A new model of form factors

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Flavour for New Physics at Present and Future Colliders
19 Jun 2025

(based on 2410.13764 with Kubis, Reboud, van Dyk)

Overview

- Motivation
- “Traditional” form factor models
- What’s new in our parameterisation?
- What new data do we want from colliders?

Why are we interested in form factors?

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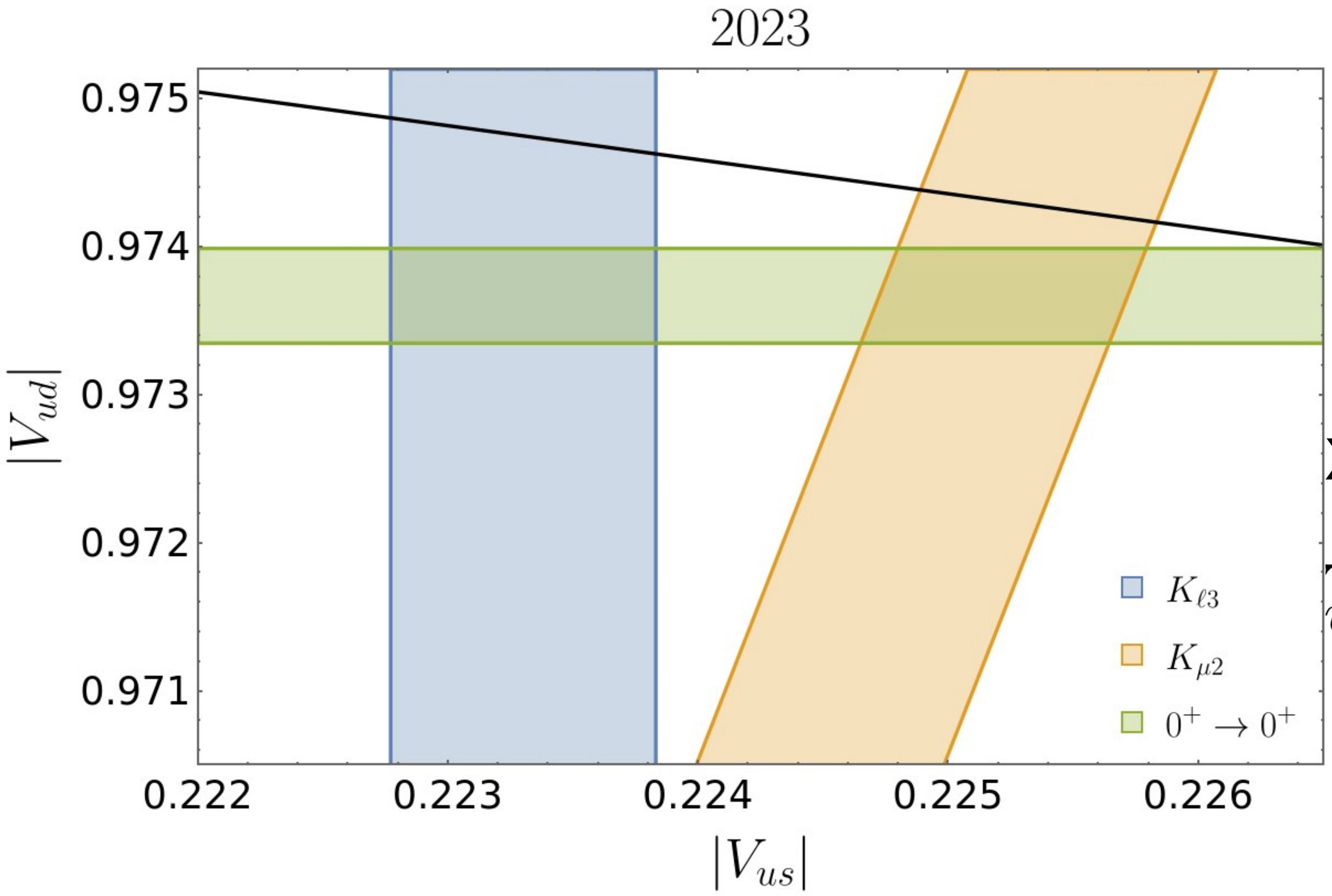
- Semi-leptonic decays are very interesting
  - E.g. for determining CKM elements, but also potential BSM
- Consider $K \to \pi \ell \nu$, which can be used to extract V_{us}
- But $\tau \to K \pi \nu$ should also give access to V_{us}

Why are we interested in form factors?

- Sem
  - E
    p
- Cor xtract V_{us}
- But V_{us}



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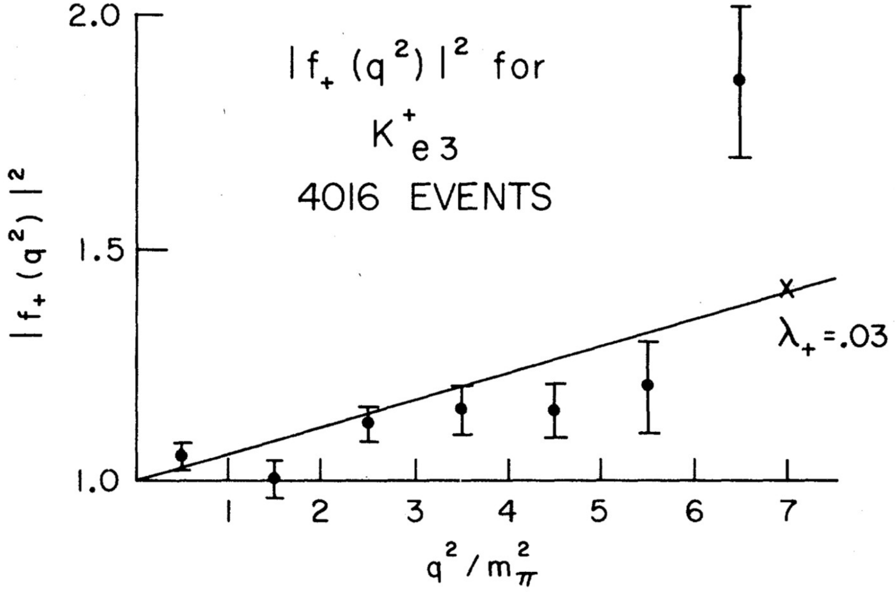
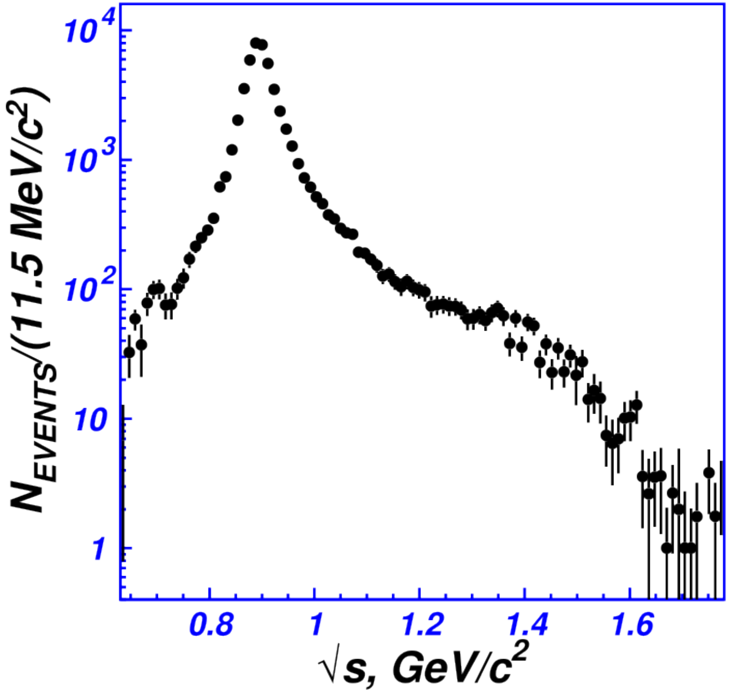
$K \to \pi e \nu$



