Universidad de Granada

Renormalization of the SMEFT to dimension eight: Fermionic interactions

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(arXiv:2409.15408)

SMEFT-Tools 2025

Motivations

Wilson coefficients

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM}^{(4)} + \sum_{d>4} \sum_{i} \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}$$

Cut-off

The **SMEFT** is (probably) the most reasonable model of new physics

If probed by experiments at very different scales

→ RGEs of the theory are needed

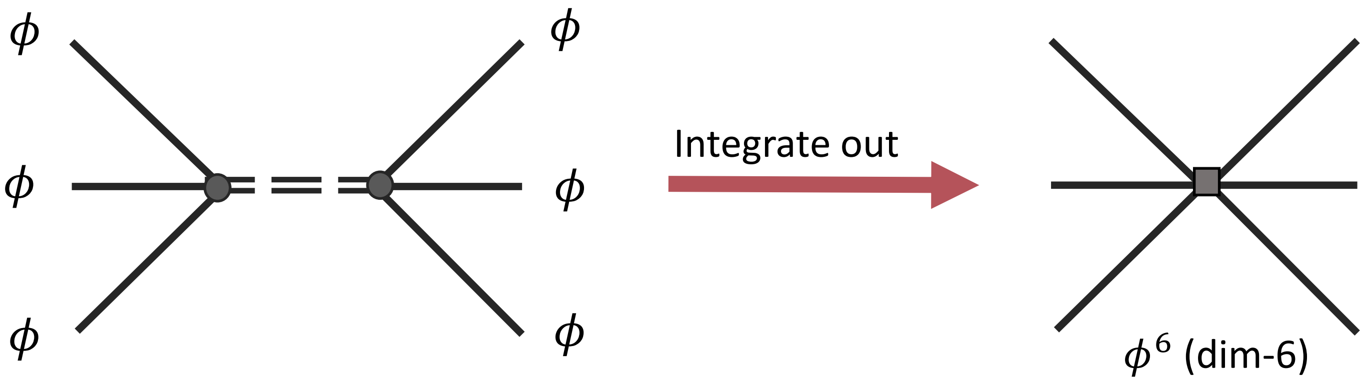
→ Higher accuracy. The dimension-8 SMEFT should be renormalized to the 1-loop level

Interesting theoretical aspects at dim-8 → Positivity bounds, test tools, phenomenological studies …

Motivation

Anomalous dimensions of dim-8 operators not always phenomenologically irrelevant

Custodial quadruplets



$\phi^6 D^2$ (dim-8)

Chala, Krause, Nardini, (2018);

Durieux, McCullough, Salvioni (2022)

Custodial symmetry violation absent at tree-level dim-6, dim-8, and 1-loop dim-6

SMEFT renormalisation status

Some partial results:

- Chala, Guedes, Ramos, Santiago; 2106.05291
- Accettulli Huber, De Angelis; 2108.03669
- Bakshi, Chala, Diaz-Carmona, Guedes; 2205.03301
- Helset, Jenkins, Manohar; 2212.03253
- Asteriadis, Dawson, Fontes; 2212.03258
- Bakshi, Diaz-Carmona; 2301.07151

→ Bosonic sector

More generally, certain aspects of the full anomalous dimension matrix well understood

Craig, Jiang, Li, Sutherland; 2001.00017

SMEFT renormalisation status

	d_5	d_5^2	d_6	d_5^3	$d_5 \times d_6$	d_7	d_5^4	$d_5^2 \times d_6$	d_6^2	$d_5 \times d_7$	d_8
$d_{\leq 4}$ (bosonic)			✓						✓		✓
$d_{\leq 4}$ (fermionic)			✓						X		X
d_5	✓				✓	✓					
d_6 (bosonic)		✓	✓					✓	✓	✓	✓
d_6 (fermionic)		✓	✓					X	X	X	X
d_7				✓	✓	✓					
d_8 (bosonic)							✓	✓	✓	✓	✓
d_8 (fermionic)							X	X	X	X	X

Blank entries vanish; ✓ $\rightarrow$ known; X $\rightarrow$ nothing, or very little, is known.

