Computing 2 loop RGEs with Matchmakereft

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MM RGEs at one loop

Matchmakereft, some basic info

- Matching and RGEs for arbitrary EFTs
- We renormalize/match off-shell 1PI amplitudes, i.e. the contributions to the Effective Action.
- We follow a diagrammatic approach, as opposed to path integral techniques

Carmona, AL, Olgoso, Santiago, 2112.10787

MM RGEs at one loop

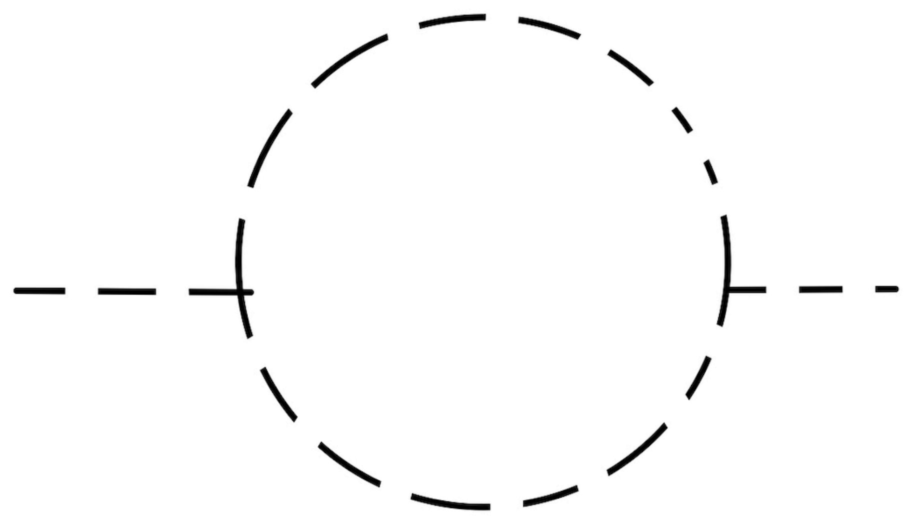
Life at one loop

- RGE calculation Based on the hard region expansion

- Hard region $k_1 \gg p_i, m_i$

- Leads to integrals that are either zero by fiat in dim-reg or to the logarithmically divergent single master integral: trivial to separate UV and IR

- No deeper poles, no ε-contributions from numerators, no evanescent structures, no problems with γ5



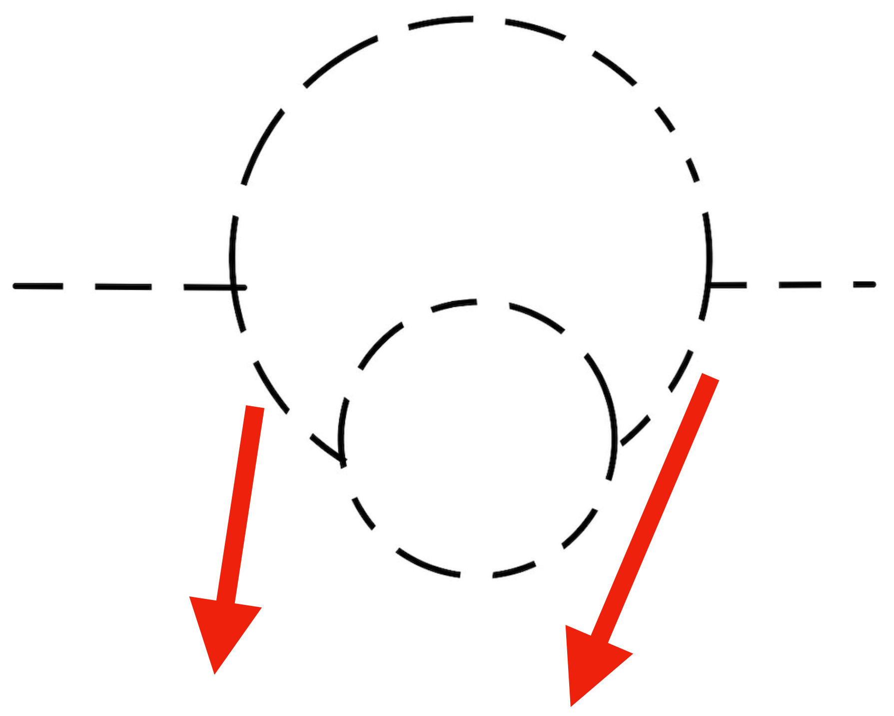
$$\int \frac{[dk_1]}{((k_1+p)^2-m^2)(k_1^2-m^2)} \to \int \frac{[dk_1]}{k_1^4} + \ldots \to \frac{i}{16\pi^2}\frac{1}{\epsilon}$$