Chiang, Rosen, Shapiro, Handler, Olsen, Pondrom 1972

$\tau \to K \pi \nu$

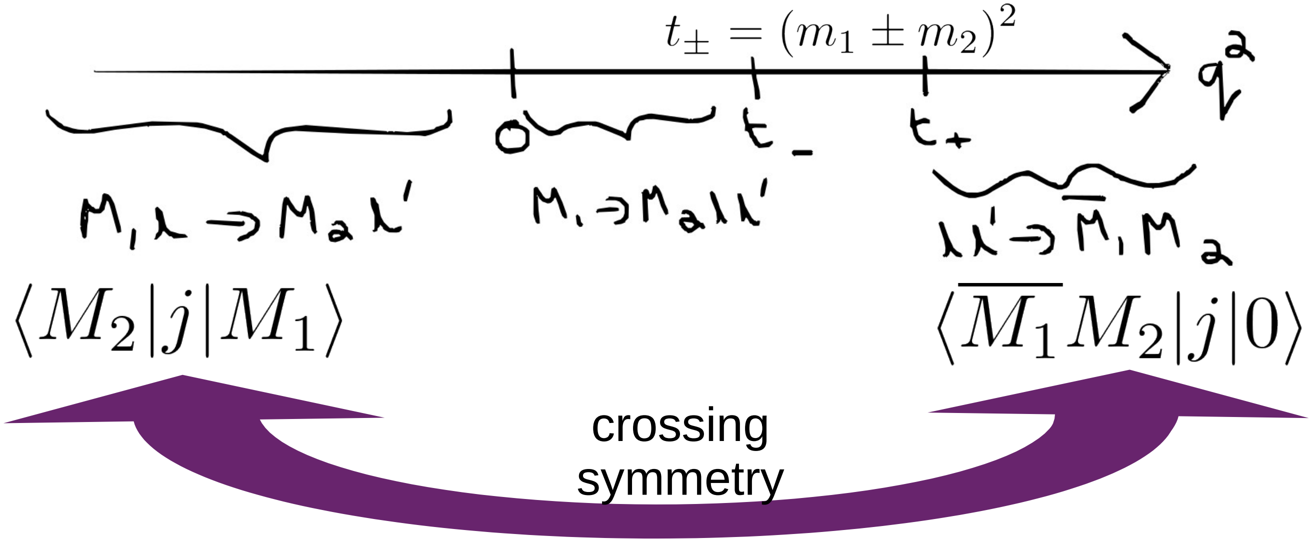
Belle 0706.2231



What is a form factor?

- Hadronic quantities

$$\langle M_2(p_2)|j|M_1(p_1)\rangle \sim F(q^2=(p_1-p_2)^2)$$



BGL / "traditional" parameterisation

- Applies in semi-leptonic decay region
- Dispersion relation, using z mapping and defining outer function
- Three slide summary…

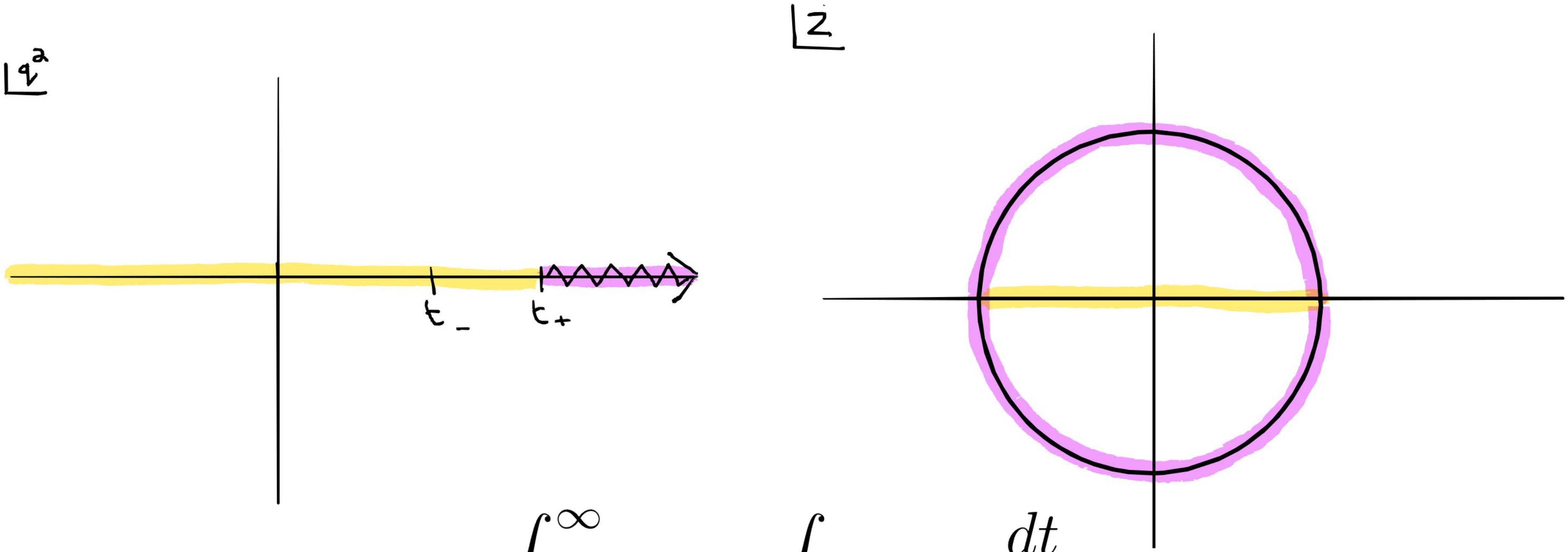
Dispersion relation (1/3)

- Consider $\Pi(q^2) =$ ~~~O~~~
- $\Pi(q^2) = \int_{t_+}^{\infty} dt \frac{\mathrm{Im}\,\Pi}{t-q^2} = \int_{t_+}^{\infty} dt \frac{1}{t-q^2} \int_{\mathrm{P.S.}} \sum_X \langle 0|j|X\rangle\langle X|j^\dagger|0\rangle$
- Sum contains $|F|^2$ plus other positive terms
- $\Pi(q^2) \geq \int_{t_+}^{\infty} dt \frac{1}{t-q^2} \frac{\sqrt{\lambda(t, M_1^2, M_2^2)}}{t} |F(t)|^2$

or

$\int_{t_+}^{\infty} dt \frac{1}{\Pi(q^2)} \frac{1}{t-q^2} \frac{\sqrt{\lambda(t, M_1^2, M_2^2)}}{t} |F(t)|^2 \leq 1$

$q^2 \to z$ conformal mapping (2/3)



$$\int_{t_+}^{\infty} dt \to \oint_{|z|=1} dz \frac{dt}{dz}$$

Define outer function (3/3)

- $\displaystyle \int_{t_+}^{\infty} dt \quad \frac{1}{\Pi(q^2)}\,\frac{1}{t-q^2}\,\frac{\sqrt{\lambda}}{t}|F(t)|^2 \leq 1$

Define outer function (3/3)

- $\oint_{|z|=1} dz \underbrace{\frac{dt}{dz} \frac{1}{\Pi(q^2)} \frac{1}{t-q^2} \frac{\sqrt{\lambda}}{t}}_{|\phi|^2} |F(t)|^2 \leq 1$

Define outer function (3/3)

- $\displaystyle \oint_{|z|=1} dz \underbrace{\frac{dt}{dz}\frac{1}{\Pi(q^2)}\frac{1}{t-q^2}\frac{\sqrt{\lambda}}{t}}_{|\phi|^2} |F(t)|^2 \leq 1$

- Then write $F = \frac{1}{\phi}\sum_i \alpha_i z^i$
- $\displaystyle \oint_{|z|=1} dz |\phi|^2 |F|^2 \leq 1 \Rightarrow \oint_{|z|=1} dz \sum_{i,j} \alpha_i \alpha_j^* z^i \bar{z}^j \leq 1 \Rightarrow \sum_i |\alpha_i|^2 \leq 1$

- Makes dispersive bound extremely simple!

