



Present and future of ALP searches in colliders

JA, G. Levati, P. Paradisi, S. Rigolin, N. Selimović [2407.18296], [2506.xxxxx]

JA, M. Fuentes Zamoro, L. Merlo, X. Ponce Díaz, S. Rigolin [2506.xxxxx]

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1 Introduction and Motivation

2 Leptophilic ALPs

Tauphilic and LFU ALPs. Direct searches and τ anomalous magnetic moment.

3 ALPs in the mesonic *terra incognita*

The ALPaca toolbox. The $B^+ \rightarrow K^+ \nu \bar{\nu}$ anomaly.

4 Closing remarks

Introduction and Motivation

$$\mathcal{L}_{\text{QCD}} \supset \theta_{\text{QCD}} \frac{\alpha_s}{8\pi} G_{\mu\nu}^a \tilde{G}^{\mu\nu,a}.$$

Neutron e.d.m. has not been observed $\implies |\bar{\theta}| < 10^{-10}$, with $\bar{\theta} = \theta_{\text{QCD}} + \arg \det(Y_U Y_D)$.

Possible solution: Peccei-Quinn mechanism

[R.D. Peccei, H.R. Quinn, Phys.Rev.Lett. 38 (1977) 1440-1443 & Phys.Rev.D 16 (1977) 1791-1797]

$$\mathcal{L}_{\text{axion}} \supset \frac{a}{f_a} \frac{\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}^{\mu\nu,a}.$$

QCD-induced axion potential: The axion acquires a vev $\langle a \rangle$ such that $\bar{\theta} \rightarrow 0$, dynamically solving the strong CP problem.

$$m_a f_a = m_\pi f_\pi \frac{\sqrt{m_u m_d}}{m_u + m_d} \sim m_\pi f_\pi.$$

Pseudoscalar a arising as pNG boson of spontaneously broken global $U(1)_{\text{PQ}}$ symmetry at scale $\Lambda_{\text{UV}} \sim f_a$.

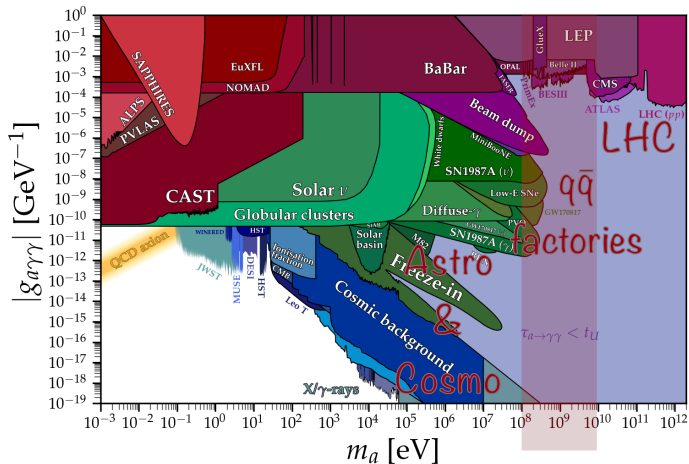
ALPs violate the relation $m_a f_a \approx m_\pi f_\pi$ and do not necessarily solve the strong CP problem via Peccei-Quinn mechanism.

Predicted by many BSM models: string theory (axiverse), flavour (familon, flaxion), neutrino masses (majoron), DM candidate, etc.

Described by dimension-5 ALP-EFT. Schematically,

$$\mathcal{L}_a = \frac{1}{2} \partial_\mu a \partial^\mu a - \frac{m_a^2}{2} a^2 + c_V \frac{a}{f_a} \frac{\alpha_V}{4\pi} V_{\mu\nu} \tilde{V}^{\mu\nu} + \frac{\partial_\mu a}{f_a} \bar{f}_i \gamma^\mu c_f^{ij} f_j.$$

[H. Georgi, D. Kaplan, L. Randall, PLB 169B(1986)73]



[C. O'Hare, AxionLimits (adapted)]

- Ideal range for charm (BESIII) and beauty (BaBar, Belle, Belle II) factories.
- Weaker constraints.
- Flavour.
- Challenges from decays into mesons.

Leptophilic ALPs

Pseudoscalar a arising as pNG boson of spontaneously broken global $U(1)_{\text{PQ}}$ symmetry.

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 - \frac{1}{2}m_a^2 a^2 - \frac{c_\ell^{ij}}{2f_a}(\partial_\mu a) \left(\chi_L \bar{\ell}_L^i \gamma^\mu \ell_L^j + \chi_R \bar{e}_R^i \gamma^\mu e_R^j \right) + c_{BB} \frac{\alpha_1}{4\pi} \frac{a}{f_a} B\tilde{B} + c_{WW} \frac{\alpha_2}{4\pi} \frac{a}{f_a} W\tilde{W}.$$

We consider 2 scenarios: **LFU** $c_\ell^{ij} = c_\ell \delta^{ij}$ and **tauphilic** $c_\ell^{33} = c_\tau$, $c^{ij} \approx 0$.

After applying integration by parts and equations of motion

$$\mathcal{L}_{\text{int}} = i \frac{c_\ell \chi_A}{f_a} a \sum_\ell m_\ell \bar{\ell} \gamma_5 \ell - \frac{\alpha_{\text{em}}}{4\pi} \frac{a}{f_a} c_{XY}^{\text{eff}} X_{\mu\nu} \tilde{Y}^{\mu\nu},$$

where $\chi_{V,A} = (\chi_R \pm \chi_L)/2$ and $XY = \gamma\gamma, \gamma Z, W^+W^-, ZZ$.

