weber Bars

2025 Update of UHF GW Living Review

Challenges and Opportunities of Gravitational Wave Searches above 10 kHz

Nancy Aggarwal^a · Odylio D. Aguiar^b · Diego Blas^{c,d} ·
Andreas Bauswein^e · Giancarlo Cella^f · Sebastian Clesse^g ·
Adrian Michael Cruise^h · Valerie Domcke^{i,*} · Sebastian Ellis^{j,*} ·
Daniel G. Figueroa^k · Gabriele Franciolini^{i,*} ·
Camilo Garcia-Cely^k · Andrew Geraci^a · Maxim Goryachev^l ·
Hartmut Grote^m · Mark Hindmarsh^{n,o} · Asuka Ito^{p,q} ·
Joachim Kopp^{i,r,*} · Sung Mook Lee^{i,*} · Killian Martineau^s ·
Jamie McDonald^t · Francesco Muia^u · Nikhil Mukund^v ·
David Ottaway^w · Marco Peloso^{x,y} · Krisztian Peters^z ·
Fernando Quevedo^{u,α} · Angelo Ricciardone^{f,β} ·
Andreas Ringwald^z · Jessica Steinlechner^{γ,δ,ε} ·
Sebastian Steinlechner^{γ,δ} · Sichun Sun^ζ · Carlos Tamarit^r ·
Michael E. Tobar^l · Francisco Torrenti^η · Caner Ünal^{θ,λ} ·
Graham White^μ

Corresponding authors: Valerie Domcke (E-mail: valerie.domcke@cern.ch) · Sebastian Ellis (E-mail: Sebastian.Ellis@unige.ch) · Gabriele Franciolini (E-mail: gabriele.franciolini@cern.ch) · Joachim Kopp (E-mail: jkopp@uni-mainz.de) · Sung Mook Lee (E-mail: sungmook.lee@cern.ch)

UHF GW initiative:

<https://www.ctc.cam.ac.uk/activities/UHF-GW.php>
Webpage, mailing list → to be updated

Living Review update:
2501.11723

Big thank you to all contributors
and in particular to the team of
coordinators: Sebastian Ellis,
Gabriele Franciolini, Joachim Kopp,
and Sung Mook Lee.

2025 Update of UHF GW Living Review

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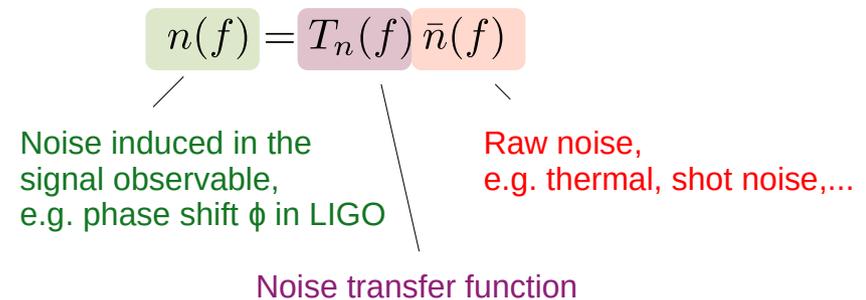
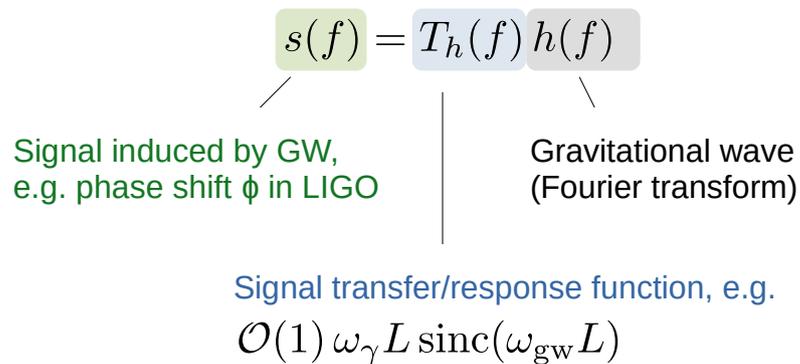
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PBH discussion summary plots notation / framework

Extended detector section, grouped by technology, with plots & tables

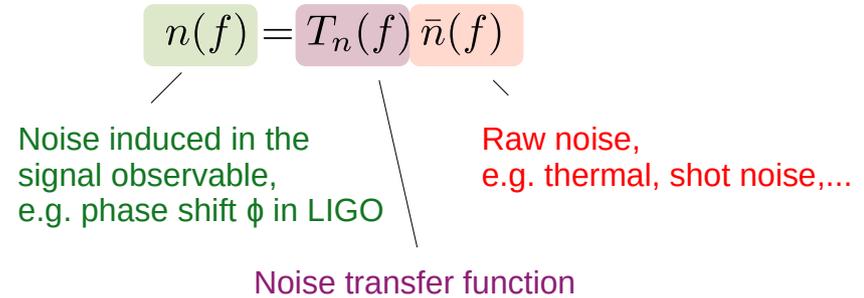
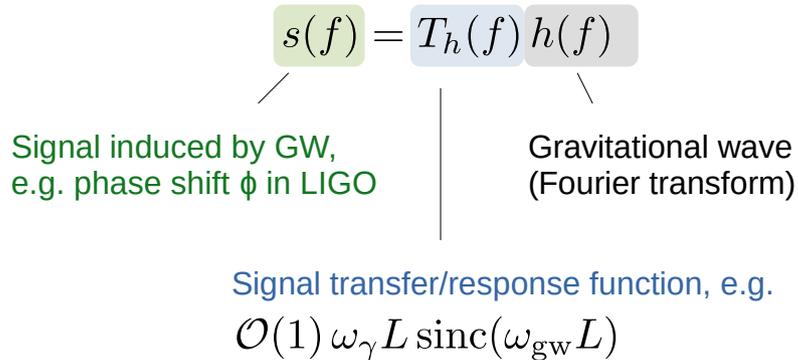
Comparing apples to apples

Signal and noise:



Comparing apples to apples

Signal and noise:



Noise-equivalent strain power spectral density:

$$S_h^{\text{noise}}(f) = S_{\bar{n}}(f) \frac{|T_n(f)|^2}{|T_h(f)|^2}$$

Noise power spectrum
in units of GW strain

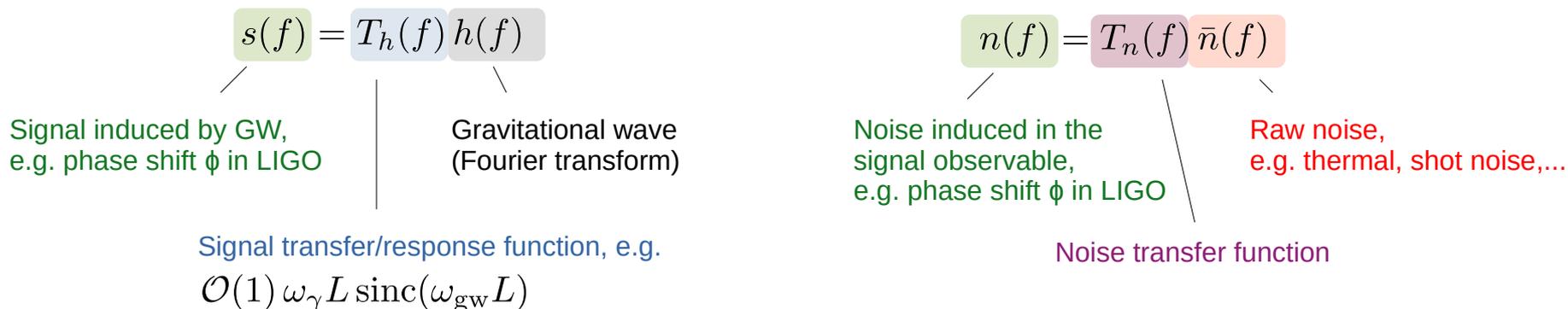
Characterization of
detector sensitivity

$$\langle X(f) X^*(f') \rangle = S_X(f) \delta(f - f')$$

PSD convention. X = h, n, s, ...

