2-loop RGEs in the LEFT

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Outline

- 2-loop RGEs
- Scheme dependences
- The LEFT in 't Hooft-Veltman
- UV extraction & theory deformation
- Some results
- Conclusion

A tower of EFTs

- 1-loop RGEs

  [Jenkins, Manohar, Trott, 2013] [Jenkins, Manohar, Stoffer, 2017]

  [Fuentes et al., 2010] [Aebischer et al., 2018]

- tree + 1-loop matchings

  [Jenkins, Manohar, Stoffer, 2017]

  [Dekens, Stoffer, 2019]

- next: dim-8 and 2-loop

  as Λ increases dim-8 effects decrease

  ↳ 2-loop becomes more important

adding a loop dominates over $\frac{1}{\Lambda^2}$

Naively in LEFT: $\frac{\alpha_s}{4\pi} \approx \frac{1}{50}$ vs. $\frac{p^2}{\Lambda^2} \approx \left(\frac{1}{100}\right)^2$

2-loop RGEs are here!

- old results for 2-loop EFT RGEs

  [Bijnens, Colangelo, Ecker, 1999] functional

  [Nihei, Arafune, 1994] diagrammatic

- more recent

  [Jenkins, Manohar, LN, Pagès, 2023] geometric

  [Born, Fuentes-Martín, Kvedaraitė, Thomsen, 2024] functional

  [Aebischer, Morell, Pesut, Virto, 2025] reconstruction

  [LN, Stoffer, 2024] LEFT dim-5 2-loop RGE diagrammatic

- soon

  [LN, Stoffer, exp. soon] LEFT ΔB 2-loop RGE

  [LN, Stoffer, exp. 2025] LEFT dim-6 2-loop RGE diagrammatic

  ...

Scheme Dependence

<u>Theory with one parameter g</u>

$$\dot{g} = b_0 g^3 + b_1 g^5 + b_2 g^7 + \dots$$

$$\dot{g} := \mu \frac{d}{d\mu} g$$

Now change scheme

$$g = \tilde{g} + a_1 \tilde{g}^3 + a_2 \tilde{g}^5 + \dots \xrightarrow{\text{invert}} \tilde{g} = g - a_1 g^3 + (3a_1 - a_2) g^5 + \dots$$

and compute RGE of $\tilde{g}$

$$\dot{\tilde{g}} = -3a_1 g^2 \dot{g} + 5(3a_1 - a_2) g^4 \dot{g} = \dots \text{ miracle } \dots = b_0 \tilde{g}^3 + b_1 \tilde{g}^5 + \mathcal{O}(\tilde{g}^7)$$

$\rightarrow$ 2-loop RGE is scheme independent**!** (but not 3-loop)

<u>With many couplings g_i</u> $\quad \dot{g}_i = A_i g_i^3 + B_{ij} g_i^3 g_j^2 \,, \quad g_i = \tilde{g}_i + X_{ij} \tilde{g}_i \tilde{g}_j^2$

$$\dot{\tilde{g}}_i = \dots = A_i \tilde{g}_i^3 + B_{ij} \tilde{g}_i^3 \tilde{g}_j^2 + \underbrace{2X_{ij}\left(A_i \tilde{g}_i^3 \tilde{g}_j^2 - A_j \tilde{g}_i \tilde{g}_j^4\right)}_{\text{generally} \neq 0}$$