SMEFT renormalisation status

	d_5	d_5^2	d_6	d_5^3	$d_5 \times d_6$	d_7	d_5^4	$d_5^2 \times d_6$	d_6^2	$d_5 \times d_7$	d_8
$d_{\leq 4}$ (bosonic)			✓						✓		✓
$d_{\leq 4}$ (fermionic)			✓						X		X
d_5	✓				✓	✓					
d_6 (bosonic)		✓	✓					✓	✓	✓	✓
d_6 (fermionic)		✓	✓					X	X	X	X
d_7				✓	✓	✓					
d_8 (bosonic)							✓	✓	✓	✓	✓
d_8 (fermionic)							X	X	X	X	X

Computation of two-fermion RGEs triggered by pairs of dimension-six terms

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \sum_i c_i^{(6)} \mathcal{O}_i^{(6)} + \frac{1}{\Lambda^4} \sum_j c_j^{(8)} \mathcal{O}_j^{(8)}$$

Can appear at tree level in UV completions of the SMEFT

$$\dot{c}_i^{(8)} \equiv 16\pi^2 \tilde{\mu} \frac{dc_i^{(8)}}{d\tilde{\mu}} = \gamma_{ij} c_j^{(8)} + \gamma'_{ijk} c_j^{(6)} c_k^{(6)}$$

Divergences to RGEs, some details:

- Compute **1-PI Loop Diagrams**

  Utilize **FeynRules**, **FeynARTs** and **FormCalc** packages
  Cross-checks with **MatchMakerEFT** arXiv:2112.10787
  - A Carmona, A Lazopoulos, P Olgoso, J Santiago

- Divergences are captured by operators of the **off-shell/Green's basis**

  New basis of dim-8 with two-fermions and two or more Higgs fields
  Explicitly-hermitian version of the one presented in arXiv:2211.01420
  - Z Rhen, J Yu

- Redundant operators removed using **on-shell relations**

  Diagrammatic on-shell matching → arXiv:2307.08745 (SMEFT-Tools 2022)

  arXiv:2411.12798
  - M Chala, J Miras, J. Santiago, F. V.

  See JLM's talk on Thursday

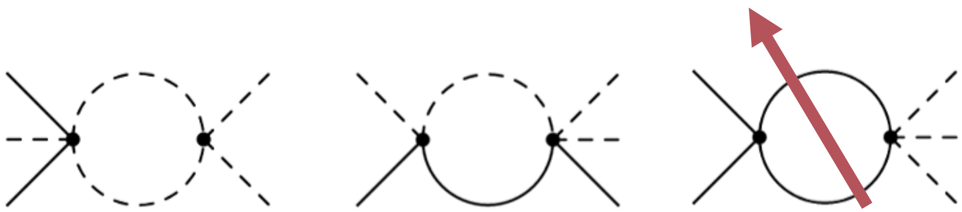
- Subset of our RGEs **cross-validated** with arXiv: 2202.09246
  - M Ardu, S Davidson, M Gorbahn

Detailed example: renormalization of $\mathcal{O}_{e\phi}$

$$\mathcal{O}_{e\phi} = (\bar{l}\phi e)\phi^\dagger\phi$$

Renormalizes *directly* through:

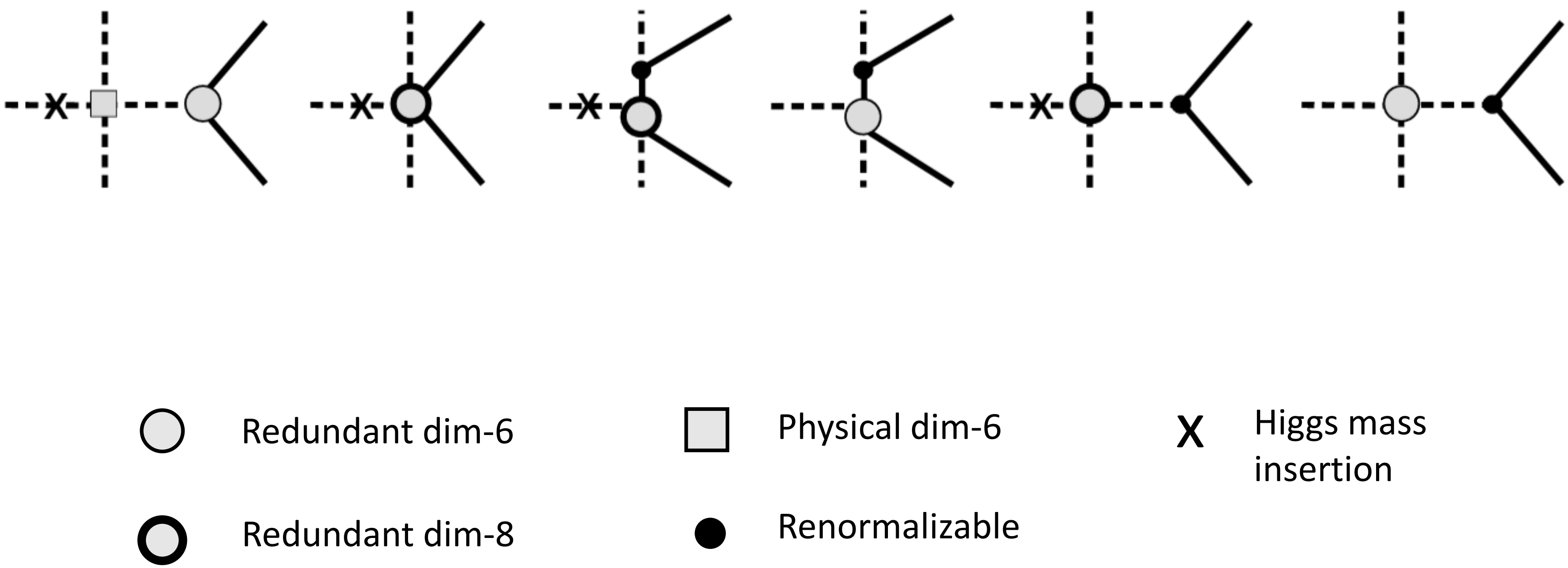
Vanish!! Dimensional analysis



$$(\dot{c}_{e\phi,mn})^{\text{dir}} = -\mu^2 \Big[8(3c_{\phi\Box} - c_{\phi D})c_{e\phi,mn} + 2c_{e\phi,mp}c_{\phi e,pn} - 2(c^{(1)}_{\phi l,mp} + 3c^{(3)}_{\phi l,mp})c_{e\phi,pn}$$
$$- 4c_{\phi D}(c^{(1)}_{\phi l,mp} + c^{(3)}_{\phi l,mp})y^e_{pn} + 4c_{\phi D}y^e_{mp}c_{\phi e,pn} - 4(c^{(1)}_{\phi l,mr} + c^{(3)}_{\phi l,mr})y^e_{rp}c_{\phi e,pn}\Big]$$

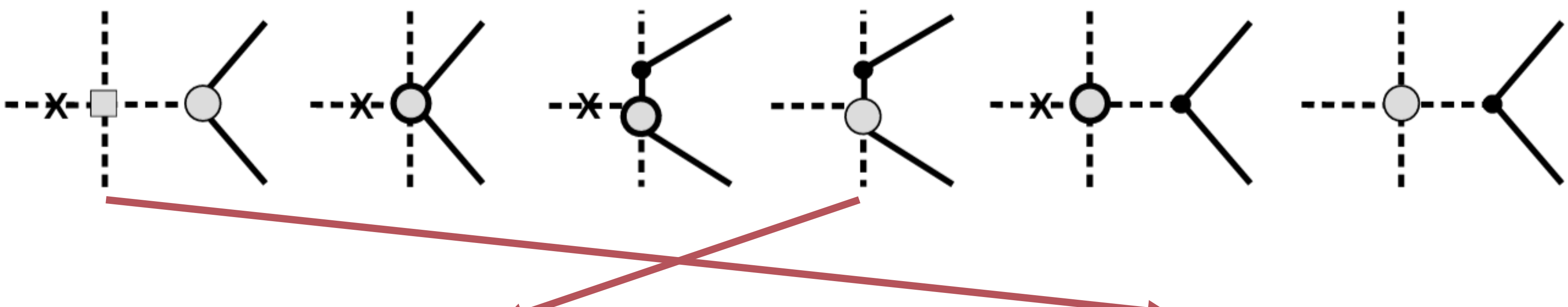
Detailed example: renormalization of $\mathcal{O}_{e\phi}$

