

# $B \rightarrow K^{(*)} l^+ l^-$ at (very) low $q^2$ and new physics

Sebastian Jäger



based on work with J Martin Camalich  
arXiv:1212.2263 (JHEP) and to appear

ERC workshop “Effective field theories for collider physics, flavor  
phenomena and electroweak symmetry breaking  
Schloss Waldthausen (Mainz/Budenheim), 12 November 2014

# Outline

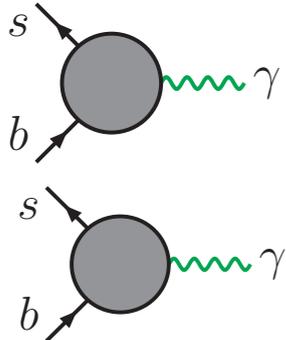
- $B \rightarrow K^* \mu^+ \mu^-$  : observables and theoretical anatomy
- power corrections and so-called clean observables
- aim to provide a transparent discussion of theory errors from which some conclusions may follow
- both form-factor power corrections and long-distance effects of the hadronic weak hamiltonian
- Helicity hierarchies survive power corrections: Clean tests of the SM and sensitivity to right-handed dipoles
- Prospects

# weak $\Delta B=\Delta S=1$ Hamiltonian

= EFT for  $\Delta B=\Delta S=1$  transitions (up to dimension six)

$$\mathcal{H}_{\text{eff}}^{\text{had}} = \frac{4G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[ C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3\dots 6} C_i P_i + C_{8g} Q_{8g} \right] \quad C_i \sim g_{\text{NP}} \frac{m_W^2}{M_{\text{NP}}^2}$$

$$\mathcal{H}_{\text{eff}}^{\text{sl}} = -\frac{4G_F}{\sqrt{2}} \lambda_t \left[ C_7 Q_{7\gamma} + C'_7 Q'_{7\gamma} + C_9 Q_{9V} + C'_9 Q'_{9V} + C_{10} Q_{10A} + C'_{10} Q'_{10A} + C_S Q_S + C'_S Q'_S + C_P Q_P + C'_P Q'_P + C_T Q_T + C'_T Q'_T \right].$$

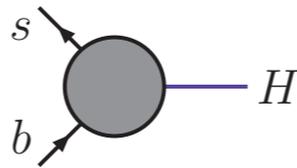


$$\mathcal{O}_7 = \frac{e}{16\pi^2} \hat{m}_b \bar{s} \sigma_{\mu\nu} P_R F^{\mu\nu} b,$$

$$\mathcal{O}_V = \frac{\alpha_{\text{em}}}{4\pi} (\bar{s} \gamma_\mu P_L b) (\bar{l} \gamma^\mu l),$$

$$\mathcal{O}_S = \frac{\alpha_{\text{em}}}{4\pi} \hat{m}_b (\bar{s} P_R b) (\bar{l} l),$$

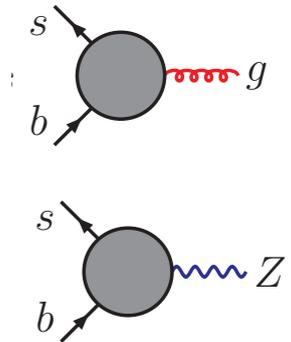
$$\mathcal{O}_T = \frac{\alpha_{\text{em}}}{4\pi} \hat{m}_b (\bar{s} \sigma_{\mu\nu} P_R b) (\bar{l} \sigma^{\mu\nu} P_R l),$$



$$\mathcal{O}_8 = \frac{g_s}{16\pi^2} \hat{m}_b \bar{s} \sigma_{\mu\nu} P_R G^{\mu\nu} b,$$

$$\mathcal{O}_A = \frac{\alpha_{\text{em}}}{4\pi} (\bar{s} \gamma_\mu P_L b) (\bar{l} \gamma^\mu \gamma^5 l)_A$$

