DIS and SIS in neutrino scattering

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MITP, Neutrino-Nucleus Interactions in the SM and Beyond

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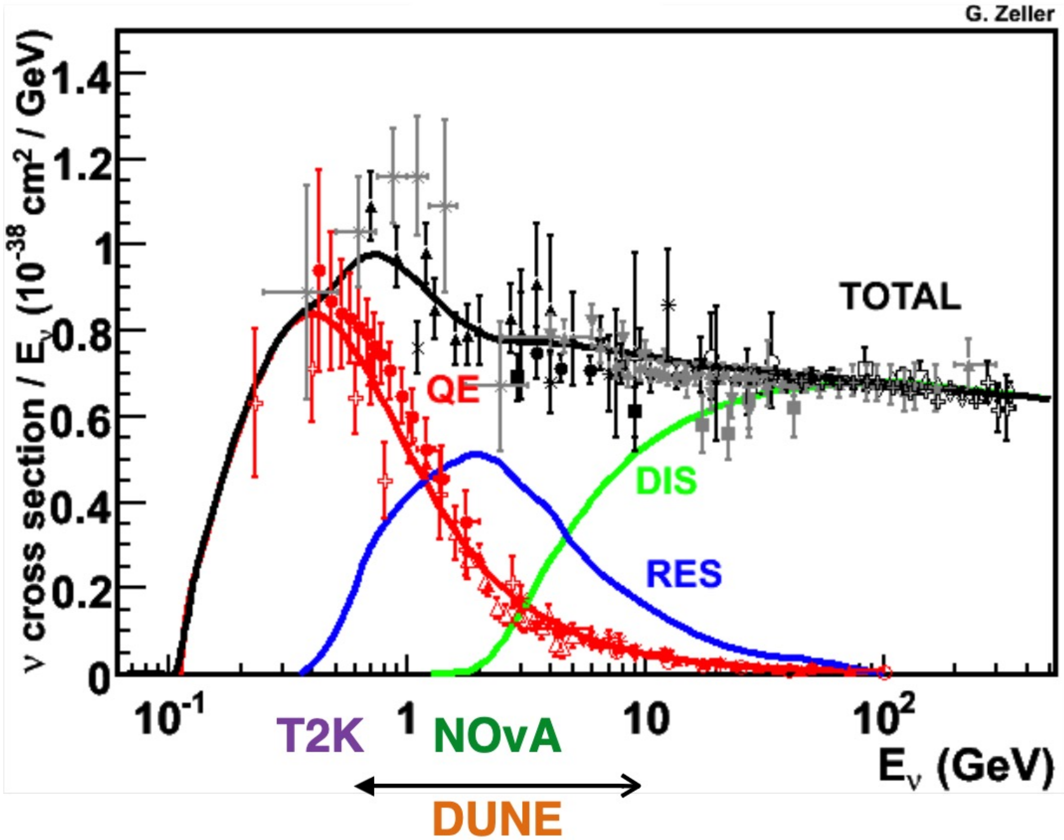
Jeong & Reno, Phys Rev D 108 (2023) 113010

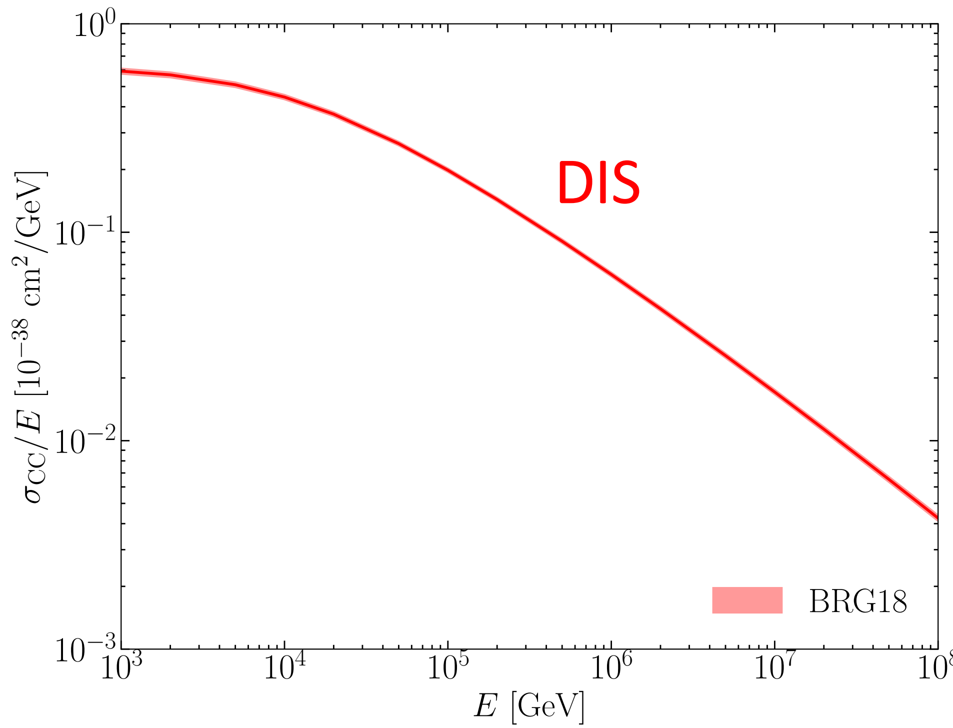
See Alfonso's talk yesterday.

Neutrino cross sections

QE=quasi-elastic, RES=resonant, DIS=deep inelastic scattering

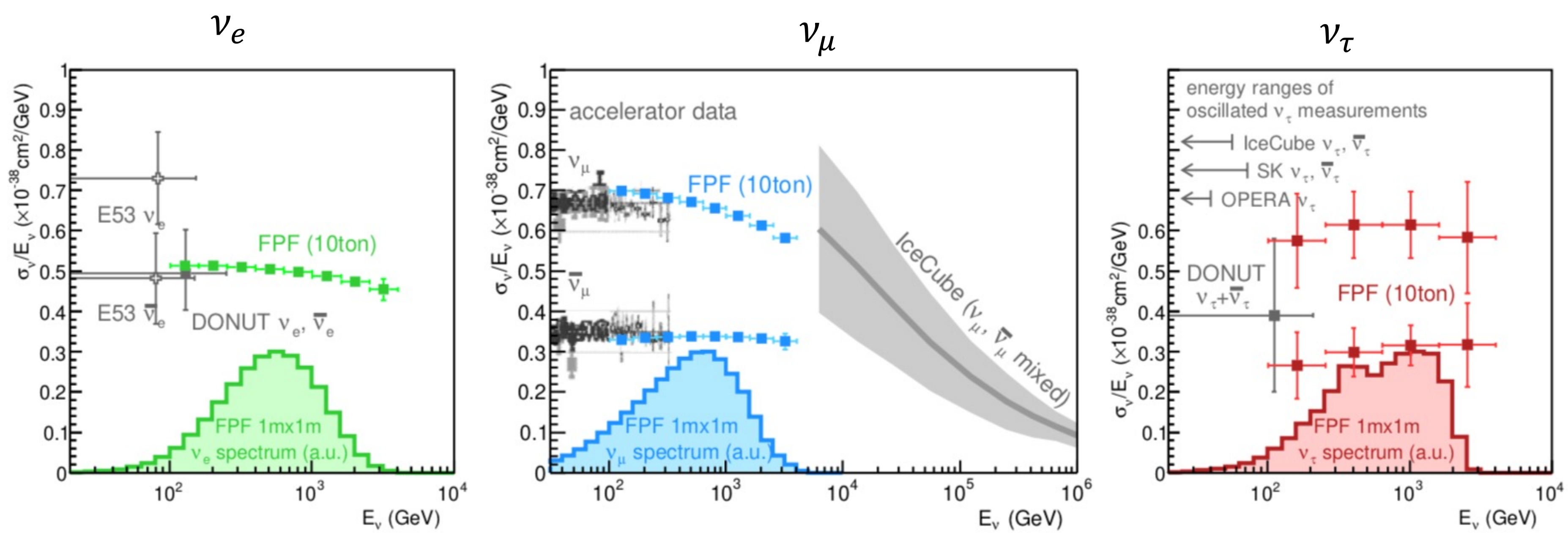


J.A. Formaggio, G. Zeller, Reviews of Modern Physics, 84 (2012)



BGR18 = Bertone, Gauld & Rojo, JHEP 01 (2019) 217
see also MHR, Ann Rev Nucl Part Sci 2023

Neutrino cross sections with hypothetical FPF



FPF Interactions:

$\sim 10^5\ \nu_e + \bar{\nu}_e$

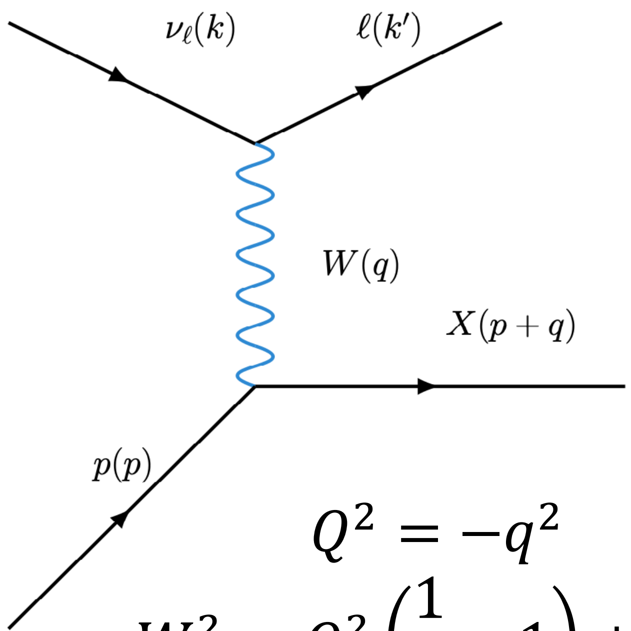
$\sim 10^6\ \nu_\mu + \bar{\nu}_\mu$

$\sim 10^3\ \nu_\tau + \bar{\nu}_\tau$

Feng et al., J Phys G 3 (2023) 030501

Shown here: current measurements, plus potential for future Forward Physics Facility (FPF) measurements (statistical uncertainty only).
FPF neutrinos mostly E>100 GeV, but still many events below 100 GeV.
What is the role of low Q?

Inelastic cross sections

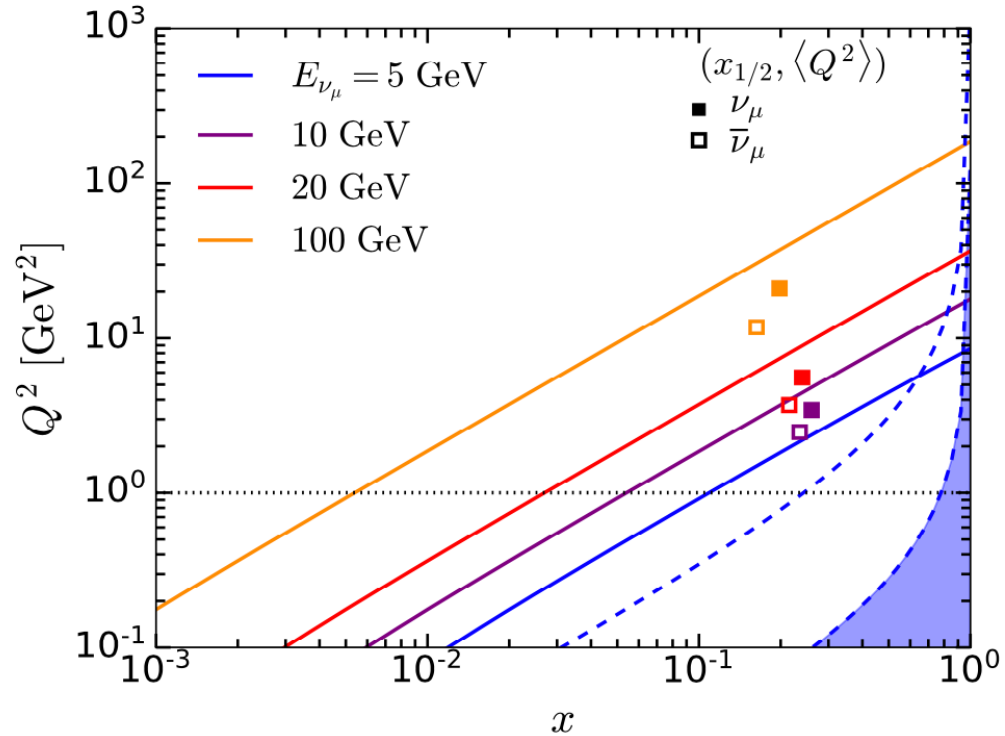


$$Q^2 = -q^2$$

$$W^2 = Q^2\left(\frac{1}{x} - 1\right) + m_N^2$$

$$x = \frac{Q^2}{2p \cdot q}$$

MHR, *Eur.Phys.J.ST* 230 (2021) 24, 4419-4431



shaded: not inelastic
dashed: *W*= 2 GeV
solid: limit for (x, Q^2)