Renormalizes *indirectly* upon using the EoM on **redundant** Green's functions



Detailed example: renormalization of $\mathcal{O}_{e\phi}$

Renormalizes *indirectly* upon using the EoM on **redundant** Green's functions

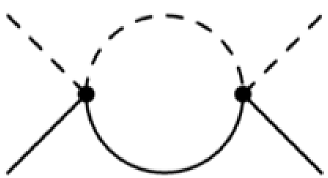


$$
\begin{aligned}
(\dot{c}_{e\phi,mn})^{\text{ind}} = {} & \frac{1}{2} r'_{\phi D} y^e_{mn} + r'_{\phi e,pn} y^e_{mp} + r'^{(1)}_{\phi l,mp} y^e_{pn} + r'^{(3)}_{\phi l,mp} y^e_{pn} - \mu^2 \Big[- 4 c_{\phi\Box} r^{(1)}_{e\phi D,mn} + c_{\phi D} r^{(1)}_{e\phi D,mn} \\
& - 2 c_{\phi\Box} r^{(2)}_{e\phi D,mn} + \frac{1}{2} c_{\phi D} r^{(2)}_{e\phi D,mn} + 2 c_{\phi\Box} r^{(4)}_{e\phi D,mn} - \frac{1}{2} c_{\phi D} r^{(4)}_{e\phi D,mn} + r^{(7)}_{le\phi^3 D^2,mn} \\
& + \mathrm{i} r^{(9)}_{le\phi^3 D^2,mn} + r^{(11)}_{le\phi^3 D^2,mn} - r^{(14)}_{le\phi^3 D^2,mn} + \frac{1}{2} r^{(15)}_{le\phi^3 D^2,mn} + \frac{3}{2} \mathrm{i} r^{(16)}_{le\phi^3 D^2,mn} - r^{(4)}_{\phi^4 D^4} y^e_{mn} \\
& + 2 r^{(8)}_{\phi^4 D^4} y^e_{mn} + r^{(10)}_{\phi^4 D^4} y^e_{mn} + r^{(11)}_{\phi^4 D^4} y^e_{mn} + \frac{1}{2} r^{(33)}_{l^2\phi^2 D^3,mp} y^e_{pn} + \frac{1}{2} r^{(35)}_{l^2\phi^2 D^3,mp} y^e_{pn} \Big]
\end{aligned}
$$

Detailed example: renormalization of $\mathcal{O}_{e\phi}$

$$
\begin{aligned}
(\dot{c}_{e\phi,mn})^{\text{ind}} = {} & \frac{1}{2} r'_{\phi D} y^e_{mn} + r'_{\phi e,pn} y^e_{mp} + r'^{(1)}_{\phi l,mp} y^e_{pn} + r'^{(3)}_{\phi l,mp} y^e_{pn} - \mu^2 \Big[-4 c_{\phi\Box} r^{(1)}_{e\phi D,mn} + c_{\phi D} r^{(1)}_{e\phi D,mn} \\
& - 2 c_{\phi\Box} r^{(2)}_{e\phi D,mn} + \frac{1}{2} c_{\phi D} r^{(2)}_{e\phi D,mn} + 2 c_{\phi\Box} r^{(4)}_{e\phi D,mn} - \frac{1}{2} c_{\phi D} r^{(4)}_{e\phi D,mn} + r^{(7)}_{le\phi^3 D^2,mn} \\
& + \mathrm{i} r^{(9)}_{le\phi^3 D^2,mn} + r^{(11)}_{le\phi^3 D^2,mn} - r^{(14)}_{le\phi^3 D^2,mn} + \frac{1}{2} r^{(15)}_{le\phi^3 D^2,mn} + \frac{3}{2} \mathrm{i} r^{(16)}_{le\phi^3 D^2,mn} - r^{(4)}_{\phi^4 D^4} y^e_{mn} \\
& + 2 r^{(8)}_{\phi^4 D^4} y^e_{mn} + r^{(10)}_{\phi^4 D^4} y^e_{mn} + r^{(11)}_{\phi^4 D^4} y^e_{mn} + \frac{1}{2} r^{(33)}_{l^2\phi^2 D^3,mp} y^e_{pn} + \frac{1}{2} r^{(35)}_{l^2\phi^2 D^3,mp} y^e_{pn} \Big]
\end{aligned}
$$

