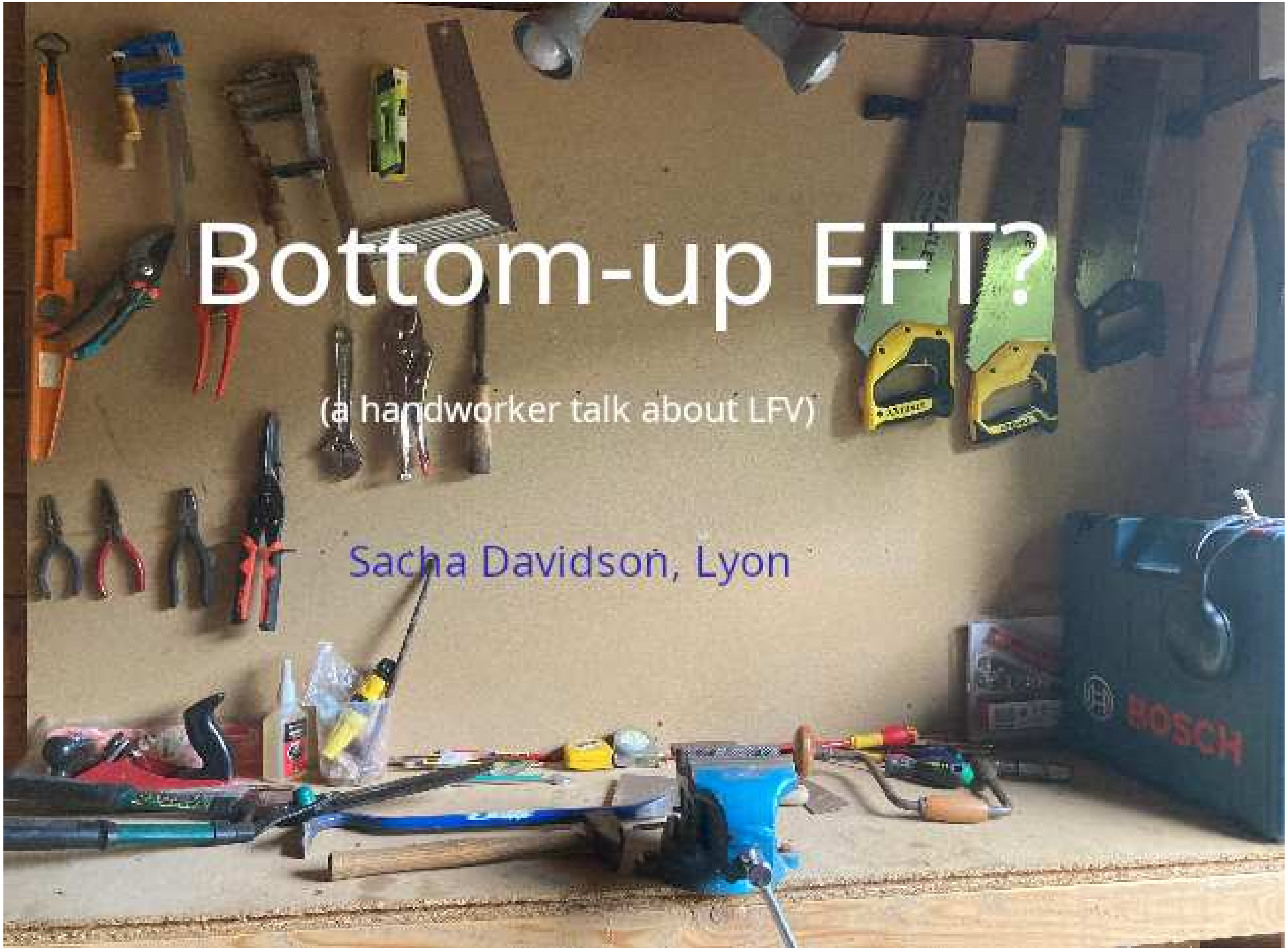
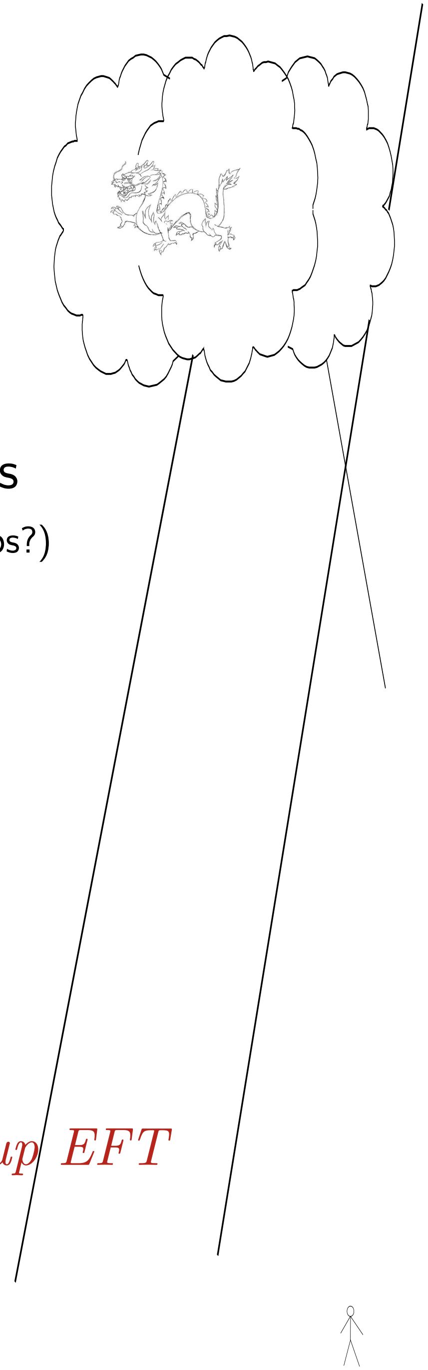
Bottom-up EFT?

(a handworker talk about LFV)

Sacha Davidson, Lyon



This talk is motivated by $\mu \leftrightarrow e$ LFV because...



1. leptons are nice: no strong interactions

2. $[m_\nu]$ tells us there is NewPhysics in the leptonic sector;
   I assume heavy NP $\Leftrightarrow$ EFT

3. diversity of unsuccessful searches for **LFV**$\equiv$ FCNC for leptons
   (colliders, ν physics, meson, τ or μ decays... $\Rightarrow$? constrain $\Lambda_{NP} \gtrsim$ few Tev for all ops?)

