Two-loop running in the SMEFT

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UNIVERSITÄT BERN

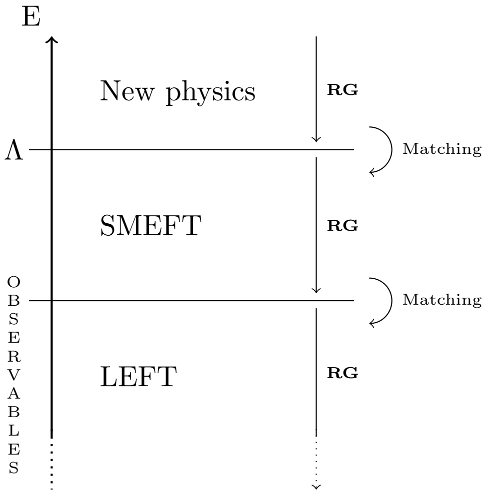
AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

with A. E. Thomsen, J. Fuentes-Martín, and S. Kvedaraitė [2410.07320]

Swiss National Science Foundation

Introduction

- Top-down approach to EFTs

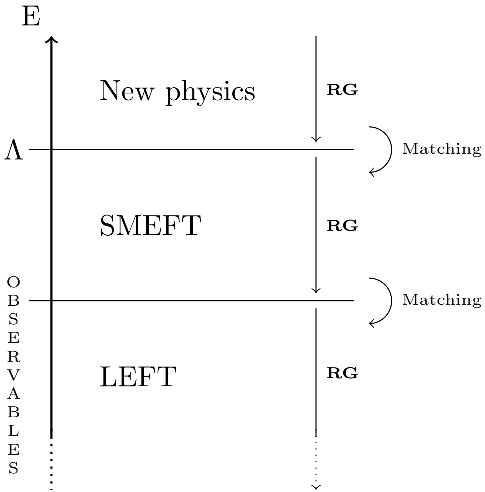


Introduction

- Top-down approach to EFTs
- Why 2-loop?
  1. Theoretical precision required
  2. Scheme independence of one-loop matching
- Build on `Matchete`

  MATCHETE

  Fuentes-Martín et al [2212.04510]
- Counterterms give β-functions



Functional methods

Effective action

$$\Gamma[\eta] = \; S^{(0)} \; + \; \hbar S^{(1)} + \tfrac{i\hbar}{2}\mathrm{STr}\left\{\log Q\right\} \; + \; \hbar^2 S^{(2)} + \tfrac{i\hbar^2}{2} Q^{-1}_{ij} B^{(1)}_{ji}$$

$$-\tfrac{\hbar^2}{8} D^{(0)}_{ijkl} Q^{-1}_{ij} Q^{-1}_{kl} + \tfrac{\hbar^2}{12} C^{(0)}_{ijk} Q^{-1}_{il} Q^{-1}_{jm} Q^{-1}_{kn} C^{(0)}_{lmn} \; + \mathcal{O}(\hbar^3)$$

Fuentes-Martín, Palavrić, Thomsen [2311.13630]

Fuentes-Martín, Moreno-Sánchez, Palavrić, Thomsen [2412.12270]

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- Tree-level

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- One-loop

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- Tree-level
- One-loop
- Two-loop

Fuentes-Martín, Palavrić, Thomsen [2311.13630]

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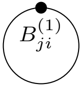
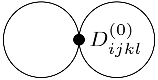
Two-loop counterterms

$$\Gamma^{(2)}[\eta] = \hbar^2 S^{(2)} + \frac{i\hbar^2}{2} Q^{-1}_{ij} B^{(1)}_{ji} - \frac{\hbar^2}{8} D^{(0)}_{ijkl} Q^{-1}_{ij} Q^{-1}_{kl} + \frac{\hbar^2}{12} C^{(0)}_{ijk} Q^{-1}_{il} Q^{-1}_{jm} Q^{-1}_{kn} C^{(0)}_{lmn}$$

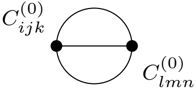
$$= \hbar^2 S^{(2)} + \frac{i\hbar^2}{2}$$



$$- \frac{\hbar^2}{8}$$



$$+ \frac{\hbar^2}{12}$$



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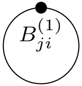

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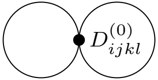

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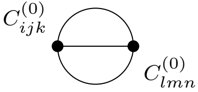


Background field method:

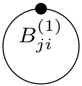
$$\Phi \to \phi + \phi$$

Two-loop counterterms: Simple example

B C D Q^{-1}

$$\mathcal{L}(\Phi) = \partial_\mu \Phi^\dagger \, \partial^\mu \Phi - m^2 \Phi^\dagger \Phi - \frac{\lambda}{4}(\Phi^\dagger \Phi)^2 - \frac{\eta}{36}(\Phi^\dagger \Phi)^3 + \ldots$$

$B_{ji}^{(1)}$: One-loop counterterm insertions



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$$C^{(0)}_{\phi^\dagger \phi \phi^\dagger}(\phi, x) = \left. \frac{\delta^3 S^{(0)}[\phi + \phi]}{\delta \phi^\dagger \, \delta \phi \, \delta \phi^\dagger} \right|_{\phi=0} = -\lambda \phi - \frac{\eta}{2} \phi \phi^\dagger \phi + \dots$$

$$C^{(0)}_{\phi \phi \phi}(\phi, x) = -\frac{\eta}{6} \phi^\dagger \phi^\dagger \phi^\dagger + \dots$$

$$\vdots$$

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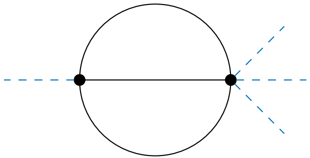

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$$D^{(0)}_{\phi\phi\phi\phi}(\phi, x) = 0$$

$$\vdots$$

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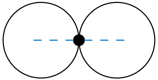

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$$Q^{-1}(\phi, x, k) = \frac{1}{(k + i\partial)^2 - X(\phi, x)} = \frac{1}{k^2} \sum_{n=0}^{\infty} \left(\frac{X(\phi, x) + \partial^2 - 2ik_\mu \partial^\mu}{k^2} \right)^n$$

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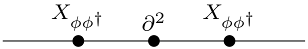
$Q^{-1}_{\phi^\dagger \phi}(\phi, x, k) \supset$



$= \dfrac{X_{\phi\phi^\dagger} \partial^2 X_{\phi\phi^\dagger}}{k^8}$

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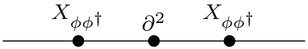

$$= \frac{X_{\phi\phi^\dagger}\partial^2 X_{\phi\phi^\dagger}}{k^8}$$

$$X_{\phi\phi^\dagger}(\phi, x) = m^2 + \lambda\phi^\dagger\phi + \frac{\eta}{4}(\phi^\dagger\phi)^2 + \dots$$

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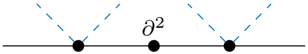
$Q^{-1}_{\phi^\dagger \phi}(\phi, x, k) \supset$ 

$= \lambda^2 \frac{(\phi^\dagger \phi)\partial^2 (\phi^\dagger \phi)}{k^8}$

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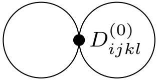
$G_{\mathrm{f8}} =$



$$= \int_x \int_{kl} D^{(0)}_{ijkl}(\phi, x) \, Q^{-1}_{ij}(\phi, x, k) \, Q^{-1}_{kl}(\phi, x, l)$$

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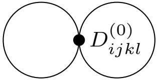
$G_{\text{f8}} =$ 

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$$G_{\text{f8}} \supset \int_x \int_{kl} D^{(0)}_{\phi^\dagger \phi \phi^\dagger \phi}(\phi, x) \left(\frac{1}{(k + i\partial)^2 - X(\phi, x)} \right)_{\phi^\dagger \phi} \left(\frac{1}{(l + i\partial)^2 - X(\phi, x)} \right)_{\phi^\dagger \phi}$$

Two-loop counterterms: Integrals

$$\int_x \int_{kl} D^{(0)}_{\phi^\dagger\phi\phi^\dagger\phi}(\phi, x) \left(\frac{1}{(k+i\partial)^2 - X(\phi, x)} \right)_{\phi^\dagger\phi} \left(\frac{1}{(l+i\partial)^2 - X(\phi, x)} \right)_{\phi^\dagger\phi}$$

- Subdivergences
- Spurious IR divergences

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- Subdivergences
- Spurious IR divergences

Solution: R^*-method

Two-loop counterterms: R^*-method

R^*-method:

$\overline{\text{MS}}$ counterterm — Extracts ϵ poles

$$\Delta_{\text{UV}}(G) = -K\bar{R}^*\mathcal{T}^{(n)}G$$

$$\bar{R}^* = \sum_{S,S'} \Delta_{\text{IR}}(S') * \Delta_{\text{UV}}(S) * G/S \setminus S'$$

Herzog, Ruijl [1703.03776]