Comparing apples to apples

Signal and noise:



Noise-equivalent strain power spectral density:

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Noise power spectrum
in units of GW strain

Characterization of
detector sensitivity

Signal-to-noise ratio:

$$\text{SNR}^{\text{lin}} = \left[2 \Delta t \int_0^\infty df \frac{S_h(f)}{S_h^{\text{noise}}(f)} \right]^{1/2}, \quad \text{Detector linear in } h$$

$$\text{SNR}^{\text{quad}} = \left[2 \Delta t \int_0^\infty df \left(\frac{S_h(f)}{S_h^{\text{noise}}(f)} \right)^2 \right]^{1/2}, \quad \text{Detector quadratic in } h$$

/

O(1)

High frequency GWs

Time and frequency regime
in which signal and detector overlap

Different types of signals

Stochastic gravitational wave backgrounds:

$$\Omega_{\text{GW}}(f) = \frac{4\pi^2}{3H_0^2} |f|^3 S_h(|f|)$$

Overlap function < 1 if multiple detectors

$$\text{SNR} \simeq \frac{3H_0^2}{4\pi^2} \left[t_{\text{int}} \int_0^\infty df \left(\frac{\Gamma(f) \Omega_{\text{GW}}(f)}{f^3 S_h^{\text{noise}}(f)} \right)^2 \right]^{1/2}$$

$$h_{c,\text{sto}} \equiv \sqrt{f S_h(f)} = \frac{H_0}{2\pi f} (3\Omega_{\text{GW}}(|f|))^{1/2}$$

» 1

→ PLS (power law integrated sensitivity) curves

Different types of signals

Stochastic gravitational wave backgrounds:

$$\Omega_{\text{GW}}(f) = \frac{4\pi^2}{3H_0^2} |f|^3 S_h(|f|)$$

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Overlap function < 1 if multiple detectors

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» 1

→ PLS (power law integrated sensitivity) curves

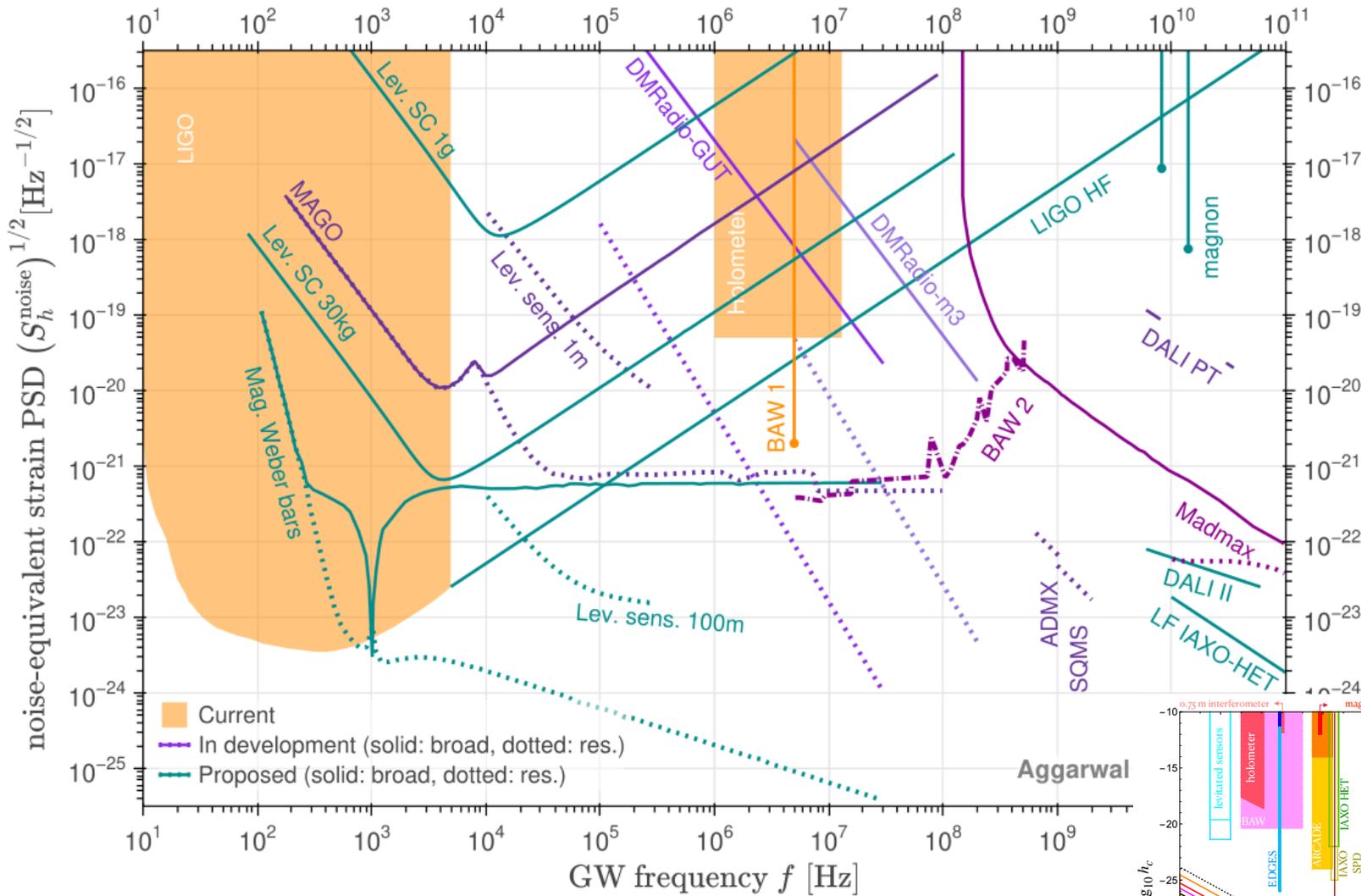
Transient gravitational waves:

$$S_h^{\text{lin(quad)}}(f) \gtrsim \text{SNR}^2 \times S_h^{\text{noise}}(f) \left(\frac{1}{\Delta f \Delta t} \right)^{1(1/2)}$$

$$h_{c,\text{insp}}(f) = 2f \tilde{h}(f) \simeq \sqrt{N_{\text{cycles}}} h_0$$

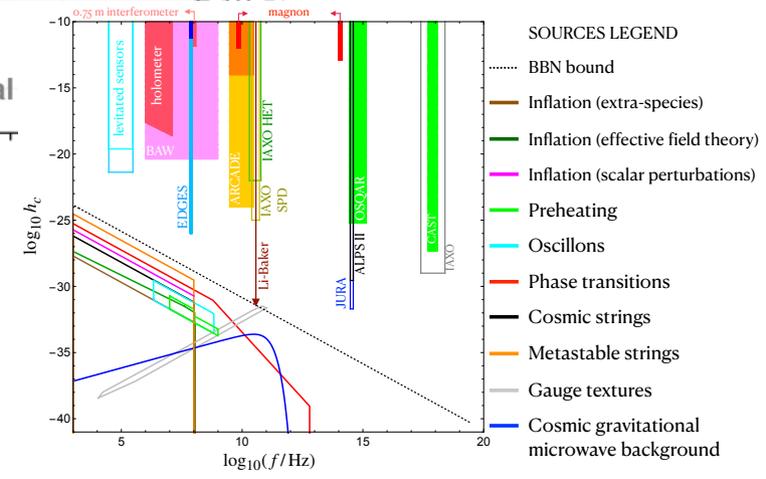
→ specifically for PBH mergers: translate to detector reach (in distance)

