The Good, the Bad, and the Evanescent

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SMEFT-Tools 2025
MITP

The Good

The Bad

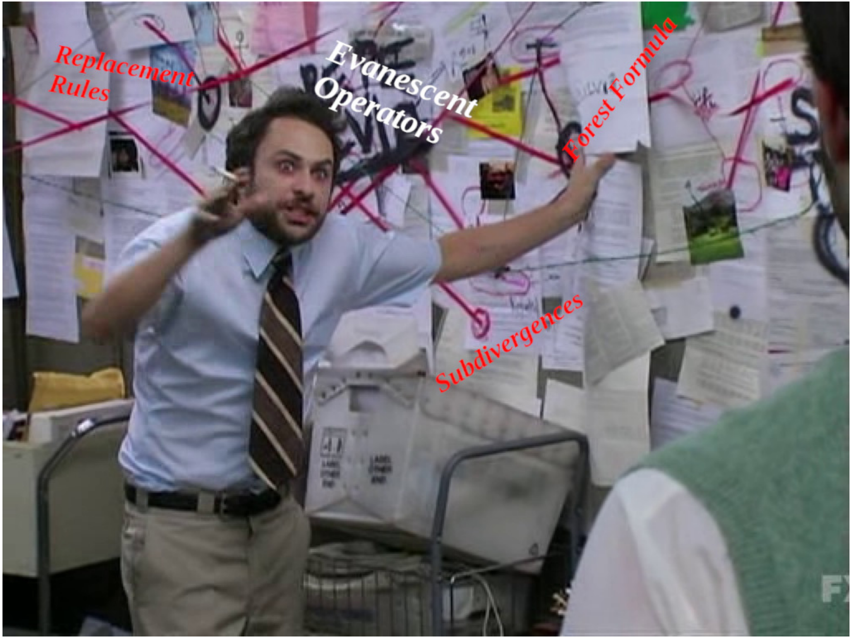
The Good

The Bad

’t Hooft-Veltmann/BMHV

The Good | **The Bad**

't Hooft-Veltmann/BMHV | Everything Else


Replacement Rules
Evanescent Operators
Forest Formula
Subdivergences

The Good

’t Hooft-Veltmann/BMHV

The Bad

Everything Else

The Good	**The Bad**
't Hooft-Veltmann/BMHV	Everything Else

Evanescent Operators

Evanescent Operators: The Standard Story

[Dugan, Grinstein; Phys.Lett.B 256 (1991) 239-244]
[Collins, *Renormalization*]
[Herrlich, Nierste; 9412375]
[Fuentes-Martìn, et al.; 2211.09144]...

- Purely four-dimensional objects (γ_5, $\epsilon^{\mu\nu\sigma\rho}$) are not well-defined in dim-reg

- Operators which coincide in $d = 4$ are no longer linearly dependent

  $\rightarrow$ Must introduce **evanscent operators**

$$E = \left(\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}P_L\right) \otimes \left(\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}P_L\right) - (16 - a_{11}\epsilon - \dots)\left(\gamma^{\mu}P_L\right) \otimes \left(\gamma_{\mu}P_L\right)$$

- Rank(ϵ): give finite (local) effects when multiplied by UV poles

- Infinitely many $\Rightarrow$ explicitly subtract finite Z_{EQ} to avoid initial conditions

The $b \to se^+e^-$ and $b \to s\gamma$ decays with next-to-leading logarithmic QCD-corrections

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What we usually require from an arbitrary regularization is that:

(i) It commutes with differentiation with respect to external momenta.

(ii) It does not affect convergent integrals.

(iii) It gives unique results independently on whether some diagram is considered separately or as a subdiagram, and independently on the order in which the subdiagrams are calculated.

What we would like to suggest here is forbidding the use of $\{\gamma_\mu, \gamma_5\} = 0$ in fermionic lines containing odd numbers of γ_5's, and not to evaluate $\mathrm{Tr}[\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5]$ at all in $d \neq 4$ dimensions. This brings to life new evanescent operators containing structures like

$$\gamma_{\mu_1}\gamma_{\mu_2}\ldots\gamma_{\mu_n}\{\gamma_\nu, \gamma_5\} \tag{A.2}$$

or

$$\gamma^\mu \mathrm{Tr}[\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5] - 4(\gamma_\nu\gamma_\rho\gamma_\sigma - g_{\nu\rho}\gamma_\sigma + g_{\nu\sigma}\gamma_\rho - g_{\rho\sigma}\gamma_\nu)\gamma_5, \tag{A.3}$$

Forest Formula (Simplified)

(See talks by Lukas and Achilleas earlier today)

