Universität Zürich UZH

Evanescent Schemes and Prescriptions*

29.01.2025, Mainz

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*in collaboration with Jason Aebischer & Zach Polonsky

Outline of the talk

I. Evanescent operators : definitions, prescription & scheme

II. Prescription vs Scheme:

- *A Simple Dirac Prescription for Two-Loop Anomalous Dimension Matrices* ([2401.16904 : J.Aebischer, M.P, Z.Polonsky])

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I. Evanescent operators : definitions, prescription & scheme

II. Prescription vs Scheme:

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➢ The «shift» perspective:

- *One-Loop Fierz Identities* ([2208.10513 : J.Aebischer & M.P]),
- *Dipole Operators in Fierz Identities* ([2211.01379 : J.Aebischer, M.P, Z.Polonsky])

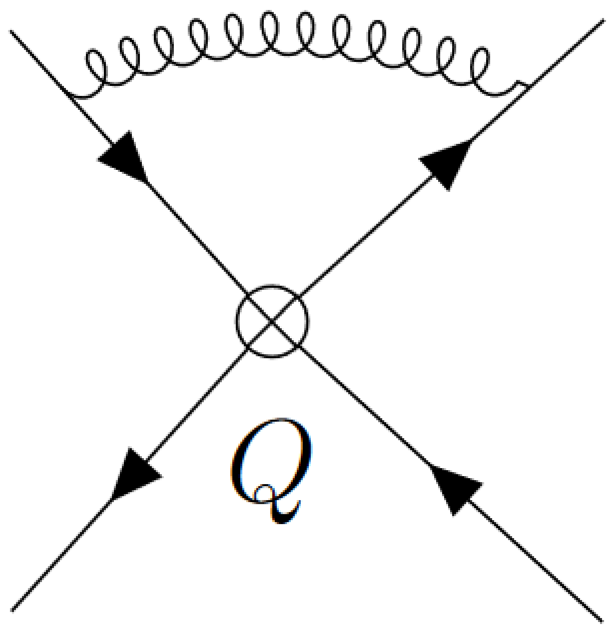
➢ Shifts in the context of change of bases of NLO ADMs:

- *Renormalization Scheme Factorization of one-loop Fierz Identities* ([2306.16449 : J.Aebischer, M.P, Z.Polonsky])

[Buras & Weisz (1990)], [Buras, Misiak, Urban (2000)], [Jenkins, Manohar, Stoffer (2018)], [Dugan & Grinstein (1991)], [Herrlich & Nierste (1995)], [Aebischer, Bobeth, Buras, Kumar (2020-2021)], [Bélusca-Maïto, Ilakovac, Mađor-Božinović, Stöckinger (2020)], ['t Hooft and Veltman (1972)], [Buras & Girrbach (2012)], [Chetyrkin, Misiak, Munz (1998)], [Dekens & Stoffer (2019)], [Grzadkowski, Iskrzynski, Misiak (2010)]...

General Context

Consider a set of physical operators $\{Q\}$, and compute one-loop matrix elements $\langle Q\rangle^{(1)}$:



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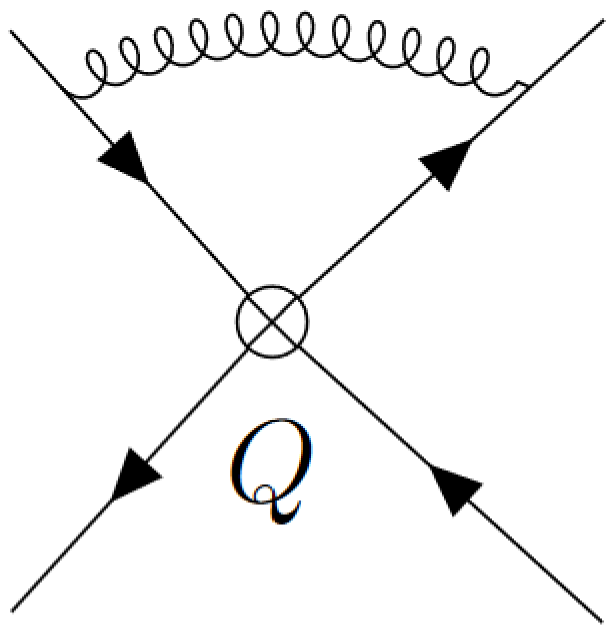

$\longrightarrow$ Dirac Structure $\{\mathcal{D}\}$. In d=4, they map back to the physical basis :

$$\mathcal{D} \overset{d=4}{=} \mathcal{F}_4 Q$$

➡ In dim. reg (d=4-2ε), no unambiguous way to continue Dirac Algebra !

Evanescent operators, Prescription and Scheme dependence

To account for this, we must :

- specify a ***prescription*** (i.e. how to *treat* Dirac Algebra: NDR, HV, Larin,…),

[See Luca's talk]

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$$E = \mathcal{D} - \mathcal{F}Q$$

$$\mathcal{F} = \mathcal{F}_4 + \sum_{n=1}^{\infty} \epsilon^n \sigma_n$$

The d=4 part is fixed.

Constants fix the *scheme dependence*.