Motivation: Example of UV completion

SM extended by **vector-like lepton doublet** $L \sim (\mathbf{1}, \mathbf{2})_{-1/2}$ with L_L charged under global $U(1)_{PQ}$, and complex scalar $\Phi \sim (\mathbf{1}, \mathbf{1})_0$ whose vev breaks PQ.

$$\mathcal{L}_{UV} = (\partial_\mu \Phi)^\dagger \partial^\mu \Phi + i \bar{L} \not{D} L - y \Phi \bar{L}_L L_R - m_3 \bar{\ell}_L^3 L_R + \text{h.c.}$$

In the broken phase $\Phi \rightarrow \frac{1}{\sqrt{2}}(f_a + \rho)e^{ia/f_a}$. We can move the pNG to the kinetic term after a chiral rotation $L_L \rightarrow e^{ia/f_a} L_L$:

$$\mathcal{L}_{UV} \supset - \left(\frac{y f_a}{\sqrt{2}} \bar{L}_L L_R + m_3 \bar{\ell}_L^3 L_R + \text{h.c.} \right) + \frac{1}{f_a} (\partial_\mu a) \bar{L}_L \gamma^\mu L_L + \frac{\alpha_1}{8\pi} \frac{a}{f_a} B_{\mu\nu} \tilde{B}^{\mu\nu} + \frac{\alpha_2}{8\pi} \frac{a}{f_a} W_{\mu\nu}^I \tilde{W}^{I\mu\nu}.$$

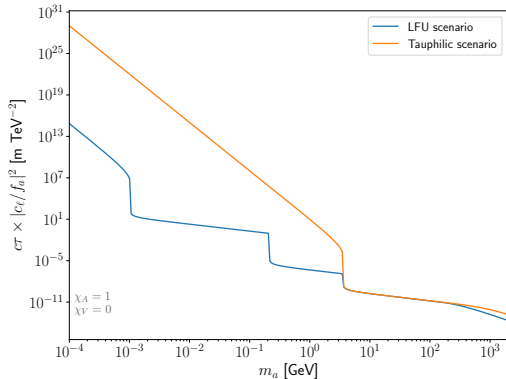
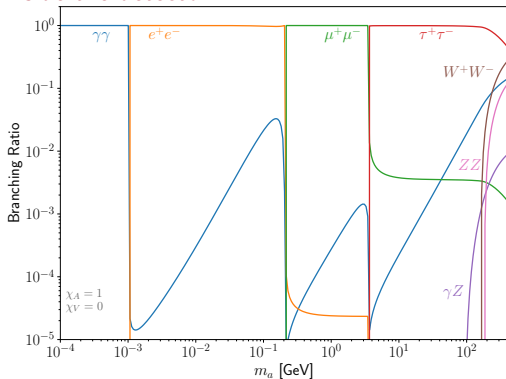
Finally, after diagonalizing the mass terms

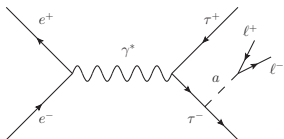
$$\mathcal{L} \supset \frac{\sin^2 \theta}{f_a} (\partial_\mu a) (\bar{\ell}_L^3 \gamma^\mu \ell_L^3), \quad \tan \theta = \frac{\sqrt{2} m_3}{y f_a},$$

which corresponds to a tauphilic ALP with

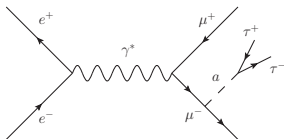
$$c_\tau = -2 \sin^2 \theta, \quad \chi_L = 1, \quad \chi_R = 0, \quad c_{BB} = c_{WW} = \frac{1}{2}.$$

LFU ALPs with $m_a < 2m_e$ (tauphilic with $m_a < 2m_\tau$) will decay into $\gamma\gamma$ typically **outside the detector**; heavier ALPs will decay mostly into a pair of the heaviest leptons available, typically **inside the detector**.

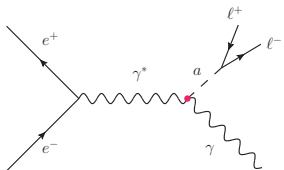




Studied by BaBar and Belle in the context of dark photon searches. Loses sensitivity for $m_a \geq 2m_\tau$.



New at Belle II. Reaches higher m_a , but is suppressed by a factor m_μ^2/m_τ^2 and experimental signature is less clear.



Proposed search: for $m_a < 2m_\tau$. Both photon and di-lepton have fixed energy. Belle II could resolve the displaced vertex of the $a \rightarrow \ell^+\ell^-$ decay. Sensitive to effective coupling to photons.

In BaBar and Belle, $e^+e^- \rightarrow \Upsilon(nS) \rightarrow \gamma a$; also $e^+e^- \rightarrow J/\psi \rightarrow \gamma a$ at BES-III.

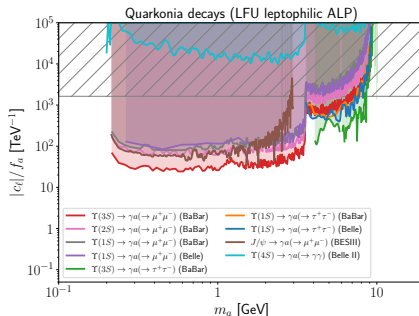
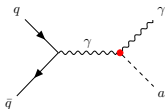
- LFU ALP: $a \rightarrow \mu^+\mu^-$ below $m_a > 2m_\tau$, $a \rightarrow \tau^+\tau^-$ above $m_a > 2m_\tau$.
- Tauphilic ALPs $a \rightarrow \gamma\gamma$ below $m_a > 2m_\tau$, $a \rightarrow \tau^+\tau^-$ above $m_a > 2m_\tau$.

