Response functions from a Chebyshev expansion

In collaboration with Joanna E. Sobczyk and Sonia Bacca

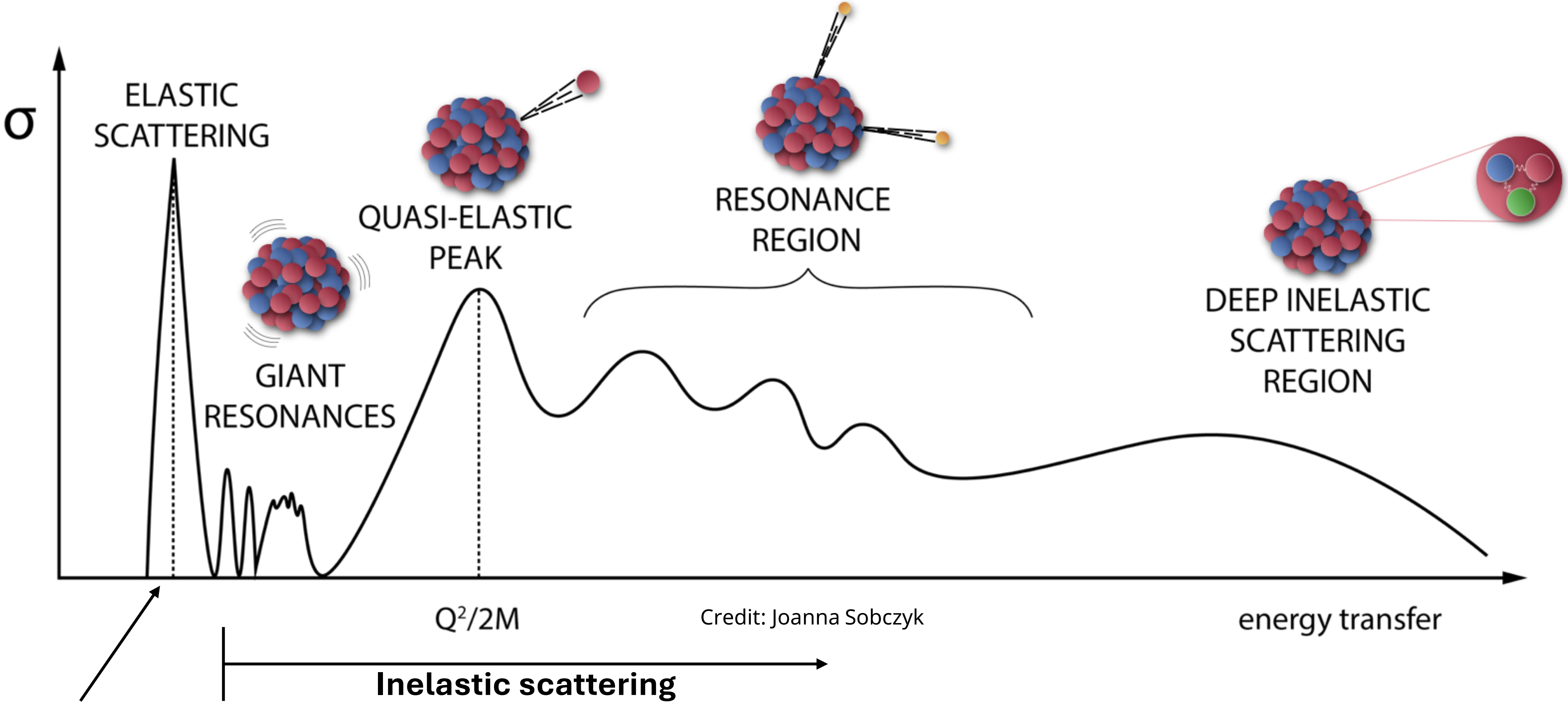
Based on: arXiv:25XX.XXXX

Immo Reis

Neutrino-Nucleus Interactions in the Standard Model and Beyond

MITP, Mainz, 20/05/2025

Motivation

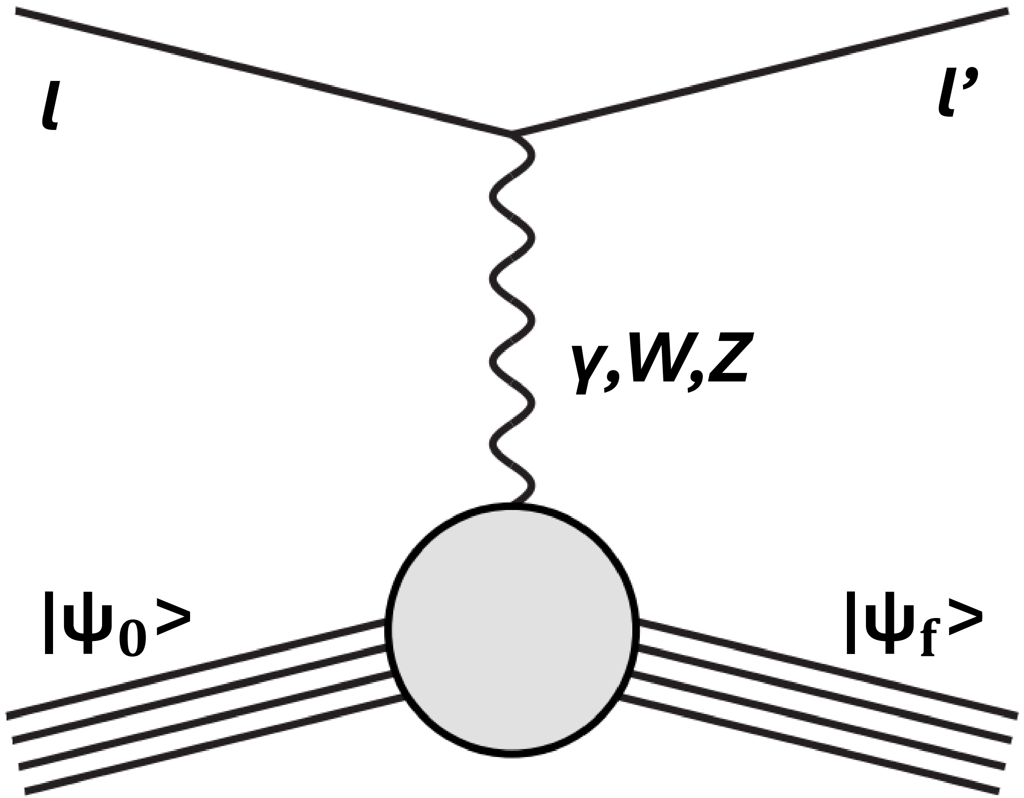


Elastic scattering: CEvNS

Supernova neutrinos

Long-baseline experiments (HyperK, DUNE)