What's new?

Above threshold form factors

- People have been thinking about extending the BGL-type formalism to τ decays for a while

JLAB-THY-98-04

New Constraints on Dispersive Form Factor Parameterizations from the Timelike Region

W. W. Buck*
The Nuclear/High Energy Physics Research Center, Hampton University, Hampton, VA 23668
and
Jefferson Lab, 12000 Jefferson Avenue, Newport News, VA 23606

Richard F. Lebed†
Jefferson Lab, 12000 Jefferson Avenue, Newport News, VA 23606
(February, 1998)

Buck Lebed 1998

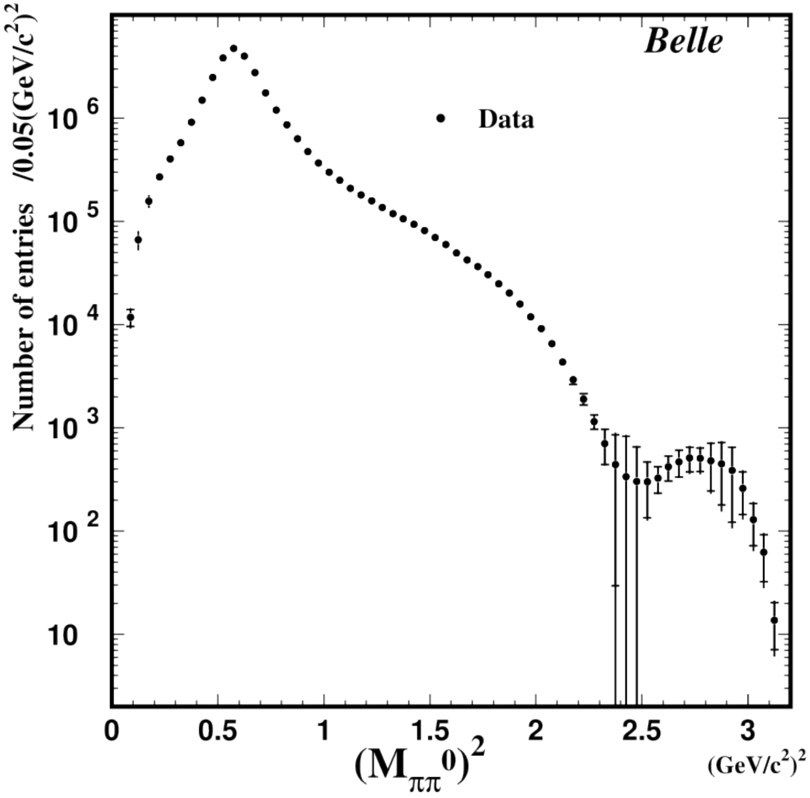
- An issue they discuss is that with $F = \frac{1}{\phi} \sum_i \alpha_i z^i$, F picks up two incorrect behaviours from ϕ
- ϕ has a zero at $q^2 = t_+$ => F blows up
- Asymptotic behaviour of ϕ as $q^2 \to \infty$ leads to $F(q^2 \to \infty) \sim (q^2)^{1/4}$

What's wrong? And how do we fix it?

- Neither is physical
  - Experiment tells us F is finite near threshold
  - And large energy QCD can be used to show $F \sim 1/q^2$
- What we do: explicitly modify the outer function to correct the behaviour in the two limits

What's new?

- We have to reproduce the ρ pole in our parameterisation
- Hard to see how a polynomial expansion can fit this behaviour
- Again make it explicit



Belle 0805.3773

The new look

$$F = \frac{W}{\phi} \frac{1}{z - z_r} \frac{1}{z - z_r^*} \sum_i b_i z^i$$

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- Finite at threshold ✅

The new look

$$F = \frac{W}{\phi}\frac{1}{z-z_r}\frac{1}{z-z_r^*}\sum_i b_i z^i$$

- Physical pole at z_r ✅
- Finite at threshold ✅
- Correct large energy limit ✅

What about the dispersive bound

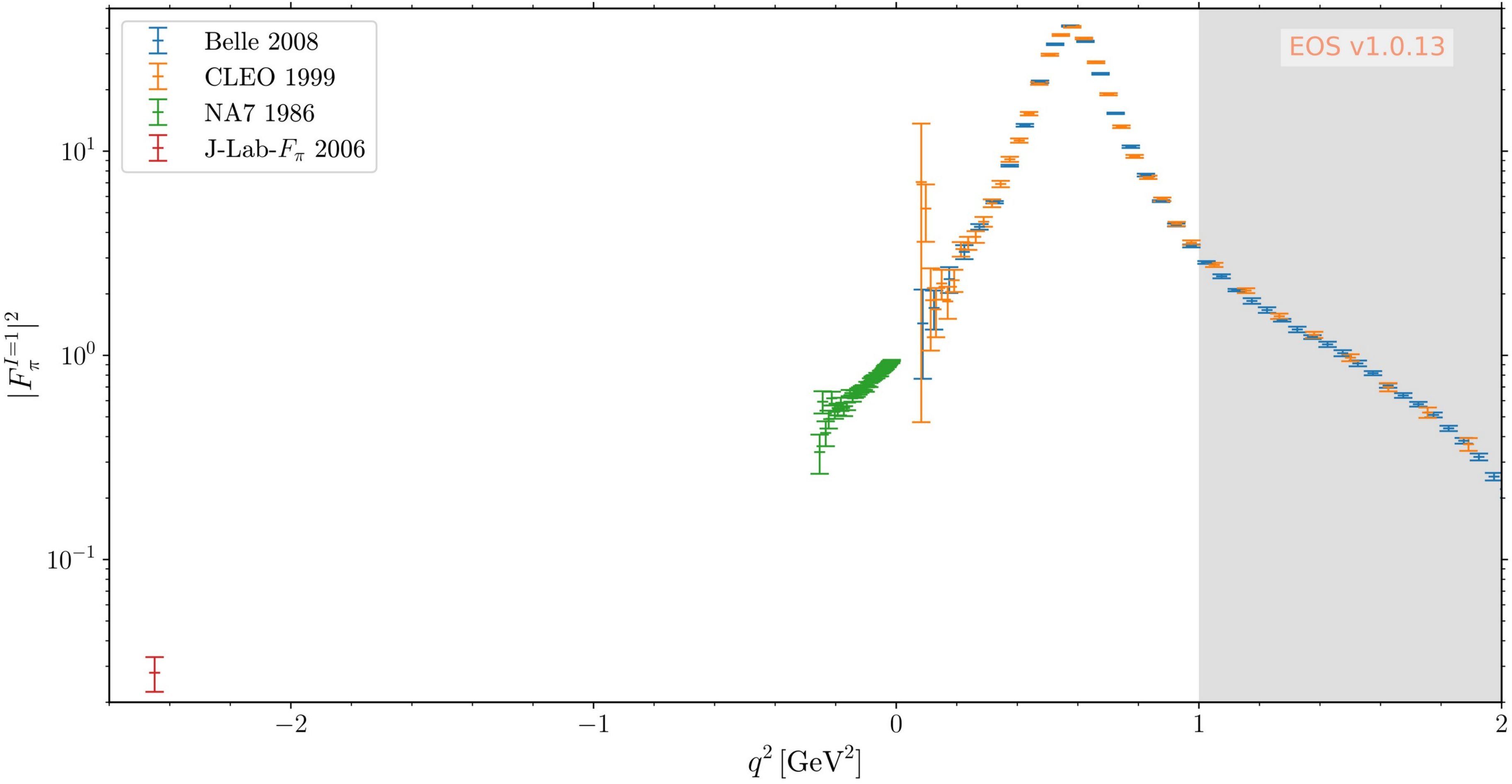
- Dispersive bound is of the form $\oint |\phi F|^2 \leq 1$
- With the standard form ($F = \frac{1}{\phi} \sum_i \alpha_i z^i$), the bound nicely simplifies to $\sum_i |\alpha_i|^2 \leq 1$
- But with our form (with explicit pole factors), doesn't simplify like that
  - We were unable to come up with a form that preserves the simple dispersive bound expression