4. $\mu \leftrightarrow e$: current reach of $\left\{ \begin{array}{c} \mu \to e\gamma \\ \mu \to e\bar{e}e \\ \mu A \to eA \end{array} \right\}$: $\Lambda_{NP} \lesssim 100$ TeV
   μ beams $\sim 10^{9\to 11}\mu$/sec
   and weak decay

   upcoming exptal sensitivity will improve:
   $$BR \lesssim 10^{-12}\Big|_{\rm now} \to \sim 10^{-(16\to 17)}\Big|_{2025\to 2030}$$

5. $\mu \leftrightarrow e$: restrictive constraints on few operators$\Rightarrow$ *do bottom-up EFT*

$$\boxed{\text{data } (\mu \to e\gamma, \mu \to e\bar{e}e, \mu A \to eA)}$$

based on work with
M Ardu+ M Gorbahn + S Lavignac

Why not do EFT bottom-up? (Please tell me...)

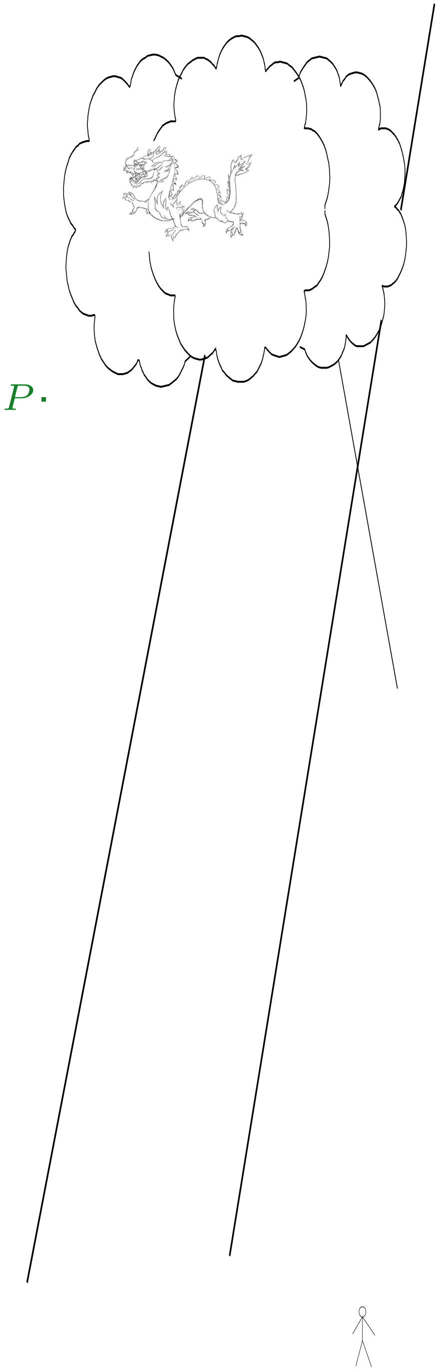
1. (I beleive that) data tells us what is true
   $\Rightarrow$ to discover NP, best to start from what we know?
   (prior to inventing a dragon, figure out colour,size and properties it should have)

2. data changes slowly $\Rightarrow$ efficient/"green" to take data $\rightarrow \Lambda_{NP}$.

3. "one-at-a-time-bounds" are (almost) constraints
   when calculated bottom-up!

4. bottom-up is motivated when exptal bds are hierarchical
   ($=$ # good exptal bds $\ll$ # operators)
   (also simple: can get $\mu \leftrightarrow e$ bds bottom-up by hand;
   nobody has obtained bds ($=$ the correlation matrix) top-down)
   But technically different:
   - in mass basis
   - want all ops$+$ anom-dims to which data is sensitive

   (quark flavour calculated "bottom-up" for decades?)



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One-at-a-time-"bounds", bottom up

recall: bound = how big could it be $\longleftrightarrow$ sensitivity = how small can one see

bounds:

$|x|, |y| \leq 1.25$

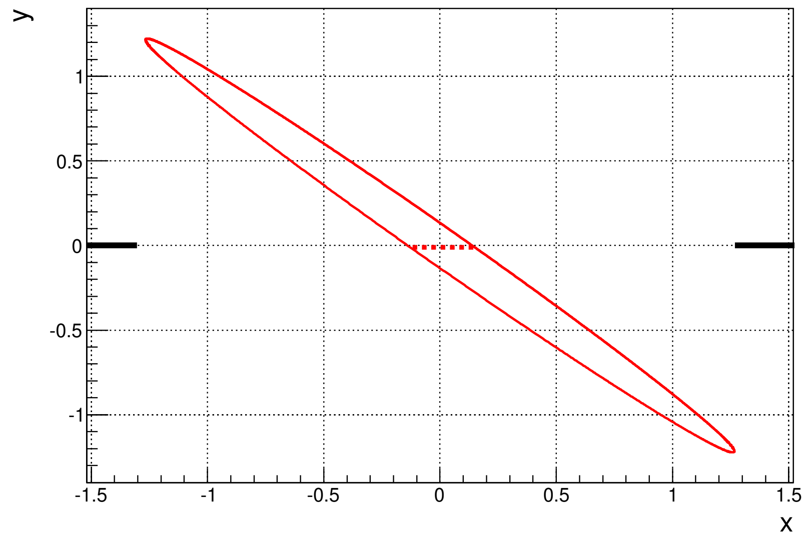
$|x|, |y|$ larger than this is excluded

1-at-at-time bds(sensitivities): ....

$|x|, |y| \leq .125$

$|x|, |y|$ undetectable if smaller than this

$\star$1-at-a-time-bd neglects possible cancellations$\star$



inside ellipse allowed

cancellations can happen: accidental, Eqns of Motion ($\Rightarrow$exact cancellations), symmetries of the model unrecognised in the EFT,...

One-at-a-time-"bounds", bottom up

Suppose a model,

1.parametrised at Λ_{NP} by $\vec{C} = \{C_1, C_2, C_3\}$

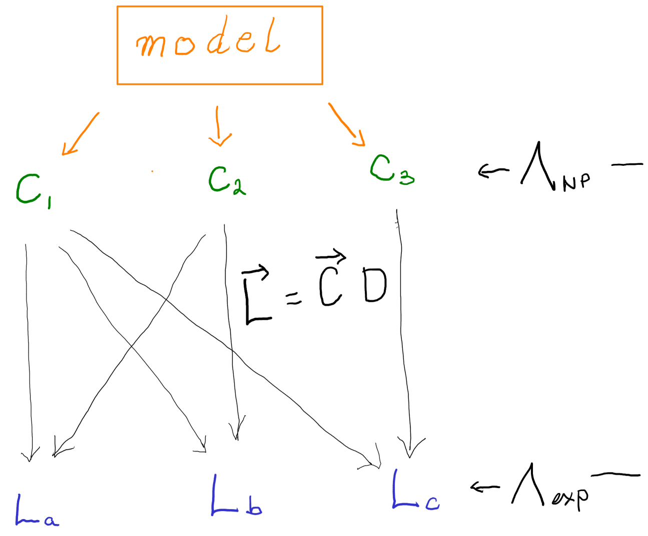
3. constrained at Λ_{exp} by rates R_A, R_B

which impose: $|L_c|^2 < \epsilon_B^2$

$$A|L_a|^2 + |L_b|^2 < \epsilon_A^2$$

2. RGEs can be solved as $\vec{L} = \vec{C}[D]$

$$L_a = C_1 D_{1a} + C_2 D_{2a}$$
$$L_b = C_1 D_{1b} + C_2 D_{2b}$$
$$L_c = C_1 D_{1c} \qquad\qquad + C_3 D_{3c}$$