Affected by UV contributions to photon coupling.

Depending on experimental setup, there is an interplay between **resonant** and **non-resonant** decays.

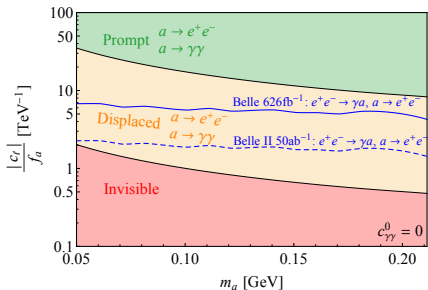
[L. Merlo, F. Pobbe, S. Rigolin, O. Sumensari.

JHEP 1906 (2019) 091]



Belle II operates exclusively at $\sqrt{s} = m_{\Upsilon(4S)}$, but the resonance is much wider than the energy spread of the beam: always non-resonant process.

Non-resonant $e^+e^- \rightarrow \gamma a$ at Belle II:



FeynRules $\rightarrow$ UFO model $\rightarrow$ MadGraph_5AMC@NLO to simulate signal and background events.

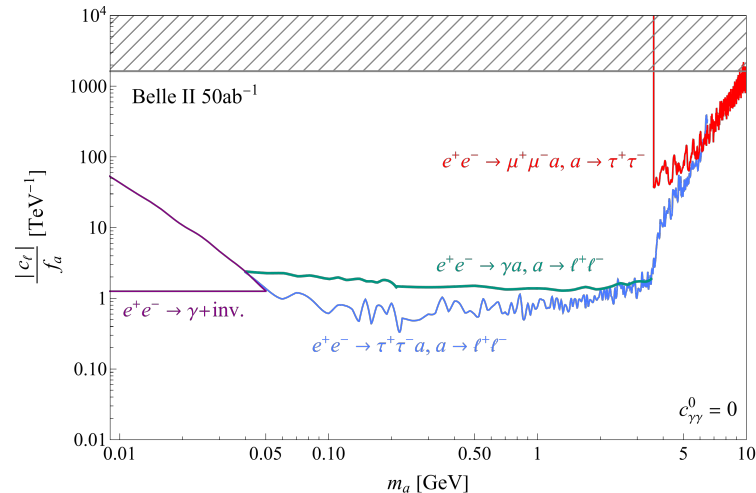
Search for events with $m_{\ell\ell}^2 = m_a^2$ and $E_\gamma = \frac{s - m_a^2}{2\sqrt{s}}$.

Belle II electromagnetic calorimeter coverage:
 $12.4^\circ < \theta < 155.1^\circ$.

The photon energy resolution averaged over the entire calorimeter is 1.7% for $E_\gamma > 5$ GeV. Di-lepton invariant mass resolution $\sigma_{m_{\ell\ell}}/m_{\ell\ell}$ reported by Belle II.

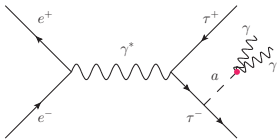
We analyze 2D bins in $E_\gamma \otimes m_{\ell\ell}^2$ assuming Poisson statistics and require $S/\sqrt{B} = 2$.

Below $m_a < 0.04$ GeV, the decay length exceeds the detector size, and the signal is mono- γ . Existing sensitivity studies for Belle II.

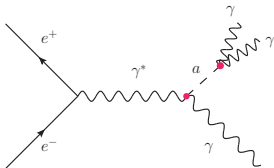


The proposed Belle II searches will

- Improve constraints in $e^+e^- \rightarrow \gamma \ell^+ \ell^-$ by one order of magnitude, complementary to $e^+e^- \rightarrow \tau^+ \tau^- \ell^+ \ell^-$.
- Extend the constraints to $m_a < 0.05 \text{ GeV}$.

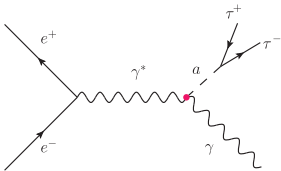


Proposed search at Belle II for $m_a < 2m_\tau$. Narrow signal at $m_{\gamma\gamma}^2 = m_a^2$ superimposed over continuous QED background.



Studied at Belle II and BESIII. Below $m_a < 0.5$ GeV, the ALP is long-lived, and the signature is $e^+e^- \rightarrow \gamma$ instead (BESIII).

Proposed search: In the region $0.5 \text{ GeV} < m_a < 1.0 \text{ GeV}$ Belle II could search for displaced vertices.



Studied at BaBar ($e^+e^- \rightarrow \Upsilon(1S, 3S) \rightarrow \gamma\tau^+\tau^-$) and Belle ($e^+e^- \rightarrow \Upsilon(1S) \rightarrow \gamma\tau^+\tau^-$).

Proposed search at Belle II, in non-resonant process.

$$e^+e^- \rightarrow \tau^+\tau^-a, a \rightarrow \gamma\gamma$$

FeynRules $\rightarrow$ UFO model $\rightarrow$ MadGraph_5AMC@NLO to simulate signal and background events.

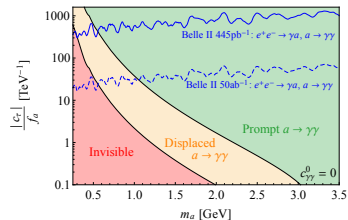
Search for events with $m_{\gamma\gamma}^2 = m_a^2$
We assume the same resolution for $m_{\gamma\gamma}^2$ as in $e^+e^- \rightarrow \gamma a, a \rightarrow \gamma\gamma$.