Lepton-Nucleus Scattering

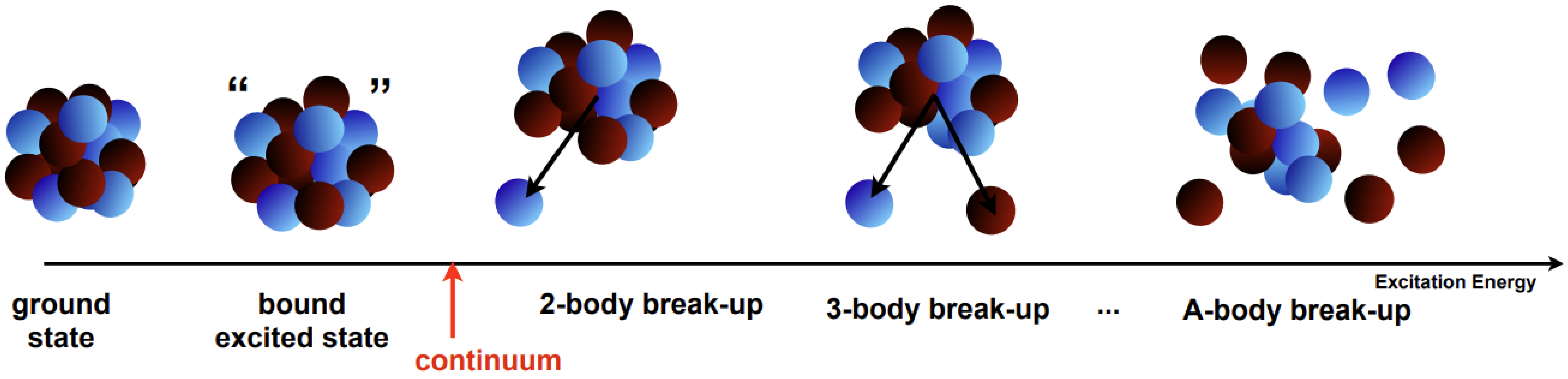


$$\sigma \propto L^{\mu\nu} \; R_{\mu\nu}$$

lepton tensor   nuclear responses

Nuclear Response Functions

$$R_\alpha(\mathbf{q},\omega) = \sum\!\!\!\!\!\!\!\int_f \left| \langle \Psi_f | O_\alpha(\mathbf{q}) | \Psi_0 \rangle \right|^2 \delta\left(E_f - E_0 - \omega\right)$$

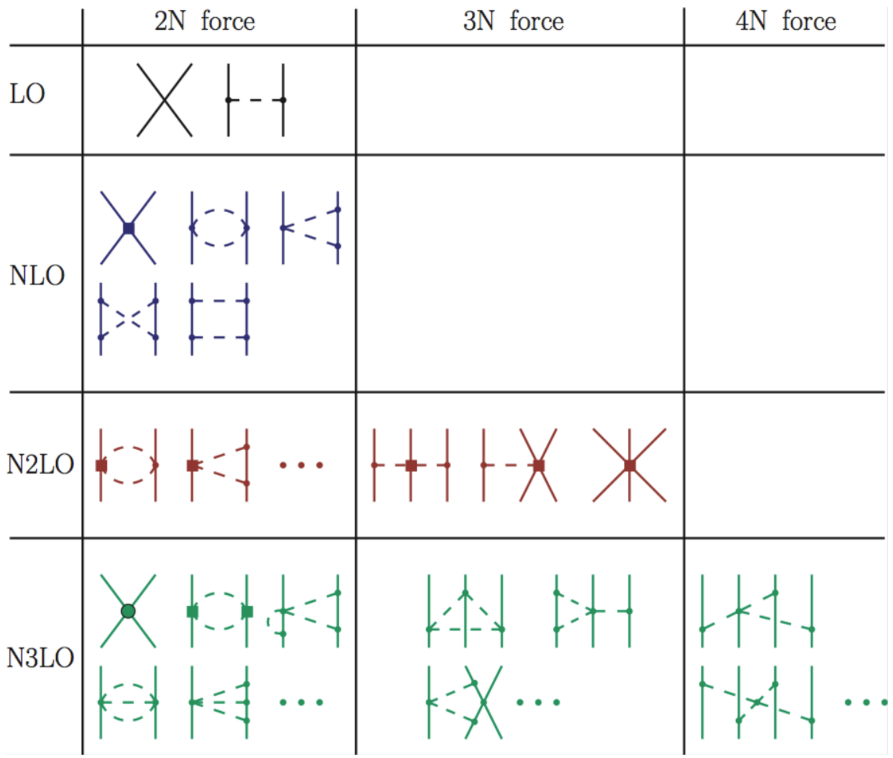


Bacca, Eur. Phys. J Plus 131, 107 (2016)

Hamiltonian & currents

EFT inspired products:

Order-by-order expansion of SM interactions in Hamiltonian and currents



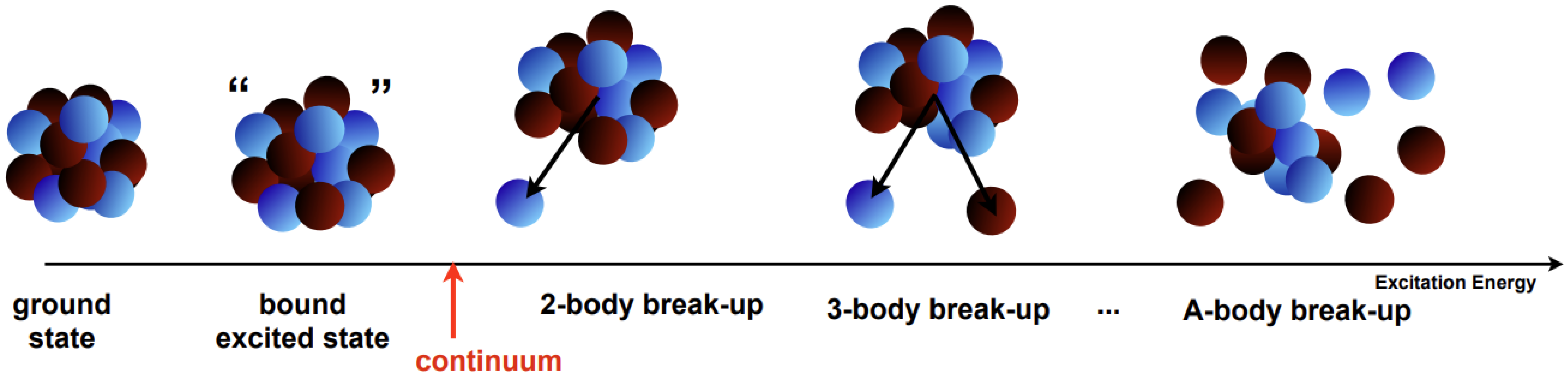
$$J = \sum_i j_i + \sum_{i<j} j_{ij} + \dots$$

Many body problem

- Solve $H|\Psi\rangle = E|\Psi\rangle$
- Finite dimensional expansion in bound basis (harmonic oscillator)

$$|\Psi_0\rangle = \sum_k c_k \, |n_k l_k ....\rangle \qquad \hat{O}_{ij} = \langle n_i l_i \ldots | \, \hat{O} \, | n_j l_j \ldots \rangle$$

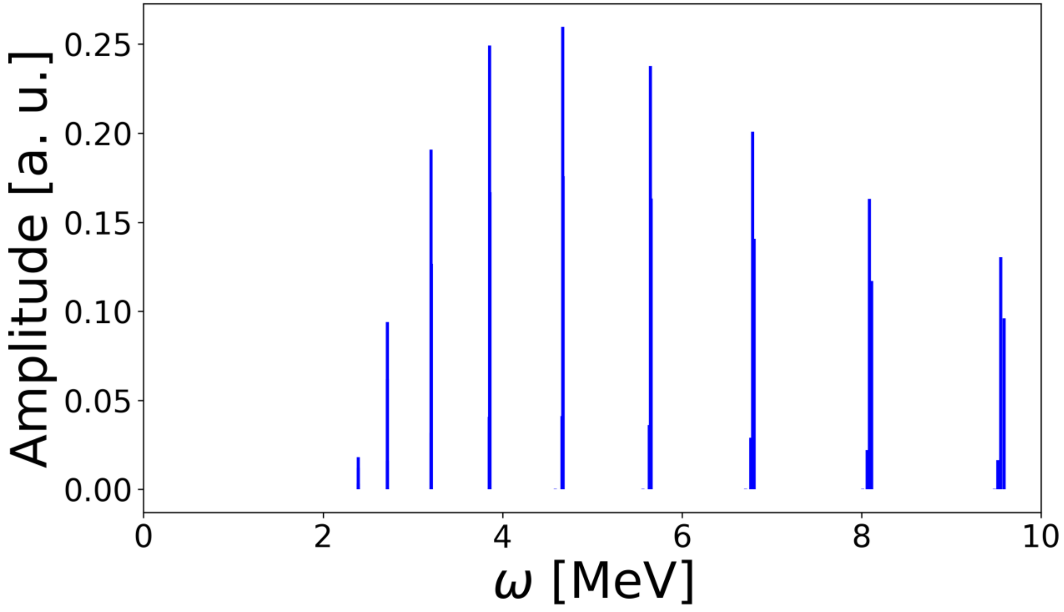
Many body problem



→Finite number of eigenvalues of H and “pseudo continuum states” with bound state boundary conditions

Response in bound state approaches

$$R_\alpha(\mathbf{q},\omega) = \sum\!\!\!\!\!\!\int_f \left| \langle \Psi_f | O_\alpha(\mathbf{q}) | \Psi_0 \rangle \right|^2 \delta\left(E_f - E_0 - \omega\right)$$



Diagonalizing H is computationally prohibitive for relevant nuclei

(but easy for our test case deuteron)

Integral transforms

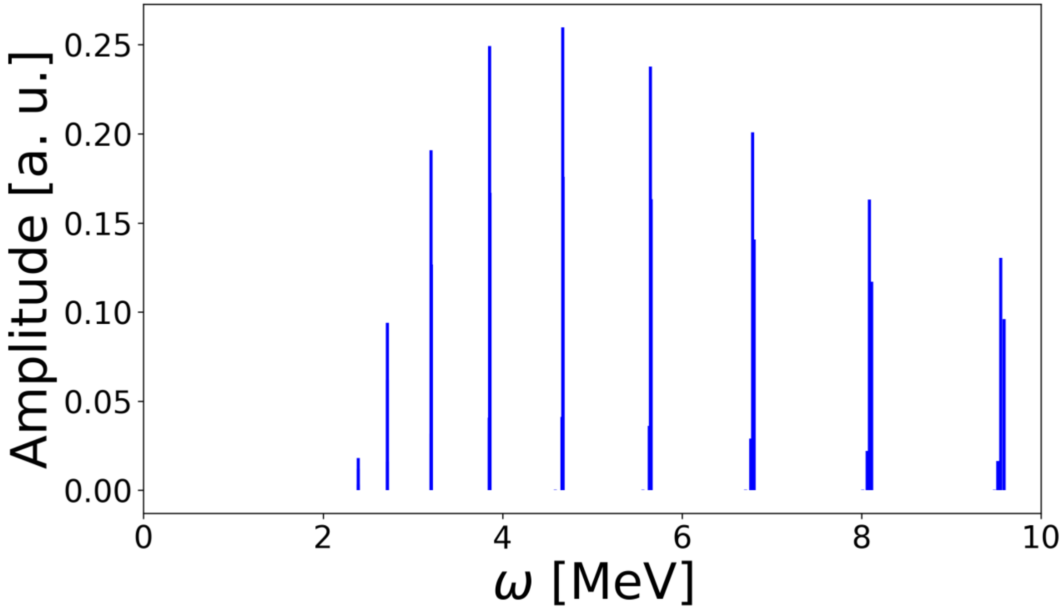
- Can avoid the final states and the discrete nature of the response

$$\Phi(\sigma) = \int d\omega\, K(\sigma,\omega) R(\omega)$$
$$= \langle \Psi_0 | O_\alpha^\dagger(\mathbf{q}) K(\sigma, H - E_0) O_\alpha(\mathbf{q}) | \Psi_0 \rangle$$