Noise-equivalent strain sensitivity



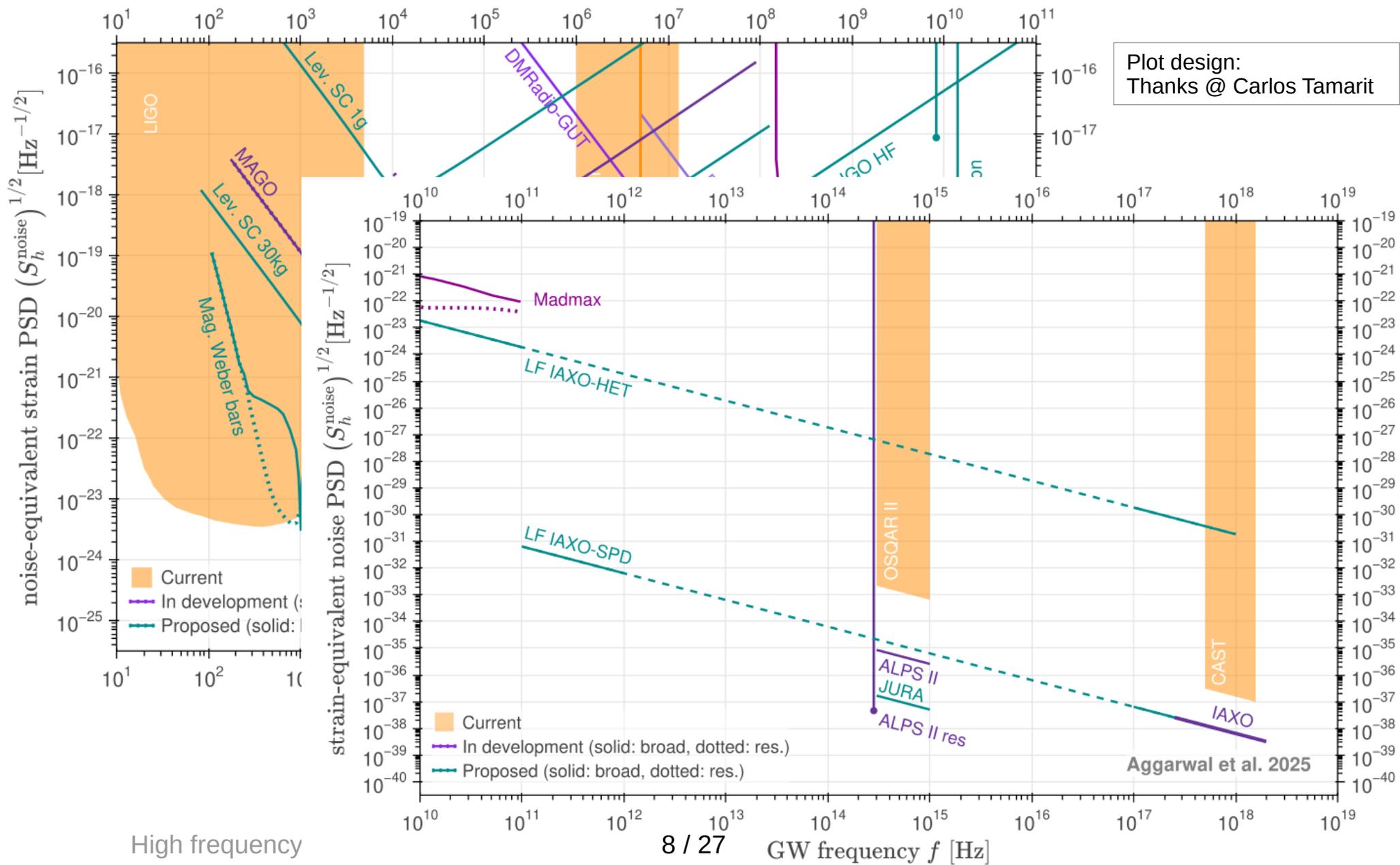
Plot design:
Thanks @ Carlos Tamarit

Previous version:

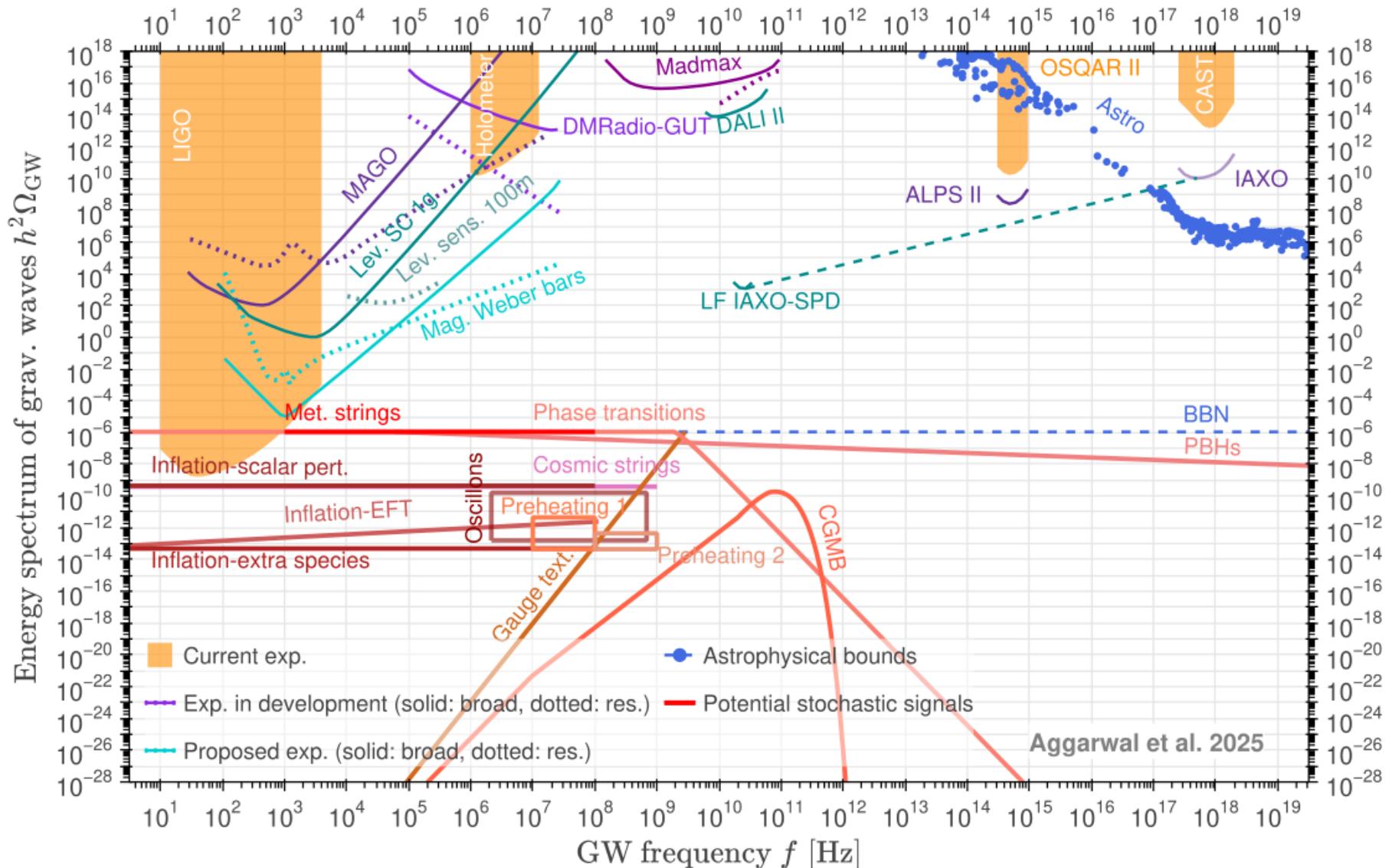


High frequency GWs

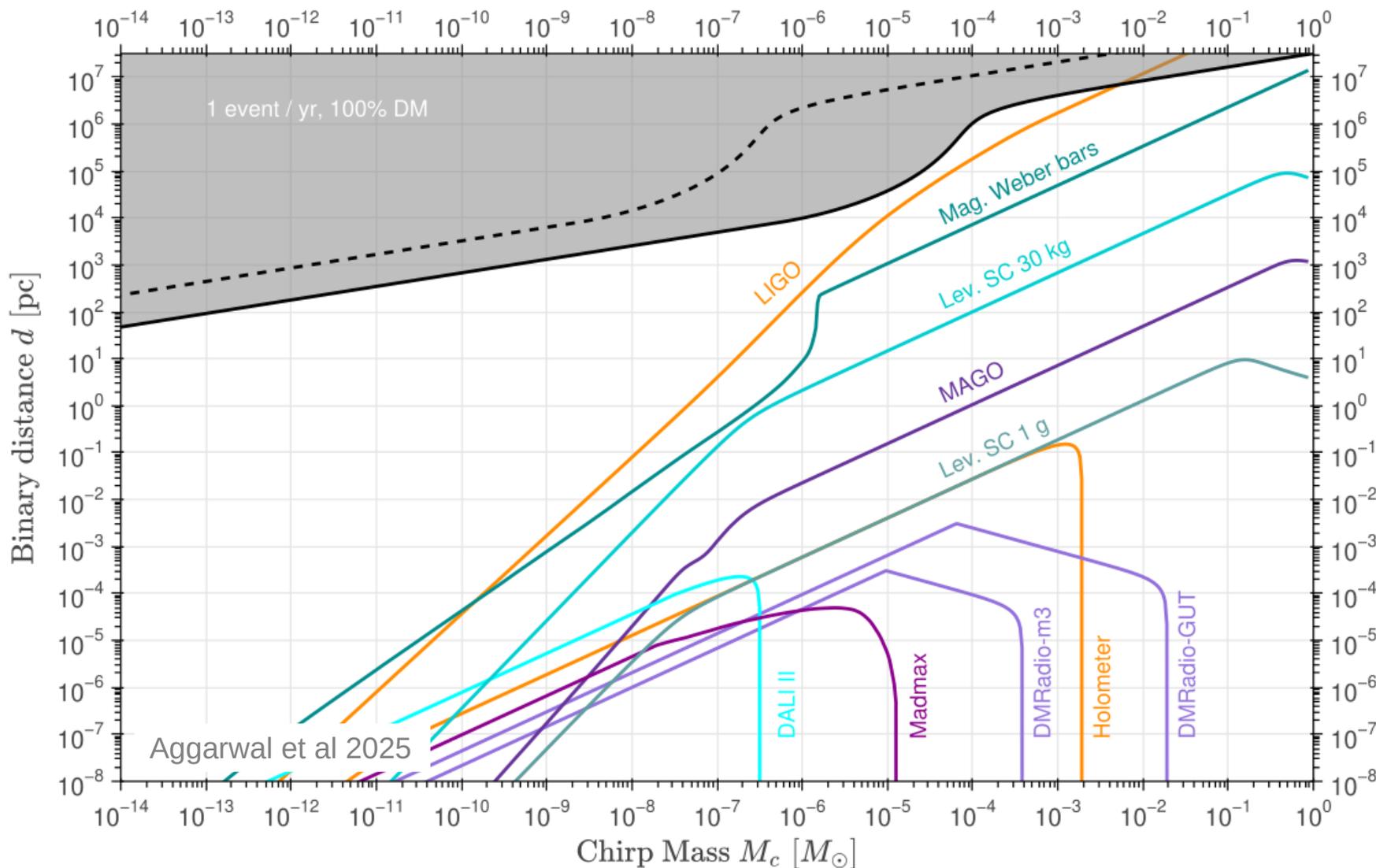
Noise-equivalent strain sensitivity



Sensitivity to stochastic GW backgrounds



Sensitivity to PBH mergers



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- 2025 Update of UHF GW Living Review
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- Madmax (dielectric axion haloscope) as a GW detector
- Magnetic Weber Bars

GW electrodynamics

Classical electrodynamics + linearized GR, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $F_{\mu\nu} = \bar{F}_{\mu\nu} + F_{\mu\nu}^h$:

$$\partial_\mu (\sqrt{-g} g^{\mu\alpha} F_{\alpha\beta} g^{\beta\nu}) = 0$$

$$\rightarrow \partial_\nu F_h^{\mu\nu} = j_{\text{eff}}^\mu = (-\nabla \cdot \mathbf{P}, \nabla \times \mathbf{M} + \partial_t \mathbf{P})$$

effective current
effective polarization vector
effective magnetization vector

with

$$P_i = -h_{ij} \bar{E}_j + \frac{1}{2} h \bar{E}_i + h_{00} \bar{E}_i - \epsilon_{ijk} h_{0j} \bar{B}_k,$$

$$M_i = -h_{ij} \bar{B}_j - \frac{1}{2} h \bar{B}_i + h_{jj} \bar{B}_i + \epsilon_{ijk} h_{0j} \bar{E}_k,$$

induced at linear order in h
in presence of external E,B field

VD, Garcia-Cely, Rodd `22

Direct analogy with axion electrodynamics

$$\mathcal{L} \supset g_{a\gamma\gamma} a \mathbf{E} \cdot \mathbf{B} \rightarrow \mathbf{P} = g_{a\gamma\gamma} a \mathbf{B}, \quad \mathbf{M} = g_{a\gamma\gamma} a \mathbf{E}$$

McAllister et al `18
Tobar, McAllister, Goryachev `19
Quellet, Bogorad `19

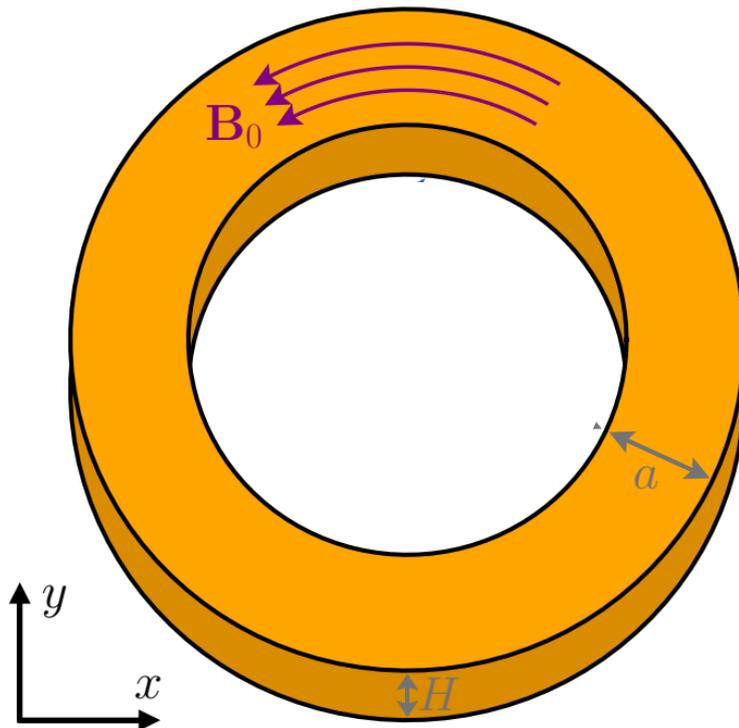
effective source terms in Maxwell's equation due to GW

GW signal in low-mass axion haloscopes

eg ABRACADABRA, SHAFT, DM Radio:

VD, Garcia-Cely, Lee, Rodd '22 + '23

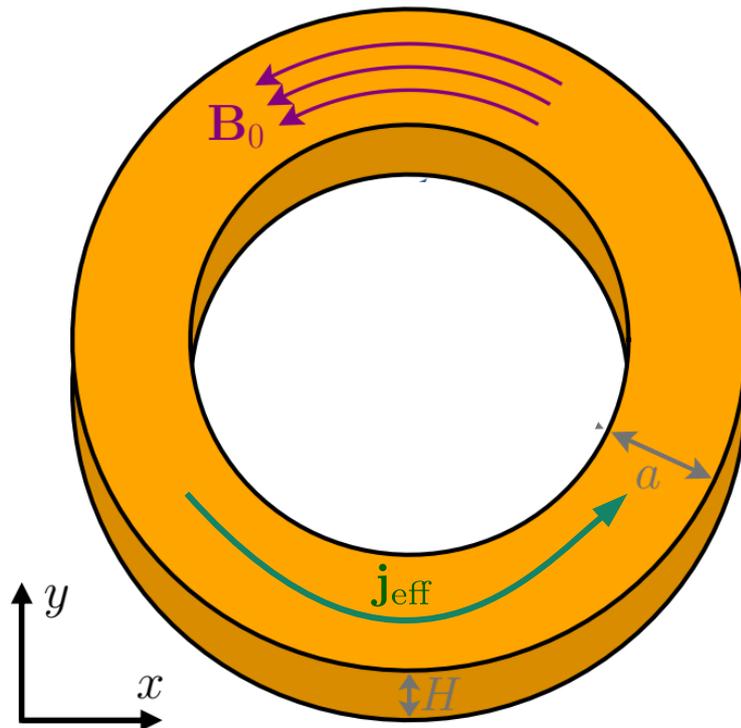
static magnetic field (i.e. rigid detector)



GW signal in low-mass axion haloscopes

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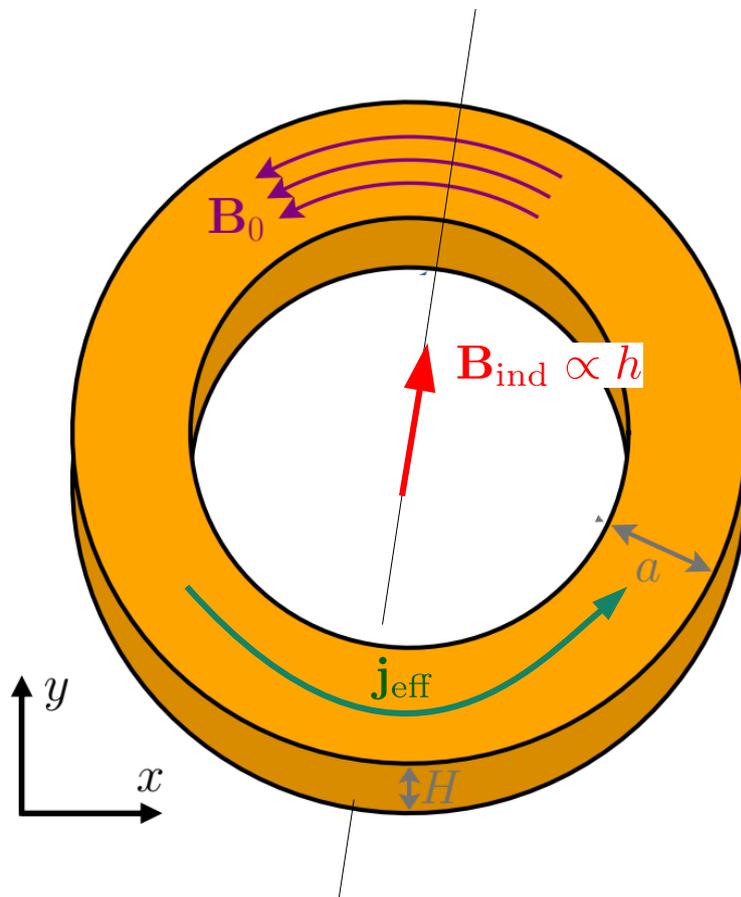
static magnetic field (i.e. rigid detector)

effective current $\sim h(\omega L)^2 B_0$

GW signal in low-mass axion haloscopes

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static magnetic field (i.e. rigid detector)