$R_A = A|L_a|^2 + |L_b^2| < \epsilon_A^2$

$R_B = |L_c|^2 < \epsilon_B^2$

1-at-a-time-bds: calculate $R_A(C_2 = C_3 = 0)$, $R_B(C_2 = C_3 = 0)$, obtain smallest values of C_1 that could have been seen. (?so what?)

coef	R_A	R_B
C_1	$\frac{\epsilon_A}{\sqrt{AD_{1a}^2 + D_{1b}^2}}$	$\frac{\epsilon_B}{\lvert D_{1c}\rvert}$
C_2	$\frac{\epsilon_A}{\sqrt{AD_{2a}^2 + D_{2b}^2}}$	
C_3		$\frac{\epsilon_B}{\lvert D_{1c}\rvert}$

One-at-a-time-"bounds", bottom up

Suppose a model,

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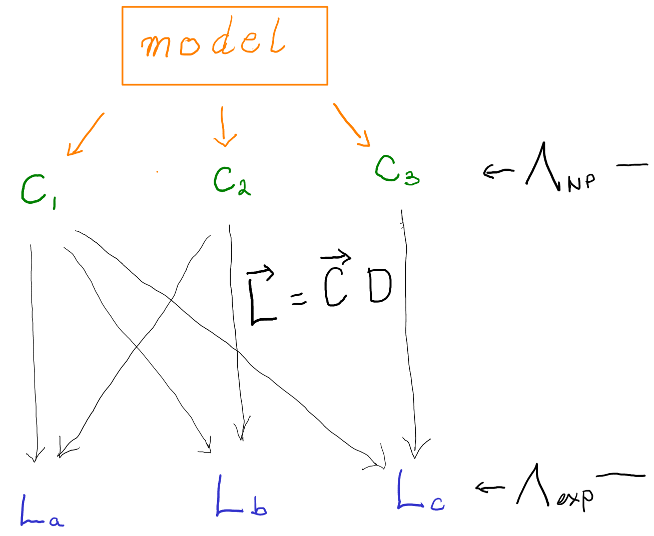


$R_A = A|L_a|^2 + |L_b^2| < \epsilon_A^2$

$R_B = |L_c|^2 < \epsilon_B^2$

bottom-up perspective: focus on constrained $\{L\}$ in terms of $\{C\}$.

exptal bd on $|L_b|$ gives "1-at-a-time-bd" $C_i < \epsilon_A / D_{ib}$. Make table $\forall\ C_i, L_\alpha$

Without absolute values(for $D_{i\alpha} \in \Re$) allows to reconstruct the correlated constraint!

coef	R_A	R_B
C_1	$\frac{\epsilon_A}{\sqrt{AD_{1a}^2 + D_{1b}^2}}$	$\frac{\epsilon_B}{\lvert D_{1c}\rvert}$
C_2	$\frac{\epsilon_A}{\sqrt{AD_{2a}^2 + D_{2b}^2}}$	
C_3		$\frac{\epsilon_B}{\lvert D_{1c}\rvert}$

coef	$\lvert L_a\rvert$	$\lvert L_b\rvert$	$\lvert L_c\rvert$
C_1	$\frac{\epsilon_A}{D_{1a}\sqrt{A}}$	$\frac{\epsilon_A}{D_{1b}}$	$\frac{\epsilon_B}{D_{1c}}$
C_2	$\frac{\epsilon_A}{D_{2a}\sqrt{A}}$	$\frac{\epsilon_A}{D_{2b}}$	
C_3			$\frac{\epsilon_B}{D_{3c}}$

First technicality: what basis are we in?

Exptal constraints are in the mass eigenstate basis of broken EW
$\Rightarrow$ do LEFT in mass basis, et tout va bien.

At m_W, match to SMEFT:

$$[m_e] = [Y_e]v - [C_{EH}]v + ... \quad , \quad (\text{for } \delta\mathcal{L} \supset \frac{C_{EH}}{v^2} Q_{EH} + h.c)$$

$\Rightarrow$ not know $[Y_e] + [C_{EH}]$, but need $[Y_e]$ for RGEs— what to do?

..?stay in mass basis, and approximate $[Y_e]$ diagonal? (Messy to match to models. But model knows Y_e and C_{EH}.)

Works for LFV : $h \to l_\alpha^\pm l_\beta^\mp$ constrains $[C_{EH}]$ to be small enough

(such that rotn mass$\to$ Yukawa eigenbasis is sufficiently small that off-diagonal Y_e do not generate observable LFV)

Technicality 2: to what order, bottom up?

If EFT takes data to models, then need all the operators and interactions to which the data is sensitive.

Because, bottom up: know exptal precision of observables $\{L_\alpha\}$.
But not which of $\{C\}$ generated it.

eg, if see $\mu A \to eA$: did model match to $\bar{e}\sigma \cdot F\mu$? or $(\bar{b}b)(\bar{e}\mu)$? or ...?

$\Rightarrow$ for $\mu \leftrightarrow e$: need all the 4-legged $\mu \leftrightarrow e$ operators below m_W eg $\bar{e}\mu GG$
+ small flock of dim8 SMEFT operators, if $\Lambda_{NP} \lesssim 30$ TeV.

$(\bar{\ell}_e H\mu)(\bar{\ell}_e He)$

+ small family of 2-loop anom dims. nicer than all 2-loop adms and d8 ops

$\star$ $\mu \leftrightarrow e$ analysis at 1-loop with dim6 operators has incomplete basis and RGEs$\star$

$[\mu \to \tau] \times [\tau \to e] = [\mu \to e] \Rightarrow$?

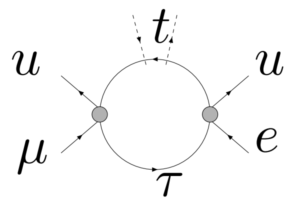
ArduDGorbahn

recall exptal reach: $\mathrm{BR}(\mu \to e) \to 10^{-(18 \to 20)} \sim [\mathrm{BR}(\tau \to l) \to 10^{-9}]^2$
? learn about $\tau \to l$ from $\mu \to e$?