$$\int \frac{[dk]}{k^2} = \frac{i}{16\pi^2}\left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}}\right)$$

MM RGEs at 2 loops

Life at two loops

- There are sub-divergences: only one of the loop momenta goes to infinity.
- UV and IR mix!
- ε-terms in numerators now affect the 1/ε coefficient
- Real and spurious IR singularities appear



$$\int \frac{[dk_1][dk_2]}{((k_1-p)^2-m^2)(k_1^2-m^2)^2(k_2^2-m^2)((k_1+k_2)^2-m^2)}$$

Note about IR

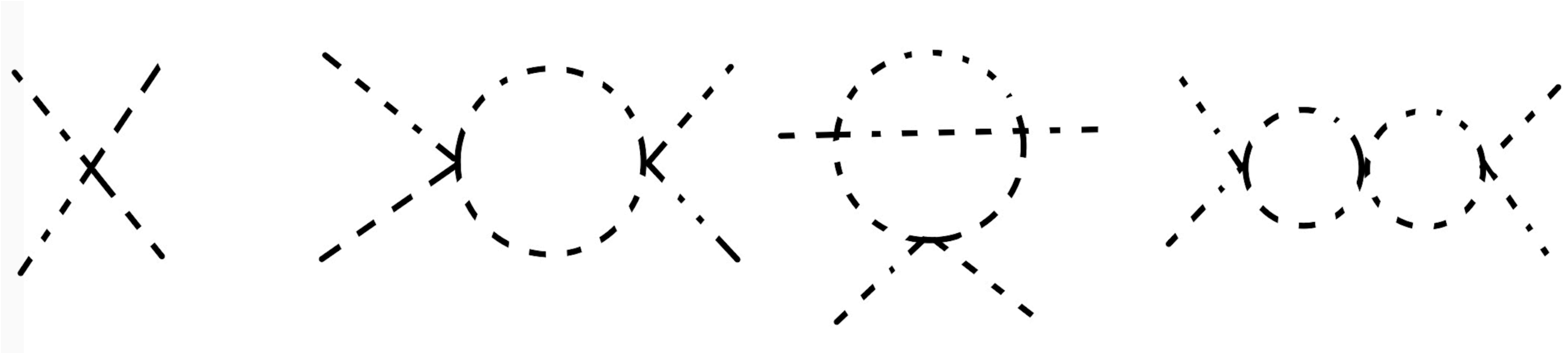
We are only interested in IR singularities in Euclidean space, i.e.

$$\frac{1}{(k+p_i)^2}$$

is never considered singular (while in Minkowski space there are soft/collinear singularities)

MM RGEs at 2 loops

One loop counterterms at two loops



$$\Gamma_{\phi^4} = -iZ_4C_4 + iZ_4^2\, C_4^2(\frac{x_1}{\epsilon} + f_1) + iC_4^3(\frac{x_{22}}{\epsilon^2} + \frac{x_{21}}{\epsilon})$$

