Non-Cancellation of Electroweak Logarithms in High-Energy Scattering

Sascha Turczyk

Work in collaboration with A. Manohar, B. Shotwell and Ch. Bauer

[arXiv:1409.1918]

ERC workshop
Schloss Waldthausen, October 13th 2014
**Example of** $pp(\text{later} \ gg) \rightarrow t\bar{t}$

- **Introduction**
- **Application to** $gg \rightarrow t\bar{t}, b\bar{b}$
- **Motivation**
  - Electroweak Corrections and SCET

**Current Status**

- Analysis extends to high energies
- $\sigma(t\bar{t})$: NNLO QCD, NNLL soft gluon resummation: top++2.0
  - [Baernreuther, Cacciari, Czakon, Fiedler, Mangano, Mitov, Nason; Beneke, Falgari, Klein, Schwinn]
- No clear sign of New Physics (yet?)
  - Electroweak effects become important for precision predictions

**Data**

- CMS
  - $19.5$ fb$^{-1} \ (8 \ TeV)$
  - $5 \times (1.75 \ TeV)$

- ATLAS
  - $\sqrt{s} = 8 \ TeV, 20.3 \ fb^{-1}$
  - $2 \ jets \ 2 \ b\text{-tags}$

**[hep-ex/1406.7830]**

**[hep-ex/1410.4103]**
Appearance of Sudakov Logs

Example of $e^+e^-$ quarks (→ KLN Theorem)

- Regularize individual contributions
- Virtual corrections are IR divergent $\propto \log^2 \frac{M^2}{s} + 3 \log \frac{M^2}{s}$
- Real emission is IR divergent $\propto -(\log^2 \frac{M^2}{s} + 3 \log \frac{M^2}{s})$
  - Fully inclusive rate is not IR sensitive: $1 + \frac{\alpha_s}{\pi}$
  - Applying phase-space cuts, Sudakov logs may survive

Sudakov Logs for Electroweak Corrections [Ciafalon et.al., Kuhn et.al., Denner et.al.]

- Presence of Sudakov double logs $\alpha_W \log^2 s/M^2$ regularized by $M_W, Z$
- Arise from collinear and soft infrared divergences
- Known to survive for
  1. Phase-space cuts [Bell, Kuhn, Rittinger]
  2. Non $SU(2)$-Singlet initial state [Ciafaloni, Comelli]
- Here: Restrict specific gauge boson emission
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Simplifying Assumptions

Consider gauge singlet current $J \rightarrow q\bar{q}$

$$\sigma_T = \frac{\alpha_W}{2\pi} (G_R - G_V) \hat{\sigma}_0 \left\{ \ln^2 \frac{M_W^2}{s} + 3 \ln \frac{M_W^2}{s} + \ldots \right\}$$

1. Fully inclusive rate
2. Any fermion, no gauge boson
3. 1 fermion, and gauge bosons
4. 1 fermion, no gauge bosons
5. 2 fermions, and gauge bosons
6. 2 fermions, and no gauge bosons
7. Any fermion, specific gauge boson

<table>
<thead>
<tr>
<th>Case</th>
<th>$G_R$</th>
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<tbody>
<tr>
<td>1</td>
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**Relevant for EW Corrections**
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- Consider gauge singlet current $J \to q\bar{q}$

$$\sigma_T = \frac{\alpha_W}{2\pi} (G_R - G_V) \hat{\sigma}_0 \left\{ \ln^2 \frac{M_W^2}{s} + 3 \ln \frac{M_W^2}{s} + \ldots \right\}$$

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Relevant for EW Corrections
Necessity for Resummation

- \( \frac{\alpha_{\text{EW}}}{4\pi \sin^2 \theta_W} \log^2 s/M_W^2 \sim 0.15 \) for \( \sqrt{s} = 4 \text{ TeV} \)
- Power corrections are less important than logarithmic ones

⇒ We have advantages to use an EFT approach: SCET

- Amplitude expansion structure
  - Fixed order: Row
  - Resummation: Column

- Exponentiated Amplitude
  - Natural EFT result
  - Non-trivial relation

\[
A = \begin{pmatrix}
1 \\
\alpha L^2 \\
\alpha^2 L^4 \\
\alpha^3 L^6 \\
\vdots
\end{pmatrix}
\quad \log A = \begin{pmatrix}
\alpha L^2 \\
\alpha^2 L^3 \\
\alpha^3 L^4 \\
\vdots
\end{pmatrix}
\]