1. if model has $(\mu \to \tau)$,$(\tau \to e)$, then no conserved flavour, so "expect" $\mu \to e$

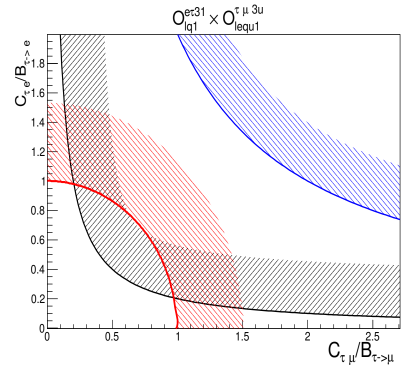
2. can one calculate anything model-independent? In SMEFT, $(\text{dim6})^2 \to$ dim8, eg $\overline{\ell}e\varepsilon\overline{q}u \times (\overline{\ell}\gamma\ell)(\overline{q}\gamma q) \to \overline{\ell}e\varepsilon\overline{q}uH^\dagger H$

$$\frac{\Delta\ ^{(8)}C^{e\mu uu}}{\Lambda^4_{NP}} \simeq \frac{\{y_t^2, g^2\}}{16\pi^2}\frac{C^{e\tau ut}_{LQ}}{\Lambda^2_{NP}}\frac{C^{\tau\mu tu}_{LEQU}}{\Lambda^2_{NP}}$$



so effective low-energy 4-fermion interaction $2\sqrt{2}G_F C_S$

$$\Delta\ ^{(6)}C^{e\mu uu}_S \propto \frac{v^4}{16\pi^2\Lambda^4_{NP}} C^{e\tau ut}C^{\tau\mu tu}$$



3. find eg, $\mu A \to eA$ sensitivity complementary to $B^- \to \{e, \mu\}\nu$ decays for some operators:

Sacha's summary of Attractions of Bottom-up EFT

Using EFT to take data to models makes phenomenological sense:
travel from known to unknown, and data changes rarely (solve RGEs less often and better?)

Does it work? (Buras doubts...)
...I think so? RGEs run up or down. Matching is defined top-down, but usually just means there could be flat directions (irrelevant: in taking exptal bds to models, unconstrained = irrelevant).

$caveat = basis$: Match $m \leftrightarrow Y_e - C_{EH}$; what about $Y_e + C_{EH}$? (not an irrelevant flat direction because Y_e is in RGEs) OK for leptons because $h \to l_\alpha^\pm l_\beta^\mp$ constrains C_{EH}.

bottom-up works differently: include next order when data is sensitive to it = only need a few dim8 operators and 2-loop anom dims...

?reduce need to scan model parameters (no measure on model parameter space), because can analytically check whether parameters match into allowed amoeba in coefficient space at Λ_{NP}

nature

Letter | Published: 14 September 2016

Discovery of species-wide tool use in the Hawaiian crow

Christian Rutz, Barbara C. Klump, Lisa Komarczy
Saskia Wischnewski, Shoko Sugasawa, Michael B. I
Clair, Richard A. Switzer & Bryce M. Masuda

Nature **537**, 403–407 (2016) | Cite this article

Subjects engaging with tool tasks during probe trial

Backup

What we know: categories of LFV constraints

$\boldsymbol{\Delta LF = 1, \Delta QF = 0}$

$\mu A \to eA$, $\tau \to 3l$, $h \to \tau^{\pm} l^{\mp}$... $(l \in \{e, \mu\})$

$\boldsymbol{\Delta LF = 2}$

$\mu\bar{e} \to e\bar{\mu}$, $\tau \to ee\bar{\mu}$...

$\boldsymbol{\Delta LF = \Delta QF = 1}$

$K \to \mu\bar{e}$

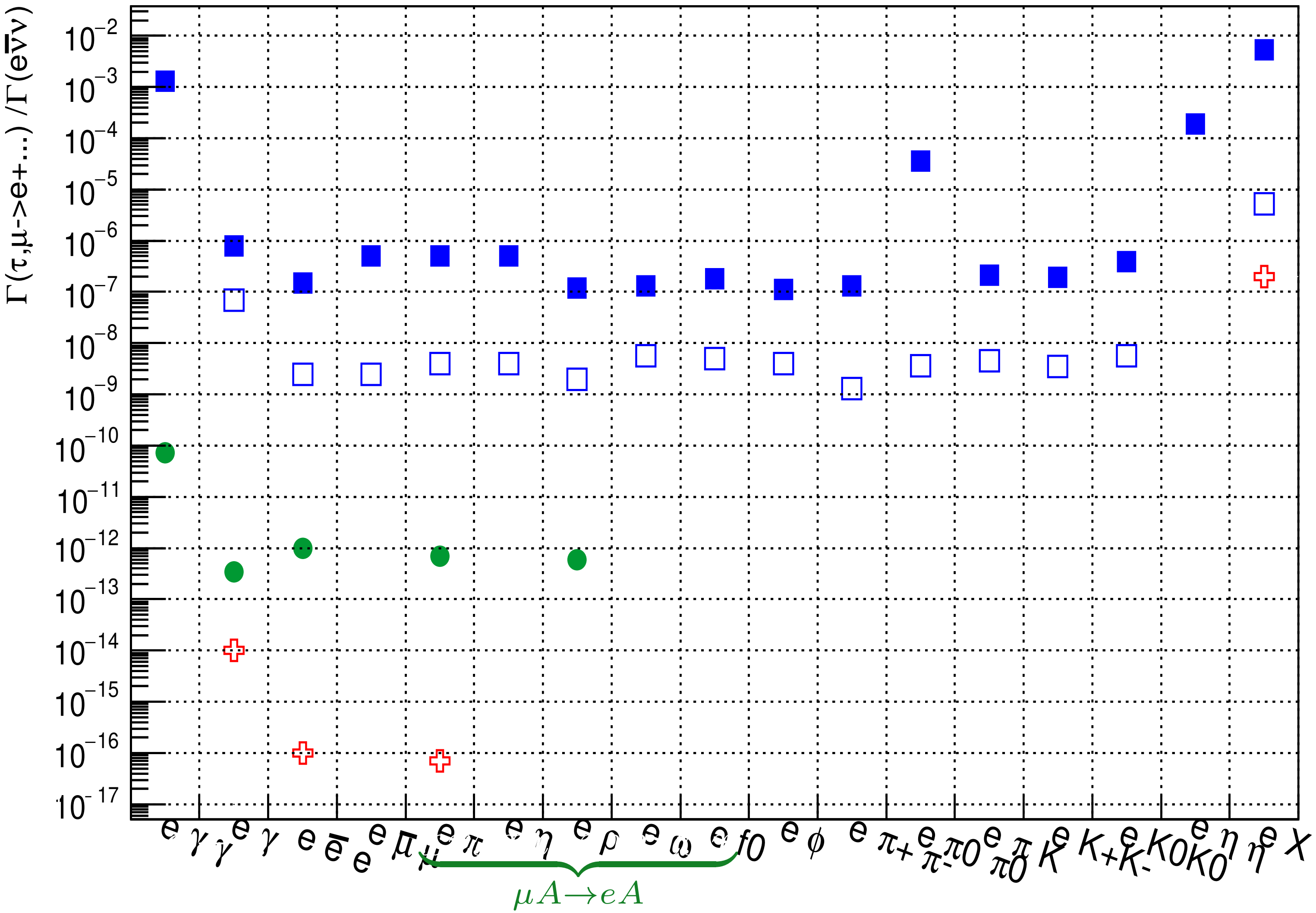
loops $\approx$ not mix categories below Λ_{NP}

what we know about LFV : bounds/upcoming reach

$\Delta LF = 1, \Delta QF = 0$ **decays:** $\tau \to e + .., \mu \to e + ...$

plagiarised
from BelleII

BanerjeeEtal Snowmass 2203.14919



upcoming $\mu \to e$

that plot in words...