- In generic graph, generate both local and non-local UV divergences
- Non-local divergences arise only from divergent sub-diagrams

  $\Rightarrow$ subtracted by lower-order CT insertions
- Remaining **local** UV divergences can be renormalized

An Inconvenient, but Transparent Approach

- Never explicitly reduce algebraic structures appearing in loop graphs

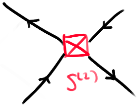
  $\rightarrow$ Add new structures to Green's basis

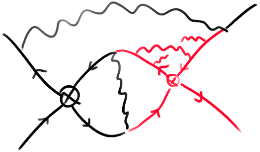
- New structures generated in divergent ℓ-loop graphs appear as sub-structures in $\ell + n$-loop graphs

  $\rightarrow$ Directly re-inserted as counterterms to cancel subdivergences

Algebraic Structure

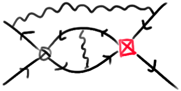
$\longrightarrow (\text{div.}) \times S^{(2)} \Longrightarrow$





$\longrightarrow$ Contains non-local divergence

$(\text{div}) \times F(p^2)\, S^{(n)}, \quad S^{(2)} \in S^{(n)}$

$+$ 

$\longrightarrow$ Exactly cancels non-local div

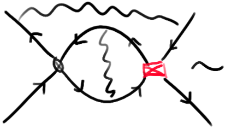
A Less Inconvenient Approach

- We would like to specify method of reducing structures back to physical basis

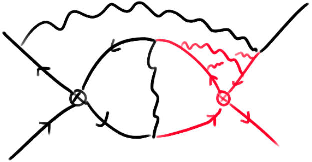
$$S_i^{(\ell)} \to M_{ij}^{(\ell)} Q_j$$

- ℓ-loop counterterms no longer necessarily cancel $(\ell + n)$-loop subdivergences!

$$\longrightarrow (\text{div.}) \times \left(M^{(2)} Q\right)$$



$$\sim M^{(2)} \left(M^{(3)} Q\right)$$



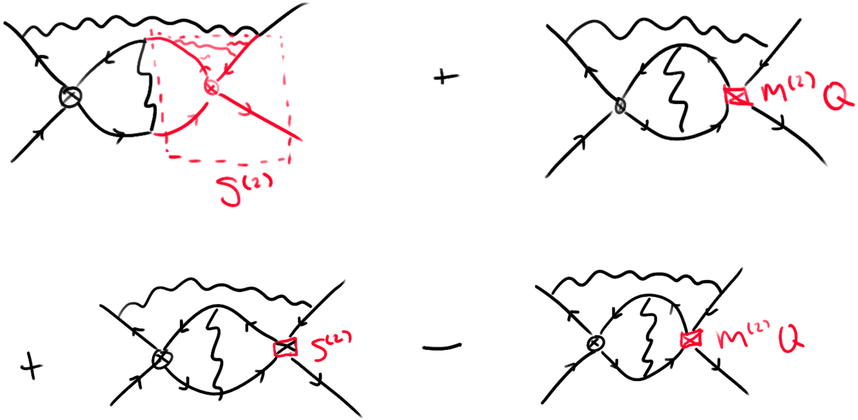
$$\longrightarrow (\text{div}) \times F(p^2) \left(M^{(5)} Q\right)$$

A Less Inconvenient Approach

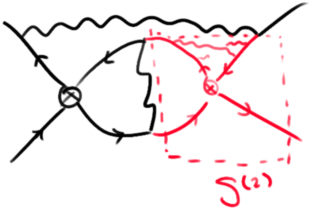
- Fix issue by hand: remove bad reductions and re-insert proper structure

$$(\text{divergent coefficient}) \times (S_i^{(\ell)} - M_{ij}^{(\ell)} Q_j)$$

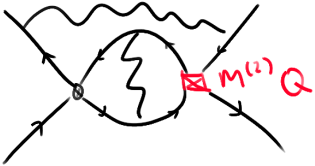
- Exactly an insertion of $Z_{QE} \times E$



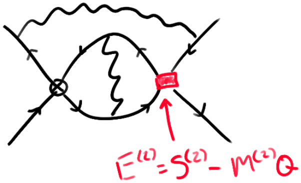
$S^{(2)}$
+
$M^{(2)}Q$
+
$S^{(2)}$
−
$M^{(2)}Q$


$S^{(2)}$

+


$M^{(2)}Q$

+


$E^{(2)} = S^{(2)} - M^{(2)}Q$

A Less Inconvenient Approach

- Fix issue by hand: remove bad reductions and re-insert proper structure

$$(\text{divergent coefficient}) \times (S_i^{(\ell)} - M_{ij}^{(\ell)} Q_j)$$

- Exactly an insertion of $Z_{QE} \times E$

- **Evanescent operators guarantee the cancellation of subdivergences when inconsistencies arise in algebraic reductions**