New parameterisation

- $F = \frac{W}{\phi} \frac{1}{z - z_r} \frac{1}{z - z_r^*} \sum_i b_i z^i$
  - Physical pole at z_r ✅
  - Finite at threshold ✅
  - Correct large energy limit ✅
  - Dispersive bound on parameters not manifest 😥
- Let's feed in some data and see what we get

Pion proof of concept

Fitting pion form factor data



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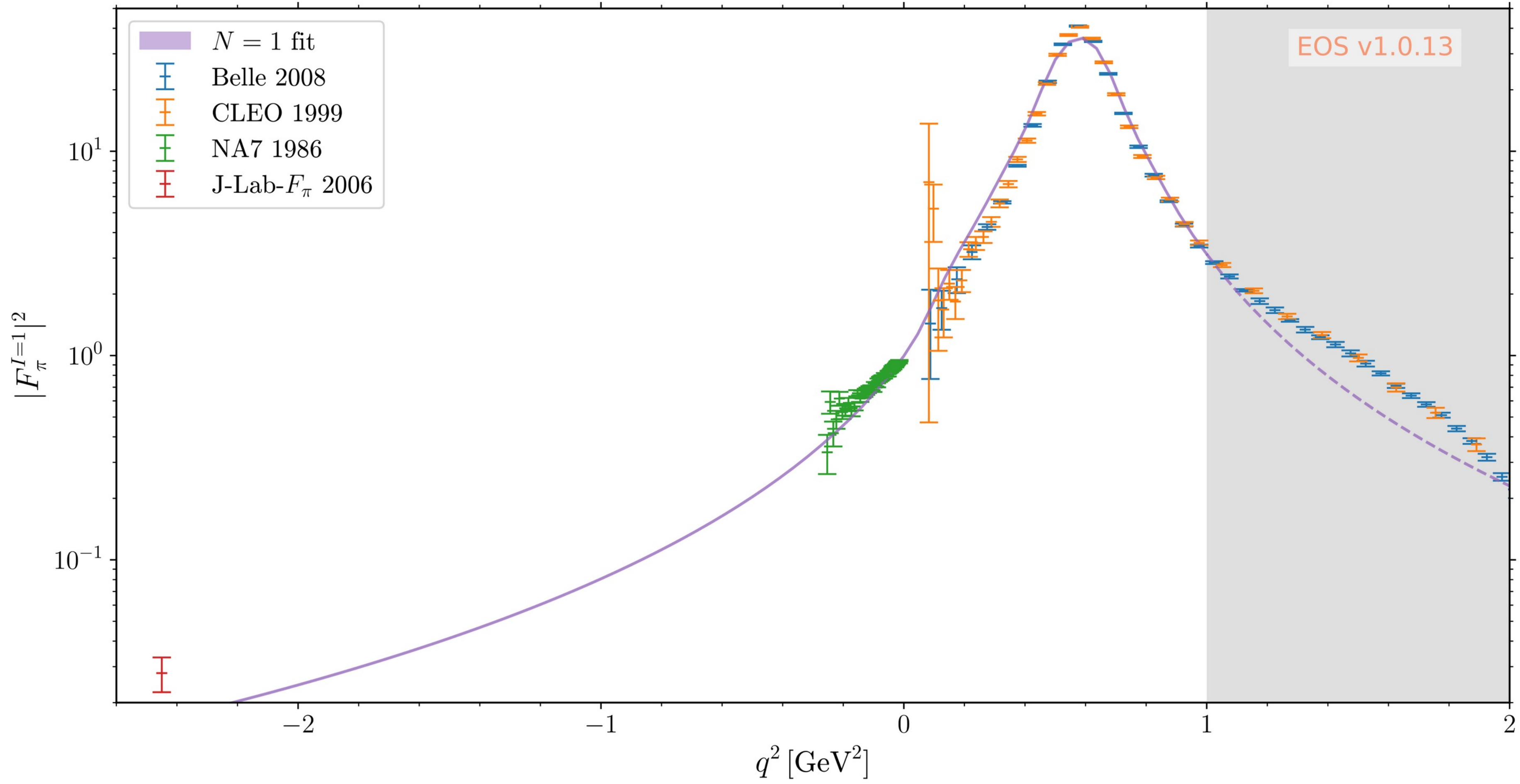

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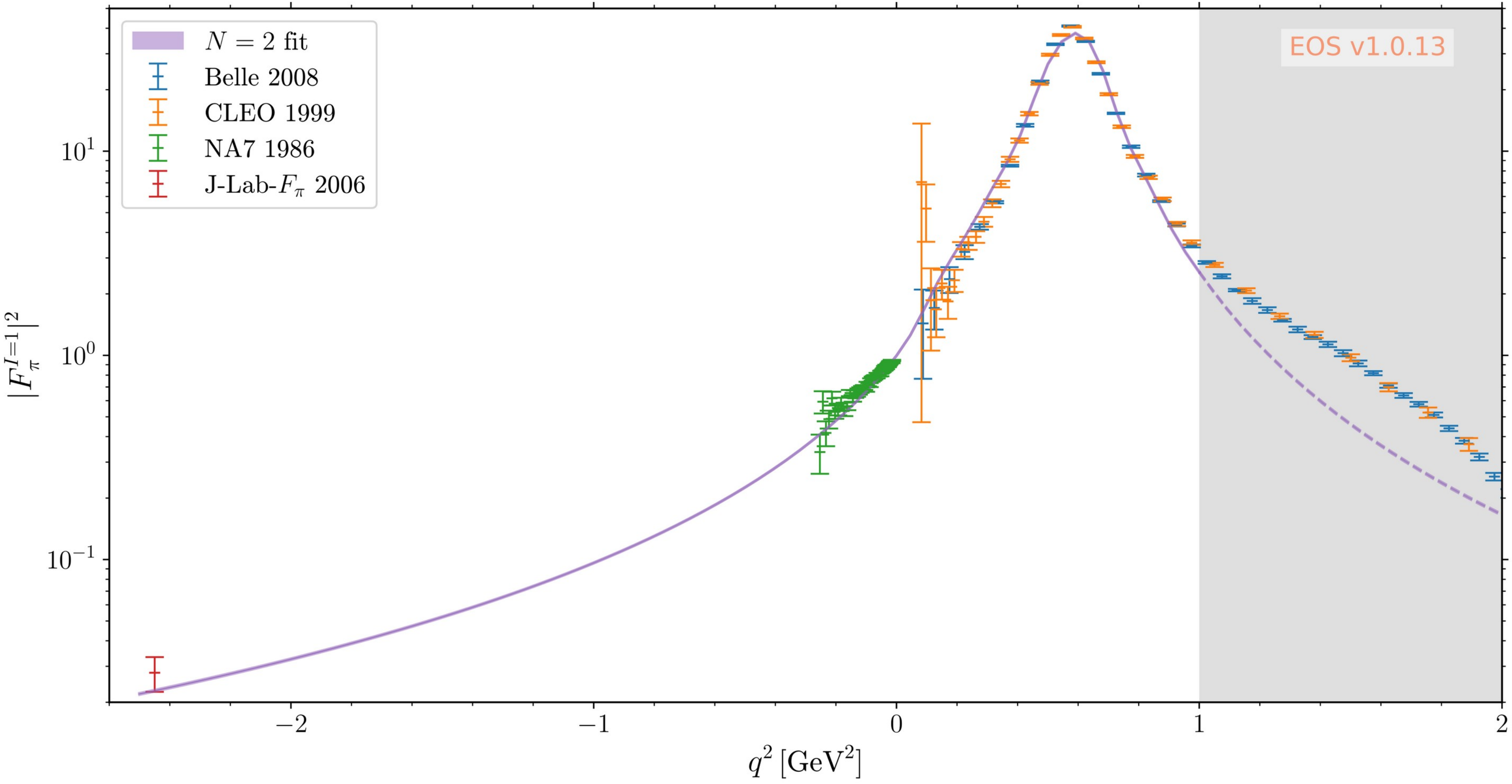