🙁

2-loop RGE: Ingredients

evanescents $K_i\,E_i$

Using $\mu \frac{d}{d\mu} L_i^{\text{bare}} = 0$ and graph theory:

$$\dot{L}_i = \underbrace{\left\{ 2L_i^{(1,1)} \right\}_1}_{\text{1-loop RGE}} + \underbrace{\left\{ 4L_i^{(2,1)} - 2L_j^{(1,0)} \frac{\partial L_i^{(1,1)}}{\partial L_j} - 2L_j^{(1,1)} \frac{\partial L_i^{(1,0)}}{\partial L_j} - 2K_j^{(1,1)} \frac{\partial L_i^{(1,0)}}{\partial K_j} \right\}_2}_{\text{2-loop RGE}}$$

where $L_i^{\text{bare}} = \mu^{n_i \epsilon} \left(L_i + \sum_{\ell,n} \frac{1}{(16\pi^2)^\ell} \frac{L_i^{(\ell,n)}}{\epsilon^n} \right)$, $\quad \ell = 1, \dots$, $\quad n = 0, \dots, n$ $\qquad \{X\}_\ell := \frac{1}{(16\pi^2)^\ell} X$

Thus we need:

- $L_i^{(1,1)}$ $\longrightarrow$ known from 1-loop RGE
- $L_i^{(1,0)}, K_i^{(1,1)}$ $\longrightarrow$ [LN, Stoffer, 2023]
- $L_i^{(2,1)}$ $\longrightarrow$ in the works
- Define a scheme!

ℓ-loop $\overline{MS}$:

$\dot{L}_i^{(\ell)} = 2\ell\, L_i^{(\ell,1)}$

Scheme Dependence

HV scheme
$\mu = \underbrace{0,1,2,3}_{\bar{\mu}}, \underbrace{\ldots, D}_{\hat{\mu}}$ $\{\gamma_5, \bar{\gamma}_\mu\} = 0$ $[\gamma_5, \hat{\gamma}_\mu] = 0$ → no problem

Through $L^{(2,1)}$, $L^{(1,0)}$ RGE depends on :

(1) γ_5 prescription. We use 't Hooft - Veltman (HV)

(2) The precise definition of the EFT basis

- physical sector : $\bar{\Psi}\sigma^{\mu\nu}\Psi F_{\mu\nu}$ vs. $\bar{\Psi}\sigma^{\bar{\mu}\bar{\nu}}\Psi \overline{F_{\mu\nu}}$
- evanescent sector : $(\bar{\Psi}\hat{\gamma}^\mu\hat{\gamma}^\nu\Psi)(\bar{\Psi}\hat{\gamma}_\mu\hat{\gamma}_\nu\Psi)$ vs. $(\bar{\Psi}\hat{\sigma}^{\mu\nu}\Psi)(\bar{\Psi}\hat{\sigma}_{\mu\nu}\Psi)$

(• class II (EOM) operators : $\bar{\Psi}Q\, i\not{D}\Psi$ etc. shouldn't matter)

(3) Renormalization scheme:

$\overline{MS}$ vs. finite renormalizations

compensate evanescents

we do $L^{(1,0)} = L^{(1,0)}_{\chi S} + L^{(1,0)}_{ev.}$

[Dugan, Grinstein, 1991]

[Fuentes-Martín, König, Pagès, Thomsen, Wilsch, 2022]

restore chiral symm.

Spurion Chiral Symmetry

E.g. mass terms violate chiral symmetry $\psi_{L,R} \to U_{L,R}\,\psi_{L,R}$

$$\mathcal{L}_{mass} = -\bar{\psi}_R M \psi_L - \bar{\psi}_L M^\dagger \psi_R$$

It can be restored by promoting parameters to spurions

$$M \to U_R M U_L^\dagger \,, \quad L_{eq} \to U_L L_{eq} U_R^\dagger$$

$$\mu = \underbrace{0,1,2,3}_{\bar{\mu}},\underbrace{\ldots,D}_{\hat{\mu}}$$

Restores symmetry, if it wasn't for

$$\bar{\psi} i \not{\partial} \psi = \bar{\psi}_L i \bar{\not{\partial}} \psi_L + \bar{\psi}_R i \bar{\not{\partial}} \psi_R + \underline{\bar{\psi}_L i \hat{\not{\partial}} \psi_R} + \underline{\bar{\psi}_R i \hat{\not{\partial}} \psi_L}$$