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➡ $E\,\big|_{d=4} = 0$

➡ $\langle E \rangle^{(1)} \neq 0$

Finite pieces to be taken into account [See Matthias' talk]

A Simple Dirac Prescription for Two-Loop Anomalous Dimension Matrices [2401.16904]

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$$\gamma^{(1)}_{Q_iQ_j} = 4\delta Z^{(2;1)}_{Q_iQ_j} - 2\delta Z^{(1;1)}_{Q_iE_k}\underset{\substack{\uparrow \\ \text{Evanescent-to-physical mixing}}}{\underline{\delta Z^{(1;0)}_{E_kQ_j}}}$$

- NLO ADM cancels one-loop matching scheme dep.
- Subtract ev-to-ph. to decouple the two sectors

[Many talks today about 2-loop running]

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Evanescent-to-physical mixing

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[Many talks today about 2-loop running]

➢ How does the choice of prescription & scheme affect NLO ADMs ?

➢ Is there a « smart » choice to simplify / remove the ev-to-ph. mixing ?

A Simple Dirac Prescription for Two-Loop Anomalous Dimension Matrices [2401.16904]

- Muon decay via a heavy scalar + QED corrections

$$\mathcal{L} \supset -y_\ell^L\big(\bar{\ell}P_L\nu_\ell\big)\phi - y_\ell^R\big(\bar{\ell}P_R\nu_\ell\big)\phi + \text{h.c.}$$

$$\Big\downarrow$$

$$Q_s = \big(\bar{\nu}_\mu P_L\mu\big)\big(\bar{e}P_L\nu_e\big), \quad Q_t = \big(\bar{\nu}_\mu\sigma^{\mu\nu}P_L\mu\big)\big(\bar{e}\sigma_{\mu\nu}P_L\nu_e\big)$$

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- One-loop matching + Two-loop running in 3 different prescriptions (GP, NP, NDR)
- Understand how different choices of prescriptions affect the ev-to-ph. mixing in the NLO ADMs

A Simple Dirac Prescription for Two-Loop Anomalous Dimension Matrices [2401.16904]

➢ Muon decay via a heavy scalar + QED corrections with 3 different prescriptions (GP, NP, NDR)

- Greek Prescription* (GP): no ev. ops., only replacement rules

$$\left(\sigma^{\mu\nu} P_L \gamma^\alpha \gamma^\beta\right) \otimes \left(\gamma_\beta \gamma_\alpha \sigma_{\mu\nu} P_L\right) = \left(48 - 80\epsilon\right)\left(P_L\right) \otimes \left(P_L\right) + \left(12 - 14\epsilon\right)\left(\sigma^{\mu\nu} P_L\right) \otimes \left(\sigma_{\mu\nu} P_L\right)$$

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Structure arising at 1-loop

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$$\overline{E}_1 = \big(\bar{\nu}_\mu P_L \gamma^\mu \gamma^\nu \mu\big)\big(\bar{e}\gamma_\nu \gamma_\mu P_L \nu_e\big) - (4 - \epsilon\overline{\sigma}_{s1})Q_s - (1 - \epsilon\overline{\sigma}_{t1})Q_t$$

$$\overline{E}_2 = \big(\bar{\nu}_\mu \sigma^{\mu\nu} P_L \gamma^\alpha \gamma^\beta \mu\big)\big(\bar{e}\gamma_\beta \gamma_\alpha \sigma_{\mu\nu} P_L \nu_e\big) - \big(48 - \epsilon\overline{\sigma}_{s2}\big)Q_s - (12 - \epsilon\overline{\sigma}_{t2})Q_t\,.$$

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New structures are generated when inserting ev. ops. -> more ev. ops. !

$$\overline{E}_1' = \left(\bar{\nu}_\mu P_L \gamma^\mu \gamma^\nu \gamma^\alpha \gamma^\beta \mu\right)\left(\bar{e}\gamma_\beta \gamma_\alpha \gamma_\nu \gamma_\mu P_L \nu_e\right) - (64 - \epsilon\underline{\overline{\sigma}'_{s1}})Q_s - (16 - \epsilon\underline{\overline{\sigma}'_{t1}})Q_t \,,$$

$$\overline{E}_2' = \left(\bar{\nu}_\mu \sigma^{\mu\nu} P_L \gamma^\alpha \gamma^\beta \gamma^\sigma \gamma^\rho \mu\right)\left(\bar{e}\gamma_\rho \gamma_\sigma \gamma_\beta \gamma_\alpha \sigma_{\mu\nu} P_L \nu_e\right) - \left(768 - \epsilon\underline{\overline{\sigma}'_{s2}}\right)Q_s - \left(192 - \epsilon\underline{\overline{\sigma}'_{t2}}\right)Q_t$$

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- NLO ADMs should *not* depend on these scheme constants!
- « pure scheme »: $Z_{\overline{E}Q}^{(1;0)} = \begin{pmatrix} -2\overline{\sigma}_{s1} + \frac{\overline{\sigma}'_{s1}}{4} - \frac{\overline{\sigma}_{s2}}{4} - 12\overline{\sigma}_{t1} & -\frac{\overline{\sigma}_{s1}}{4} - 4\overline{\sigma}_{t1} + \frac{\overline{\sigma}'_{t1}}{4} - \frac{\overline{\sigma}_{t2}}{4} \\ -12\overline{\sigma}_{s1} - 4\overline{\sigma}_{s2} + \frac{\overline{\sigma}'_{s2}}{4} - 12\overline{\sigma}_{t2} & -\frac{\overline{\sigma}_{s2}}{4} - 12\overline{\sigma}_{t1} - 6\overline{\sigma}_{t2} + \frac{\overline{\sigma}'_{t2}}{4} \end{pmatrix}$