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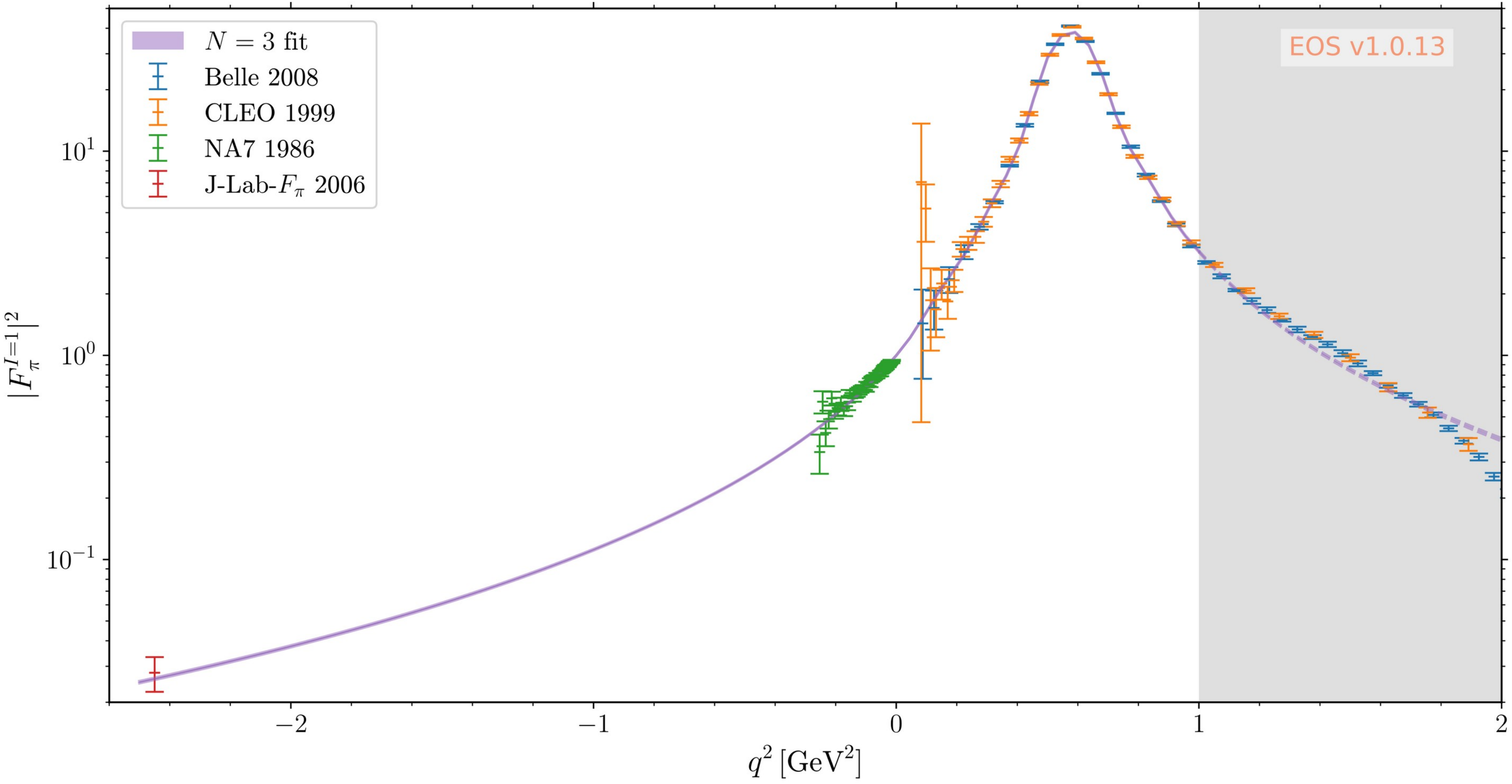

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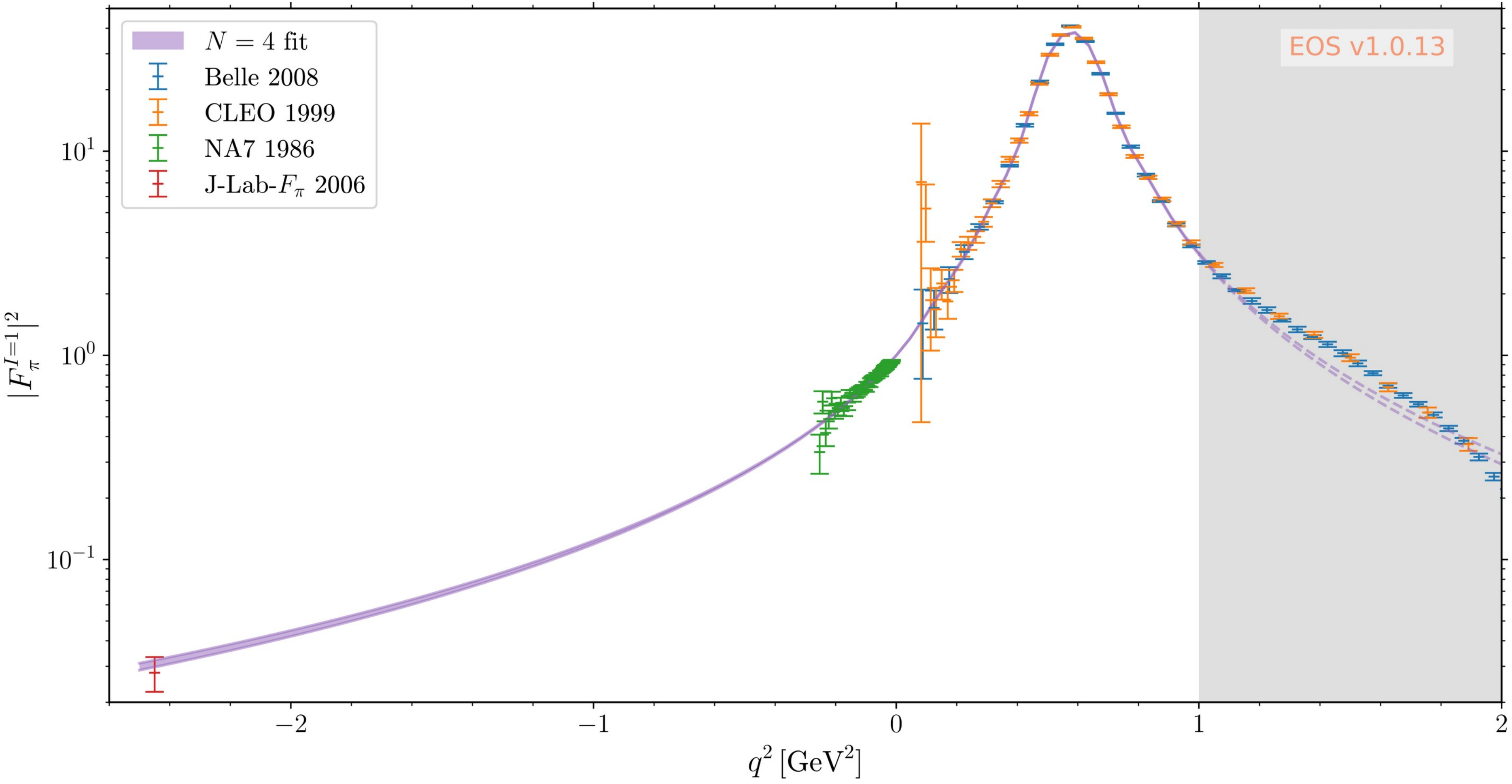