Two-loop counterterms: R^*-method

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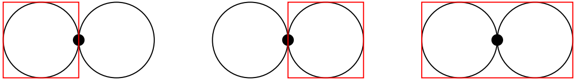

Herzog, Ruijl [1703.03776]

Two-loop counterterms: Integrals

$$\Delta_{\mathrm{UV}}(G_{\mathrm{f8}}) \supset -K\bar{R}^*\mathcal{T}^{(4)} \int_x \int_{kl} D_{\phi^\dagger\phi\phi^\dagger\phi}(\phi, x) \left(\frac{1}{(k+i\partial)^2 - X(\phi,x)}\right)_{\phi^\dagger\phi} \left(\frac{1}{(l+i\partial)^2 - X(\phi,x)}\right)_{\phi^\dagger\phi}$$

$$\supset -K\bar{R}^* \int_x \int_{kl} D_{\phi^\dagger\phi\phi^\dagger\phi}(\phi, x) \, \frac{X_{\phi^\dagger\phi}(\phi,x)}{k^4} \, \frac{X_{\phi^\dagger\phi}(\phi,x)}{l^4}$$

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$$\supset -K\bar{R}^* \int_x \int_{kl} D_{\phi^\dagger\phi\phi^\dagger\phi}(\phi, x) \frac{X_{\phi^\dagger\phi}(\phi, x)}{k^4} \frac{X_{\phi^\dagger\phi}(\phi, x)}{l^4} \propto -K\bar{R}^* \int_{kl} \frac{1}{(k^2 - a)^2} \frac{1}{(l^2 - a)^2}$$

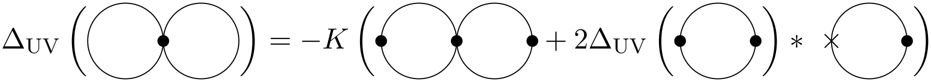
- IR rearrangement removes spurious IR divergences

Two-loop counterterms: Integrals

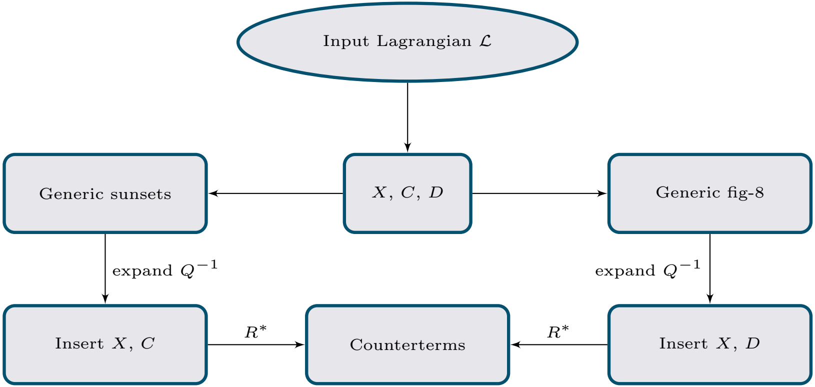
$$\Delta_{\mathrm{UV}}(G_{\mathrm{f8}}) \supset -K\bar{R}^*\mathcal{T}^{(4)} \int_x \int_{kl} D_{\phi^\dagger\phi\phi^\dagger\phi}(\phi, x) \left(\frac{1}{(k+i\partial)^2 - X(\phi,x)} \right)_{\phi^\dagger\phi} \left(\frac{1}{(l+i\partial)^2 - X(\phi,x)} \right)_{\phi^\dagger\phi}$$

$$\supset -K\bar{R}^* \int_x \int_{kl} D_{\phi^\dagger\phi\phi^\dagger\phi}(\phi, x) \frac{X_{\phi^\dagger\phi}(\phi,x)}{k^4} \frac{X_{\phi^\dagger\phi}(\phi,x)}{l^4} \propto -K\bar{R}^* \int_{kl} \frac{1}{(k^2-a)^2} \frac{1}{(l^2-a)^2}$$

- IR rearrangement removes spurious IR divergences
- UV subdivergences: $\int_k \frac{1}{(k^2-a)^2}$ and $\int_l \frac{1}{(l^2-a)^2}$