squares: avg values
@ fixed energies

SIS, DIS, soft DIS

Interest here: low Q^2 structure functions for inclusive cross sections

E = 7 GeV

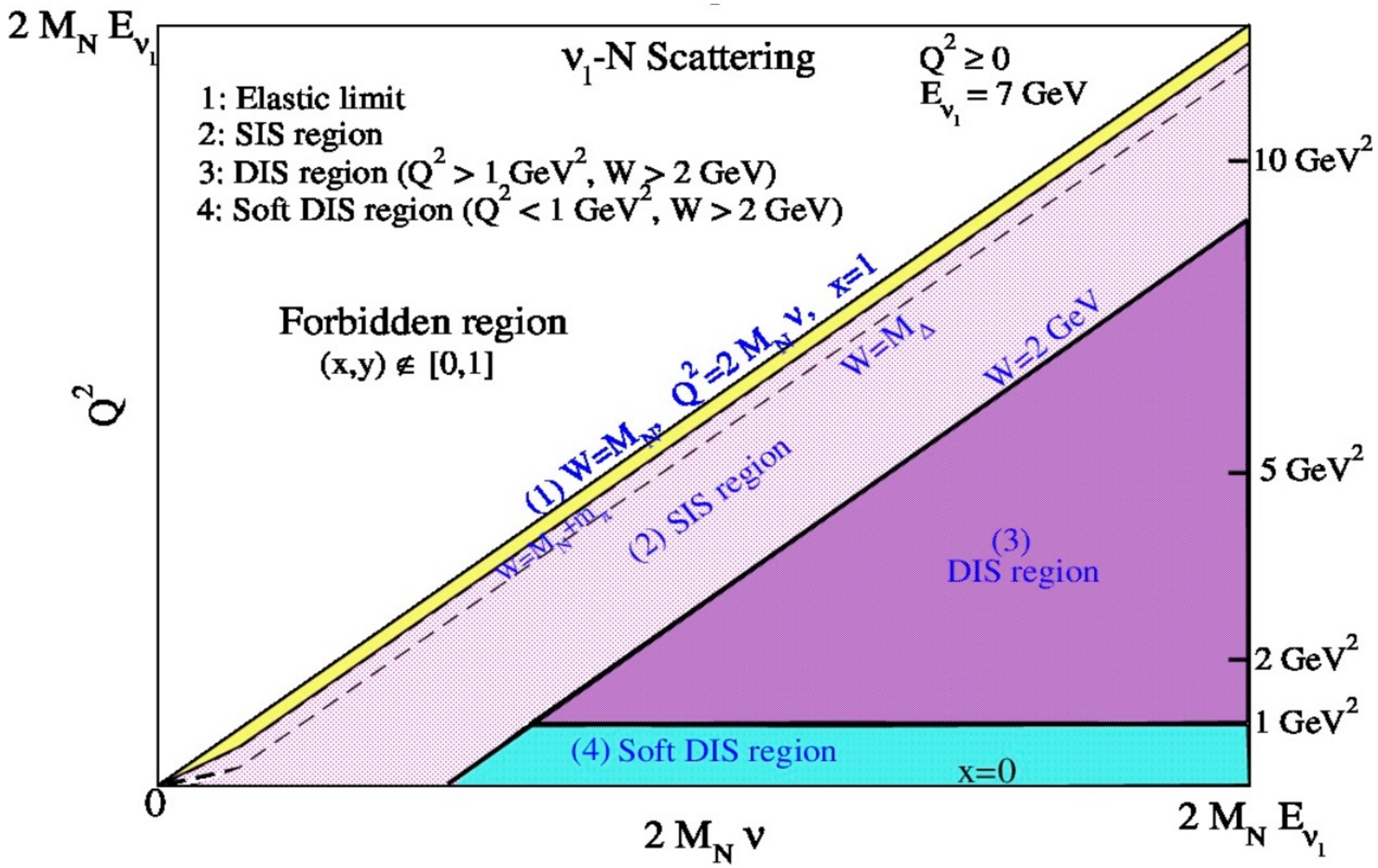


Fig: Morfin & Athar, 2006.08603

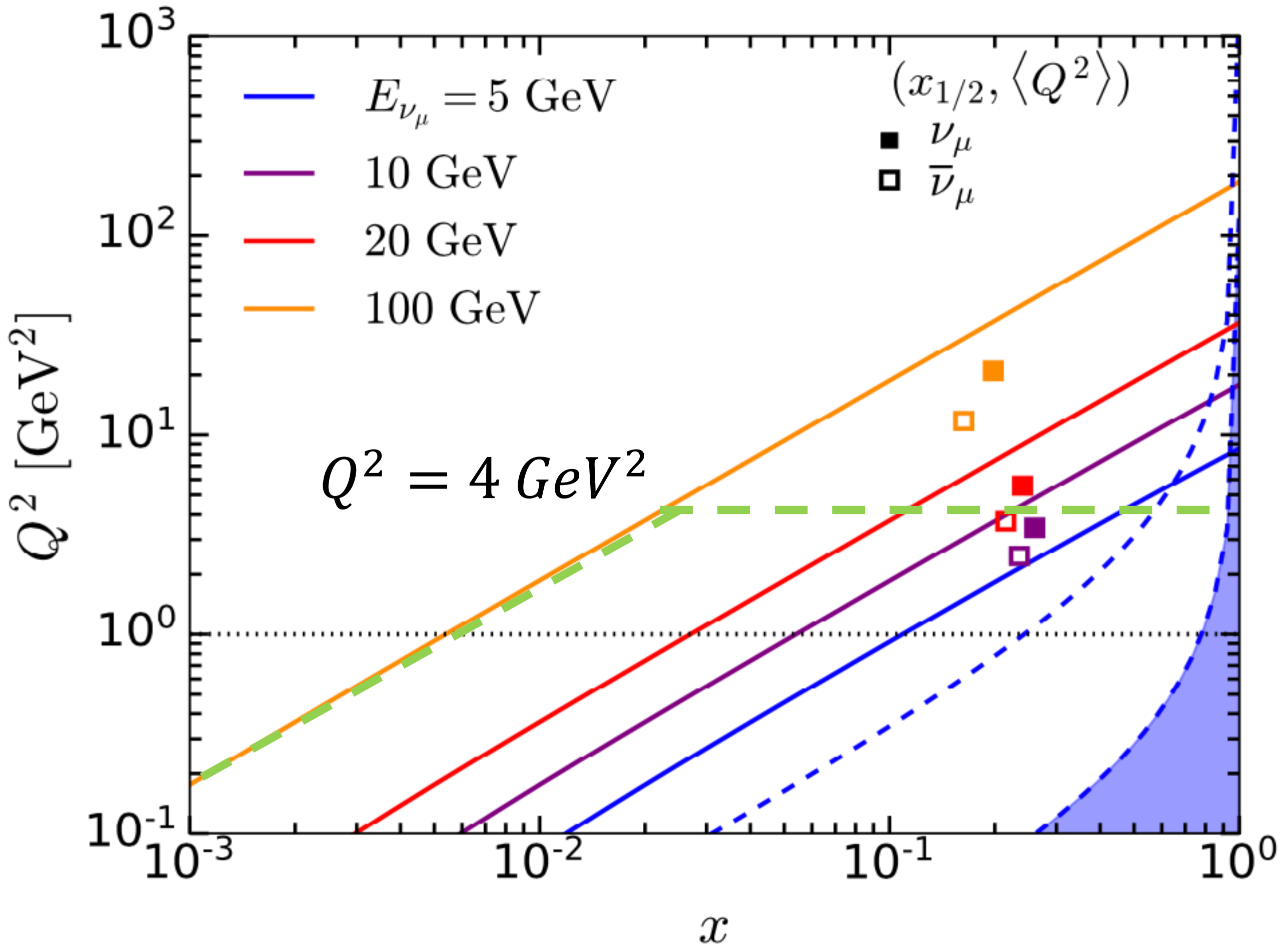
Look at inclusive meson production, Morfin and Athar have:

- All $Q^2, W < 2$ GeV – SIS
- $Q^2 > 1$ GeV^2, $W > 2$ GeV – DIS
- $Q^2 < 1$ GeV^2, $W > 2$ GeV – "soft" DIS

Where:

- SIS – resonant and non-resonant pion production
- DIS – (more or less) pert QCD with partons
- soft DIS – not scattering with quarks

Low Q^2 structure functions



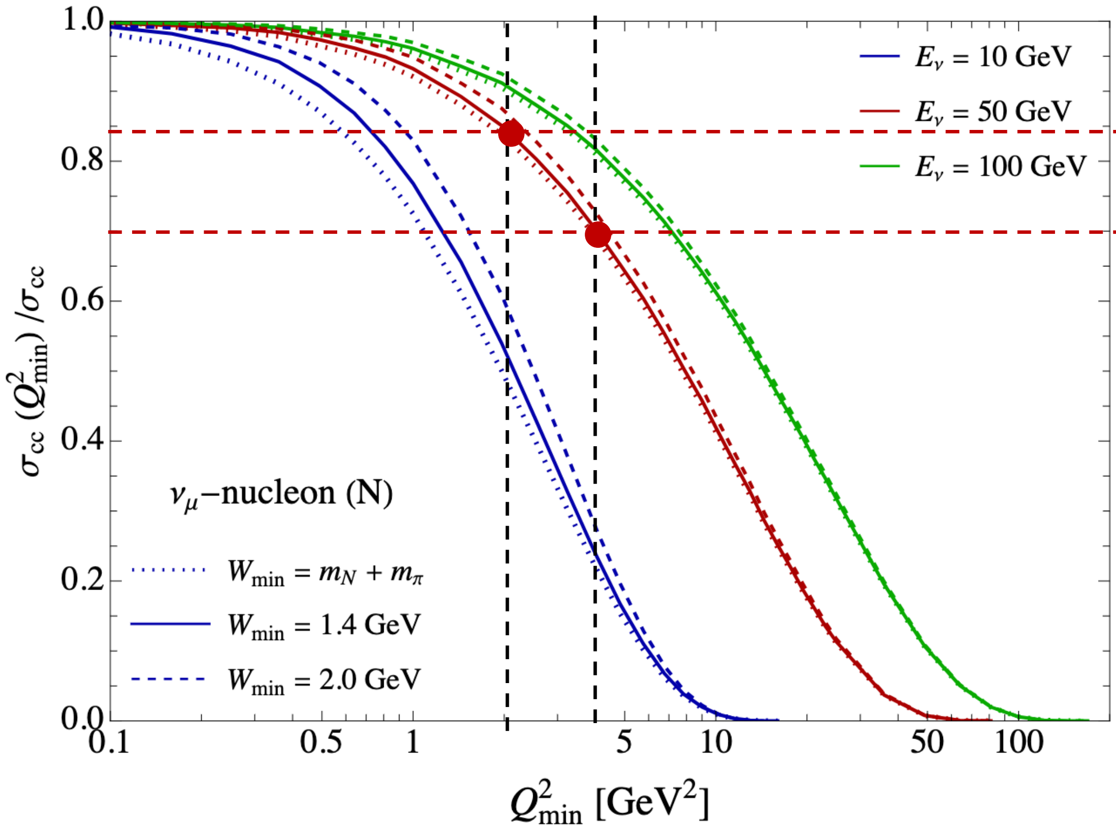
shaded: not inelastic
dashed: W= 2 GeV
solid: limit for (x, Q^2)