A Simple Dirac Prescription for Two-Loop Anomalous Dimension Matrices [2401.16904]

- Muon decay via a heavy scalar + QED corrections with 3 different prescriptions (GP, NP, NDR)

- Naive dimensional regularization: d-dim Dirac algebra + scheme-dependent ev. ops.

$$E_{\mathrm{NDR}} = \left(\bar{\nu}_\mu \gamma^\mu \gamma^\nu \gamma^\alpha \gamma^\beta P_L \mu\right)\left(\bar{e} \gamma_\mu \gamma_\nu \gamma_\alpha \gamma_\beta P_L \nu_e\right) - (64 - \epsilon\sigma_s) Q_s - (-16 - \epsilon\sigma_t) Q_t$$

$$\longrightarrow \left(\bar{\nu}_\mu \sigma^{\mu\nu} P_L \gamma^\alpha \gamma^\beta \mu\right)\left(\bar{e} \gamma_\beta \gamma_\alpha \sigma_{\mu\nu} P_L \nu_e\right) = \left(48 - (\underline{-16} + \sigma_s)\epsilon\right) Q_s + \left(12 - (\underline{22} + \sigma_t)\epsilon\right) Q_t + E_{\mathrm{NDR}}$$

Re-ordering of Dirac matrices to project on the chosen ev. ops. -> ev-to-ph. is ***not*** pure scheme anymore!

$$Z^{(1;0)}_{EQ} = \begin{pmatrix} 160 + 5\sigma_s - 12\sigma_t - \frac{\sigma_s'}{4} & -40 - \frac{\sigma_s}{4} + 3\sigma_t - \frac{\sigma_t'}{4} \end{pmatrix}$$

A Simple Dirac Prescription for Two-Loop Anomalous Dimension Matrices [2401.16904]

All NLO ADMs agree (up to scheme choice)…

$$\gamma^{(1)}_{\mathrm{NDR}} = \begin{pmatrix} -21 - \frac{\sigma_s}{8} & -\frac{23}{18} - \frac{\sigma_t}{8} \\ \frac{520}{3} + \frac{\sigma_s}{3} + 6\sigma_t & \frac{103}{9} + \frac{\sigma_s}{8} + \frac{4\sigma_t}{3} \end{pmatrix}$$

$$\gamma^{(1)}_{\mathrm{NP}} = \begin{pmatrix} -\frac{77}{3} + \frac{4}{3}\overline{\sigma}_{s1} + 6\overline{\sigma}_{t1} - \frac{\overline{\sigma}_{s2}}{8} & \frac{11}{9} + \frac{\overline{\sigma}_{s1}}{8} + \frac{7\overline{\sigma}_{t1}}{3} - \frac{\overline{\sigma}_{t2}}{8} \\ \frac{176}{3} - 6\overline{\sigma}_{s1} + \frac{\overline{\sigma}_{s2}}{3} + 6\overline{\sigma}_{t2} & -\frac{143}{9} - 6\overline{\sigma}_{t1} + \frac{\overline{\sigma}_{s2}}{8} + \frac{4\overline{\sigma}_{t2}}{3} \end{pmatrix}$$

$$\gamma^{(1)}_{\mathrm{GP}} = \begin{pmatrix} -33 & -\frac{5}{18} \\ \frac{472}{3} & \frac{115}{9} \end{pmatrix}$$

A Simple Dirac Prescription for Two-Loop Anomalous Dimension Matrices [2401.16904]

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What about GP that has **no** ev. ops. ? Can we forget about them ?

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(By construction)

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(By construction)

➡ GP seems to feature a *smart* choice of replacement rules -> *no* need to insert ev. ops. !

What is the «general»* condition to have a prescription *free* of Evanescent-to-Physical mixing ?

*More on this later!

A Simple Dirac Prescription for Two-Loop Anomalous Dimension Matrices [2401.16904]

$$\langle Q_{ij;a}\rangle^{(1)} \propto (\bar{\psi}_{b_1}\Gamma_{\alpha_1}\Gamma_i^a\Gamma_{\alpha_2}\psi_{b_2})(\bar{\psi}_{b_3}\Gamma_{\alpha_3}\Gamma_j^a\Gamma_{\alpha_4}\psi_{b_4})$$

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$$\longrightarrow \quad E^{(1);a\to b}_{\alpha_1 i\alpha_2;\alpha_3 j\alpha_4} = (\bar{\psi}_{b_1}\Gamma_{\alpha_1}\Gamma^{a}_{i}\Gamma_{\alpha_2}\psi_{b_2})(\bar{\psi}_{b_3}\Gamma_{\alpha_3}\Gamma^{a}_{j}\Gamma_{\alpha_4}\psi_{b_4}) - C^{k\ell;c}_{\alpha_1 i\alpha_2;\alpha_3 j\alpha_4;a\to b}\, Q_{k\ell;c}$$

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This ev. ops. can mix with physical ops! -> compute finite subtractions