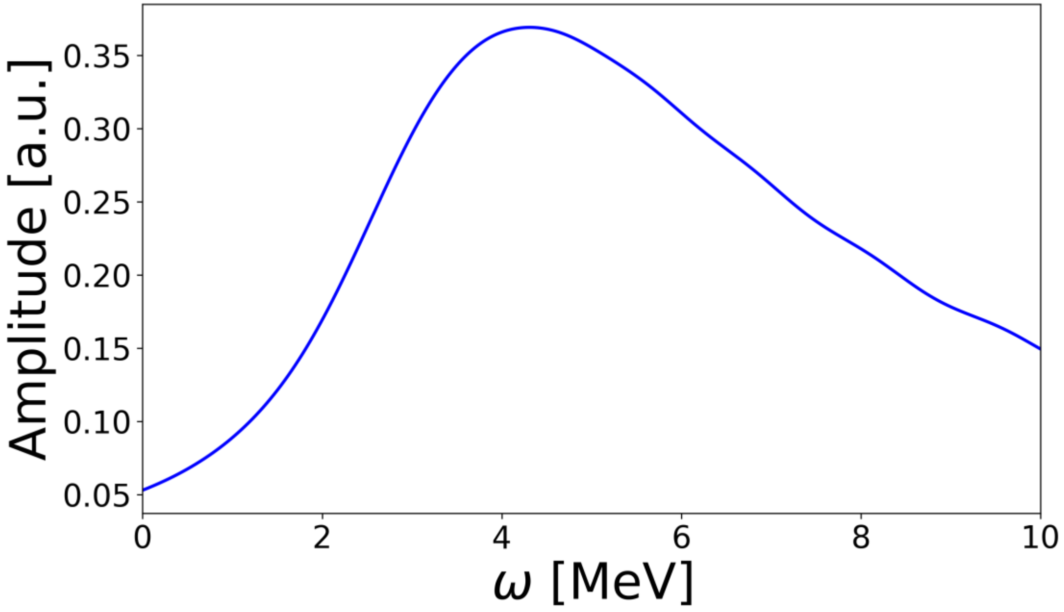
- Kernels are usually some representation of delta function

Integral transforms

Response

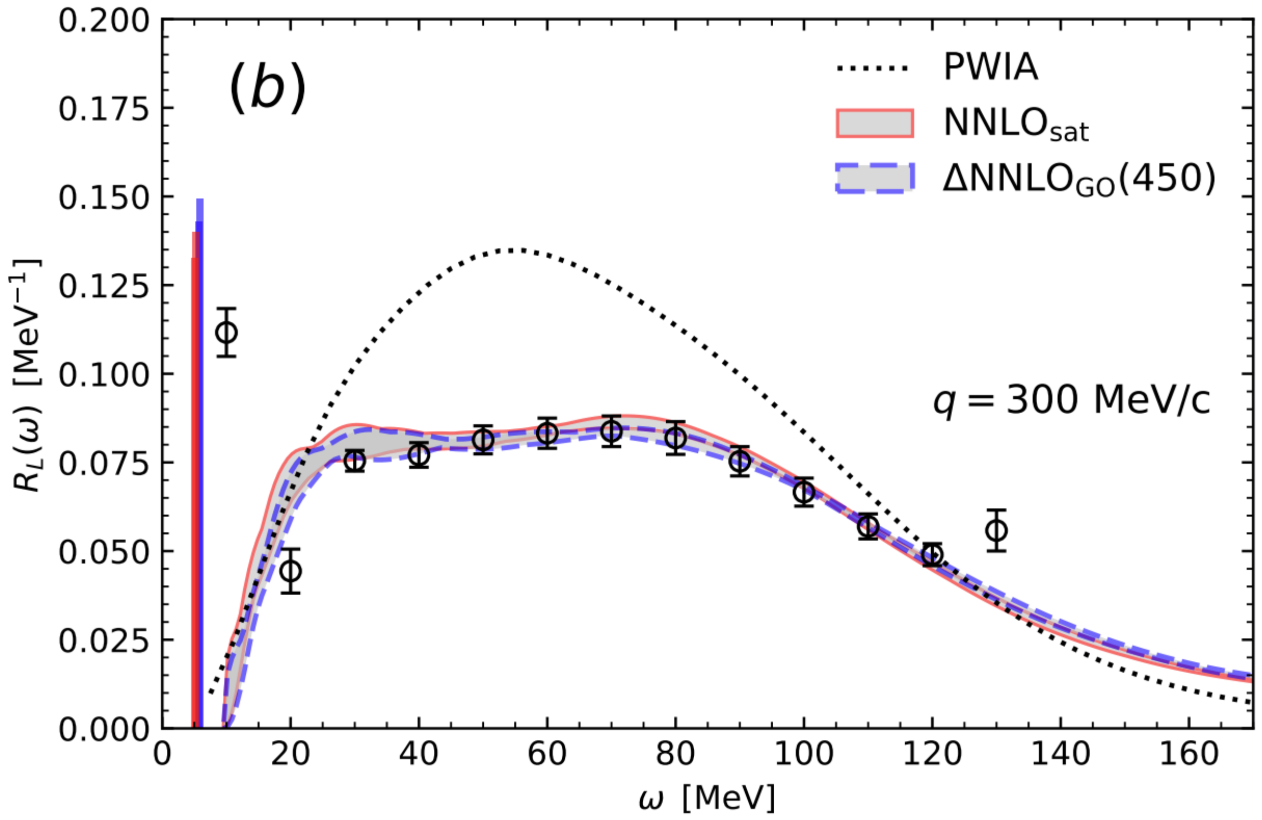


LIT(width 2.5 MeV)



Inversion

- We impose smoothness assumptions
- The inversion is not unique
  →How do we quantify errors?