Belle II electromagnetic calorimeter coverage:
 $37.3^\circ < \theta_\gamma < 123.7^\circ$; separation between photons of $\Delta\theta_{\gamma\gamma} > 0.014$ rad and $\Delta\phi_{\gamma\gamma} > 0.400$ rad to reduce background from photon conversions outside of the tracking detectors.

At least one τ decays leptonically.

We analyze bins in $m_{\gamma\gamma}^2$ assuming Poisson statistics and require $S/\sqrt{B} = 2$.

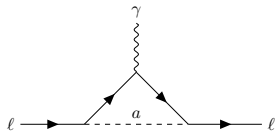
$$e^+e^- \rightarrow \gamma a, a \rightarrow \gamma\gamma$$



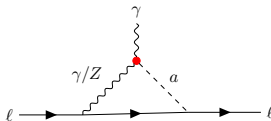
$$e^+e^- \rightarrow \gamma a, a \rightarrow \tau^+\tau^-$$

FeynRules $\rightarrow$ UFO model $\rightarrow$ MadGraph_5AMC@NLO to simulate signal and background events.

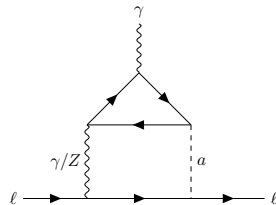
Peak in $m_{\tau\tau}^2 = s - 2\sqrt{s}E_\gamma$.
Both τ required to decay leptonically.



Negative contribution.
Dominant term for small m_a .



The contribution from the
effective coupling is negative,
and dominant for large m_a .
UV contributions may have
either sign.

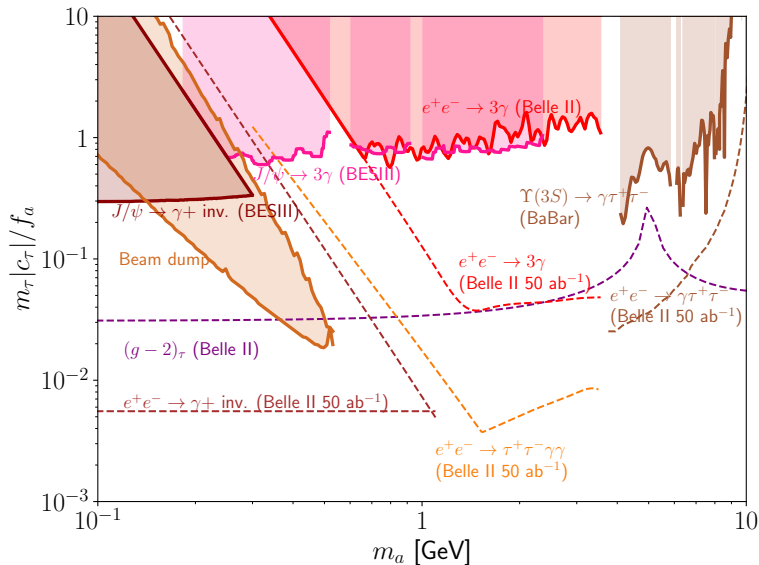


Barr-Zee term:
Additional suppression m_f^2/m_a^2 .

Belle II will be sensitive to the $\tau^+\tau^-\gamma^*$ form factor $F_2(s)$,
with experimental resolution $\mathcal{O}(10^{-6})$.

Belle II will significantly improve all the bounds.

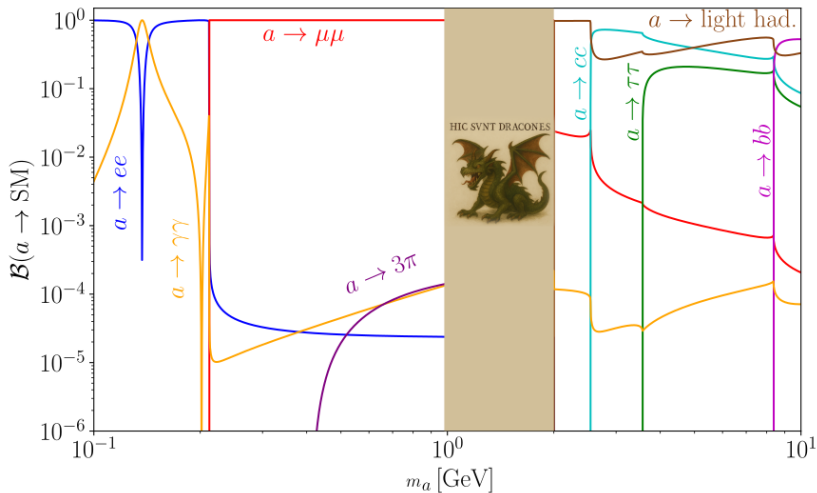
Complementarity between direct searches and anomalous magnetic moment.



ALPs in the mesonic *terra incognita*

ALP decays: the *terra incognita*

Example: Universal couplings to fermions $c_{q_L}^{ij} = c_{d_R}^{ij} = c_{u_R}^{ij} = c_{\ell_L}^{ij} = c_{e_R}^{ij} \equiv c_f \delta^{ij}$.