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$$\begin{aligned}\left\langle E^{(1);a\to b}_{\alpha_1 i\alpha_2;\alpha_3 j\alpha_4}\right\rangle^{(1)} \propto\, &(\bar{\psi}_{c_1}\Gamma_{\beta_1}\Gamma_{\alpha_1}\Gamma_i^a\Gamma_{\alpha_2}\Gamma_{\beta_2}\psi_{c_2})(\bar{\psi}_{c_3}\Gamma_{\beta_3}\Gamma_{\alpha_3}\Gamma_j^a\Gamma_{\alpha_4}\Gamma_{\beta_4}\psi_{c_4})\\ &- C^{k\ell;d}_{\alpha_1 i\alpha_2;\alpha_3 j\alpha_4;a\to b}(\bar{\psi}_{c_1}\Gamma_{\beta_1}\Gamma_k^d\Gamma_{\beta_2}\psi_{c_2})(\bar{\psi}_{c_3}\Gamma_{\beta_3}\Gamma_\ell^d\Gamma_{\beta_4}\psi_{c_4})\end{aligned}$$

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$$\begin{aligned}E^{(2);a\to c}_{\beta_1\alpha_1 i\alpha_2\beta_2;\beta_3\alpha_3 j\alpha_4\beta_4} =\, &(\bar\psi_{c_1}\Gamma_{\beta_1}\Gamma_{\alpha_1}\Gamma^a_i\Gamma_{\alpha_2}\Gamma_{\beta_2}\psi_{c_2})(\bar\psi_{c_3}\Gamma_{\beta_3}\Gamma_{\alpha_3}\Gamma^a_j\Gamma_{\alpha_4}\Gamma_{\beta_4}\psi_{c_4}) \\ &- K^{k\ell;d}_{\beta_1\alpha_1 i\alpha_2\beta_2;\beta_3\alpha_3 j\alpha_4\beta_4;a\to c}\, Q_{k\ell;d}\,.\end{aligned}$$

①

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$$\left\langle E^{(1);a\to b}_{\alpha_1 i\alpha_2;\alpha_3 j\alpha_4}\right\rangle^{(1)} \propto (\bar{\psi}_{c_1}\Gamma_{\beta_1}\Gamma_{\alpha_1}\Gamma^a_i\Gamma_{\alpha_2}\Gamma_{\beta_2}\psi_{c_2})(\bar{\psi}_{c_3}\Gamma_{\beta_3}\Gamma_{\alpha_3}\Gamma^a_j\Gamma_{\alpha_4}\Gamma_{\beta_4}\psi_{c_4})$$
$$- C^{k\ell;d}_{\alpha_1 i\alpha_2;\alpha_3 j\alpha_4;a\to b}\underline{(\bar{\psi}_{c_1}\Gamma_{\beta_1}\Gamma^d_k\Gamma_{\beta_2}\psi_{c_2})(\bar{\psi}_{c_3}\Gamma_{\beta_3}\Gamma^d_\ell\Gamma_{\beta_4}\psi_{c_4})}$$

$$(\bar{\psi}_{c_1}\Gamma_{\beta_1}\Gamma^d_k\Gamma_{\beta_2}\psi_{c_2})(\bar{\psi}_{c_3}\Gamma_{\beta_3}\Gamma^d_\ell\Gamma_{\beta_4}\psi_{c_4}) = E^{(1);d\to c}_{\beta_1 k\beta_2;\beta_3\ell\beta_4} + C^{mn;f}_{\beta_1 k\beta_2;\beta_3\ell\beta_4;d\to c}Q_{mn;f}$$

②

(one-loop physical operator insertion)

A Simple Dirac Prescription for Two-Loop Anomalous Dimension Matrices [2401.16904]

$$\longrightarrow \quad \left\langle E^{(1);a\to b}_{\alpha_1 i\alpha_2;\alpha_3 j\alpha_4}\right\rangle^{(1)} \propto (\bar\psi_{c_1}\Gamma_{\beta_1}\Gamma_{\alpha_1}\Gamma^a_i\Gamma_{\alpha_2}\Gamma_{\beta_2}\psi_{c_2})(\bar\psi_{c_3}\Gamma_{\beta_3}\Gamma_{\alpha_3}\Gamma^a_j\Gamma_{\alpha_4}\Gamma_{\beta_4}\psi_{c_4})$$

$$- C^{k\ell;d}_{\alpha_1 i\alpha_2;\alpha_3 j\alpha_4;a\to b}(\bar\psi_{c_1}\Gamma_{\beta_1}\Gamma^d_k\Gamma_{\beta_2}\psi_{c_2})(\bar\psi_{c_3}\Gamma_{\beta_3}\Gamma^d_\ell\Gamma_{\beta_4}\psi_{c_4})$$

① $\big\downarrow$ ②

$$\left\langle E^{(1);a\to b}_{\alpha_1 i\alpha_2;\alpha_3 j\alpha_4}\right\rangle^{(1)} \propto E^{(2);a\to c}_{\beta_1\alpha_1 i\alpha_2\beta_2;\beta_3\alpha_3 j\alpha_4\beta_4} - C^{k\ell;d}_{\alpha_1 i\alpha_2;\alpha_3 j\alpha_4;a\to b}E^{(1);d\to c}_{\beta_1 k\beta_2;\beta_3\ell\beta_4}$$

$$+ \left(K^{mn;f}_{\beta_1\alpha_1 i\alpha_2\beta_2;\beta_3\alpha_3 j\alpha_4\beta_4;a\to c} - C^{k\ell;d}_{\alpha_1 i\alpha_2;\alpha_3 j\alpha_4;a\to b}C^{mn;f}_{\beta_1 k\beta_2;\beta_3\ell\beta_4;d\to c}\right)Q_{mn;f}$$

