Universidad de Granada
FTAE
High Energy Theory

Efficient on-shell matching

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[2411.12798]

SMEFT-Tools|2025

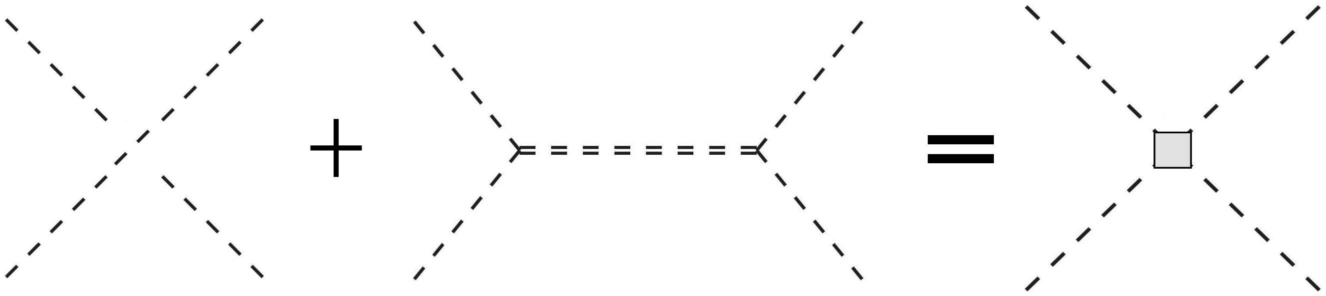
Junta de Andalucía

Matching a theory: *off-shell vs on-shell*

OFF-SHELL	VS	ON-SHELL
Small number of diagrams (one-light-particle-irreducible)	1 – 0	All diagrams (light bridges too)
Large set of operators (Green's basis)	1 – 1	Smaller set of operators (physical basis)
Reduction to the physical basis	1 – 2	No reduction is needed
Contribution directly local in momenta		Delicate cancelation of non-local contributions

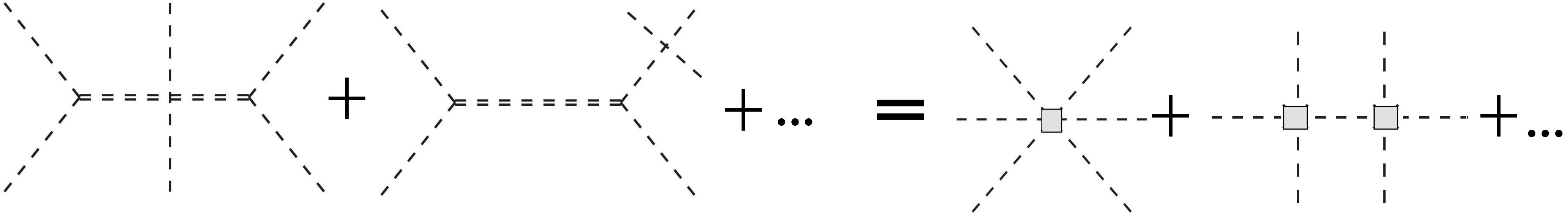
Matching a theory: *off-shell vs on-shell*

4-pt:



$$\lambda + \frac{\kappa^2}{M^2} = C_4$$

6-pt:



$$\frac{\eta\kappa^2}{M^4} + \frac{\lambda\kappa^2}{M^2}\frac{i}{(p_{456}^2 - m^2)} + \ldots = C_6 + \frac{iC_4^2}{(p_{456}^2 - m^2)} + \ldots$$

cancel

Matching a theory: *off-shell vs on-shell*

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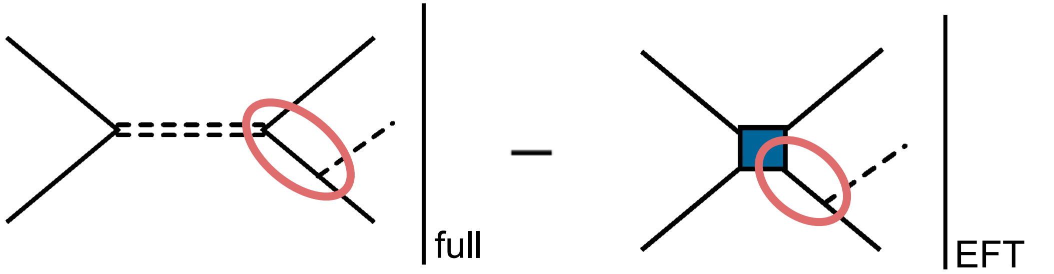
On-shell matching: *non-localities*

Light bridges →



local (polynomial in external momenta)

non-local non-local

- Difficult to follow this cancelation analytically → Substitution of **random-generated kinematics**
- The procedure is to be **numerical** but exact → **Rational kinematics**

Spinor Helicity Formalism
[*arXiv:2304.01589, arXiv:2202.02681*]

Rational values for { momenta, polarizations, spinors } with symbolic masses m_i

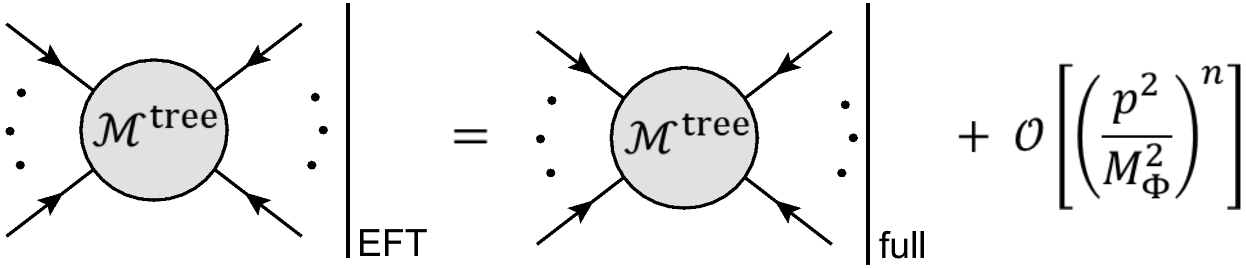
Satisfying...
Momentum conservation
On-shell condition
Dirac equation
Transversality

$$\mathcal{M} = \alpha p_2 \cdot p_3 + \frac{\beta^2}{(p_1+p_4)^2 - m^2} = \frac{9044503}{1681920} m^2 \alpha - \frac{840960}{8203543} \frac{\beta^2}{m^2}$$

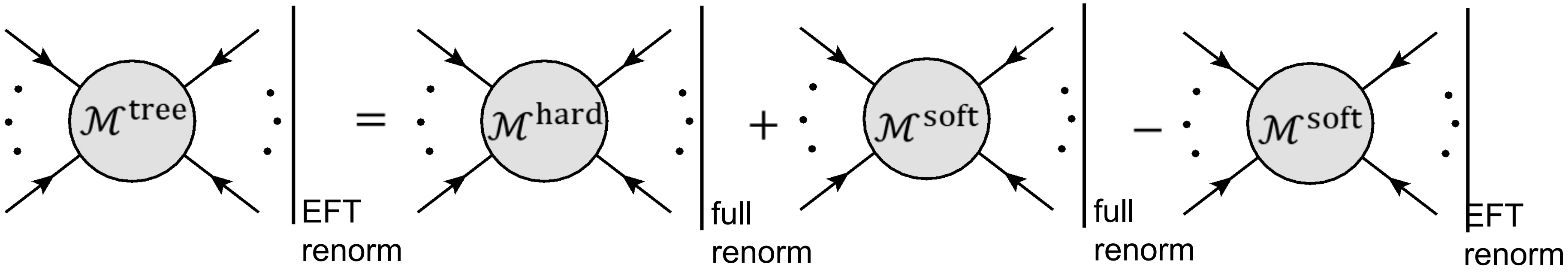
On-shell matching: *the matching condition*

$$\boxed{\frac{p^2}{M_\Phi^2} \ll 1}$$

- Tree-level matching:



- One-loop matching: (method of regions)
  - hard: $k \sim M_\Phi \gg p, m$
  - soft: $M_\Phi \gg k \sim p, m$