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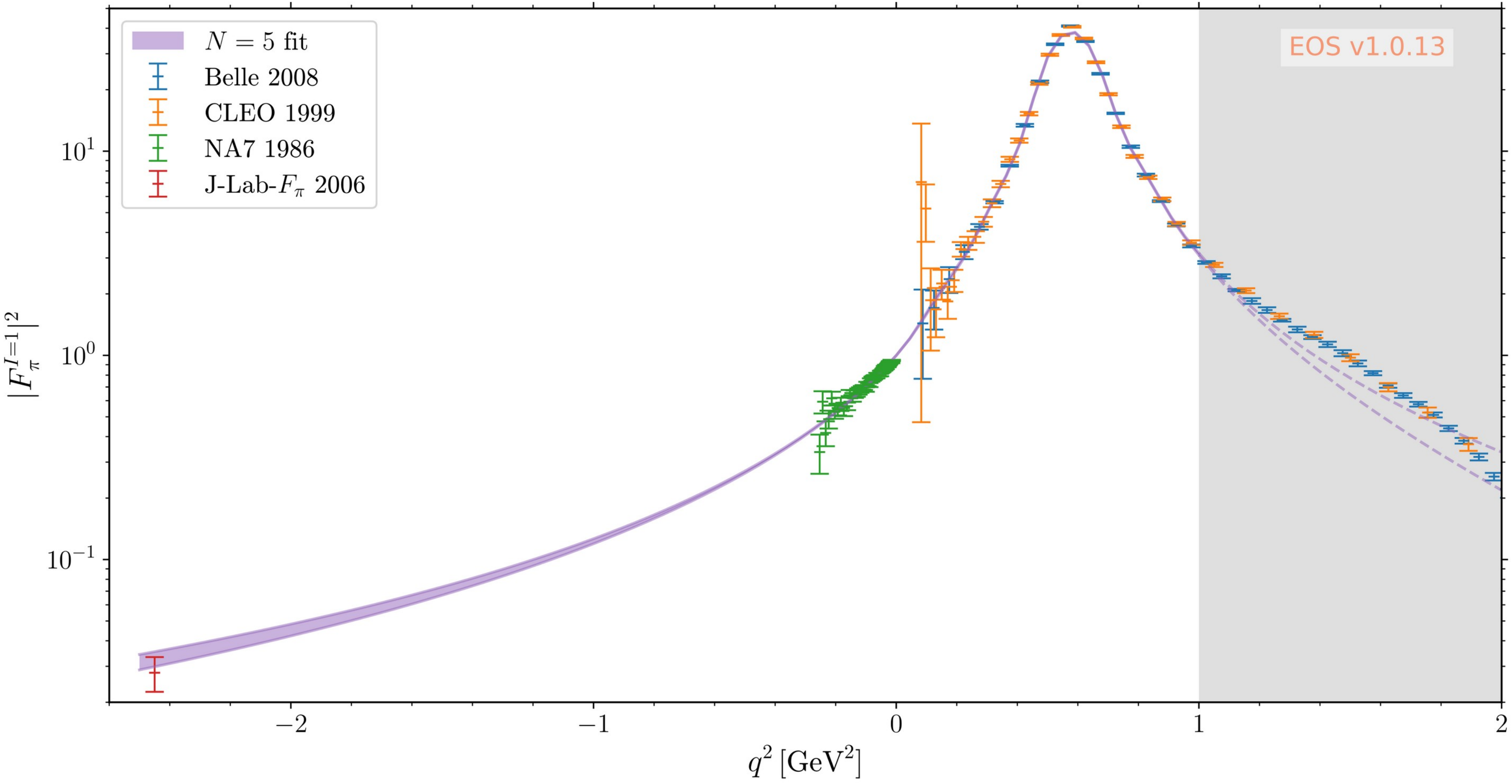
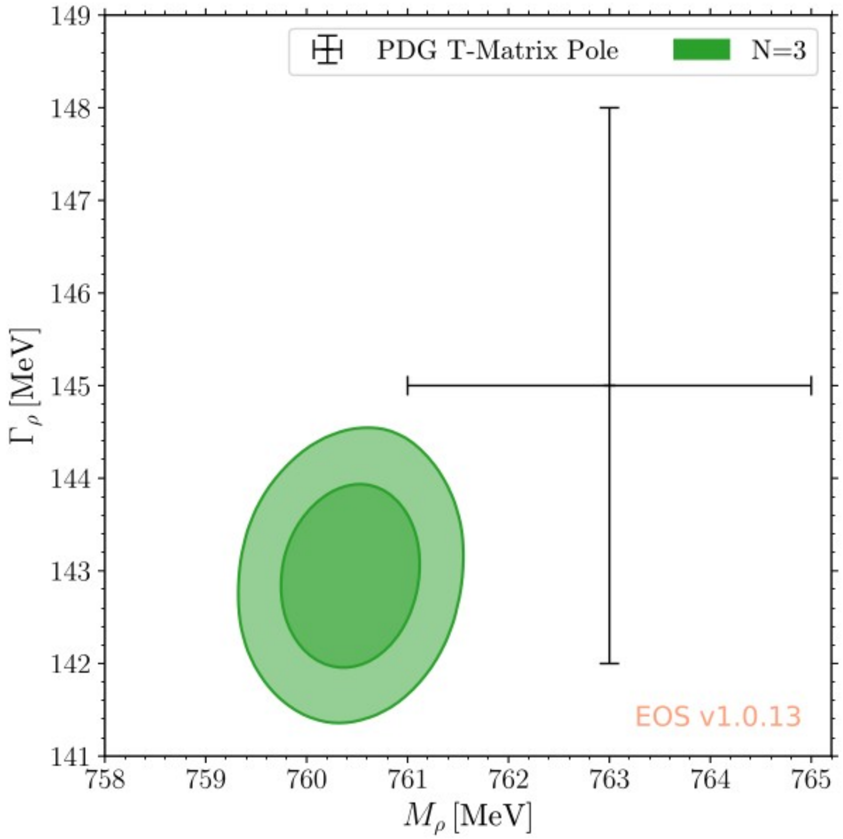