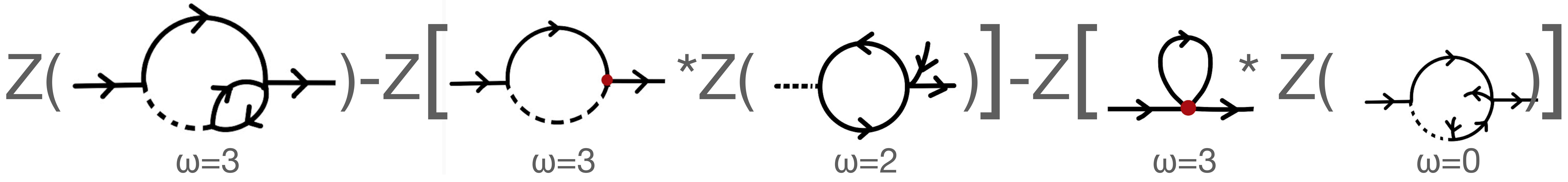
- Fully diagrammatic approach: we don't actually compute the Z_i's by making the 1PI amplitudes finite
- Just computing the UV poles of the two loop diagrams results in non-local poles ~ $log(p^2)$ that cancel with one loop FINITE terms times one loop counterterms.
- Would need to be careful about mass renormalization due to $m^{-2\epsilon}$

MM RGEs at 2 loops

BPHZ, forests and all that

- Classical approach in the absence of IR divergences: BPHZ formalism
- Each diagram, Γ, contributes by a counterterm, Δ(Γ)
- the sum of counterterms gives Z_i order by order.

$$\Delta(\Gamma) = Z(\Gamma) - \sum_i Z(\Gamma/\gamma_i * Z(\gamma_i))$$



Z(Γ) computes the overall divergence of the (sub)diagram, i.e. the UV singularity when all loop momenta are going to infinity