$\longrightarrow$ spurions symmetry-violating terms in intermediate steps

$\longrightarrow$ SB effects are local $\longrightarrow$ absorb in δL

$\longrightarrow$ obtain SCS scheme

Evanescent operators

$\mathcal{E}_i$ are created during renormalization. Starting with just $\mathcal{O}_i$

$$= \frac{1}{\epsilon}\mathcal{O}_j + \frac{1}{\epsilon}\mathcal{E}_j + \text{finite}$$

$\longrightarrow$ need $K_i \mathcal{E}_i$ to renormalize theory

But for $L_i^{(2,1)}$ and RGE formula we also need

$$= \frac{\epsilon}{\epsilon}\mathcal{O}_i + \underbrace{\frac{1}{\epsilon}\mathcal{E}_i}_{\text{not needed}}$$

[LN, Stoffer, 2023]

Families of evanescent Operators

In HV scheme

- Operators with explicit $\hat{\gamma}_\mu$:

$$E_1 = \bar{\psi}\, i\hat{\not{\partial}}\,\psi, \quad E_2 = \bar{\psi}\hat{\gamma}_\mu \bar{\gamma}_\nu \psi F^{\mu\nu}, \quad E_3 = (\bar{\psi}\hat{\gamma}_\mu \psi)(\bar{\psi}\hat{\gamma}^\mu \psi), \ \ldots$$

- Fierz - evanescent operators:

$$(\bar{\gamma}^\mu P_L) \otimes [\bar{\gamma}_\mu P_R] \neq -[\bar{\gamma}^\mu P_L) \otimes (\bar{\gamma}_\mu P_L]$$

$$E_1^{\text{Fierz}} = (\bar{\psi}_p \bar{\gamma}^\mu P_L \psi_r)(\bar{\psi}_s \bar{\gamma}_\mu P_L \psi_t) - (\bar{\psi}_p \bar{\gamma}^\mu P_L \psi_t)(\bar{\psi}_s \bar{\gamma}_\mu P_L \psi_r), \ \ldots$$

In NDR scheme

- Fierz - evanescents

$$E_1^{\text{Fierz}} = (\bar{\psi}_p \gamma^\mu P_L \psi_r)(\bar{\psi}_s \gamma_\mu P_L \psi_t) - (\bar{\psi}_p \gamma^\mu P_L \psi_t)(\bar{\psi}_s \gamma_\mu P_L \psi_r), \ \ldots$$

- Chisholm - evanescents

$$E^{\text{Chis}} = (\bar{\psi}_p \gamma^\mu \gamma^\nu \gamma^\rho \psi_r)(\bar{\psi}_s \gamma_\mu \gamma_\nu \gamma_\rho \psi_t) + 4(4-\epsilon)(\bar{\psi}_p \gamma^\mu \psi_r)(\bar{\psi}_s \gamma_\mu \psi_t)$$

(• Gauss - Bonnet - Evanescents)

Our Scheme

Choosing a scheme means :

(A) basis choice

$$\longrightarrow \quad \mathcal{L}_{\text{LEFT}} = \underbrace{\mathcal{L}_{\text{QED}+\text{QCD}}}_{d\text{-dim}} + \underbrace{\sum_i L_i \mathcal{O}_i}_{4\text{-dim}} + \left(\underbrace{\sum_i L_i^{\text{red}} \mathcal{O}_i^{\text{red}}}_{\text{redundant}} \right) + \underbrace{\sum_i K_i E_i}_{\text{evanescent}}$$

(B) renormalization scheme

$$\longrightarrow \quad L^{\text{bare}} = \mu^{4\epsilon} \Big(L^{\text{ren}} + \underbrace{\delta L^{\text{div}}}_{\text{mandatory}} + \underbrace{\delta L^{\text{finite}}_{\text{ev}} + \delta L^{\text{finite}}_{\not{S}B}}_{\text{choice}} \Big)$$