$\boldsymbol{\mu \leftrightarrow e}$: restrictive bounds, but only on three processes

$$\frac{\Gamma(\mu \to e...)}{\Gamma(\mu \to e\bar{\nu}\nu)} < 10^{-12} \to 10^{-(16\to 17)} \Rightarrow \Lambda_{LFV} > 10^3 v \to 10^4 v$$

$\boldsymbol{\tau \leftrightarrow l}$: $\approx$ as many bds as Wilson coefficients(!), excluded up to $\Lambda_{NP} \gtrsim 70v \to 200v$

interpretations

$\boldsymbol{\mu \leftrightarrow e}$ **afficcionado:** discover LFV in $\mu \leftrightarrow e$, distinguish models with $\tau \leftrightarrow l$
$\boldsymbol{\tau \leftrightarrow l}$ **afficcionado:** NP couples preferentially to 3rd generation

(How preferential do those couplings need to be ?

$$\mathcal{A}(l_\alpha \to l_\beta + ...) \quad \propto \quad y_\alpha \quad , \quad y_\alpha^2 \quad , \quad y_\alpha y_\beta$$

$$\frac{\Gamma(\mu \to e...)}{\Gamma(\tau \to l...)} \sim \frac{m_\mu^2}{m_\tau^2} \quad , \quad \frac{m_\mu^4}{m_\tau^4} \quad , \quad \frac{m_e^2}{m_\tau^2}$$

$$3{\times}10^{-3} \, , \, 10^{-5} \, , \, 10^{-7} \text{ vs } \frac{\Gamma(\mu \to e...)}{\Gamma(\tau \to l...)} \sim 10^{-5}\Big|_{now} \to 10^{-7}\Big|_{soon}$$

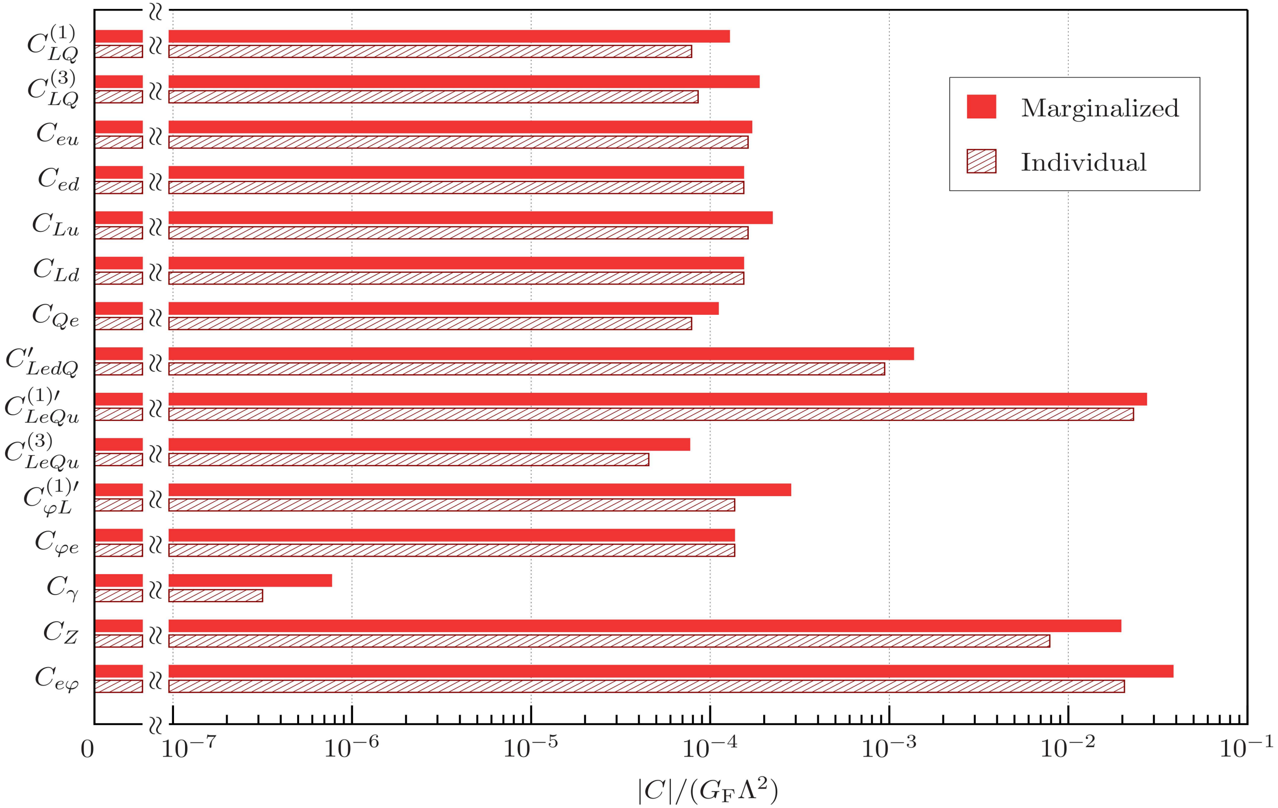
$\Rightarrow$ need strong preference for 3rd gen ? (and remember to reproduce $[m_\nu]$)...)

The $\tau \leftrightarrow l$ sector : *marvellous place to distinguish models*

many processes: current data give indep bounds on magnitude of (almost) all operator coeffs, with $\Lambda_{NP} \gtrsim 10$ TeV (quid flavour indices)?

$\Rightarrow$ promising for distinguishing models (+insensitive to most loops≈theoretically simple)

expected sensitivity of BelleII: BR$\lesssim 10^{-9} \to 10^{-10} \Leftrightarrow \Lambda_{NP} \sim 30$ TeV :



(taken from BanerjeeEtal, Snowmass WPaper 2203.14919) dipole as $C_\gamma v \mathcal{O}_D = C_D m_\tau \mathcal{O}_D$!