MM RGEs at 2 loops

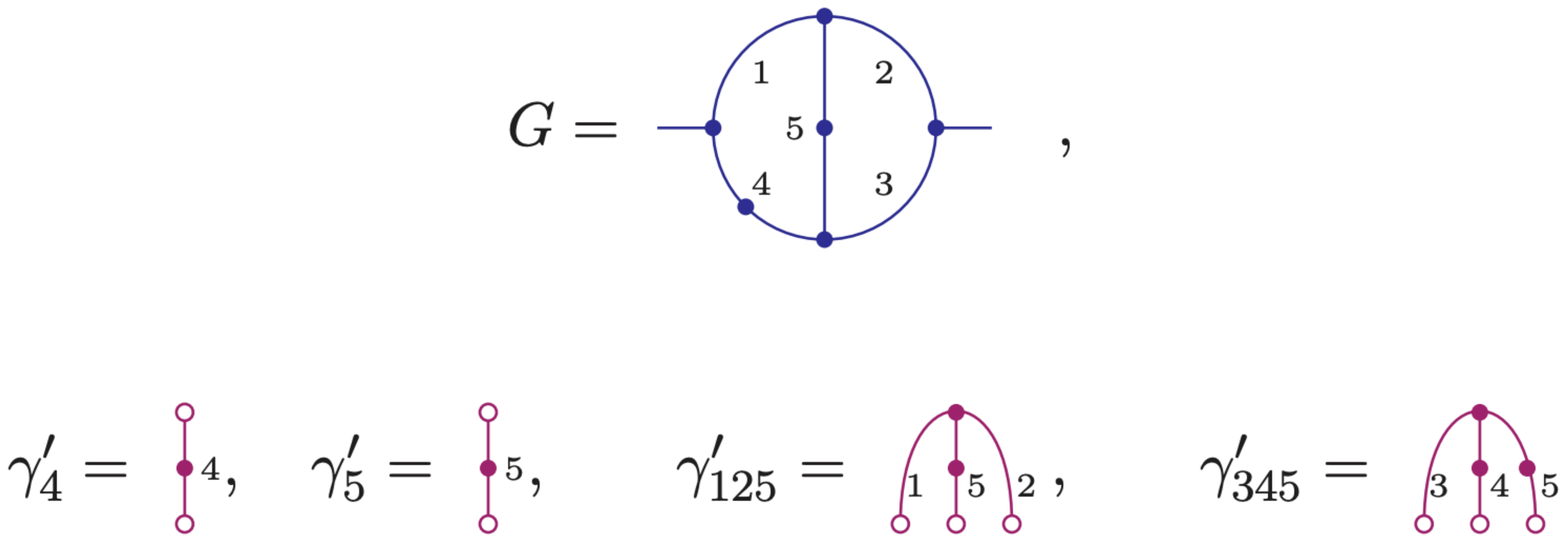
BPHZ, forests and all that

- Z(γ): since the overall divergence is always polynomial in external momenta and masses, taking enough derivatives would turn each contribution into a constant
- Derivatives crucially commute with the Z operation. We first take the derivatives and then compute the Z.
- Taylor expansion, similar to the hard region
- Results in “master” integrals that are independent of p_i and m_i: tadpoles, with arbitrary powers of propagators - can be reduced by IBPs, known to 5 loops Luthe and Schröder 1609.06786

MM RGEs at 2 loops

Dealing with IR, via R*

- Infrared divergences complicate Rbar.
- Solution 1: the R* operation (for a good explanation see Herzog and Ruijl 2017).
- Subtract IR singularities: define IR divergent sub-graphs (not loops) and perform a soft region expansion on them to get the IR pole.
- Much more complicated forest formulas.
- Used as a cross-check in collaboration with Franz Herzog and Sam Teale.



from Herzog and Ruijl, 1703.03776

MM RGEs at 2 loops

What we actually implement

- Solution 2: Rbar with an IR regulator mass.
- Implementing a modification of rQFT library by B. Ruijl Ruijl, Hirschi, Capati 2203.11038

$$\gamma \to \gamma^\lambda$$

$$\gamma^\lambda = \frac{\mathcal{N}\left(\{\lambda p_i\}_{i=1}^{n}, \{\lambda m_j\}_{j=1}^{l}, \{k_m\}_{m=1}^{L}\right)}{\prod_{e\in\mathbf{e}} D_e^\lambda},$$

$$D_e^\lambda = k_e^2 - m_{\mathrm{UV}}^2 + 2\lambda k_e \cdot p_e + \lambda^2 p_e^2 - \lambda^2 (m_e^2 - m_{\mathrm{UV}}^2)$$

$$Z(\gamma) = K\left(\mathcal{T}_{\omega;\{k_i\}}(\gamma^\lambda)\right)$$

Take the pole part

Taylor expand in λ, up to ω (exactly)!