$\psi^2\phi^2 D$

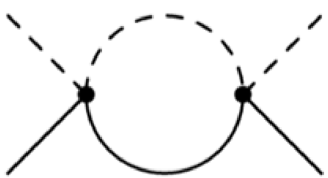


$$r'_{\phi e,mn} = -2\mu^2 c_{\phi e,mp} c_{\phi e,pn}$$

Detailed example: renormalization of $\mathcal{O}_{e\phi}$

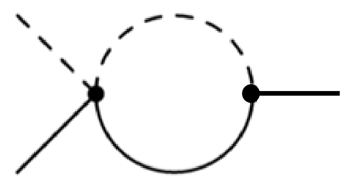
$$
\begin{aligned}
(\dot{c}_{e\phi,mn})^{\text{ind}} = {} & \frac{1}{2} r'_{\phi D} y^e_{mn} + r'_{\phi e,pn} y^e_{mp} + r'^{(1)}_{\phi l,mp} y^e_{pn} + r'^{(3)}_{\phi l,mp} y^e_{pn} - \mu^2 \Big[-4 c_{\phi\Box} r^{(1)}_{e\phi D,mn} + c_{\phi D} r^{(1)}_{e\phi D,mn} \\
& - 2 c_{\phi\Box} r^{(2)}_{e\phi D,mn} + \frac{1}{2} c_{\phi D} r^{(2)}_{e\phi D,mn} + 2 c_{\phi\Box} r^{(4)}_{e\phi D,mn} - \frac{1}{2} c_{\phi D} r^{(4)}_{e\phi D,mn} + r^{(7)}_{le\phi^3 D^2,mn} \\
& + \mathrm{i} r^{(9)}_{le\phi^3 D^2,mn} + r^{(11)}_{le\phi^3 D^2,mn} - r^{(14)}_{le\phi^3 D^2,mn} + \frac{1}{2} r^{(15)}_{le\phi^3 D^2,mn} + \frac{3}{2} \mathrm{i} r^{(16)}_{le\phi^3 D^2,mn} - r^{(4)}_{\phi^4 D^4} y^e_{mn} \\
& + 2 r^{(8)}_{\phi^4 D^4} y^e_{mn} + r^{(10)}_{\phi^4 D^4} y^e_{mn} + r^{(11)}_{\phi^4 D^4} y^e_{mn} + \frac{1}{2} r^{(33)}_{l^2\phi^2 D^3,mp} y^e_{pn} + \frac{1}{2} r^{(35)}_{l^2\phi^2 D^3,mp} y^e_{pn} \Big]
\end{aligned}
$$

$\psi^2\phi^2 D$



$$r'_{\phi e,mn} = -2\mu^2 c_{\phi e,mp} c_{\phi e,pn}$$

$\psi^2 \phi D^2$



$$
\begin{aligned}
r^{(1)}_{e\phi D,mn} = {} & -2 c_{\phi e,pn} y^e_{mp} + 4 c_{le,mprn} y^e_{pr} \\
& - 6 c_{ledq,mnpr} y^d_{rp} + 6 c^{(1)}_{lequ,mnpr} y^{u*}_{pr}
\end{aligned}
$$

Dimension-eight operators

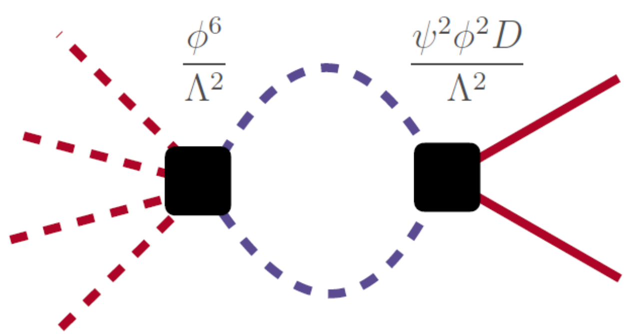
	$\phi^4 D^2$	$\psi^2\phi^2 D$	$\psi^2\phi D^2$	$X\phi^2 D^2$	$\phi^4 D^4$	$\phi^6 D^2$	$\psi^2\phi^2 D^3$	$\psi^2\phi^3 D^2$	$\psi^2\phi^4 D$	$X\psi^2\phi^2 D$	$X\phi^4 D^2$
$\psi^2\phi^2 D$							✓				
$\psi^2\phi^3$	✓	✓	✓		✓		✓	✓			
$\psi^2\phi^2 D^3$							✓				
$\psi^2\phi^3 D^2$			✓		✓		✓	✓			
$\psi^2\phi^4 D$	✓	✓	✓	✓			✓	✓		✓	✓
$\psi^2\phi^5$	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓
$X\psi^2\phi^2 D$							✓			✓	
$X\psi^2\phi^3$							✓	✓		✓	