(C) algebraic prescription in $D = 4 - 2\epsilon$

$\longrightarrow$ HV scheme for γ_5, $\epsilon_{\mu\nu\rho\sigma}$

Our Scheme

The upshot is:

- Green's functions are free of κ_i and χSB terms
- Same for RGEs, matching relations!

proof in appendix [Jenkins, Manohar, LN, Pagès]

E.g. we find in $\overline{\text{MS}}$ (turning off quark fields):

$$\dot{e}^{(2)} = \left\{ 4e^5 n_f + 48 e^4 \left(\text{tr}(L_{e\gamma} M_e^\dagger) + \text{tr}(L_{e\gamma}^\dagger M_e) \right) - 224 e^4 \left(\text{tr}(L_{e\gamma} M_e) + \text{tr}(L_{e\gamma}^\dagger M_e^\dagger) \right) \right\}_2$$

$$\dot{L}_{e\gamma}^{(2)} = \left\{ \left(-\frac{80}{9} e^4 n_f - 47 e^4 \right) L_{e\gamma} - \frac{4}{3} L_{e\gamma}^\dagger \right\}_2$$

And in our scheme:

$$\dot{e}^{(2)} = \left\{ 4e^5 n_f - 80 e^4 \left(\text{tr}(L_{e\gamma} M_e) + \text{tr}(L_{e\gamma}^\dagger M_e^\dagger) \right) \right\}_2$$

$$\dot{L}_{e\gamma}^{(2)} = \left\{ \left(-\frac{80}{9} e^4 n_f - 51 e^4 \right) L_{e\gamma} \right\}_2$$

Extraction of UV divergences

Global Renormalization $\longrightarrow$ Operation level: Green's function

$$(2\text{-loop}) + (1\text{-loop}) \times \mathrm{CT}_s = \frac{\text{local}}{\epsilon^2} + \frac{\text{local}}{\epsilon}$$

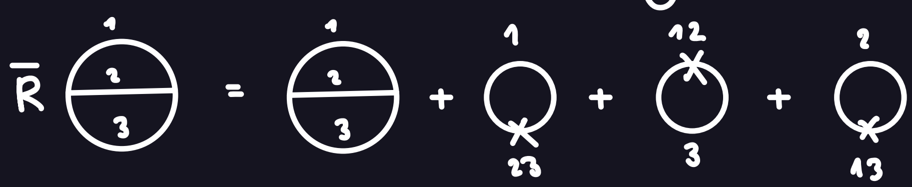
\+ few counterterm diagrams

\+ checks on $\mathcal{L}_{CT}$

gauge-variant EOM

− Requires $\mathcal{L}_{CT}$ at 1 loop, including class II_b (unless ...)

Local Renormalization $\longrightarrow$ Operation level: diagram

Local R Operation 

\+ $\mathcal{L}_{CT}$ automatically generated and inserted

\+ Individual CT-subtr. diagrams are local

must respect scheme!

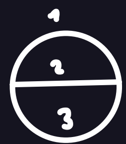
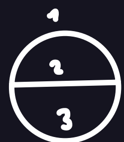
− fewer checks on $\mathcal{L}_{CT}$

Extraction of UV divergences

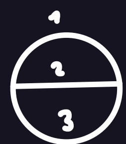
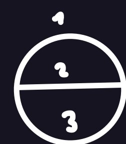
- Taylor Expansion T leaves UV poles unchanged
- Introduction of dummy mass $\hat{m}$ regulates away IR poles

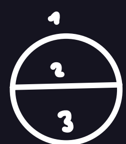
$\underline{i}$ has loop momentum k_i
$k_3 = k_1 + k_2$

For UV divergent terms:

$\bar{R}$  =  +  +  +  $\overset{\text{BPHZ}}{=}$ local

T acts directly on integrand

= $T\bar{R}$  − IR poles = $\hat{m}T\bar{R}$  − (logs of m)