The $b \to se^+e^-$ and $b \to s\gamma$ decays with next-to-leading logarithmic QCD-corrections

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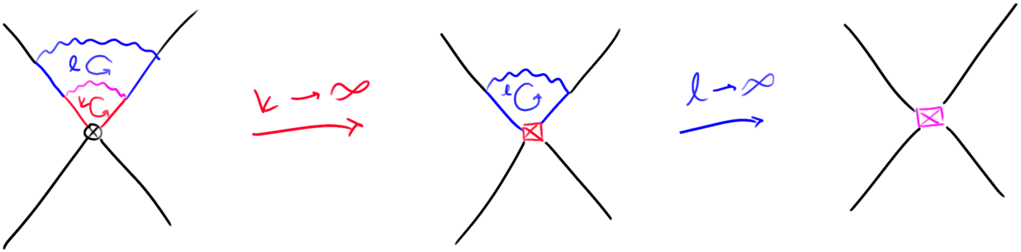
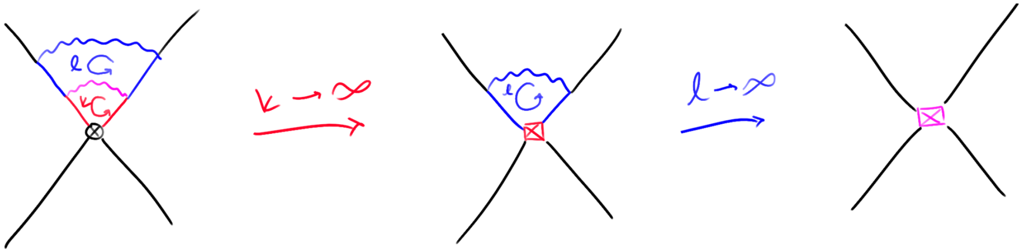
Evanescent operators don't *cause* violations of (iii), but *resolve* them

Example I: Nested Subdivergences



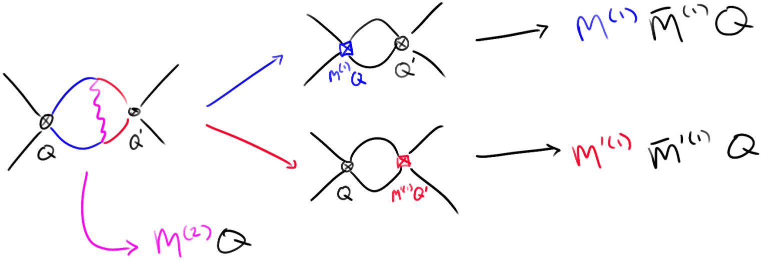
Example I: Nested Subdivergences



- Clear ordering of structure reductions $\Rightarrow$ can eliminate evanescent op.

  (see Marko's talk for more details)

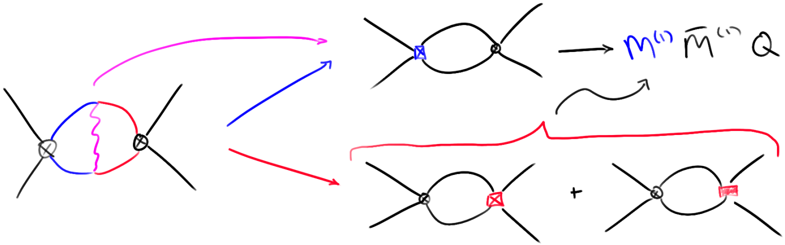
Example II: Overlapping Subdivergences



- Can eliminate evanescent operators by relating $O(\epsilon^n)$ parts of reductions

  $\rightarrow$ Linear algebra problem: enough degrees of freedom?

Example II: Overlapping Subdivergences



- Commit to one reduction

  $\rightarrow$ only need introduce evanescent for other reductions

Conservation of Trouble

- Generic reductions do not preserve gauge invariance [Jegerlehner; 0005255]
- Can't resolve inconsistencies if no evanescent operators introduced

  $\Rightarrow$ Traces in NDR still can be problematic if not treated with care
- Avoid initial Dirac reductions: worse tensor integal reductions

Summary

- Evanescent ops. understood as ensuring cancellation of subdivergences when using inconsistent reductions of algebraic structures
- Smart choices of reductions can greatly reduce the number of evanescent operators/effects
- **Not magic:** Not eliminating all evanescents, and possible issues still arise
- Allows greater flexibility for automation of higher-order RGEs