Sobczyk et al., Phys.Rev.Lett. 127 (2021) 7, 072501

Expansion of the integral transform

$$K(\omega,\sigma) = \sum_{k}^{\cancel{\infty}\, N} c_k(\sigma) T_k(\omega)$$

$$\Phi(\sigma) = \int d\omega\, K(\omega,\sigma) R(\omega) = \sum_{k}^{\cancel{\infty}\, N} c_k(\sigma) m_k$$

$$m_k = \int d\omega\, T_k(\omega) R(\omega) = \langle \Psi_0 | O^\dagger T_k(H) O | \Psi_0 \rangle$$

The Chebyshev approach

$$T_0(x) = 1; \quad T_{-1}(x) = T_1(x) = x;$$
$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x) \ .$$

$$h(\eta, \Delta) = \int d\omega R(\mathbf{q}, \omega) f(\omega, \eta; \Delta) \quad \tilde{h}^{\Lambda}(\eta; \Delta) = \int\int d\sigma d\omega \, K(\omega, \sigma; \lambda) R(\mathbf{q}, \omega) f(\omega, \eta; \Delta)$$

$$\tilde{h}^{\Lambda}(\eta; \Delta - \Lambda) - Q^0 \Sigma \leq h(\eta, \Delta) \leq \tilde{h}^{\Lambda}(\eta; \Delta + \Lambda) + Q^0 \Sigma$$

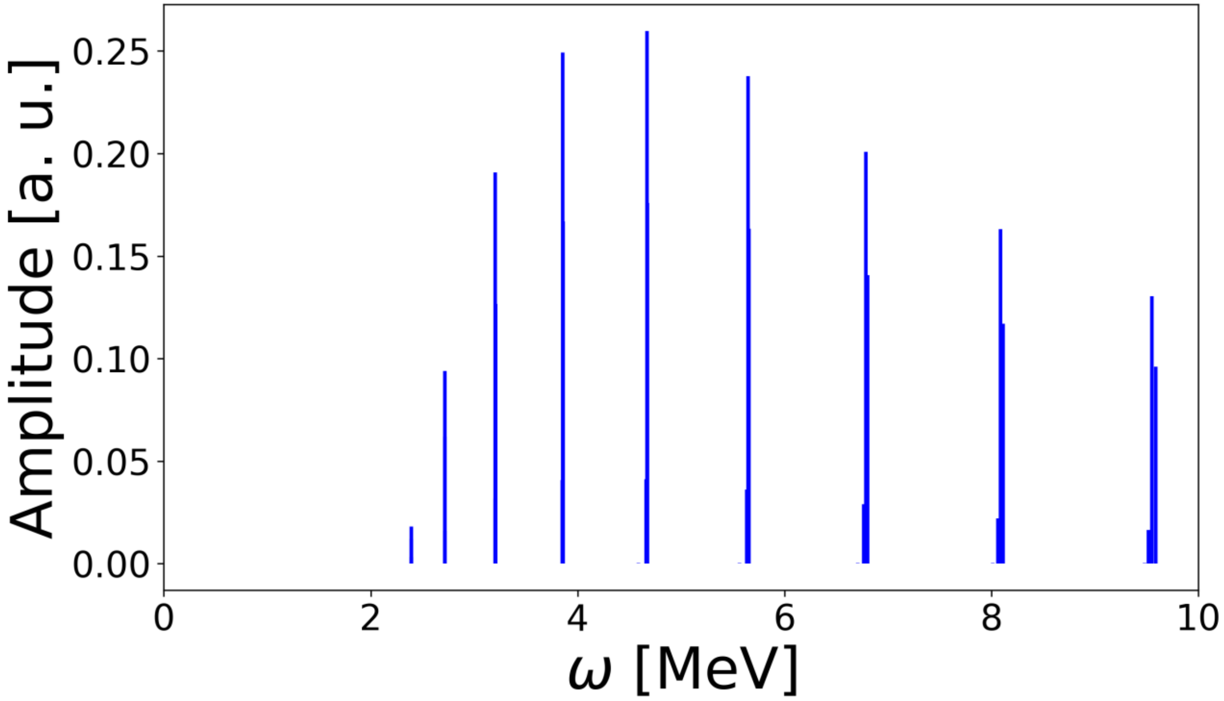
Sobczyk and Roggero, Phys. Rev. E **105** (2022) 5, 055310

To minimize errors:

→Keep things as local as possible: Kernel width O(10-100 keV)

→Need to establish a well motivated binning

Binning Strategy



- Each bin should contain a similar number of eigenvalues (>0)
- Bin edges should be in between eigenvalue clusters to minimize error