Table 2. Green's functions (columns) that, on-shell, contribute to the renormalization of the different physical operators (rows) are indicated with ✓. Dimension-six and eight interactions are separated by vertical and horizontal lines.

Dimension-eight operators

	$\phi^4 D^2$	$\psi^2\phi^2 D$	$\psi^2\phi D^2$	$X\phi^2 D^2$	$\phi^4 D^4$	$\phi^6 D^2$	$\psi^2\phi^2 D^3$	$\psi^2\phi^3 D^2$	$\psi^2\phi^4 D$	$X\psi^2\phi^2 D$	$X\phi^4 D^2$
$\psi^2\phi^2 D$							✓				
$\psi^2\phi^3$	✓	✓	✓		✓		✓	✓			
$\psi^2\phi^2 D^3$							✓				
$\psi^2\phi^3 D^2$			✓		✓		✓	✓			
$\psi^2\phi^4 D$	✓	✓	✓	✓			✓	✓		✓	✓
$\psi^2\phi^5$	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓
$X\psi^2\phi^2 D$							✓			✓	
$X\psi^2\phi^3$							✓	✓		✓	

Table 2. Green's functions (columns) that, on-shell, contribute to the renormalization of the different physical operators (rows) are indicated with ✓. Dimension-six and eight interactions are separated by vertical and horizontal lines.



- **Contributes to a redundant operator.**
- **Translates to** $\psi^2\phi^5$

Dimension-eight operators

Classes of tree-generated fermionic operators

- Largest contribution from each operator class is shown
- $\textcolor{blue}{\text{Blue}}$ entries contribute larger than what expected from naive dimensional analysis

$\psi^2\phi^2D^3$	$C_{\phi^4D^2}$	C_{ϕ^6}	$C_{\psi^2\phi^2D}$	$C_{\psi^2\phi^3}$	C_{ψ^4}
$C_{\phi^4D^2}$	0	0	$\ldots$	0	0
C_{ϕ^6}		0	0	0	0
$C_{\psi^2\phi^2D}$			$\ldots$	0	$\textcolor{blue}{\ldots}$
$C_{\psi^2\phi^3}$				0	0
C_{ψ^4}					0

$X\psi^2\phi^2D$	$C_{\phi^4D^2}$	C_{ϕ^6}	$C_{\psi^2\phi^2D}$	$C_{\psi^2\phi^3}$	C_{ψ^4}
$C_{\phi^4D^2}$	0	0	g	0	0
C_{ϕ^6}		0	0	0	0
$C_{\psi^2\phi^2D}$			$\textcolor{blue}{g}$	0	$\textcolor{blue}{g}$
$C_{\psi^2\phi^3}$				0	0
C_{ψ^4}					0

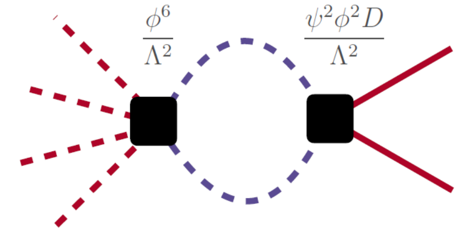
$\psi^2\phi^3D^2$	$C_{\phi^4D^2}$	C_{ϕ^6}	$C_{\psi^2\phi^2D}$	$C_{\psi^2\phi^3}$	C_{ψ^4}
$C_{\phi^4D^2}$	$\textcolor{blue}{y}$	0	$\textcolor{blue}{y}$	$\textcolor{blue}{\ldots}$	$\textcolor{blue}{y}$
C_{ϕ^6}		0	0	0	0
$C_{\psi^2\phi^2D}$			$\textcolor{blue}{y}$	$\ldots$	$\textcolor{blue}{y}$
$C_{\psi^2\phi^3}$				0	$\textcolor{blue}{\ldots}$
C_{ψ^4}					0