[L. Di Luzio, A. Guerrero, X. Ponce Díaz, S. Rigolin. JHEP 06 (2024) 217 (adapted)]

The degrees of freedom are no longer quarks $q = (u, d, s)^T$, but mesons $U = \exp\left(\frac{i}{F_0} \lambda^a \pi^a\right)$.

$$\mathcal{L} = -\frac{1}{4} G_{\mu\nu}^a G^{\mu\nu,a} + i\bar{q}\not{D}q - \bar{q}(s - i\gamma_5 p)q + \bar{q}\gamma^\mu(r_\mu P_R + l_\mu P_L)q - \frac{\alpha_s}{8\pi}\theta(x)G_{\mu\nu}^a\tilde{G}^{\mu\nu,a}.$$

$$\Downarrow \quad \mu \lesssim 1.6 \text{ GeV}$$

$$\mathcal{L}_{\chi\text{PT}}^{(p^2)} = \frac{F_0^2}{4} \langle (\partial_\mu U + iU r_\mu - i l_\mu U) (\partial^\mu U^\dagger - i r^\mu U^\dagger + i U^\dagger l^\mu) \rangle + \frac{F_0^2 B_0}{2} \langle U^\dagger (s + i p) + U (s^\dagger - i p^\dagger) \rangle.$$

[J. Gasser, H. Leutwyler. *Annals Phys.* 158 (1984) 142 & *Nucl.Phys.B* 250 (1985) 465-516]

In the ALP case,

$$s = m_q, \quad p = 0, \quad r_\mu = c_q^R \frac{\partial_\mu a}{f_a}, \quad l_\mu = c_q^L \frac{\partial_\mu a}{f_a}, \quad \theta = -2c_G \frac{a}{f_a}.$$

The chiral Lagrangian describes interactions with pseudoscalar mesons: $a \rightarrow 3\pi$, $a \rightarrow \eta^{(\prime)}\pi\pi$.

How to deal with the $\theta(x)$ term? Two equivalent approaches:

- 1 Eliminate via chiral rotation $q(x) \rightarrow \exp(i\theta(x)\kappa_q\gamma_5/2)q(x)$.

[D. Aloni, Y. Soreq, M. Williams. Phys.Rev.Lett. 123 (2019) 3, 031803]

- 2 Promote the meson matrix to $U(3)$

$$\det U_\theta = \exp(-i\theta(x)) \implies U_\theta = \exp(-i\theta(x)\kappa_q/2)U \exp(-i\theta(x)\kappa_q/2).$$

[C. Cornella, A. M. Galda, M. Neubert, D. Wyler. JHEP 06 (2024) 029]

with $\langle \kappa_q \rangle = 1$. Physical observables must be independent of the components of κ_q .

Adding other mesons: **Recent theoretical developments!**

- **Vector and axial-vector:** $a \rightarrow \gamma\pi\pi$, $a \rightarrow \gamma\omega$, $a \rightarrow \omega\omega, \dots$

VMD [M. Ovchinnikov, A. Zaporozhchenko. 2501.04525],

Wess-Zumino-Witten [Y. Bai, T.-K. Chen, J. Liu, X. Ma. 2505.24822].

- **Scalar and tensor:** Resonant contributions to $a \rightarrow 3\pi$, $a \rightarrow \eta^{(\prime)}\pi\pi$ above 1 GeV. Phenomenological Lagrangians. [M. Ovchinnikov, A. Zaporozhchenko. 2501.04525]

[R. Balkin, T. Coren, Y. Soreq, M. Williams. 2506.15637]



ALP automatic computing algorithm

<https://github.com/alp-aca/alp-aca>

1 At the **UV scale** Λ_{UV} define the ALP-EFT couplings or match one of the benchmark models: **DFSZ**, **KSVZ**, **flaxion**...

2 Run the RGEs and integrate out heavy particles.

[M. Bauer, M. Neubert, S. Renner, M. Schnubel, A. Thamm. JHEP 04 (2021) 063]

3 ALP production: $M_1 \rightarrow M_2 a$, $V \rightarrow \gamma a$.

ALP decays (including mesons).

Probability of decaying in/outside the detector.

[T. Ferber, A. Filimonova, R. Schäfer, S. Westhoff. JHEP 04 (2023) 131]