Density of states estimation

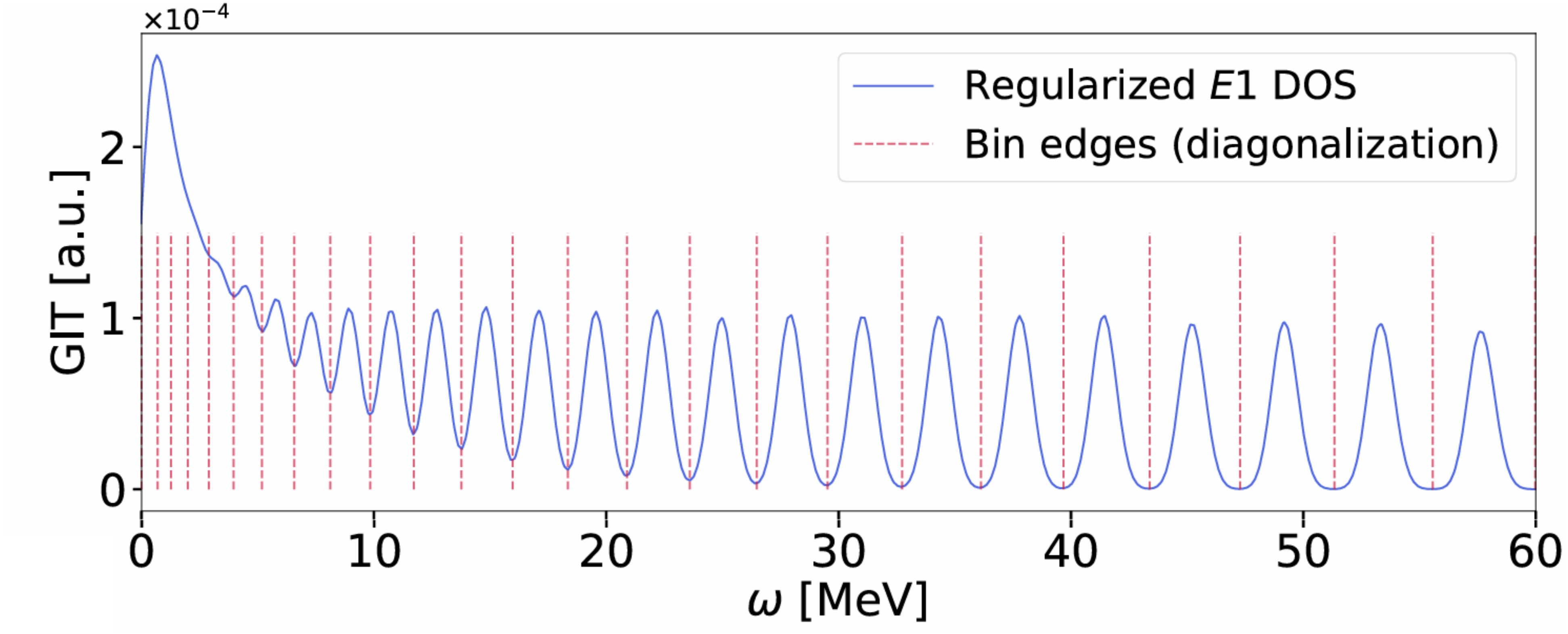
We compute moments of the form $\langle \Psi_0 | O^\dagger H^k O | \Psi_0 \rangle$

$$O | \Psi_0 \rangle = \sum_n d_n | \Psi_n \rangle \qquad H^k O | \Psi_0 \rangle = \sum_n d_n E_n^k | \Psi_n \rangle$$

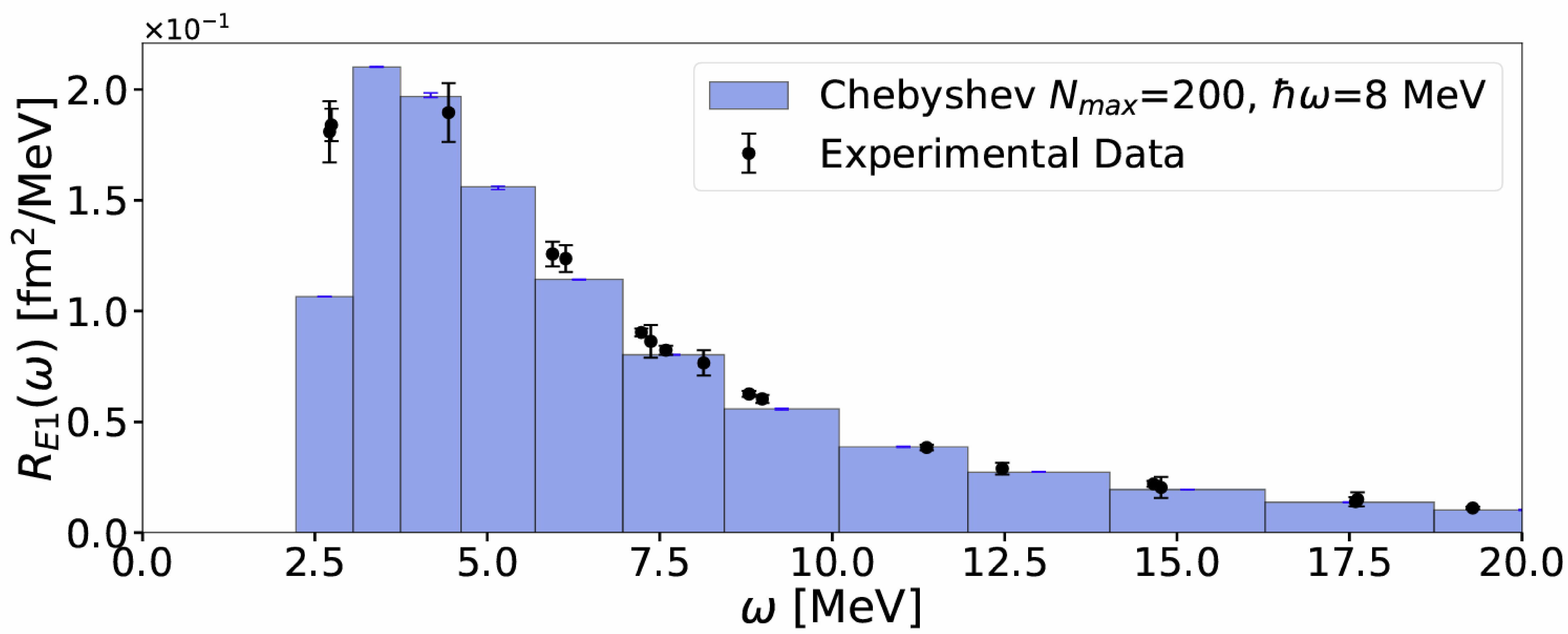
Draw $O | \Psi_0 \rangle$ randomly so each eigenstate contributes equally to the moments