$X\psi^2\phi^3$	$C_{\phi^4D^2}$	C_{ϕ^6}	$C_{\psi^2\phi^2D}$	$C_{\psi^2\phi^3}$	C_{ψ^4}
$C_{\phi^4D^2}$	0	0	gy	0	0
C_{ϕ^6}		0	0	0	0
$C_{\psi^2\phi^2D}$			gy	g	0
$C_{\psi^2\phi^3}$				0	0
C_{ψ^4}					0

$\psi^2\phi^4D$	$C_{\phi^4D^2}$	C_{ϕ^6}	$C_{\psi^2\phi^2D}$	$C_{\psi^2\phi^3}$	C_{ψ^4}
$C_{\phi^4D^2}$	y^2	0	$\textcolor{blue}{y^2}$	y	g^2
C_{ϕ^6}		0	0	0	0
$C_{\psi^2\phi^2D}$			$\textcolor{blue}{y^2}$	$\textcolor{blue}{y}$	$\textcolor{blue}{y^2}$
$C_{\psi^2\phi^3}$				$\ldots$	$\textcolor{blue}{y^2}$
C_{ψ^4}					0

$\psi^2\phi^5$	$C_{\phi^4D^2}$	C_{ϕ^6}	$C_{\psi^2\phi^2D}$	$C_{\psi^2\phi^3}$	C_{ψ^4}
$C_{\phi^4D^2}$	$\textcolor{blue}{y^3}$	$\textcolor{blue}{y}$	$\textcolor{blue}{y^3}$	$\textcolor{blue}{y^2}$	$\textcolor{blue}{y\lambda}$
C_{ϕ^6}		0	$\textcolor{blue}{y}$	$\textcolor{blue}{\ldots}$	$\textcolor{blue}{y}$
$C_{\psi^2\phi^2D}$			y^3	$\textcolor{blue}{y^2}$	0
$C_{\psi^2\phi^3}$				$\textcolor{blue}{y}$	y^2
C_{ψ^4}					0

Dimension-eight operators



Classes of tree-generated fermionic operators

- Largest contribution from each operator class is shown
- Blue entries contribute larger than what expected from naive dimensional analysis

$\psi^2\phi^2D^3$	$C_{\phi^4D^2}$	C_{ϕ^6}	$C_{\psi^2\phi^2D}$	$C_{\psi^2\phi^3}$	C_{ψ^4}
$C_{\phi^4D^2}$	0	0	$\dots$	0	0
C_{ϕ^6}		0	0	0	0
$C_{\psi^2\phi^2D}$			$\dots$	0	$\dots$
$C_{\psi^2\phi^3}$				0	0
C_{ψ^4}					0

$X\psi^2\phi^2D$	$C_{\phi^4D^2}$	C_{ϕ^6}	$C_{\psi^2\phi^2D}$	$C_{\psi^2\phi^3}$	C_{ψ^4}
$C_{\phi^4D^2}$	0	0	g	0	0
C_{ϕ^6}		0	0	0	0
$C_{\psi^2\phi^2D}$			g	0	g
$C_{\psi^2\phi^3}$				0	0
C_{ψ^4}					0

$\psi^2\phi^3D^2$	$C_{\phi^4D^2}$	C_{ϕ^6}	$C_{\psi^2\phi^2D}$	$C_{\psi^2\phi^3}$	C_{ψ^4}
$C_{\phi^4D^2}$	y	0	y	$\dots$	y
C_{ϕ^6}		0	0	0	0
$C_{\psi^2\phi^2D}$			y	$\dots$	y
$C_{\psi^2\phi^3}$				0	$\dots$
C_{ψ^4}					0