Code structure



Code structure

```
Lag // NiceForm
```

$$-\frac{1}{4} B^{\mu\nu 2} - \frac{1}{4} G^{\mu\nu A 2} - \frac{1}{4} W^{\mu\nu I 2} + D_\mu \bar{H}_i D_\mu H^i + \mu^2 \bar{H}_i H^i - \frac{1}{2} \lambda \bar{H}_i \bar{H}_j H^i H^j + cH \bar{H}_i \bar{H}_j \bar{H}_k H^i H^j H^k$$

```
CountertermLagrangian[Lag, EFTOrder → 6] // NiceForm
```

$$\left(\frac{1}{24} \hbar \frac{1}{\epsilon} gY^2 + \frac{1}{16} \hbar^2 \frac{1}{\epsilon} \left(3\, gL^2 gY^2 + gY^4\right)\right) B^{\mu\nu 2} + \left(-\frac{7}{2} \hbar \frac{1}{\epsilon} gs^2 - \frac{183}{16} \hbar^2 \frac{1}{\epsilon} gs^4\right) G^{\mu\nu A 2} + \left(-\frac{55}{24} \hbar \frac{1}{\epsilon} gL^2 + \frac{1}{16} \hbar^2 \frac{1}{\epsilon} \left(-77\, gL^4 + gL^2 gY^2\right)\right) W^{\mu\nu I 2} -$$
$$\left(\frac{1}{2} \hbar \frac{1}{\epsilon} \left(-3\, gL^2 - gY^2\right) + \hbar^2 \left(\frac{3}{16} \frac{1}{\epsilon^2} \left(29\, gL^4 - 4\, gL^2 gY^2 - gY^4\right) + \frac{1}{192} \frac{1}{\epsilon} \left(-1179\, gL^4 + 54\, gL^2 gY^2 + 31\, gY^4 + 144\, \lambda^2\right)\right)\right) D_\mu \bar{H}_i D_\mu H^i +$$
$$\left(-\frac{1}{4} \hbar \frac{1}{\epsilon} \mu^2 \left(3\, gL^2 + gY^2 - 12\, \lambda\right) + \hbar^2 \left(\frac{3}{32} \frac{1}{\epsilon} \mu^2 \left(5\, gL^4 + 7\, gY^4 + 32\, \lambda\, gY^2 - 48\, \lambda^2 + 2\, gL^2 \left(gY^2 + 48\, \lambda\right)\right) + \frac{1}{32} \frac{1}{\epsilon^2} \mu^2 \left(105\, gL^4 + 19\, gY^4 + 6\, gL^2 \left(7\, gY^2 - 48\, \lambda\right) - 96\, \lambda\, gY^2 + 144 \left(3\, \lambda^2 + 8\, cH\, \mu^2\right)\right)\right)\right) \bar{H}_i H^i +$$
$$\left(\frac{1}{16} \hbar \frac{1}{\epsilon} \left(-9\, gL^4 - 3\, gY^4 - 6\, gL^2 \left(gY^2 - 2\, \lambda\right) + 4\, \lambda\, gY^2 - 48 \left(\lambda^2 + 4\, cH\, \mu^2\right)\right) + \hbar^2 \left(\frac{1}{64} \frac{1}{\epsilon^2} \left(225\, gL^6 - 5\, gY^6 + gL^4 \left(57\, gY^2 - 336\, \lambda\right) - 76\, \lambda\, gY^4 + 240\, gY^2 \left(\lambda^2 + 4\, cH\, \mu^2\right) - 1152 \left(\lambda^3 + 14\, \lambda\, cH\, \mu^2\right) + gL^2 \left(-17\, gY^4 - 168\, \lambda\, gY^2 + 720 \left(\lambda^2 + 4\, cH\, \mu^2\right)\right)\right) +\right.\right.$$
$$\left.\left.\frac{1}{192} \frac{1}{\epsilon} \left(-1449\, gL^6 + 59\, gY^6 + 3\, gL^4 \left(37\, gY^2 - 102\, \lambda\right) - 198\, \lambda\, gY^4 - 144\, gY^2 \left(3\, \lambda^2 + 16\, cH\, \mu^2\right) + 288 \left(7\, \lambda^3 + 48\, \lambda\, cH\, \mu^2\right) + gL^2 \left(239\, gY^4 - 180\, \lambda\, gY^2 - 432 \left(3\, \lambda^2 + 16\, cH\, \mu^2\right)\right)\right)\right)\right) \bar{H}_i \bar{H}_j H^i H^j +$$
$$\left(-\frac{3}{4} \hbar \frac{1}{\epsilon} cH \left(3\, gL^2 + gY^2 - 36\, \lambda\right) + \hbar^2 \left(\frac{9}{32} \frac{1}{\epsilon^2} cH \left(77\, gL^4 + 19\, gY^4 + 6\, gL^2 \left(7\, gY^2 - 72\, \lambda\right) - 144\, \lambda\, gY^2 + 1584\, \lambda^2\right) + \frac{9}{32} \frac{1}{\epsilon} cH \left(29\, gL^4 + 15\, gY^4 + 64\, \lambda\, gY^2 - 816\, \lambda^2 + 6\, gL^2 \left(3\, gY^2 + 32\, \lambda\right)\right)\right)\right) \bar{H}_i \bar{H}_j \bar{H}_k H^i H^j H^k -$$
$$6\, \hbar^2 \frac{1}{\epsilon} \lambda\, cH \left(\bar{H}_i D^2 \bar{H}_j H^i H^j + \bar{H}_i \bar{H}_j D^2 H^i H^j\right)$$