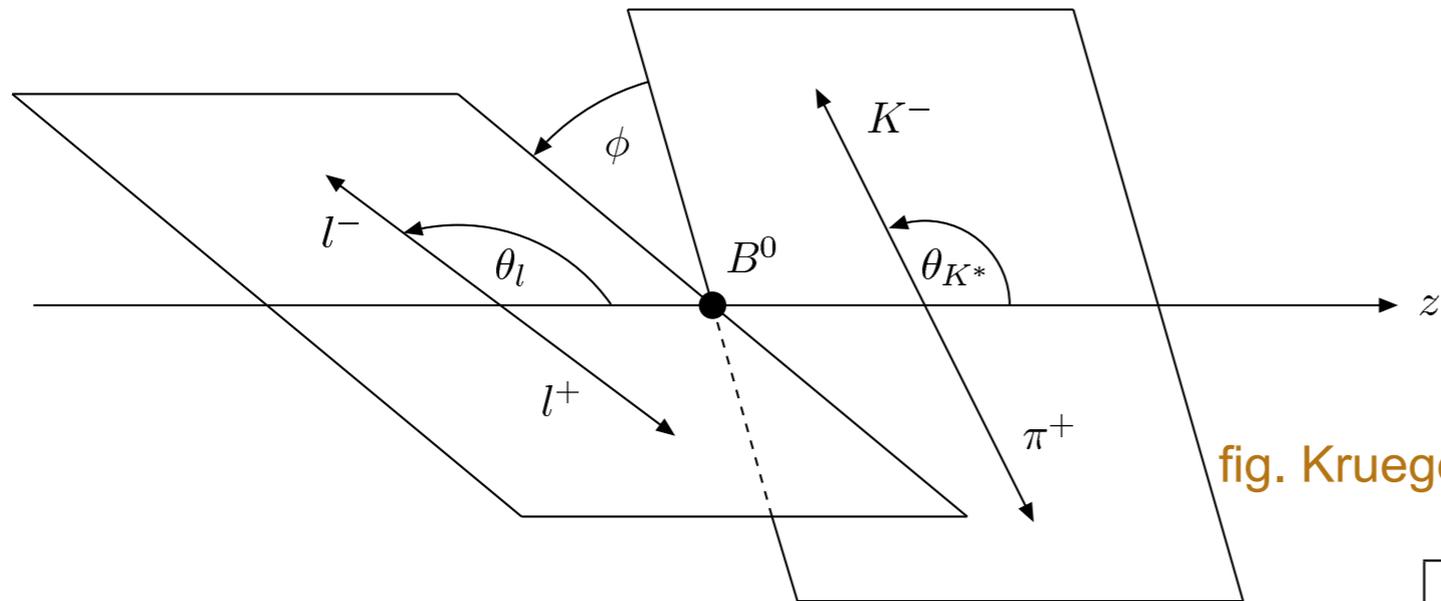
$$\mathcal{O}_P = \frac{\alpha_{\text{em}}}{4\pi} \hat{m}_b (\bar{s} P_R b) (\bar{l} \gamma^5 l),$$



+ chirality-flipped operators with  $P_R \leftrightarrow P_L$ : **suppressed in SM by  $m_s/m_b$**

look for observables sensitive to  $C_i$ 's, specifically those that are suppressed in the SM

# B → K\* l l: angular distribution



$\theta_K$  in  $K^*$  rest frame

$\theta_l$  in dilepton cm frame

$\phi$  boost-invariant (w.r.t. z axis)

fig. Krueger, Matias 2002

$$\frac{d^{(4)}\Gamma}{dq^2 d(\cos \theta_l) d(\cos \theta_k) d\phi} = \frac{9}{32\pi}$$

$$\begin{aligned} & \times \left( I_1^s \sin^2 \theta_k + I_1^c \cos^2 \theta_k + (I_2^s \sin^2 \theta_k + I_2^c \cos^2 \theta_k) \cos 2\theta_l \right. \\ & + I_3 \sin^2 \theta_k \sin^2 \theta_l \cos 2\phi + I_4 \sin 2\theta_k \sin 2\theta_l \cos \phi \\ & + I_5 \sin 2\theta_k \sin \theta_l \cos \phi + (I_6^s \sin^2 \theta_k + I_6^c \cos^2 \theta_k) \cos \theta_l \\ & \left. + I_7 \sin 2\theta_k \sin \theta_l \sin \phi + I_8 \sin 2\theta_k \sin 2\theta_l \sin \phi + I_9 \sin^2 \theta_k \sin^2 \theta_l \sin 2\phi \right) \end{aligned}$$

Expt.	$\sim$ # events
CDF	100 <a href="#">PRL106(2011)161801</a>
BaBar	150 <a href="#">PRD86(2012)032012</a>
Belle	200 <a href="#">PRL103(2009)171801</a>
CMS	400 <a href="#">PLB727(2013)77</a>
ATLAS	500 <a href="#">arXiv:1310.4213</a>
LHCb ( $\mu$ )	1000 ( $1 \text{ fb}^{-1}$ ) <a href="#">JHEP 1308 (2013) 131</a>
LHCb (e)	128 ( $[0.0004, 1] \text{ GeV}^2$ ) <a href="#">M Borsato (LHCb)</a>

Each angular coefficient is a function of the Wilson coefficients and the dilepton invariant mass  $q^2$

and can be used to probe for new physics in various bins

Theoretical expressions for  $I_i$  quadratic functions of **helicity amplitudes**

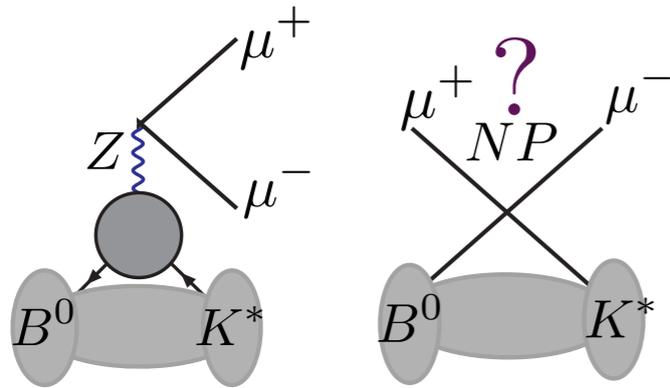
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Two mechanisms to produce dilepton in & beyond SM