= $\hat{m}T$  + $\hat{m}T$  + $\hat{m}T$  + $\hat{m}T$  − (logs of m)

Result has $\dfrac{\cdots}{(k_1^2-m^2)^a\,(k_2^2-m^2)^b\,(k_3^2-m^2)^c} \xrightarrow[\text{IBP}]{\text{tensor red.}} \dfrac{1}{(k_1^2-m^2)(k_2^2-m^2)(k_3^2-m^2)}$

easy

[Chetyrkin, Misiak, Muenz, 1997]

Extraction of UV divergences

- $\hat{m}$ deforms theory by $\Delta\mathcal{L}^{cT} \sim m$. Result doesn't change. $\Delta\mathcal{L}^{cT}$ automatic
- $\hat{m}, T$ must act consistenty across terms
- For T order compute mass dimension of (sub-) graph
- For $\hat{m}$

$\longrightarrow$ act at a precisely defined step of calculation

otherwise dangerous ambiguity in cancellation of k:

$$\frac{k^2}{k^2} = 1 \quad \text{vs.} \quad \frac{k^2}{k^2 - m^2} = 1 + \frac{m^2}{k^2 - m^2}$$

$\longrightarrow$ need precription for fermion propagators

our choice

$$\frac{i}{\not{k}} \xrightarrow{\hat{m}} \frac{i(\not{k}+m)}{k^2 - m^2} \quad \text{vs.} \quad \frac{i}{\not{k}} \xrightarrow{\hat{m}} \frac{i\not{k}}{k^2 - m^2}$$

Steps in the calculation

(1) Generate diagrams

→ ~ 10k 2-loop diagrams

→ ~1k at dimension five

QGraf

(2) R Operation → subgraphs and CT diagrams

(3) Apply Feynman Rules

Mathematica

(4) Color algebra

(5) Dirac Algebra

(6) Tensor Reduction

(7) Dirac Algebra

(8) Evaluation of Integrals

Form and Symbolica

(9) Absorb $L_i^{(2,n)}$

(10) Field Redefinition

Mathematica

Conclusions

- We define the LEFT in HV, including ε:
- We propose a scheme, restoring CS and compensating ε:
- We derive the 2-loop RGE of the LEFT, in $\overline{\text{MS}}$ and our scheme
- Results are part of an effort towards NLL accuracy

Acknowledgements

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Ben Ruijl, Dominik Stöckinger

Next steps

- Build into tools?
- Same for SMEFT?
- Play with $\hat{m}$ prescriptions?

?

Backup

Backup: EOM Operators

Consider a toy $U(1)$ EFT

redundant

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi + L\,\bar{\psi}\sigma_{\mu\nu}F^{\mu\nu}\psi + R\,\bar{\psi}(i\not{D} - m)^2\psi + \bar{J}\psi + \bar{\psi}J$$

$$Z[J] = e^{iW[J]} = \int \mathcal{D}\psi\,\mathcal{D}A\,\exp\left\{ i\int d^Dx\,\mathcal{L} \right\}$$ generates Green's functions

$$\langle \psi(x_1)\bar{\psi}(x_2)\dots\rangle = \frac{1}{Z[0]}\,\frac{-i\delta}{\delta\bar{J}(x_1)}\,\frac{i\delta}{\delta\bar{J}(x_2)}\dots Z[J]\Big|_{J=0}$$

Field redefinition is just a change of variables $\rightarrow$ leaves $\langle\dots\rangle$ invariant:

$$\psi \rightarrow \psi - \frac{R}{2}(i\not{D} - m)\psi$$

want not

$$\mathcal{L} \rightarrow \mathcal{L}' = \mathcal{L} - R\bar{\psi}(i\not{D} - m)\psi - \frac{R}{2}\bar{\psi}(i\not{D} - m)J - \frac{R}{2}\bar{J}(i\not{D} - m)\psi$$