ρ pole parameters

- We extract the ρ mass and width from our fit



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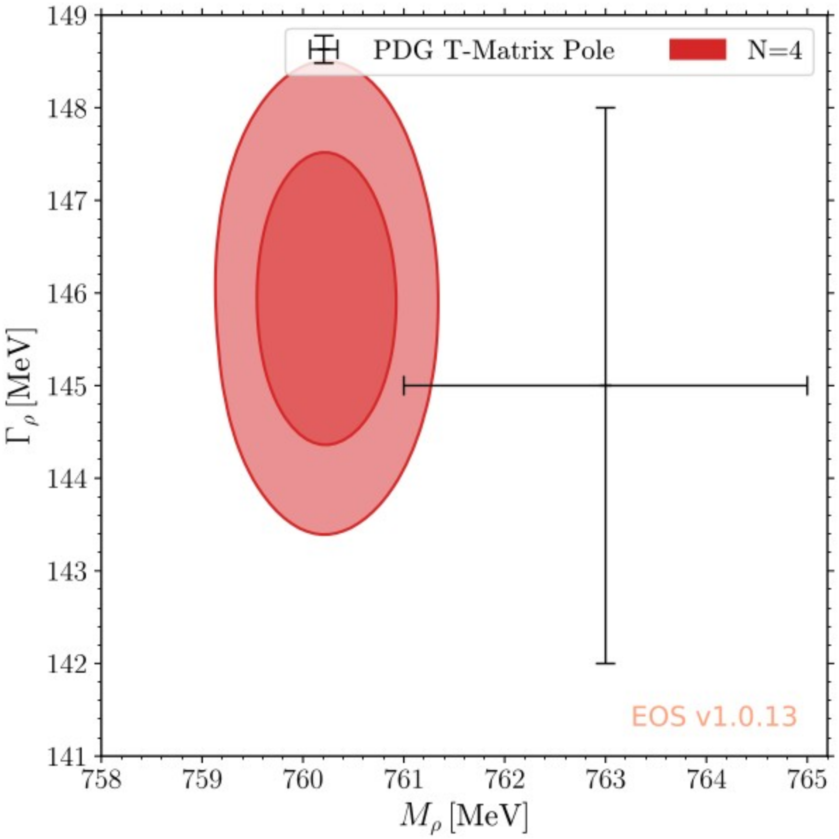
- Stable under increasing order of the expansion



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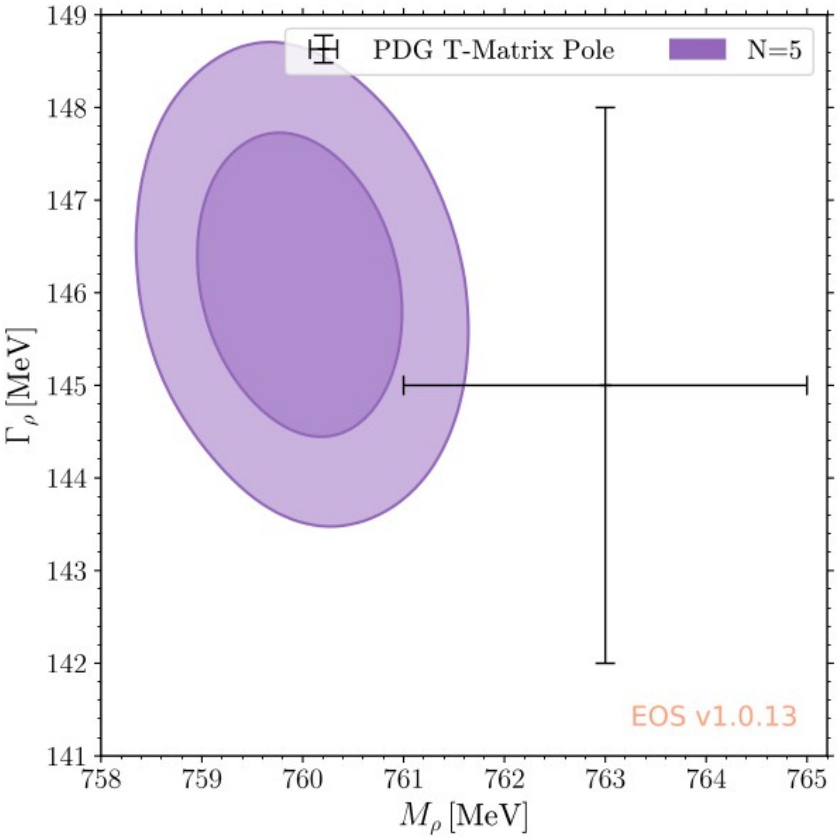