interested in Q^2< 4 GeV^2, away from the resonance region

$$W^2 = Q^2\left(\frac{1}{x} - 1\right) + m_N^2$$

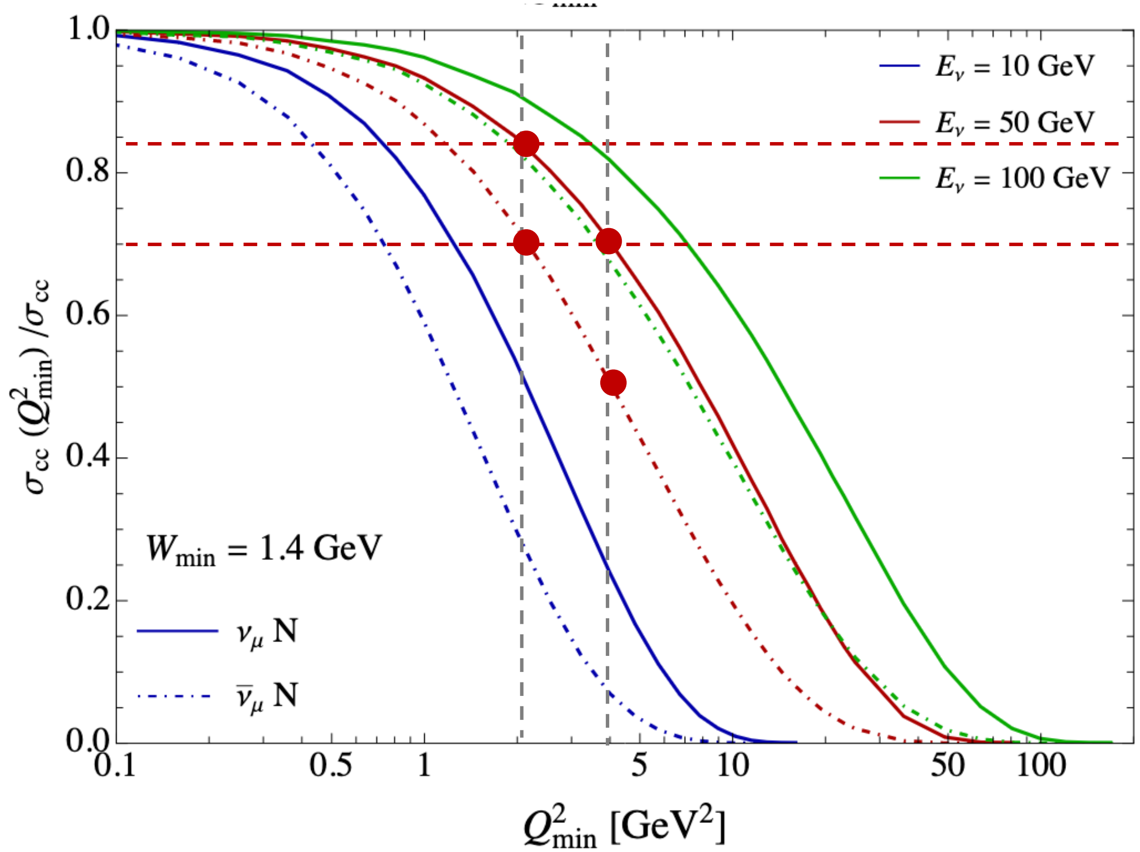
Q^2 dependence of CC cross section

different W_{min}



Low Q^2 important even when $W > W_{min}$.

ν_μ & $\bar{\nu}_\mu$



$\bar{\nu}_\mu$ more sensitive to low Q^2 than ν_μ.

Jeong & Reno, Phys Rev D 108 (2023) 113010

Differential cross section

$$\frac{d^2\sigma^{\nu(\bar\nu)}}{dx\ dy} = \frac{G_F^2 M E_\nu}{\pi(1+Q^2/M_W^2)^2}\Big(y^2 x F_1(x,Q^2) + (1-y)F_2(x,Q^2) \pm\ xy(1-\frac{y}{2})F_3(x,Q^2)\Big)\,,$$

Structure function strategies:

- Must match PDF approach at high Q^2.
- Go directly to neutrino data and make fits (Candido… Garcia Soto et al. NNSFν)
- Use EM scattering as a guide, correct EM to CC, and for PCAC.
    - Bodek-Yang (BY) more directly tied to PDFs, standard implementation without PCAC.
    - We “patch and match” with parameterized (but theory-guided) structure functions from Capella et al. (CKMT).

Plan

- Brief review of Bodek-Yang prescription.
- Walk through what we have done with CKMT approach to get neutrino structure functions.
- Discussion points.

Bodek-Yang prescription

- Keep parton picture.
  - Advantage: use parton composition to go from EM to weak structure functions
  - Disadvantage: parton picture not relevant at low Q^2
- GRV98 go to low Q^2 – modify with new x and Q^2 variables:

$$F_2^{\mathrm{EM}} = \sum e_q^2 \xi_w (\tilde{q}(\xi_w, Q^2) + \tilde{\bar{q}}(\xi_w, Q^2))$$

$$\begin{aligned} \xi_w &= \frac{2x(Q^2+B)}{Q^2(1+\rho)+2Ax} \\ \xi_{wc} &= \frac{2x(Q^2+B+m_c^2)}{Q^2(1+\rho)+2Ax} \\ A &= 0.538 \text{ GeV}^2 \\ B &= 0.305 \text{ GeV}^2 \\ m_c &= 1.5 \text{ GeV} \\ \rho &= (1+4M_N^2x^2/Q^2)^{1/2} \end{aligned}$$

$$\begin{aligned} \tilde{u}_v &= \frac{(1-G_D^2)\cdot(Q^2+C_{2vu})}{Q^2+C_{1vu}} u_v \\ \tilde{d}_v &= \frac{(1-G_D^2)\cdot(Q^2+C_{2vd})}{Q^2+C_{1vd}} d_v \\ \tilde{\bar{u}} &= \frac{Q^2}{Q^2+C_{su}} \bar{u} \text{ (and sim. for } \tilde{\bar{d}}, \tilde{\bar{s}}) \end{aligned}$$

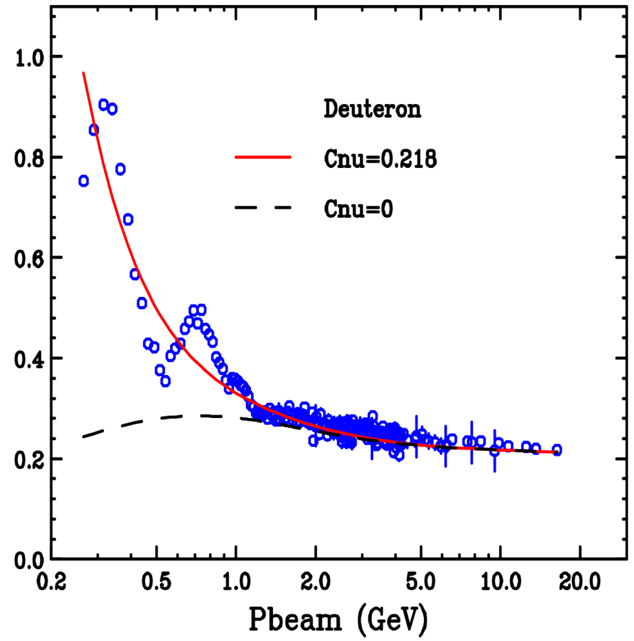
(also extra valence factors)

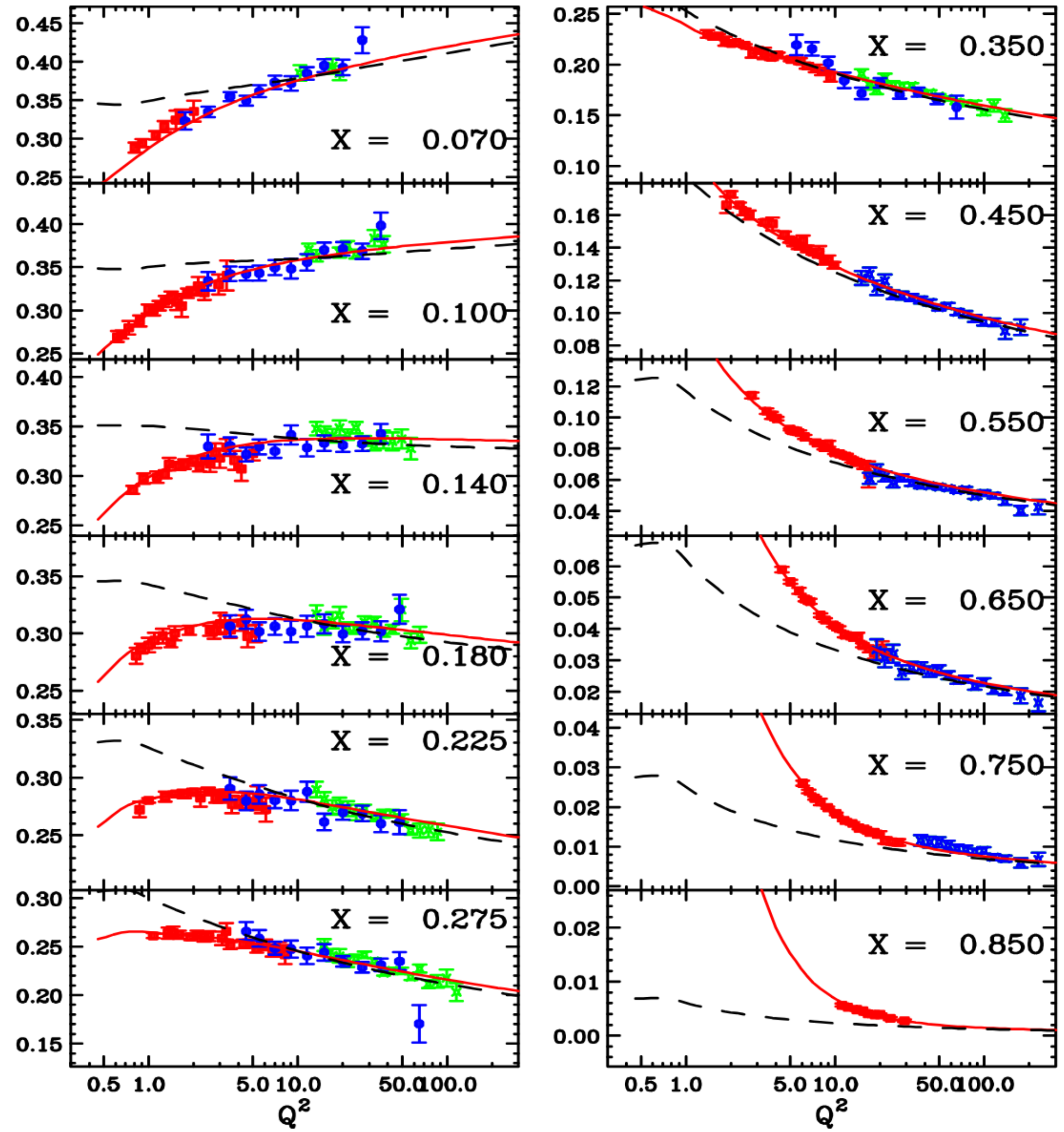
GRV98 LO PDFs, modified Nachtmann variable (more below), freeze below Q^2_0 = 0.8 GeV^2, then multiplied by Q^2 dependent K-factors). See Bodek, Park, Yang, Nucl. Phys. B Proc. Suppl. 139, 113 (2005)