MM RGEs at 2 loops

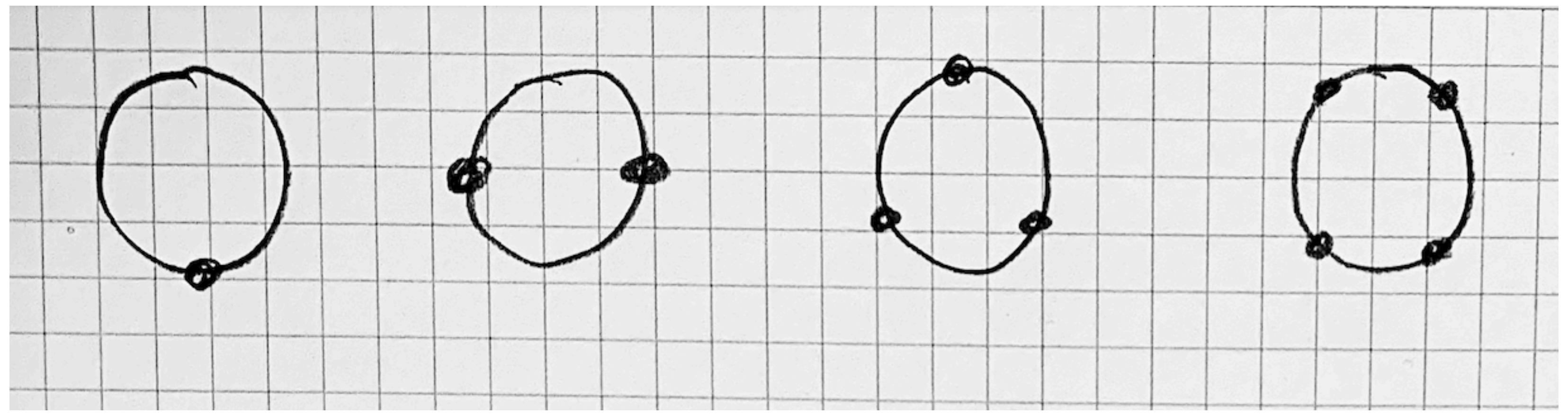
What we actually implement

- Solution 2: Rbar with an IR regulator mass.
- Implementing a modification of rQFT library by B. Ruijl Ruijl, Hirschi, Capati 2203.11038
- IR sub-divergences are regulated, master integrals are massive tadpoles.
- Various terms in $\Delta(\Gamma)$ will depend in the regulator M_uv, but their sum will not (if the prescription, and in particular the expansion depth is followed religiously)
- Extra care with tensor reduction and tensor structures in numerators: Z and tensor reduction do not commute!!!

MM RGEs at 2 loops

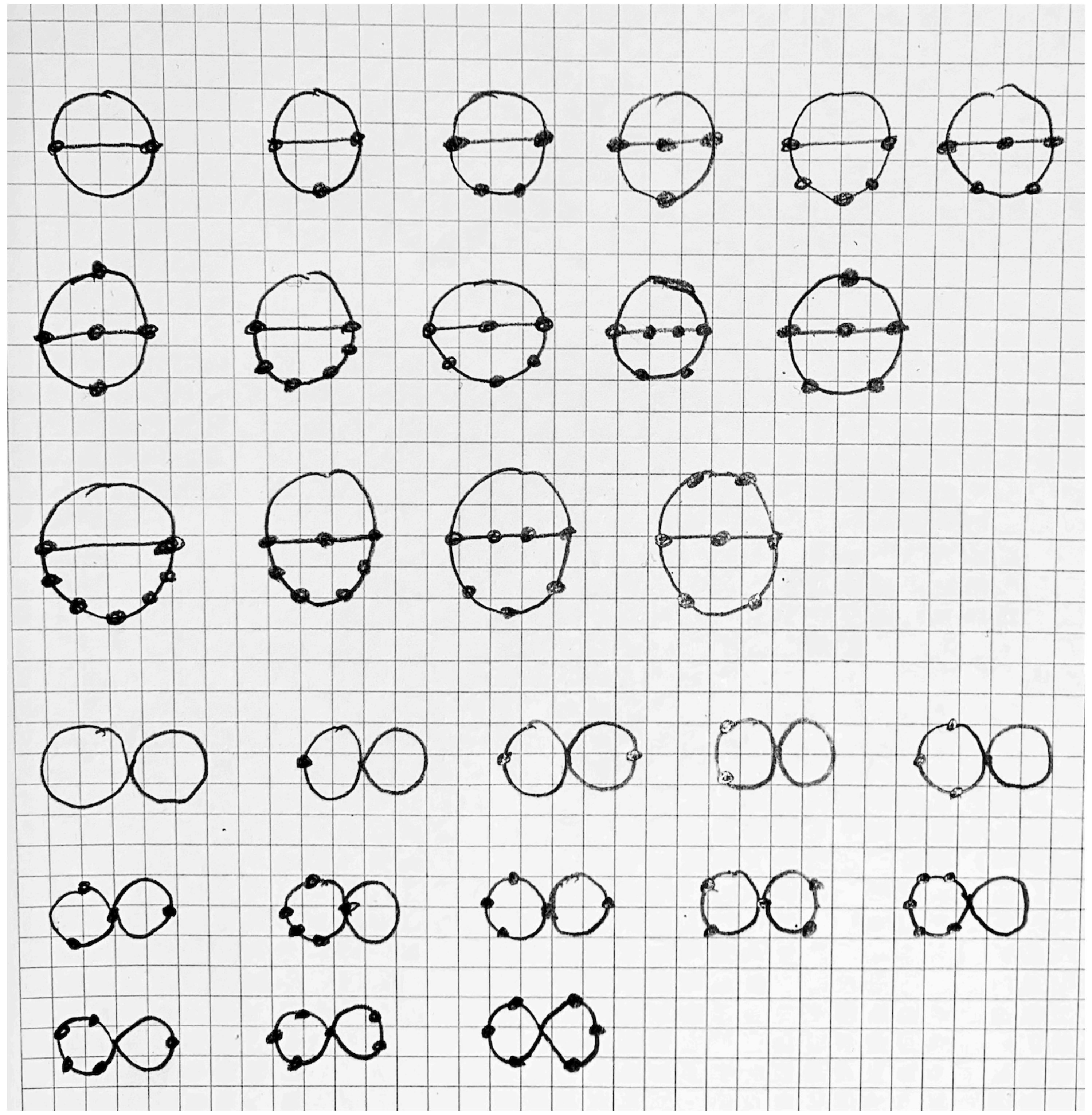
Aside: we could list all two loop diagrams that can be singular