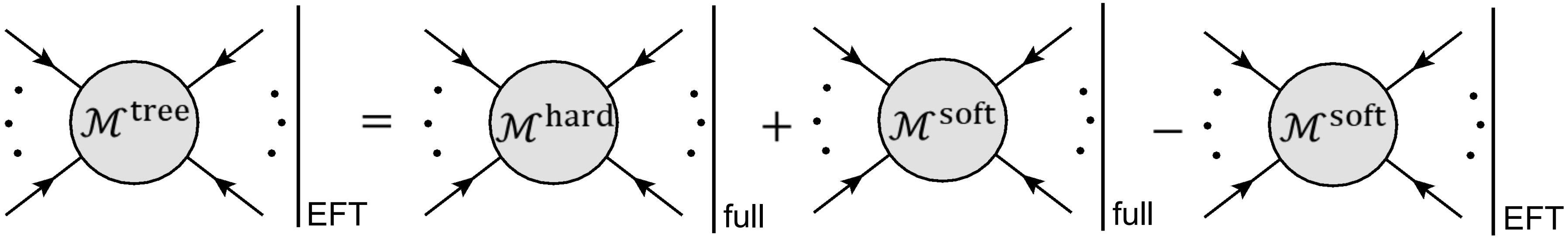
Almost cancel
evanescent contributions

On-shell condition: $p^2 = m_{\text{phys}}^2$

Wavefunction factors: $\mathcal{M} = \left(\sqrt{Z}\right)^n \mathcal{M}_{\text{amp}}$

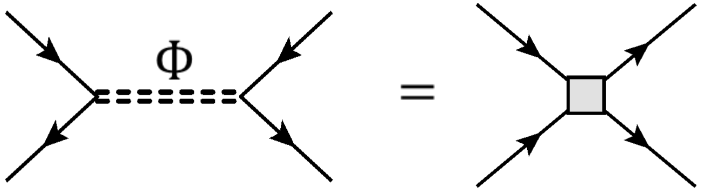
On-shell matching: *evanescent shifts*

- One-loop matching:
  (method of regions)
  - hard: $k \sim M_\Phi \gg p, m$
  - soft: $M_\Phi \gg k \sim p, m$

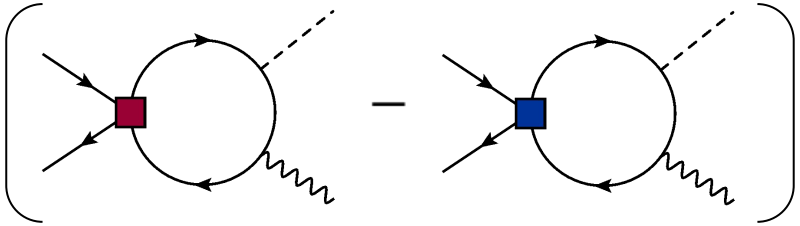


Almost cancel
evanescent contributions

Copy of SM Higgs: $\mathcal{L} = \mathcal{L}_{\text{SM}} + D_\mu \Phi^\dagger D^\mu \Phi - M^2 \Phi^\dagger \Phi - \left(\mathcal{Y}^{pr} \bar{\ell}^p \Phi e^r + \text{h.c.} \right)$



$$(\bar{\ell}^p e^r)(\bar{e}^s \ell^t) = -\frac{1}{2} (\bar{\ell}^p \gamma^\mu \ell^r)(\bar{e}^s \gamma_\mu e^t) \quad \text{(d=4)}$$



$\neq 0$ ΔC_{eB}

Only finite parts coming from $\epsilon \times \frac{1}{\epsilon_{\text{UV}}}$

[J. Fuentes-Martín, M. König, J. Pagès, A. Eller Thomsen, F. Wilsch: 2211.09144]

An example: *finite one-loop matching to SMEFT*

We solve for effective couplings perturbatively in the EFT order:

$$\lambda \to \lambda + \frac{g_2^4 m_H^2}{960\pi^2 M^2}\,,$$

$$y_e^{pr} \to y_e^{pr} - \frac{1}{128\pi^2}\left(\mathcal{Y}\mathcal{Y}^\dagger y_e + 2y_e\mathcal{Y}^\dagger\mathcal{Y}\right)^{pr} + \frac{m_H^2}{32\pi^2 M^2}\mathcal{Y}^{pr}\left(\mathcal{Y}^\dagger\right)^{st} y_e^{ts}\,,$$

$$c_{HD} \to -\frac{g_1^4}{1920\pi^2 M^2}\,,$$

$$c_{H\Box} \to -\frac{1}{7680\pi^2 M^2}\left(g_1^4 + 3g_2^4\right)\,,$$

$$c_{He}^{pr} \to \frac{g_1^4}{1920\pi^2 M^2}\delta^{pr} + \frac{7g_1^2}{576\pi^2 M^2}\left(\mathcal{Y}^\dagger\mathcal{Y}\right)^{pr} + \frac{1}{192\pi^2 M^2}\left(6\mathcal{Y}^\dagger y_e y_e^\dagger\mathcal{Y} + y^\dagger\mathcal{Y}\mathcal{Y}^\dagger y_e\right)^{pr}\,,$$

$$c_{Hl}^{(1)pr} \to \frac{g_1^4}{3840\pi^2 M^2}\delta^{pr} + \frac{17g_1^2}{1152\pi^2 M^2}\left(\mathcal{Y}\mathcal{Y}^\dagger\right)^{pr} - \frac{1}{192\pi^2 M^2}\left(6\mathcal{Y}y_e^\dagger y_e\mathcal{Y}^\dagger + y_e\mathcal{Y}^\dagger\mathcal{Y}y_e^\dagger\right)^{pr}\,,$$

$$c_{Hl}^{(3)pr} \to -\frac{g_2^4}{3840\pi^2 M^2}\delta^{pr} + \frac{g_2^2}{1152\pi^2 M^2}\left(\mathcal{Y}\mathcal{Y}^\dagger\right)^{pr} - \frac{1}{192\pi^2 M^2}\left(y_e\mathcal{Y}^\dagger\mathcal{Y}y_e^\dagger\right)^{pr}\,,$$

$$c_{eB}^{pr} \to -\frac{g_1}{768\pi^2 M^2}\left(5\mathcal{Y}\mathcal{Y}^\dagger y_e + 2y_e\mathcal{Y}^\dagger\mathcal{Y}\right)^{pr} + \frac{3g_1}{128\pi^2 M^2}\mathcal{Y}^{pr}\left(\mathcal{Y}^\dagger\right)^{st} y_e^{ts}\,,$$

$$c_{eW}^{pr} \to -\frac{5g_2}{768\pi^2 M^2}\left(\mathcal{Y}\mathcal{Y}^\dagger y_e\right)^{pr} - \frac{g_2}{128\pi^2 M^2}\mathcal{Y}^{pr}\left(\mathcal{Y}^\dagger\right)^{st} y_e^{ts}\,,$$

$$c_{eH}^{pr} \to -\frac{g_2^4}{3840\pi^2 M^2}y_e^{pr} + \frac{1}{192\pi^2 M^2}\left(y_e y_e^\dagger\mathcal{Y}\mathcal{Y}^\dagger y_e + 2y_e\mathcal{Y}^\dagger\mathcal{Y}y^\dagger y - 3\mathcal{Y}y_e^\dagger y_e\mathcal{Y}^\dagger y_e - 3y_e\mathcal{Y}^\dagger y_e y_e^\dagger\mathcal{Y}\right)^{pr} + \frac{1}{16\pi^2 M^2}\mathcal{Y}^{pr}y_e^{st}\left(y_e^\dagger\right)^{tu}y_e^{uv}\left(\mathcal{Y}^\dagger\right)^{vs} - \frac{\lambda}{32\pi^2 M^2}\mathcal{Y}^{pr}\left(\mathcal{Y}^\dagger\right)^{st}y_e^{ts}$$

Cross-check with

`Matchete` [arXiv:2212.04510]

and

`MatchMakerEFT` [arXiv:2112.10787]

A second example: *reduction of Green's basis*

Green's basis: minimal set of operators to match amplitudes **off-shell.**

Physical basis: minimal set of operators to match amplitudes **on-shell.**

Field redefinitions

Tedious
Non systematic
Hard to automate

$\mathcal{L}_{\text{full}} \supset$ operators in Green's basis

$\mathcal{L}_{\text{phys}} \supset$ operators in physical basis

Match both theories on-shell at tree level

» We find the couplings of $\mathcal{L}_{\text{phys}}$ in terms of those in $\mathcal{L}_{\text{full}}$.