Evanescent-to-physical mixing vanishes if:

$$\boxed{K^{mn;f}_{\beta_1\alpha_1 i\alpha_2\beta_2;\beta_3\alpha_3 j\alpha_4\beta_4;a\to c} = C^{k\ell;d}_{\alpha_1 i\alpha_2;\alpha_3 j\alpha_4;a\to b}C^{mn;f}_{\beta_1 k\beta_2;\beta_3\ell\beta_4;d\to c}}$$

A Simple Dirac Prescription for Two-Loop Anomalous Dimension Matrices [2401.16904]

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In our simple example, the effect of the evanescent operators on ADMs is equivalent to that of replacement rules

$$(\Gamma_{\alpha_1}\Gamma_i\Gamma_{\alpha_2})\otimes(\Gamma_{\alpha_3}\Gamma_j\Gamma_{\alpha_4}) \to C^{k\ell}_{\alpha_1 i\alpha_2;\alpha_3 j\alpha_4}(\Gamma_k)\otimes(\Gamma_\ell)\,,$$

$$(\Gamma_{\beta_1}\Gamma_{\alpha_1}\Gamma_i\Gamma_{\alpha_2}\Gamma_{\beta_2})\otimes(\Gamma_{\beta_3}\Gamma_{\alpha_3}\Gamma_j\Gamma_{\alpha_4}\Gamma_{\beta_4}) \to C^{k\ell}_{\alpha_1 i\alpha_2;\alpha_3 j\alpha_4} C^{mn}_{\beta_1 k\beta_2;\beta_3\ell\beta_4}(\Gamma_m)\otimes(\Gamma_n)$$

2nd line = recursive application of the first line, as is required for such a prescription to be *self-consistent*

A Simple Dirac Prescription for Two-Loop Anomalous Dimension Matrices [2401.16904]

Lessons:

I. Fixing

$$(P_L\gamma^\mu\gamma^\nu)\otimes(\gamma_\nu\gamma_\mu P_L)\rightarrow(4-\epsilon\overline{\sigma}_{s1})(P_L)\otimes(P_L)+(1-\epsilon\overline{\sigma}_{t1})(\sigma^{\mu\nu}P_L)\otimes(\sigma_{\mu\nu}P_L)\,,$$
$$(\sigma^{\mu\nu}P_L\gamma^\alpha\gamma^\beta)\otimes(\gamma_\beta\gamma_\alpha\sigma_{\mu\nu}P_L)\rightarrow(48-\epsilon\overline{\sigma}_{s2})(P_L)\otimes(P_L)+(12-\epsilon\overline{\sigma}_{t2})(\sigma^{\mu\nu}P_L)\otimes(\sigma_{\mu\nu}P_L)$$

\+ $\{\gamma^\mu,\gamma^\nu\}=2\eta^{\mu\nu}$

➡ Completely specifies the Dirac structure reduction at NLO:

$$\begin{aligned}(P_L\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta)\otimes(\gamma_\beta\gamma_\alpha\gamma_\nu\gamma_\mu P_L)\rightarrow&\Big(64-\underline{\{8\overline{\sigma}_{s1}+\overline{\sigma}_{s2}+48\overline{\sigma}_{t1}\}}\epsilon\Big)(P_L)\otimes(P_L)\\&+\Big(16-\underline{\{\overline{\sigma}_{s1}+16\overline{\sigma}_{t1}+\overline{\sigma}_{t2}\}}\epsilon\Big)(\sigma^{\mu\nu}P_L)\otimes(\sigma_{\mu\nu}P_L)\,,\\(\sigma^{\mu\nu}P_L\gamma^\alpha\gamma^\beta\gamma^\sigma\gamma^\rho)\otimes(\gamma_\rho\gamma_\sigma\gamma_\beta\gamma_\alpha\sigma_{\mu\nu}P_L)\rightarrow&\Big(768-\underline{\{48\overline{\sigma}_{s1}+16\overline{\sigma}_{s2}+48\overline{\sigma}_{t2}\}}\epsilon\Big)(P_L)\otimes(P_L)\\&+\Big(192-\underline{\{\overline{\sigma}_{s2}+48\overline{\sigma}_{t1}+24\overline{\sigma}_{t2}\}}\epsilon\Big)(\sigma^{\mu\nu}P_L)\otimes(\sigma_{\mu\nu}P_L)\,.\end{aligned}$$

NLO scheme constants are fixed!