List of un-dressed diagrams at 1 loop:



For every graph generate all the possible field-dressed graphs with fermions, scalars or gauge bosons, such that the resulting graph has ω>=0

List of un-dressed diagrams at 2 loops:



MM RGEs at 2 loops

Simplest non-trivial example

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m_\phi^2\phi^2 + i\bar{\psi}\partial\!\!\!/\psi - m_\psi\bar{\psi}\psi - \frac{C_3}{3!}\phi^3 - \frac{C_4}{4!}\phi^4 - C_{13}\phi\bar{\psi}\psi$$
$$- \frac{C_5}{5!\Lambda}\phi^5 - \frac{C_{23}}{2\Lambda}\bar{\psi}\psi\phi^2 - \frac{C_6}{6!\Lambda^2}\phi^6 - \frac{C_{33}}{3!\Lambda^2}\bar{\psi}\psi\phi^3$$
$$- \frac{C_{60}}{\Lambda^2}\bar{\psi}\psi\bar{\psi}\psi - \frac{C_{61}}{\Lambda^2}\bar{\psi}\gamma^\mu\psi\bar{\psi}\gamma_\mu\psi + \frac{C_{62}}{\Lambda^2}\bar{\psi}\sigma^{\mu\nu}\psi\bar{\psi}\sigma_{\mu\nu}\psi$$

Amplitude	# of diags
ϕ^2	22
ϕ^3	187
ϕ^4	1597
ϕ^5	15670
ϕ^6	197405
$\bar{\psi}\psi$	27
$\bar{\psi}\psi\phi$	160
$\bar{\psi}\psi\phi^2$	1011
$\bar{\psi}\psi\phi^3$	9945
$\bar{\psi}\psi\bar{\psi}\psi$	785

- Diagram isomorphisms to the rescue (197405 -> 902)
- reproduced decades old result for the renormalizable part see e.g. Machacek & Vaughn, 1983
- Double checking against R* code of Herzog and Teale