REDUCTION OF GREEN'S BASIS

- **Example I:** Real scalar singlet with Z_2 symmetry up to dimension 8
- **Example II:** Bosonic sector in the SMEFT up to dimension 8

A second example: *reduction of Green's basis*

BOSONIC SECTOR

Cross-check with [2003.12525v5]

X^3		X^2H^2		H^2D^4	
$\mathcal{O}_{3G}$	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{HG}$	$G_{\mu\nu}^{A}G^{A\mu\nu}(H^\dagger H)$	$\mathcal{O}_{DH}$	$(D_\mu D^\mu H)^\dagger(D_\nu D^\nu H)$
$\mathcal{O}_{3\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{H\widetilde{G}}$	$\widetilde{G}_{\mu\nu}^{A}G^{A\mu\nu}(H^\dagger H)$	H^4D^2	
$\mathcal{O}_{3W}$	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{HW}$	$W_{\mu\nu}^{I}W^{I\mu\nu}(H^\dagger H)$	$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
$\mathcal{O}_{3\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{H\widetilde{W}}$	$\widetilde{W}_{\mu\nu}^{I}W^{I\mu\nu}(H^\dagger H)$	$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^\dagger(H^\dagger D_\mu H)$
X^2D^2		$\mathcal{O}_{HB}$	$B_{\mu\nu}B^{\mu\nu}(H^\dagger H)$	$\mathcal{O}'_{HD}$	$(H^\dagger H)(D_\mu H)^\dagger(D^\mu H)$
$\mathcal{O}_{2G}$	$-\frac{1}{2}(D_\mu G^{A\mu\nu})(D^\rho G_{\rho\nu}^{A})$	$\mathcal{O}_{H\widetilde{B}}$	$\widetilde{B}_{\mu\nu}B^{\mu\nu}(H^\dagger H)$	$\mathcal{O}''_{HD}$	$(H^\dagger H)D_\mu(H^\dagger i\overleftrightarrow{D}^\mu H)$
$\mathcal{O}_{2W}$	$-\frac{1}{2}(D_\mu W^{I\mu\nu})(D^\rho W_{\rho\nu}^{I})$	$\mathcal{O}_{HWB}$	$W_{\mu\nu}^{I}B^{\mu\nu}(H^\dagger\sigma^I H)$	H^6	
$\mathcal{O}_{2B}$	$-\frac{1}{2}(\partial_\mu B^{\mu\nu})(\partial^\rho B_{\rho\nu})$	$\mathcal{O}_{H\widetilde{W}B}$	$\widetilde{W}_{\mu\nu}^{I}B^{\mu\nu}(H^\dagger\sigma^I H)$	$\mathcal{O}_{H}$	$(H^\dagger H)^3$
		H^2XD^2			
		$\mathcal{O}_{WDH}$	$D_\nu W^{I\mu\nu}(H^\dagger i\overleftrightarrow{D}_\mu^I H)$		
		$\mathcal{O}_{BDH}$	$\partial_\nu B^{\mu\nu}(H^\dagger i\overleftrightarrow{D}_\mu H)$		

V. Gherardi, D. Marzocca y E. Venturini (2021) [2003.12525v5]

X^2H^2

$$c_{HB} \to c_{HB} \boxed{- 2m_0^2 c_{HB} r_{DH}}$$

$$c_{H\widetilde{B}} \to c_{H\widetilde{B}} \boxed{- 2m_0^2 c_{H\widetilde{B}} r_{DH}}$$

H^4D^2

$$c_{H\Box} \to c_{H\Box} - \frac{1}{8}g'^2 r_{2B} + \frac{1}{2}g' r_{BDH} \boxed{- m_0^2(4c_{H\Box}r_{DH} + g' r_{BDH} r_{DH} + 2r_{DH}r'_{HD})}$$

$$c_{HD} \to c_{HD} - \frac{1}{2}g'^2 r_{2B} + 2g' r_{BDH} \boxed{- m_0^2(4c_{HD}r_{DH} + 4g' r_{BDH} r_{DH})}$$

Reduction of dim. 6 operators up to dim. 8!!

H^6

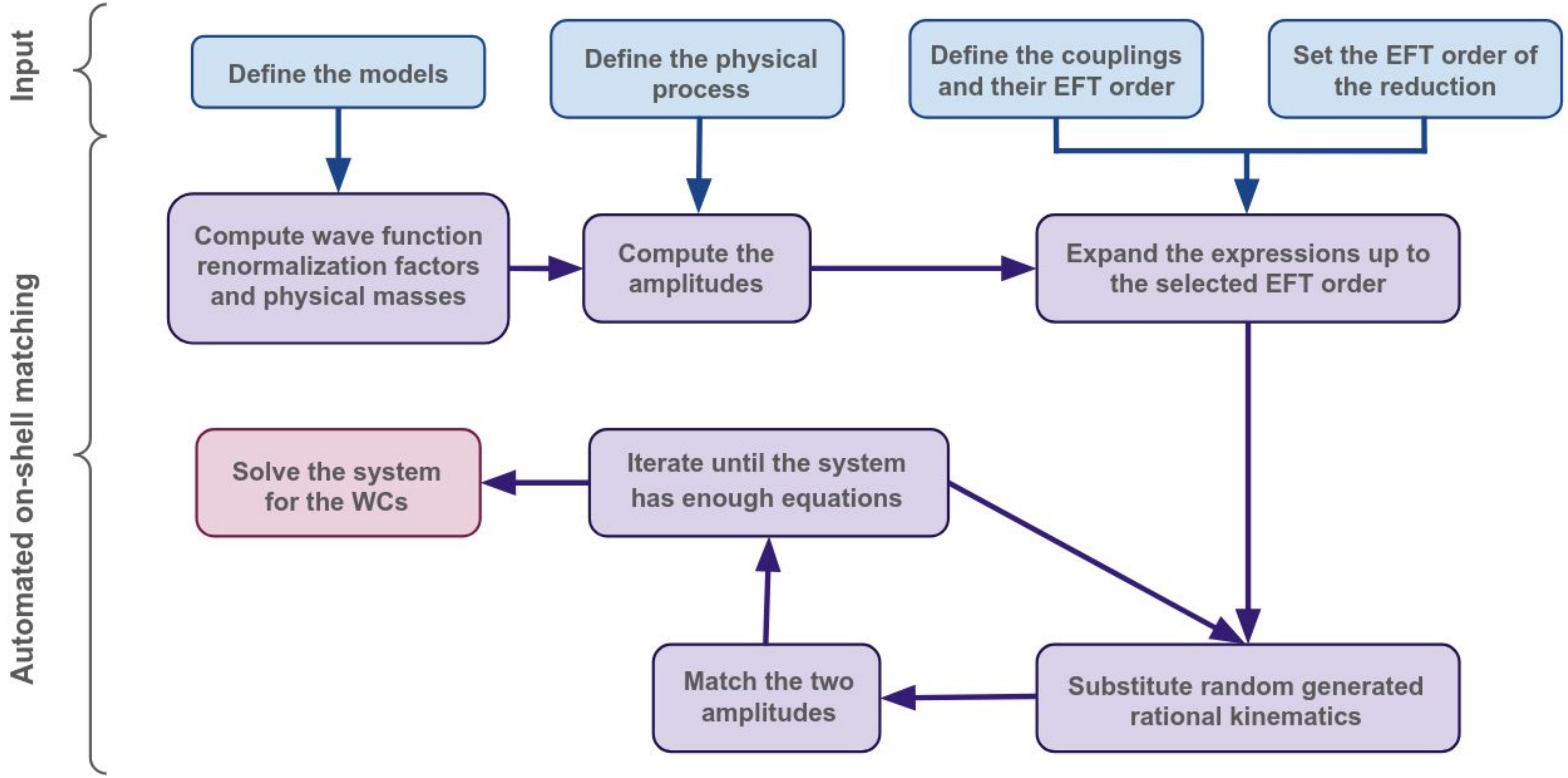
$$c_H \to c_H + \lambda^2 r_{DH} + \lambda r'_{HD} + m_0^2\Big(\frac{1}{4}g'^2 c_{HD} r_{2B} - \frac{1}{16}g'^4 r_{2B}^2 - \frac{1}{2}g' c_{HD} r_{BDH} + \frac{1}{2}g'^3 r_{2B} r_{BDH}$$

$$- \frac{3}{4}g'^2 r_{BDH}^2 - 6c_H r_{DH} - \lambda c_{HD} r_{DH} + 8\lambda c_{H\Box} r_{DH} + g'\lambda r_{BDH} r_{DH} - 11\lambda^2 r_{DH}^2$$

$$-\frac{1}{2}c_{HD} r'_{HD} + 4c_{H\Box} r'_{HD} + \frac{1}{2}g' r_{BDH} r'_{HD} - 9\lambda r_{DH} r'_{HD} - \frac{1}{4}r'^2_{HD} - r''^2_{HD}\Big)$$