ρ pole parameters

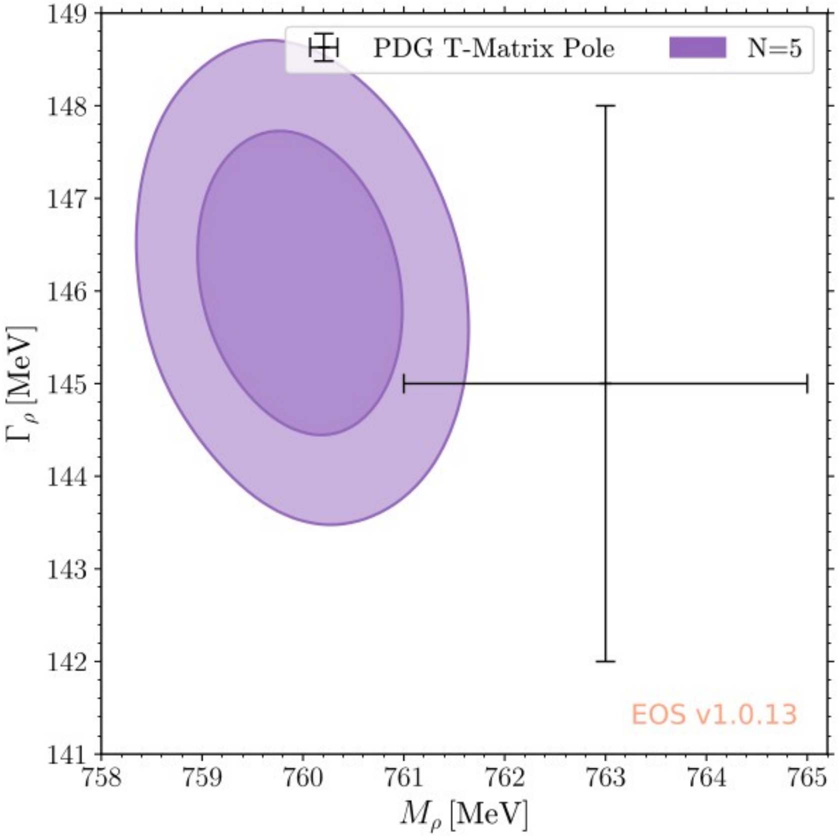
- Our N=5 fit gives

$$M_\rho = (760.0 \pm 0.6)\,\mathrm{MeV}$$

$$\Gamma_\rho = (146.1 \pm 0.9)\,\mathrm{MeV}$$

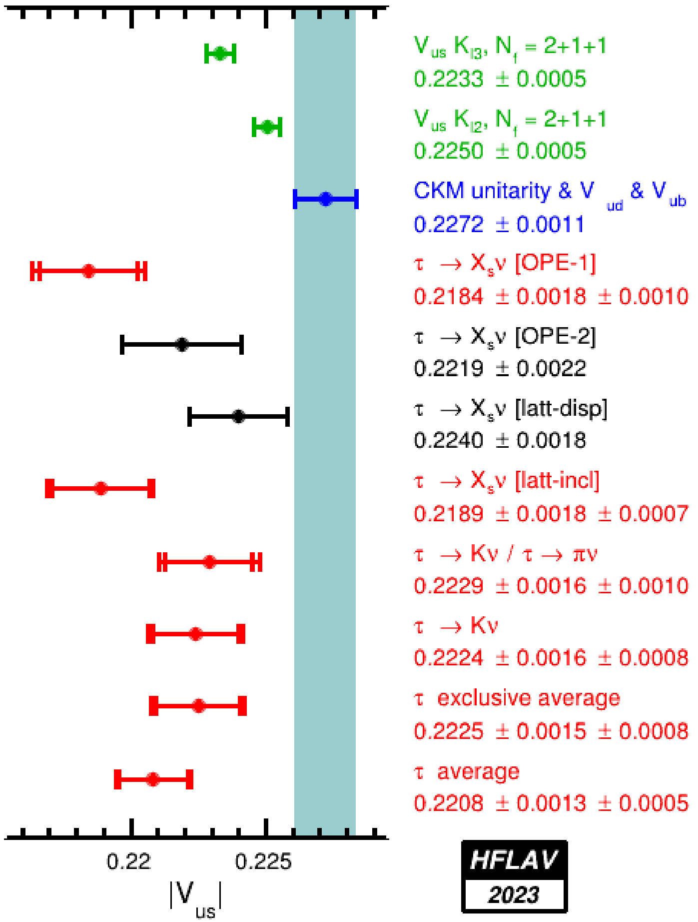
for ρ pole location

- Reasonable agreement with PDG which comes from other methods

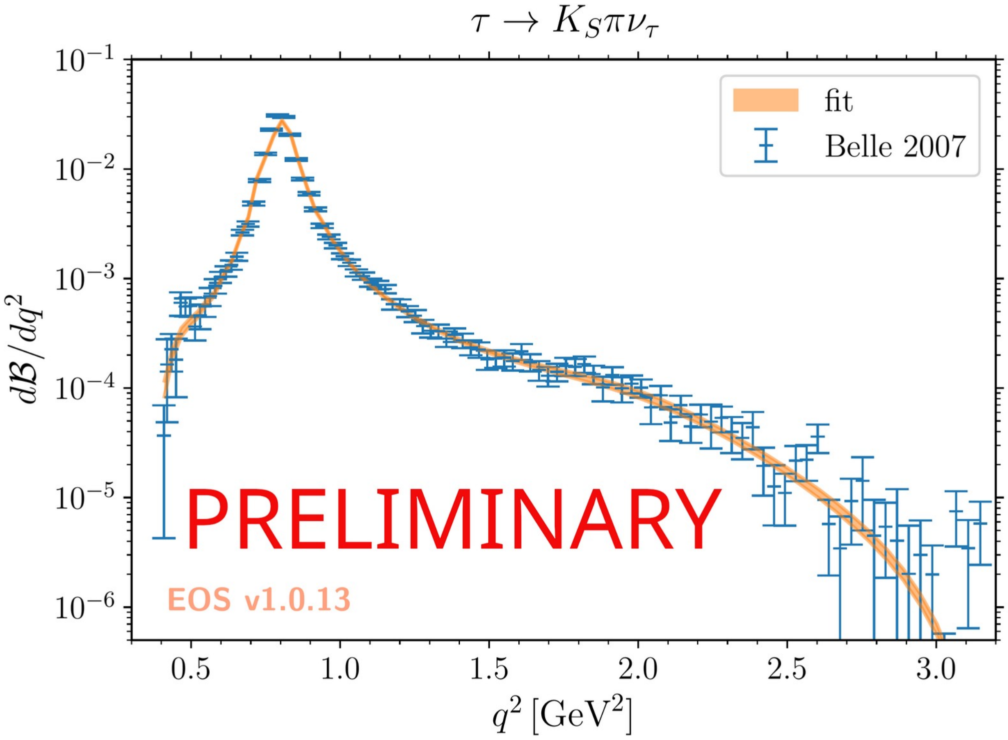


Going forward

- Now we have successful proof of concept, we are working on the $K \to \pi$ form factor
- Will enable a "global" fit to V_{us}, covering $K_{\ell 3}$, $K_{\ell 2}$, and $\tau \to K\pi\nu$



Sneak peak at $\tau \to K\pi\nu$



- Now 2 form factors (scalar and vector), each with two above threshold resonances: $K^*(890), K^*(1410)$, $K_0^*(700), K_0^*(1430)$
- We can fit them!

What do we want from future colliders (/current colliders in the future)?

Semi-leptonic kaon decays

- Current data has internal inconsistencies, e.g. for charged kaon decay:

CONSTRAINED FIT INFORMATION

show precise values?

An overall fit to mean life, decay rate, and 15 branching ratios uses 35 measurements and one constraint to determine 8 parameters. The overall fit has a $\chi^2 = 53.4$ for 28 degrees of freedom.

	Mode	Rate	Scale factor
Γ_2	$K^+ \to \mu^+ \nu_\mu$	$(63.56 \pm 0.11) \times 10^{-2}$	1.2
Γ_3	$K^+ \to \pi^0 e^+ \nu_e$	$(5.07 \pm 0.04) \times 10^{-2}$	2.1
Γ_4	$K^+ \to \pi^0 \mu^+ \nu_\mu$	$(3.352 \pm 0.034) \times 10^{-2}$	1.9
Γ_5	$K^+ \to \pi^0 \pi^0 e^+ \nu_e$	$(2.55 \pm 0.04) \times 10^{-5}$	1.1
Γ_{10}	$K^+ \to \pi^+ \pi^0$	$(20.67 \pm 0.08) \times 10^{-2}$	1.2
Γ_{11}	$K^+ \to \pi^+ \pi^0 \pi^0$	$(1.760 \pm 0.023) \times 10^{-2}$	1.1
Γ_{12}	$K^+ \to \pi^+ \pi^+ \pi^-$	$(5.583 \pm 0.024) \times 10^{-2}$	
$\Gamma_{K^\pm}$	$K^\pm$ MEAN LIFE	$(1.2380 \pm 0.0020) \times 10^{-8}$ (s#sup{#n{-1}})	1.8

Semi-leptonic kaon decays

- Current data has internal inconsistencies, e.g. for neutral kaon decay:

$\Gamma(K_L^0 \to \pi^\pm e^\mp \nu_e)/\Gamma_{\text{total}}$ Γ_1/Γ

VALUE		EVTS	DOCUMENT ID		TECN
0.4055 ± 0.0011	OUR FIT Error includes scale factor of 1.7.				
0.4047 ± 0.0028	OUR AVERAGE Error includes scale factor of 3.1.				
0.4007 ±0.0005 ±0.0015		13M	[1] AMBROSINO	2006	KLOE
0.4067 ±0.0011			[2] ALEXOPOULOS	2004	KTEV

$K_{\ell 3}$ decays at BESIII or STCF?

- Potentially…
- Very rough BESIII numbers:
  - TBD, potentially comparable to KLOE dataset
- And of course more from some kind of future charm factory

Current data on $\tau \to K\pi\nu$ decay

- Belle differential data only partially publicly available
  - 2007 paper gives data file
  - 2014 has 3x the data, but only a figure
- BaBar data not publicly available, again only as a figure in their paper

e^+e^- colliders

- FCC-ee will produce ~ 6 trillion Z bosons, CEPC ~ 4.5 trillion
- $\mathcal{B}(Z \to \tau\tau) \sim 3\%$ => 150 billion tau pairs
  - vs ~ 45 billion tau pairs at Belle II, 0.6 billion at Belle
- $\mathcal{B}(\tau \to K\pi\nu) \sim 1\%$ => 1.5 billion decays
- Clearly huge potential with respect to current data

BACKUP

References

- Talk by Alberto Lusiani @ CEPC Flavor Physics Workshop 2023 (https://indico.ihep.ac.cn/event/19839/contributions/138731/)

experimental conditions of tau pairs much better at Z peak compared to lower energies

- better momentum resolution and vertexing because less multiple scattering with higher track momenta
- better higher momentum muon id (much lower pion-to-muon misidentification)
- much better $\tau^+\tau^-$ separation from $q\bar{q}$ background because of higher $q\bar{q}$ multiplicity
- LHC produces more tau leptons, but with much less favourable experimental conditions

Current Vus from tau