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T30f
Theoretische Teilchen-
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Exploring the SMEFT Flavour Structure with AutoEFT

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AutoEFT

arXiv:2309.15783

- automatic basis construction for generic (SMEFT-like) EFTs
- operators free of redundancies (IBP, EOM, Fierz, *etc.*)
- based on algorithm by Hao-Lin Li *et al.* (SMEFT @ $d = 8, 9$)
- implemented in `Python`, using `SageMath` & `FORM`
- open source: `https://gitlab.com/auto_eft/autoeft`

Redundancies

1. Completeness relations: $g_{\mu\nu}\sigma^{\mu}_{\alpha\dot{\alpha}}\sigma^{\nu}_{\beta\dot{\beta}} = 2\epsilon_{\alpha\beta}\tilde{\epsilon}_{\dot{\alpha}\dot{\beta}}$

2. Field redefinitions: $\mathcal{O} + \frac{\delta S_0}{\delta \Phi_i}\mathcal{O}' \sim \mathcal{O}$

3. Topological terms: $\mathcal{O} + \partial\mathcal{O}' \sim \mathcal{O}$

4. Repeated fields: $\pi_{\Phi} \circ \mathcal{O} = \mathcal{O}$

Completeness Relations

Lorentz group: $\mathrm{SO}^+(1,3)$

$$\phi \in (0,0)$$

$$\psi_\alpha \in (1/2,0) \qquad \psi^{\dagger\,\dot\alpha} \in (0,1/2)$$

$$F_{\mathrm{L}\,\alpha\beta} = \frac{i}{2} F_{\mu\nu}\sigma^{\mu\nu}_{\alpha\beta} \in (1,0) \qquad F_{\mathrm{R}}^{\dot\alpha\dot\beta} = -\frac{i}{2} F^{\mu\nu}\bar\sigma^{\dot\alpha\dot\beta}_{\mu\nu} \in (0,1)$$

$$D_\alpha^{\dot\alpha} = D_\mu \sigma^{\mu}{}_{\alpha}^{\dot\alpha} \in (1/2,1/2)$$

Gauge group: $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$

$$G_{abc} = \epsilon_{acd}(\lambda^A)^d_b G^A \qquad W_{ij} = \epsilon_{jk}(\tau^I)^k_i W^I$$

Field Redefinitions

Fields

$$\phi \in (0,0) \qquad \psi_\mathrm{L} \in (1/2,0) \qquad F_\mathrm{L} \in (1,0)$$

Equations of motion

$$D^2\phi \in \underbrace{(0,0)}_{\sim\phi} \oplus \underbrace{(1,0) \oplus (0,1)}_{\sim F} \oplus (1,1)$$

$$D\psi_\mathrm{L} \in \underbrace{(0,1/2)}_{\sim\psi_\mathrm{R}} \oplus (1,1/2)$$

$$DF_\mathrm{L} \in \underbrace{(1/2,1/2)}_{\sim A} \oplus (3/2,1/2)$$

Field Redefinitions

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$$DF_\mathrm{L} \in \underbrace{(1/2,1/2)}_{\sim A} \oplus (3/2,1/2)$$

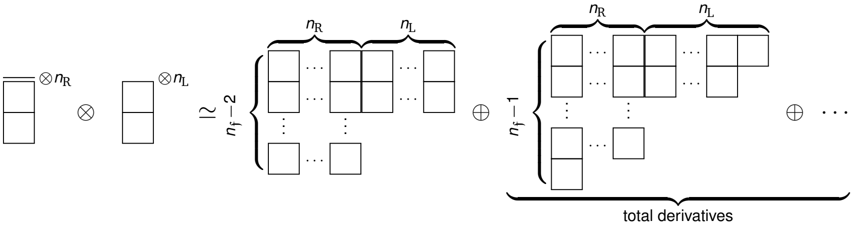
$\Rightarrow$ **only keep symmetric combinations**

Topological Terms

Invariant tensor

$$T_{\text{Lorentz}} = (\epsilon_{\dot{\alpha}_k \dot{\alpha}_l})^{\otimes n_{\mathrm{R}}} (\epsilon^{\alpha_i \alpha_j})^{\otimes n_{\mathrm{L}}}$$