A Simple Dirac Prescription for Two-Loop Anomalous Dimension Matrices [2401.16904]

Lessons:

II. Treatment of Dirac structures at LO and NLO should be *consistent*

$$E_{\mathrm{NDR}} = \left(\bar{\nu}_\mu \gamma^\mu \gamma^\nu \gamma^\alpha \gamma^\beta P_L \mu\right)\left(\bar{e} \gamma_\mu \gamma_\nu \gamma_\alpha \gamma_\beta P_L \nu_e\right) - (64 - \epsilon \sigma_s) Q_s - (-16 - \epsilon \sigma_t) Q_t$$

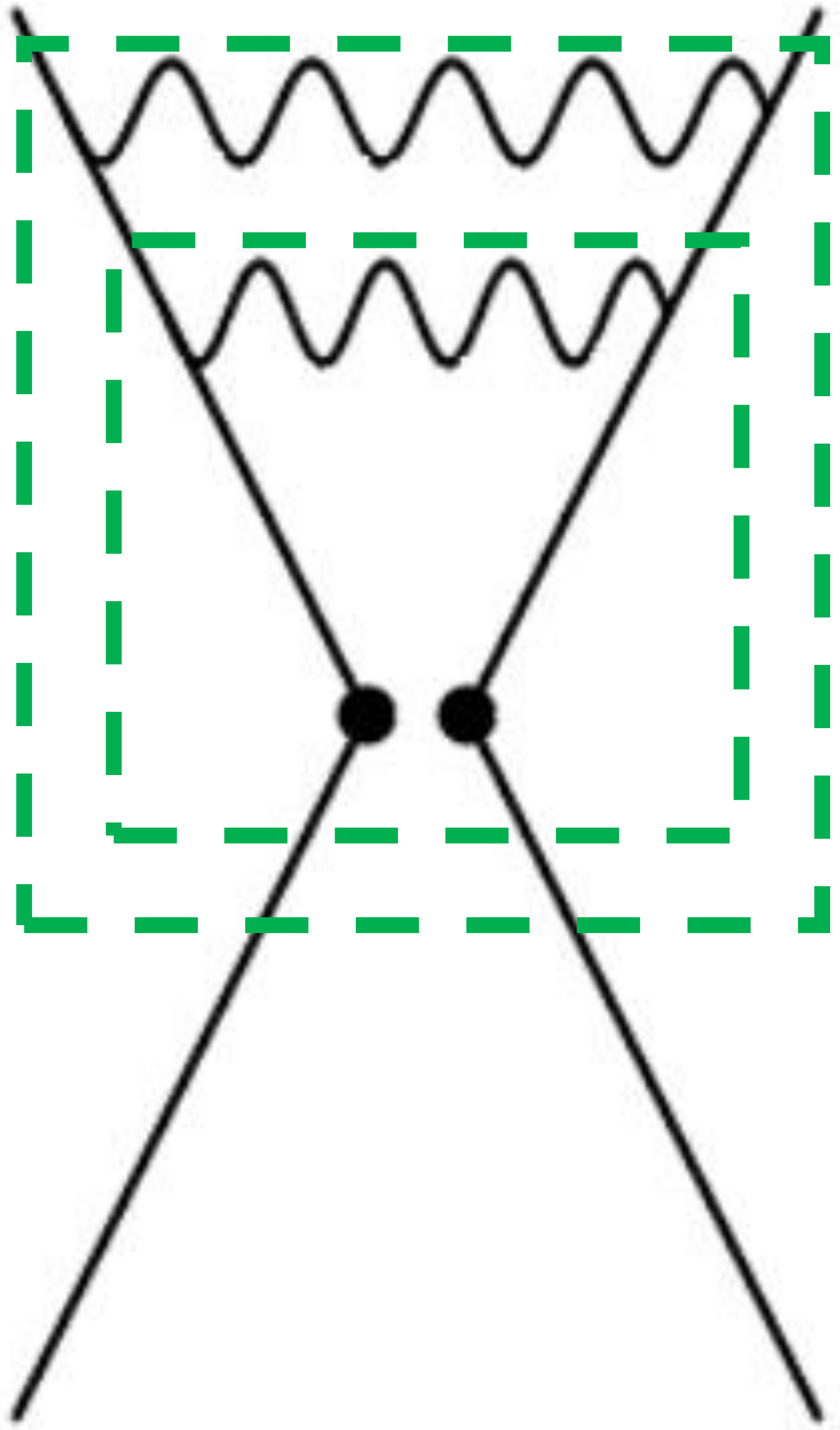
$\longrightarrow$ $(\gamma^\mu \gamma^\nu \gamma^\alpha \gamma^\beta P_L \gamma^\sigma \gamma^\rho) \otimes (\gamma_\rho \gamma_\sigma \gamma_\mu \gamma_\nu \gamma_\alpha \gamma_\beta P_L)$ + E_{NDR} to fix E'_{NDR} ✓

$\xrightarrow{\textit{(Re-order)}}$ $(\gamma^\mu \gamma^\nu \gamma^\alpha \gamma^\beta \gamma^\sigma \gamma^\rho P_L) \otimes (\gamma_\mu \gamma_\nu \gamma_\alpha \gamma_\beta \gamma_\sigma \gamma_\rho P_L)$ + E_{NDR} spoils the ev-to-ph. cancellation!