- Leading Log (LL) Regime: \( L \sim \frac{1}{\alpha} \)
- Leading Log squared (LL^2) Regime: \( L \sim \frac{1}{\sqrt{\alpha}} \)
Necessity for Resummation

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\[
A = \begin{pmatrix}
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\alpha^2 L^4 & \alpha^2 L^3 & \alpha^2 L & \alpha^2 \\
\alpha^3 L^6 & \ldots & \ldots \\
\vdots
\end{pmatrix}
\]

\[
\log A = \begin{pmatrix}
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\alpha^2 L^3 & \alpha^2 L^2 & \alpha^2 L & \alpha^2 \\
\alpha^3 L^4 & \alpha^3 L^3 & \alpha^3 L^2 & \alpha^3 L & \alpha^3 \\
\alpha^4 L^5 & \ldots & \ldots \\
\vdots
\end{pmatrix}
\]

- Leading Log (LL) Regime: \( L \sim \frac{1}{\alpha} \)
- Leading Log squared (LL^2) Regime: \( L \sim \frac{1}{\sqrt{\alpha}} \)
Comparing Fixed Order vs EFT

Legend

- Red: Real radiation
- Blue: Virtual corrections
- Black: Sum of contributions
- Solid: Full result
- Dashed: EFT result
Scales of the Problem

- **Needed:** High-Scale matching $C$
- **No large logs!**
- **Known analytic results**
  1. Collinear running $\gamma$ in $\text{SCET}_{\text{EW},\gamma}$ for all fields within the SM
  2. Soft running $\gamma$ in $\text{SCET}_{\text{EW},\gamma}$ for all field representations within the SM
  3. Soft matching $D$ at weak scale $\Rightarrow$ symmetry breaking effects
  4. Collinear matching $D$ at weak scale $\Rightarrow$ Corrections are available analytically for an arbitrary process (up to high scale matching)

[ Chiu, Fuhrer, Kelley, Manohar, 0909.0012[hep-ph] ]
**The Wilson Coefficient**

### Matrices for EW Corrections in SCET

\[ \mathcal{M}_{\text{low}} = \exp \left[ \int_{\mu}^{M_z} \frac{d\mu}{\mu} R^{\text{soft+col}}_{U(1)_{EM}} \right] \]

\[ \mathcal{M}_{\text{col}} = \exp \left[ M_{U(1)_{EM}}^{\text{col}} \otimes SU(2)_W \right] \exp \left[ \int_{M_z}^{\sqrt{s}} \frac{d\mu}{\mu} R^{\text{col}}_{U(1)_{EM}} \otimes SU(2)_W \right] \]

\[ \mathcal{M}_{\text{soft}} = \exp \left[ \int_{M_z}^{\sqrt{s}} \frac{d\mu}{\mu} R^{\text{soft}}_{U(1)_{EM}} \right] M_{SU(2)}^{\text{break}} \exp \left[ \int_{M_z}^{\sqrt{s}} \frac{d\mu}{\mu} R^{\text{soft}}_{SU(2)_{W}} \right] \]

- \( R \) is running matrix (anomalous dimension)
- \( M \) is a matching matrix

### Wilson Coefficients

\[ C_{\text{low}} = \mathcal{M}_{\text{low}} \mathcal{M}_{\text{col}} \mathcal{M}_{\text{soft}} \]

\[ C_{\text{high}} = M_{SU(3)}^T \]
The Wilson Coefficient

Matrices for EW Corrections in SCET [Chiu, Fuhrer, Kelley, Manohar, Hoang, Golf]

\[
\mathcal{M}_{\text{low}} = \exp \left[ \int_{\mu}^{M_z} \frac{d\mu}{\mu} R_{U(1)_{EM}}^{\text{soft+col}} \right]
\]

\[
\mathcal{M}_{\text{col}} = \exp \left[ M_{U(1)_{Y} \otimes SU(2)_{W}}^{\text{col}} \right] \exp \left[ \int_{M_z}^{\sqrt{s}} \frac{d\mu}{\mu} R_{U(1)_{Y} \otimes SU(2)_{W}}^{\text{col}} \right]
\]

\[
\mathcal{M}_{\text{soft}} = \exp \left[ \int_{M_z}^{\sqrt{s}} \frac{d\mu}{\mu} R_{U(1)_{Y}}^{\text{soft}} \right] \mathcal{M}_{SU(2)}^{\text{break}} \exp \left[ \int_{M_z}^{\sqrt{s}} \frac{d\mu}{\mu} R_{SU(2)_{W}}^{\text{soft}} \right]
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\[
\mathcal{C}_{\text{low}} = \mathcal{M}_{\text{low}} \mathcal{M}_{\text{col}} \mathcal{M}_{\text{soft}} \mathcal{C}_{\text{high}} \mathcal{M}_{SU(3)}^{T}
\]
Goals