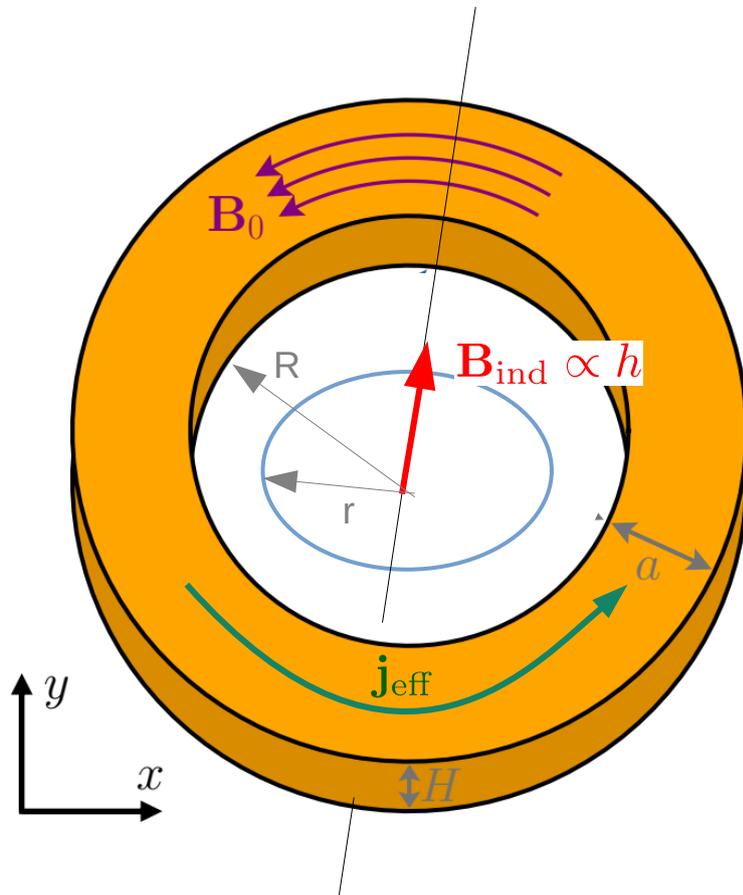
effective current $\sim h(\omega L)^2 B_0$

induced oscillating magnetic field

GW signal in low-mass axion haloscopes

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static magnetic field (i.e. rigid detector)

effective current $\sim h(\omega L)^2 B_0$

induced oscillating magnetic field

measure magnetic flux ($\sim h$)
through pickup loop

at leading order in (ωR) :

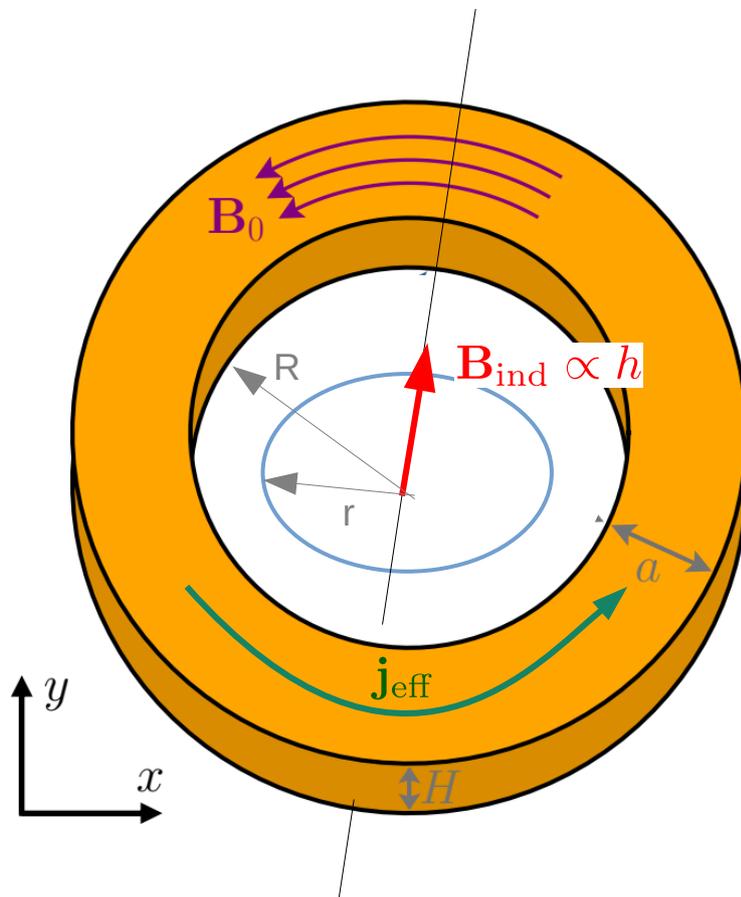
$$\Phi_{\text{gw}} = \frac{i e^{-i\omega t}}{16\sqrt{2}} h \times \omega^3 B_0 \pi r^2 R a (a + 2R) s_{\theta_h}^2$$

$$\sim (\omega L)^3 h B_0 L^2 \rightarrow (\omega L)^2 h B_0 L^2 \text{ fig-8 loop}$$

GW signal in low-mass axion haloscopes

eg ABRACADABRA, SHAFT, DM Radio:

VD, Garcia-Cely, Lee, Rodd '22 + '23



match to axion induced flux to estimate sensitivity to GW signals

static magnetic field (i.e. rigid detector)

effective current $\sim h(\omega L)^2 B_0$

induced oscillating magnetic field

measure magnetic flux ($\sim h$) through pickup loop

at leading order in (ωR) :

$$\Phi_{\text{gw}} = \frac{i e^{-i\omega t}}{16\sqrt{2}} h \times \omega^3 B_0 \pi r^2 R a (a + 2R) s_{\theta_h}^2$$

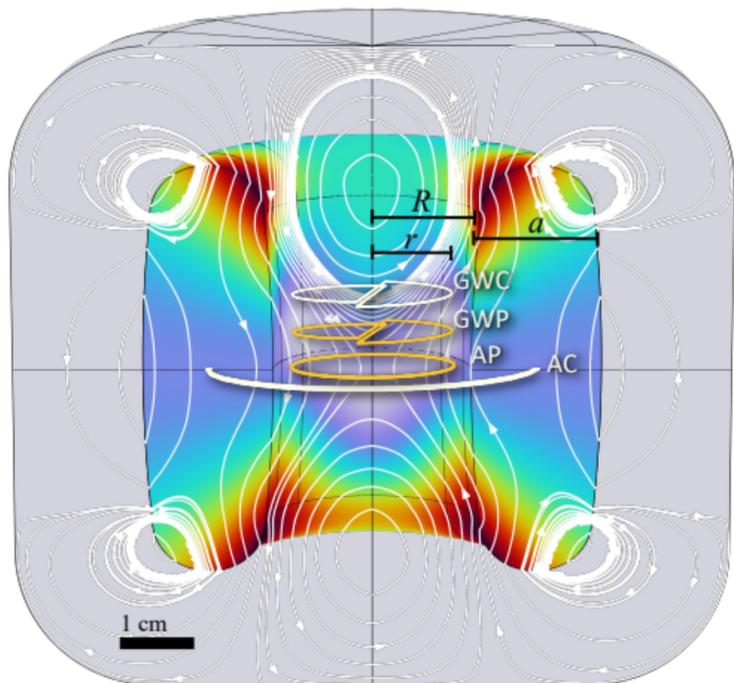
$$\sim (\omega L)^3 h B_0 L^2 \rightarrow (\omega L)^2 h B_0 L^2 \text{ fig-8 loop}$$

$$\Phi_a = e^{-i\omega t} g_{a\gamma\gamma} \sqrt{2\rho_{\text{DM}}} B_0 \pi r^2 R \ln(1 + a/R)$$