Bodek-Yang prescription

$F_2(x,Q2)$ for deuteron, fit with solid red line.
Dashed – GRV98LO

Q^2=0 limit,
extra valence factor

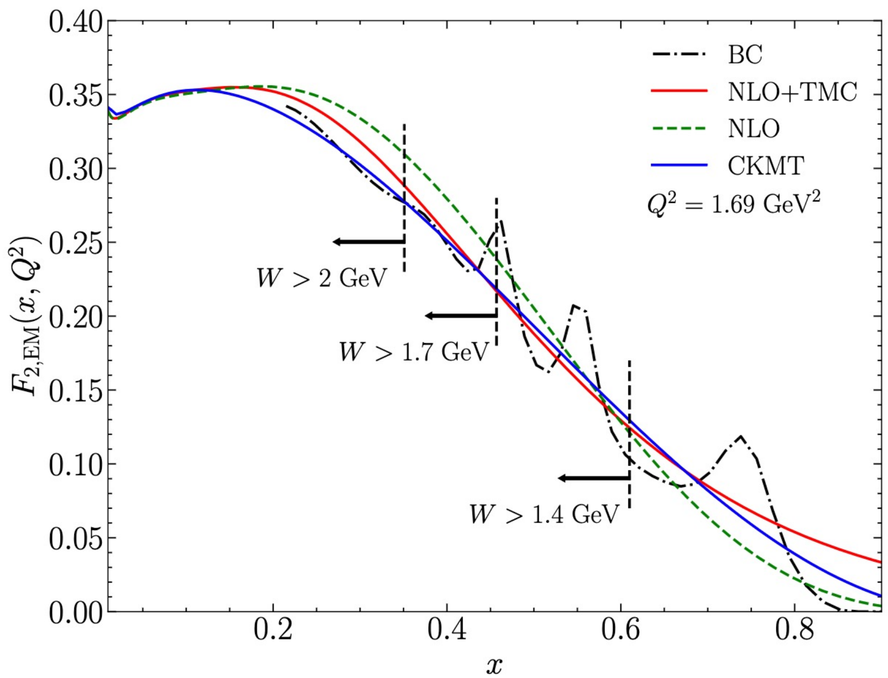




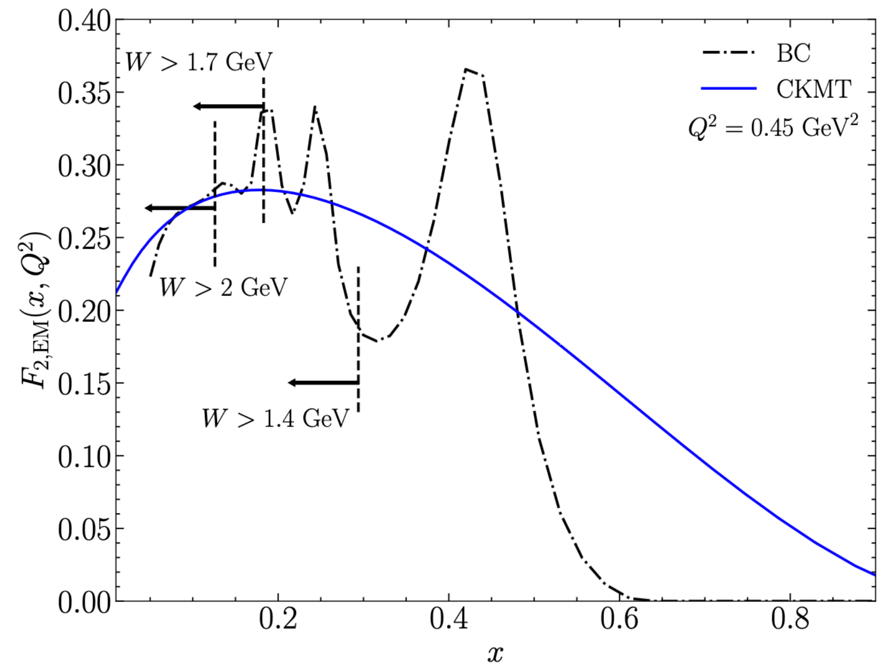
https://arxiv.org/pdf/1011.6592

CKMT to ν: start with EM structure functions

$Q^2 = 1.69\ GeV^2$



$Q^2 = 0.45\ GeV^2$



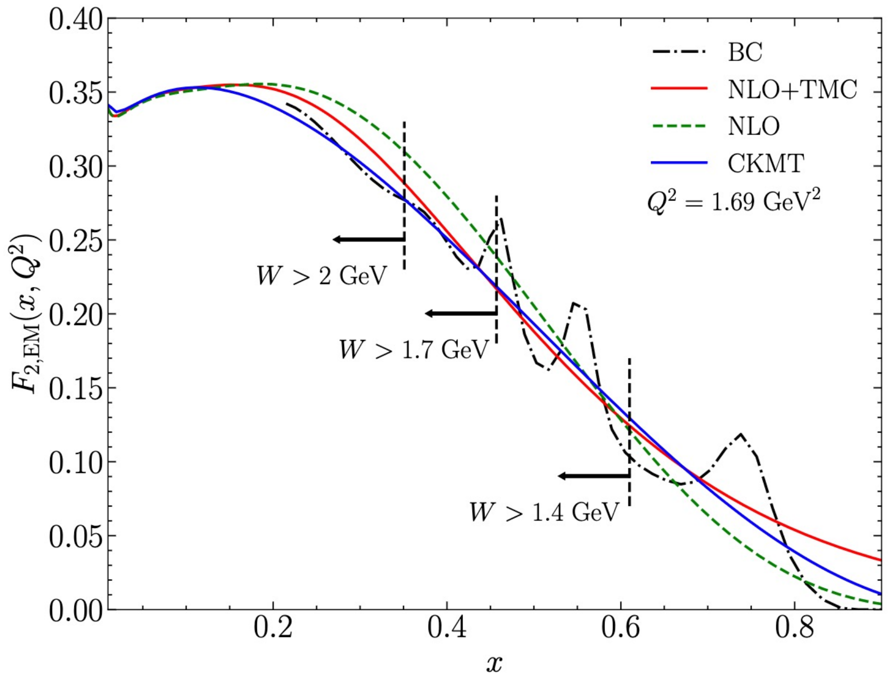
$$W^2 = Q^2\left(\frac{1}{x} - 1\right) + m_N^2$$

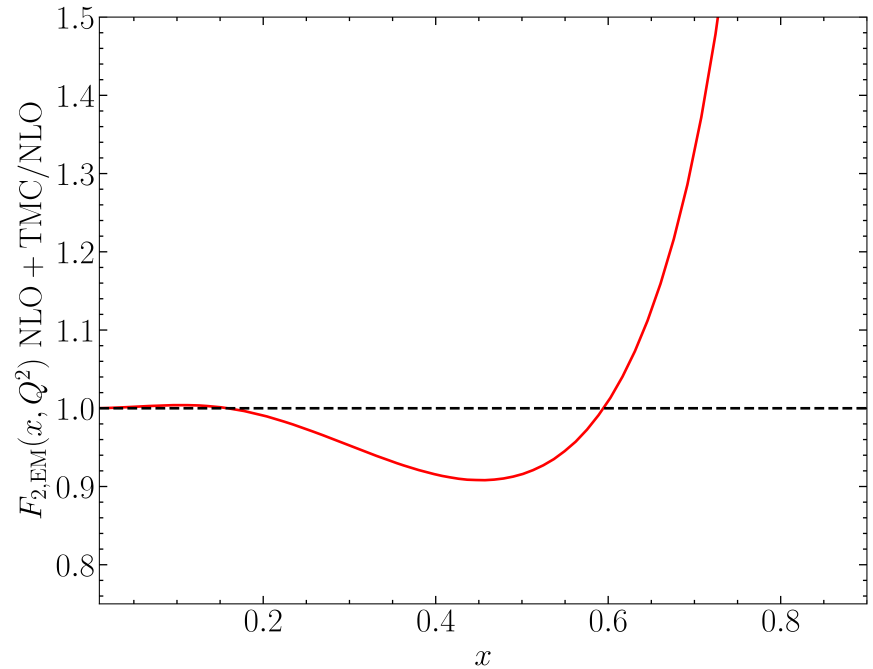
- BC=Bosted, Christy, Phys. Rev. C 81, 055213 (2010) parametrization of EM data.
- TMC is parton distribution function (PDF) approach that includes target mass (nucleon target) corrections, see e.g., Schienbein et al. J Phys G 35 (2008) 053101. See also Ruiz et al, Prog Part Nucl Phys 136 (2024) 104096
- CKMT= Capella et al. Phys. Lett. B 337, 358 (1994) phenomenological fit to averaged EM data.

NLO, NLO+TMC

- kinematic corrections
- kT<M corrections (collinear) Ellis, Furmanski, Petronzio, NPB212 (1983) 24

$$F_2^{\mathrm{TMC}}(x,Q^2)=\frac{x^2}{\xi^2 r^3}F_2^{(0)}(\xi,Q^2)+\frac{6M^2x^3}{Q^2r^4}h_2(\xi,Q^2)+\frac{12M^4x^4}{Q^4r^5}g_2(\xi,Q^2)$$





Nachtmann variable: $\xi=\frac{2x}{1+\sqrt{1+4x^2M^2/Q^2}}$ $\quad r=\sqrt{1+\frac{4x^2M^2}{Q^2}}$

Bodek-Yang:

$$\xi_w=\frac{2x(Q^2+B)}{Q^2(1+r)+2Ax}$$

CKMT EM structure functions

Currently looking at improvements.

$$
\begin{aligned}
F_{2,\mathrm{EM}}^{\mathrm{CKMT}}(x,Q^2) &= F_2^{sea}(x,Q^2) + F_2^{val}(x,Q^2) \\
&= Ax^{-\Delta(Q^2)}(1-x)^{n(Q^2)+4} \\
&\times \left(\frac{Q^2}{Q^2+a}\right)^{1+\Delta(Q^2)} \\
&+ Bx^{1-\alpha_R}(1-x)^{n(Q^2)}\left(\frac{Q^2}{Q^2+b}\right)^{\alpha_R} \\
&\times \Big(1+f(1-x)\Big)\ .
\end{aligned}
$$

$$
\begin{aligned}
n(Q^2) &= \frac{3}{2}\left(1+\frac{Q^2}{Q^2+c}\right) ; \\
\Delta(Q^2) &= \Delta_0\left(1+\frac{2Q^2}{Q^2+d}\right)
\end{aligned}
$$

Δ_0	α_R	a [GeV^2]	b [GeV^2]	c [GeV^2]	d [GeV^2]
0.07684	0.4150	0.2631	0.6452	3.5489	1.1170

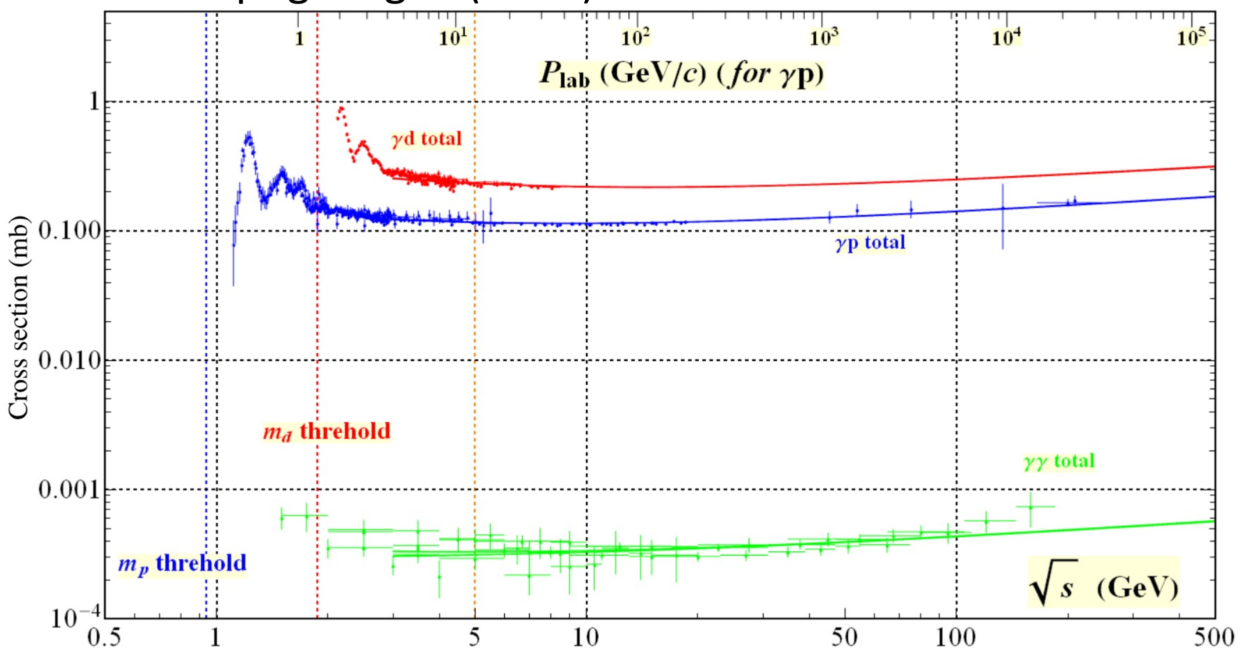
Process	A	B	f
EM F_2	0.1502	1.2064	0.15

B and f from valence counting rules.