$X\psi^2\phi^3$	$C_{\phi^4D^2}$	C_{ϕ^6}	$C_{\psi^2\phi^2D}$	$C_{\psi^2\phi^3}$	C_{ψ^4}
$C_{\phi^4D^2}$	0	0	gy	0	0
C_{ϕ^6}		0	0	0	0
$C_{\psi^2\phi^2D}$			gy	g	0
$C_{\psi^2\phi^3}$				0	0
C_{ψ^4}					0

$\psi^2\phi^4D$	$C_{\phi^4D^2}$	C_{ϕ^6}	$C_{\psi^2\phi^2D}$	$C_{\psi^2\phi^3}$	C_{ψ^4}
$C_{\phi^4D^2}$	y^2	0	y^2	y	g^2
C_{ϕ^6}		0	0	0	0
$C_{\psi^2\phi^2D}$			y^2	y	y^2
$C_{\psi^2\phi^3}$				$\dots$	y^2
C_{ψ^4}					0

$\psi^2\phi^5$	$C_{\phi^4D^2}$	C_{ϕ^6}	$C_{\psi^2\phi^2D}$	$C_{\psi^2\phi^3}$	C_{ψ^4}
$C_{\phi^4D^2}$	y^3	y	y^3	y^2	$y\lambda$
C_{ϕ^6}		0	y	$\dots$	y
$C_{\psi^2\phi^2D}$			y^3	y^2	0
$C_{\psi^2\phi^3}$				y	y^2
C_{ψ^4}					0

Summary and Outlook

- We have computed the **one-loop RGEs** of the **two-fermion operators** of the SMEFT to $\mathcal{O}(1/\Lambda^4)$ triggered by **pairs of dimension-6 terms**

  These corrections do not spoil the validity of tree-level positivity bounds

  Full result in a Mathematica notebook at: https://github.com/SMEFT-Dimension8-RGEs

- **Future directions**

  Renormalization of two-fermion dim-8 operators by loops involving single dim-8 terms

  Completion of the running for four-fermion interactions

- **Applications**

  Improved description of **universality breaking** within SMEFT

  Analysis of deviations from different **flavour assumptions** due to quantum corrections

  Impact on **electroweak precision parameters**

  Results enable **higher precision** confrontation of SMEFT with experimental data

THANK YOU VERY MUCH!!!

- **Restrictions** on Wilson coefficients using unitarity, analyticity, and crossing symmetry

From M. Chala, X. Li (2309.16611) :

One flavour limit

$$\dot{c}^{(1)}_{e^2\phi^2 D^3} + \dot{c}^{(2)}_{e^2\phi^2 D^3} \geq 0$$

$$\dot{c}^{(1)}_{l^2\phi^2 D^3} + \dot{c}^{(2)}_{l^2\phi^2 D^3} + \dot{c}^{(3)}_{l^2\phi^2 D^3} + \dot{c}^{(4)}_{l^2\phi^2 D^3} \geq 0$$

$$\dot{c}^{(1)}_{l^2\phi^2 D^3} + \dot{c}^{(2)}_{l^2\phi^2 D^3} - \dot{c}^{(3)}_{l^2\phi^2 D^3} - \dot{c}^{(4)}_{l^2\phi^2 D^3} \geq 0$$

In our RGEs :

$$\dot{c}^{(1)}_{e^2\phi^2 D^3} + \dot{c}^{(2)}_{e^2\phi^2 D^3} = 4c^2_{\phi e}\,,$$

$$\dot{c}^{(1)}_{l^2\phi^2 D^3} + \dot{c}^{(2)}_{l^2\phi^2 D^3} + \dot{c}^{(3)}_{l^2\phi^2 D^3} + \dot{c}^{(4)}_{l^2\phi^2 D^3} = 4(c^{(1)}_{\phi l} + c^{(3)}_{\phi l})^2 + 8(c^{(3)}_{\phi l})^2\,,$$

$$\dot{c}^{(1)}_{l^2\phi^2 D^3} + \dot{c}^{(2)}_{l^2\phi^2 D^3} - \dot{c}^{(3)}_{l^2\phi^2 D^3} - \dot{c}^{(4)}_{l^2\phi^2 D^3} = 4(c^{(1)}_{\phi l} - c^{(3)}_{\phi l})^2 + 8(c^{(3)}_{\phi l})^2\,.$$