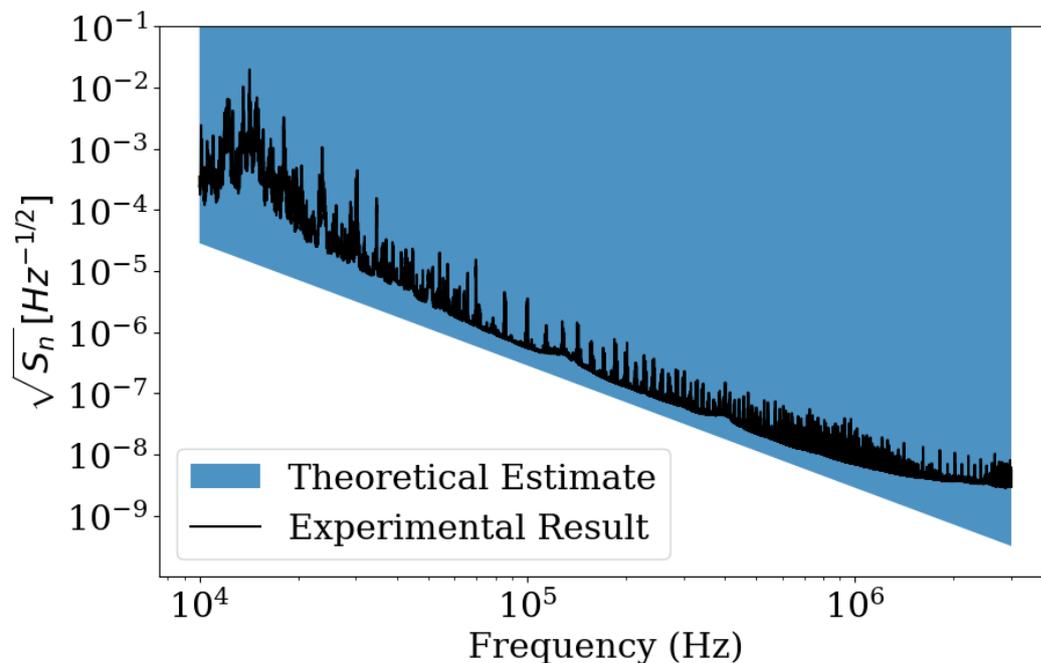
$$\sim (\omega L) g_{a\gamma\gamma} a B_0 L^2$$

Demonstrator: ABRA-GW results

Kaliroe Pappas et al, 2505.02821



Comsol simulation for calibration & signal



Noise measurement in agreement with theory projection

Some lessons learned:

- No 0.01 Msol PBH merger found withing 46 km of MIT
- Scaling to DM RadioGUT, gives a reach of 3 pc
- Data analysis: match filtering at MHz is a computational challenge

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Dielectric GW haloscope

Maxwell equations:

$$\begin{aligned}\nabla \cdot \mathbf{E} &= 0, & \nabla \times \mathbf{B} - \underline{\epsilon} \dot{\mathbf{E}} &= \underline{j_{\text{eff}}}, \\ \nabla \cdot \mathbf{B} &= 0, & \nabla \times \mathbf{E} + \dot{\mathbf{B}} &= 0,\end{aligned}$$

effective current induced by Gws
dielectric constant

Dielectric GW haloscope

Maxwell equations:

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effective current induced by Gws
dielectric constant

particular solutions in medium and vacuum

$$\mathbf{E}_m^p = \frac{c_\theta B_0}{\epsilon - 1} (h_\times \hat{\mathbf{p}} + h_+ \hat{\mathbf{s}}) e^{-i\omega(t - \hat{\mathbf{k}} \cdot \mathbf{x})} \sim h B_0$$

$$\mathbf{E}_v^p = -\frac{B_0}{2} \left[i\omega x (h_\times \hat{\mathbf{p}} + h_+ \hat{\mathbf{s}}) + h_\times s_\theta \hat{\mathbf{k}} \right] e^{-i\omega(t - \hat{\mathbf{k}} \cdot \mathbf{x})} \sim h B_0 \omega x$$

Dielectric GW haloscope

Maxwell equations:

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ALPs, IAXO, ...
as GW detectors

[Ejlli et al `19]



resonant conversion
in vacuum

Dielectric GW haloscope

Maxwell equations:

$$\begin{aligned}\nabla \cdot \mathbf{E} &= 0, & \nabla \times \mathbf{B} - \epsilon \dot{\mathbf{E}} &= \mathbf{j}_{\text{eff}}, \\ \nabla \cdot \mathbf{B} &= 0, & \nabla \times \mathbf{E} + \dot{\mathbf{B}} &= 0,\end{aligned}$$

effective current induced by Gws
dielectric constant

particular solutions in medium and vacuum

$$\mathbf{E}_m^p = \frac{c_\theta B_0}{\epsilon - 1} (h_\times \hat{\mathbf{p}} + h_+ \hat{\mathbf{s}}) e^{-i\omega(t - \hat{\mathbf{k}} \cdot \mathbf{x})} \sim h B_0$$

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ALPs, IAXO, ...
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[Ejlli et al `19]



