From inclusive to exclusive processes

- Alexis Nikolakopoulos
- Neutrino Theory Network (NTN) Fellow
- University of Washington

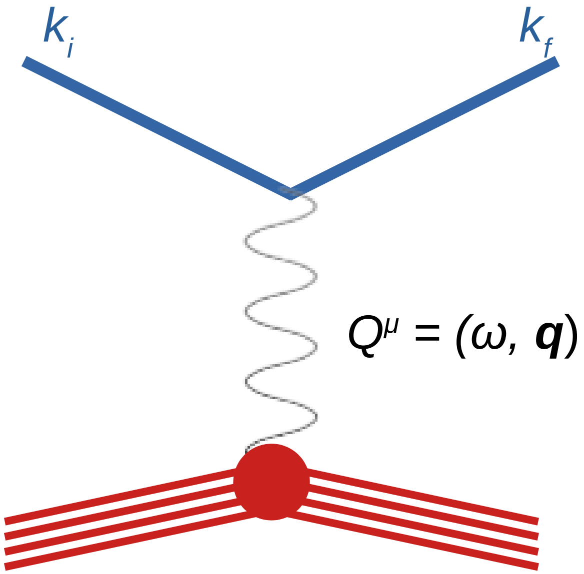
Outline

1. **Flux-averaged neutrino cross section from the Euclidian response**
   w. Noemi Rocco (FNAL)

2. **Semi-Exclusive nucleon knockout: RDWIA and INC**
   w. R. Gonzalez-Jimenez, N. Rocco, J. Isaacson, K. Niewczas,
   J.M. Udias, F. Sanchez, A. Ershova, N. Jachowicz

3. **What data do we need to constrain the exclusive cross section?**

Inclusive nuclear response functions



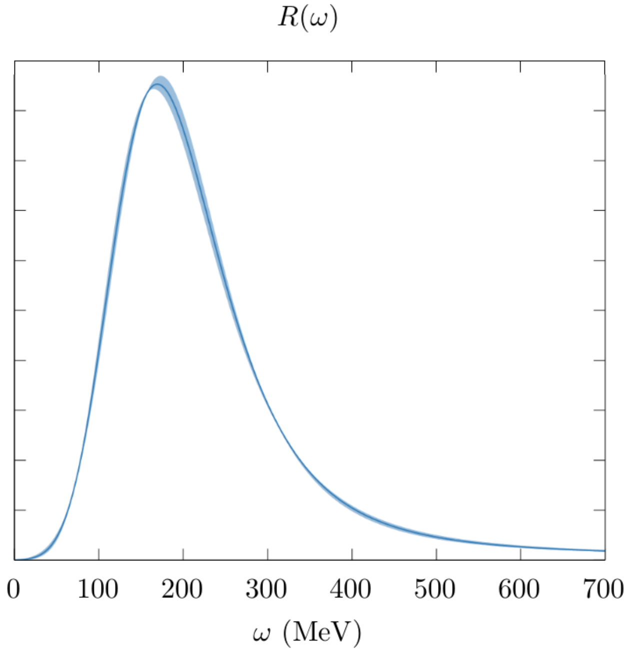
$$\frac{\mathrm{d}\sigma(E_i)}{\mathrm{d}E_f \, \mathrm{d}\cos\theta_f} = v_{CC}(E_i, E_f, \cos\theta_f) R_{CC}(\omega, q) + v_{CL}(E_i, E_f, \cos\theta_f) R_{CL}(\omega, q) + v_{LL}(E_i, E_f, \cos\theta_f) R_{LL}(\omega, q) + v_T(E_i, E_f, \cos\theta_f) R_T(\omega, q) + v_{T'}(E_i, E_f, \cos\theta_f) R_{T'}(\omega, q)$$

$$M_{if} = J^\mu_{lepton} \, J_{\mu,\, hadron}$$

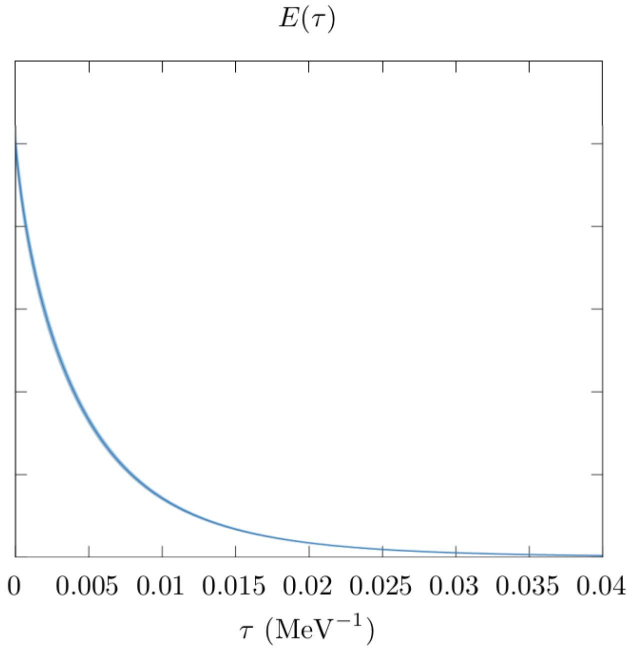
Euclidian response function

$$E(\tau, q) \equiv \mathcal{L}\left[R(\omega, q)\right](\tau, q) = \int_0^\infty R(\omega, q) e^{-\tau\omega} \mathrm{d}\omega$$

Calculated in Green's function Monte-Carlo (see A. Lovato's talk)



Laplace transform



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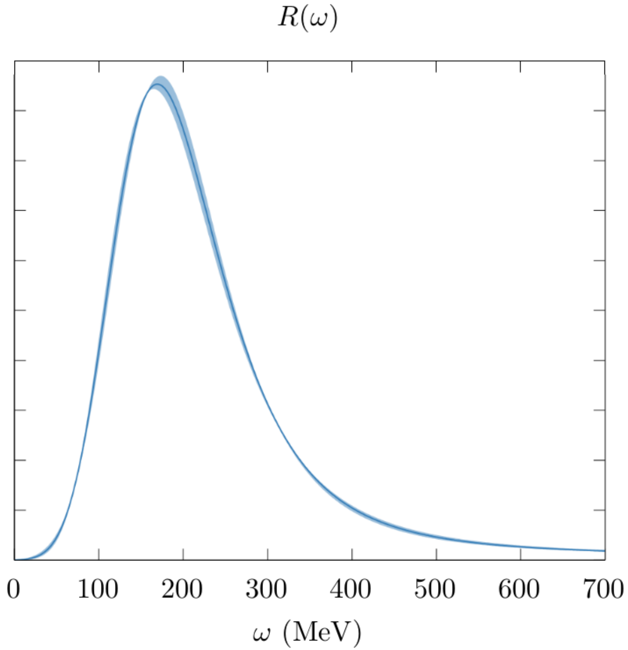
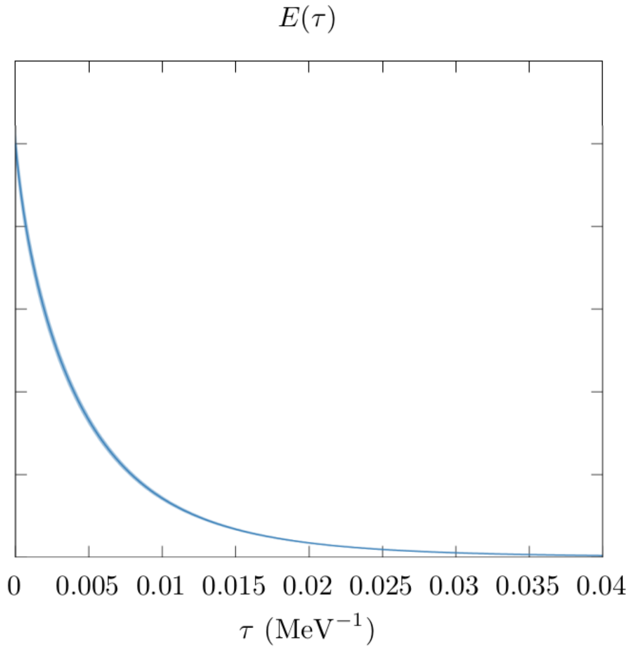


Laplace transform

Inverse Laplace

(yuk!!)



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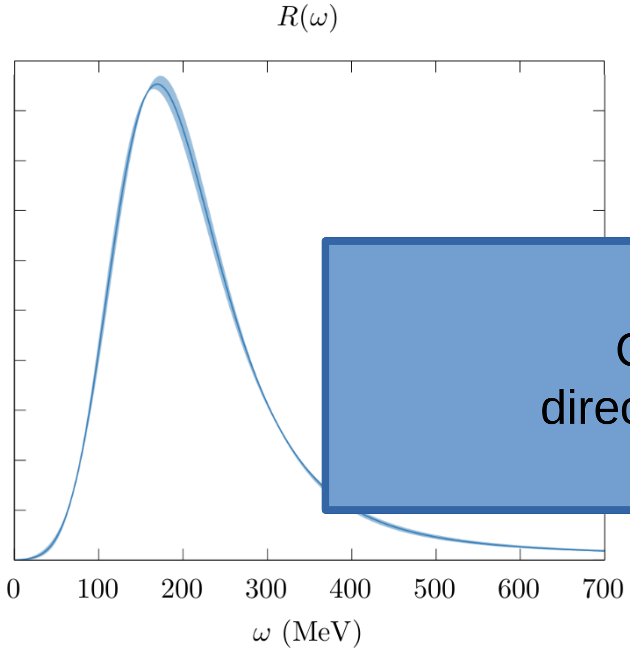
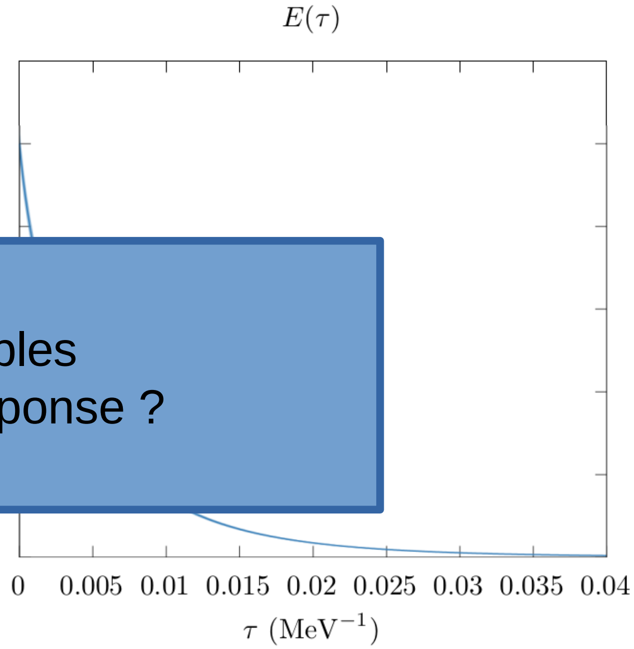


Can we calculate observables
directly from the Euclidian response ?



Energy-averaged neutrino cross section

$E_i = \omega + E_f$

$$\langle \frac{\mathrm{d}\sigma}{\mathrm{d}E_f \mathrm{d}q} \rangle = G^2 q \int \mathrm{d}\omega \frac{E_f}{E} \phi(\omega + E_f) \sum_i [v_i(\omega, E_f, q) R_i(\omega, q)]$$

Energy-averaged neutrino cross section

$$\langle\frac{\mathrm{d}\sigma}{\mathrm{d}E_f\mathrm{d}q}\rangle = G^2 q \int \mathrm{d}\omega \frac{E_f}{E}\phi(\omega+E_f)\sum_i [v_i(\omega, E_f, q) R_i(\omega, q)]$$

Property of the Laplace transform

$$\int_0^\infty f(\omega) g(\omega) \mathrm{d}\omega = \int_0^\infty \mathcal{L}[f](\tau)\mathcal{L}^{-1}[g](\tau)\mathrm{d}\tau$$

Energy-averaged neutrino cross section

$$\left\langle \frac{d\sigma}{dE_f dq} \right\rangle = G^2 q \int d\omega \boxed{\frac{E_f}{E}\phi(\omega+E_f)\sum_i [v_i(\omega, E_f, q)} \boxed{R_i(\omega, q)]}$$

$$\int_0^\infty \boxed{f(\omega)}\boxed{g(\omega)}d\omega = \int_0^\infty \underline{\mathcal{L}[f](\tau)}\mathcal{L}^{-1}[g](\tau)d\tau$$

Is the Euclidian response

→ **Can compute weighted integrals of the response from the Euclidian response**

→ **The energy-averaged cross section is a weighted integral of the response**

Energy-dependence of $v_i(\omega, E_f, q)$

All *v*-factors can be decomposed in **five functions of ω**

$$\mathcal{D}_0(\omega) = 1 = \omega^0, \quad \mathcal{D}_1(\omega) = \omega, \quad \mathcal{D}_2(\omega) = \omega^2,$$

$$\mathcal{E}_1(\omega) = \frac{1}{E_f + \omega}, \quad \mathcal{E}_2(\omega) = \frac{1}{(E_f + \omega)^2}.$$

With known inverse Laplace transforms

Energy-dependence of $v_i(\omega, E_f, q)$

The energy-averaged cross section is given by **five integrals of the Responses**

Energy weighted sum rules:

$$I_i\left[\mathcal{D}_n\right] \equiv \int_0^\infty \mathrm{d}\omega \; \omega^n R_i(\omega, q) = E_i^{(n)}(0)$$

For n = (0,1,2)

E_f – dependent integrals:

$$I_i\left[\mathcal{E}_n\right] \equiv \int_0^\infty \mathrm{d}\omega \; \frac{R_i(\omega, q)}{(E_f + \omega)^n} = \int_0^\infty \mathrm{d}\tau E_i(\tau) \tau^{n-1} e^{-E_f \tau}$$

For n = (1,2)

Energy-dependence of $v_i(\omega, E_f, q)$

The energy-averaged cross section is given by **five integrals of the Responses**