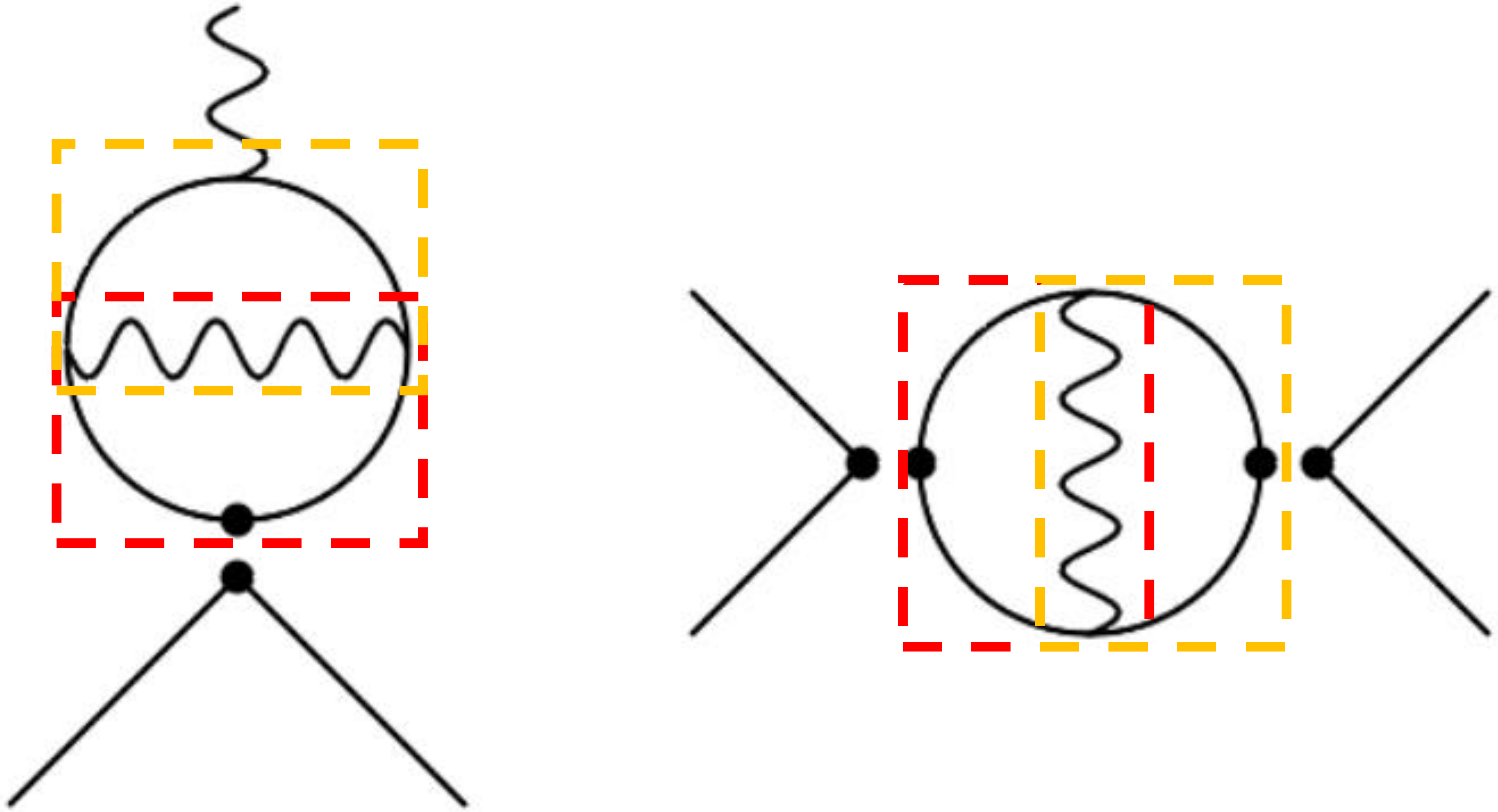
A Simple Dirac Prescription for Two-Loop Anomalous Dimension Matrices [2401.16904]

Disclaimer

[More on this in Zach's talk]



Nested divergences ✓



Overlapping divergences ?

Conclusion

- Recursive and well-suited for automation
- Ensures vanishing of evanescent-to-physical mixing -> no need to insert ev. ops.
- γ_5 traces still problematic -> needs to be treated separately
- Scheme transformation might be complicated [2306.16449 : J.Aebischer, M.P, Z.Polonsky]
- Generalization / applicability -> Zach's talk

Backup Slides

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

Relating two bases with different schemes :

$$\vec{\tilde{Q}}_{\underbrace{\tilde{\Sigma};\tilde{S}}_{\text{Scheme \& Prescription}}} = (R_0 + \underset{\uparrow}{\Delta})\vec{Q}_{\Sigma;S}$$

Shifts (« Generalized » R_1)

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

Relating two bases with different schemes :

$$\vec{\tilde{Q}}_{\underbrace{\tilde{\Sigma};\tilde{S}}} = (R_0 + \Delta)\vec{Q}_{\Sigma;S}$$

Scheme &
Prescription

Shifts (« Generalized » R_1)

Scheme dependence of E

$$\Delta\langle\vec{Q}\rangle^{(0)} = P_{Q;\Sigma}\left(\langle\vec{\tilde{Q}}\rangle^{(1)}_{\tilde{\Sigma};\tilde{S}}\right) - (R_0 + \epsilon\Sigma)\langle\vec{Q}\rangle^{(1)}_{\Sigma;S}$$