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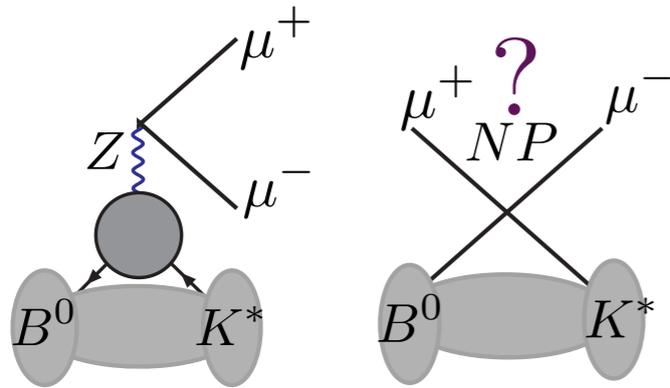
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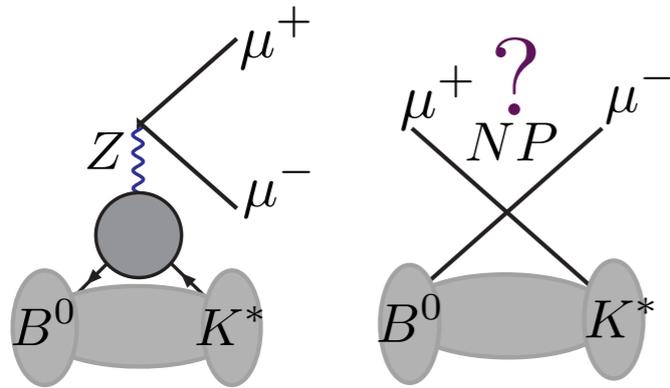


$$H_A(\lambda) \propto \tilde{V}_\lambda(q^2) C_{10} - V_{-\lambda}(q^2) C'_{10}$$

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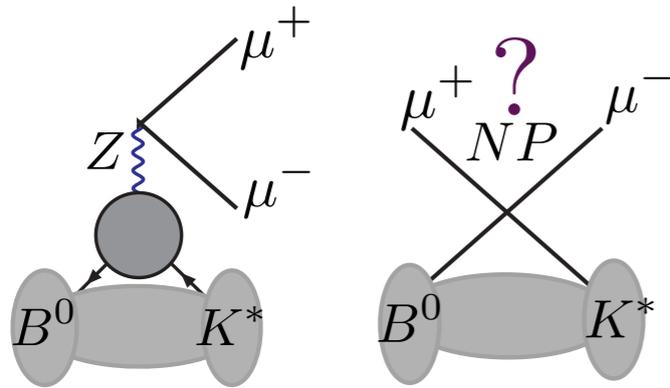
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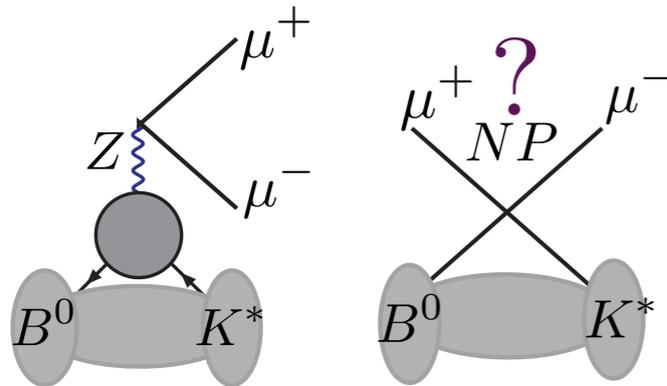
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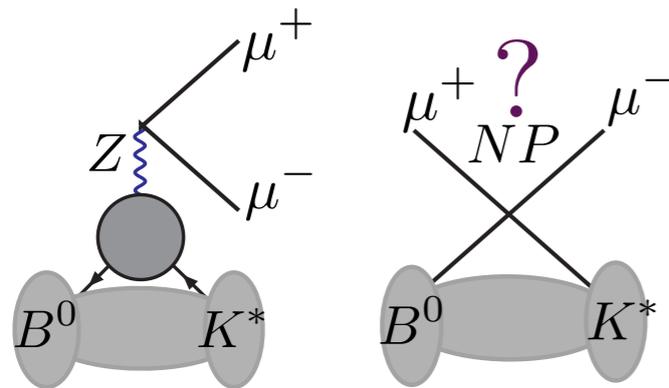
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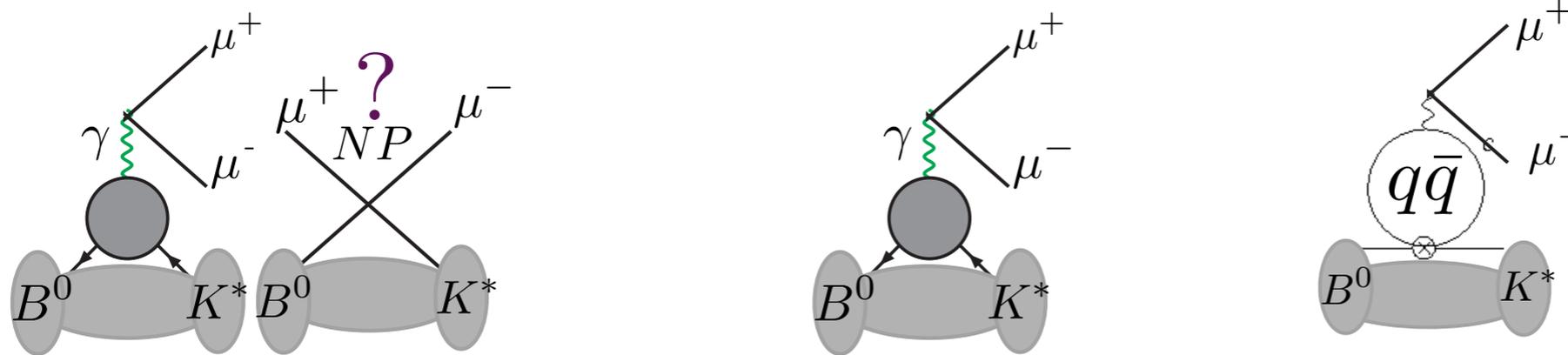
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- via vector lepton current (in SM: (mainly) photon)