- Numerical estimate for $t\bar{t}$ production
  - Effect for final state gauge bosons larger $C_F \rightarrow C_A$
  - Enhancement by phase-space cuts and non-singlet initial states

- Consider three scenarios for the top
  1. $q = u, d$
  2. $q = t, b$ with $m_b=100 \text{ GeV}$ and $m_t = 173 \text{ GeV}$
  3. $q = t, b$ with $m_b=4.7 \text{ GeV}$ and $m_t = 173 \text{ GeV}$.

Simplifications

- Partonic level
- Unbroken $SU(2)$ theory
- Singlet initial state $\Rightarrow gg \rightarrow t\bar{t}$
- No Decay, shower or hadronization effects

- To avoid $t$-channel singularity demand
  1. $|\eta| < 1.3$ for highest transverse momentum particle
  2. $|\eta| < 5$ for second highest transverse momentum particle
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Virtual and Real Corrections

Virtual Corrections

- We use the results obtained in SCET
- Expand out to order $\alpha_W$ to demonstrate cancellation

\[
\sigma_V(gg \rightarrow t\bar{t}) = \sigma_{0,t} \{v_W + 3v_t + v_b\}
\]
\[
\sigma_V(gg \rightarrow b\bar{b}) = \sigma_{0,b} \{v_W + v_t + 3v_b\}
\]

\[
v_W = \frac{C_F \alpha_W}{4\pi} \left[-L^2 + 3L\right], \quad v_t = -\frac{y_t^2}{32\pi^2}L, \quad v_b = -\frac{y_b^2}{32\pi^2}L
\]

Real Radiation

- To our considered order: Tree Level process
- We use MadGraph5_aMC@NLO for calculating this numerically
Subtlety for Decaying Top

- Usually done: Narrow width approximation

\[
\frac{1}{p^2 - m_t^2 + i\epsilon} \rightarrow \frac{1}{p^2 - m_t^2 + i m_t \Gamma_t}
\]

- Equivalent to sum all imaginary part of

\[
\begin{align*}
W & \quad t & & t & & b & & t \\
\rightarrow & & & & & & + \ldots
\end{align*}
\]

- Same as one cut of the $J \rightarrow t\bar{t}$ example
- Cuts cannot be treated separately
- Not gauge invariant
- Mixes different orders in $\alpha_W$: $\Gamma_t$ is $\mathcal{O}(\alpha_W m_t)$
- If $t \rightarrow Wb$ is kinematically allowed: $\mathcal{O}(\alpha_W)$ piece is wrong
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\[ \text{W} + t + t + b + t + \ldots \]

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\[
\begin{array}{c}
\text{W} \\
\rightarrow \\
\text{t} \\
\text{t} \\
\text{b} \\
\text{t} \\
\rightarrow \\
\text{t} \\
\end{array}
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### Practical Solution

![Graph](https://via.placeholder.com/150)

\[ \sqrt{s} = 0.5 \text{ TeV} \]

Divide into resonant \((A)\) and non-resonant \((A')\) region

\[
I = \int_{m_t^2 - \Delta}^{m_t^2 + \Delta} dm_{bW}^2 \frac{f(m_{bW}^2)}{(m_{bW}^2 - m_t^2)^2 + \epsilon^2}
\]

\(\epsilon\) regulator from \(i\epsilon\) prescription

- Need to subtract singular piece in region \(A\) by expanding

\[
f(m_{bW}^2) = f_0 + (m_{bW}^2 - m_t^2)f_1 + (m_{bW}^2 - m_t^2)^2 f_2 + \ldots
\]

\[
I = \frac{i}{\epsilon} f_0 + 2\Delta f_2 + \ldots
\]

- Size is suppressed by \(\Delta\)

\(\Rightarrow\) Practically ignoring the region \(A\) is a good approximation
Divide into resonant ($A$) and non-resonant ($A'$) region

$$I = \int_{m_t^2 - \Delta}^{m_t^2 + \Delta} dm_{bW}^2 \frac{f(m_{bW}^2)}{(m_{bW}^2 - m_t^2)^2 + \epsilon^2}$$