Capella et al. Phys. Lett. B 337, 358 (1994)
Also Kaidalov & Merino, Eur.Phys. J. C10, 153 (1999)

Photon-proton scattering – low Q^2

pdg.lbl.gov (2018)



$$W^2 = Q^2\left(\frac{1}{x} - 1\right) + m_N^2$$

$$\frac{Q^2}{x} \to W^2$$

$$\sigma_{\gamma * p}(W^2, Q^2) = \sigma_T + \sigma_L \simeq \frac{4\pi^2\alpha}{Q^2} F_2(x, Q^2)$$

F_2 scales as Q^2 as Q^2 to 0.

$$\sigma_{\gamma p} = A_{\Delta_0}(W^2)^{\Delta_0} + B_{\alpha_R}(W^2)^{\alpha_R - 1}$$

CKMT:

$$\Delta_0 \simeq 0.077$$

$$\alpha_R = 0.415$$

Large *x*

$$
\begin{aligned}
F_{2,\mathrm{EM}}^{\mathrm{CKMT}}(x,Q^2) &= F_2^{sea}(x,Q^2) + F_2^{val}(x,Q^2) \\
&= Ax^{-\Delta(Q^2)}(1-x)^{n(Q^2)+4} \\
&\times \left(\frac{Q^2}{Q^2+a}\right)^{1+\Delta(Q^2)} \\
&+ Bx^{1-\alpha_R}(1-x)^{n(Q^2)}\left(\frac{Q^2}{Q^2+b}\right)^{\alpha_R} \\
&\times \Big(1+f(1-x)\Big) \, . \qquad (6)
\end{aligned}
$$

$n(0) = \frac{3}{2}$ (dual parton model power)

$n(4\ GeV^2) = 2.3$

$n(10\ GeV^2) = 2.6$

one extra power of *(1-x)* for down quark

naïve counting rules for partons:

n= 3 for valence

n = 7 for sea

Conversion to neutrino CC scattering

- Valence and sea normalization factors:
  - Conserved vector current dictates the low Q behavior of structure function for EM. Adapt *A, B* and *f* for neutrino scattering. *B* & *f* from valence counting at 2 GeV^2, *A* from sea relations.
- Ad hoc implementation of PCAC correction, so far.

$$\sigma_{\gamma p}^{\rm tot}(W^2) = \frac{4\pi^2\alpha_{\rm EM}}{Q^2} F_{2,\rm EM}^{\rm CKMT}(x,Q^2)|_{Q^2\to 0}$$
$$\simeq 4\pi^2\alpha_{\rm EM}\left[\frac{A}{a}\left(\frac{W^2}{a}\right)^{\Delta_0} + \frac{B(1+f)}{b}\left(\frac{W^2}{b}\right)^{\alpha_R-1}\right],$$

Capella et al. Phys. Lett. B 337, 358 (1994)

$$F_L^{\rm PCAC} = \frac{f_\pi^2\sigma_\pi(W^2)}{\pi} f_{\rm PCAC}(Q^2)$$
$$f_{\rm PCAC}(Q^2) = \left(1+\frac{Q^2}{M_{\rm PCAC}^2}\right)^{-2}$$
$$\sigma_\pi \simeq X(W^2)^\epsilon + Y(W^2)^{-\eta_1}$$

Kulagin & Petti, Phys. Rev. D 76, 094023 (2007)

Similar behavior to CKMT $p\gamma$

$$\epsilon \simeq \Delta_0, \qquad \eta_1 \simeq \alpha_R - 1$$

Conversion to neutrino CC scattering

Δ_0	α_R	a [GeV^2]	b [GeV^2]	c [GeV^2]	d [GeV^2]
0.07684	0.4150	0.2631	0.6452	3.5489	1.1170

Process	A	B	f
EM F_2	0.1502	1.2064	0.15
νN F_2	0.5967	2.7145	0.5962
νN xF_3	9.3955×10^{-3}	2.4677	0.5962
$\bar{\nu} N$ xF_3	9.3955×10^{-3}	-2.4677	0.5962

B_ν and f_ν, from valence counting rules.
For A_ν, pick to match NLO+TMC at Q^2=10 GeV^2.
F_3: valence plus small (strange) sea component, normalized to match NLO+TMC, GLS sum rule.
Crude longitudinal structure function.

From pion scattering, functional form, match at Q^2 = 0:

$$A^{\rm PCAC} = 0.147$$

$$B^{\rm PCAC} = 0.256$$

$$\begin{aligned} F_{2,\rm CC}{}^{\rm PCAC} &= \Bigg[A^{\rm PCAC} x^{-\Delta(Q^2)}(1-x)^{n(Q^2)+4} \\ &\times \left(\frac{Q^2}{Q^2+a}\right)^{\Delta(Q^2)} \\ &+ B^{\rm PCAC} x^{1-\alpha_R}(1-x)^{n(Q^2)}\left(\frac{Q^2}{Q^2+b}\right)^{\alpha_R-1} \\ &\times \Big(1+f(1-x)\Big)\Bigg] f_{\rm PCAC}(Q^2) \end{aligned} \qquad (16)$$

Weak structure functions

For $Q^2 > Q_0^2$ use PDFs, NLO QCD and target mass corrections (TMC):

$$F_{2,\mathrm{CC}}(x,Q^2) \;=\; \sum_{q,q'} 2x\big(q(x,Q^2) + \bar{q}'(x,Q^2)\big) \qquad \to F_{2,\mathrm{CC}}^{\mathrm{NLO+TMC}}(x,Q^2)$$

For $Q^2 < Q_0^2$ use phenomenological parameterization:

$$F_{2,\mathrm{CC}}(x,Q^2) \;=\; \left[F_{2,\mathrm{CC}}^{\mathrm{CKMT}}(x,Q^2) + F_{2,\mathrm{CC}}^{\mathrm{PCAC}}(x,Q^2)\right]$$ times normalization at Q_0^2

Weak structure functions

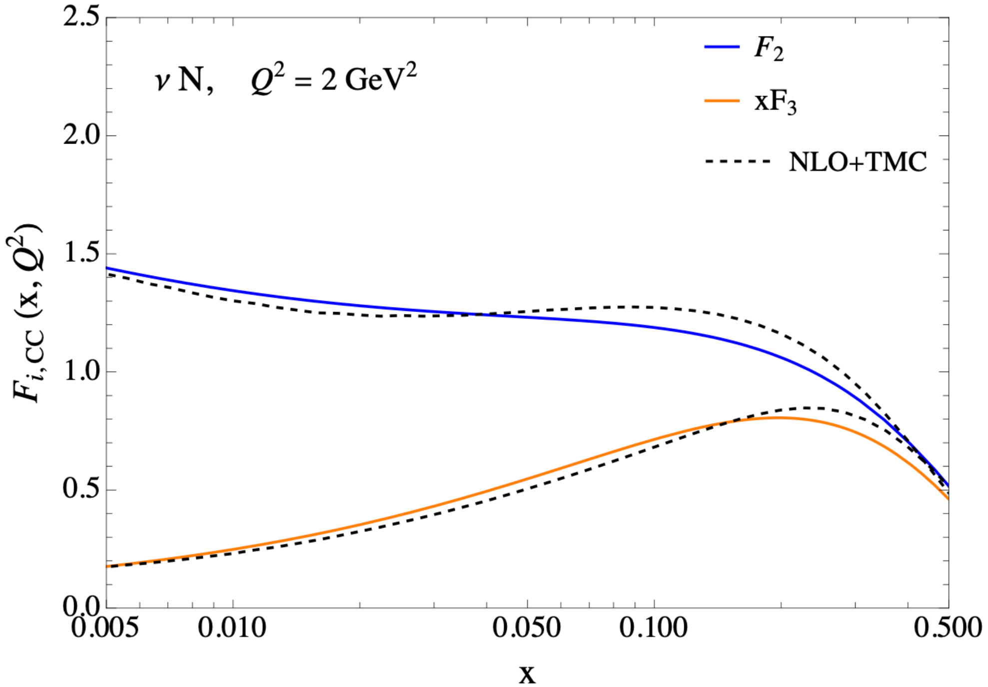
Patch and match at $Q_0^2 = 4\ GeV^2$, for $Q^2 < Q_0^2$:

$$F_{2,\mathrm{CC}}(x,Q^2) = \left[F_{2,\mathrm{CC}}^{\mathrm{CKMT}}(x,Q^2) + F_{2,\mathrm{CC}}^{\mathrm{PCAC}}(x,Q^2) \right] \times \frac{F_{2,\mathrm{CC}}^{\mathrm{NLO+TMC}}(x,Q_0^2)}{F_{2,\mathrm{CC}}^{\mathrm{CKMT}}(x,Q_0^2)} \,,$$

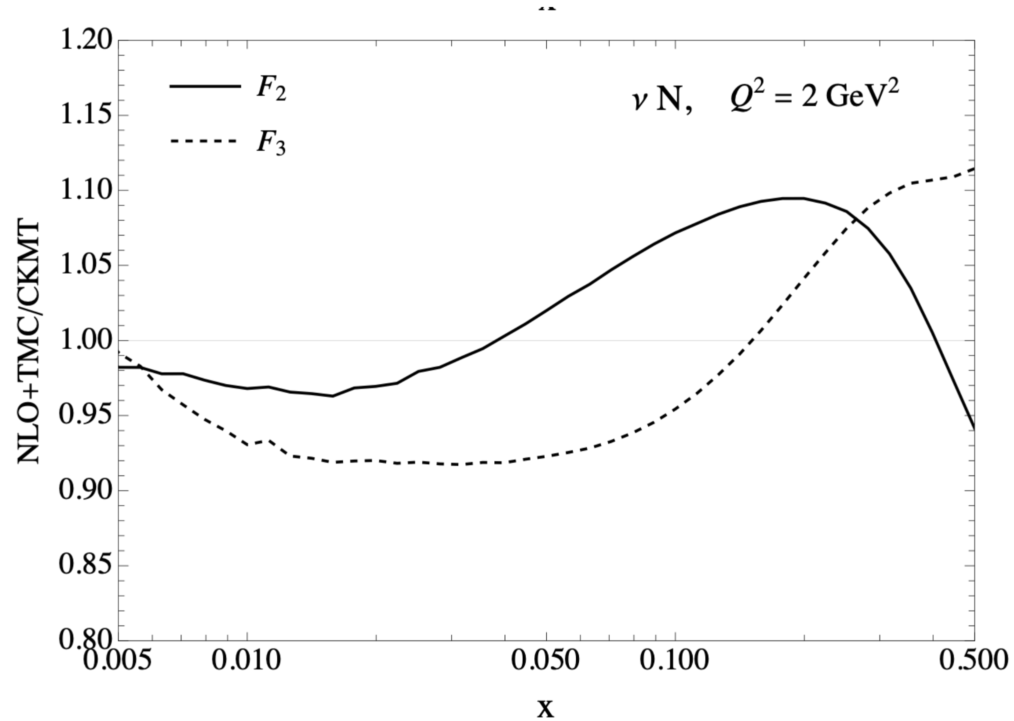
"CKMT+PCAC-NT"
no PCAC: "CKMT-NT"

- Compare with Bodek-Yang prescription
- Compare with NNSFν structure function inputs, where NNPDF fitting methodology for low Q structure functions is used. See Candido et al., JHEP 05 (2023) 149 and Alfonso Garcia Soto (here!).
- We have started with isoscalar nucleons.