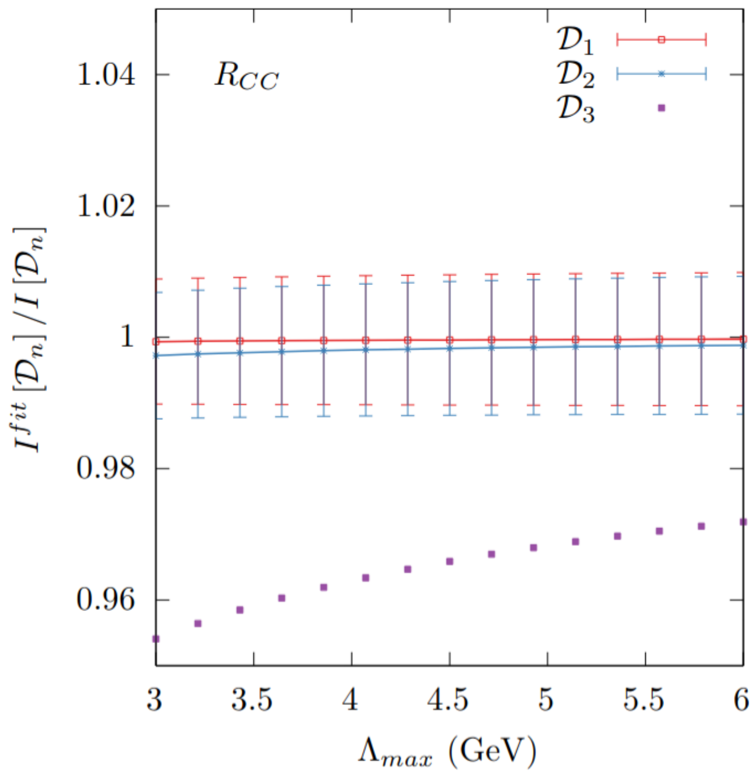
Only the *n*-th moments of the response:

$$\int_0^\infty \mathrm{d}\omega \; \omega^n R_i(\omega, q) = E_i^{(n)}(0)$$

Are non-trivial to compute

Numerical evaluation of n-th moment of the Response

We develop a robust method to compute **higher moments with uncertainty from *E(τ)***



Sub-percent accuracy for n=1,2
With toy-model response

Numerical evaluation of n-th moment of the Response

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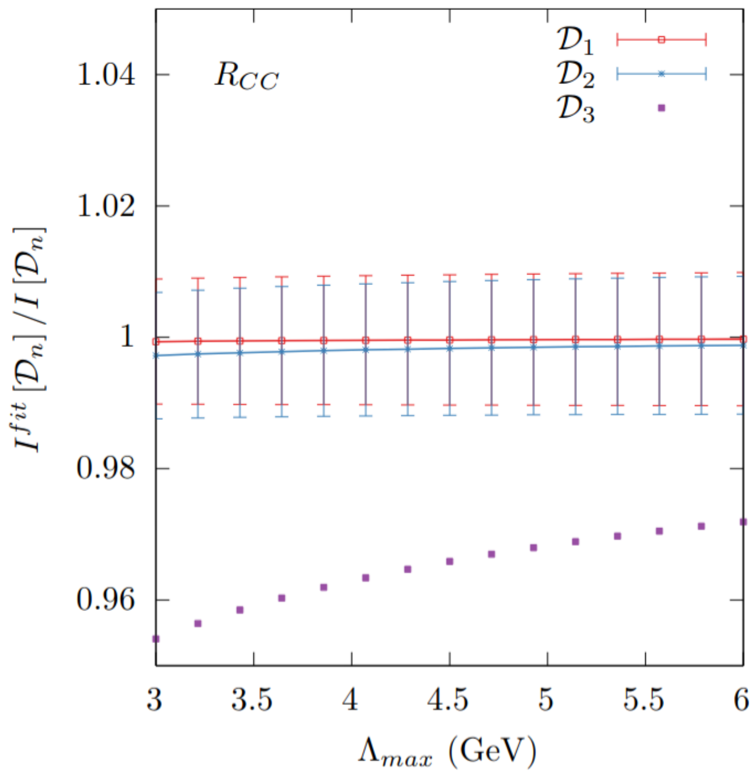

Sub-percent accuracy for n=1,2
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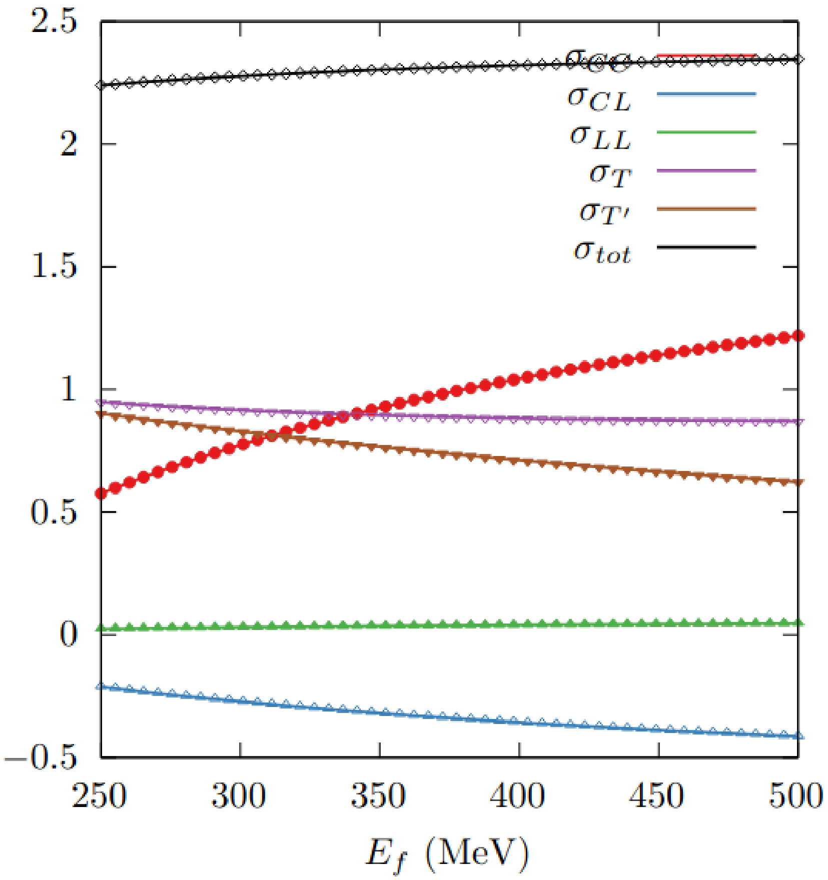
Energy-averaged cross section: toy model response



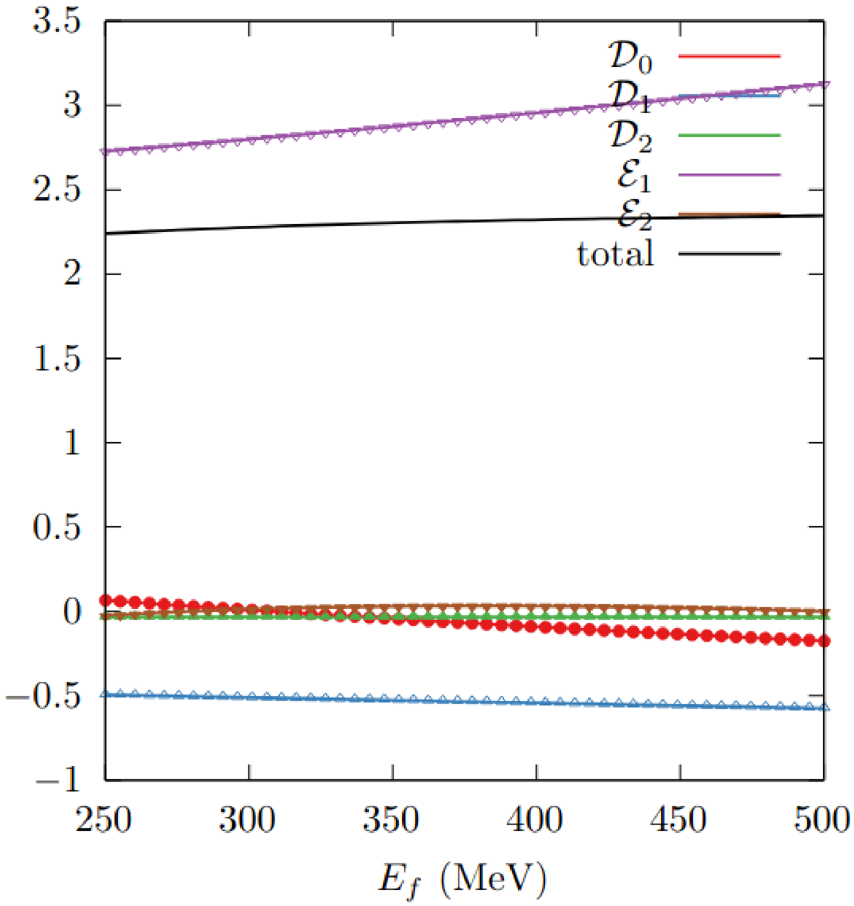
Contributions from ω-dependence

Dominated by 2 integrals:

$\omega R(\omega)$ and $R(\omega)/(\omega+E_f)$

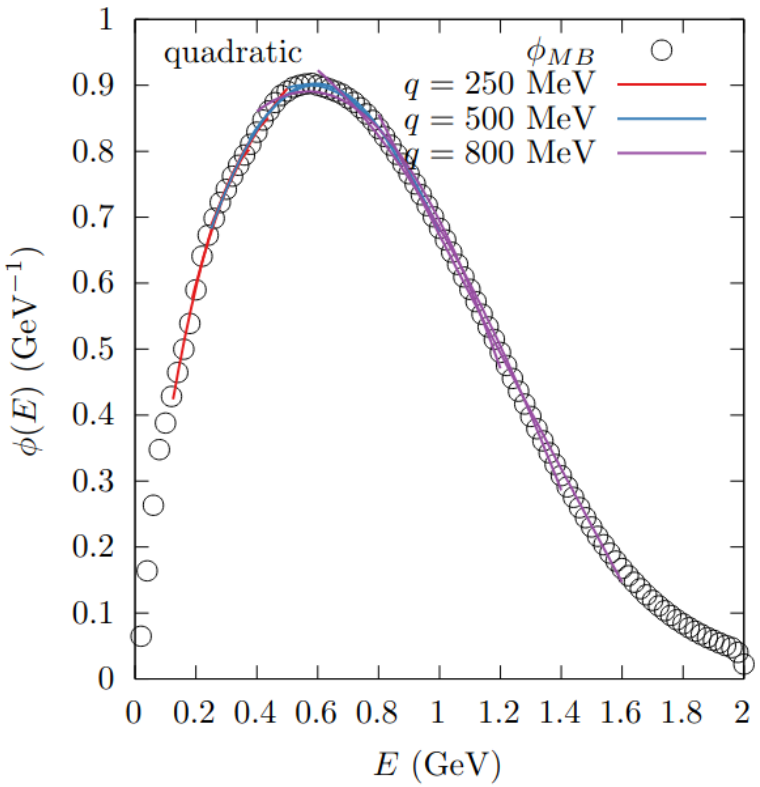
Second moment is negligible:

Dependence enters as ω^2/q^2



Flux-averaged cross sections

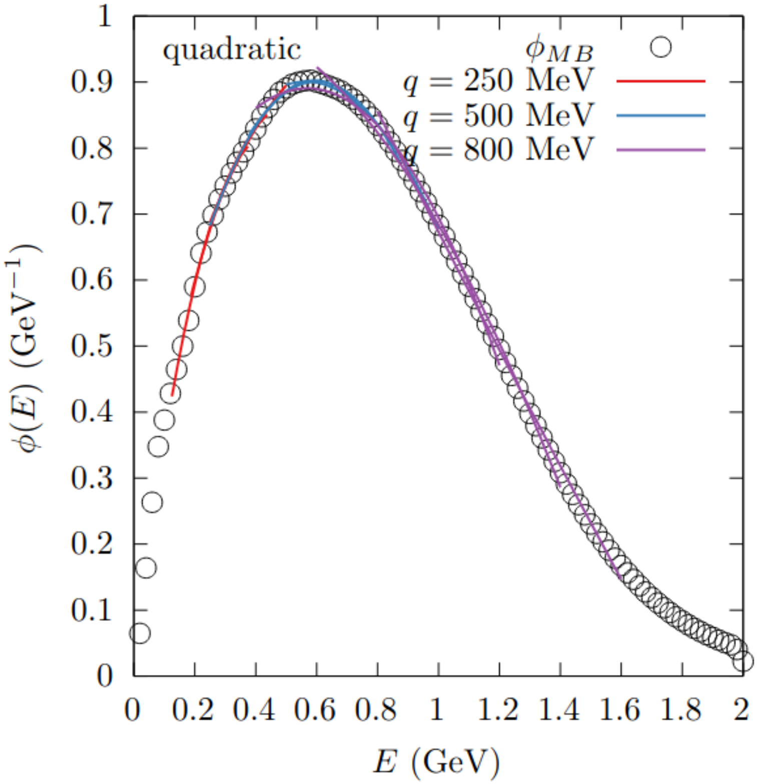
Replace $v(\omega, E_f, q) \rightarrow v(\omega, E_f, q)\, \boldsymbol{\Phi(\omega+E_f)}$



Fit with 2th order polynomial

Flux-averaged cross sections

Replace $v(\omega, E_f, q) \rightarrow v(\omega, E_f, q)\,\boldsymbol{\Phi(\omega+E_f)}$



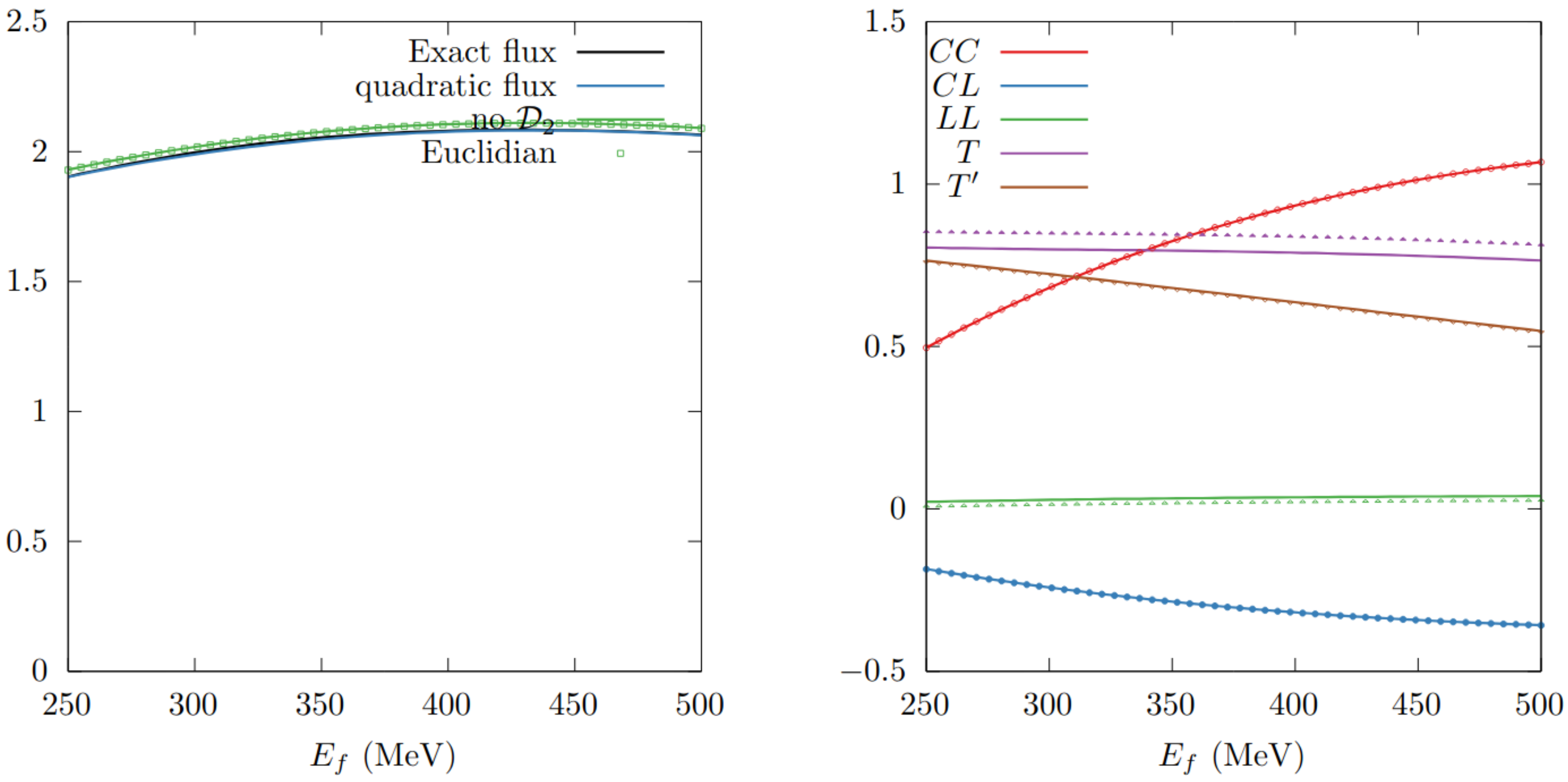
Fit with 2^{th} order polynomial

Need up to ~~4^{th} moment~~ of the Response

3^{rd} moment

Since 2^{th} moment is negligible

Flux-averaged cross sections



Excellent results with toy-model response!