Creating a tool: *MOSCA*

MOSCA: A MATCHING ON–SHELL CALCULATOR



Creating a tool: *MOSCA*

MOSCA: A MATCHING ON–SHELL CALCULATOR

Let us consider a Green's basis for a Z_2-symmetric scalar theory up to dimension 8. A suitable choice is given by

$$\mathcal{L} = \mathcal{L}_4 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \frac{1}{\Lambda^4}\mathcal{L}_8, \tag{10}$$

where

$$\mathcal{L}_4 = -\frac{1}{2}\phi(\partial^2 + m^2)\phi - \lambda\phi^4, \tag{11}$$

$$\mathcal{L}_6 = \alpha_{61}\phi^6 + \beta_{61}\partial^2\phi\partial^2\phi + \beta_{62}\phi^3\partial^2\phi, \tag{12}$$

$$\mathcal{L}_8 = \alpha_{81}\phi^8 + \alpha_{82}\phi^2\partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi + \beta_{81}\phi\partial^6\phi + \beta_{82}\phi^3\partial^4\phi + \beta_{83}\phi^2\partial^2\phi\partial^2\phi + \beta_{84}\phi^5\partial^2\phi. \tag{13}$$

β are the redundant WCs (they are in the Green's basis but not in the physical one)

Conclusions

- On-shell matching is a **diagrammatic alternative** that allows to compute the **coefficients directly into the physical basis**.
- **More diagrams** to be computed in a single amplitude, but **several operators can be matched** at the **same time**.
- **No** need of defining **evanescent operators** or using **background field method** for finite matching.
- Useful for **renormalization** and computing **beta functions** (see [arXiv:2409.15408]).
- Can be used to compute the **reduction to any physical basis** from any redundant basis.

↳ Code in `Mathematica` in progress (***MOSCA***) based in `FeynRules`+`FeynArts`+`FeynCalc` (with F. Vilches)

FIXED BUGS

- $(p_i \cdot p_j)^n$ structures
- $\epsilon^{\mu\nu\rho\sigma}\gamma_\mu$ structures
- Duplicate components in 'FFSSS'

Universidad de Granada

High Energy Theory

THANKS FOR YOUR ATTENTION !

Green's basis vs physical basis

EFT Lagrangian : $$\mathcal{L} = \mathcal{L}^{(4)} + \sum_{d>4} \sum_i \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}$$

Valid operators

- Local operators
- Preserve the symmetries of the Lagrangian

Finite number of operators → Integration by parts, Fierz identities (4D), etc. → **Green's basis**

Some operators are still **redundant** on-shell → **Physical basis**

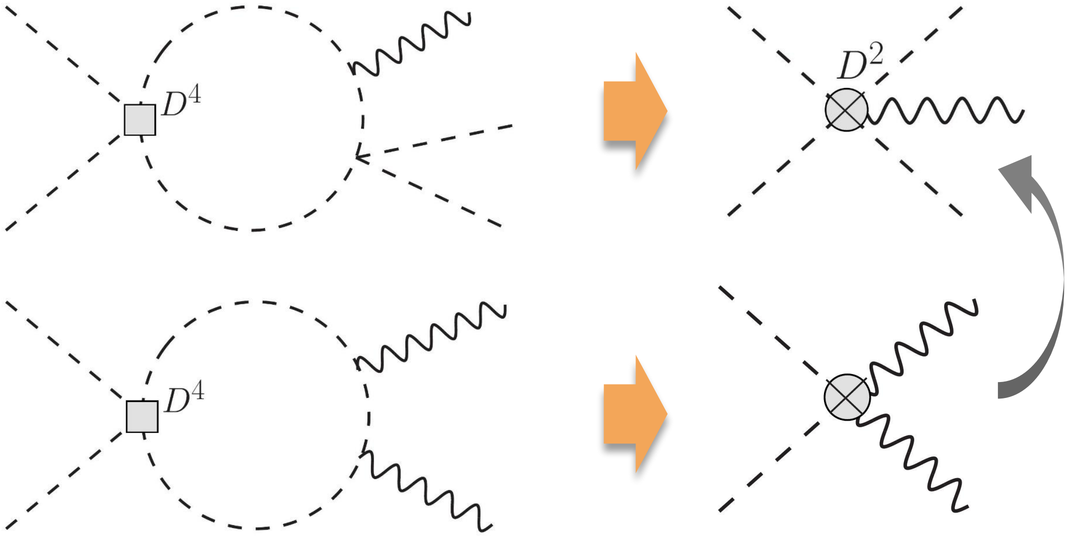
Applications: on-shell RGEs

A theory expressed in terms of the physical basis can still generate redundant operators at the loop level through RGEs.

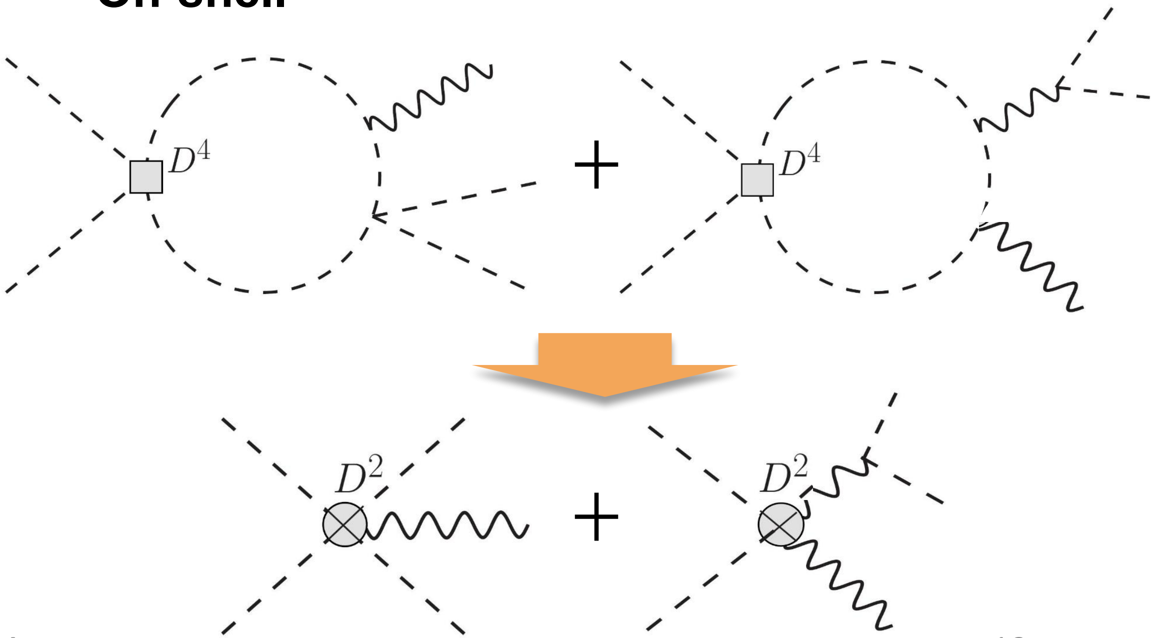
$$\mathcal{L}^{\text{div}} = \sum_j \frac{1}{16\pi^2\epsilon} c'_j(c)\mathcal{O}_j \quad \Rightarrow \quad 16\pi^2\mu\frac{\mathrm{d}c_i}{\mathrm{d}\mu} = -2c'_i \qquad \textit{(RGEs)}$$

Mixing of H^4D^4 into H^4BD^2

Off-shell



On-shell



Some results in the SMEFT

Dimension 4 up to dimension 8 (H and B)

$$m_0^2 \to m_0^2 - m_0^4 r_{DH} + 2m_0^6 r_{DH}^2$$

$$\lambda \to \lambda - m_0^2(4\lambda r_{DH} + 2r'_{HD}) + m_0^4(16\lambda r_{DH}^2 + 10 r_{DH} r'_{HD})$$

$$y_E \to y_E(1 - m_o^2 r_{DH} + \frac{5}{2} m_0^4 r_{DH}^2)$$

Some results in the SMEFT

Dimension 6 up to dimension 8 (H and B)

X^2H^2

$$c_{HB} \to c_{HB} - 2m_0^2 c_{HB} r_{DH}$$
$$c_{H\tilde{B}} \to c_{H\tilde{B}} - 2m_0^2 c_{H\tilde{B}} r_{DH}$$