MM RGEs at 2 loops

Sample results from $\overline{\psi}\psi$

- Note that we renormalize off-shell
- All O(p^2, p^3) terms will be absorbed by redundant operators.
- After redundant operator contributions are mapped to physical, we can compute the RGEs for couplings and masses.

```
res =
    + gamma[s4,p1,s2]*poles[ - 9/8*ep^(-1)]*i_*mL^2*C13^2*C60
    + gamma[s4,p1,s2]*poles[ - ep^(-1)]*i_*mpsi^2*C13^2*C61
    + gamma[s4,p1,s2]*poles[ - ep^(-1)]*p1.p1*i_*C13^2*C62
    + gamma[s4,p1,s2]*poles[ - 13/16*ep^(-1)]*i_*C13^4
    + gamma[s4,p1,s2]*poles[ - 3/8*ep^(-1)]*i_*C3*C13*C23
    + gamma[s4,p1,s2]*poles[ - 3/8*ep^(-1)]*i_*mpsi*C13^2*C23
    + gamma[s4,p1,s2]*poles[ - 1/3*ep^(-1)]*p1.p1*i_*C13^2*C61
    + gamma[s4,p1,s2]*poles[ - 1/8*ep^(-1)]*i_*mpsi^2*C23^2
    + gamma[s4,p1,s2]*poles[ - 1/8*ep^(-1)]*i_*mL^2*C23^2
    + gamma[s4,p1,s2]*poles[1/24*ep^(-1)]*p1.p1*i_*C23^2
    + gamma[s4,p1,s2]*poles[1/4*ep^(-1)]*p1.p1*i_*C13^2*C60
    + gamma[s4,p1,s2]*poles[ep^(-1)]*i_*mL^2*C13^2*C62
    + gamma[s4,p1,s2]*poles[ep^(-1)]*i_*mL^2*C13^2*C61
    + gamma[s4,p1,s2]*poles[3*ep^(-1)]*i_*mpsi^2*C13^2*C60
    + gamma[s4,p1,s2]*poles[9*ep^(-1)]*i_*mpsi^2*C13^2*C62
    + gamma[s4,s2]*poles[ - 14*ep^(-1)]*i_*mL^2*mpsi*C13^2*C62
    + gamma[s4,s2]*poles[ - 27/2*ep^(-1)]*i_*mpsi^3*C13^2*C60
    + gamma[s4,s2]*poles[ - 2*ep^(-1)]*i_*mL^2*mpsi*C13^2*C61
    + gamma[s4,s2]*poles[ - 3/4*ep^(-1)]*p1.p1*i_*mpsi*C13^2*C60
    + gamma[s4,s2]*poles[ - 1/4*ep^(-1)]*p1.p1*i_*C13^2*C23
    + gamma[s4,s2]*poles[ - 1/8*ep^(-1)]*p1.p1*i_*mpsi*C23^2
    + gamma[s4,s2]*poles[1/8*ep^(-1)]*i_*C3^2*C23
    + gamma[s4,s2]*poles[1/4*ep^(-1)]*i_*mpsi^3*C23^2
    + gamma[s4,s2]*poles[1/4*ep^(-1)]*i_*mL^2*C3*C33
    + gamma[s4,s2]*poles[1/2*ep^(-1)]*i_*C3*C13^3
    + gamma[s4,s2]*poles[1/2*ep^(-1)]*i_*mpsi*C3*C13*C23
    + gamma[s4,s2]*poles[1/2*ep^(-1)]*i_*mL^2*mpsi*C23^2
    + gamma[s4,s2]*poles[ep^(-1)]*p1.p1*i_*mpsi*C13^2*C61
    + gamma[s4,s2]*poles[3/2*ep^(-1)]*i_*mpsi*C13^4
    + gamma[s4,s2]*poles[3/2*ep^(-1)]*i_*mpsi^2*C13^2*C23
    + gamma[s4,s2]*poles[3/2*ep^(-1)]*i_*mL^2*C13^2*C23
    + gamma[s4,s2]*poles[3*ep^(-1)]*p1.p1*i_*mpsi*C13^2*C62
    + gamma[s4,s2]*poles[27*ep^(-1)]*i_*mpsi^3*C13^2*C61
    + gamma[s4,s2]*poles[61*ep^(-1)]*i_*mpsi^3*C13^2*C62
   ;
```

MM RGEs at 2 loops and beyond?

From non-chiral to chiral

- Chiral theories, the plague of gamma5 in dim reg
- Currently under investigation, probably the BMHV scheme is the way to go forward
- Much progress in this issue recently

see Cornella, Feruglio, Vecchi ’22,
Di Noi et al 2023,
D. Stöckinger, M. Weißwange et al.’24,
Olgoso, Vecchi ’24

$$\{\gamma_\mu, \gamma_5\} = \begin{cases} 0 & \text{for NDR,} \\ 2\gamma_{\hat{\mu}}\gamma_5 & \text{for BMHV.} \end{cases}$$

MM RGEs at 2 loops and beyond?

Outlook

- Automated two loop RGEs for any theory, including those with chiral fermions within the Matchmakereft framework.
- All the usual input/output interface of Matchmaker (Feynrules file support, mathematica output) kept.
- Full results for SMEFT still some (but not that much) time away.
- 3-loop extension: probably unnecessary but straightforward in principle (with the usual caveats about $\gamma 5$).