Realistic responses: PWIA with realistic spectral function

We address **two main uncertainties**

1. Integral is restricted to the physical region ω < q

$$\langle \frac{d\sigma}{dE_f dq} \rangle = G^2 q \boxed{\int_0^q} d\omega \frac{E_f}{E} \phi(\omega + E_f) \sum_i [v_i(\omega, E_f, q) R_i(\omega, q)]$$

Upper limit q < ∞ ! Need to **correct for contribution from ω ϵ [q,∞]**

$$\omega + M_N \sim E_N = \sqrt{(q + p_m)^2 + M_N^2}$$

The large-ω dependence comes from high-p_m contributions (SRC)

Realistic responses: PWIA with realistic spectral function

We address **two main uncertainties**

1. Integral is restricted to the physical region $\omega < q$

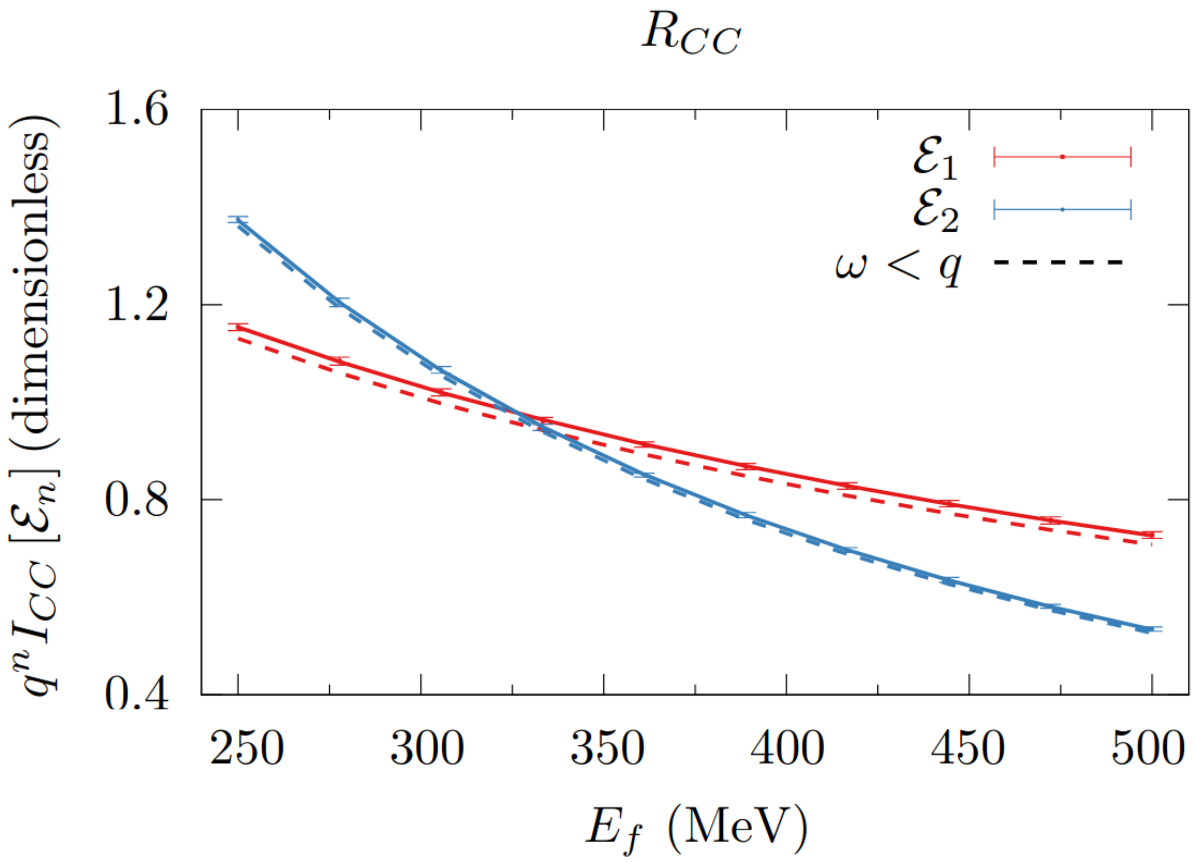
2. Euclidian response has uncertainty, is noisy

We assign a realistic statistical uncertainty based on the magnitude:

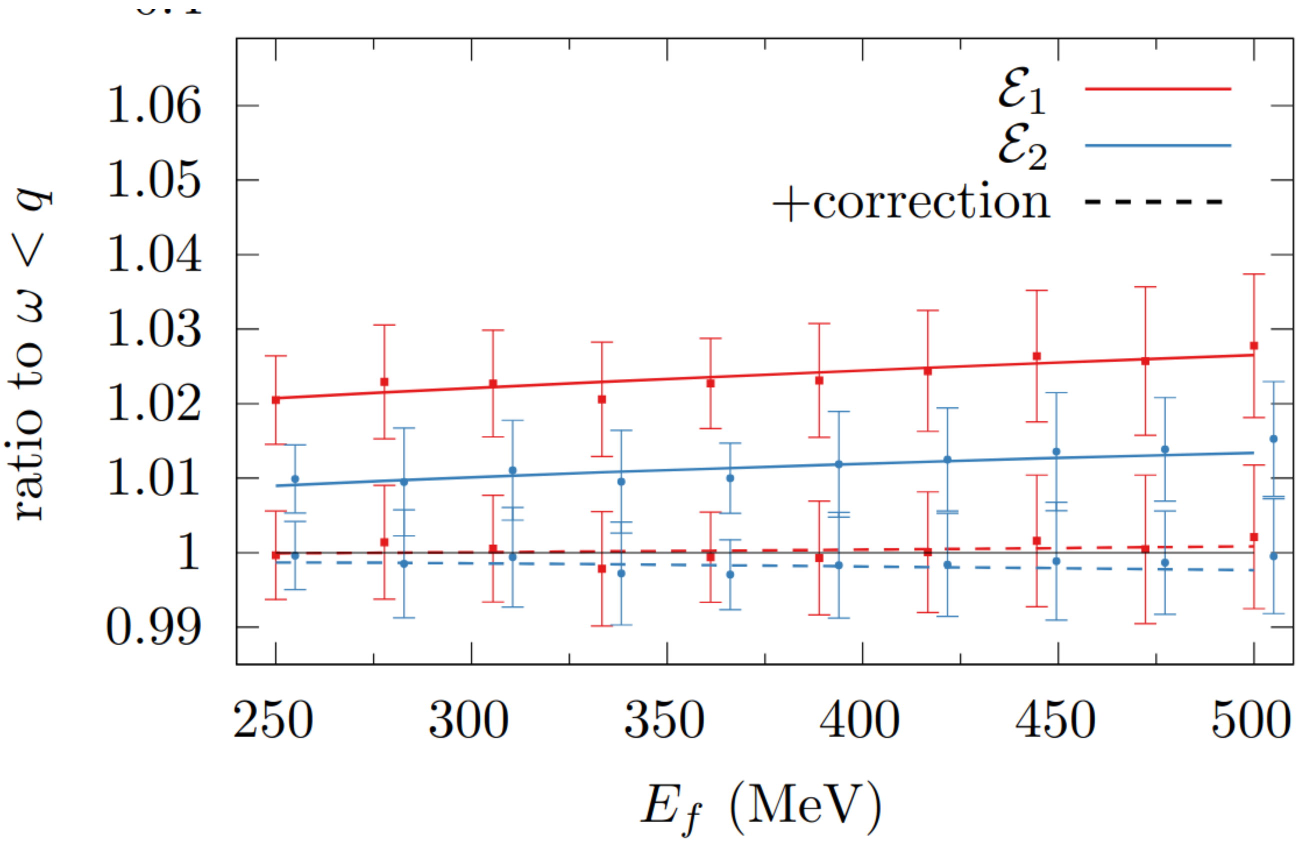
$$Var\left[E(\tau)\right] = f_0\sqrt{E(0)E(\tau)},$$

→ Calculate observables with ensembles of *E(τ)* sampled within uncertainty
To **propagate uncertainty to observables**

Energy-dependent integrals



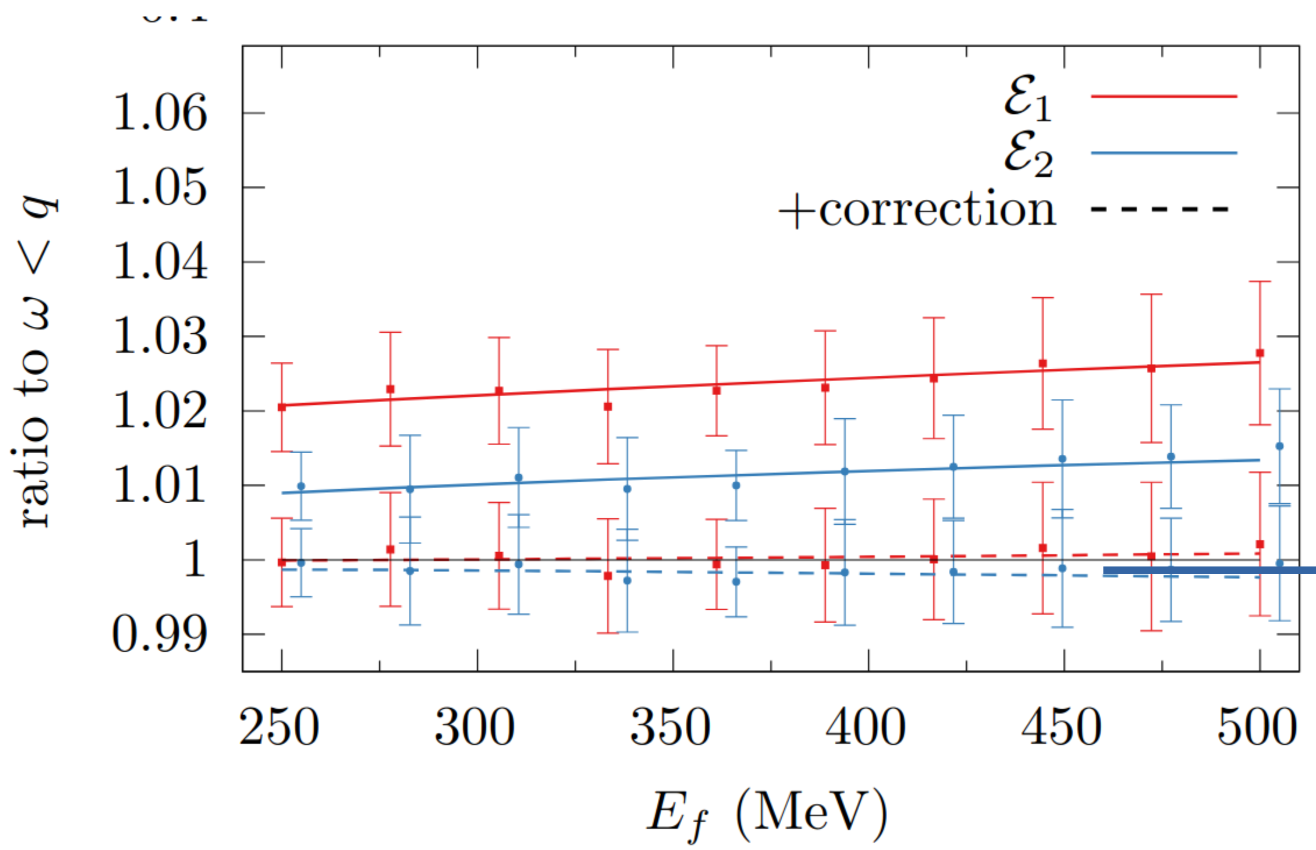
Energy-dependent integrals



Fractional uncertainty from noise

$F_{shape} \sim 0.15\ F_0$

Energy-dependent integrals



Fractional uncertainty from noise

$F_{shape} \sim 0.15\ F_0$

Universal correction From momentum distribution

Of percent-level

Realistic responses: uncertainties small and under control

1. Euclidian response has uncertainty: is noisy
→ percent-level shape uncertainties under control

2. Integral is restricted to the physical region $\omega < q$
→ Corrections come from large p_m region

Inversion of Laplace = nucleus specific, sensitive to low-ω region
Instead : need estimation of large-ω behavior:
== Small corrections & driven by SRC ~ universal !

Paper and code soon: [A.N. & N. Rocco, arxiv:2506.xxxx]

From inclusive to (semi)-exclusive cross sections

Electron experiments A(e,e'p)B*: Measure 1 specific kinematic (usually)
Strict kinematic cuts : **specific state** B*
→ This is **exclusive**

Neutrino experiments A(ν,μp) X: Measure a range of kinematics
The residual system is **many states** X
→ This is **semi-inclusive/semi-exclusive**

From inclusive to (semi)-exclusive cross sections

Electron experiments A(e,e'p)B*: Measure 1 specific kinematic (usually)
Strict kinematic cuts : **specific state** B*
→ This is **exclusive**

Neutrino experiments A(ν,μp) X: Measure a range of kinematics
The residual system is **many states** X

Need information on all accessible final-states X

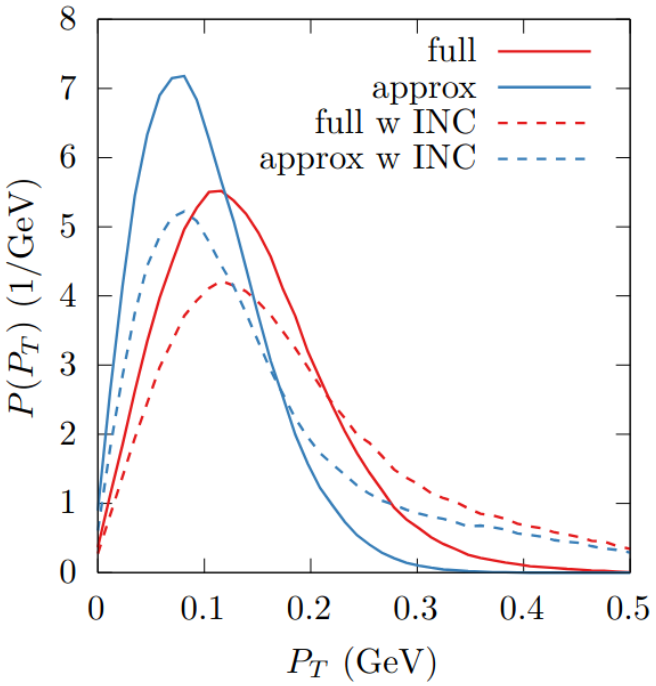
(impossible?)