$\epsilon$ regulator from $i\epsilon$ prescription

Need to subtract singular piece in region $A$ by expanding

$$f(m_{bW}^2) = f_0 + (m_{bW}^2 - m_t^2)f_1 + (m_{bW}^2 - m_t^2)^2f_2 + \ldots$$

$$I = \frac{\pi}{\epsilon}f_0 + 2\Delta f_2 + \ldots$$

Size is suppressed by $\Delta$

$\Rightarrow$ Practically ignoring the region $A$ is a good approximation
Divide into resonant (A) and non-resonant (A') region

\[ I = \int_{m_t^2 - \Delta}^{m_t^2 + \Delta} dm_{bW}^2 \frac{f(m_{bW}^2)}{(m_{bW}^2 - m_t^2)^2 + \epsilon^2} \]

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⇒ Practically ignoring the region A is a good approximation

\( \sqrt{s} = 0.5 \text{ TeV} \)
Results for Massless Quarks

- Tree level: \( gg \rightarrow u\bar{u} \) and \( gg \rightarrow d\bar{d} \)
- Real radiation: \( gg \rightarrow u\bar{u}Z, \) \( gg \rightarrow d\bar{d}Z, \) \( gg \rightarrow u\bar{d}W^- \) and \( gg \rightarrow d\bar{u}W^+ \)
- \( SU(2) \): \( \sigma(u\bar{u}) = \sigma(d\bar{d}), \) \( \sigma(u\bar{d}W^-) = \sigma(d\bar{u}W^+) = 2\sigma(u\bar{u}Z) = 2\sigma(d\bar{d}Z) \)
- Cancellation \( 3\sigma(u\bar{d}W) + 2\nu_W\sigma(u\bar{u}) \rightarrow 0 \)
- No full cancellation for experimental observables!
Introduction
Application to $gg \rightarrow t\bar{t}, b\bar{b}$

Results for Massive and Stable Top: Remarks

- Use equivalence theorem for longitudinal polarization of $W$
  \[
  \sigma(t\bar{b}W^-) \rightarrow \sigma(u\bar{d}W^-) + 2(y_t^2 + y_b^2)\sigma_S \\
  \sigma(t\bar{t}Z) \rightarrow \frac{1}{2}\sigma(u\bar{d}W^-) + 2y_t^2\sigma_S \\
  \sigma(b\bar{b}Z) \rightarrow \frac{1}{2}\sigma(u\bar{d}W^-) + 2y_b^2\sigma_S \\
  \sigma(t\bar{t}H) \rightarrow 2y_t^2\sigma_S, \quad \sigma(b\bar{b}H) \rightarrow 2y_b^2\sigma_S \\
  \sigma_V(t\bar{t}) \rightarrow (\nu_W + 3\nu_t + \nu_b)\sigma(u\bar{u}) \\
  \sigma_V(b\bar{b}) \rightarrow (\nu_W + \nu_t + 3\nu_b)\sigma(u\bar{u})
  \]

- Total real radiation
  \[
  \sigma_R = 2\sigma(t\bar{b}W^-) + \sigma(t\bar{t}Z) + \sigma(b\bar{b}Z) + \sigma(t\bar{t}H) + \sigma(b\bar{b}H) \\
  \rightarrow 3\sigma(u\bar{d}W^-) + 8(y_t^2 + y_b^2)\sigma_S
  \]

- Total virtual correction
  \[
  \sigma_V = \sigma_V(t\bar{t}) + \sigma_V(b\bar{b}) = (2\nu_W + 4\nu_t + 4\nu_b)\sigma(u\bar{u})
  \]
**Numerical Result for Massive and Stable Top**

- $\nu_{t,b}$ linear in Log
- Gauge and Higgs part cancel separately

$$8(y_t^2 + y_b^2)\sigma_S + (4\nu_t + 4\nu_b)\sigma(\bar{u}\bar{u}) \to 0.$$

- No full cancellation for experimental observables!
Numerical Results for Physical Top Quark Case

- Same as before, Yukawa coupling for bottom much smaller
- Used $t\bar{t}$ tag for MadGraph5_aMC@NLO
  ⇒ Excludes a region of width $15\Gamma_t$ around the on-shell $t$-quark
- No full cancellation for experimental observables!
Summary