Normalization factors



Before matching, at $Q^2 = 2$ GeV^2.



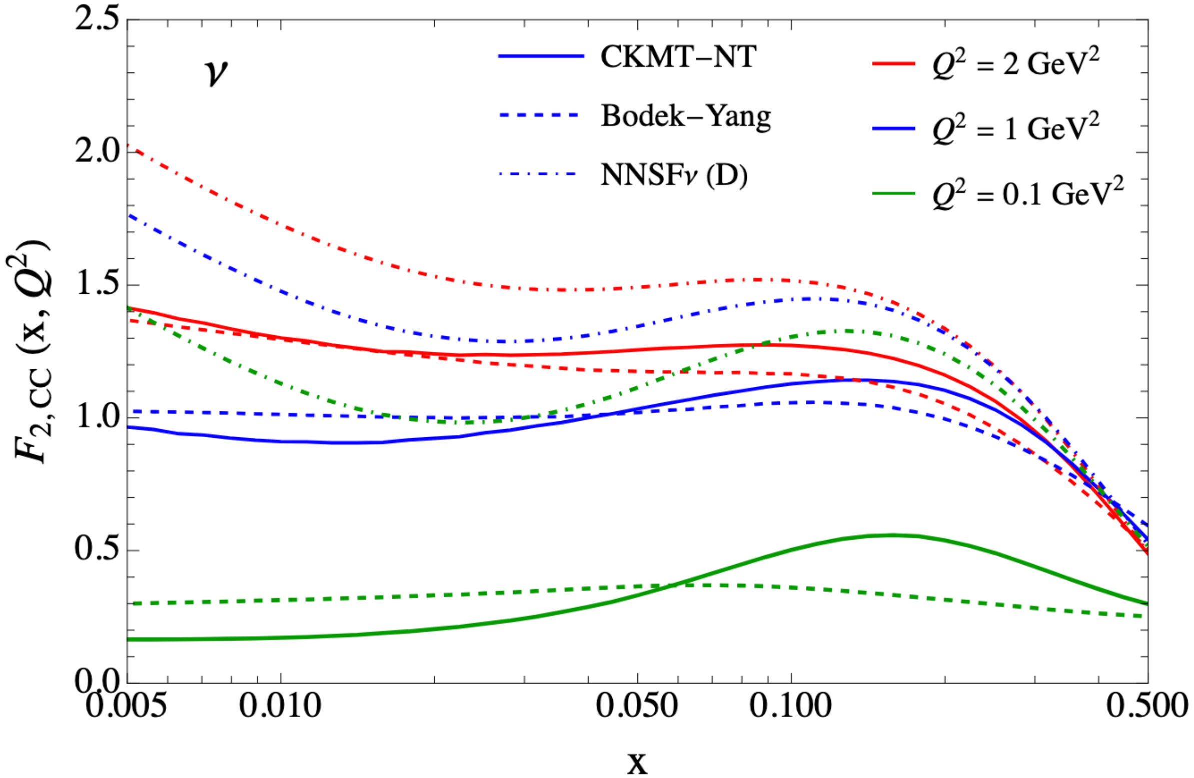
Relative normalization at $Q^2 = 2$ GeV^2.

Weak structure functions

isoscalar nucleon targets

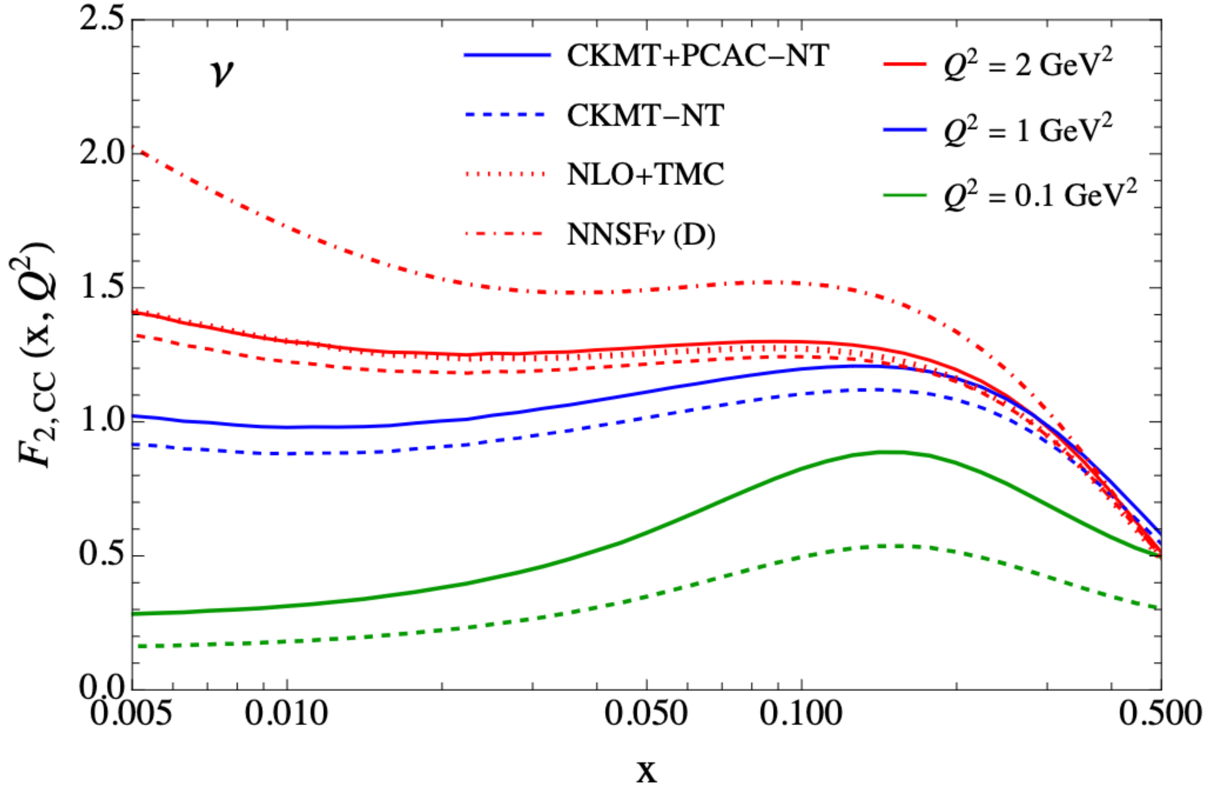
Q_0^2 = 2 GeV^2

comparisons of approaches



Q_0^2 = 4 GeV^2

with and without PCAC correction



NNSFnu so different? Evidence of PCAC in the data? No.

Jeong & Reno, Phys Rev D 108 (2023) 113010

Weak structure functions

CKMT+PCAC-NT

$$F_{2,\mathrm{CC}}(x,Q^2) = \left[F_{2,\mathrm{CC}}^{\mathrm{CKMT}}(x,Q^2) + F_{2,\mathrm{CC}}^{\mathrm{PCAC}}(x,Q^2)\right] \times \frac{F_{2,\mathrm{CC}}^{\mathrm{NLO+TMC}}(x,Q_0^2)}{F_{2,\mathrm{CC}}^{\mathrm{CKMT}}(x,Q_0^2)}, \quad (17)$$

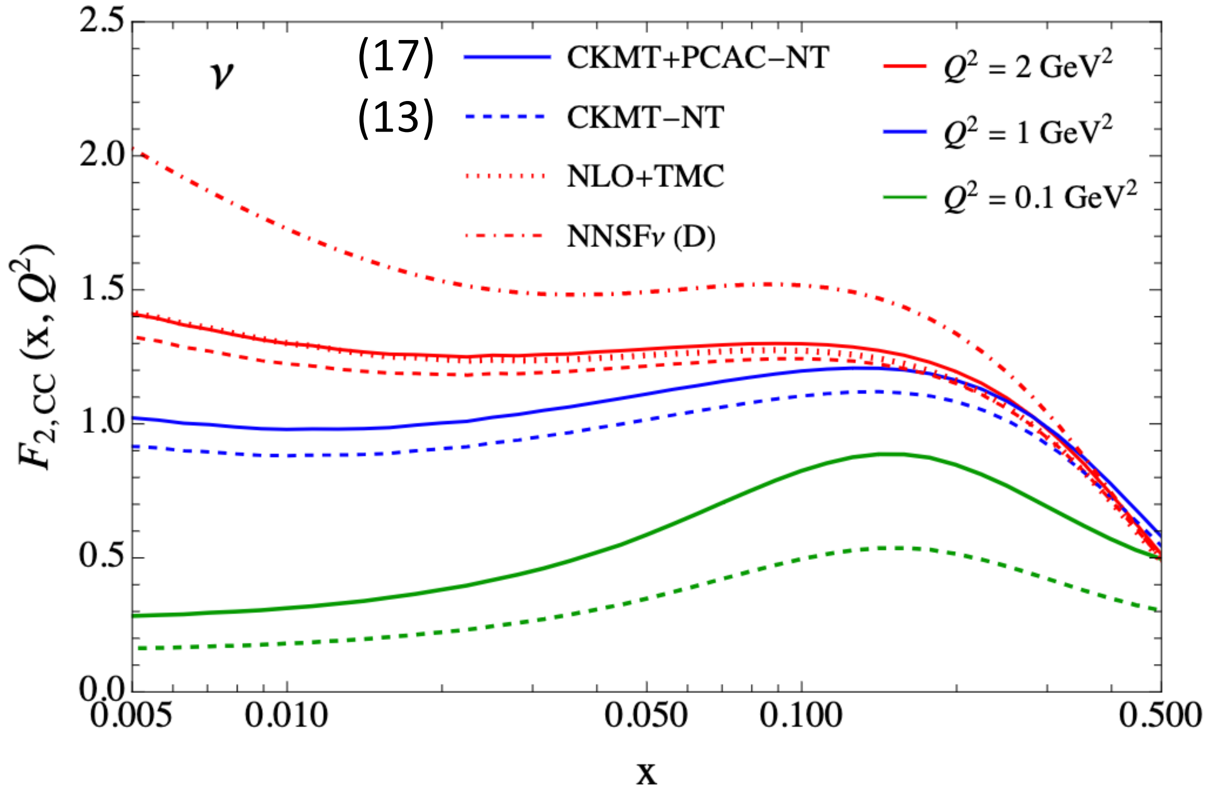
CKMT-NT

$$F_{i,\mathrm{CC}}(x,Q^2) = F_{i,\mathrm{CC}}^{\mathrm{CKMT}}(x,Q^2)\frac{F_{i,\mathrm{CC}}^{\mathrm{NLO+TMC}}(x,Q_0^2)}{F_{i,\mathrm{CC}}^{\mathrm{CKMT}}(x,Q_0^2)}, \quad (13)$$