4 Statistical χ^2 analysis with **> 100 observables**.

5 Plot the exclusion significance.

6 Generate the list of bibliographical references.



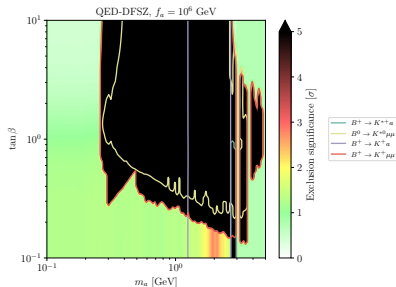
} NWA approximation

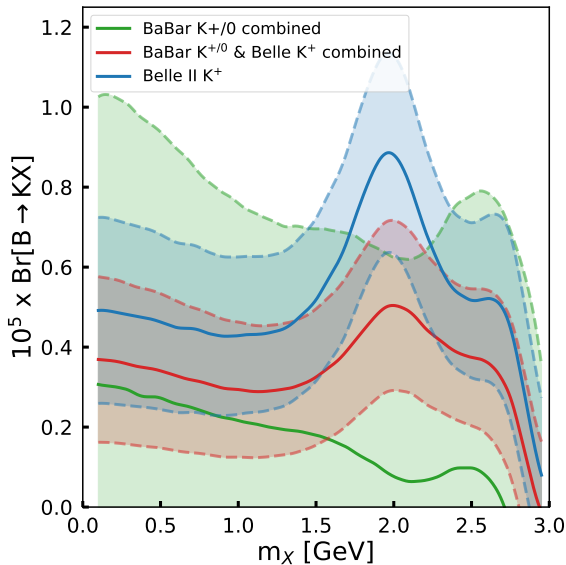
```

1 from alpaca.models import QED_DFSZ, beta
2 from alpaca.statistics import get_chi2
3 from alpaca.sectors import default_sectors
4 from alpaca.plotting.mpl import exclusionplot
5 import numpy as np
6
7 fa = 1e6 #GeV
8 tanbeta = np.logspace(-1, 1, 100)
9 ma = np.logspace(-1, np.log10(5), 100) #GeV
10 x_ma, y_tanbeta = np.meshgrid(ma, tanbeta)
11 couplings = [QED_DFSZ.get_couplings({beta: np.arctan(tb)}, 4*np.pi*fa).match_run(10, 'VA_below') for tb in tanbeta]
12 x_ma, y_couplings = np.meshgrid(ma, couplings)
13
14 chi2 = get_chi2(default_sectors['bsa_lfu'], x_ma, y_couplings, fa, 0, integrator='leadinglog')
15 chi2_obs = chi2[0].split_observables()
16
17 exclusionplot(x_ma, y_tanbeta, chi2_obs, r'$m_a$ [GeV]', r'$\tan\beta$', r'QED-DFSZ, $f_a = 10^6$ GeV')

```

Written completely in Python
Easy to use, concise syntax





Belle II has found a $\sim 2.7\sigma$ excess in $\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})$.

[Belle II collaboration. Phys.Rev.D 109 (2024) 11, 112006]

Compatible with two-body decay with a $\sim 2\text{ GeV}$ invisible particle.

[W. Altmannshofer, A. Crivellin, H. Haigh, G. Inguglia, J. Martín Camalich. Phys.Rev.D 109 (2024) 7, 075008]

Could it be **an ALP???**
Let's find out with ALPaca!

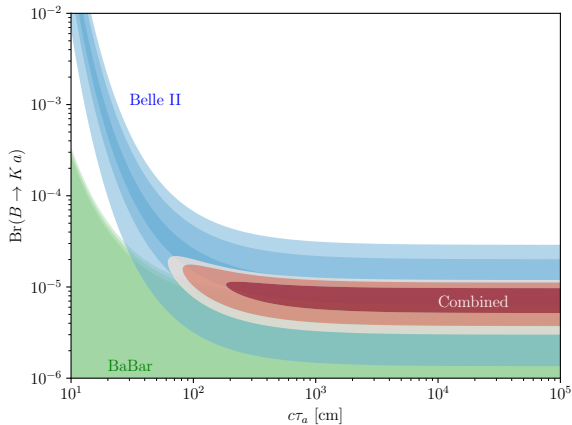
Invisible decay

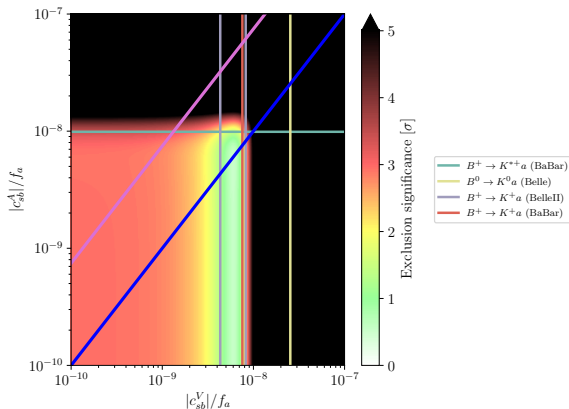
$$\mathcal{B}(B^+ \rightarrow K^+ + \text{inv}) = \mathcal{B}(B^+ \rightarrow K^+ a) \times P_{\text{out}}.$$

$$P_{\text{out}} = \exp\left(-\frac{L_{\text{det}}}{c\tau_a \beta_a \gamma_a}\right).$$

The difference in detector size and boost between BaBar and Belle II allows us to disentangle $\mathcal{B}(B^+ \rightarrow K^+ a)$ and P_{out} .

For $m_a = 2 \text{ GeV}$, preference for longer-lived ALP and $\mathcal{B}(B^+ \rightarrow K^+ a) \sim 10^{-5}$.

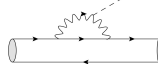
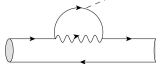




Effective Lagrangian at $\mu = m_a = 2 \text{ GeV}$:

$$\mathcal{L}_{\text{eff}} \supset \frac{\partial_\mu a}{f_a} (c_{sb}^V \bar{s} \gamma^\mu b + c_{sb}^A \bar{s} \gamma^\mu \gamma_5 b) + \text{h.c.}$$

If flavour violation is induced by top loops, $c_{sb}^V = -c_{sb}^A$, compatible with $B^+ \rightarrow K^+ \nu \bar{\nu}$ and $B^+ \rightarrow K^{*+} \nu \bar{\nu}$. See Xiyuan Gao's talk

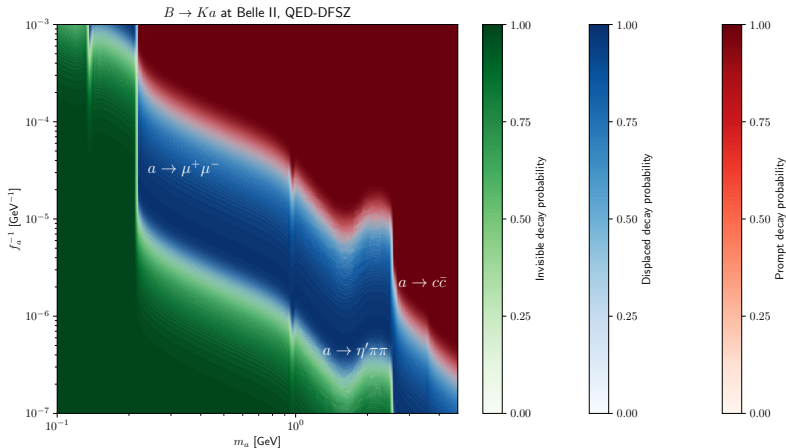


Models with tree-level flavour violation (e.g. **flaxion**) populate all the $c_{sb}^V - c_{sb}^A$ plane.