H^4D^2

$$c_{H\Box} \to c_{H\Box} - \frac{1}{8} g'^2 r_{2B} + \frac{1}{2} g' r_{BDH} - m_0^2 (4 c_{H\Box} r_{DH} + g' r_{BDH} r_{DH} + 2 r_{DH} r'_{HD})$$
$$c_{HD} \to c_{HD} - \frac{1}{2} g'^2 r_{2B} + 2 g' r_{BDH} - m_0^2 (4 c_{HD} r_{DH} + 4 g' r_{BDH} r_{DH})$$

H^6

$$c_H \to c_H + \lambda^2 r_{DH} + \lambda r'_{HD} + m_0^2 \left(\frac{1}{4} g'^2 c_{HD} r_{2B} - \frac{1}{16} g'^4 r_{2B}^2 - \frac{1}{2} g' c_{HD} r_{BDH} + \frac{1}{2} g'^3 r_{2B} r_{BDH} \right.$$
$$- \frac{3}{4} g'^2 r_{BDH}^2 - 6 c_H r_{DH} - \lambda c_{HD} r_{DH} + 8 \lambda c_{H\Box} r_{DH} + g' \lambda r_{BDH} r_{DH} - 11 \lambda^2 r_{DH}^2$$
$$\left. - \frac{1}{2} c_{HD} r'_{HD} + 4 c_{H\Box} r'_{HD} + \frac{1}{2} g' r_{BDH} r'_{HD} - 9 \lambda r_{DH} r'_{HD} - \frac{1}{4} r'^2_{HD} - r''^2_{HD} \right)$$

Some results in the SMEFT

Dimension 8 (H and B)

XH^4D^2

$$c^{(1)}_{BH^4D^2} \to c^{(1)}_{BH^4D^2} - 4g' c_{HB} r_{2B} + \frac{1}{2} g'^3 r_{2B}^2 + 8 c_{HB} r_{BDH} - 2 g'^2 r_{2B} r_{BDH} + 2 g' r_{BDH}^2$$

$$c^{(2)}_{BH^4D^2} \to -4g' c_{H\tilde{B}} r_{2B} + 8 c_{H\tilde{B}} r_{BDH}$$

$X^2H^2D^2$

$$c^{(1)}_{B^2H^2D^2} \to 0$$

$$c^{(2)}_{B^2H^2D^2} \to 0$$

$$c^{(3)}_{B^2H^2D^2} \to 0$$

X^4

$$c^{(1)}_{B^4} \to 0$$

$$c^{(2)}_{B^4} \to 0$$

$$c^{(3)}_{B^4} \to 0$$

X^2H^4

$$c^{(1)}_{B^2H^4} \to -c_{HB} g'^2 r_{2B} + \frac{1}{16} g'^4 r_{2B}^2 + 2 c_{HB} g' r_{BDH} - \frac{1}{4} g'^3 r_{2B} r_{BDH} + \frac{1}{4} g'^2 r_{BDH}^2 - 2 c_{HB} \lambda r_{DH} - c_{HB} r'_{HD}$$

$$c^{(2)}_{B^2H^4} \to -g'^2 c_{H\tilde{B}} r_{2B} + 2 g' c_{H\tilde{B}} r_{BDH} - 2\lambda c_{H\tilde{B}} r_{BDH} - c_{H\tilde{B}} r'_{HD}$$

Some results in the SMEFT

Dimension 8 (H and B)

H^4D^4

$$c_{H^4}^{(1)} \to \frac{1}{2}g'^2 r_{2B}^2 - 2g' r_{2B} r_{BDH} + 2r_{BDH}^2$$

$$c_{H^4}^{(2)} \to -\frac{1}{2}g'^2 r_{2B}^2 + 2g' r_{2B} r_{BDH} - 2r_{BDH}^2$$

$$c_{H^4}^{(3)} \to 0$$

H^6D^2

$$\begin{aligned} c_{H^6}^{(1)} \to & -\frac{3}{4}g'^2 c_{HD} r_{2B} + \frac{3}{16}g'^4 r_{2B}^2 + \frac{3}{2}g' c_{HD} r_{BDH} - \frac{3}{2}g'^3 r_{2B} r_{BDH} + \frac{9}{4}g'^2 r_{BDH}^2 - \lambda c_{HD} r_{DH} \\ & - 8\lambda c_{H\Box} r_{DH} - 3g'\lambda r_{BDH} r_{DH} + \lambda^2 r_{DH}^2 - \frac{1}{2}c_{HD} r'_{HD} - 4c_{H\Box} r'_{HD} - \frac{3}{2}g' r_{BDH} r'_{HD} \\ & - 3\lambda r_{DH} r'_{HD} - \frac{7}{4}r'^2_{HD} + r''^2_{HD} \end{aligned}$$

$$\begin{aligned} c_{H^6}^{(2)} \to & -\frac{1}{2}g'^2 c_{HD} r_{2B} + \frac{1}{8}g'^4 r_{2B}^2 + g' c_{HD} r_{BDH} - g'^3 r_{2B} r_{BDH} + \frac{3}{2}g'^2 r_{BDH}^2 - 2\lambda c_{HD} r_{DH} \\ & - 2g'\lambda r_{BDH} r_{DH} - c_{HD} r'_{HD} - g' r_{BDH} r'_{HD} \end{aligned}$$

Generation of random momenta

$$SL(2,\mathbb{C}) \cong SU(2)_L \times SU(2)_R \quad \begin{cases} \lambda \in SU(2)_L \\ \tilde{\lambda} \in SU(2)_R \end{cases} \quad \begin{aligned} \lambda^{\alpha} &= \varepsilon^{\alpha\beta}\lambda_{\beta} \\ \tilde{\lambda}_{\dot{\alpha}} &= \varepsilon_{\dot{\alpha}\dot{\beta}}\tilde{\lambda}^{\dot{\beta}} \end{aligned}$$

Massless momenta :

$$P_{\alpha\dot{\alpha}} = \lambda_{\alpha}\tilde{\lambda}_{\dot{\alpha}} \Rightarrow P = p_{\mu}\sigma^{\mu} = \begin{pmatrix} p_0 + p_3 & p_1 - ip_2 \\ p_1 + ip_2 & p_0 - p_3 \end{pmatrix}$$

Massive momenta :

$$P^{\mu} := q^{\mu} + \frac{m^2}{2q\cdot k}k^{\mu}$$

$$\begin{aligned} q^2, k^2 &= 0 \\ q_{\alpha\dot{\alpha}} &= \lambda_{\alpha}\tilde{\lambda}_{\dot{\alpha}} \\ k_{\alpha\dot{\alpha}} &= \mu_{\alpha}\tilde{\mu}_{\dot{\alpha}} \end{aligned}$$

Evanescent operators

$$\mathcal{R} = \alpha\,\mathcal{O} \xrightarrow{d=4-2\epsilon} \mathcal{R} = \alpha\mathcal{O} + \mathcal{E}$$

$\mathcal{O}(\epsilon)$

Additional finite local contributions in loop amplitudes

$$\boldsymbol{IR^{(0)} + IR^{(1)} = UV^{(0)} + UV^{(1)}}$$

$$\boldsymbol{IR^{(0)} + IR^{(1)}_{soft} = UV^{(0)} + UV^{(1)}_{hard} + UV^{(1)}_{soft}}$$

We take the hard region

$$\int \mathcal{O} = \frac{1}{\epsilon}(a + b_{\mathcal{O}}\epsilon)$$

$$\int \mathcal{R} = \frac{1}{\epsilon}(a + b_{\mathcal{R}}\epsilon)$$

$$\int \mathcal{R} - \mathcal{O} = \frac{1}{\epsilon}(b_{\mathcal{R}}\epsilon - b_{\mathcal{O}}\epsilon) = b$$

Matching a theory: *off-shell vs on-shell*

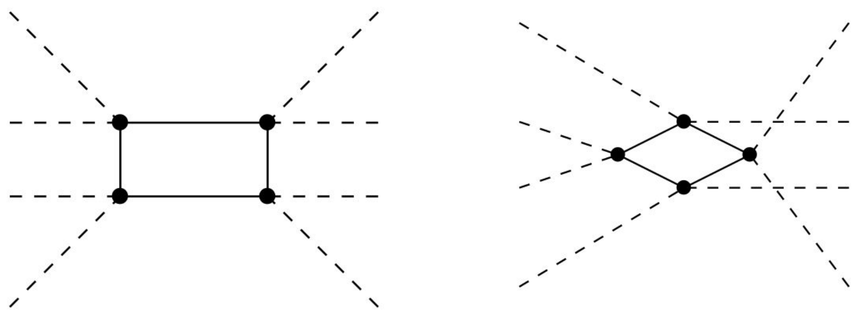
OFF-SHELL	VS	ON-SHELL
Small number of diagrams (1lPI in UV, 1PI in IR)		All diagrams (light bridges too)