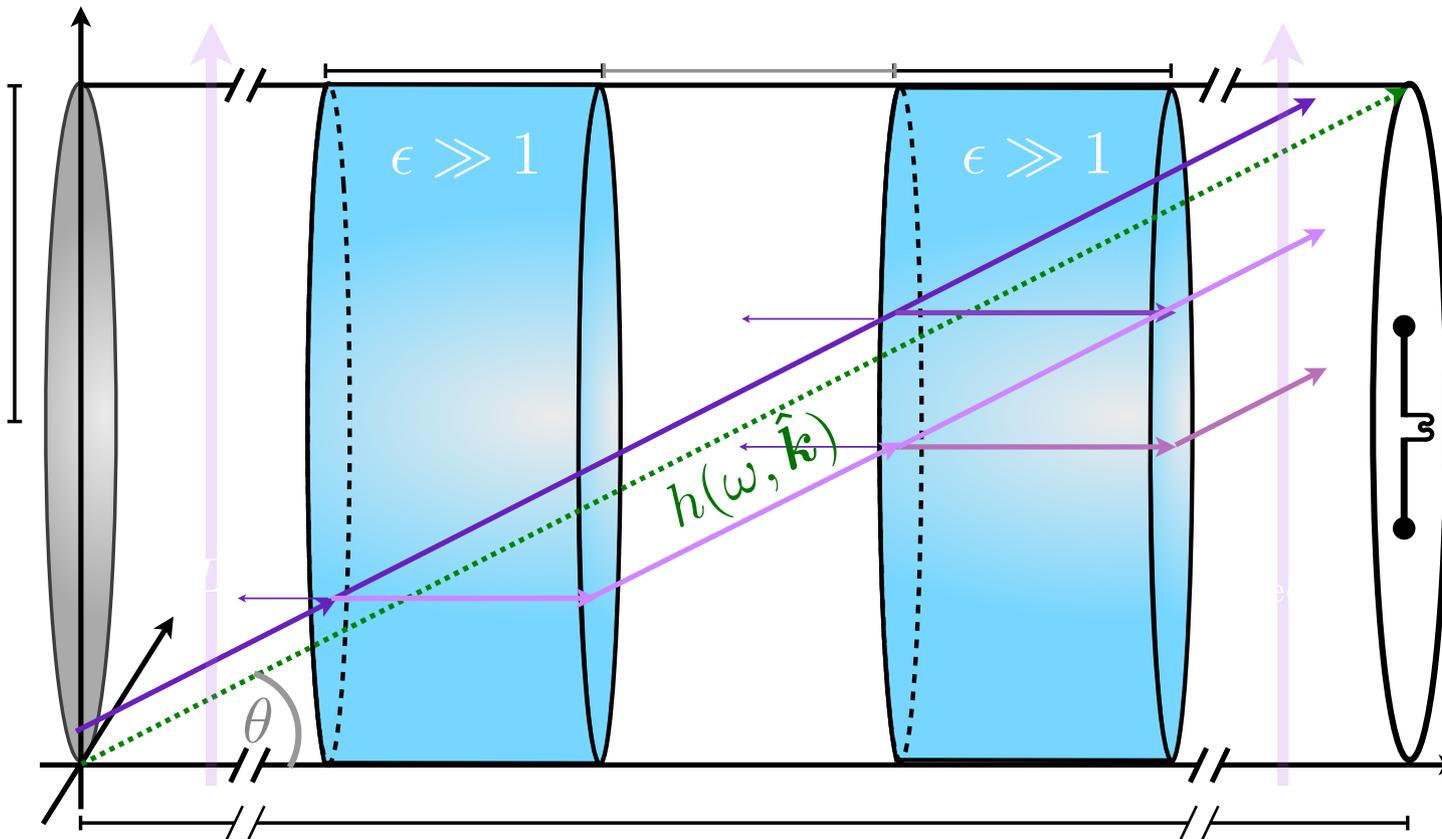
resonant conversion
in vacuum

Boundary conditions at surface of dielectric medium

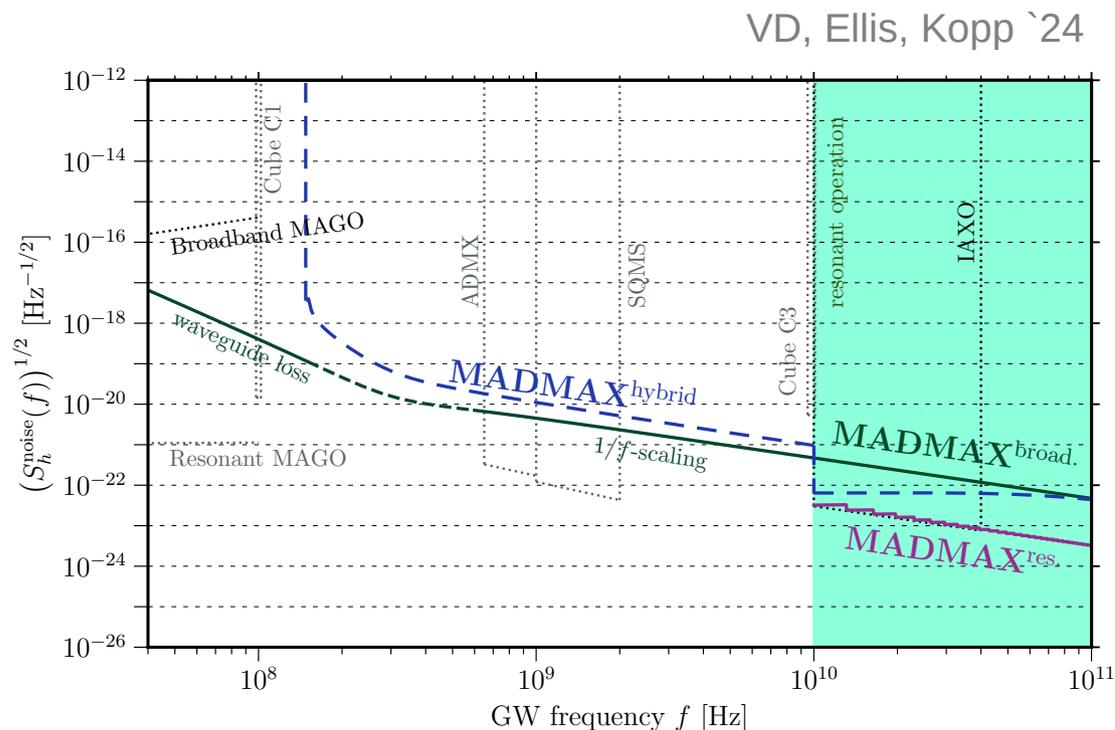
→ EM waves sourced at surfaces of dielectric disks

Dielectric GW haloscope

Madmax prototype
@ CERN



Dielectric GW Haloscope



GWs vs axions

GWs are relativistic:

- + resonant conversion in vacuum
- + relaxed requirement on disk surface
- new requirement on effective disk width

- MADMAX can be operated as GW detector
- Hybrid resonant / broadband mode particularly interesting

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mechanical coupling

mechanical response to GWs

$$\omega_n \sim n v_s L^{-1} \sim 10^{-5} L^{-1} \quad \text{mech. eigenmodes}$$

$$\omega_{\text{gw}} \ll \omega_n$$

rigid limit

$$\omega_{\text{gw}} \sim \omega_n$$

on resonance
(Weber Bars)

$$\omega_n \ll \omega_{\text{gw}} \ll L^{-1}$$

free-falling limit

$$L^{-1} \ll \omega_{\text{gw}}$$

response function
may be suppressed
by oscillation pattern

mechanical coupling

mechanical response to GWs

$$\omega_n \sim n v_s L^{-1} \sim 10^{-5} L^{-1} \quad \text{mech. eigenmodes}$$

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on resonance
(Weber Bars)

$$\omega_n \ll \omega_{\text{gw}} \ll L^{-1}$$

free-falling limit

$$L^{-1} \ll \omega_{\text{gw}}$$

mechanical deformations less stiff than
Maxwell's equations by factor $v_s/c \sim 10^{-5}$

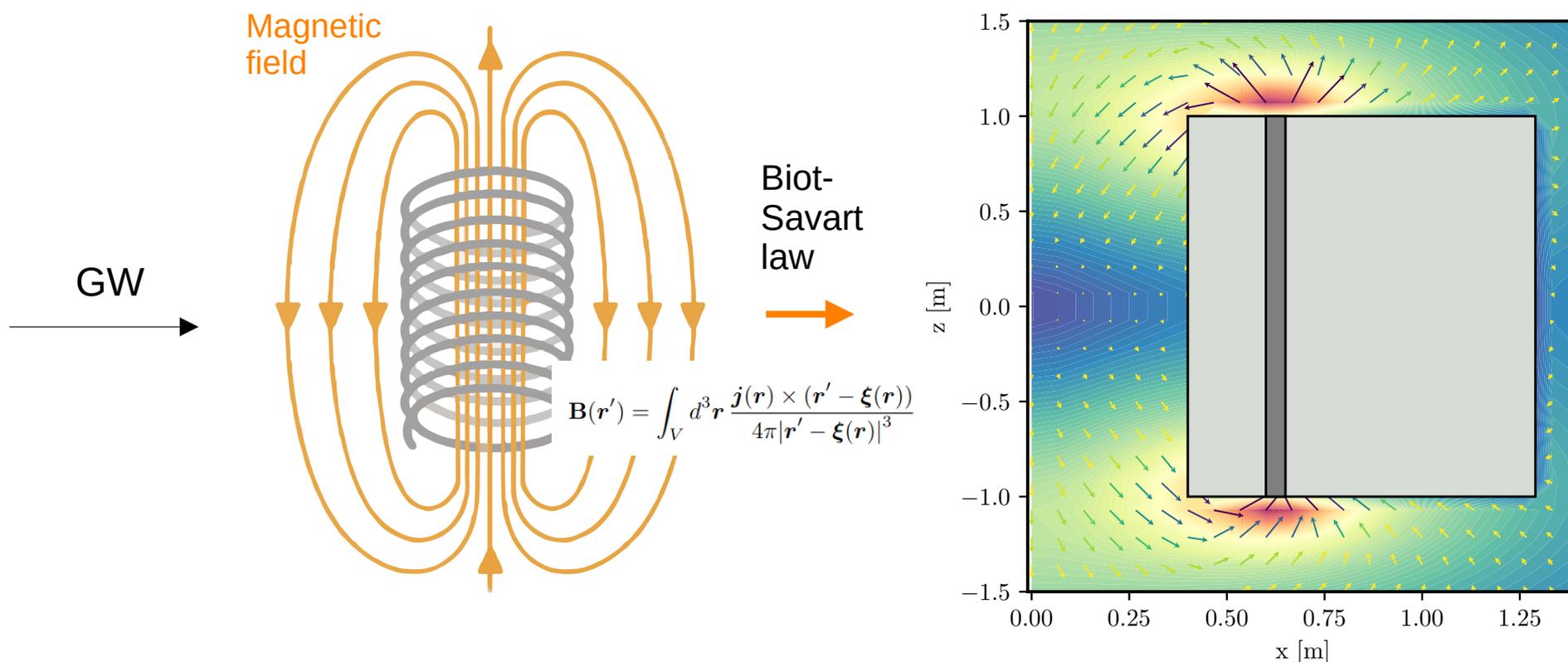
response function
may be suppressed
by oscillation pattern

- mechanical coupling can be significantly more efficient
- challenge of transducing mechanical deformation to EM signal for quantum readout

Magnetic Weber Bar

VD, Ellis, Rodd '24

GW acts as a mechanical force on (current-carrying) wires:

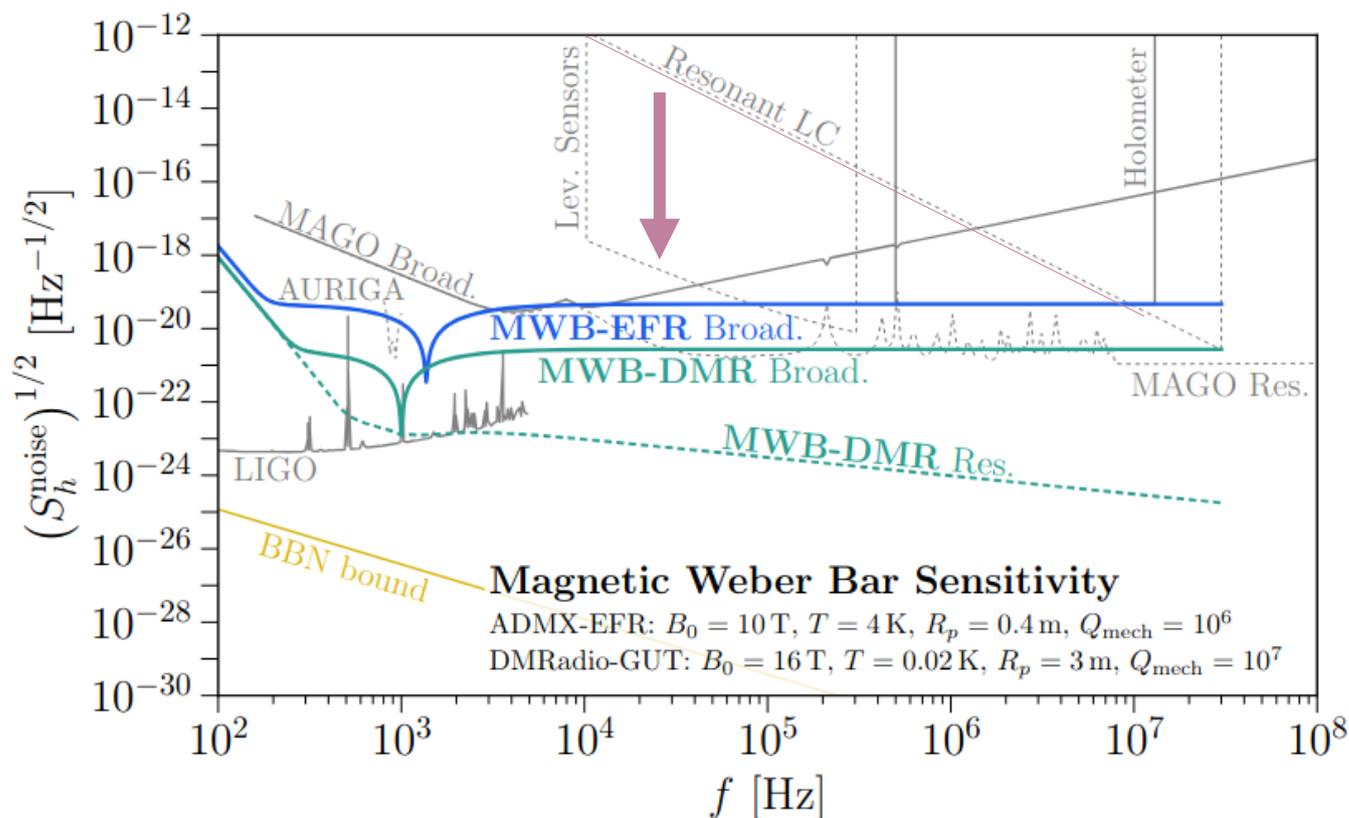


➔ Induced AC magnetic field $\sim h B$, read out with pickup loop + SQUID

Magnetic Weber Bar

VD, Ellis, Rodd '24

mechanical vs
EM coupling



effects at $O(h)$:

- deformation of magnet coil
- motion of pickup loop

Noise contributions:

SQUID, thermal mechanical, seismic, thermal noise of resonant readout

Very competitive broadband sensitivity !

Conclusions and Outlook

UHF GW searches as an interdisciplinary challenge with an active community

- Precision measurements and quantum sensing
- New ideas and prototype development
- Experiment, theory and data analysis

Some recent progress:

- ABRA-GW demonstrator: first results
- Operating dielectric axion haloscopes as GW detectors
- Promising estimates for Magnetic Weber Bars → needs testing / demonstrator

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„such detectors [laser interferometers] have so low sensitivity that they are of little experimental interest“ [Misner, Thorne, Wheeler 1974]

↙ nobel prize 2016 for detection of GWs with LIGO

... and an advertisement:

CERN TH visitor program

<https://theory.cern/visitor-info>

short-term visits typically O(week)

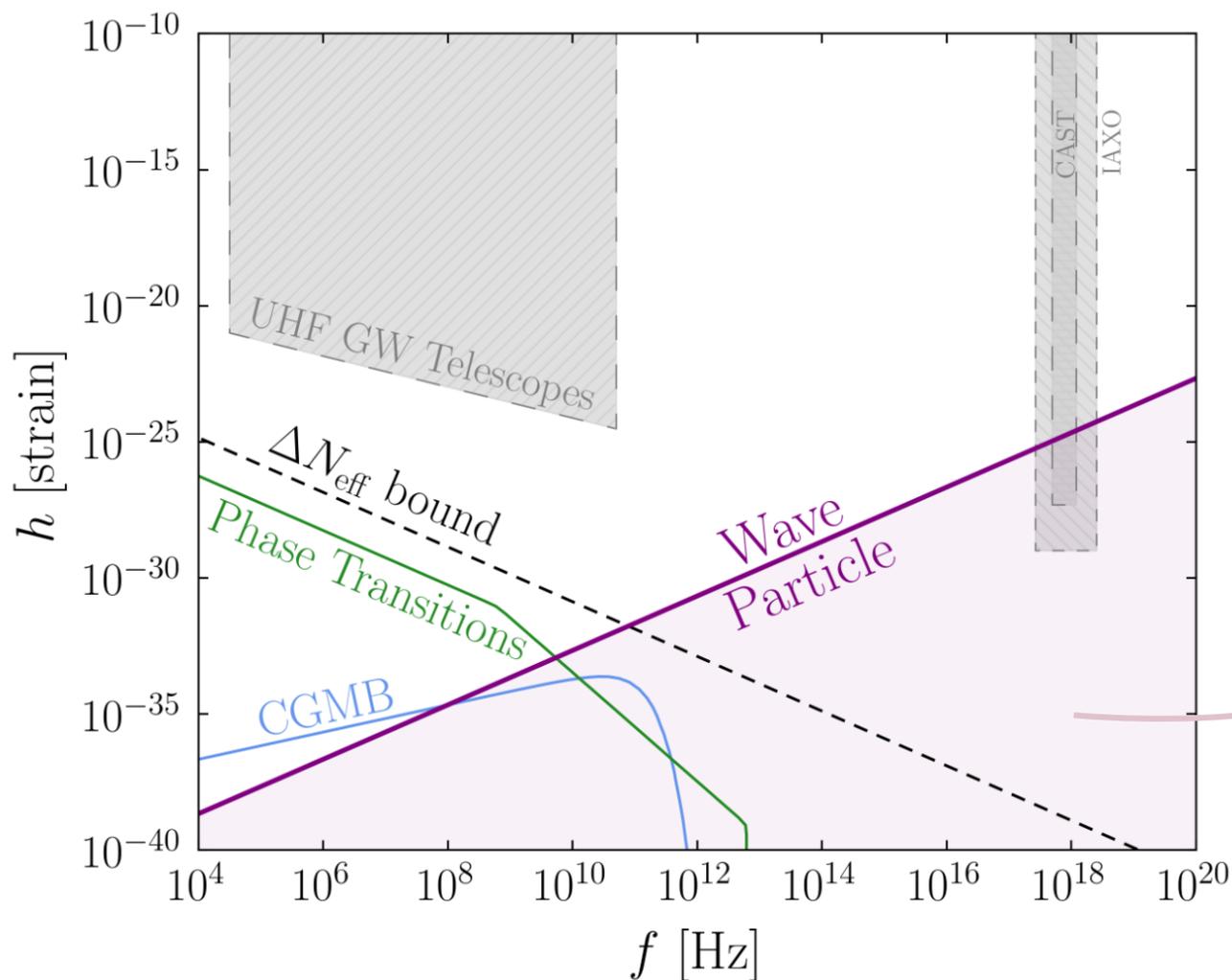
long term visits (> 3 months, usually sabbaticals)

consider applying!

backup slides

tests of quantum gravity ?

Carney, VD, Rodd `23



- dilute graviton gas vs classical GW
- CAST has the sensitivity to detect single gravitons (the source is the issue)
- Rigorous test of quantisation de facto impossible

see also F. Dyson `13

wave versus particle regime

energy density of GW:

$$\rho \sim h^2 \omega^2 M_{\text{pl}}^2$$

number of GW 'quanta' in de-Broglie volume:

$$n = \rho/\omega, \quad \lambda_{\text{dB}} \sim 1/\omega \quad \Rightarrow \quad n \lambda_{\text{dB}}^3 \sim h^2 M_{\text{pl}}^2 / \omega^2$$

single graviton limit:

$$N = n \lambda_{\text{dB}}^3 < 1 \quad \Rightarrow \quad h \lesssim \omega / M_{\text{pl}}$$

(at LIGO, $N \sim 10^{37} (h/10^{-22})^2$)