[Y. Ema, K. Hamaguchi, T. Moroi, and K. Nakayama, JHEP 01 (2017) 096]

$B \rightarrow Ka$: is the ALP invisible?

The previous ALP-EFT analysis fails because $c_{sb}^{V,A}$ do not generate ALP decays.
In more complete scenarios, a 2 GeV ALP will have large Γ to muons and/or mesons.
The ALP will, in general, decay very promptly.

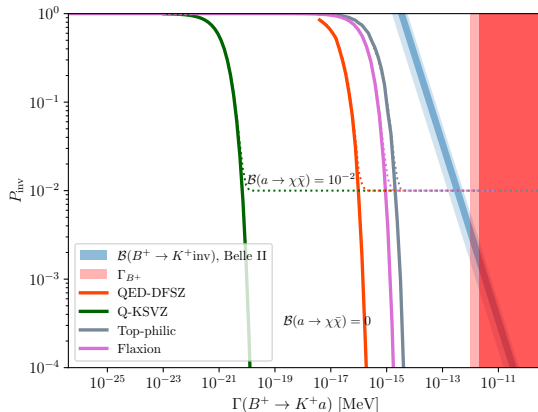
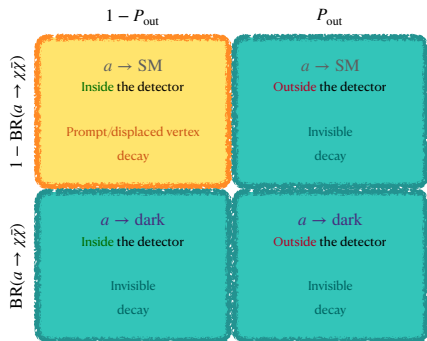


Solution 1: ALP portal to dark sector

If the ALP can decay into a dark particle, e.g.

$$\mathcal{L}_\chi \supset c_\chi \frac{\partial_\mu a}{f_a} \bar{\chi} \gamma^\mu \gamma_5 \chi,$$

it is possible to achieve large P_{inv} and $\mathcal{B}(B^+ \rightarrow K^+ a)$ at the same time.



Solution 2: Flavour violation at tree level

Is it possible to find a model with sizeable c_{sb}^V while suppressing decays to muons and mesons?

Yes!, but fine-tuning is required...

Example:

Non-universal PQ-2HDM with

$$\mathcal{X}_{qL} = \text{diag}(0, 0, 1),$$

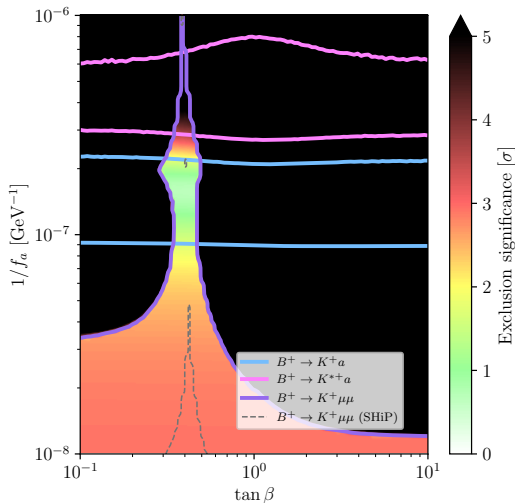
$$\mathcal{X}_{\ell L} = -\mathcal{X}_{qL},$$

$$\mathcal{X}_{uR} = \sin^2 \beta \mathbb{1},$$

$$\mathcal{X}_{dR} = \cos^2 \beta \mathbb{1},$$

$$\mathcal{X}_{eR} = -\mathcal{X}_{uR}.$$

[L. Di Luzio, F. Mescia, E. Nardi, P. Panci, R. Ziegler. Phys.Rev.Lett. 120 (2018) 26, 261803]



Closing remarks

- ALPs with $m_a \sim \mathcal{O}(\text{GeV})$ display a rich phenomenology.
- ALP-EFT allows for a systematic study of all possible observables.
- Belle II will significantly improve the constraints for ALP couplings with τ leptons.
- We have created ALPaca to facilitate the full analysis of ALP models, with special emphasis on mesonic processes.
- We showcased the capabilities of ALPaca with the study of the excess in $B^+ \rightarrow K^+ \nu \bar{\nu}$ found by Belle II.
It could be explained by a $m_a = 2 \text{ GeV}$ ALP, albeit requiring additional hypothesis or fine-tuning.

Thank you for your attention!