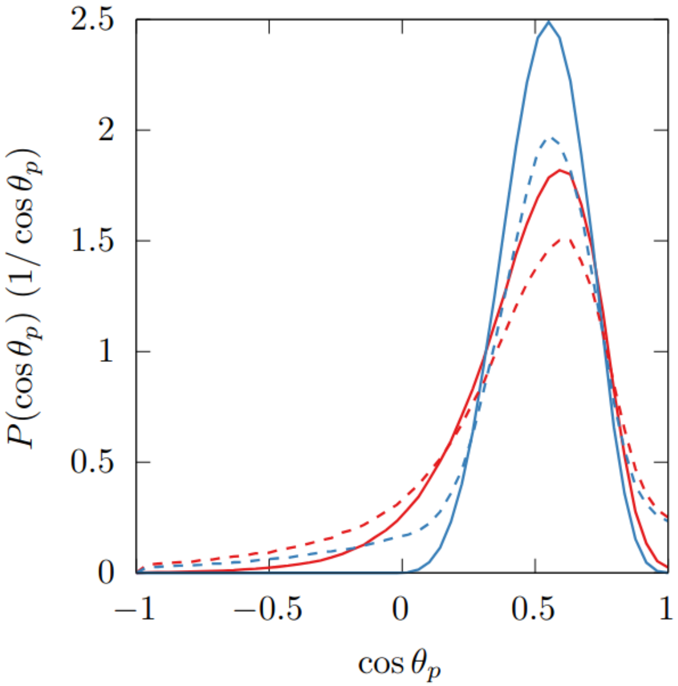
Use the Intranuclear Cascade (INC) approximation

$$|\mathcal{M}(\{i\} \to \{f\})|^2 \approx \int \mathrm{d}p' |\mathcal{M}(\{i\} \to \{p'\})|^2 |\mathcal{P}(\{p'\} \to \{f\})|^2$$

Input to the INC in generators

Some implementations in generators: **Only inclusive** cross section
→ The generator invents the nucleon final-state





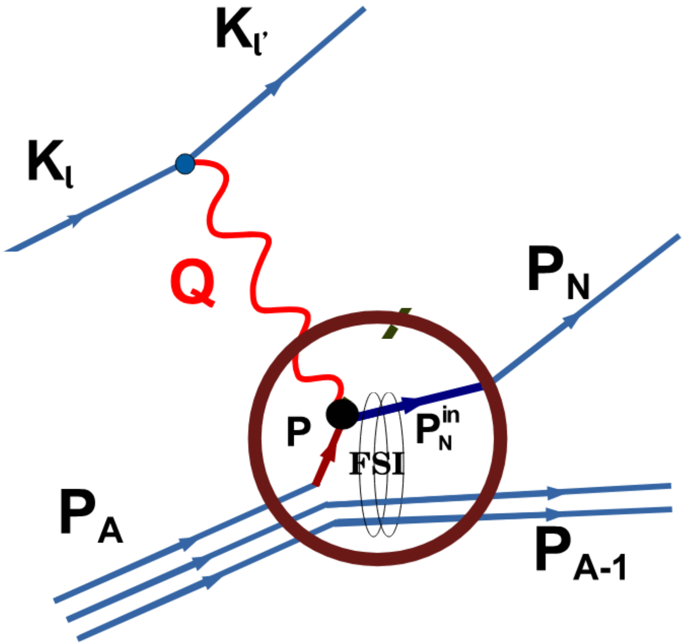
[A.N., S. Gardiner, A. Papadopoulo, S. Dolan, R. Gonzalez-Jimenez, arxiv:2302.12182]

Terminology : RDWIA RPWIA & PWIA

-Relativistic Distorted Wave Impulse Approximation (RDWIA)

$$\mathcal{J}_{\kappa}^{m_j}(Q, P_N) = \int \mathrm{d}\mathbf{p}\ \overline{\psi}(\mathbf{p}+\mathbf{q}, \mathbf{k}_N, s_N)\ \mathcal{O}^{\mu}\ \psi_{\kappa}^{m_j}(\mathbf{p})$$

Distorted wave function for final-state



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- Relativistic Plane Wave Impulse Approximation (RPWIA)

$$\mathcal{J} = (2\pi)^{3/2}\ \overline{u}(\mathbf{k}_N, s_N)\ \mathcal{O}^{\mu}\ \psi_{\kappa}^{m_j}(\mathbf{k}_N - \mathbf{q})$$

By treating the final-state wavefunction as a plane-wave:

$$\overline{\psi}(\mathbf{p}, \mathbf{k}_N, s_N) \to (2\pi)^{3/2}\delta(\mathbf{p}-\mathbf{k}_N)\overline{u}(\mathbf{k}_N, s_N)$$

→ Neglect all final-state interactions

Terminology : RDWIA RPWIA & PWIA

-Relativistic Distorted Wave Impulse Approximation (RDWIA)

$$\mathcal{J}_{\kappa}^{m_j}(Q, P_N) = \int \mathrm{d}\mathbf{p}\ \overline{\psi}(\mathbf{p}+\mathbf{q}, \mathbf{k}_N, s_N)\ \mathcal{O}^{\mu}\ \psi_{\kappa}^{m_j}(\mathbf{p})$$

- Plane-Wave Impulse Approximation (PWIA)

The initial state is assumed proportional to a positive-energy spinor:

$$\psi_{\kappa}^{m_j}(\mathbf{p}) \propto f(|\mathbf{p}|)u(\mathbf{p})$$

One obtains a factorized expression (‘spectral function approach’)

$$\frac{d\sigma(E_\nu)}{dp_\mu d\Omega_\mu d\Omega_p dp_N} = \frac{G_F^2 \cos^2\theta_c}{(2\pi)^2}\frac{p_\mu^2 p_N^2}{E_\nu E_\mu}\frac{M_N^2}{E_N \overline{E}} L_{\mu\nu} h^{\mu\nu}_{s.n.} S(E_m, p_m)$$

Terminology : RDWIA RPWIA & PWIA

-Relativistic Distorted Wave Impulse Approximation (RDWIA)

↓ Remove elastic FSI

- Relativistic Plane Wave Impulse Approximation (RPWIA)

↓ Project onto particle spinors

- Plane-Wave Impulse Approximation (PWIA)

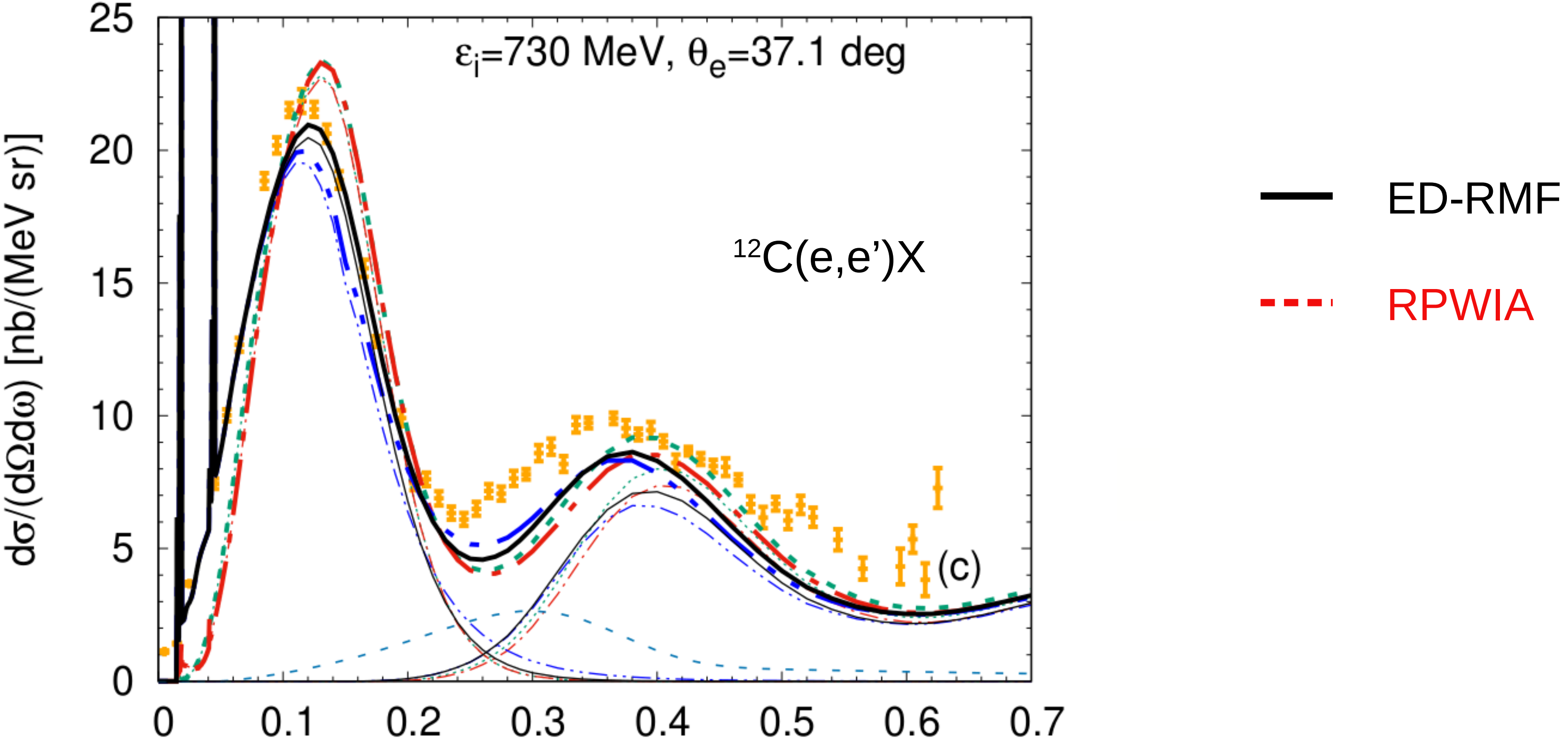
Remember

- All results use **the same spectral function** but different final-state
  → **can consistently check effect of FSI**

RDWIA with real potential

- Energy-Dependent Relativistic Mean-Field (ED-RMF)

Final-state in **real potential** → suitable for **FSI in inclusive** cross section



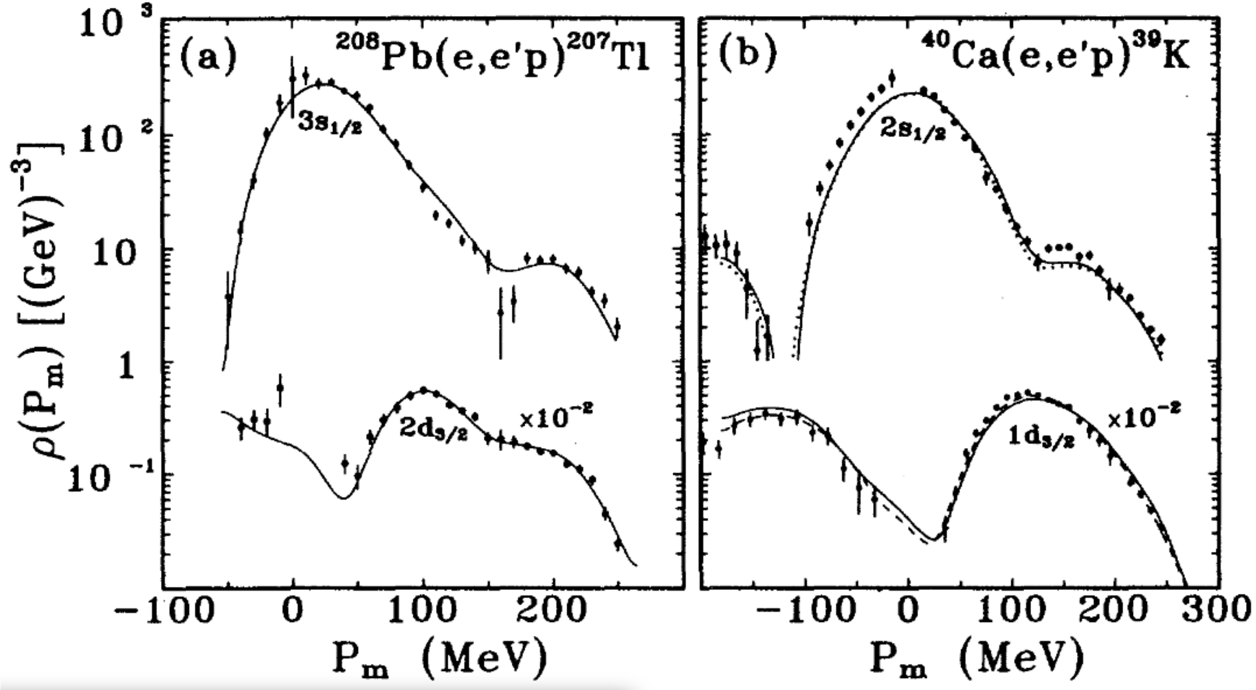
[R. Gonzalez-Jimenez et al Phys. Rev. C 100, 045501 (2019)]

RDWIA with optical (complex) potential

- Relativistic Optical Potential (ROP)

Final-state in **complex potential** → suitable for **FSI in exclusive** cross section

[Udias et al. PRC48, 2731]



-'Standard' approach for FSI in exclusive (e,e'p) analysis

E.g. recent Jlab analyses of ^{40}Ar & ^{48}Ti
[PRD 107, 012005]
[PRD 105, 112002]

RDWIA with optical (complex) potential

- Relativistic Optical Potential (ROP)

Final-state in **complex potential** → suitable for **FSI in exclusive** cross section

