Renormalization Group Equations of the General Gauge EFT at Dimension Six: Bosonic Sector

SMEFT-Tools 2025

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Based on:

[25xx.xxxxx] Jason Aebischer, LCB, Nudžeim Selimović

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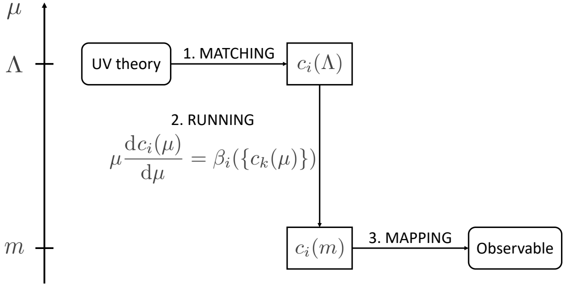
EFT & Running

- **EFT Approach:** Standard Model as the low-energy description of a more fundamental theory emerging at a large energy scale Λ

$$\mathcal{L}_{\rm EFT} = \sum_i \frac{c_i}{\Lambda^{[\mathcal{O}_i]-4}} \mathcal{O}_i \, .$$

- **Running:** The Wilson coefficients c_i need to be evolved from the scale Λ down to the experimental scale.

- **EFT Anomalous Dimensions** are crucial for interpreting experimental results.



General Gauge EFT

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- Easy to extend RGEs with new degrees of freedom interacting with SM fields (*e.g.* axions, axion-like particles, light Z', $\ldots$)
- Easy to extend RGEs with new gauge sectors (*e.g.* non-universal gauge interactions, unification, $\ldots$)
- Cross-check of literature
  - SMEFT [Jenkins, Manohar, Trott (13)]^3
  - LEFT [Jenkins, Manohar, Stoffer (17)]
  - ALP-SMEFT / ALP-LEFT [Chala, Guedes, Ramos, Santiago (20)], [Bonilla, Brivio, Gavela, Sanz (21)], [Bauer, Neubert, Renner, Schnubel, Thamm (20)], [Galda, Neubert, Renner (21)], [Di Luzio, Gröber, Paradisi (20)], [LCB, Brunello, Levati, Mastrolia, Paradisi (24)], $\ldots$
  - General gauge EFT **[Fonseca, Olgoso, Santiago (25)], [Misiak, Nałęcz (25)]**
  - $\ldots$

Bosonic General Gauge EFT

- **Gauge group**: $G = \prod_{\alpha=1}^{N_G} G_\alpha$ [R. Fonseca's talk]
- Arbitrary number of **real scalars** (ϕ_a) and **gauge bosons** ($A_\mu^{A_\alpha}$)
- Renormalizable Lagrangian

$$\mathcal{L}_{\mathrm{LO}} = -\frac{1}{4}\sum_{\alpha=1}^{N_G} F_{\mu\nu}^{A_\alpha} F^{A_\alpha \mu\nu} + \frac{1}{2}(D_\mu \phi)_a (D^\mu \phi)_a$$

$$- \Lambda - t_a \phi_a - \frac{m_{ab}^2}{2!}\phi_a\phi_b - \frac{h_{abc}}{3!}\phi_a\phi_b\phi_c - \frac{\lambda_{abcd}}{4!}\phi_a\phi_b\phi_c\phi_d$$

$$(D_\mu \phi)_a = \partial_\mu \phi_a - i \sum_{\alpha=1}^{N_G} g_\alpha A_\mu^{A_\alpha} \theta_{ab}^{A_\alpha} \phi_b$$

$$F_{\mu\nu}^{A_\alpha} = \partial_\mu A_\nu^{A_\alpha} - \partial_\nu A_\mu^{A_\alpha} + g_\alpha f^{A_\alpha B_\alpha C_\alpha} A_\mu^{B_\alpha} A_\nu^{C_\alpha}$$

Dimension-5 Operators

Physical operators are identified through the **Amplitude-Operator Correspondence** [Li, Ren, Xiao, Yu, Zheng (22)] [M. Schaaf's talk]

Name	Operator	Symmetry	Contact Amplitude
$\mathcal{O}_{\phi^5}$	$\phi_a\phi_b\phi_c\phi_d\phi_e$	$[C_{\phi^5}]_{abcde} = [C_{\phi^5}]_{(abcde)}$	$F_{\phi^5}(1_a, 2_b, 3_c, 4_d, 5_e) = 5!\left[C_{\phi^5}\right]_{abcde}$
$\mathcal{O}_{\phi F^2}$	$\phi_a F^{A_\alpha}_{\mu\nu} F^{B_\beta\,\mu\nu}$	$\left[C_{\phi F^2}\right]^{A_\alpha B_\beta}_a = \left[C_{\phi F^2}\right]^{(A_\alpha B_\beta)}_a$	$F_{\phi F^2_{\alpha\beta}}(1_a, 2^-_{A_\alpha}, 3^-_{B_\beta}) = -\mathcal{S}_{\alpha\beta}\left[C_{\phi F^2}\right]^{A_\alpha B_\beta}_a \langle 23\rangle^2$
$\mathcal{O}_{\phi\widetilde{F}^2}$	$\phi_a F^{A_\alpha}_{\mu\nu} \widetilde{F}^{B_\beta\,\mu\nu}$	$\left[C_{\phi\widetilde{F}^2}\right]^{A_\alpha B_\beta}_a = \left[C_{\phi\widetilde{F}^2}\right]^{(A_\alpha B_\beta)}_a$	$F_{\phi\widetilde{F}^2_{\alpha\beta}}(1_a, 2^-_{A_\alpha}, 3^-_{B_\beta}) = -i\mathcal{S}_{\alpha\beta}\left[C_{\phi\widetilde{F}^2}\right]^{A_\alpha B_\beta}_a \langle 23\rangle^2$

