The t'Hooft-Veltman Scheme in the Functional Approach

Adrián Moreno-Sánchez

with J. Fuentes-Martín
and A.E. Thomsen

Junta de Andalucía

FTAE High Energy Theory

UNIVERSIDAD DE GRANADA

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The t’Hooft-Veltman / BHMV Scheme

The t'Hooft-Veltman/ BHMV Scheme

THE GOOD

γ_5 in Dimensional Regularization

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- In Dimensional Regularization we must extend our action to d = 4 - 2ϵ dimensions

- For chiral theories this implies extending the meaning of

$$\gamma_5 = \frac{i}{4!}\varepsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$$

NDR

$$\{\gamma^\mu, \gamma_5\} = 0$$

t'Hooft-Veltman

$$\gamma^\mu = \gamma^{\bar{\mu}} + \gamma^{\hat{\mu}}$$

$$[\gamma^{\hat{\mu}}, \gamma_5] = 0$$

$$\{\gamma^{\bar{\mu}}, \gamma_5\} = 0$$

The Fermionic Extension

We must extend the chiral-fermionic kinetic lagrangian to regularize our theory

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$$\mathcal{L} \supset i\bar{\psi}\gamma^{\bar{\mu}} P_L D_{\bar{\mu}}\psi + i\bar{\psi}\gamma^{\hat{\mu}} P_R \partial_{\hat{\mu}}\psi^{ev} + \text{h.c.}$$

- Explicit breaking of chiral gauge-invariance in the evanescent part.
- BRST invariance also broken by the regulator: include finite counterterms to recover the symmetry.
- Case-by-case problem (never done for SMEFT).

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Scheme Choice

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See Luca's Talk!

Spurious Gauge-Invariance

[P. Olgoso Ruiz, L. Vecchi]

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- Formally maintain gauge invariance introducing a spurion field

$$\mathcal{L} \supset \bar{\psi}\gamma^{\bar{\mu}} P_L D_{\bar{\mu}}\psi + \bar{\psi}\Omega\gamma^{\hat{\mu}} P_R \partial_{\hat{\mu}}\psi^{ev} + \text{h.c.}$$

- It transforms in the same representation and is unitary

$$\psi \to U\psi \qquad \Omega \to U\Omega \qquad \Omega\Omega^\dagger = \mathcal{I}$$

Symmetry-Restoring Counterterms [P. Olgoso Ruiz, L. Vecchi]

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We can compute the symmetry restoring counterterms at each order through the quantum effective action

$$\Delta S_{\mathrm{ct}}^{\mathrm{Fin}}\Big|^{(n)} = -\lim_{\Omega \to \mathcal{I}} \Gamma_{\Omega}\Big|^{(n)}_{\mathrm{hard}}$$

- Performing the Identity Limit will give you the finite counterterms

Diagrammatic Approach

[P. Olgoso Ruiz, L. Vecchi]

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- Cumbersome Propagator: Consider a constant spurion

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- Difficult to systematize to EFTs

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Perfect problem for functional methods!

Functional Approach

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THE `Matchete` THING

Quantum Effective Action

[Fuentes-Martín, AMS, Palavrić , Eller Thomsen]

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We are going to work with the path integral itself (BFM)

$$e^{i\Gamma_{\rm UV}} = \int [D\Phi][D\phi]\exp\left(\int \mathrm{d}^d x \mathcal{L}_{\rm UV}[\Phi,\phi]\right)$$

See Lukas' Talk!

- "Functionally invert" the kinetic term

$$\Gamma^{(1)}_{\Omega} = \frac{i}{2}\mathrm{STr}\log\mathbf{\Delta} - \frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\mathrm{STr}\left(\mathbf{\Delta}^{-1}X\right)^n$$

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Only Chiral Fermions

Spurion Counterterms

- Only modify the kinetic term for chiral fermions

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$$\boldsymbol{\Delta}_{\psi^{ev}\psi^{ev}} = i\gamma^{\bar{\mu}}\partial_{\bar{\mu}}$$

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4-dimensional gauge transformations

$$D_{\hat{\mu}}\psi = \partial_{\hat{\mu}}\psi \rightarrow [D_{\hat{\mu}}\Omega] = 0$$

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Log-Trace

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- Contains all the chiral-fermionic loops

- The evanescent Fermion will also contribute

$$\log \Delta(P+k) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n} \frac{1}{k^{2n}} \times$$

$$\times \prod_{i=1}^{n} \begin{pmatrix} (\gamma^{\bar\mu}\gamma^{\bar\nu} + \gamma^{\hat\mu}\gamma^{\hat\nu})P_\mu P_\nu + 2k^\mu P_\mu & (\gamma^{\bar\mu}\gamma^{\hat\nu} + \gamma^{\hat\mu}\gamma^{\bar\nu})\Omega P_\mu P_\nu + \gamma^{\bar\mu}\gamma^{\hat\nu}[P_\mu \Omega](P_\nu + k_\nu) \\ (\gamma^{\bar\mu}\gamma^{\hat\nu} + \gamma^{\hat\mu}\gamma^{\bar\nu})\Omega^\dagger P_\mu P_\nu + \gamma^{\bar\mu}\gamma^{\hat\nu}[P_\mu \Omega^\dagger](P_\nu + k_\nu) & (\gamma^{\bar\mu}\gamma^{\bar\nu} + \gamma^{\hat\mu}\gamma^{\hat\nu})P_\mu P_\nu + 2k^\mu P_\mu \end{pmatrix}$$

Power-Trace

- No interaction terms with the evanescent fermion

$$\Delta^{-1}(P+k) = \sum_{n=0}^{\infty} \frac{(-1)^n}{k^{2(n+1)}} k_{\nu_1} \cdots k_{\nu_{n+1}} \times$$
$$\times \prod_{i=1}^{n} \begin{pmatrix} (\gamma^{\bar{\nu}_i}\gamma^{\bar{\mu}} + \gamma^{\hat{\nu}_i}\gamma^{\hat{\mu}})P_\mu & (\gamma^{\bar{\nu}_i}\gamma^{\hat{\mu}} + \gamma^{\hat{\nu}_i}\gamma^{\bar{\mu}})\Omega P_\mu \\ (\gamma^{\bar{\nu}_i}\gamma^{\hat{\mu}} + \gamma^{\hat{\nu}_i}\gamma^{\bar{\mu}})\Omega^\dagger P_\mu & (\gamma^{\bar{\nu}_i}\gamma^{\bar{\mu}} + \gamma^{\hat{\nu}_i}\gamma^{\hat{\mu}})P_\mu \end{pmatrix} \begin{pmatrix} \gamma^{\bar{\nu}_{n+1}} & \gamma^{\hat{\nu}_{n+1}}\Omega \\ \gamma^{\hat{\nu}_{n+1}}\Omega^\dagger & \gamma^{\bar{\nu}_{n+1}} \end{pmatrix}$$

Matchete

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A `Mathematica` Package for functional matching in EFTs

Current Modification

MATCHETE

- Inclusion of the t'Hooft-Veltman Scheme to treat γ_5
- Change of the functional propagators for chiral fermions
- Implemented an automatic function to compute the finite symmetry restoring counterterms for any EFT theory

Matchete

A `Mathematica` Package for functional matching in EFTs

Future directions:

- Compute the counterterms for the SMEFT (almost there)
- Implement the automatic one-loop matching in the t'Hooft-Veltman scheme
- Going beyond one-loop

To take home

- We need to automatize BHMV for future multiloop computations
- Symmetry restoring counterterms obtained through spurion insertions.
- Functional methods is a powerful tool here.
- Well suited for automation: `Matchete`.

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Thank you!