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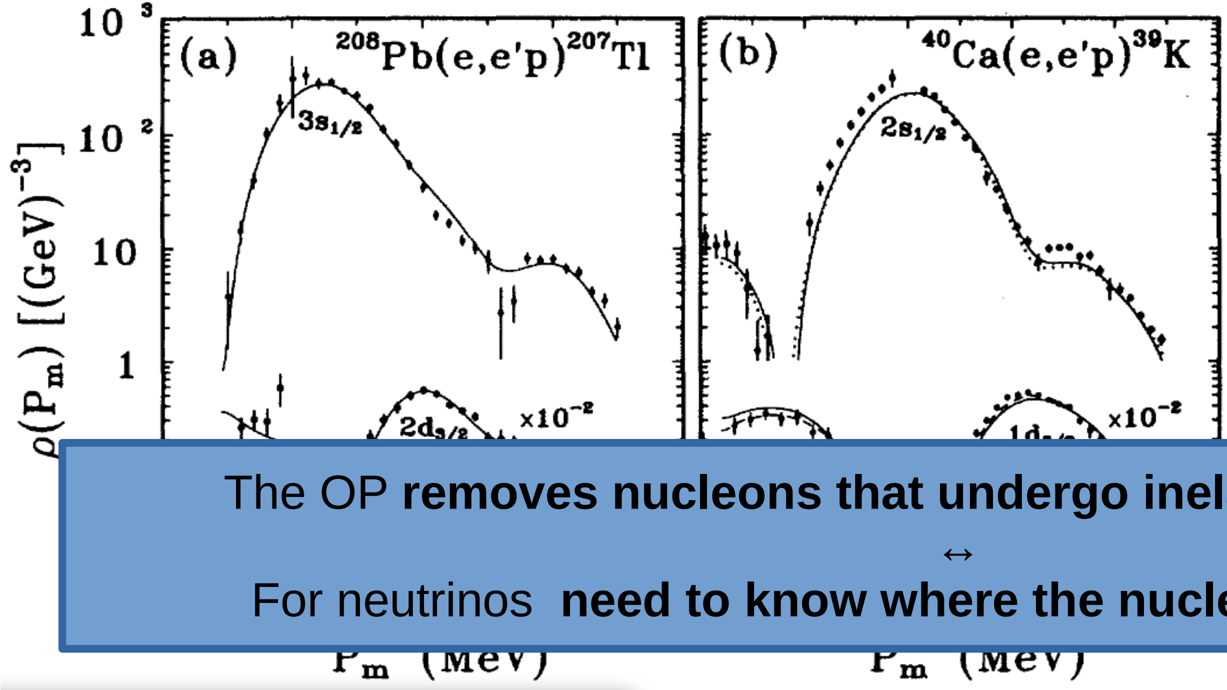


-'Standard' approach for FSI in exclusive (e,e'p) analysis

E.g. recent Jlab analyses of ^{40}Ar & ^{48}Ti [PRD 107, 012005]

The OP **removes nucleons that undergo inelastic FSI = A(e,e'p)B***

↔

For neutrinos **need to know where the nucleon goes = A(ν,μp)X**

Where do the nucleons go ? : Intranuclear Cascade model (INC)

Production of final-state $|X\rangle = |p\rangle|^{39}\mathrm{Ar}^*\rangle$

$$|\mathcal{M}|^2 \approx |\sum_\alpha \langle\Psi_0|T_{1b}|\psi_\alpha\rangle\langle\psi_\alpha|X\rangle|^2,$$

→ Restrict to 1-body operator

$$\approx \sum_\alpha |\langle\Psi_0|T_{1b}|\psi_\alpha\rangle|^2|\langle\psi_\alpha|X\rangle|^2$$

→ Classical approximation

$$\approx \sum_\alpha |\langle\Psi_0|T_{1b}|\psi_\alpha\rangle|^2 P(X|\alpha).$$

→ Intranuclear Cascade

Where do the nucleons go ? : Intranuclear Cascade model (INC)

Production of final-state $|X\rangle = |p\rangle|^{39}\mathrm{Ar}^*\rangle$

$$|\mathcal{M}|^2 \approx \boxed{\underbrace{|\sum_\alpha \langle \Psi_0|T_{1b}|\psi_\alpha\rangle\langle\psi_\alpha|X\rangle|^2,}_{\text{ROP}}}$$

→ Restrict to 1-body operator

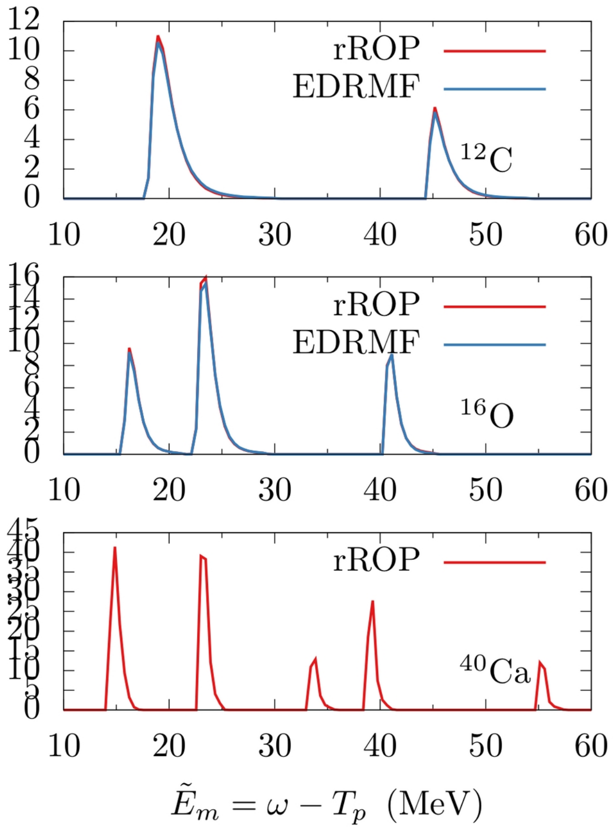
$$\approx \sum_\alpha |\langle \Psi_0|T_{1b}|\psi_\alpha\rangle|^2|\langle\psi_\alpha|X\rangle|^2$$

→ Classical approximation

$$\approx \boxed{\underbrace{\sum_\alpha |\langle \Psi_0|T_{1b}|\psi_\alpha\rangle|^2}_{\text{ED-RMF}}}\boxed{\underbrace{P(X|\alpha)}_{\text{INC}}}$$

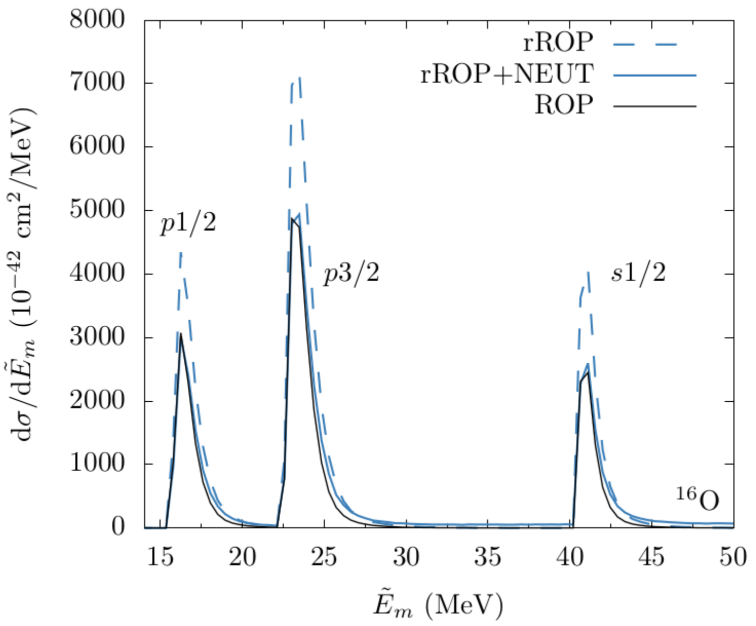
→ Intranuclear Cascade

Results from NEUT INC with simple spectral function



Input : **EDRMF with Simple spectral functions**

Results from NEUT : simple spectral function



[A.N. et al. PRC 105, 054603]

Input : **EDRMF with Simple spectral functions**

After INC:

NEUT result matches exclusive Calculation in the peaks!

PHYSICAL REVIEW C 110, 054611 (2024)

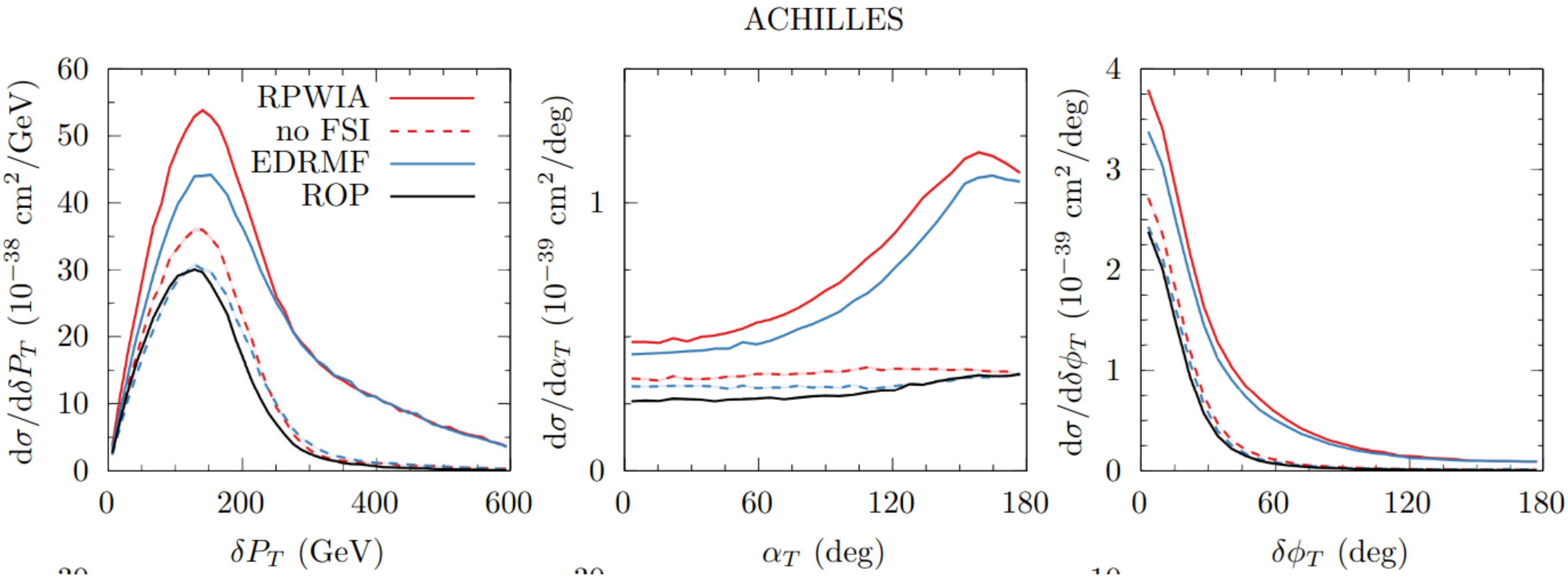
Final-state interactions in neutrino-induced proton knockout from argon in MicroBooNE

A. Nikolakopoulos,[1,*] A. Ershova,[2] R. González-Jiménez,[3] J. Isaacson,[1] A. M. Kelly,[1]
K. Niewczas,[4] N. Rocco,[1] and F. Sánchez[5]

Extended study:

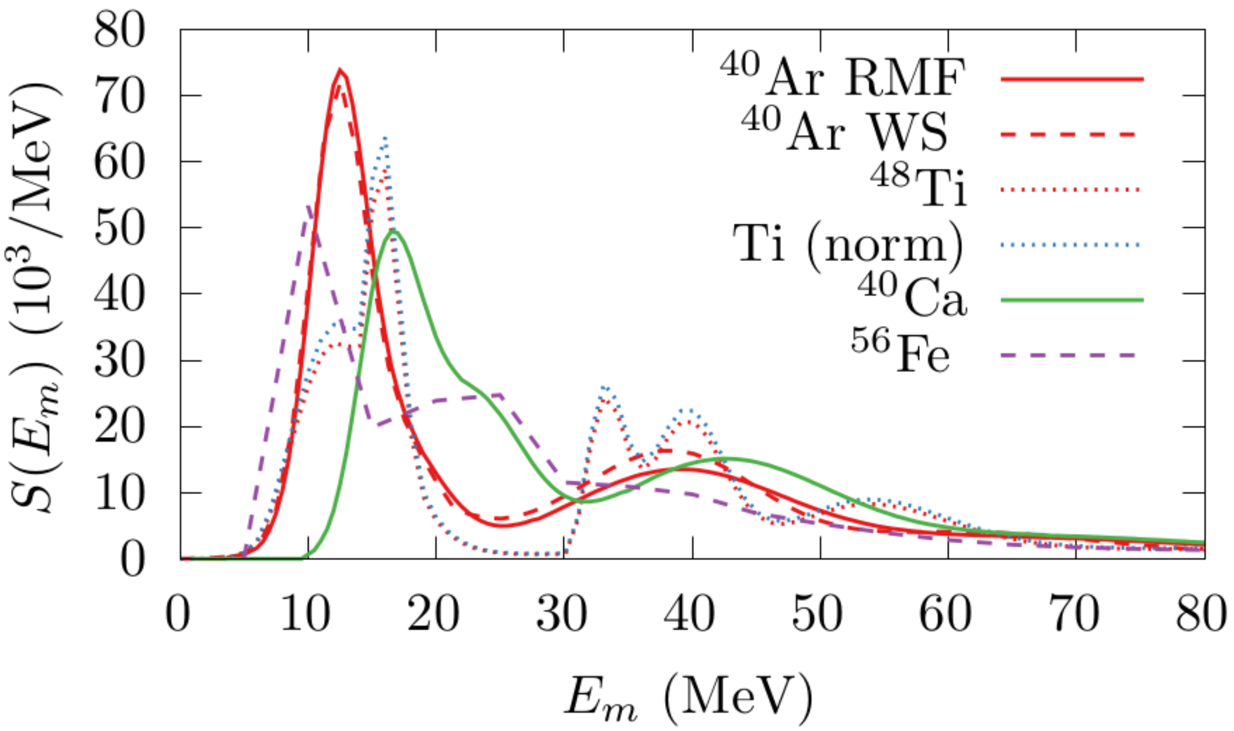
1. **Argon target & MicroBooNE data**
2. **Realistic spectral functions**
3. **Comparison of many INC: ACHILLES, NEUT, NuWro, INCL**
4. **Many kinematic observables**