The $\mu \leftrightarrow e$ sector

process	current bd on BR	future sensitivities
$\mu \to e\gamma$	$< 3.1 \times 10^{-13}$ (MEGII,now) $\to$	6×10^{-14} (MEGII) $\to \ldots$
$\mu \to e\bar{e}e$	$< 1.0 \times 10^{-12}$(SINDRUM)	2×10^{-15} (2025, Mu3e)$\to 10^{-16}$
$\mu Ti \to eTi$	$< 6 \times 10^{-13}$, (SINDRUMII)	$\lesssim 10^{-(16 \to ?)}$ (Mu2eI,COMETI+II, ?2025-30)
$\mu Au \to eAu$	$< 7 \times 10^{-13}$, (SINDRUMII)	$10^{-(18 \to ?)}$ (PRISM/PRIME/ENIGMA)
$(\mu \to e\gamma\gamma$	$< 7.2 \times 10^{-11})$ (CrystalBox)	

1. current data constrains (≈ measures?) **12 complex operator coefficients**

3. if we see something, what can we learn?
≈ could observations rule out models?

2. is that enough?
≈ if μ-e LFV is there, will we see it?

(**4.** does one need all three processes,
and how to illustrate that?)

What can be measured in $\mu \to e\gamma$ or $\mu \to e\bar{e}e$?

KunoOkada

$$\delta\mathcal{L}_{\substack{\mu\to e\gamma \\ \mu\to e\bar{e}e}}\Big|_{m_\mu} = \frac{1}{v^2}\Big[C_{DR}(m_\mu\overline{e}\sigma^{\alpha\beta}\mu_R)F_{\alpha\beta} + C_{SRR}(\overline{e}P_R\mu)(\overline{e}P_R e) + C_{VLR}(\overline{e}\gamma^\alpha\mu_L)(\overline{e}\gamma_\alpha e_R)$$

$$+C_{VLL}(\overline{e}\gamma^\alpha P_L\mu)(\overline{e}\gamma_\alpha P_L e)\Big] + \frac{1}{v^2}\Big[R \leftrightarrow L\Big] \quad , \quad \frac{1}{v^2} = 2\sqrt{2}G_F$$

What can be measured in $\mu \to e\gamma$ or $\mu \to e\bar{e}e$? (review from KunoOkada)

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KunoOkada

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$\mu \to e\gamma$ with μ-polarisation fraction P_μ, θ_e= angle between μ-spin and $\vec{p}_e$

$$\frac{dBR(\mu \to e\gamma)}{d\cos\theta_e} = 192\pi^2\Big[|C_{DR}|^2(1 - P_\mu\cos\theta_e) + |C_{DL}|^2(1 + P_\mu\cos\theta_e)\Big]$$

KunoOkada

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KunoOkada

$\mu \to e\bar{e}e$: (e relativistic $\Rightarrow$ negligeable interference between e_L, e_R)

$$BR = \frac{|C_{S,LL}|^2}{8} + 2|C_{V,RR} + 4eC_{D,L}|^2 + (64\ln\frac{m_\mu}{m_e} - 136)|eC_{D,L}|^2$$
$$+ |C_{V,RL} + 4eC_{D,L}|^2 + \{L \leftrightarrow R\}$$

OkadaOkumuraShimizu

μ pol. + e angular distributions $\Rightarrow$ measure $4l$ coefficients + some phases

(but S indistinguishable from V)

$\Rightarrow$ **measure magnitude of** $\{C_{DR}, C_{VLL}, C_{VLR}, C_{SRR}, +[L \leftrightarrow R]\}$

If observe $\mu A \to eA$ — what can be measured?

KunoNagamineYamazaki

- μ^- captured by nucleus, falls to $1s$. ($r_\mu \lesssim r_A$, can obtain some μ polarisation)
- in SM: muon "capture" $\mu + p \to \nu + n$, or decay-in-orbit

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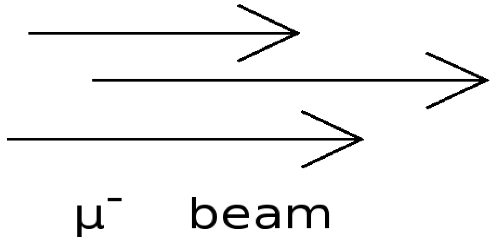
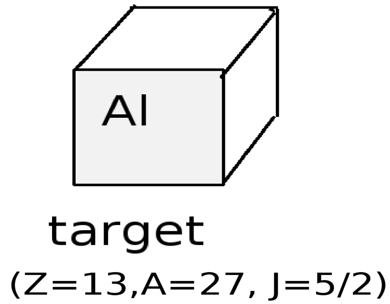
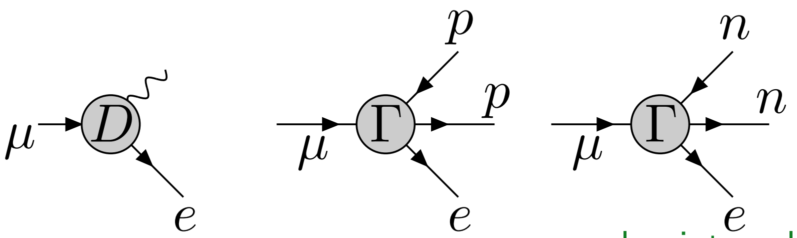




KunoNagamineYamazaki

- μ^- captured by nucleus, falls to $1s$. ($r_\mu \lesssim r_A$, can obtain some μ polarisation)
- $\mu \leftrightarrow e$ via dipole (with $\vec{E}$) or $\widetilde{C}^N_{\Gamma,X}(\bar{e}\Gamma P_X\mu)(\bar{N}\Gamma N)$ or interacting with pion(s)...



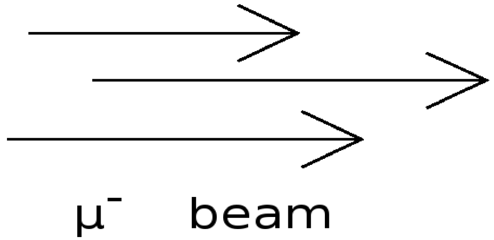
$\Gamma = \{I, \gamma^\alpha, ...\}$

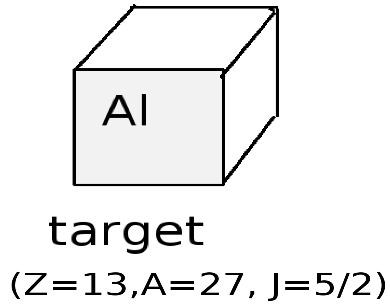
- D,S,V amplitude coherent, grows with A (∼ Spin Indep) KitanoKoikeOkada 2002

$$\frac{32G_F^2 m_\mu^5}{\Gamma_{cap}}\left[\Big|\sum_\Gamma \int_{\text{nucleus A}} \overbrace{(e,\mu \text{ wavefns}) \times \text{nucleon densities}}^{\text{overlap integral}} \times C_{\tilde{\Gamma}}\Big|^2 + |L \leftrightarrow R|^2\right]$$

⋆ much work to include SpinDep, better nucleon distributions, more isotopes, NLO χPT, ...