- Electroweak corrections for $gg \rightarrow t\bar{t}$
- Virtual corrections are around $-10\%$ for $E_{cm} \sim 2$ GeV
- If part of real radiation can be excluded, there are large electroweak radiative corrections
- Importance grows with energy and become measurable at LHC energies

Comments

- Corrections for $q\bar{q} \rightarrow t\bar{t}$ expected to be twice as large
- Explored minimal effect
- Total corrections to NLL can be written as a product $R_{QCD}R_{EW}$
  $\Rightarrow$ EW can be included by reweighting QCD result
  $\Rightarrow$ Inclusion of EW corrections possible with SCET approach
  $\Rightarrow$ Full study with proton PDF, shower, exp. cuts, ... necessary
Summary

- Electroweak corrections for $gg \rightarrow t\bar{t}$
- Virtual corrections are around $-10\%$ for $E_{cm} \sim 2$ GeV
- If part of real radiation can be excluded, there are large electroweak radiative corrections
- Importance grows with energy and become measurable at LHC energies

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Backup Slides
**SCET Operators**

- \( g(p_1) + g(p_2) \rightarrow q(p_3) + \bar{q}(p_4) \)
  - \( O_1 = \bar{q}_4 q_3 A_2^A A_1^A \)
  - \( O_2 = d^{ABC} \bar{q}_4 T^C q_3 A_2^A A_1^B \)
  - \( O_3 = i f^{ABC} \bar{q}_4 T^C q_3 A_2^A A_1^B \).

- High Scale Matching contains EW and QCD corrections

**Running in SCET**

- First run down to Electroweak scale
- Then integrate out gauge bosons \( \Rightarrow \) low scale matching
  \( \Rightarrow \) Breaks up \( SU(2) \) doubletts into individual fields

\[
\mathcal{M} = \exp \left[ D_C(\mu_l, L_M, \bar{n} \cdot p) \right] d_S(\mu_l, L_M) \\
\times P \exp \left[ \int_{\mu_h}^{\mu_l} \frac{d\mu}{\mu} \gamma(\mu, \bar{n} \cdot p) \right] C(\mu_h, L_Q)
\]
Anomalous Dimension Matrices

\[ \gamma(\mu, \vec{n} \cdot p) = \gamma_C(\mu, \vec{n} \cdot p) + \gamma_S(\mu) \]

- Soft matrix: \( \hat{=} \) interactions between particles

\[ \gamma_S(\mu) = - \sum_{(rs, i)} \frac{\alpha_i(\mu)}{\pi} T_r^{(i)} \cdot T_s^{(i)} \ln \frac{-n_r \cdot n_s + i0^+}{2} \]

- Colinear Part: Knows about species (Diagonal!)

\[ \gamma_C(\mu, \vec{n} \cdot p) = \sum_r \left[ A_r(\mu) \ln \frac{2E_r}{\mu} + B_r(\mu) \right] \]

- \( A_r(\mu) \) and \( B_r(\mu) \) have a perturbative expansion in \( \alpha_i(\mu) \)
Idea of SCET

Ansatz

- Describing **energetic** particles with multiple scales
  1. Scattering process $\mathcal{O}(Q^2)$
  2. Collinear component
  3. Soft component
  4. Systematic power-counting $\lambda \ll 1$
- Propagating around light-cone: $p^2 \ll Q^2$
  $\Rightarrow$ Factorization of perturbative and non-perturbative effects

Kinematics

- Light-cone vectors $n^\mu = (1, n)$ and $\bar{n}^\mu = (1, -n)$
  $\Rightarrow$ Collinear field: $p^- = \bar{n} \cdot p \sim Q$, $p^+ = n \cdot p \sim \lambda^2 Q$, $p_\perp \sim \lambda Q$
- Ultrasoft field: All components scale as $\lambda^2 Q$
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High-Scale Operator in Full SM

**Full Standard Model**
- Respect full \( SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \) gauge symmetry
- All masses set to zero
⇒ Infrared divergencies

**Matching onto SCET\(_{\text{ew}}\)**
- Describing fields with collinear Wilson lines
  \[
  \bar{Q} \Gamma Q \rightarrow \exp C(\mu) [\bar{\xi}_{n,p_1} \mathcal{W}_n] \Gamma [\mathcal{W}_{\bar{n}}^\dagger \xi_{\bar{n},p_2}]
  \]
- Ultraviolett match onto infrared ones from full theory
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### Simplified Problem