The $\mathrm{SU}(n_{\mathfrak{f}})$ trick



Installation

PyPI

```
$ pip install autoeft
```

or

```
$ sage -pip install autoeft
```

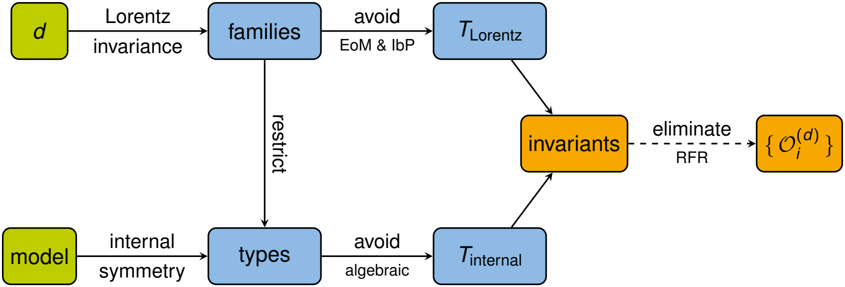
conda-forge

```
$ conda install autoeft -c conda-forge
```

or

```
$ mamba install autoeft
```

Automated Construction



The Model File

```yaml
# AutoEFT model file
name: QED-EFT
description: Effective field theory of QED interactions

symmetries:
  u1_groups:
    QED: {}

fields:
  eL:
    representations:
      Lorentz: -1/2
      QED: -1
  eR:
    representations:
      Lorentz: 1/2
      QED: -1
  FL:
    representations:
      Lorentz: -1
```

The Model File

```yaml
# AutoEFT model file
name: QCD-EFT
description: Effective field theory of QCD interactions

symmetries:
  sun_groups:
    QCD: {N: 3}

fields:
  qL:
    representations:
      Lorentz: -1/2
      QCD: [1]
  qR:
    representations:
      Lorentz: 1/2
      QCD: [1]
  GL:
    representations:
      Lorentz: -1
      QCD: [2,1]
```

Outlook

Work in progress

- add flavour spurions
- add off-shell operators
- express operators in common format
- automatic Hermitian conjugation
- operator projection

Wish list

- full basis transformation ($D = 4$)
- non-linear symmetries (*eg.* HEFT)
- evanescent operators (requested by Luca)

Backup

SMEFT Operators

General operator

$$\mathcal{O} = T^{\{g\}}_{\mathrm{SU(3)}} T^{\{h\}}_{\mathrm{SU(2)}} T^{\{l\}}_{\mathrm{Lorentz}} \times \prod_{i=1}^{N} (D^{n_i}\Phi_i)_{\{g\},\{h\},\{l\}}$$

Invariant tensors

$$T_{\mathrm{SU(3)}} \in \langle f^{ABC}, d^{ABC}, \delta^{AB}, (\lambda^A)^b_a, \epsilon_{abc}, \epsilon^{abc} \rangle$$
$$T_{\mathrm{SU(2)}} \in \langle \epsilon^{IJK}, \delta^{IJ}, (\tau^I)^j_i, \epsilon_{ij}, \epsilon^{ij} \rangle$$
$$T_{\mathrm{Lorentz}} \in \langle \sigma^{\mu\nu}_{\alpha\beta}, \bar{\sigma}^{\mu\nu}_{\dot{\alpha}\dot{\beta}}, \sigma^{\mu}_{\alpha\dot{\alpha}}, \bar{\sigma}^{\mu\,\dot{\alpha}\alpha}, \epsilon^{\alpha\beta}, \tilde{\epsilon}_{\dot{\alpha}\dot{\beta}} \rangle$$