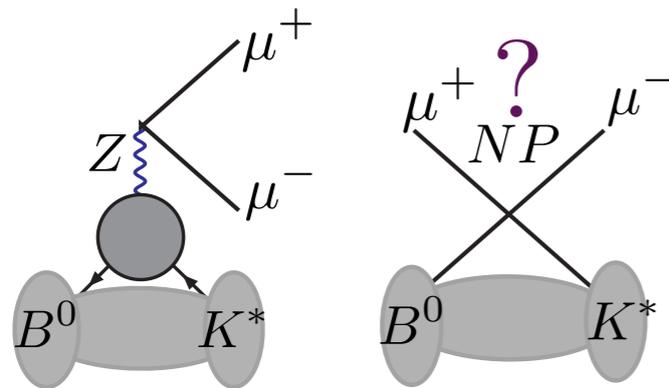


$$H_V(\lambda) \propto \tilde{V}_\lambda(q^2) C_9 - V_{-\lambda}(q^2) C'_9 + \frac{2 m_b m_B}{q^2} \left( \tilde{T}_\lambda(q^2) C_7 - \tilde{T}_{-\lambda}(q^2) C'_7 \right) - \frac{16 \pi^2 m_B^2}{q^2} h_\lambda(q^2)$$

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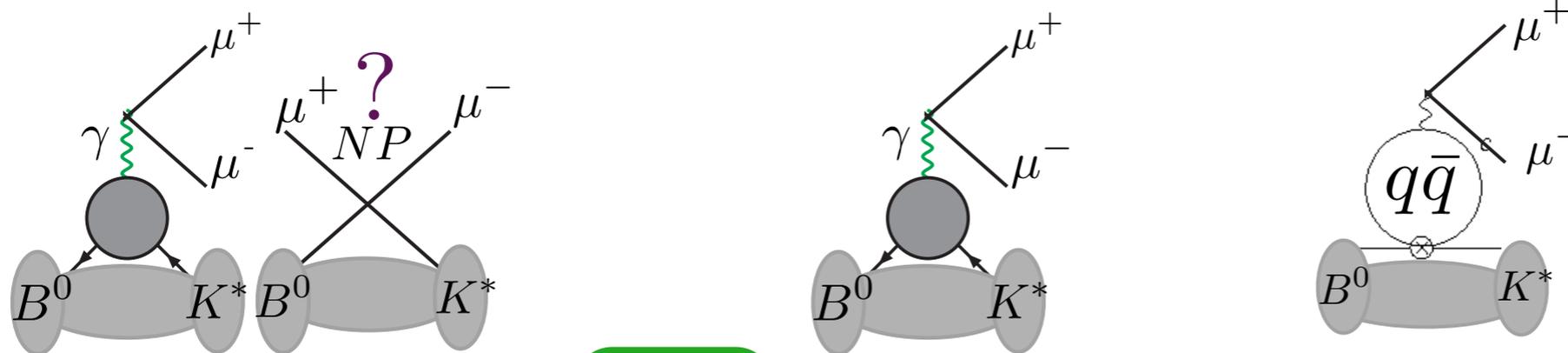
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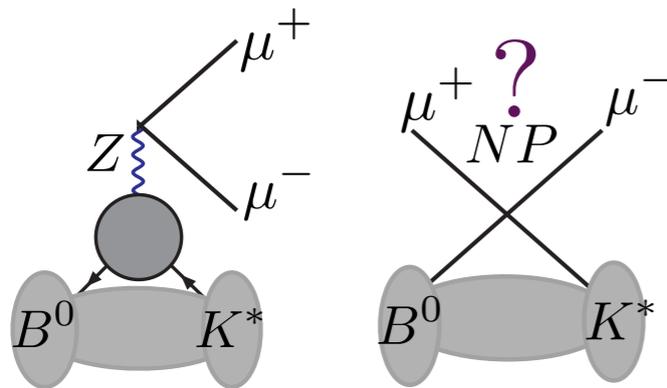