isoscalar nucleon targets

$Q_0^2 = 4\ \mathrm{GeV}^2$

with and without PCAC correction

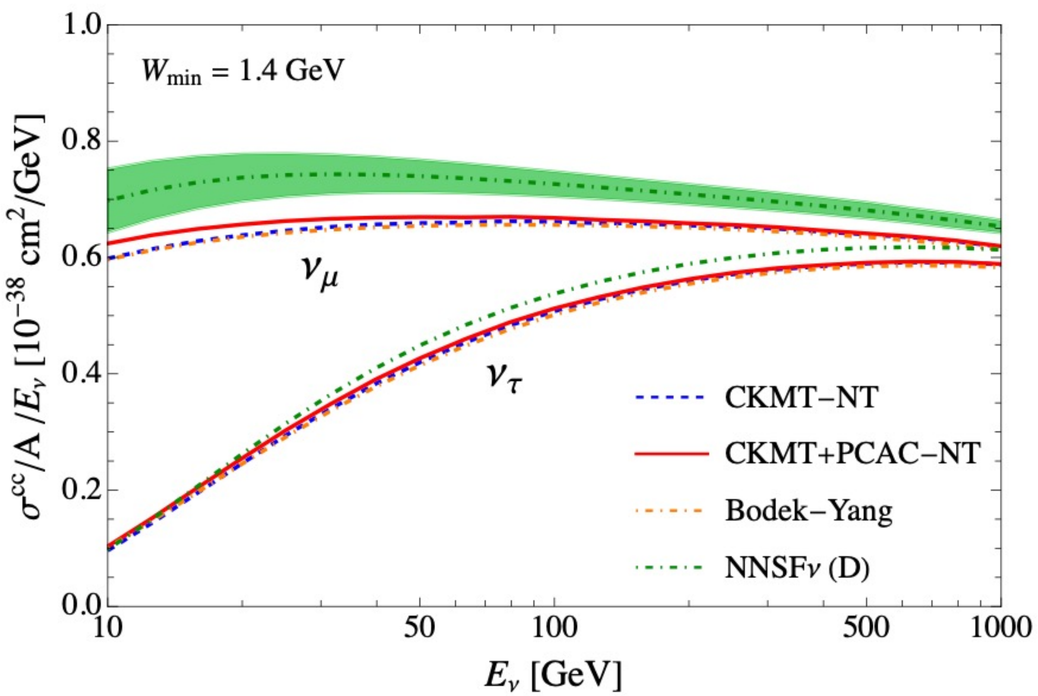


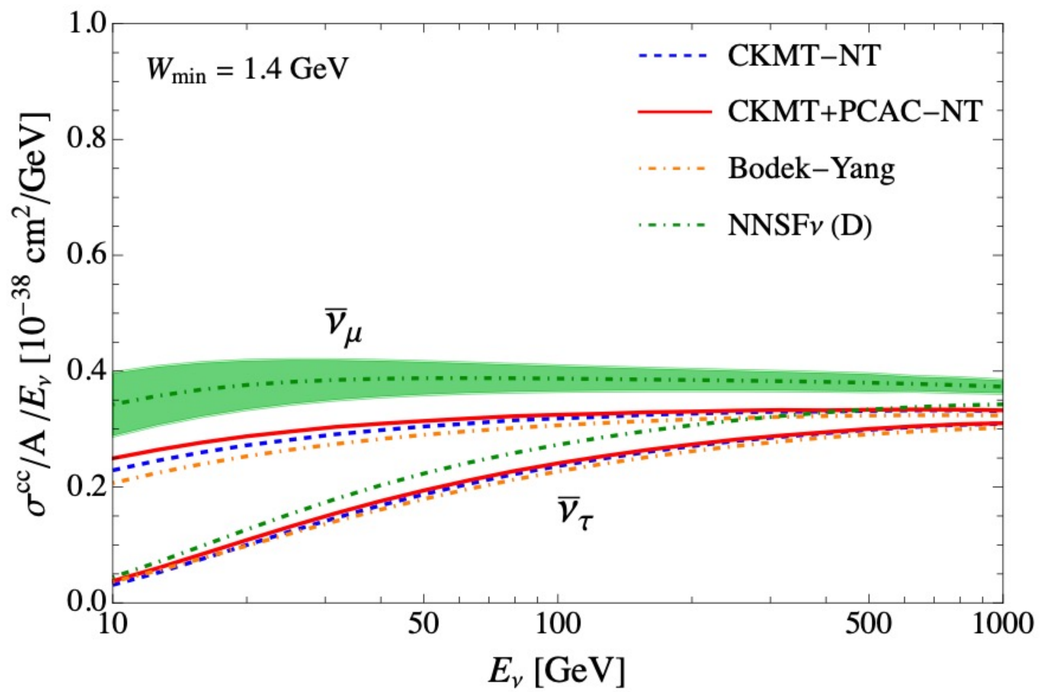
NNSFnu so different? Evidence of PCAC in the data? No.

Jeong & Reno, Phys Rev D 108 (2023) 113010

DIS CC for $\nu_\mu, \nu_\tau, \bar{\nu}_\mu, \bar{\nu}_\tau$ for isoscalar targets

- CKMT+PCAC-NT extrapolation of EM fit plus PCAC corrections Jeong & Reno, Phys Rev D 108 (2023) 113010
- NNSFν using NNPDF fitting methodology for low *Q* structure functions with green error band Candido … Garcia … et al *JHEP* 05 (2023) 149
- Bodek-Yang prescription Bodek, Park, Yang, Nucl. Phys. B Proc. Suppl. 139, 113 (2005)