Projects the matrix
elements on the Q-basis
using the Σ-scheme

The double scheme-dependence
appearing in the shifts factorizes

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

Another example* : basis change

$$\vec{\tilde{Q}} = R_0(\vec{Q} + W\vec{E})$$

Change of bases formula for LO and NLO ADMs :

$$Z^{(1,0)}_{\tilde{Q}\tilde{Q}} = R_0 \left[W Z^{(1,0)}_{EQ} \right] R_0^{-1}$$

$$\tilde{\gamma}^{(0)} = R_0 \gamma^{(0)} R_0^{-1}$$

$$\tilde{\gamma}^{(1)} = R_0 \gamma^{(1)} R_0^{-1} - \left[Z^{(1,0)}_{\tilde{Q}\tilde{Q}}, \tilde{\gamma}^{(0)} \right] - 2\beta^{(0)} Z^{(1,0)}_{\tilde{Q}\tilde{Q}}$$

These are our shifts :

$$\boxed{R_1 = -R_0 W Z^{(1,0)}_{EQ}}$$

*Gorbahn, Jäger, Nierste, Trine (2009)

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

Another example* : basis change

$$\tilde{\vec{Q}} = \cancel{R_0(\vec{Q} + W\vec{E})} \Rightarrow \tilde{\vec{Q}} = R_0(\vec{Q} + W\vec{E}) \quad \& \quad \tilde{\vec{E}} = M(\epsilon U\vec{Q} + (\mathbb{1} + \epsilon UW)\vec{E})$$

Change of bases formula for LO and NLO ADMs :

$$Z^{(1;0)}_{\tilde{Q}\tilde{Q}} = R_0\left[WZ^{(1;0)}_{EQ} - \underbrace{\left(Z^{(1;1)}_{QE} + WZ^{(1;1)}_{EE} - Z^{(1;1)}_{QQ}W\right)U}_{\text{ev-to-ev}}\right]R_0^{-1}$$

$$\tilde{\gamma}^{(0)} = R_0\gamma^{(0)}R_0^{-1}$$

$$\tilde{\gamma}^{(1)} = R_0\gamma^{(1)}R_0^{-1} - \left[Z^{(1,0)}_{\tilde{Q}\tilde{Q}}, \tilde{\gamma}^{(0)}\right] - 2\beta^{(0)}Z^{(1,0)}_{\tilde{Q}\tilde{Q}}$$

$\Sigma;S$

$\tilde{\Sigma};\tilde{S}$

*Gorbahn, Jäger, Nierste, Trine (2009)
Chetyrkin, Misiak, Münz (1998)
Gorbahn, Haisch (2005)
Brod, Gorbahn (2010)

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

Another example* : basis change

$$\cancel{\tilde{\vec{Q}} = R_0(\vec{Q} + W\vec{E})} \Rightarrow \tilde{\vec{Q}} = R_0(\vec{Q} + W\vec{E}), \quad \tilde{\vec{E}} = M(\epsilon U\vec{Q} + (\mathbb{1} + \epsilon U W)\vec{E})$$

Change of bases formula for LO and NLO ADMs :

$$Z^{(1;0)}_{\tilde{Q}\tilde{Q}} = R_0 \left[W Z^{(1;0)}_{EQ} - \underbrace{\left(Z^{(1;1)}_{QE} + W Z^{(1;1)}_{EE} - Z^{(1;1)}_{QQ} W \right) U}_{\text{ev-to-ev}} \right] R_0^{-1}$$

$$\tilde{\gamma}^{(0)} = R_0 \gamma^{(0)} R_0^{-1}$$

$$\tilde{\gamma}^{(1)} = R_0 \gamma^{(1)} R_0^{-1} - \left[Z^{(1,0)}_{\tilde{Q}\tilde{Q}}, \tilde{\gamma}^{(0)} \right] - 2\beta^{(0)} Z^{(1,0)}_{\tilde{Q}\tilde{Q}}$$

➡ Need to **relate** ev. ops. of the two bases (M and U matrices)

➡ Need to **compute** 1-loop matrix elements ev. ops.

*Gorbahn, Jäger, Nierste, Trine (2009)
Chetyrkin, Misiak, Münz (1998)
Gorbahn, Haisch (2005)
Brod, Gorbahn (2010)

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

Instead : use shifts !

$$\tilde{\gamma}^{(0)} = R_0 \gamma^{(0)} R_0^{-1}$$

$$\tilde{\gamma}^{(1)} = R_0 \gamma^{(1)} R_0^{-1} - \left[\Delta R_0^{-1}, \tilde{\gamma}^{(0)}\right] - 2\beta^{(0)} \Delta R_0^{-1}$$

$$\boxed{\Delta\langle\vec{Q}\rangle^{(0)} = P_{Q;\Sigma}\left(\langle\vec{\tilde{Q}}\rangle^{(1)}_{\tilde{\Sigma};\tilde{S}}\right) - (R_0 + \epsilon\Sigma)\langle\vec{Q}\rangle^{(1)}_{\Sigma;S}}$$

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

Instead : use shifts !

$$\tilde{\gamma}^{(0)} = R_0 \gamma^{(0)} R_0^{-1}$$

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$$\boxed{\Delta\langle\vec{Q}\rangle^{(0)} = P_{Q;\Sigma}\left(\langle\vec{\tilde{Q}}\rangle^{(1)}_{\tilde{\Sigma};\tilde{S}}\right) - (R_0 + \epsilon\Sigma)\langle\vec{Q}\rangle^{(1)}_{\Sigma;S}}$$

- No need to relate anything accross different bases,
- 1-loop matrix elements of physical operators only,
- the shift **factorises** the schemes : erases S - dependence & restores $\tilde{S}$ - dependence