$$\mathcal{S}_{\alpha\beta} = 1 + \delta_{\alpha\beta} = \begin{cases} 1 & \alpha \neq \beta \\ 2 & \alpha = \beta \end{cases}$$

Symmetry factor

Dimension-6 Operators

Name	Operator	Symmetry
$\mathcal{O}_{\phi^6}$	$\phi_a\phi_b\phi_c\phi_d\phi_e\phi_f$	$[C_{\phi^6}]_{abcdef} = [C_{\phi^6}]_{(abcdef)}$
$\mathcal{O}_{D^2\phi^4}$	$(D_\mu\phi)_a(D^\mu\phi)_b\phi_c\phi_d$	$[C_{D^2\phi^2}]_{abcd} = [C_{D^2\phi^2}]_{(ab)cd} = [C_{D^2\phi^2}]_{ab(cd)}$
$\mathcal{O}_{\phi^2F^2}$	$\phi_a\phi_b F^{A_\alpha}_{\mu\nu} F^{B_\beta\,\mu\nu}$	$[C_{\phi^2F^2}]^{A_\alpha B_\beta}_{ab} = [C_{\phi^2F^2}]^{(A_\alpha B_\beta)}_{ab} = [C_{\phi^2F^2}]^{A_\alpha B_\beta}_{(ab)}$
$\mathcal{O}_{\phi^2\widetilde{F}^2}$	$\phi_a\phi_b F^{A_\alpha}_{\mu\nu} \widetilde{F}^{B_\beta\,\mu\nu}$	$\left[C_{\phi^2\widetilde{F}^2}\right]^{A_\alpha B_\beta}_{ab} = \left[C_{\phi^2\widetilde{F}^2}\right]^{(A_\alpha B_\beta)}_{ab} = \left[C_{\phi^2\widetilde{F}^2}\right]^{A_\alpha B_\beta}_{(ab)}$
$\mathcal{O}_{F^3}$	$F^{A_\alpha\,\nu}_{\mu} F^{B_\alpha\,\rho}_{\nu} F^{C_\alpha\,\mu}_{\rho}$	$[C_{F^3}]^{A_\alpha B_\alpha C_\alpha} = [C_{F^3}]^{[A_\alpha B_\alpha C_\alpha]}$
$\mathcal{O}_{\widetilde{F}^3}$	$F^{A_\alpha\,\nu}_{\mu} F^{B_\alpha\,\rho}_{\nu} \widetilde{F}^{C_\alpha\,\mu}_{\rho}$	$[C_{\widetilde{F}^3}]^{A_\alpha B_\alpha C_\alpha} = [C_{\widetilde{F}^3}]^{[A_\alpha B_\alpha C_\alpha]}$