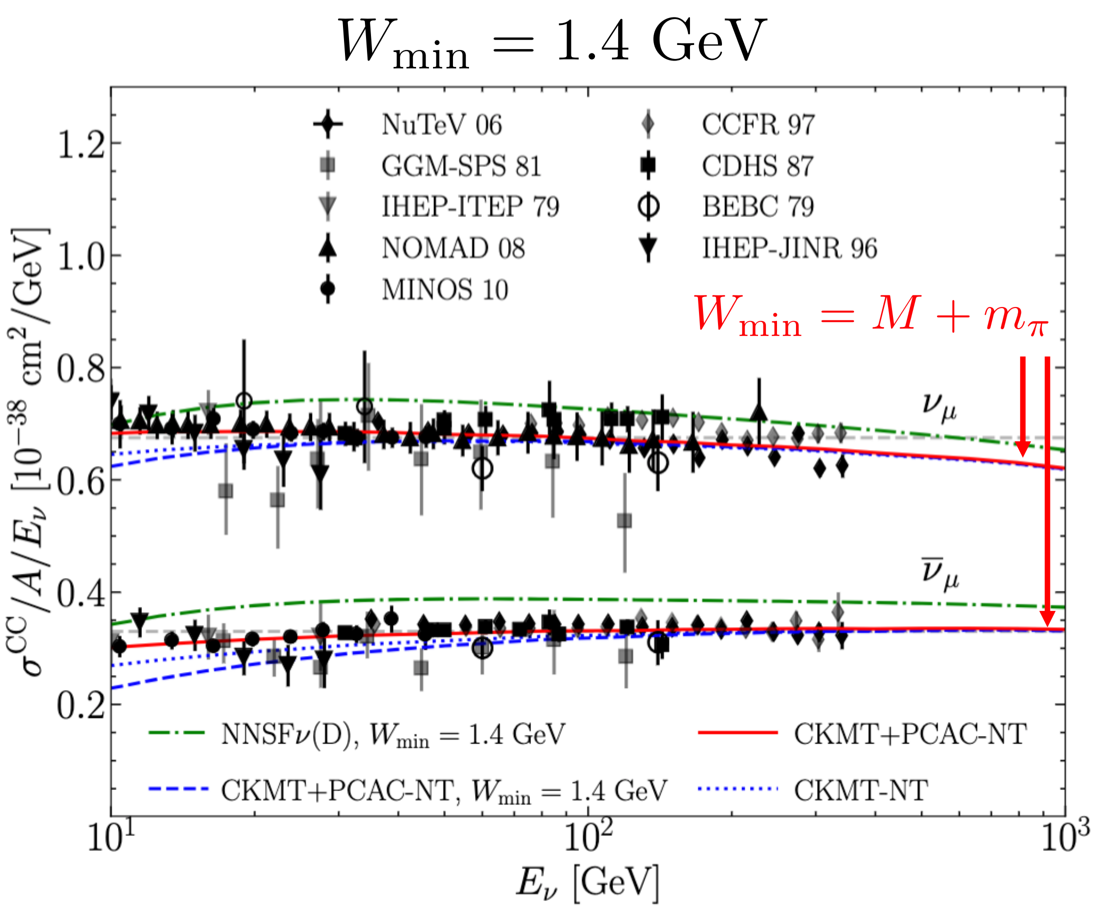


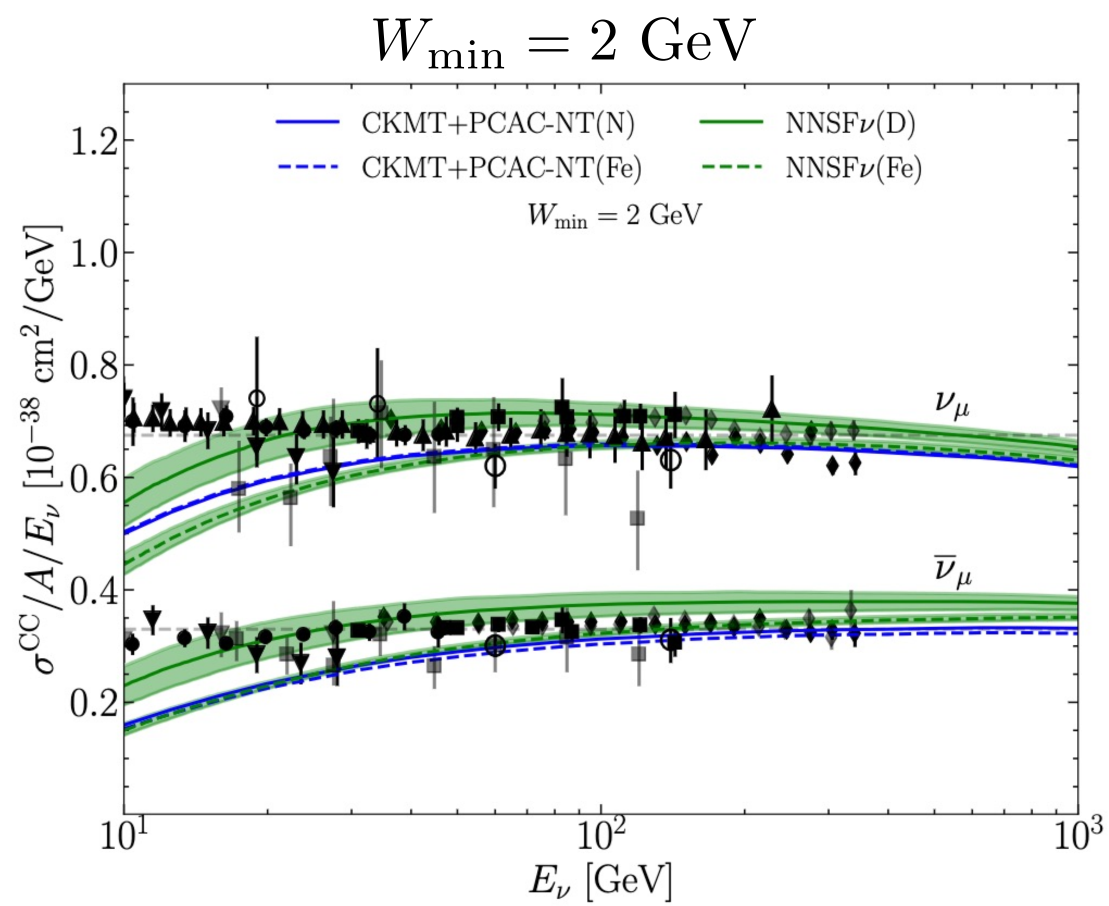


DIS CC , $\nu_\mu, \bar{\nu}_\mu$ for D, Fe

note: many nuclear targets for experiments

better agreement between approaches with Fe

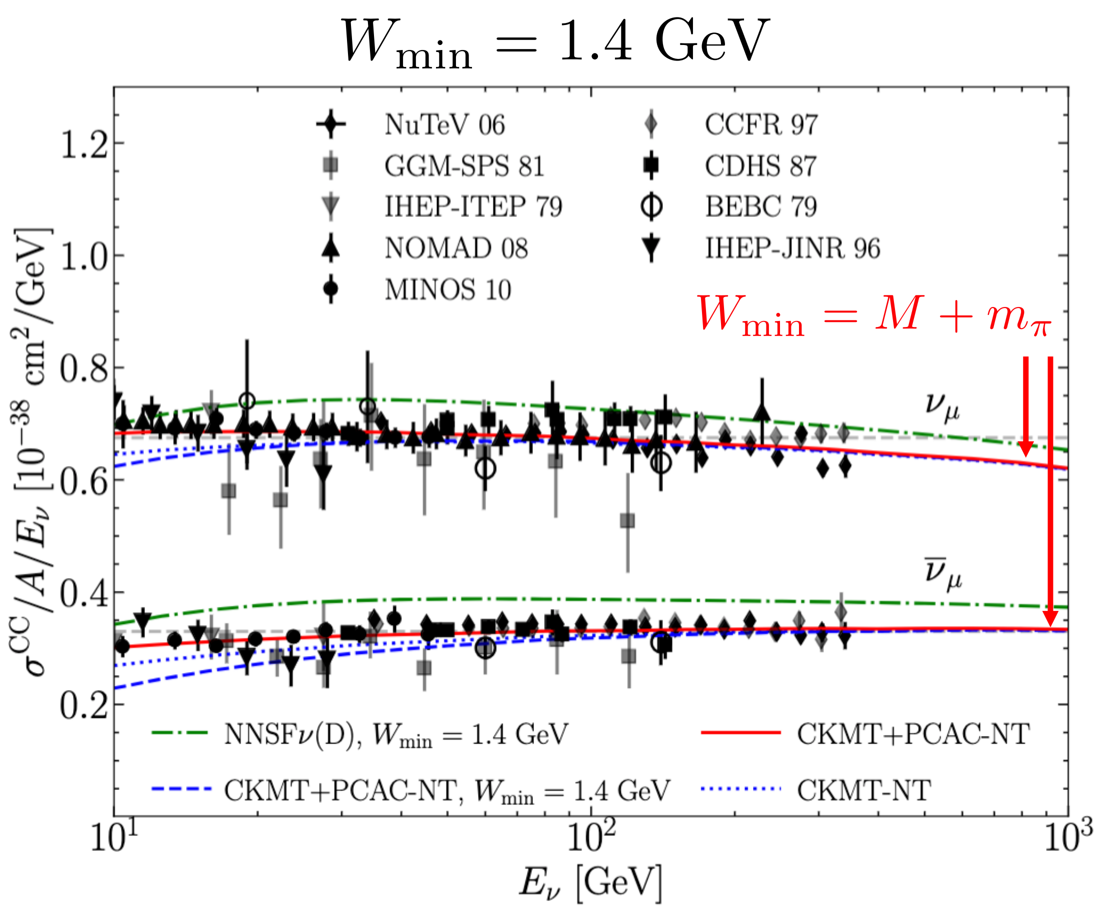




Jeong & Reno, Phys Rev D 108 (2023) 113010

DIS CC , $\nu_\mu, \bar{\nu}_\mu$ for D

note: many nuclear targets for experiments



Difference between blue-dashed and red is averaged "SIS" contribution.
Dotted blue – no PCAC but otherwise equivalent to the red curve.

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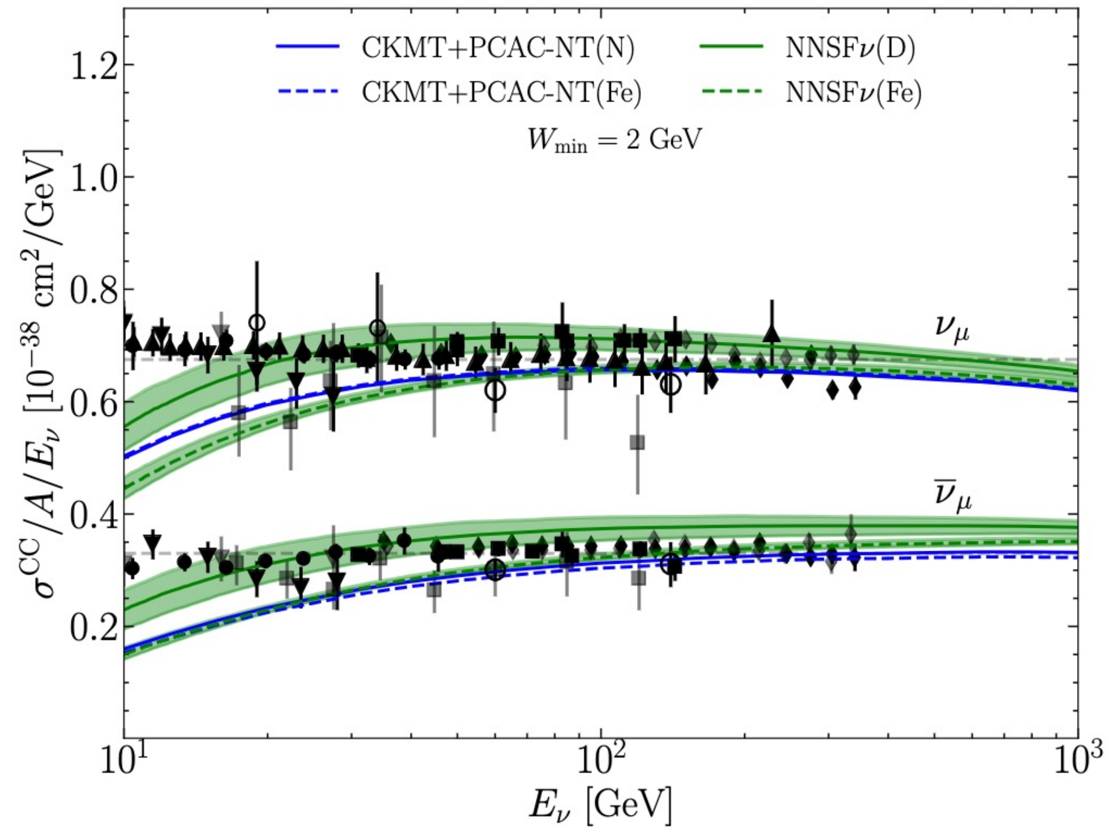
DIS CC , $\nu_\mu, \bar{\nu}_\mu$ for Fe

- Blue solid and dashed – nCTEQ15 PDFs for D, Fe
  - Very little change with Fe.
- Upper green – D, lower green – Fe
  - Much bigger difference.
  - Neutrino data – not with light nuclei.

better agreement between approaches with Fe

$W_{\rm min} = 2$ GeV



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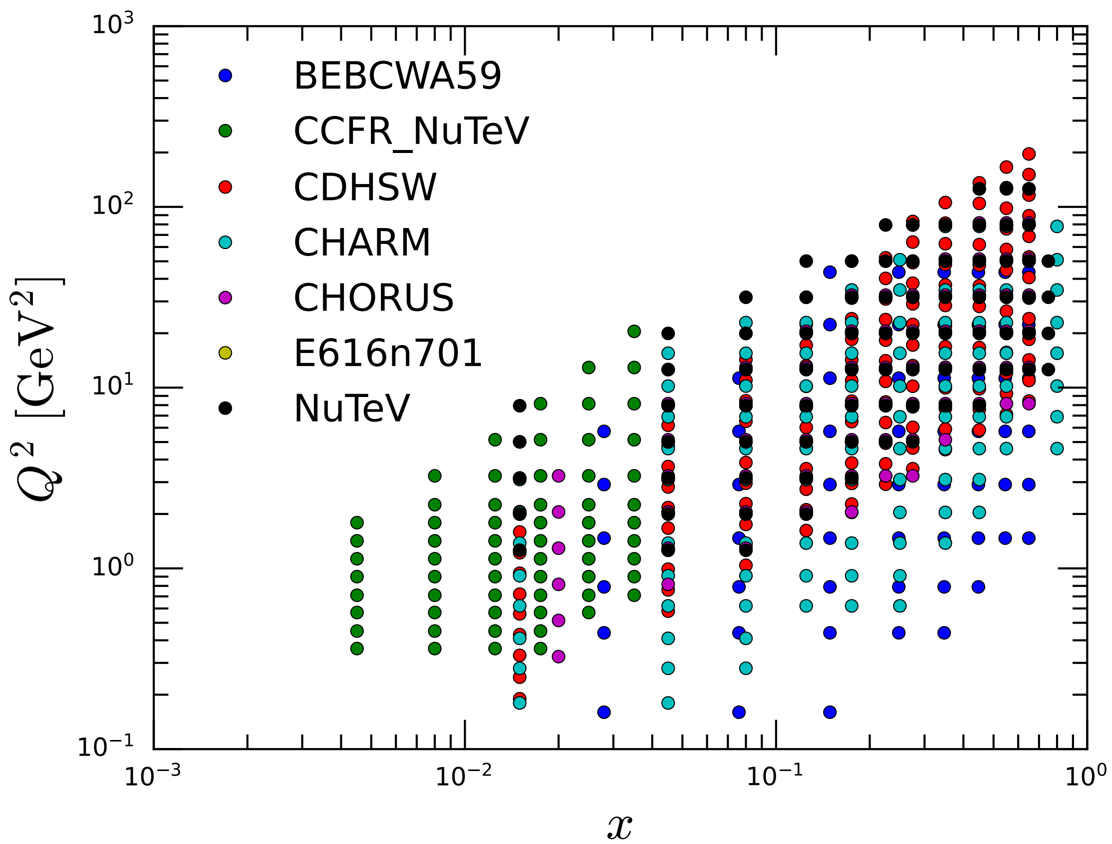
Neutrino data



Nuclear targets:

Ne – BEBC

Fe - CDHS, CCFR, NuTeV

CuCO3 – CHARM (A~20)

Remarks

- Based on our CKMT+PCAC (patched and matched) extrapolation, the fraction contribution of low Q, W< 2 GeV for E>100 GeV in CC cross section is small, but is larger for low E, depends on W.
  - E = 50 GeV, W_{min}=1.4 GeV, Q^2<2 GeV^2 accounts for ~15% of the ν_μ CC cross section, ~30% of the $\bar{\nu}_\mu$ CC cross section.
- Thus, even above W=1.4, 2 GeV, modeling low Q structure functions for E<100 GeV is useful for FPF experiments, can have implications for DUNE. There will be 10's of thousands of $\nu_\mu + \bar{\nu}_\mu$ CC events at a potential future FPF in the energy range of 50-100 GeV in the high luminosity era.

More remarks

- The NNSFv structure function extrapolations to deuterium don't agree with our inputs.
  - Are nuclear corrections different for weak and EM scattering?
  - Small-x behavior is very different!
- CKMT+PCAC as implemented can be improved, or is there a better way?
  - Extensive fits to HERA data (ALLM), but hard to see how to convert to weak structure functions.
  - What is the right way to implement PCAC? We did a crude job.

ALLM

$$
\begin{aligned}
F_2 &= \frac{Q^2}{Q^2 + m_0}(F_2^{I\!P} + F_2^{I\!R}) \\
F_2^{I\!P} &= c_{I\!P} * x_{I\!P}^{a_{I\!P}}(1 - x_{Bj})^{b_{I\!P}} \\
F_2^{I\!R} &= c_{I\!R} * x_{I\!R}^{a_{I\!R}}(1 - x_{Bj})^{b_{I\!R}} \\
\frac{1}{x_{I\!P}} &= 1 + \frac{W^2 - m_p^2}{Q^2 + p_1} \quad \text{where } m_p \text{ is the proton mass} \\
\frac{1}{x_{I\!R}} &= 1 + \frac{W^2 - m_p^2}{Q^2 + p_2} \\
t &= \ln\left(\frac{\ln \frac{Q^2+p_3}{p_4}}{\ln \frac{p_3}{p_4}}\right)
\end{aligned}
$$

$$
\begin{aligned}
c_{I\!P} &= p_5 + (p_5 - p_6)\left[\frac{1}{1 + t^{p_7}} - 1\right] \\
a_{I\!P} &= p_8 + (p_8 - p_9)\left[\frac{1}{1 + t^{p_{10}}} - 1\right] \\
b_{I\!P} &= p_{11} + p_{12}t^{p_{13}} \\
c_{I\!R} &= p_{14} + p_{15}t^{p_{16}} \\
a_{I\!R} &= p_{17} + p_{18}t^{p_{19}} \\
b_{I\!R} &= p_{20} + p_{21}t^{p_{22}} \ .
\end{aligned}
$$

HHT-ALLM

see https://arxiv.org/pdf/1704.03187

ALLM: H. Abramowicz et al., Phys. Lett. B 269, 465 (1991);
H. Abramowicz and A. Levy (1997), [hep-ph/9712415].

HHT-ALLM: New fit to combined HERA data.