SMEFT Operators

General operator

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Equations of Motion

$$(D^{r-|h|}\Phi)_{\alpha^{(1)}\ldots\alpha^{(r-h)}\dot\alpha^{(1)}\ldots\dot\alpha^{(r+h)}} \equiv \begin{cases} D_{\alpha^{(1)}\dot\alpha^{(1)}}\ldots D_{\alpha^{(r+h)}\dot\alpha^{(r+h)}}\Phi_{\alpha^{(r+h+1)}\ldots\alpha^{(r-h)}}\,, & \text{h<0} \\ D_{\alpha^{(1)}\dot\alpha^{(1)}}\ldots D_{\alpha^{(r-h)}\dot\alpha^{(r-h)}}\Phi_{\dot\alpha^{(r-h+1)}\ldots\dot\alpha^{(r+h)}}\,, & \text{h>0} \end{cases}$$

$$\begin{aligned} D_{[\alpha\dot\alpha}D_{\beta]\dot\beta}\phi &= -\epsilon_{\alpha\beta}\tilde\epsilon_{\dot\alpha\dot\beta}D^2\phi + \frac{i}{2}\epsilon_{\alpha\beta}\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta}[D_\mu, D_\nu]\phi \\ D_{[\alpha\dot\alpha}\psi_{\beta]} &= -\epsilon_{\alpha\beta}(\not{D}\psi)_{\dot\alpha} \\ D_{[\alpha\dot\alpha}F_{L\,\beta]\gamma} &= 2\epsilon_{\alpha\beta}\sigma^{\nu}_{\gamma\dot\alpha}D^\mu F_{\mu\nu} \end{aligned}$$

$$(D^{r-|h|}\Phi)^{(\dot\alpha^{(1)}\ldots\dot\alpha^{(r+h)})}_{(\alpha^{(1)}\ldots\alpha^{(r-h)})} \equiv (D^{r-|h|}\Phi)^{\dot\alpha^{r+h}}_{\alpha^{r-h}} \in \left(\frac{r-h}{2}, \frac{r+h}{2}\right)$$

Irreducible Representations

Integration by parts

$$-D\phi_1\,\phi_2\phi_3 D\phi_4 = \phi_1 D\phi_2\,\phi_3 D\phi_4 + \phi_1\phi_2 D\phi_3\,D\phi_4$$

Irreducible representation of $\mathrm{SU}(N)$

$$-\ \begin{array}{|c|c|}\hline 2 & 1 \\ \hline 3 & 4 \\ \hline\end{array}\ =\ -\ \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline\end{array}\ +\ \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline\end{array}$$

Irreducible Representations

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$$-D\phi_1\,\phi_2\phi_3 D\phi_4 = \phi_1 D\phi_2\,\phi_3 D\phi_4 + \phi_1\phi_2 D\phi_3\,D\phi_4$$

Irreducible representation of $\mathsf{SU}(N)$

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$\Rightarrow$ **only keep Semi-Standard Young Tableaux**

Irreducible Representations

Gauge group: $SU(3)_C \times SU(2)_W \times U(1)_Y$

$$\mathrm{SU}(3): \qquad \mathbf{3} \sim \ydiagram{1} \quad \overline{\mathbf{3}} \sim \ydiagram{1,1} \quad \mathbf{8} \sim \ydiagram{2,1}$$

$$\mathrm{SU}(2): \qquad \mathbf{2} \sim \ydiagram{1} \quad \mathbf{3} \sim \ydiagram{2}$$

Irreducible Representations

Invariant contractions

$$L_i \sim \boxed{i} \qquad Q_j \sim \boxed{j} \qquad Q_k \sim \boxed{k} \qquad Q_l \sim \boxed{l}$$

$$\boxed{i} \to \begin{array}{|c|c|}\hline i & j \\ \hline \end{array} \to \begin{array}{|c|c|}\hline i & j \\ \hline k \\ \cline{1-1} \end{array} \to \begin{array}{|c|c|}\hline i & j \\ \hline k & l \\ \hline \end{array} \sim \epsilon^{ik}\epsilon^{jl} L_i Q_j Q_k Q_l$$

$$\boxed{i} \to \begin{array}{|c|}\hline i \\ \hline j \\ \hline \end{array} \to \begin{array}{|c|c|}\hline i & k \\ \hline j \\ \cline{1-1} \end{array} \to \begin{array}{|c|c|}\hline i & k \\ \hline j & l \\ \hline \end{array} \sim \epsilon^{ij}\epsilon^{kl} L_i Q_j Q_k Q_l$$

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$\Rightarrow$ **tensors according to Littlewood-Richardson rule**

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