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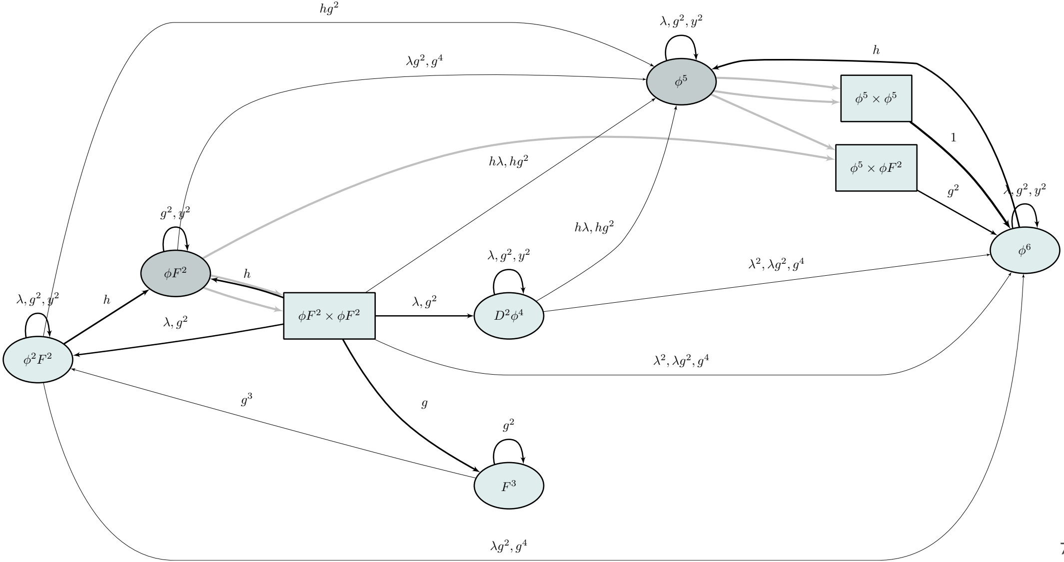
$\mathcal{O}_{\phi^6}$	$F_{\phi^5}(1_a, 2_b, 3_c, 4_d, 5_e, 6_f) = 6! \left[C_{\phi^6}\right]_{abcdef}$
$\mathcal{O}_{D^2\phi^4}$	$F_{D^2\phi^4}(1_a, 2_b, 3_c, 4_d) = -2\left(\left[\hat{C}_{D^2\phi^4}\right]_{abcd} s_{12} + \left[\hat{C}_{D^2\phi^4}\right]_{acbd} s_{13}\right)$
$\mathcal{O}_{\phi^2 F^2}$	$F_{\phi^2 F^2_{\alpha\beta}}(1_a, 2_b, 3^-_{A_\alpha}, 4^-_{B_\beta}) = -2\mathcal{S}_{\alpha\beta} \left[C_{\phi^2 F^2}\right]^{A_\alpha B_\beta}_{ab} \langle 34\rangle^2$
$\mathcal{O}_{\phi^2 \widetilde{F}^2}$	$F_{\phi^2 \widetilde{F}^2_{\alpha\beta}}(1_a, 2_b, 3^-_{A_\alpha}, 4^-_{B_\beta}) = -2i\mathcal{S}_{\alpha\beta} \left[C_{\phi^2 \widetilde{F}^2}\right]^{A_\alpha B_\beta}_{ab} \langle 34\rangle^2$
$\mathcal{O}_{F^3}$	$F_{F^3_\alpha}(1^-_{A_\alpha}, 2^-_{B_\alpha}, 3^-_{C_\alpha}) = -3i\sqrt{2} \left[C_{F^3}\right]^{A_\alpha B_\alpha C_\alpha} \langle 12\rangle\langle 23\rangle\langle 31\rangle$
$\mathcal{O}_{\widetilde{F}^3}$	$F_{\widetilde{F}^3_\alpha}(1^-_{A_\alpha}, 2^-_{B_\alpha}, 3^-_{C_\alpha}) = 3\sqrt{2} \left[C_{\widetilde{F}^3}\right]^{A_\alpha B_\alpha C_\alpha} \langle 12\rangle\langle 23\rangle\langle 31\rangle$

$$\left[\hat{C}_{D^2\phi^4}\right]_{abcd} = \left[C_{D^2\phi^4}\right]_{abcd} + \left[C_{D^2\phi^4}\right]_{cdab} - \left[C_{D^2\phi^4}\right]_{adbc} - \left[C_{D^2\phi^4}\right]_{bcad}$$

Operator Mixing



On-Shell Methods for Renormalization

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Core Features:

- **Unitarity Cuts:** Anomalous dimensions are derived from discontinuities of scattering amplitudes.
- **Phase-Space Integration:** Lorentz-invariant phase-space integrals replace full Feynman integrals.
- **Advantages:**
  - Avoid complexities of standard loop calculations by focusing on physical, on-shell states.
  - No need to consider the Green's basis and reduce it to the physical one.
  - Gauge invariance is automatic.
  - Explain zeros in anomalous dimensions $\rightsquigarrow$ Nonrenormalization Theorems based on
    - HELICITY; [Cheung, Shen (15)]
    - LENGTH; [Bern, Parra-Martinez, Sawyer (20)]
    - ANGULAR MOMENTUM. [Jiang, Shu, Xiao, Zheng (21)]

Limitations and Generalizations

- Originally applied only to massless particles and operators with same dimensions.
- **Generalized** [LCB, Levati, Mastrolia, Paradisi (23)] to
  - include **Leading Mass Effects** via the *Higgs low-energy Theorem*:

$$\mathcal{L}_h^{\text{int}} = -\left(1 + \frac{h}{v}\right) \sum_f m_f \bar{f} f$$

$$\lim_{\{p_h\}\to 0} \mathcal{M}(A \to B + Nh) = \sum_f \left(\frac{m_f}{v} \frac{\partial}{\partial m_f}\right)^N \mathcal{M}(A \to B)$$

  - handle the most **General Operator Mixing**:

$$\mu \frac{\mathrm{d}c_i}{\mathrm{d}\mu} = \sum_{n>0} \frac{1}{n!} \gamma_{i\leftarrow j_1,\ldots,j_n} c_{j_1} \cdots c_{j_n} = \gamma_{i\leftarrow j} c_j + \frac{1}{2} \gamma_{i\leftarrow j,k} c_j c_k + \cdots$$

$$\gamma_{i\leftarrow j_1,\ldots,j_n} = \left.\frac{\partial^n \beta_i}{\partial c_{j_1} \cdots \partial c_{j_n}}\right|_*$$