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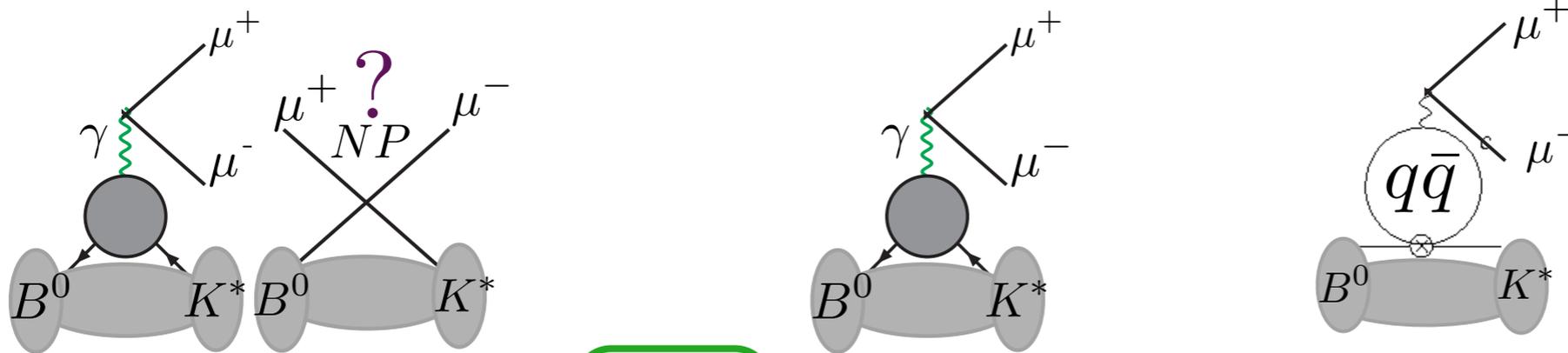
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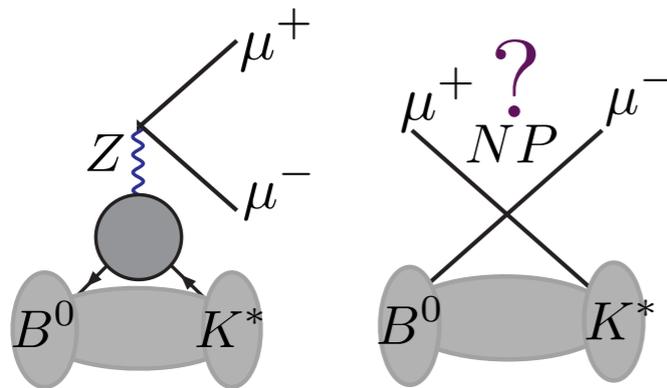
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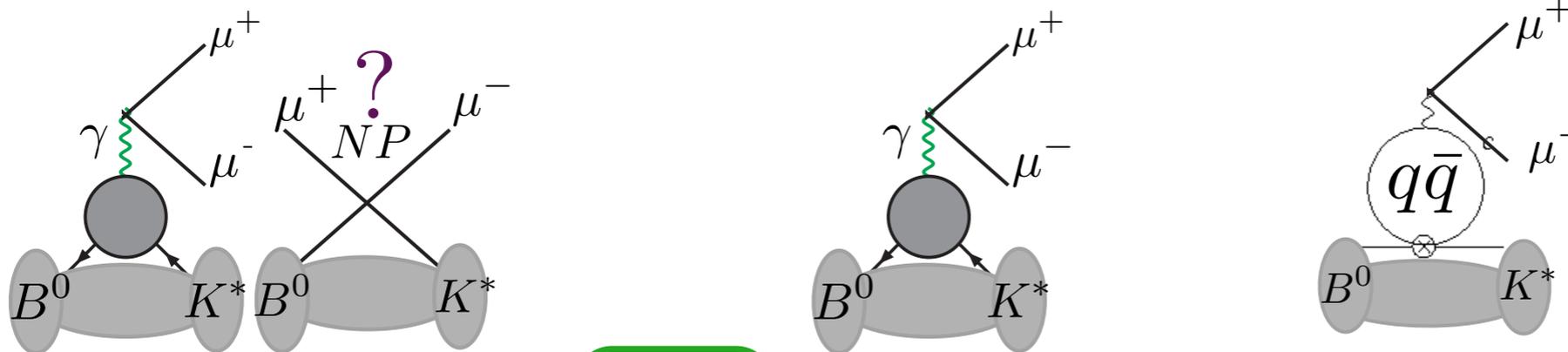
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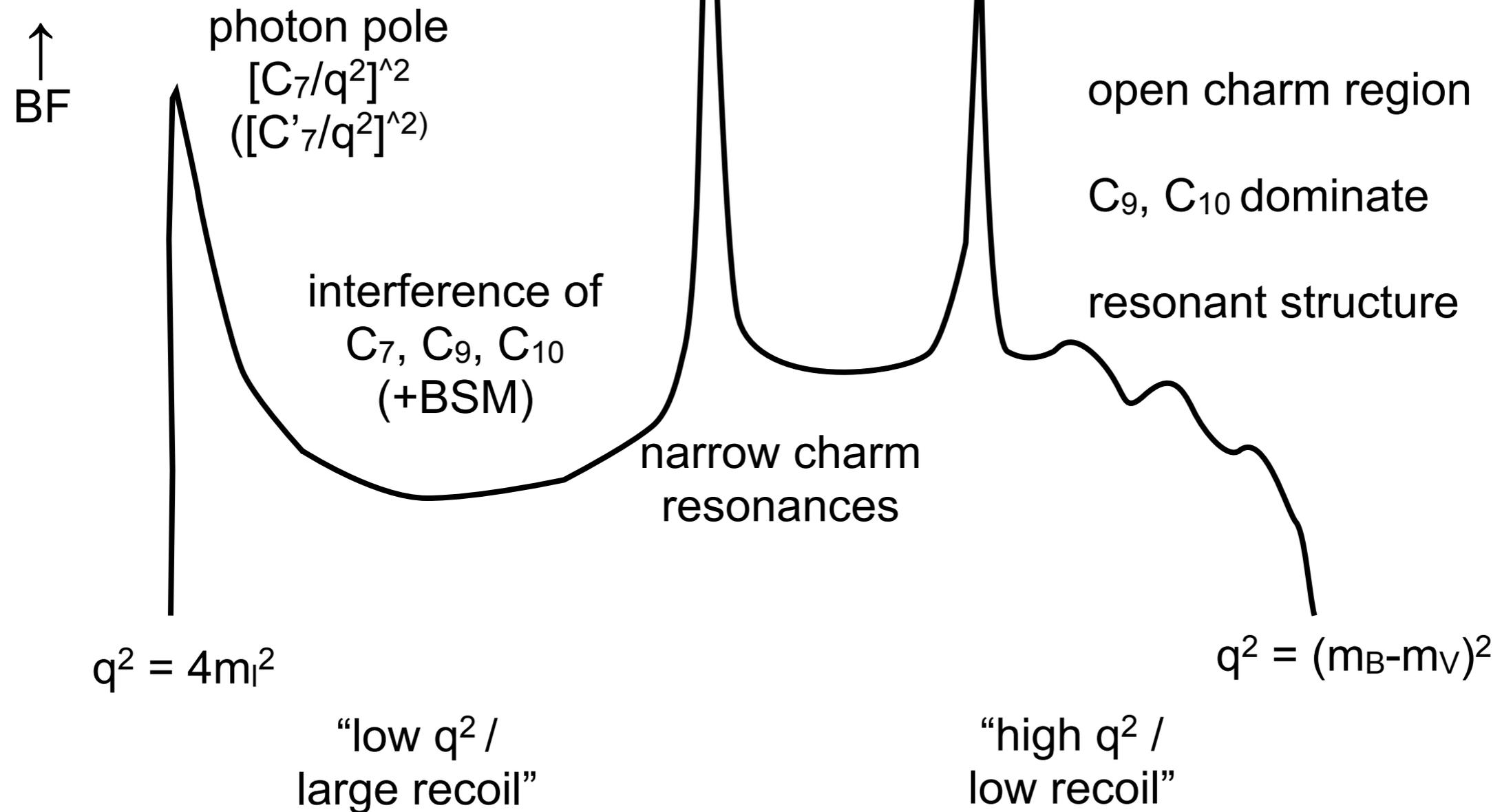
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only corrections:

- higher orders in electromagnetism
- finite-width effects (would need dealing with K pi final state)

**no** tensor or scalar operators if  $M_{NP} \gg M_Z$  Alonso, Grinstein, Martin Camalich 2014

# Rate: $q^2$ dependence (qualitative)



Note - artist's impression only.

LHCb has not yet published sufficiently fine binning to show the resonant features [open charm resonances are however visible in published  $B \rightarrow K \ell \ell$  data]

# Form factors

Helicity amplitudes naturally involve helicity form factors

$$-im_B \tilde{V}_{L(R)\lambda}(q^2) = \langle M(\lambda) | \bar{s} \not{\epsilon}^*(\lambda) P_{L(R)} b | \bar{B} \rangle,$$

$$m_B^2 \tilde{T}_{L(R)\lambda}(q^2) = \epsilon^{*\mu}(\lambda) q^\nu \langle M(\lambda) | \bar{s} \sigma_{\mu\nu} P_{R(L)} b | \bar{B} \rangle \quad \sim \text{Bharucha et al 2010}$$

$$im_B \tilde{S}_{L(R)}(q^2) = \langle M(\lambda = 0) | \bar{s} P_{R(L)} b | \bar{B} \rangle.$$

(& rescale helicity-0 form factors by kinematic factor.)