→Regularized estimate for the density of states

Regularized density of states

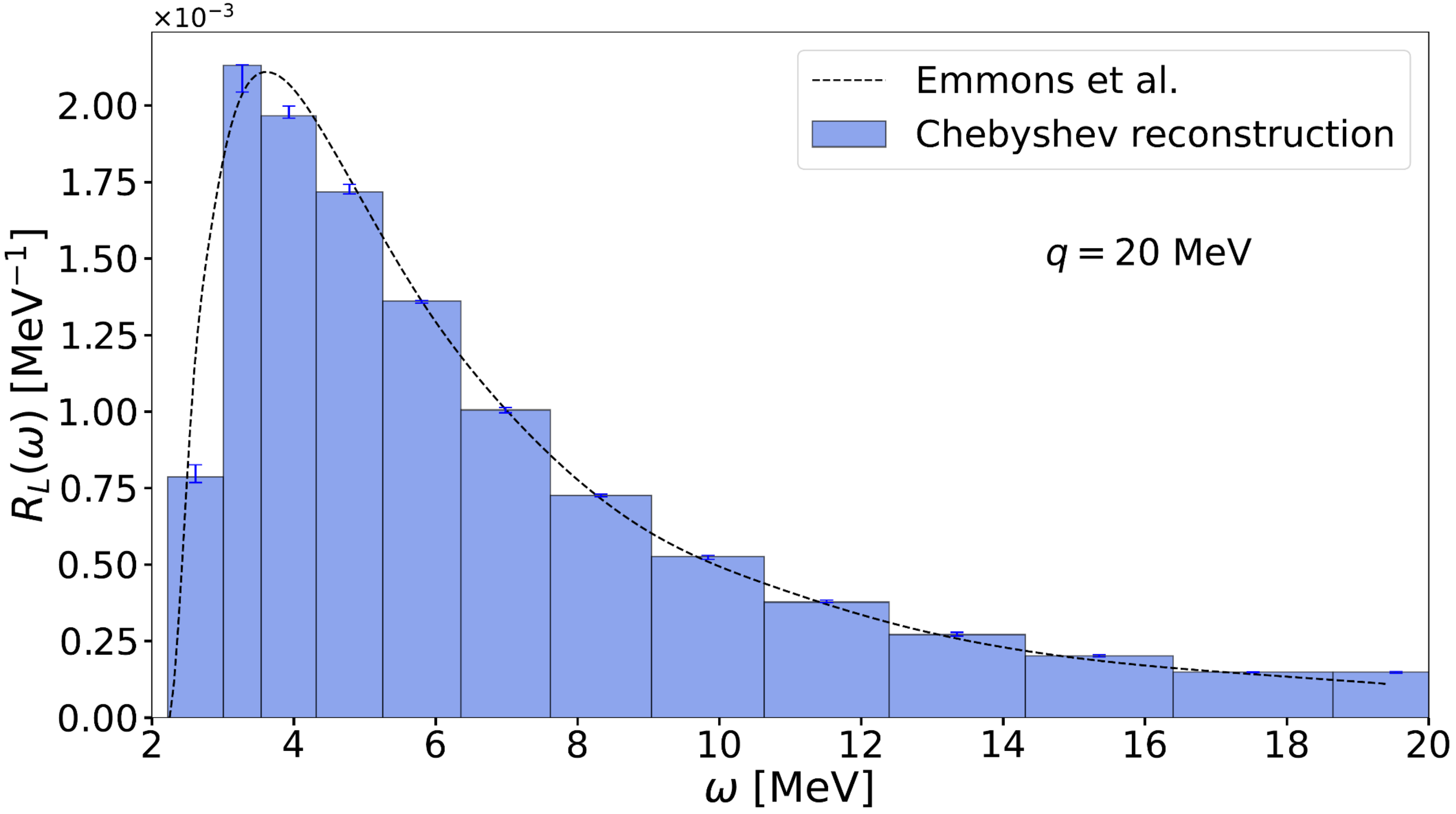


Results for E1



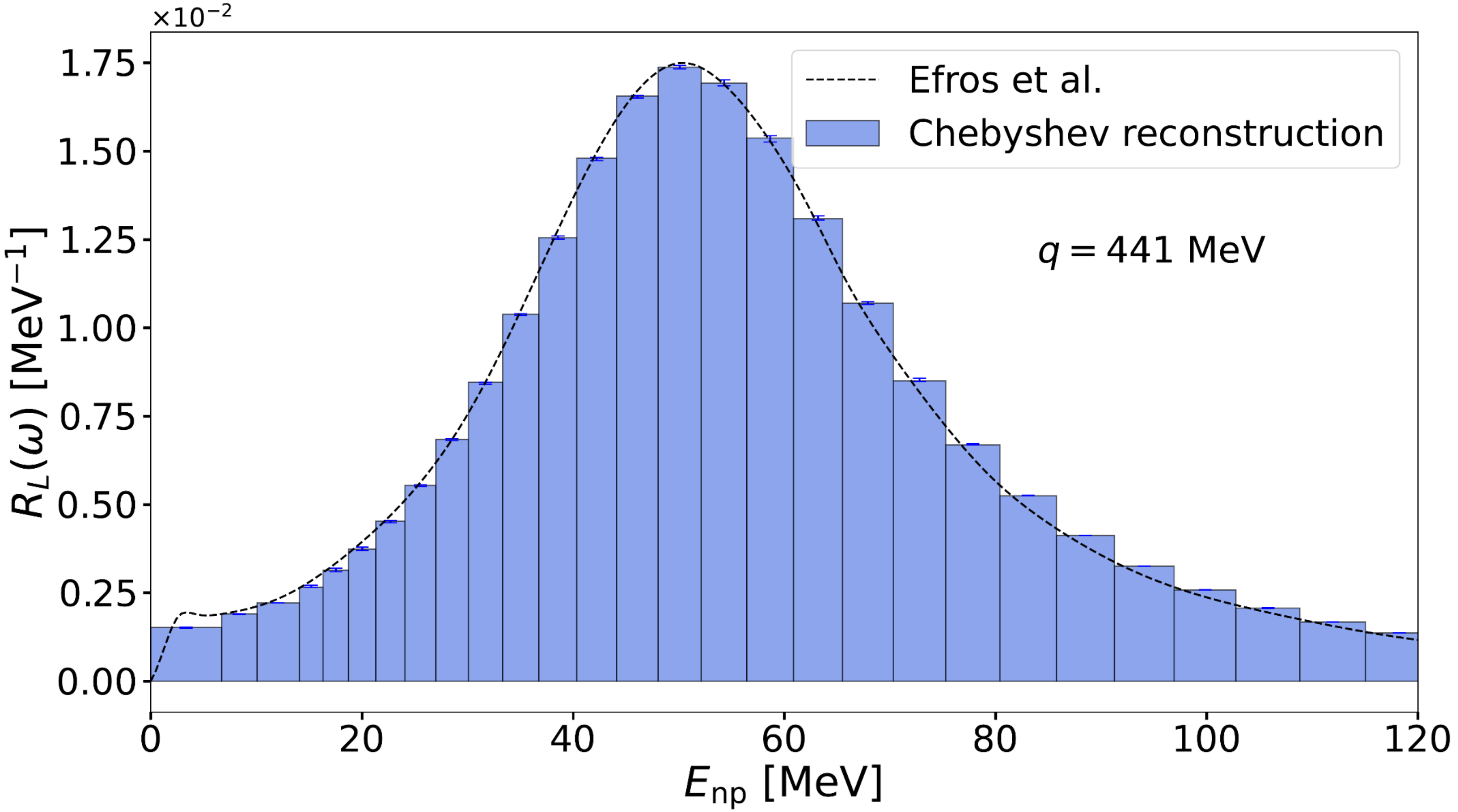
Data from H. Arenhövel and M. Sanzone, Few Body Syst. Suppl. 3, 1 (1991)