S-Matrix and Dilatation Operator

- **Form Factor** associated with a local, gauge-invariant operator $\mathcal{O}_i$:

$$F_i(\vec{n}; q) = \langle \vec{n} | \mathcal{O}_i(q) | 0 \rangle \ .$$

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- Exploiting the fundamental relations [Elias-Miró, Ingoldby, Riembau (20)]
  - ◇ **Analyticity:** $F_i^*(\{s_{ij}-i\epsilon\}) = F_i(\{s_{ij}+i\epsilon\})$
  - ◇ **Unitarity:** $\sum_{\vec{n}} \int d\mathrm{LIPS}_n \ |\vec{n}\rangle\langle\vec{n}| = \mathbb{1}\,, \quad d\mathrm{LIPS}_n = \prod_{i=1}^{n} \frac{d^3p_i}{(2\pi)^3}\frac{1}{2E_i}$
  - ◇ **CPT Theorem:** $\langle \vec{n};\mathrm{out}|\mathcal{O}_i(x)|0\rangle = \langle 0|\mathcal{O}_i^\dagger(-x)|\vec{n};\mathrm{in}\rangle$

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it is possible to show that [Caron-Huot, Wilhelm (16)]

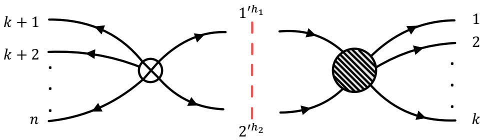
$$e^{-i\pi D} F_i^*(\vec{n}) = (S F_i^*)(\vec{n}) \left(= \sum_{\vec{m}} \int d\mathrm{LIPS}_m \ \langle \vec{n} | S | \vec{m} \rangle \ F_i^*(\vec{m}) \right)$$

where $S = \mathbb{1} + i\mathcal{M}$ is the **S-matrix** and $D = \sum_i p_i \cdot \partial/\partial p_i$ is the **Dilatation Operator**.

Master Formulae

Linear operator mixing

$$\left(\gamma^{(1)}_{i\leftarrow j} - \delta_{ij}\gamma^{(1)}_{i,\mathrm{IR}}\right) F_i\big|^{(0)}_{*} = -\frac{1}{\pi}(\mathcal{M}F_j)\big|^{(1)}_{*}$$

$$(\mathcal{M}F_j)^{(1)}(\vec{n}) = \sum_{k}\sum_{h_1,h_2}$$  $$+ \ \text{permutations}$$

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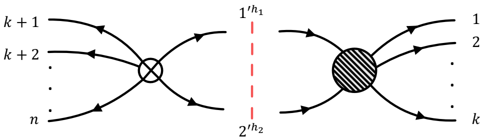


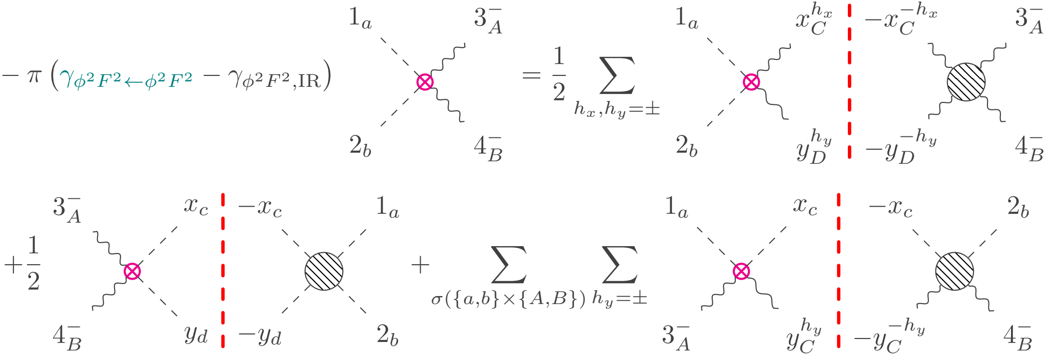
$$+ \ \text{permutations}$$

Nonlinear operator mixing

$$\gamma^{(1)}_{i\leftarrow j,k} F_i\big|^{(0)}_* = -\frac{1}{\pi}\frac{\partial}{\partial c_k}\bigg|_* (\mathcal{M}F_j)^{(1)}$$