Redundant operator is gone, but have weird sources. Now

$$\psi \rightarrow \psi + \frac{R}{2}J$$

??

$$\mathcal{L}' \rightarrow \mathcal{L}'' = \mathcal{L} - R\bar{\psi}(i\not{D} - m)\psi + R\bar{J}J$$

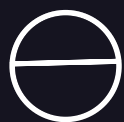
$R\bar{J}J$ contributes only at tree-level, a term $\otimes\otimes = -iR \xrightarrow{\text{amp}} iR(\not{p} - m)^2$

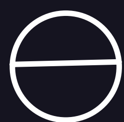
$\longrightarrow$ find "true" loop-level results using $\mathcal{L}''$, which has no EOM operator!

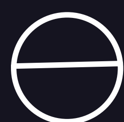
Backup: R operation in our scheme

R := Renormalization Operator in $\overline{MS}$

R_f := Renormalization Operator in our scheme

R_f  $=$  $+$  $\cdot\, \delta Z^{(1)} + \cdots + \times \cdot \delta Z^{(2)}$

$=$  $+$  $\cdot \left(\delta Z_d^{(1)} + \delta Z_f^{(1)}\right) + \times \cdot \left(\delta Z^{(2)} + \left(\delta Z_d^{(1)} + \delta Z_f^{(1)}\right)^2\right)$

$= R$  $+$  $\cdot\, \delta Z_f^{(1)} + 2 \cdot Z_d^{(1)} Z_f^{(1)} + \text{finite}$

↑ divergent part

→ obtain correction globally from tree-level

Scheme Dependence

Theory with one parameter g

$$\dot{g} = b_0 g^3 + b_1 g^5 + b_2 g^7 + \ldots$$

Now change scheme

$$g = \tilde{g} + a_1 \tilde{g}^3 + a_2 \tilde{g}^5 + \ldots \quad \xrightarrow{\text{invert}} \quad \tilde{g} = g - a_1 g^3 + (3a_1 - a_2) g^5 + \ldots$$

and compute RGE of $\tilde{g}$

$$\dot{\tilde{g}} = -3a_1 g^2 \dot{g} + 5(3a_1 - a_2) g^4 \dot{g} = \ldots \text{ miracle } \ldots = b_0 \tilde{g}^3 + b_1 \tilde{g}^5 + \mathcal{O}(\tilde{g}^7)$$

$\rightarrow$ 2-loop RGE is scheme independent ! (but not 3-loop)

With many couplings g_i $\quad \dot{g}_i = A_i g_i^3 + B_{ij} g_i^3 g_j^2 , \quad g_i = \tilde{g}_i + X_{ij} \tilde{g}_i \tilde{g}_j^2$

$$\dot{\tilde{g}}_i = \ldots = A_i \tilde{g}_i^3 + B_{ij} \tilde{g}_i^3 \tilde{g}_j^2 + \underbrace{2X_{ij} \left(A_i \tilde{g}_i^3 \tilde{g}_j^2 - A_j \tilde{g}_i \tilde{g}_j^4 \right)}_{\text{generally } \neq 0}$$

🙁

Dimensionality of Operators

Defining $\mathcal{O}_i$ in d dimensions :

$$\sigma^{\mu\nu} \sim [\gamma^\mu, \gamma^\nu] \sim [\bar{\gamma}^\mu + \hat{\gamma}^\mu, \bar{\gamma}^\nu + \hat{\gamma}^\nu]$$

$$\sim \bar{\sigma}^{\mu\nu} + \sigma^{\bar{\mu}\hat{\nu}} + \sigma^{\hat{\mu}\bar{\nu}} + \sigma^{\hat{\mu}\hat{\nu}}$$

... changes k_i. same for e.g.