Matching a theory: *off-shell vs on-shell*

$$\mathcal{L}_{\text{int}} = \eta\phi^2\Phi^2 + \lambda\phi^4$$

OFF-SHELL

$$\phi \quad \phi \quad \phi \quad \phi \to \phi \quad \phi \quad \phi \quad \phi$$

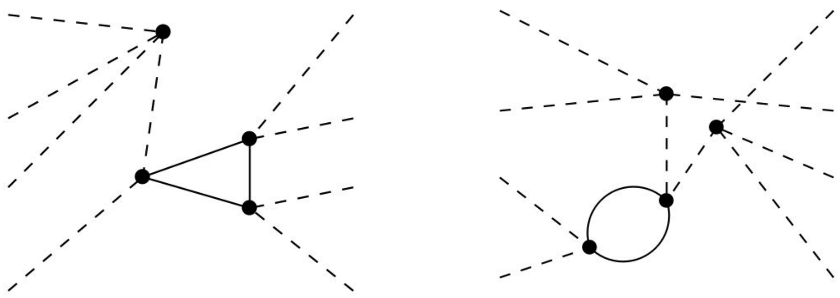


In[66]:= `Length[diags1lPIHeavy]`

Out[66]= 315

ON-SHELL

$$\phi \quad \phi \quad \phi \quad \phi \to \phi \quad \phi \quad \phi \quad \phi$$



In[65]:= `Length[diagsHeavy]`

Out[65]= 6475

Matching a theory: *off-shell vs on-shell*

OFF-SHELL	VS	ON-SHELL
Small number of diagrams (one-light-particle-irreducible)	1 – 0	All diagrams (light bridges too)

Matching a theory: *off-shell vs on-shell*

OFF-SHELL	VS	ON-SHELL
Small number of diagrams (one-light-particle-irreducible)	1 – 0	All diagrams (light bridges too)
Large set of operators (Green's basis)		Smaller set of operators (physical basis)

Matching a theory: *off-shell vs on-shell*

[2003.12525]

$\psi^2 D^3$	
$\mathcal{O}_{qD}$	$\frac{i}{2}\overline{q}\left\{D_\mu D^\mu, \not{D}\right\} q$
$\mathcal{O}_{uD}$	$\frac{i}{2}\overline{u}\left\{D_\mu D^\mu, \not{D}\right\} u$
$\mathcal{O}_{dD}$	$\frac{i}{2}\overline{d}\left\{D_\mu D^\mu, \not{D}\right\} d$
$\mathcal{O}_{\ell D}$	$\frac{i}{2}\overline{\ell}\left\{D_\mu D^\mu, \not{D}\right\} \ell$
$\mathcal{O}_{eD}$	$\frac{i}{2}\overline{e}\left\{D_\mu D^\mu, \not{D}\right\} e$

$\psi^2 H D^2 + \text{h.c.}$	
$\mathcal{O}_{uHD1}$	$(\overline{q}u)D_\mu D^\mu \widetilde{H}$
$\mathcal{O}_{uHD2}$	$(\overline{q}i\sigma_{\mu\nu}D^\mu u)D^\nu \widetilde{H}$
$\mathcal{O}_{uHD3}$	$(\overline{q}D_\mu D^\mu u)\widetilde{H}$
$\mathcal{O}_{uHD4}$	$(\overline{q}D_\mu u)D^\mu \widetilde{H}$
$\mathcal{O}_{dHD1}$	$(\overline{q}d)D_\mu D^\mu H$
$\mathcal{O}_{dHD2}$	$(\overline{q}i\sigma_{\mu\nu}D^\mu d)D^\nu H$
$\mathcal{O}_{dHD3}$	$(\overline{q}D_\mu D^\mu d)H$
$\mathcal{O}_{dHD4}$	$(\overline{q}D_\mu d)D^\mu H$
$\mathcal{O}_{eHD1}$	$(\overline{\ell}e)D_\mu D^\mu H$
$\mathcal{O}_{eHD2}$	$(\overline{\ell}i\sigma_{\mu\nu}D^\mu e)D^\nu H$
$\mathcal{O}_{eHD3}$	$(\overline{\ell}D_\mu D^\mu e)H$
$\mathcal{O}_{eHD4}$	$(\overline{\ell}D_\mu e)D^\mu H$

$\psi^2 X H + \text{h.c.}$	
$\mathcal{O}_{uG}$	$(\overline{q}T^A\sigma^{\mu\nu}u)\widetilde{H}G^A_{\mu\nu}$
$\mathcal{O}_{uW}$	$(\overline{q}\sigma^{\mu\nu}u)\sigma^I\widetilde{H}W^I_{\mu\nu}$
$\mathcal{O}_{uB}$	$(\overline{q}\sigma^{\mu\nu}u)\widetilde{H}B_{\mu\nu}$
$\mathcal{O}_{dG}$	$(\overline{q}T^A\sigma^{\mu\nu}d)HG^A_{\mu\nu}$
$\mathcal{O}_{dW}$	$(\overline{q}\sigma^{\mu\nu}d)\sigma^I HW^I_{\mu\nu}$
$\mathcal{O}_{dB}$	$(\overline{q}\sigma^{\mu\nu}d)HB_{\mu\nu}$
$\mathcal{O}_{eW}$	$(\overline{\ell}\sigma^{\mu\nu}e)\sigma^I HW^I_{\mu\nu}$
$\mathcal{O}_{eB}$	$(\overline{\ell}\sigma^{\mu\nu}e)HB_{\mu\nu}$