Example: $\phi^2 F^2 \leftarrow \phi^2 F^2$

Master formula



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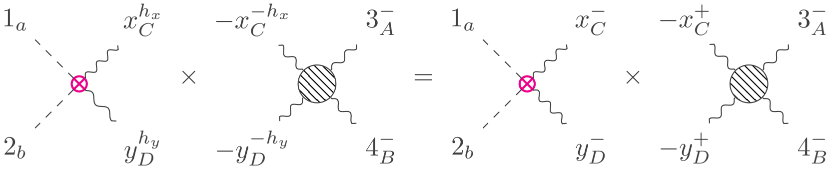
First term

$$g_1 = \sum_{h_x, h_y = \pm}$$



$$= \left[-4\left[C_{\phi^2 F^2}\right]_{ab}^{CD} \langle xy\rangle^2\right]\left[-2g^2\langle 34\rangle^4\left(\frac{f^{ACE}f^{BDE}}{\langle 3x\rangle\langle 3y\rangle\langle 4x\rangle\langle 4y\rangle} - \frac{f^{ABE}f^{CDE}}{\langle 34\rangle\langle 3y\rangle\langle 4x\rangle\langle xy\rangle}\right)\right]$$

$$= 8\frac{1+z\bar{z}}{z\bar{z}}g^2\left[C_{\phi^2F^2}\right]_{ab}^{CD}\langle 34\rangle^2\left[z\bar{z}f^{ABE}f^{CDE} - (1+z\bar{z})f^{ACE}f^{BDE}\right]$$

$$\begin{pmatrix}|x\rangle \\ |y\rangle\end{pmatrix} = \frac{1}{\sqrt{1+z\bar{z}}}\begin{pmatrix}1 & \bar{z} \\ -z & 1\end{pmatrix}\begin{pmatrix}|3\rangle \\ |4\rangle\end{pmatrix}$$

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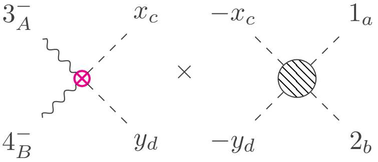
Second term

$g_2 =$



$$= \left[-4\left[C_{\phi^2 F^2}\right]^{AB}_{cd}\langle 34\rangle^2\right]\left[g^2\left(\frac{s_{1y}-s_{1x}}{s_{12}}\theta^C_{ab}\theta^C_{cd} - \frac{s_{12}+s_{1y}}{s_{1x}}\theta^C_{ac}\theta^C_{bd} - \frac{s_{12}+s_{1x}}{s_{1y}}\theta^C_{ad}\theta^C_{bc}\right) - \lambda_{abcd}\right]$$

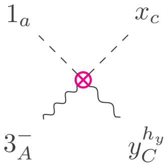
$$= 4\left[C_{\phi^2 F^2}\right]^{AB}_{cd}\langle 34\rangle^2\left[g^2\left(\frac{z\bar{z}-1}{1+z\bar{z}}\theta^C_{ab}\theta^C_{cd} + \frac{z\bar{z}+2}{z\bar{z}}\theta^C_{ac}\theta^C_{bd} + (1+2z\bar{z})\theta^C_{ad}\theta^C_{bc}\right) + \lambda_{abcd}\right]$$

$$\begin{pmatrix}|x\rangle\\|y\rangle\end{pmatrix} = \frac{1}{\sqrt{1+z\bar{z}}}\begin{pmatrix}1 & \bar{z}\\ -z & 1\end{pmatrix}\begin{pmatrix}|1\rangle\\|2\rangle\end{pmatrix}$$

Example: $\phi^2 F^2 \leftarrow \phi^2 F^2$

Third term

$$g_3 = \sum_{h_y=\pm}$$



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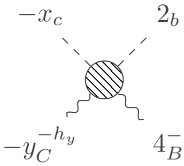

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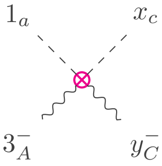

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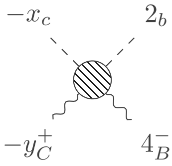

$$= \left[-4\left[C_{\phi^2F^2}\right]^{AC}_{ac}\langle 3y\rangle^2\right]\left[2g^2\langle 4x\rangle[yx]\frac{s_{24}\theta^C_{bd}\theta^B_{dc} - s_{2y}\theta^B_{bd}\theta^C_{dc}}{\langle 2x\rangle\langle 2y\rangle[42][x2]}\right]$$

$$= 8g^2\left[C_{\phi^2F^2}\right]^{AC}_{ac}\langle 34\rangle^2\frac{(z\langle 23\rangle + \langle 34\rangle)^2}{z\bar{z}(1+z\bar{z})\langle 34\rangle^2}\left[(1+z\bar{z})\theta^C_{bd}\theta^B_{dc} - \theta^B_{bd}\theta^C_{dc}\right]$$

$$\begin{pmatrix} |x\rangle \\ |y\rangle \end{pmatrix} = \frac{1}{\sqrt{1+z\bar{z}}}\begin{pmatrix} 1 & \bar{z} \\ -z & 1 \end{pmatrix}\begin{pmatrix} |2\rangle \\ |4\rangle \end{pmatrix}$$

Stokes' integration

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1. Integrate in $\bar{z}$, keeping only rational contributions:

$$G(z, \bar{z}) = \text{Rational}[\int d\bar{z} \, \frac{g(z, \bar{z})}{(1 + z\bar{z})^2}]$$

$$g(z, \bar{z}) = \frac{1}{2} g_1(z, \bar{z}) + \frac{1}{2} g_2(z, \bar{z}) + \sum_{\sigma(\{A_\alpha, B_\beta\} \times \{a,b\})} g_3(z, \bar{z})$$