Kinematic distributions : ACHILLES INC



Flux-folded with MicroBooNE flux

Realistic spectral functions

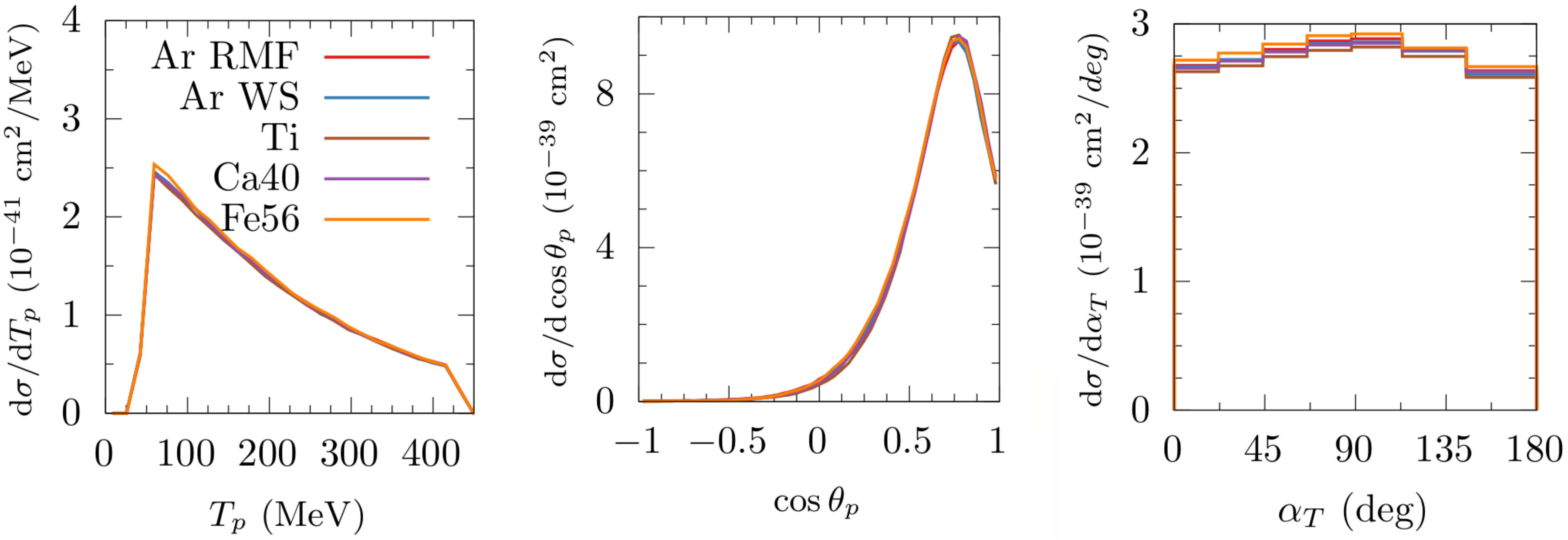


- ^{40}Ar spectral functions
  [Butkevich PRC 85, 065501]
  & [Jlab, PRD 107, 012005]
- ^{48}Ti from Jlab
  [PRD 107, 012005]
- ^{56}Fe
  [Benhar et al. NPA 579, 493]
- ^{40}Ca
  [Butkevich PRC 85, 065501]

Large variation in E_m profiles to check sensitivity of observables

Sensitivity to the spectral function

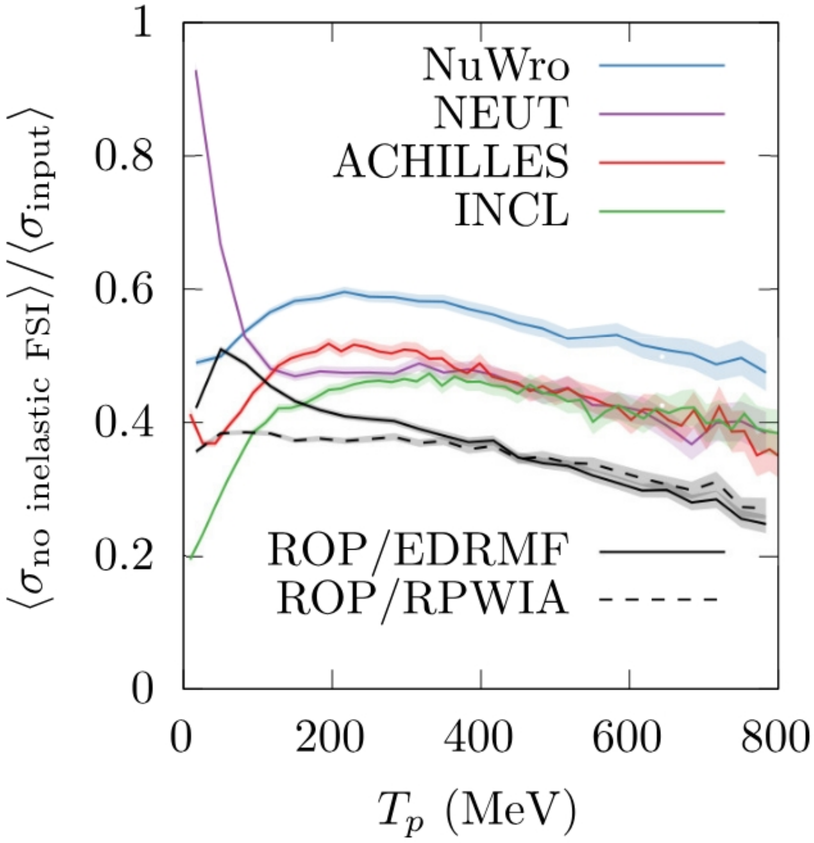
Observables for MicroBooNE flux-averaged signal



Observables that do not correlate p_p and p_μ in flux-averaged data**: no sensitivity**

Find **no sensitivity to missing energy,** only to momentum distribution

Significant variation between different INCs: ‘transparency’

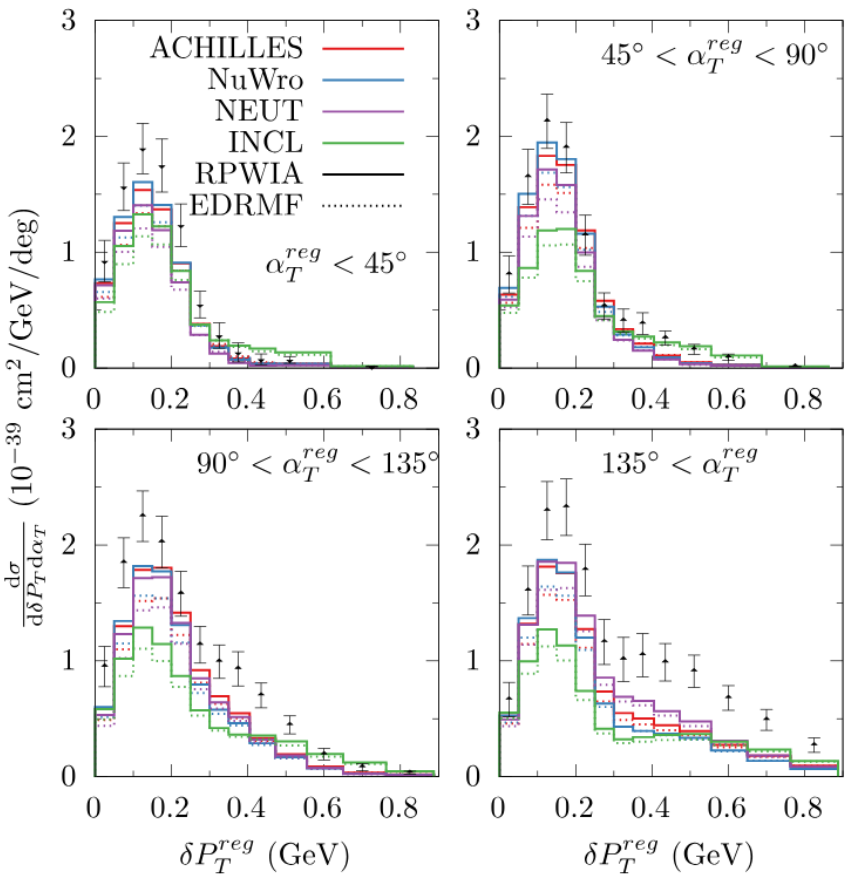


We understand (mostly) the INC variation (you’ll need to read the paper)

Less ‘transparent’:

> The difference in absorption between **ROP and INC** as function of T_p ?

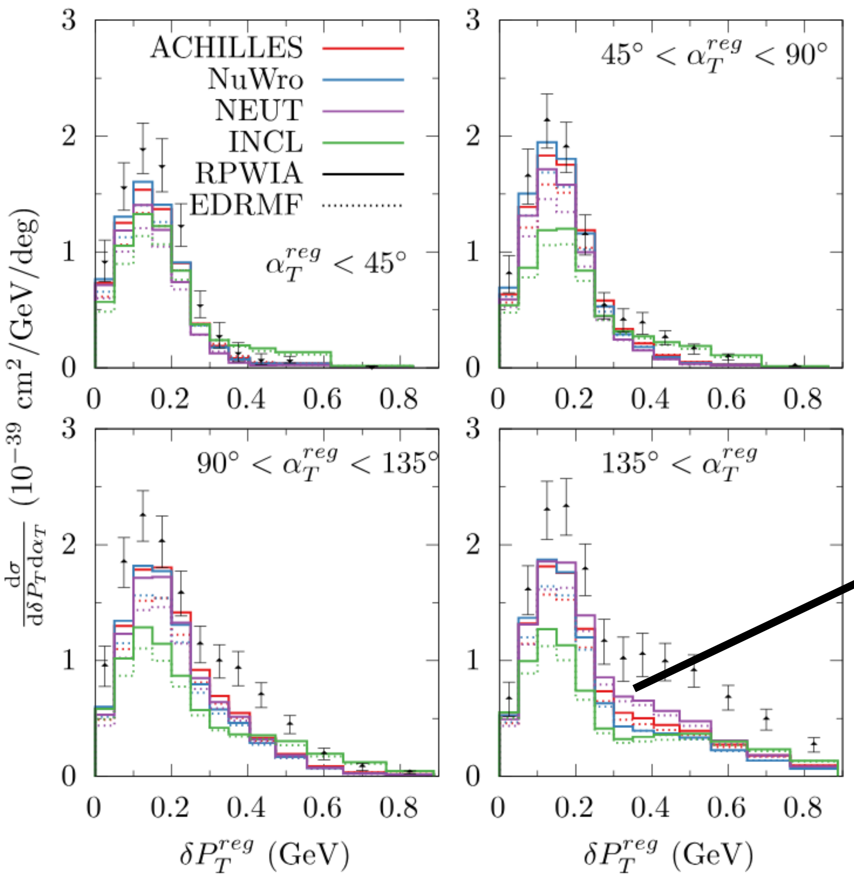
MicroBooNE data



-Difference RPWIA vs RDWIA
~ 10 %

-Variation in INCs
~10 % ACHILLES, NuWro, NEUT
Large difference in INCL

MicroBooNE data



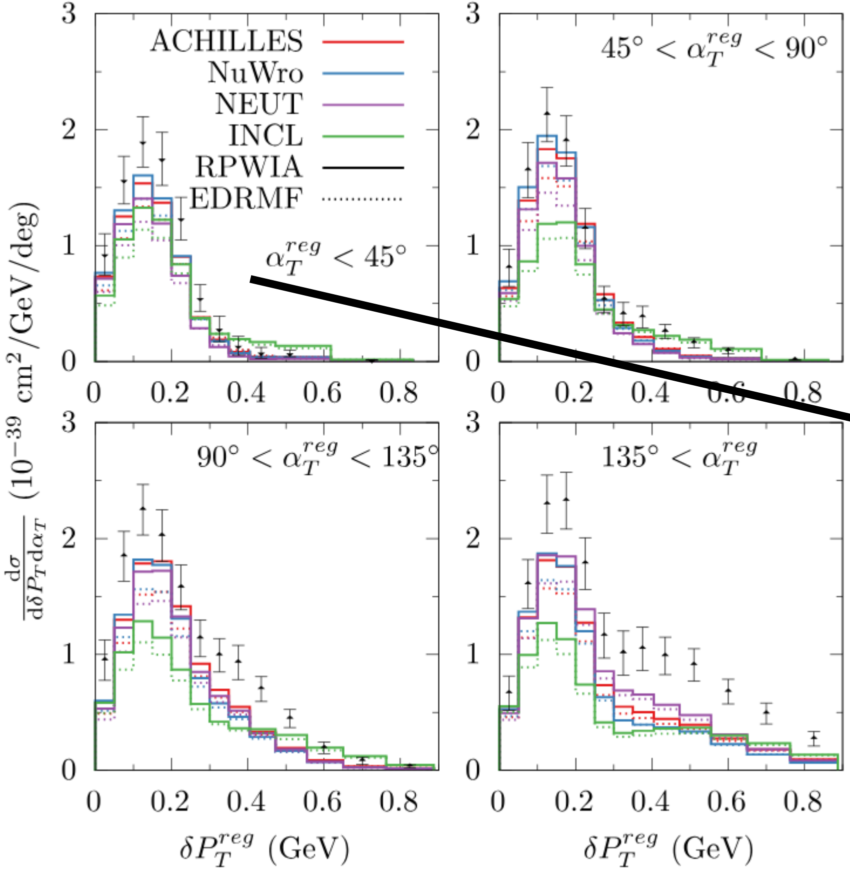
-Difference RPWIA vs RDWIA
~ 10 %

-Variation in INCs
~10 % ACHILLES, NuWro, NEUT
Large difference in INCL

Phase space populated by INC & multi-nucleon knockout

Should be underpredicted

MicroBooNE data



-Difference RPWIA vs RDWIA
~ 10 %

-Variation in INCs
~10 % ACHILLES, NuWro, NEUT
Large difference in INCL

Phase space dominated by
Direct knockout
An increase is needed ?

Interference with 2-body ?
[T Franco-Munoz et al. PRC 108 064608]
[Lovato et al arxiv:2312.12545]

Increase axial form factor ?

Misinterpretation of the data ?

Summary: INCs and all that stuff

- The INC relies on a classical factorized approximation

→ Agreement with exclusive calculations to 0.5th order

→ Unclear **when this approximation is valid** → further study needed

- Can use realistic spectral functions + distorted waves as input

→ **[PhysRevC.110.054611]** and **[PhysRevC.105.054603]**

→ **Recently implemented in NEUT : [J. McKean et al. Arxiv:2502.10629]**

→ **Can use ACHILLES for electron/neutrino scattering studies consistently**

- Comparison to data might need enhancement in 'direct knockout'

→ **Interference 1-2 body ?**

→ **Increase axial form factor ?**

→ **Constraints from (e,e'p) ?**

Constraints from data: What data do we need ?