Can be expressed in terms of traditional “transversity” FFs

$$V_\pm(q^2) = \frac{1}{2} \left[ \left( 1 + \frac{m_V}{m_B} \right) A_1(q^2) \mp \frac{\lambda^{1/2}}{m_B(m_B + m_V)} V(q^2) \right],$$

$$V_0(q^2) = \frac{1}{2m_V \lambda^{1/2} (m_B + m_V)} \left[ (m_B + m_V)^2 (m_B^2 - q^2 - m_V^2) A_1(q^2) - \lambda A_2(q^2) \right]$$

$$T_\pm(q^2) = \frac{m_B^2 - m_V^2}{2m_B^2} T_2(q^2) \mp \frac{\lambda^{1/2}}{2m_B^2} T_1(q^2),$$

$$T_0(q^2) = \frac{m_B}{2m_V \lambda^{1/2}} \left[ (m_B^2 + 3m_V^2 - q^2) T_2(q^2) - \frac{\lambda}{(m_B^2 - m_V^2)} T_3(q^2) \right],$$

$$S(q^2) = A_0(q^2),$$

The form factors satisfy two exact relations:

$$T_+(q^2 = 0) = 0,$$

$$S(q^2 = 0) = V_0(0)$$

note - M can be multiparticle state. Eg for a two-pseudoscalar state

$$\tilde{V}_{L\lambda} = -\eta(-1)^L \tilde{V}_{R,-\lambda} \equiv \tilde{V}_\lambda,$$

$$\tilde{T}_{L\lambda} = -\eta(-1)^L \tilde{T}_{R,-\lambda} \equiv \tilde{T}_\lambda,$$

$$\tilde{S}_L = -\eta(-1)^L \tilde{S}_R \equiv \tilde{S},$$

L = angular momentum

$\eta$  = intrinsic parity

+ invariant mass dependence

SJ, J Martin Camalich 2012

# Heavy-quark limit and corrections

$$F(q^2) = \underbrace{F^\infty(q^2)}_{\text{heavy quark limit}} + \underbrace{a_F + b_F q^2/m_B^2 + \mathcal{O}([q^2/m_B^2]^2)}_{\text{Power corrections - parameterise}}$$

At most 1-2%  
over entire 0..6  
GeV<sup>2</sup> range ->  
ignore

$$F^\infty(q^2) = F^\infty(0)/(1 - q^2/m_B^2)^p + \Delta_F(\alpha_s; q^2)$$

(Charles et al)

(Beneke, Feldmann)

$q^2$  dependence in heavy-quark limit not known  
(model by a power  $p$ , and/or a pole model)

Corrections are  
calculable in terms of perturbation  
theory, decay constants, light cone  
distribution amplitudes

$$\begin{aligned} V_+^\infty(0) = 0 & \quad T_+^\infty(0) = 0 & \text{from heavy-quark/} \\ V_-^\infty(0) = T_-^\infty(0) & & \text{large energy} \\ V_0^\infty(0) = T_0^\infty(0) & & \text{symmetry} \end{aligned}$$

$$V_+^\infty(q^2) = 0 \quad T_+^\infty(q^2) = 0$$

hence

$$T_+(q^2) = \mathcal{O}(q^2) \times \mathcal{O}(\Lambda/m_b)$$

$$V_+(q^2) = \mathcal{O}(\Lambda/m_b).$$

[SJ @ LHCb 2013, Aspen 2014, ...]

- “naively factorizing” part of the helicity amplitudes  $H_{V,A}^+$  strongly suppressed as a consequence of chiral SM weak interactions
- We see the suppression is particularly strong near low- $q^2$  endpoint
- Form factor relations imply reduced uncertainties in suitable observables

Burdman, Hiller 1999  
(quark picture)

Beneke, Feldmann,  
Seidel 2001 (QCDF)

# “Clean” angular observables

Useful to consider functions of the angular coefficients for which form factors drop out in the heavy quark limit if perturbative QCD corrections neglected.

E.g.

neglecting strong phase differences  
[tiny; take into account in numerics]

$$P_1 \equiv \frac{I_3 + \bar{I}_3}{2(I_{2s} + \bar{I}_{2s})} = \frac{-2 \operatorname{Re}(H_V^+ H_V^{-*} + H_A^+ H_A^{-*})}{|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2}$$

$$P_3^{CP} \equiv -\frac{I_9 - \bar{I}_9}{4(I_{2s} + \bar{I}_{2s})} = -\frac{\operatorname{Im}(H_V^+ H_V^{-*} + H_A^+ H_A^{-*})}{|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2}$$

$$P_5' = \frac{\operatorname{Re}[(H_V^- - H_V^+) H_A^{0*} + (H_A^- - H_A^+) H_V^{0*}]}{\sqrt{(|H_V^0|^2 + |H_A^0|^2)(|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2)}}$$

Becirevic, Schneider 2011  
Matias, Mescia, Ramon, Virto 2012  
Descotes-Genon et al 2012  
(also Krueger, Matias 2005; Egede et al 2008)

$$= 0 \quad \left. \begin{array}{l} \text{(Melikhov 1998)} \\ \text{Krueger, Matias 2002} \\ \text{Lunghi, Matias 2006} \\ \text{Becirevic, Schneider 2011} \end{array} \right\}$$

$$= 0$$

$$= \frac{C_{10} (C_{9,\perp} + C_{9,\parallel})}{\sqrt{(C_{9,\parallel}^2 + C_{10}^2)(C_{9,\perp}^2 + C_{10}^2)}}$$

in SM, neglecting power corrections and pert. QCD corrections

where  $C_{9,\perp} = C_9^{\text{eff}}(q^2) + \frac{2 m_b m_B}{q^2} C_7^{\text{eff}}$

$$C_{9,\parallel} = C_9^{\text{eff}}(q^2) + \frac{2 m_b E}{q^2} C_7^{\text{eff}}$$