$\psi^2 X D$	
$\mathcal{O}_{Gq}$	$(\overline{q}T^A\gamma^\mu q)D^\nu G^A_{\mu\nu}$
$\mathcal{O}'_{Gq}$	$\frac{1}{2}(\overline{q}T^A\gamma^\mu i\overleftrightarrow{D}^\nu q)G^A_{\mu\nu}$
$\mathcal{O}'_{\widetilde{G}q}$	$\frac{1}{2}(\overline{q}T^A\gamma^\mu i\overleftrightarrow{D}^\nu q)\widetilde{G}^A_{\mu\nu}$
$\mathcal{O}_{Wq}$	$(\overline{q}\sigma^I\gamma^\mu q)D^\nu W^I_{\mu\nu}$
$\mathcal{O}'_{Wq}$	$\frac{1}{2}(\overline{q}\sigma^I\gamma^\mu i\overleftrightarrow{D}^\nu q)W^I_{\mu\nu}$
$\mathcal{O}'_{\widetilde{W}q}$	$\frac{1}{2}(\overline{q}\sigma^I\gamma^\mu i\overleftrightarrow{D}^\nu q)\widetilde{W}^I_{\mu\nu}$
$\mathcal{O}_{Bq}$	$(\overline{q}\gamma^\mu q)\partial^\nu B_{\mu\nu}$
$\mathcal{O}'_{Bq}$	$\frac{1}{2}(\overline{q}\gamma^\mu i\overleftrightarrow{D}^\nu q)B_{\mu\nu}$
$\mathcal{O}'_{\widetilde{B}q}$	$\frac{1}{2}(\overline{q}\gamma^\mu i\overleftrightarrow{D}^\nu q)\widetilde{B}_{\mu\nu}$
$\mathcal{O}_{Gu}$	$(\overline{u}T^A\gamma^\mu u)D^\nu G^A_{\mu\nu}$
$\mathcal{O}'_{Gu}$	$\frac{1}{2}(\overline{u}T^A\gamma^\mu i\overleftrightarrow{D}^\nu u)G^A_{\mu\nu}$
$\mathcal{O}'_{\widetilde{G}u}$	$\frac{1}{2}(\overline{u}T^A\gamma^\mu i\overleftrightarrow{D}^\nu u)\widetilde{G}^A_{\mu\nu}$
$\mathcal{O}_{Bu}$	$(\overline{u}\gamma^\mu u)\partial^\nu B_{\mu\nu}$
$\mathcal{O}'_{Bu}$	$\frac{1}{2}(\overline{u}\gamma^\mu i\overleftrightarrow{D}^\nu u)B_{\mu\nu}$
$\mathcal{O}'_{\widetilde{B}u}$	$\frac{1}{2}(\overline{u}\gamma^\mu i\overleftrightarrow{D}^\nu u)\widetilde{B}_{\mu\nu}$
$\mathcal{O}_{Gd}$	$(\overline{d}T^A\gamma^\mu d)D^\nu G^A_{\mu\nu}$
$\mathcal{O}'_{Gd}$	$\frac{1}{2}(\overline{d}T^A\gamma^\mu i\overleftrightarrow{D}^\nu d)G^A_{\mu\nu}$
$\mathcal{O}'_{\widetilde{G}d}$	$\frac{1}{2}(\overline{d}T^A\gamma^\mu i\overleftrightarrow{D}^\nu d)\widetilde{G}^A_{\mu\nu}$
$\mathcal{O}_{Bd}$	$(\overline{d}\gamma^\mu d)\partial^\nu B_{\mu\nu}$
$\mathcal{O}'_{Bd}$	$\frac{1}{2}(\overline{d}\gamma^\mu i\overleftrightarrow{D}^\nu d)B_{\mu\nu}$
$\mathcal{O}'_{\widetilde{B}d}$	$\frac{1}{2}(\overline{d}\gamma^\mu i\overleftrightarrow{D}^\nu d)\widetilde{B}_{\mu\nu}$
$\mathcal{O}_{W\ell}$	$(\overline{\ell}\sigma^I\gamma^\mu \ell)D^\nu W^I_{\mu\nu}$
$\mathcal{O}'_{W\ell}$	$\frac{1}{2}(\overline{\ell}\sigma^I\gamma^\mu i\overleftrightarrow{D}^\nu \ell)W^I_{\mu\nu}$
$\mathcal{O}'_{\widetilde{W}\ell}$	$\frac{1}{2}(\overline{\ell}\sigma^I\gamma^\mu i\overleftrightarrow{D}^\nu \ell)\widetilde{W}^I_{\mu\nu}$
$\mathcal{O}_{B\ell}$	$(\overline{\ell}\gamma^\mu \ell)\partial^\nu B_{\mu\nu}$
$\mathcal{O}'_{B\ell}$	$\frac{1}{2}(\overline{\ell}\gamma^\mu i\overleftrightarrow{D}^\nu \ell)B_{\mu\nu}$
$\mathcal{O}'_{\widetilde{B}\ell}$	$\frac{1}{2}(\overline{\ell}\gamma^\mu i\overleftrightarrow{D}^\nu \ell)\widetilde{B}_{\mu\nu}$
$\mathcal{O}_{Be}$	$(\overline{e}\gamma^\mu e)\partial^\nu B_{\mu\nu}$
$\mathcal{O}'_{Be}$	$\frac{1}{2}(\overline{e}\gamma^\mu i\overleftrightarrow{D}^\nu e)B_{\mu\nu}$
$\mathcal{O}'_{\widetilde{B}e}$	$\frac{1}{2}(\overline{e}\gamma^\mu i\overleftrightarrow{D}^\nu e)\widetilde{B}_{\mu\nu}$

$\psi^2 D H^2$	
$\mathcal{O}^{(1)}_{Hq}$	$(\overline{q}\gamma^\mu q)(H^\dagger i\overleftrightarrow{D}_\mu H)$
$\mathcal{O}'^{(1)}_{Hq}$	$(\overline{q}i\overleftrightarrow{\not{D}} q)(H^\dagger H)$
$\mathcal{O}''^{(1)}_{Hq}$	$(\overline{q}\gamma^\mu q)\partial_\mu(H^\dagger H)$
$\mathcal{O}^{(3)}_{Hq}$	$(\overline{q}\sigma^I\gamma^\mu q)(H^\dagger i\overleftrightarrow{D}^I_\mu H)$
$\mathcal{O}'^{(3)}_{Hq}$	$(\overline{q}i\overleftrightarrow{\not{D}}^I q)(H^\dagger\sigma^I H)$
$\mathcal{O}''^{(3)}_{Hq}$	$(\overline{q}\sigma^I\gamma^\mu q)D_\mu(H^\dagger\sigma^I H)$
$\mathcal{O}_{Hu}$	$(\overline{u}\gamma^\mu u)(H^\dagger i\overleftrightarrow{D}_\mu H)$
$\mathcal{O}'_{Hu}$	$(\overline{u}i\overleftrightarrow{\not{D}} u)(H^\dagger H)$
$\mathcal{O}''_{Hu}$	$(\overline{u}\gamma^\mu u)\partial_\mu(H^\dagger H)$
$\mathcal{O}_{Hd}$	$(\overline{d}\gamma^\mu d)(H^\dagger i\overleftrightarrow{D}_\mu H)$
$\mathcal{O}'_{Hd}$	$(\overline{d}i\overleftrightarrow{\not{D}} d)(H^\dagger H)$
$\mathcal{O}''_{Hd}$	$(\overline{d}\gamma^\mu d)\partial_\mu(H^\dagger H)$
$\mathcal{O}_{Hud}$	$(\overline{u}\gamma^\mu d)(\widetilde{H}^\dagger iD_\mu H)$
$\mathcal{O}^{(1)}_{H\ell}$	$(\overline{\ell}\gamma^\mu \ell)(H^\dagger i\overleftrightarrow{D}_\mu H)$
$\mathcal{O}'^{(1)}_{H\ell}$	$(\overline{\ell}i\overleftrightarrow{\not{D}} \ell)(H^\dagger H)$
$\mathcal{O}''^{(1)}_{H\ell}$	$(\overline{\ell}\gamma^\mu \ell)\partial_\mu(H^\dagger H)$
$\mathcal{O}^{(3)}_{H\ell}$	$(\overline{\ell}\sigma^I\gamma^\mu \ell)(H^\dagger i\overleftrightarrow{D}^I_\mu H)$
$\mathcal{O}'^{(3)}_{H\ell}$	$(\overline{\ell}i\overleftrightarrow{\not{D}}^I \ell)(H^\dagger\sigma^I H)$
$\mathcal{O}''^{(3)}_{H\ell}$	$(\overline{\ell}\sigma^I\gamma^\mu \ell)D_\mu(H^\dagger\sigma^I H)$
$\mathcal{O}_{He}$	$(\overline{e}\gamma^\mu e)(H^\dagger i\overleftrightarrow{D}_\mu H)$
$\mathcal{O}'_{He}$	$(\overline{e}i\overleftrightarrow{\not{D}} e)(H^\dagger H)$
$\mathcal{O}''_{He}$	$(\overline{e}\gamma^\mu e)\partial_\mu(H^\dagger H)$

$\psi^2 H^3 + \text{h.c.}$	
$\mathcal{O}_{uH}$	$(H^\dagger H)\overline{q}\widetilde{H}u$
$\mathcal{O}_{dH}$	$(H^\dagger H)\overline{q}Hd$
$\mathcal{O}_{eH}$	$(H^\dagger H)\overline{\ell}He$

X^3	
$\mathcal{O}_{3G}$	$f^{ABC}G^{A\nu}_\mu G^{B\rho}_\nu G^{C\mu}_\rho$
$\mathcal{O}_{3\widetilde{G}}$	$f^{ABC}\widetilde{G}^{A\nu}_\mu G^{B\rho}_\nu G^{C\mu}_\rho$
$\mathcal{O}_{3W}$	$\epsilon^{IJK}W^{I\nu}_\mu W^{J\rho}_\nu W^{K\mu}_\rho$
$\mathcal{O}_{3\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}^{I\nu}_\mu W^{J\rho}_\nu W^{K\mu}_\rho$

$X^2 D^2$	
$\mathcal{O}_{2G}$	$-\frac{1}{2}(D_\mu G^{A\mu\nu})(D^\rho G^A_{\rho\nu})$
$\mathcal{O}_{2W}$	$-\frac{1}{2}(D_\mu W^{I\mu\nu})(D^\rho W^I_{\rho\nu})$
$\mathcal{O}_{2B}$	$-\frac{1}{2}(\partial_\mu B^{\mu\nu})(\partial^\rho B_{\rho\nu})$