Longitudinal response



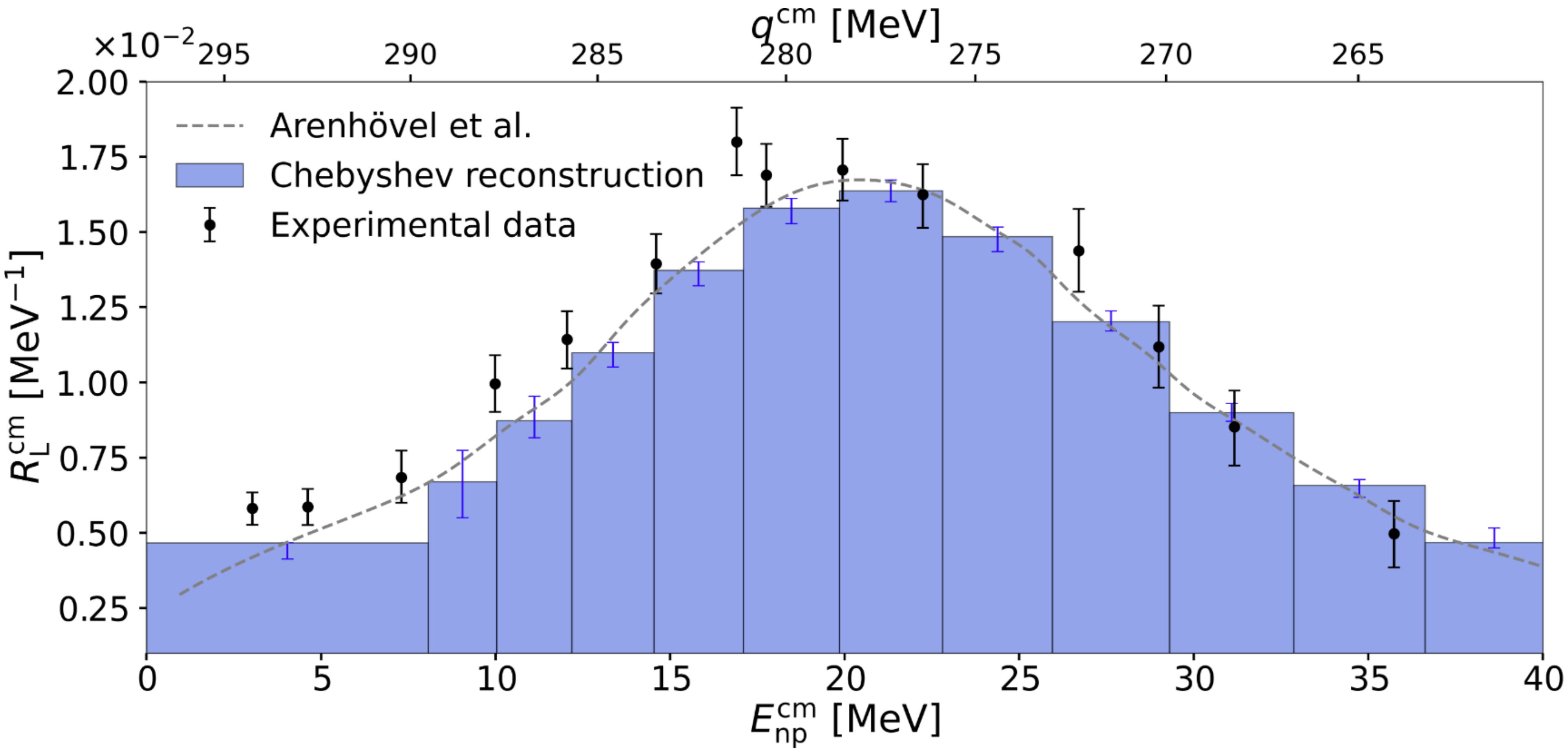
Emmons et al., J. Phys. G 48, 035101 (2021)

Longitudinal response



Efros et al., Few Body Syst. 14, 151 (1993)

Longitudinal response



B. P. Quinn et al., Phys. Rev. C 37, 1609 (1988)

Summary

- Chebyshev expansion produces responses with errors
- Regularized density of states can be obtained in the same framework
- Works well in the deuteron (test case) and should be extended to relevant nuclei

Questions for discussion

- How does rebinning work?
- Which BSM scenarios would be interesting to investigate? Which nuclei?
- What is most useful to calculate? Responses, flux-folded cross sections, ...