Focus here on **electron scattering data**

- **Mainz Microtron (MaMI) :** capability for precision experiments
  - → See talk R. Gonzalez-Jimenez : **$(e,e'p\pi^-)$ experiment**
  - → See talk S. Bacca : current and future planned data

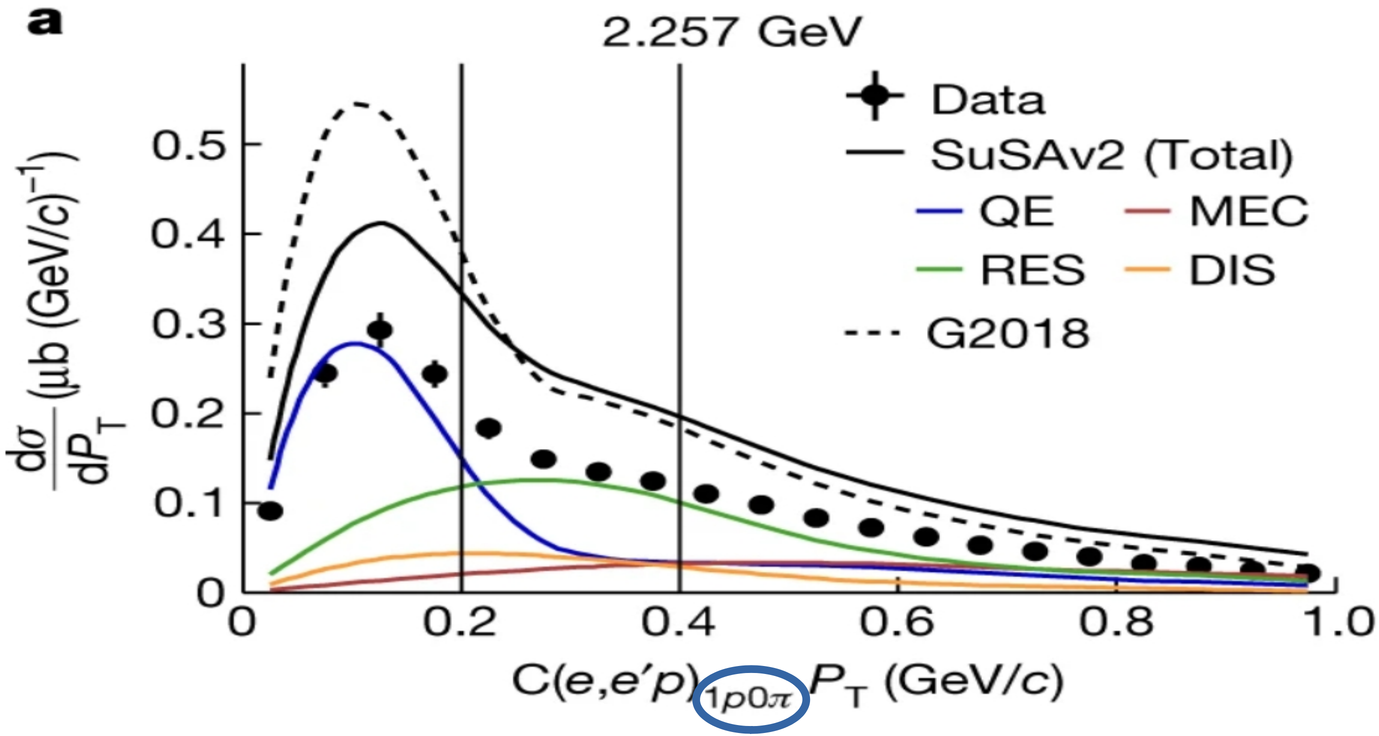
I will discuss **CLAS**

→ Have inclusive & semi-inclusive measurements from **e4nu**

I give my view on how we can learn more from CLAS data

CLAS data: The ‘full’ problem

The data mimic a neutrino experiment



[M. Khachatryan et al. (e4nu) Nature 599, 565 (2021)]

CLAS data: The 'full' problem

The data mimic a neutrino experiment

↓

Interpreting the data is very hard!

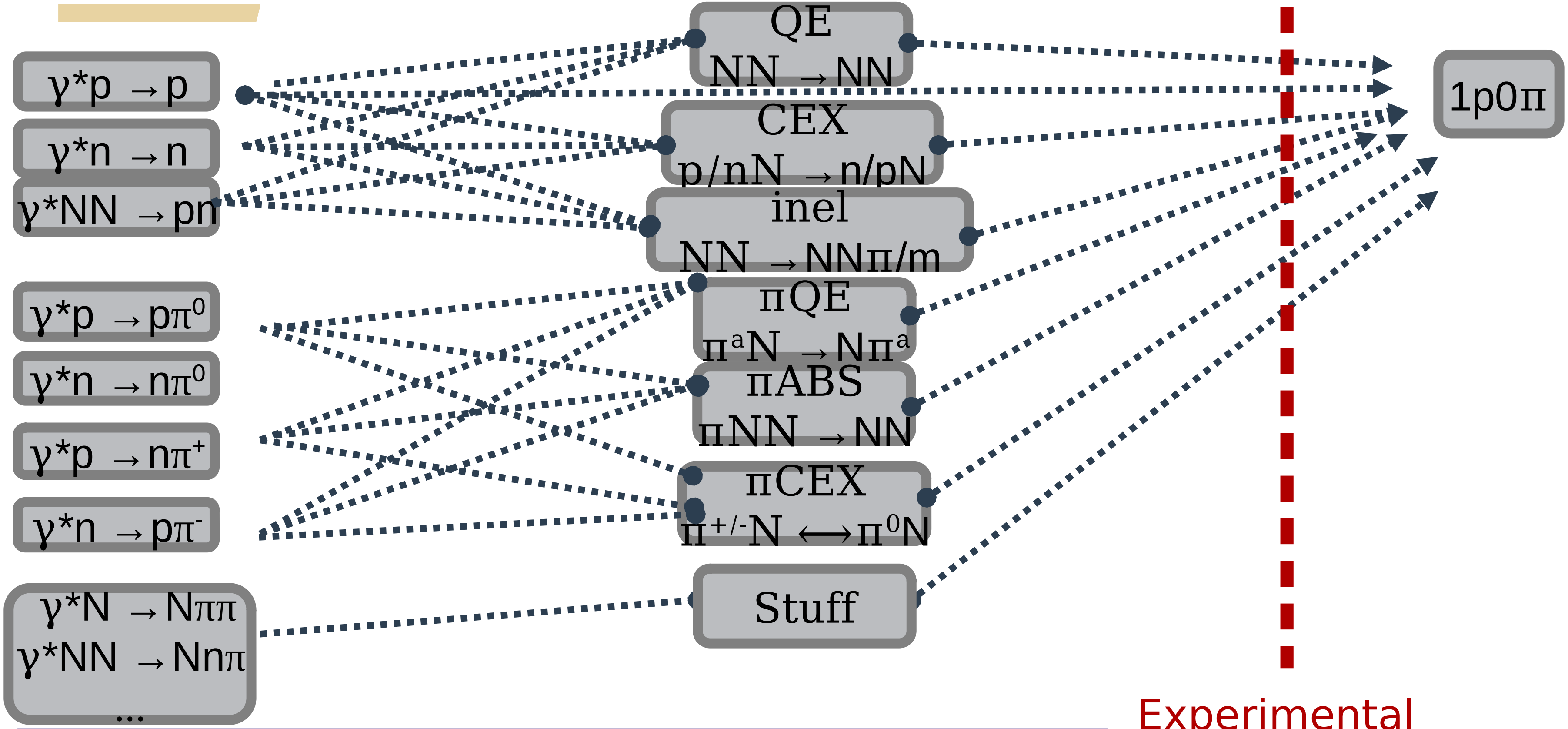
Should use the capabilities of electron
Beams to get more information!

<u>Can do a 0π experiment truly without pions</u>

<u>Can do a true 1p experiment</u>

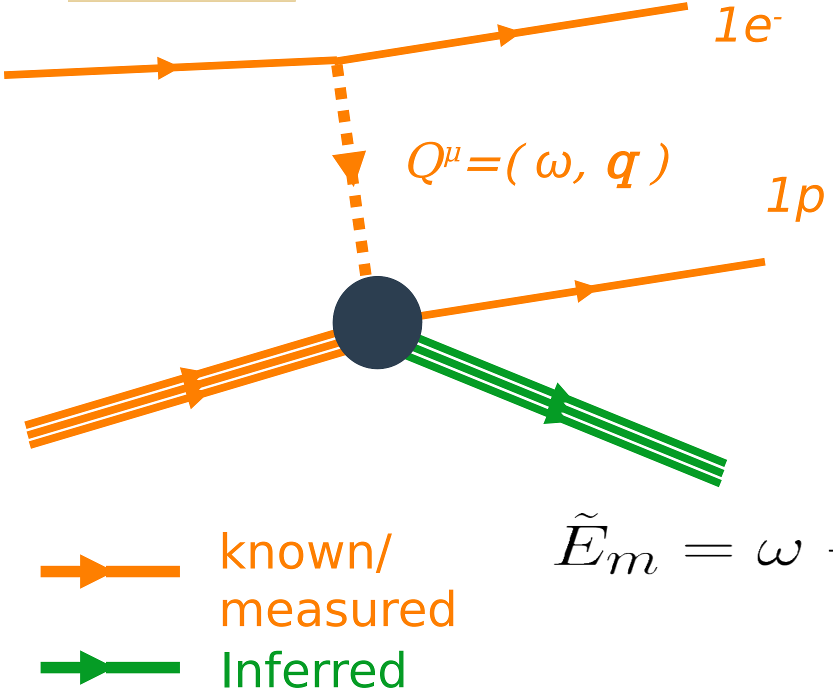
[M. Khachatryan et al. (e4nu) Nature 599, 565 (2021)]

Where does the complexity come from



Need to describe all channels simultaneously!

Controlling missing energy



$$\tilde{E}_m = \omega - T_p$$

$$Q^\mu + P_A^\mu = P_p^\mu + P_B^\mu$$

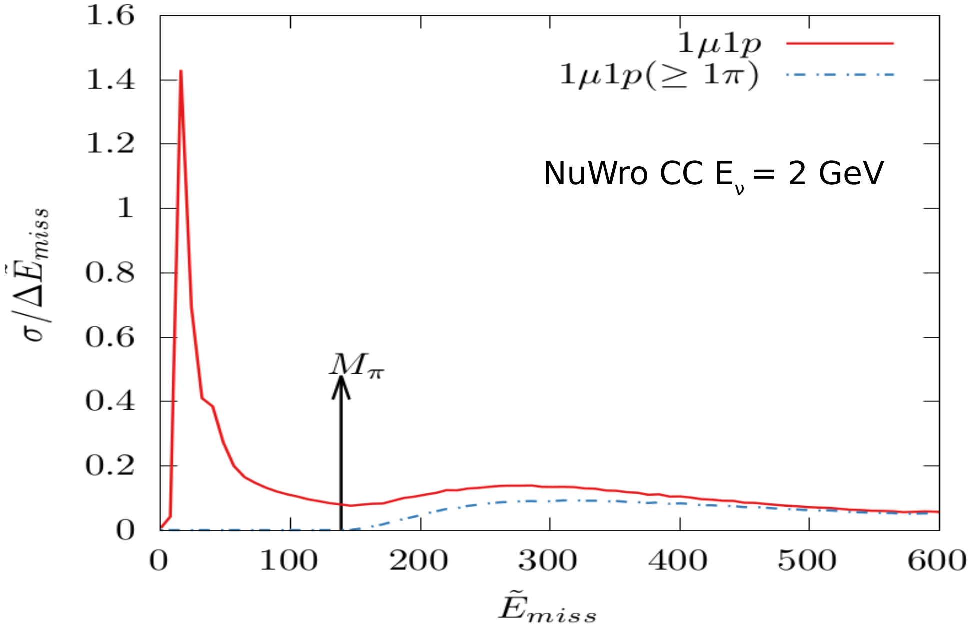
1-proton 1-electron measurement

→ Events in full lepton/proton
Phase space
(like a neutrino experiment)

Missing energy to control the Content of the full final-state
(not like a neutrino experiment!)