$X^2 H^2$	
$\mathcal{O}_{HG}$	$G^A_{\mu\nu}G^{A\mu\nu}(H^\dagger H)$
$\mathcal{O}_{H\widetilde{G}}$	$\widetilde{G}^A_{\mu\nu}G^{A\mu\nu}(H^\dagger H)$
$\mathcal{O}_{HW}$	$W^I_{\mu\nu}W^{I\mu\nu}(H^\dagger H)$
$\mathcal{O}_{H\widetilde{W}}$	$\widetilde{W}^I_{\mu\nu}W^{I\mu\nu}(H^\dagger H)$
$\mathcal{O}_{HB}$	$B_{\mu\nu}B^{\mu\nu}(H^\dagger H)$
$\mathcal{O}_{H\widetilde{B}}$	$\widetilde{B}_{\mu\nu}B^{\mu\nu}(H^\dagger H)$
$\mathcal{O}_{HWB}$	$W^I_{\mu\nu}B^{\mu\nu}(H^\dagger\sigma^I H)$
$\mathcal{O}_{H\widetilde{W}B}$	$\widetilde{W}^I_{\mu\nu}B^{\mu\nu}(H^\dagger\sigma^I H)$

$H^2 X D^2$	
$\mathcal{O}_{WDH}$	$D_\nu W^{I\mu\nu}(H^\dagger i\overleftrightarrow{D}^I_\mu H)$
$\mathcal{O}_{BDH}$	$\partial_\nu B^{\mu\nu}(H^\dagger i\overleftrightarrow{D}_\mu H)$

$H^2 D^4$	
$\mathcal{O}_{DH}$	$(D_\mu D^\mu H)^\dagger(D_\nu D^\nu H)$

$H^4 D^2$	
$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^\dagger(H^\dagger D_\mu H)$
$\mathcal{O}'_{HD}$	$(H^\dagger H)(D_\mu H)^\dagger(D^\mu H)$
$\mathcal{O}''_{HD}$	$(H^\dagger H)D_\mu(H^\dagger i\overleftrightarrow{D}^\mu H)$

H^6	
$\mathcal{O}_H$	$(H^\dagger H)^3$

Four-quark	
$\mathcal{O}^{(1)}_{qq}$	$(\overline{q}\gamma^\mu q)(\overline{q}\gamma_\mu q)$
$\mathcal{O}^{(3)}_{qq}$	$(\overline{q}\gamma^\mu\sigma^I q)(\overline{q}\gamma_\mu\sigma^I q)$
$\mathcal{O}_{uu}$	$(\overline{u}\gamma^\mu u)(\overline{u}\gamma_\mu u)$
$\mathcal{O}_{dd}$	$(\overline{d}\gamma^\mu d)(\overline{d}\gamma_\mu d)$
$\mathcal{O}^{(1)}_{ud}$	$(\overline{u}\gamma^\mu u)(\overline{d}\gamma_\mu d)$
$\mathcal{O}^{(8)}_{ud}$	$(\overline{u}\gamma^\mu T^A u)(\overline{d}\gamma_\mu T^A d)$
$\mathcal{O}^{(1)}_{qu}$	$(\overline{q}\gamma^\mu q)(\overline{u}\gamma_\mu u)$
$\mathcal{O}^{(8)}_{qu}$	$(\overline{q}\gamma^\mu T^A q)(\overline{u}\gamma_\mu T^A u)$
$\mathcal{O}^{(1)}_{qd}$	$(\overline{q}\gamma^\mu q)(\overline{d}\gamma_\mu d)$
$\mathcal{O}^{(8)}_{qd}$	$(\overline{q}\gamma^\mu T^A q)(\overline{d}\gamma_\mu T^A d)$
$\mathcal{O}^{(1)}_{quqd}$	$(\overline{q}^r u)\epsilon_{rs}(\overline{q}^s d)$
$\mathcal{O}^{(8)}_{quqd}$	$(\overline{q}^r T^A u)\epsilon_{rs}(\overline{q}^s T^A d)$

Four-lepton	
$\mathcal{O}_{\ell\ell}$	$(\overline{\ell}\gamma^\mu \ell)(\overline{\ell}\gamma_\mu \ell)$
$\mathcal{O}_{ee}$	$(\overline{e}\gamma^\mu e)(\overline{e}\gamma_\mu e)$
$\mathcal{O}_{\ell e}$	$(\overline{\ell}\gamma^\mu \ell)(\overline{e}\gamma_\mu e)$

Semileptonic	
$\mathcal{O}^{(1)}_{\ell q}$	$(\overline{\ell}\gamma^\mu \ell)(\overline{q}\gamma_\mu q)$
$\mathcal{O}^{(3)}_{\ell q}$	$(\overline{\ell}\gamma^\mu\sigma^I \ell)(\overline{q}\gamma_\mu\sigma^I q)$
$\mathcal{O}_{eu}$	$(\overline{e}\gamma^\mu e)(\overline{u}\gamma_\mu u)$
$\mathcal{O}_{ed}$	$(\overline{e}\gamma^\mu e)(\overline{d}\gamma_\mu d)$
$\mathcal{O}_{qe}$	$(\overline{q}\gamma^\mu q)(\overline{e}\gamma_\mu e)$
$\mathcal{O}_{\ell u}$	$(\overline{\ell}\gamma^\mu \ell)(\overline{u}\gamma_\mu u)$
$\mathcal{O}_{\ell d}$	$(\overline{\ell}\gamma^\mu \ell)(\overline{d}\gamma_\mu d)$
$\mathcal{O}_{\ell edq}$	$(\overline{\ell}e)(\overline{d}q)$
$\mathcal{O}^{(1)}_{\ell equ}$	$(\overline{\ell}^r e)\epsilon_{rs}(\overline{q}^s u)$
$\mathcal{O}^{(3)}_{\ell equ}$	$(\overline{\ell}^r\sigma^{\mu\nu} e)\epsilon_{rs}(\overline{q}^s\sigma_{\mu\nu} u)$

Physical basis ops: 59

Green's basis ops: 129

Matching a theory: *off-shell vs on-shell*

OFF-SHELL	VS	ON-SHELL
Small number of diagrams (one-light-particle-irreducible)	1 – 0	All diagrams (light bridges too)
Large set of operators (Green's basis)	1 – 1	Smaller set of operators (physical basis)

Matching a theory: *off-shell vs on-shell*

OFF-SHELL	VS	ON-SHELL
Small number of diagrams (one-light-particle-irreducible)	1 – 0	All diagrams (light bridges too)
Large set of operators (Green's basis)	1 – 1	Smaller set of operators (physical basis)
Reduction to the physical basis		No reduction is needed

Matching a theory: *off-shell vs on-shell*

Elimination of redundant operators in Green's basis via:

EOMs:

$$
\begin{aligned}
(D_\mu D^\mu H)^a &= m^2 H^a - \lambda(H^\dagger H)H^a - \bar{e} y_E^\dagger \ell^a + i(\sigma_2)^{ab}\bar{q}^b y_U u - \bar{d} y_D^\dagger q^a \ , \\
(D^\nu G_{\mu\nu})^A &= g_s(\bar{q}_i\gamma_\mu T^A q_i + \bar{u}_i\gamma_\mu T^A u_i + \bar{d}_i\gamma_\mu T^A d_i) \ , \\
(D^\nu W_{\mu\nu})^I &= \frac{g}{2}(H^\dagger i \overset{\leftrightarrow}{D}{}^I_\mu H + \bar{\ell}_\alpha\gamma_\mu\sigma^I\ell_\alpha + \bar{q}_i\gamma_\mu\sigma^I q_i) \ , \\
(\partial^\nu B_{\mu\nu}) &= g'(Y_H H^\dagger i \overset{\leftrightarrow}{D}_\mu H + \sum_f Y_f \bar{f}\gamma_\mu f) \ ,
\end{aligned}
$$

$$
\begin{aligned}
i\not{D}\ell &= y_E e H \ , \\
i\not{D}e &= y_E^\dagger H^\dagger \ell \ , \\
i\not{D}q &= y_U u \widetilde{H} + y_D d H \ , \\
i\not{D}u &= y_U^\dagger \widetilde{H}^\dagger q \ , \\
i\not{D}d &= y_D^\dagger H^\dagger q \ .
\end{aligned}
$$

Straightfoward *but* ...
only valid up to linear order!
[1811.09413]

Modified EOMs: [2210.14761]

Field redefinitions

Non-trivial process

Hard to program it in a systematic way

Matching a theory: *off-shell vs on-shell*

OFF-SHELL	VS	ON-SHELL
Small number of diagrams (one-light-particle-irreducible)	1 – 0	All diagrams (light bridges too)
Large set of operators (Green's basis)	1 – 1	Smaller set of operators (physical basis)
Reduction to the physical basis	1 – 2	No reduction is needed