If observe $\mu A \to eA$ — what can be measured?





KunoNagamineYamazaki

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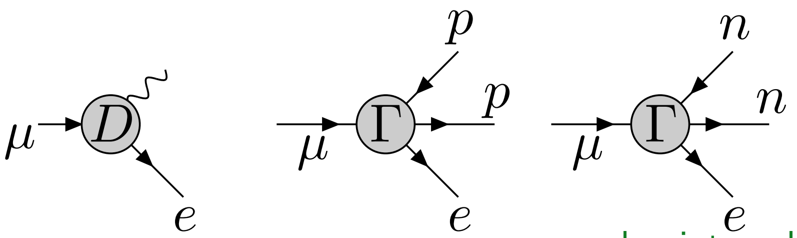

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... probe different combo of coefficients by changing target A KitanoKoikeOkada 2002

- count that $(\mu A \to eA)_{SI}$ (now) constrains coefficient on $p{+}n$ and p-n for $\{e_L, e_R\}$:
  $\{\boldsymbol{C_{Al,L}, C_{Al,R}, C_{Au\perp,L}, C_{Au\perp,R}}\}$) DKunoYamanada

$\star$ much work to include SpinDep, better nucleon distributions, more isotopes, NLO χPT, ...

operators + RGEs: *everything to which data could be sensitive*

operator basis: below m_W, all gauge invariant operators with $\leq$ 4 legs$\approx$ 100 ops.
add to $\mathcal{L}_{SM}$ as $\delta\mathcal{L} = 2\sqrt{2}G_F C^{e\mu ee}_{V,LL}(\overline{e}\gamma\mu)(\overline{e}\gamma e) + ...$

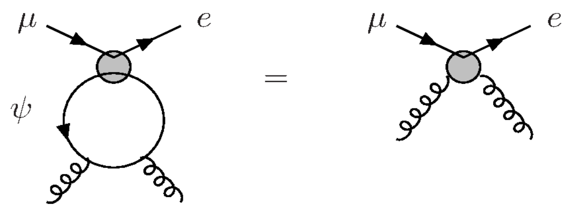
(not dim6: bottom-up perspective/ operator dim. not preserved in matching)

above m_W: dim 6 + selected dim 8 (guess by powercounting)

ArduDavidson

ex: $(\bar{e}\mu)G_{\alpha\beta}G^{\alpha\beta}$ is dim7 $< m_W$, dim8 in SMEFT. But

- dim6 heavy quark scalar ops $(\bar{e}\mu)(\bar{Q}Q)$ match to $(\bar{e}\mu)GG$ at m_Q (coef.$C_{QQ}/(m_Q\Lambda^2_{NP})$):



- gluons contribute most of the mass of the nucleon ShifmanVainshteinZahkarov

$$\langle N|m_N\overline{N}N|N\rangle = \sum_{q\in\{u,d,s\}}\langle N|m_q\overline{q}q|N\rangle - \frac{\alpha_s}{8\pi}\beta_0\langle N|GG|N\rangle$$

$\Rightarrow$ dim7 $(\bar{e}\mu)GG$ contributes significantly to $\mu A \to eA$ via scalar $\mu \to e$ interactions with nucleons N.

CiriglianoKitanoOkadaTuscon

operators + RGEs: *everything to which data could be sensitive*

operator basis: below m_W, all gauge invariant operators with $\leq$ 4 legs$\approx$ 100 ops.

add to $\mathcal{L}_{SM}$ as $\delta\mathcal{L} = 2\sqrt{2}G_F C^{e\mu ee}_{V,LL}(\overline{e}\gamma\mu)(\overline{e}\gamma e) + ...$

(not dim6: bottom-up perspective/ operator dim. not preserved in matching)

above m_W: dim 6 + selected dim 8 (guess by powercounting)

ArduDavidson

RGEs+matching: at "leading order"$\equiv$ largest contribution of each operator to each observable. (2GeV$\to m_W$:resum LL QCD, $\alpha_e \log$, some $\alpha_e^2 \log^2$, $\alpha_e^2 \log$)

why not just 1-loop RGEs?

- expand in loops, hierarchical Yukawas, $1/\Lambda^2_{NP}$,... largest effect maybe not 1-loop (ex: Barr-Zee)
- sometimes 1-loop vanishes...eg: 2-loop $\Delta a_\mu|_{EW} \simeq$ 1-loop $\Delta a_\mu|_{EW}$.
  Because 2-loop log-enhanced
  $=$ mixing vector ops to dipole in 2-loop RGEs.

if see $\mu \to e\gamma$, $\mu \to e\bar{e}e$, or $\mu A \to eA$...?can distinguish models?

...model predictions studied for decades...

EFT recipe to study this: (not scan model space—no measure)

• current bounds give a "12-d" ellipse/box in coefficient-space (in an ideal theorist's world)

• with RGEs, can take ellipse to Λ_{NP}

• are there parts of ellipse that a model *cannot* fill?
If yes, model can be distinguished/ruled out by future $\mu \leftrightarrow e$ data.

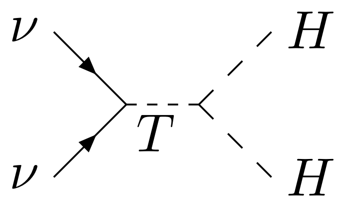
Apply recipe to some TeV-scale models: (we thought all models would fill ellipse and would need colliders to distinguish...)

1) type II seesaw
2) inverse seesaw
3)(singlet LQ for R_D^*)

Type II seesaw — add SU(2) triplet scalar $\vec{T}$

$$\mathcal{L} \supset \left([Y]_{\alpha\beta}\, \overline{\ell^c_\alpha} \varepsilon \vec{\tau} \cdot \vec{T} \ell_\beta + M_T \lambda_H \;\; H \varepsilon \vec{\tau} \cdot \vec{T}^* H + \text{h.c.} \right) + \ldots$$

get $[m_\nu]$ in matching, at tree (NB two mass scales):



$$[m_\nu]_{\alpha\beta} \sim \frac{[Y]_{\alpha\beta} \lambda_H M_T v^2}{M_T^2} \sim 0.03 \text{ eV} \times [Y]_{\alpha\beta} \frac{\lambda_H}{10^{-12}} \frac{\text{TeV}}{M_T}$$

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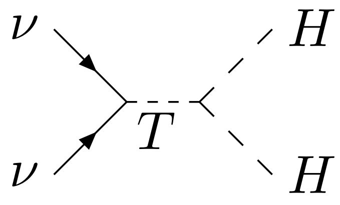
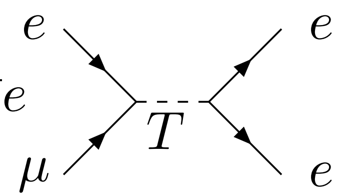
get $[m_\nu]$ at tree (NB: 2 mass scales, so unclear notion of Λ_{NP}):



$$[m_\nu]_{\alpha\beta} \sim \frac{[Y]_{\alpha\beta} \lambda_H M_T v^2}{M_T^2} \sim 0.03 \text{ eV} \times [Y]_{\alpha\beta} \frac{\lambda_H}{10^{-12}} \frac{\text{TeV}}{M_T}$$

expect $\mu \to e\bar{e}e$ at tree (can vanish via unknown Majorana phases ϕ_i):