Bosonic SMEFT

15 effective operators in the bosonic SMEFT:

	X^2H^2
C_{HB}	$H^\dagger H\, B^{\mu\nu}\, B_{\mu\nu}$
C_{HW}	$H^\dagger H\, W^{I\,\mu\nu}\, W^I_{\mu\nu}$
C_{HG}	$H^\dagger H\, G^{A\,\mu\nu}\, G^A_{\mu\nu}$
$C_{H\widetilde{B}}$	$H^\dagger H\, B^{\mu\nu}\, \widetilde{B}_{\mu\nu}$
$C_{H\widetilde{W}}$	$H^\dagger H\, W^{I\,\mu\nu}\, \widetilde{W}^I_{\mu\nu}$
$C_{H\widetilde{G}}$	$H^\dagger H\, G^{A\,\mu\nu}\, \widetilde{G}^A_{\mu\nu}$
C_{HWB}	$H^\dagger H\, \tau^I\, B^{\mu\nu}\, W^I_{\mu\nu}$
$C_{H\widetilde{W}B}$	$H^\dagger H\, \tau^I\, B^{\mu\nu}\, \widetilde{W}^I_{\mu\nu}$

	X^3
C_W	$f^{IJK}\, W^{I\,\nu}_\mu\, W^{J\,\rho}_\nu\, W^{K\,\mu}_\rho$
C_G	$f^{ABC}\, G^{A\,\nu}_\mu\, G^{B\,\rho}_\nu\, G^{C\,\mu}_\rho$
$C_{\widetilde{W}}$	$f^{IJK}\, \widetilde{W}^{I\,\nu}_\mu\, W^{J\,\rho}_\nu\, W^{K\,\mu}_\rho$
$C_{\widetilde{G}}$	$f^{ABC}\, \widetilde{G}^{A\,\nu}_\mu\, G^{B\,\rho}_\nu\, G^{C\,\mu}_\rho$
	H^4D^2 and H^6
C_H	$\left(H^\dagger H\right)^3$
$C_{H\Box}$	$H^\dagger H \Box \left(H^\dagger H\right)$
C_{HD}	$\left(H^\dagger D_\mu H\right)^\dagger \left(H^\dagger D^\mu H\right)$

Bosonic SMEFT

15 effective operators in the bosonic SMEFT:

	Runtime [s]
C_{HB}	1100
C_{HW}	2300
C_{HG}	1200
$C_{H\widetilde{B}}$	1100
$C_{H\widetilde{W}}$	1800
$C_{H\widetilde{G}}$	1500
C_{HWB}	1800
$C_{H\widetilde{W}B}$	1700

	Runtime [s]
C_{W}	10000
C_{G}	6500
$C_{\widetilde{W}}$	24800
$C_{\widetilde{G}}$	13700
	Runtime [s]
C_{H}	940
$C_{H\Box}$	1300
C_{HD}	2700

Validation

(i) Yang-Mills + Weinberg operator[1]

(ii) SMEFT scalar sector[2]

(iii) Scalar $O(N)$ model[2]

Consistency relation:

Loop-counting operator

$$\zeta\, \delta g_2^I = \beta_J\, \partial^J\, \delta g_1^I$$

[1] de Vries, Falcioni, Herzog, Ruijl [1907.04923]

[2] Jenkins, Manohar, Naterop, Pagès [2310.19883]

Full SMEFT

In total: 2499 baryon number conserving operators in the Warsaw basis

Challenges (compared to bosonic part):

- Dirac algebra $\rightarrow$ Good foundation in `Matchete`
- Be careful when to contract chains of γ-matrices in R^*
- NDR scheme + reading point ambiguities
- Evanescent operators

Conclusion & outlook

- Functional formalism works well for computers.
- So far: Able to reproduce the two-loop running of the full SM.
- Next: Running of the SMEFT dim. 6 operators.