- $1 \rightarrow 1$ process problem
- Simple coordinates
- $n$ and $\bar{n}$ are orthogonal

$$\log F_E(Q^2, \mu) = C(\mu=Q) + \int_Q^{M_Z} \frac{d\mu}{\mu} \gamma_1(\mu) + D_{Z,W}(\mu=M_Z) + \int_{M_Z}^{\mu} \frac{d\mu}{\mu} \gamma_2(\mu)$$

### Extension to More Particles

- $r$ particles $\Rightarrow$ Set of $n_i$, $i = 1, \ldots, r$
- $n_i \cdot n_j \neq 0$ if $i \neq j$
- Two types of corrections
  1. Field dependent corrections
  2. Corrections between different fields
Sudakov Logarithm

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Comment about Regulator for divergencies

- **Analytic regulator:** 
  \[ \frac{1}{p_i^2 - m^2} \rightarrow \frac{1}{(p_i^2 - m^2)^{1+\delta_i}} \]
  
  \[ \Rightarrow \text{Breaks color Ward identity} \]

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Factorization into Soft and Collinear

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**Soft Corrections**

### Definition

- **Universal soft function** $U_S(n_i, n_j) = \log \frac{-\mathbf{n}_i \cdot \mathbf{n}_j - i 0^+}{2}$
- **Anomalous dimension** $\gamma_s = \Gamma(\alpha(\mu)) \left[ -\sum_{\langle ij \rangle} \mathbf{T}_i \mathbf{T}_j U_S(n_i, n_j) \right]$
- **Low-scale matching** $D_s = J(\alpha(\mu), L_m) \left[ -\sum_{\langle ij \rangle} \mathbf{T}_i \mathbf{T}_j U_S(n_i, n_j) \right]$
  1. **Cusp anomalous dimension** $\Gamma(\alpha(\mu)) = \frac{\alpha(\mu)}{4\pi} 4$
  2. **Matching** $J(\alpha(\mu), L_m) = \frac{\alpha(\mu)}{4\pi} 2 \log \frac{M^2}{\mu^2}$

### Comments

- Contains all information about kinematics of process
- Assumes Casimir scaling (gauge singlett operator)
  $\Rightarrow$ 3-Loop Cusp anomalous dimension is used
Collinear Corrections

Definition

- Regulator choice: $n_i$ Wilson line interactions only with $i$ particle
  - Particle dependent corrections
  - Sum over all particles

Comments

- Anomalous dimension contains cusp and non-cusp part
  - Cusp: 3-loop K factor
  - Non-cusp: 2 loop K factor
- Matching contains wave-function renormalization
- Matching contains $Z - \gamma$ mixing
- Additional matching in $SU(3)_C$, because top is integrated out
- All functions listed in [0909.0947]
Collinear Corrections

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Breaking of the Electroweak Symmetry

**Standard Model**

- Scalar field (Higgs) obtains vacuum expectation value (VEV)
- Couples to other fields
- VEV breaks symmetry spontaneously

\[ \text{SU}(3)_C \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y \rightarrow \text{SU}(3)_C \otimes \text{U}(1)_{\text{em}} \]

**Consequences**

- Mixing of fields:
  \[ Z = \cos \theta_W W^3 - \sin \theta_W B \]
  \[ A = \sin \theta_W W^3 + \cos \theta_W B \]
- Mass splitting \( M_W \neq M_Z \)
- Goldston Boson \( \equiv \) Longitudinal polarization
- Top Yukawa coupling non-negligible
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Effect of Symmetry Breaking

- Operator basis is extended: \((M_{\text{break} SU(2)}^{\text{break}})_{ij}\) non-square matching matrix
- Gauge structure is changed \(SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{\text{em}}\)
  \(\Rightarrow\) EW symmetry breaking described by soft matrix

Matching at 1-Loop

\[
R_{S,W}^{(1)} = \frac{\alpha_W}{4\pi} 2 \log M_W^2 \mu^2 \left[ R^{(0)} O_{SU(2)} + \sum_{\langle ij \rangle} T_{3,i} T_{3,j} U_S(n_i, n_j) \right]
\]

\[
R_{S,Z}^{(1)} = \frac{\alpha_Z}{4\pi} 2 \log M_W^2 \mu^2 \left[ - \sum_{\langle ij \rangle} T_{Z,i} T_{Z,j} U_S(n_i, n_j) \right]
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\[
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