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2. Apply Residue Theorem by summing over the z-poles $\mathcal{P}_G$ of G:

$$\int d\text{LIPS}_2\, g = -\frac{1}{8\pi} \sum_{z_0\in\mathcal{P}_G} \text{Res}_{(z,\bar{z})=(z_0,z_0^*)} G(z,\bar{z})$$

Example: $\phi^2 F^2 \leftarrow \phi^2 F^2$

Final result

$$\left[\dot{C}_{\phi^2F^2}\right]^{A_\alpha B_\beta}_{ab} = \left[\lambda_{abcd} + 2\sum_{\gamma=1}^{N_G} g_\gamma^2 \theta^{C_\gamma}_{ac}\theta^{C_\gamma}_{db}\right]\left[C_{\phi^2F^2}\right]^{A_\alpha B_\beta}_{cd}$$
$$+ 2\sum_{\sigma(\{A_\alpha,B_\beta\}\times\{a,b\})}\sum_{\gamma=1}^{N_G} g_\alpha g_\gamma \Big[\delta_{\alpha\gamma}\left(3\theta^{A_\alpha}_{ad}\theta^{C_\gamma}_{dc} - 2\theta^{C_\gamma}_{ad}\theta^{A_\alpha}_{dc}\right)$$
$$+ (1-\delta_{\alpha\gamma})\mathcal{S}^{-1}_{\alpha\beta}\mathcal{S}_{\gamma\beta}\theta^{A_\alpha}_{ad}\theta^{C_\gamma}_{dc}\Big]\left[C_{\phi^2F^2}\right]^{C_\gamma B_\beta}_{cb}$$
$$+ \left(\gamma^{A_\alpha C_\alpha}_{c,v} + \gamma^{B_\beta D_\beta}_{c,v} + \gamma^{ac}_{c,s} + \gamma^{bd}_{c,s}\right)\left[C_{\phi^2F^2}\right]^{C_\alpha D_\beta}_{cd}$$

with the collinear anomalous dimensions given by

$$\gamma^{A_\alpha B_\alpha}_{c,v} = -g_\alpha^2\left[\frac{11}{3}f^{A_\alpha C_\alpha D_\alpha}f^{B_\alpha C_\alpha D_\alpha} - \frac{1}{6}\mathrm{Tr}\left(\theta^{A_\alpha}\theta^{B_\alpha}\right)\right] \qquad \gamma^{ab}_{c,s} = -4\sum_{\alpha=1}^{N_G} g_\alpha^2\theta^{A_\alpha}_{ac}\theta^{A_\alpha}_{cb}$$

SMEFT Cross-Check

Renormalizable Lagrangian

$$\mathcal{L}_H = (D_\mu H)^\dagger (D^\mu H) - \lambda \left(H^\dagger H - \frac{1}{2} v^2 \right)^2 \qquad H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_3 \\ \phi_2 + i\phi_4 \end{pmatrix}$$

$$= \frac{1}{2} (D_\mu \phi)_a (D^\mu \phi)_a + \frac{\lambda v^2}{2} \delta_{ab}\, \phi_a \phi_b - \frac{\lambda}{4} \delta_{(ab}\delta_{cd)}\, \phi_a\phi_b\phi_c\phi_d$$

$$m^2_{ab} = -\lambda v^2 \delta_{ab} \qquad \lambda_{abcd} = \frac{4!}{4} \lambda\, \delta_{(ab}\delta_{cd)} = 2\lambda(\delta_{ab}\delta_{cd} + \delta_{ad}\delta_{bc} + \delta_{ac}\delta_{bd})$$

SMEFT Cross-Check

Renormalizable Lagrangian

$$\mathcal{L}_H = (D_\mu H)^\dagger (D^\mu H) - \lambda \left(H^\dagger H - \frac{1}{2} v^2 \right)^2 \qquad H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_3 \\ \phi_2 + i\phi_4 \end{pmatrix}$$

$$= \frac{1}{2} (D_\mu \phi)_a (D^\mu \phi)_a + \frac{\lambda v^2}{2} \delta_{ab}\, \phi_a \phi_b - \frac{\lambda}{4} \delta_{(ab}\delta_{cd)}\, \phi_a \phi_b \phi_c \phi_d$$

$$m^2_{ab} = -\lambda v^2 \delta_{ab} \qquad \lambda_{abcd} = \frac{4!}{4} \lambda\, \delta_{(ab}\delta_{cd)} = 2\lambda(\delta_{ab}\delta_{cd} + \delta_{ad}\delta_{bc} + \delta_{ac}\delta_{bd})$$

$$D_\mu H = \left(\partial_\mu - i g_2 \frac{\tau^I}{2} W^I_\mu - i g_1 y_h B_\mu \right) H \longrightarrow (D_\mu \phi)_a = \left(\partial_\mu \delta_{ab} - i g_2 \theta^I_{ab} W^I_\mu - i g_1 \theta^h_{ab} B_\mu \right) \phi_b$$

$$\theta^h = \frac{i}{2} \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix} \qquad \theta^I = \frac{i}{2} \begin{pmatrix} \mathrm{Im}(\tau^I) & \mathrm{Re}(\tau^I) \\ -\mathrm{Re}(\tau^I) & \mathrm{Im}(\tau^I) \end{pmatrix}$$