$$f^{ABC} \bar{G}^A_{\mu\nu} \bar{G}^B_{\nu\rho} \bar{G}^C_{\rho\mu} \quad \text{versus} \quad f^{ABC} G^A_{\mu\nu} G^B_{\nu\rho} G^C_{\rho\mu}$$

$\longrightarrow$ scheme choice

Backup: The LEFT in HV

dipole operators:

$$L_{e\gamma}\, \bar{e}_L \bar{\sigma}^{\mu\nu} e_R F_{\mu\nu} + \text{h.c.}$$

$$L_{u\gamma}\, \bar{u}_L \bar{\sigma}^{\mu\nu} u_R F_{\mu\nu} + \text{h.c.}$$

$$L_{uG}\, \bar{u}_L \bar{\sigma}^{\mu\nu} T^A u_R G^A_{\mu\nu} + \text{h.c.}$$

$$\vdots$$

3-gluon operators:

$$L_G\, f^{ABC}\, \overline{G^A_{\mu\nu} G^B_{\nu\rho} G^C_{\rho\mu}}$$

$$L_{\tilde{G}}\, f^{ABC}\, \overline{\tilde{G}^A_{\mu\nu} G^B_{\nu\rho} G^C_{\rho\mu}}$$

ψ^4 operators $\Delta L = \Delta B = 0$:

$$L^{V,LL}_{ee} (\bar{e}_L \bar{\gamma}^\mu e_L)(\bar{e}_L \bar{\gamma}^\mu e_L)$$

$$L^{V,LR}_{ee} (\bar{e}_L \bar{\gamma}^\mu e_L)(\bar{e}_R \bar{\gamma}^\mu e_R)$$

$$\vdots$$

ψ^4 operators $\Delta L, \Delta B \neq 0$:

$$L^{S,LL}_{\nu e} (\bar{\nu}_L^T C \nu_L)(\bar{e}_R e_L)$$

$$L^{T,LL}_{\nu e} (\bar{\nu}_L^T C \bar{\sigma}^{\mu\nu} \nu_L)(\bar{e}_R \bar{\sigma}_{\mu\nu} e_L)$$

$$\vdots$$

Backup : LEFT Evanescents

→ full basis ~10 x larger than physical basis

dimension 4 :

$$K_{eD}\, \bar{e}_L\, i\hat{\not{D}}\, e_R + h.c.$$

$$K_{\gamma'}\, \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} + h.c.$$

$$\vdots$$

power counting?

dimension 5 :

$$K^{L}_{eD}\, \bar{e}_L [\, i\hat{\not{D}}, i\bar{\not{D}}\,]\, e_L$$

$$K^{LR}_{eD}\, \bar{e}_L\, i\hat{\not{D}}\, i\hat{\not{D}}\, e_R + h.c.$$

$$K^{LR}_{e\gamma}\, \bar{e}_L\, \hat{\sigma}^{\mu\nu} e_R F_{\mu\nu} + h.c.$$

$$\vdots$$

+ many at dimension six ...

Backup: Facts about evanescents

① Coefficients of evanescents can start at tree-level

But have in basis $K \bar{\Psi} i \hat{\not{\partial}} \Psi$

$\longrightarrow$ needs loop or EFT suppression

to have well-defined perturbative expansion

② Evanescents are not $\mathcal{O}(\epsilon)$

They have rank $\sim \epsilon$.

Double insertion effects on $\mathcal{O}_1$ produces ϵ, not ϵ^2.

Backup: γ_5 issues

$$\gamma_5 = \frac{i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma \longrightarrow \{\gamma^5, \gamma^\mu\} = 0 \quad \text{in} \quad D = 4$$

In $D \neq 4$

$$\{\gamma^5, \gamma^\mu\} = 0 \; + \text{cyclicity} \longrightarrow \mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5) = 0$$

But in $D = 4$ must find

$$\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5) = 4i\epsilon^{\mu\nu\rho\sigma}$$

$\longrightarrow$ spoils analytic continuation

[Jegerlehner, 2000]