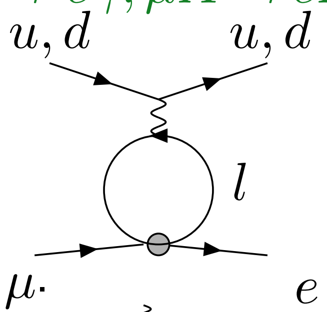
$\mu \to e\bar{e}e$



$$C^{e\mu ee}_{V,LL} \sim \frac{[Y]_{\mu e}[Y^*]_{ee} v^2}{M_T^2}$$

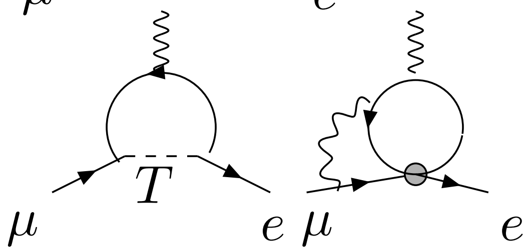
and $\mu \to e\gamma, \mu A \to eA$ at loop

$\mu A \to eA$



$C^{e\mu qq}_{Al,L} \sim$ known from m_ν+phase dep

$\mu \to e\gamma$



$$C_{D,L} \sim \frac{y_\mu [YY^\dagger]_{\mu e} v^2}{128\pi^2 M_T^2} + \mathit{tiny}\ f(\textstyle\sum m_{\nu_i}, \phi_i)$$

Type II seesaw: predictions

recall 12 (complex) operator coefficients $\left\{ \begin{array}{l} C_{DR},\ C^{e\mu ee}_{VLL},\ C^{e\mu ee}_{VLR},\ C^{e\mu ee}_{SRR},\ C_{AlightL},\ C_{AheavyR} \\ C_{DL},\ C^{e\mu ee}_{VRL},\ C^{e\mu ee}_{VRR},\ C^{e\mu ee}_{SLL},\ C_{AlightL},\ C_{AheavyR} \end{array} \right.$

- 7 Cs $\propto y_e$ (LFV-involving-singlet-leptons)

(predicted by all m_ν models where NP interacts with doublets); test by polarising μ. Kuno Okada

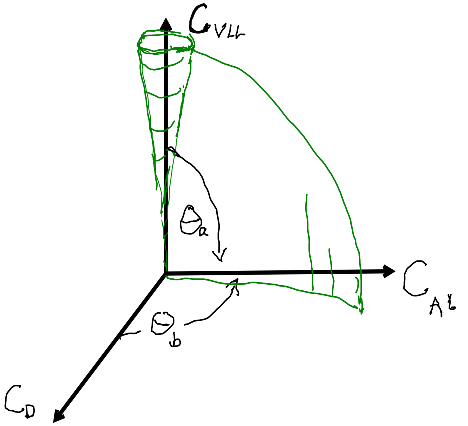
- 3 Cs arise via penquin so are $\propto$ each other

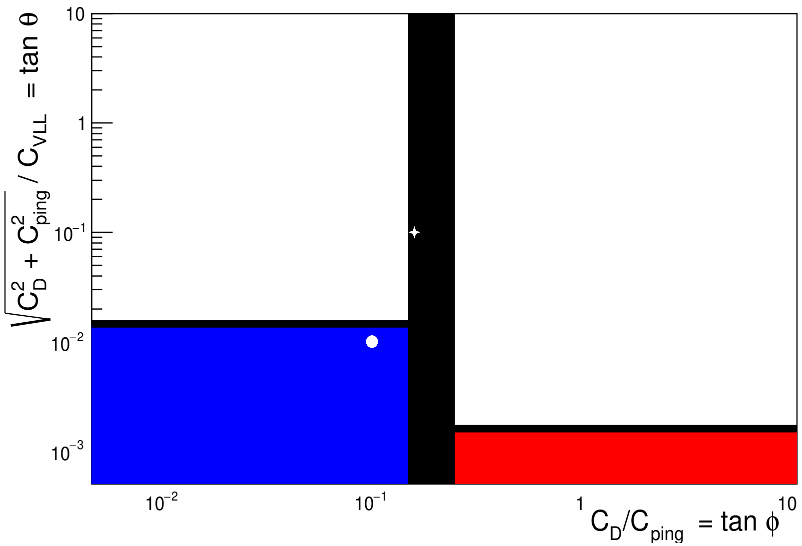
- Remain three "unpredicted": $C_{VLL}(\mu \to e\bar{e}e), C_{Al,L}(\mu A \to eA), C_{DR}(\mu \to e\gamma)$

  (depend on m_ν scale and Majorana phases),

  but if one vanishes there are correlations among other two = combos of BR($\mu \to e\gamma$), BR($\mu A \to eA$), BR($\mu \to e\bar{e}e$) that model cannot reproduce.

$\star$? did not find vanishing dipole in literature?(2-loop EW $(g-2)$ not included for $\mu \to e\gamma$...)





model lives in coloured area, expt can probe whole plot.

"Inverse" Seesaw @ TeV

TypeI seesaw model with extra singlets: add n gen. of singlet Dirac fermions $\psi_D^T = (S_L, N_R)$, and tiny Majorana $[\mu]$

$$\delta\mathcal{L} = = i\overline{N}\not\partial N + i\overline{S}\not\partial S - \left(Y_\nu^{\alpha a}(\overline{\ell}_\alpha \tilde{H} N_a) + M_{ab}\overline{S}_a N_b + \frac{1}{2}\mu_{ab}\overline{S_a} S_b^c + \mathrm{h.c}\right),$$

⋆ gives $m_\nu \sim Y_\nu v M^{-1}\mu[M^T]^{-1}Y_\nu^T v$, $=$ can obtain m_ν by ajusting $\mu(Y_\nu)$
$\Leftrightarrow$ Y_ν indep of m_ν, "can be large", generically LFV
⋆ we take $M \sim$ TeV

obtain observable LFV-coeffs via EW loops, (no QED-RG effects cut off by charged-lepton masses)

★ despite no info about Y_ν, M from m_ν, single-scale model is *more predictive*
★ than type II seesaw because all $\mu \leftrightarrow e$ LFV $\propto [Y_\nu Y_\nu^\dagger]_{e\mu}, [Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger]_{e\mu}$
$\Rightarrow$ occupies plane in ellipse (for real Cs)