SMEFT Cross-Check

$\phi^2 F^2$ Operators

$$\mathcal{L}_{\text{SMEFT}} \supset C_{HG} G^A_{\mu\nu} G^{A\mu\nu} (H^\dagger H) + C_{HW} W^I_{\mu\nu} W^{I\mu\nu} (H^\dagger H) + C_{HB} B_{\mu\nu} B^{\mu\nu} (H^\dagger H) + C_{HWB} B_{\mu\nu} W^{I\mu\nu} (H^\dagger \tau^I H)$$

$$[C_{HG}]^{AB}_{ab} = \frac{1}{2} C_{HG} \delta_{ab} \delta^{AB} \quad [C_{HW}]^{IJ}_{ab} = \frac{1}{2} C_{HW} \delta_{ab} \delta^{IJ} \quad [C_{HB}]_{ab} = \frac{1}{2} C_{HB} \delta_{ab} \quad [C_{HWB}]^{I}_{ab} = \frac{1}{2} C_{HWB} \Sigma^I_{ab}$$

$$\Sigma^1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \Sigma^2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad \Sigma^3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

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Collinear anomalous dimensions

$$\gamma_{c,H} = -g_1^2 - 3g_2^2 \qquad \gamma_{c,G} = -g_3^2 b_{0,3} \qquad \gamma_{c,W} = -g_2^2 b_{0,2} \qquad \gamma_{c,B} = -g_1^2 b_{0,1}$$

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Group algebra $\Longrightarrow$

$$\dot{C}_{HG} = \left(12\lambda - \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2 - 2b_{0,3}g_3^2\right) C_{HG} \qquad \dot{C}_{HW} = \left(12\lambda - \frac{3}{2}g_1^2 - \frac{5}{2}g_2^2 - 2\, b_{0,2}g_2^2\right) C_{HW} + g_1 g_2\, C_{HWB}$$

$$\dot{C}_{HB} = \left(12\lambda + \frac{1}{2}g_1^2 - 2\, b_{0,1}g_1^2 - \frac{9}{2}g_2^2\right) C_{HB} + 3g_1 g_2\, C_{HWB}$$

$$\dot{C}_{HWB} = \left(4\lambda - \frac{1}{2}g_1^2 - b_{0,1}g_1^2 + \frac{9}{2}g_2^2 - b_{0,2}g_2^2\right) C_{HWB} + 2g_1 g_2\, C_{HB} + 2g_1 g_2\, C_{HW} \qquad \checkmark$$

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  - On-shell methods have already proven to be efficient at 2 loops [Bern, Parra-Martinez, Sawyer (20)], [Elias Miro, Fernandez, Gumus, Pomarol, (21)], ...
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  - General gauge EFT is not much more complicated than specific EFTs
  - (Bonus: we can recycle all tree-level amplitudes computed so far)
- **Automatize** calculation of RGEs
  - **[Gauge group]** + **[Particle content]** $\overset{\text{Group algebra}}{\rightsquigarrow}$ **[RGEs]**

Thank you for your attention!

Q&A

Selection Rules: Dimension-6 Operators

	F^3	$\phi^2 F^2$	$F\phi\psi^2$	$D^2\phi^4$	$D\phi^2\psi^2$	ψ^4	$\phi^3\psi^2$	ϕ^6
F^3		$\times_1$	(2)	$\times_2$	$\times_2$	$\times_2$	$\times_3$	$\times_3$
$\phi^2 F^2$							(2)	$\times_2$
$F\phi\psi^2$							$\times_1$	$\times_3$
$D^2\phi^4$							$\times_1$	$\times_2$
$D\phi^2\psi^2$							$\times_1$	(3)
ψ^4							(2)	(4)
$\phi^3\psi^2$								(2)
ϕ^6								

Table: From [Bern, Parra-Martinez, Sawyer (20)]. Dimension-6 operator mixing pattern. Operators labeling the rows are renormalized by the operators labeling the columns.