Simulation: CLAS kinematics

$$Q^{\mu} + P_A^{\mu} = P_p^{\mu} + P_B^{\mu}$$



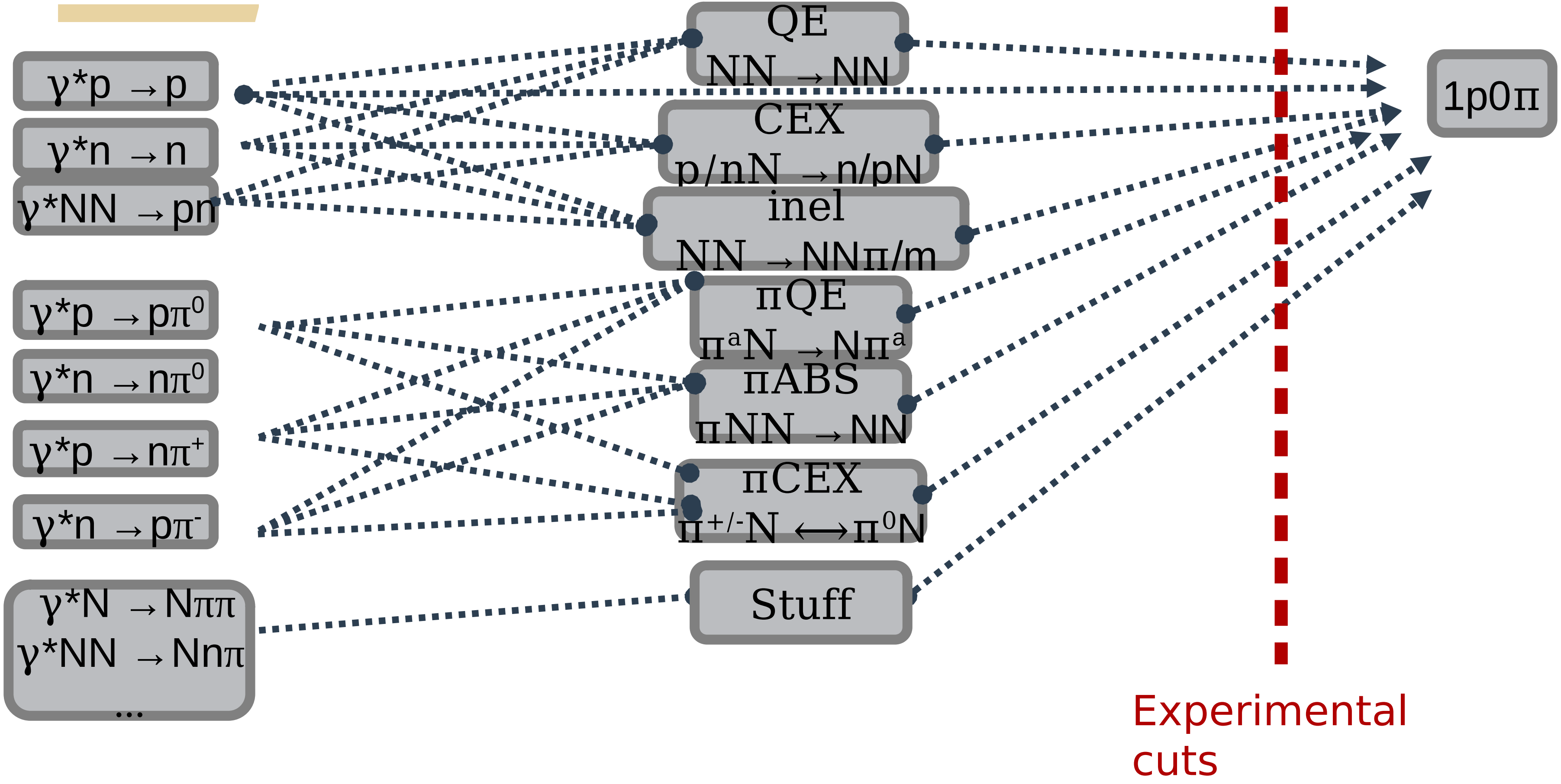
$$\tilde{E}_m = \omega - T_p$$

1-proton 1-electron measurement

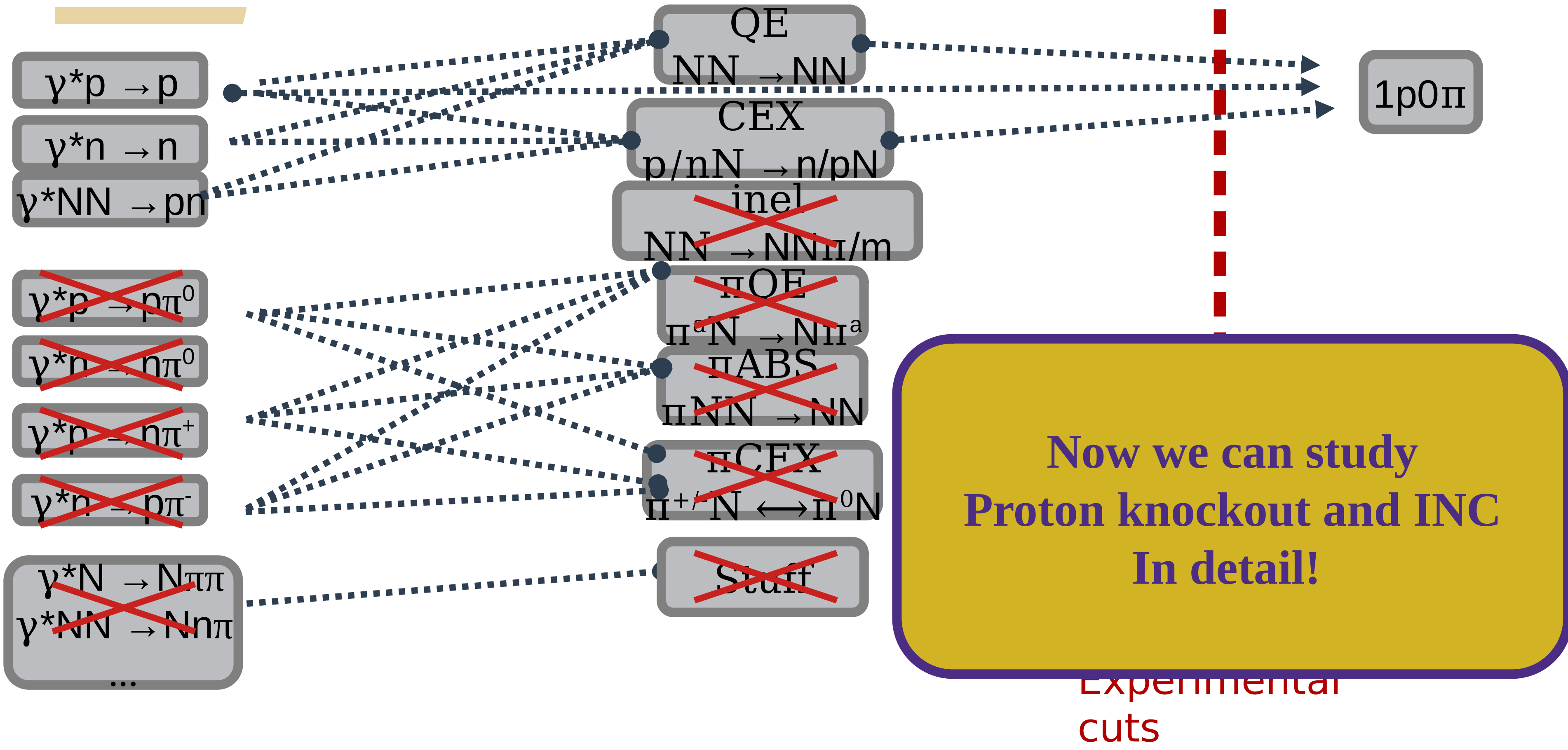
Missing energy to control the Content of the full final-state

Can do a '0π' measurement Without detecting/subtracting pions

The full problem

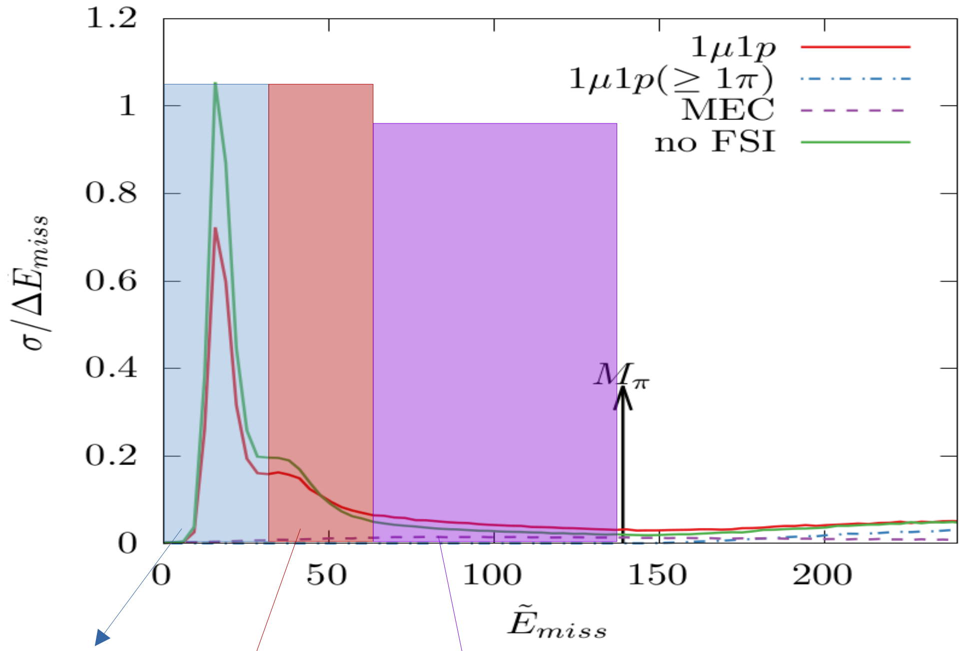


Cuts in missing energy : $E_m < M_\pi$



Now we can study
Proton knockout and INC
In detail!

Setting increasingly difficult constraints

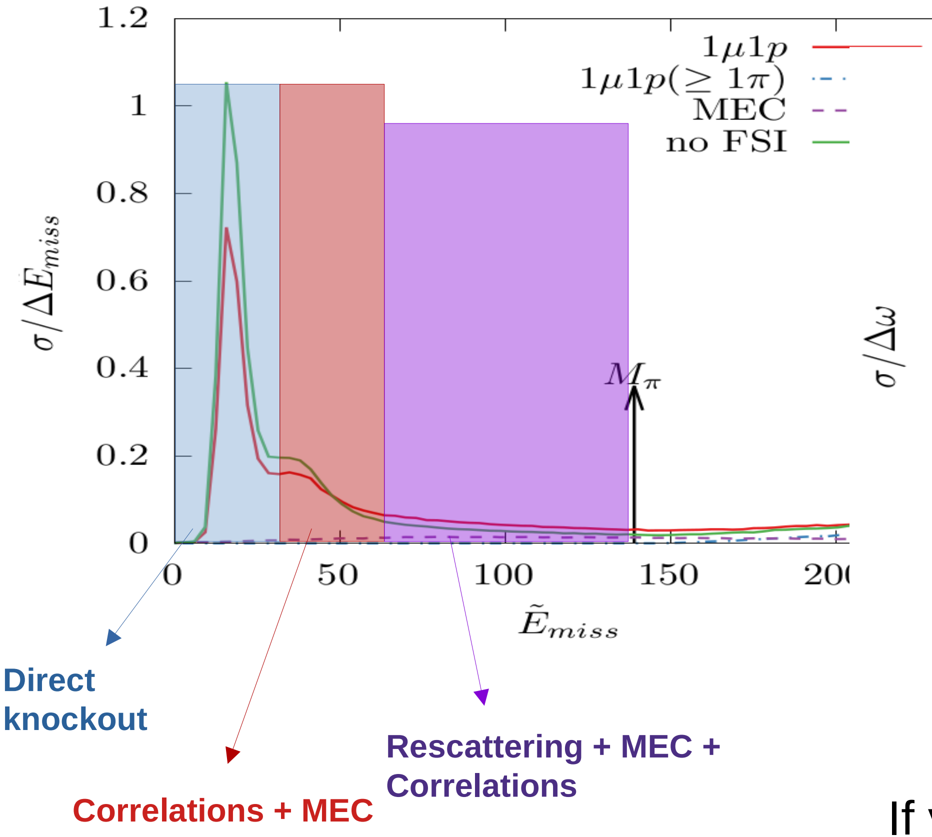


Direct knockout

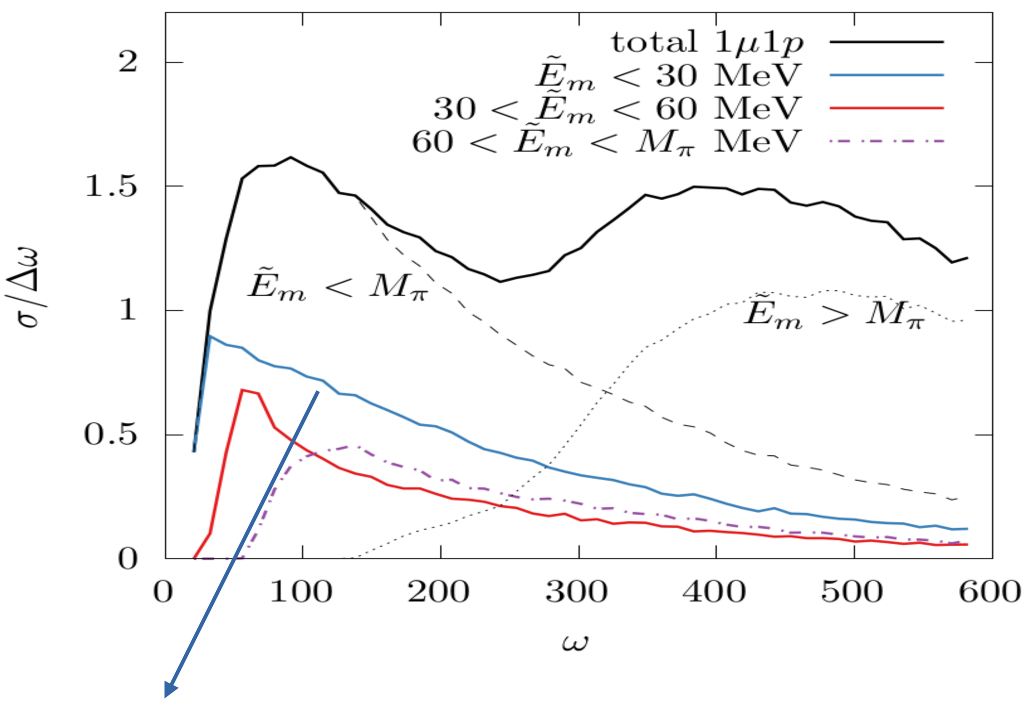
Correlations + MEC

Rescattering + MEC + Correlations

Setting increasingly difficult constraints

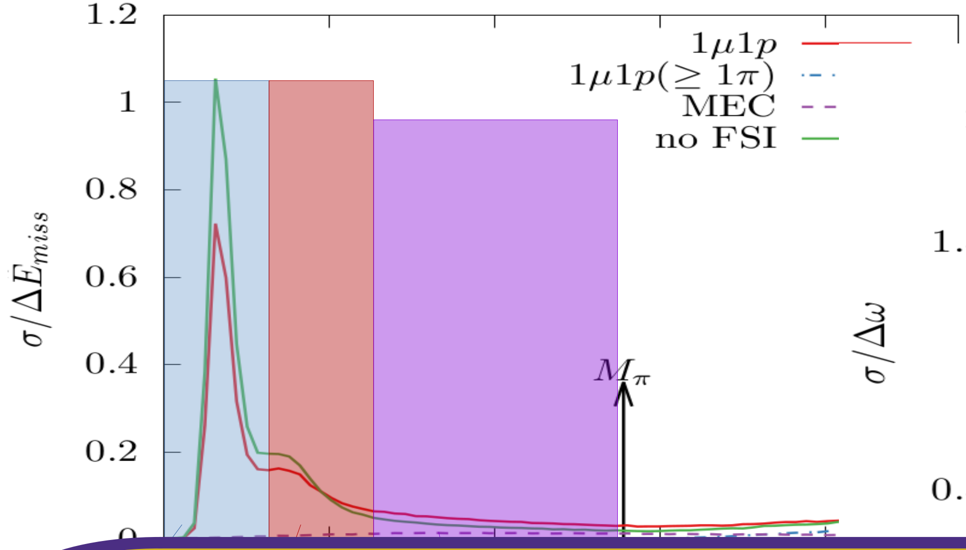


Excitation energy distributions

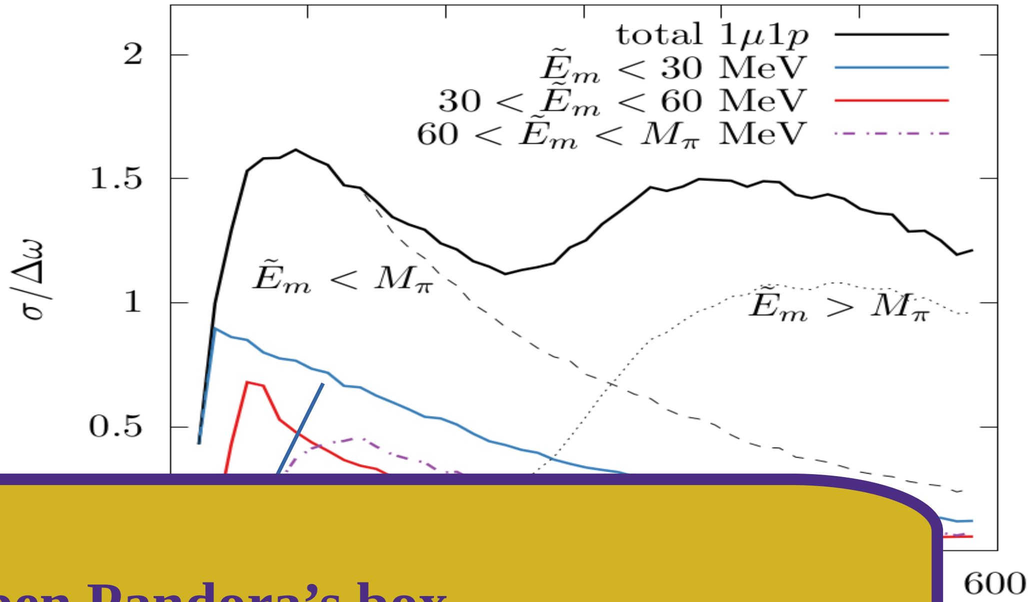


If you can't match **direct knockout,**
No chance for **rescattering**

Setting increasingly difficult constraints



Excitation energy distributions



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We need to open Pandora's box
But maybe step-by-step...

No chance for rescattering