Backup slides

Two on-shell photons:

$$c_{\gamma\gamma}^{\text{eff},0} = c_{\gamma\gamma}^0 + c_\ell \sum B_1\left(\frac{4m_\ell^2}{m_a^2}\right) .$$

One on-shell and one off-shell photon

$$c_{\gamma\gamma}^{\text{eff}}(s) = c_{\gamma\gamma}^0 + c_\ell \sum B_3\left(\frac{4m_\ell^2}{m_a^2}, \frac{4m_\ell^2}{s}\right) .$$

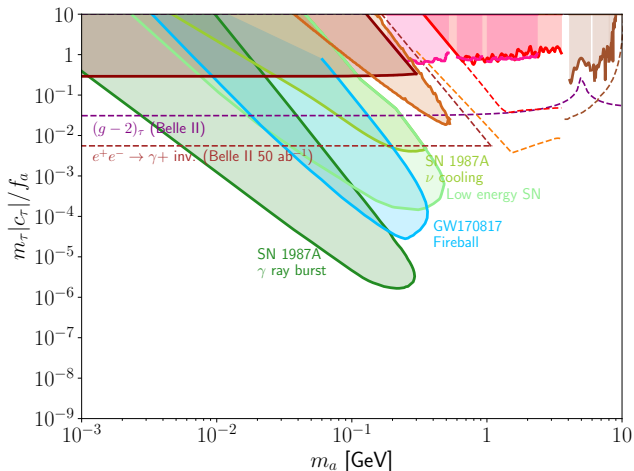
with loop functions

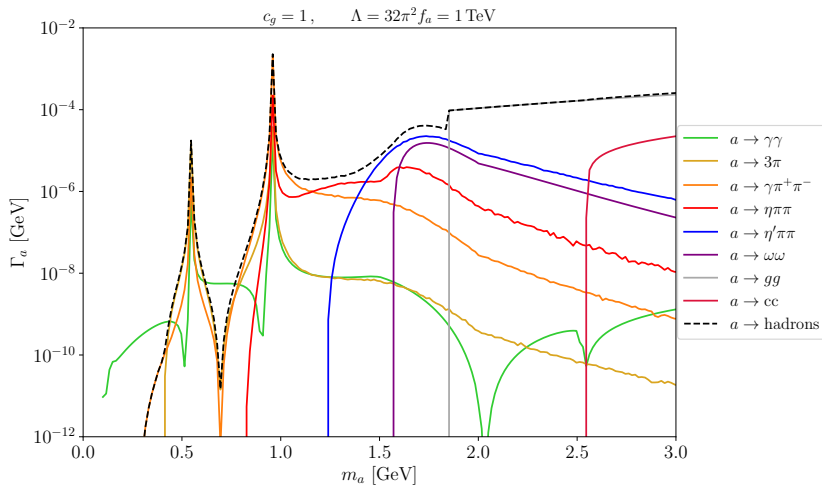
$$B_3(x, y) = 1 + \frac{xy}{x - y} [f^2(x) - f^2(y)] ,$$

$$f(x) = \begin{cases} \arcsin\left(\frac{1}{\sqrt{x}}\right) & x \geq 1 \\ \frac{\pi}{2} + \frac{i}{2} \log \frac{1+\sqrt{1-x}}{1-\sqrt{1-x}} & x < 1 \end{cases}$$

$$B_1(x) = B_3(x, y \gg 1) \simeq 1 - x f^2(x) .$$

For $m_a \lesssim 0.4 \text{ GeV}$, supernovae and neutron star mergers impose strong bounds to tauphilic ALPs via $c_{\gamma\gamma}^{\text{eff}}$.



ALP hadronic (and $\gamma\gamma$) decays

- **Q-KSVZ:** Heavy VLF charged under PQ, $Q \sim (\mathbf{3}, \mathbf{1}, 0, \pm 1/2)$:
At the matching scale $c_G = -1/2$.
- **QED-DFSZ:** $-\mathcal{L} \supset \bar{q}_L Y_u \tilde{H} q_R + \bar{q}_L Y_d H q_R + \bar{\ell}_L Y_e \tilde{H} e_R + \lambda_{H\phi} H_e^\dagger H_q \phi^2$.

$$a = \frac{1}{f_a} \sum_{i=q,e,\phi} \mathcal{X}_{H_i} v_i a_i, \quad \text{with} \quad f_a^2 = \sum_i \mathcal{X}_{H_i}^2 v_i^2,$$

At the matching scale

$$c_u^A = -c_d^A = 2 \sin^2 \beta \mathbb{1}, \quad c_e^A = 2 \cos^2 \beta \mathbb{1}, \quad c_\gamma = 6, \quad c_G = 0.$$

- **Flaxion:** Froggatt-Nielsen model

$$\mathcal{L}_{\text{FN}} = \bar{q}_{Li} y_{u,ij} \tilde{H} u_{Rj} \left(\frac{\phi}{\Lambda} \right)^{\mathcal{X}_{qL}^i - \mathcal{X}_{uR}^j} + \bar{q}_{Li} y_{d,ij} H d_{Rj} \left(\frac{\phi}{\Lambda} \right)^{\mathcal{X}_{qL}^i - \mathcal{X}_{dR}^j} + \bar{\ell}_{Li} y_{e,ij} H e_{Rj} \left(\frac{\phi}{\Lambda} \right)^{\mathcal{X}_{\ell L}^i - \mathcal{X}_{eR}^j},$$

$$\mathcal{X}_{qL} = \begin{pmatrix} 3 & 2 & 0 \end{pmatrix}, \quad \mathcal{X}_{uR} = \begin{pmatrix} -5 & -1 & 0 \end{pmatrix}, \quad \mathcal{X}_{dR} = \begin{pmatrix} -4 & -3 & -3 \end{pmatrix}, \\ \mathcal{X}_{\ell L} = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}, \quad \mathcal{X}_{eR} = \begin{pmatrix} -8 & -5 & -3 \end{pmatrix}.$$