- $\times_L$: length selection rules apply at L-loop order
- (L): no diagrams before L loops, but renormalization is possible at that order
- Light-gray: zero at one loop due to helicity selection rules

Spinor-Helicity Formalism

The 4-momentum of an on-shell state is mapped onto a 2×2 matrix

$$p^\mu = (p^0, \vec{p}) \qquad \longrightarrow \qquad p^{\dot{\alpha}\alpha} = \bar{\sigma}_\mu^{\dot{\alpha}\alpha} p^\mu = \begin{pmatrix} p^0 + p^3 & p^1 - ip^2 \\ p^1 + ip^2 & p^0 - p^3 \end{pmatrix},$$

where $\bar{\sigma}^{\mu\,\dot{\alpha}\alpha} = (\mathbb{1}, -\vec{\sigma})^{\dot{\alpha}\alpha}$. If the particle is massless then

$$p^2 = \det\left(p^{\dot{\alpha}\alpha}\right) = m^2 = 0 \qquad \Longrightarrow \qquad p^{\dot{\alpha}\alpha} = \tilde{\lambda}^{\dot{\alpha}} \lambda^\alpha ,$$

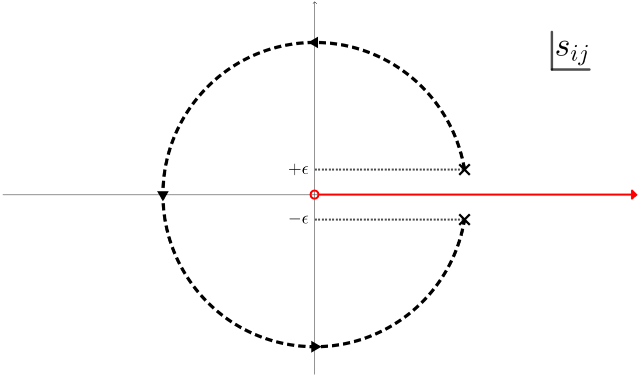
where $\lambda, \tilde{\lambda}$ are commuting Weyl spinors known as helicity spinors.
The angle and square inner products are Lorentz invariant

$$\langle i\, j\rangle \equiv \lambda_i^\alpha \lambda_{j\,\alpha} = \epsilon_{\alpha\beta}\lambda_i^\alpha \lambda_j^\beta = -\langle j i\rangle , \qquad [i\, j] \equiv \tilde{\lambda}_{i\,\dot{\alpha}} \tilde{\lambda}_j^{\dot{\alpha}} = -\epsilon_{\dot{\alpha}\dot{\beta}} \tilde{\lambda}_i^{\dot{\alpha}} \tilde{\lambda}_j^{\dot{\beta}} = -[j i] .$$

The Mandelstam invariants can thus be written as

$$s_{ij} \equiv (p_i + p_j)^2 = 2 p_i \cdot p_j = \langle i j\rangle [j i] .$$

Dilatation Operator & Complex Rotations



The **Dilatation Operator**

$$D = \sum_i p_i \cdot \frac{\partial}{\partial p_i}$$

generates the **Complex Rotations**:

$$p_i \to e^{i\alpha} p_i \implies F_{\mathcal{O}} \to e^{i\alpha D} F_{\mathcal{O}} \, .$$

For $\alpha = \pi$ their infinitesimal imaginary part ϵ changes sign:

$$F_{\mathcal{O}}(\{s_{ij} - i\epsilon\}) = e^{i\pi D} F_{\mathcal{O}}(\{s_{ij} + i\epsilon\}) \, .$$

Nonperturbative Relations

$S\&D$ Relation

$$(e^{-i\pi D} - 1)F_i^* = i(\mathcal{M}F_i^*)$$

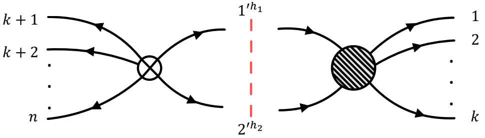
- In dimensional regularization and in absence of masses, $D \simeq -\mu\,\partial/\partial\mu$, which implies

Callan-Symanzik Equation

$$DF_j = \left(\frac{\partial\beta_i}{\partial c_j} - \delta_{ij}\gamma_{i,\mathrm{IR}} + \delta_{ij}\beta_g\frac{\partial}{\partial g}\right)F_i$$

- Can be combined and expanded, *e.g.* at one-loop

$$\left(\frac{\partial\beta_i^{(1)}}{\partial c_j} - \delta_{ij}\gamma_{i,\mathrm{IR}}^{(1)} + \delta_{ij}\beta_g^{(1)}\frac{\partial}{\partial g}\right)F_i^{(0)} = -\frac{1}{\pi}(\mathcal{M}F_j)^{(1)}$$



IR Anomalous Dimensions

- In theories with massless fields, IR singularities originate from configurations where loop momenta become soft or collinear.
- The IR anomalous dimension only depends on the external state $\langle\vec{n}|$ [Becher, Neubert (09)]

$$\gamma_{\mathrm{IR}}^{(1)}(\{s_{ij}\};\mu) = \frac{g^2}{4\pi^2}\sum_{i<j} T_i^a T_j^a \log\frac{\mu^2}{-s_{ij}} + \sum_i \gamma_{i,\mathrm{coll.}}^{(1)} \,.$$

- Since the stress-energy tensor $T_{\mu\nu}$ is UV protected, γ_{IR} can be computed as

$$\gamma_{\mathrm{IR}}^{(1)} F_{T_{\mu\nu}}^{(0)}(\vec{n}) = \frac{1}{\pi}(\mathcal{M}F_{T_{\mu\nu}})^{(1)}(\vec{n})\,.$$