$C_7$  and  $C_9$  opposite sign

destructive interference enhances vulnerability to anything that violates the large-energy form factor relations

much more relevant to  $P_5'$  (and others)  
than to  $P_1$  or  $P_3^{CP}$

# LHCb anomaly

PRL 111, 191801 (2013)

PHYSICAL REVIEW LETTERS

week ending  
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DOI: [10.1103/PhysRevLett.111.191801](https://doi.org/10.1103/PhysRevLett.111.191801)

PACS numbers: 13.20.He, 11.30.Rd, 12.60.-i

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interpretation in NP models: Altmannshofer&Straub, Gault,Goertz,Haisch;  
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BSM physics?

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PACS numbers: 13.20.He, 11.30.Rd, 12.60.-i

Descotes-Genon, Matias, Virto e PRD 88,074002 claim 3.9 *global*

further model-independent fits: Altmannshofer&Straub; Beaujean, Bobeth, van Dyk

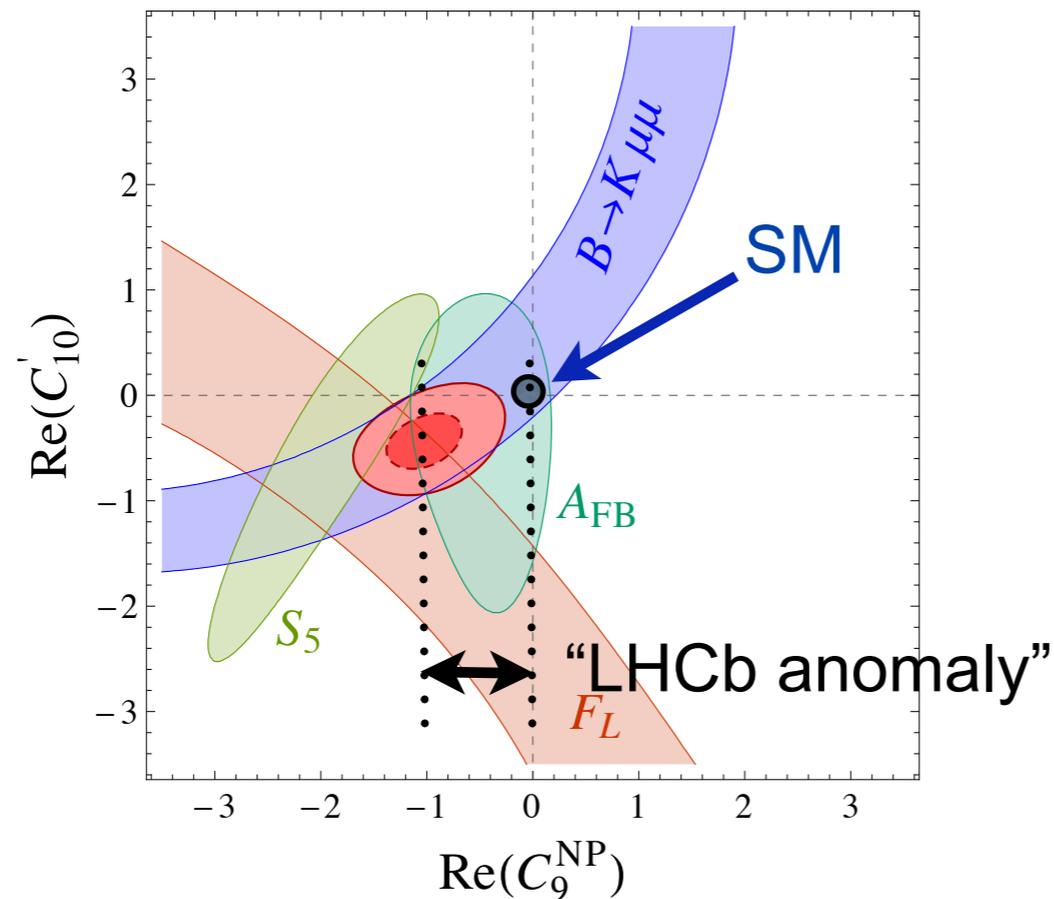
interpretation in NP models: Altmannshofer&Straub, Gault, Goertz, Haisch;  
Buras&Girrbach; Buras, DeFazio, Girrbach; ...

BSM physics?

Or not quite form-factor independent?

# BSM interpretation

Global fits to (mostly)  $B \rightarrow K^* \ell \ell$  angular distribution.



Altmannshofer, Straub 2013

[also Descotes-Genon, Matias, Virto 2013;  
Beaujean, Bobeth, van Dyk 2013, ...]

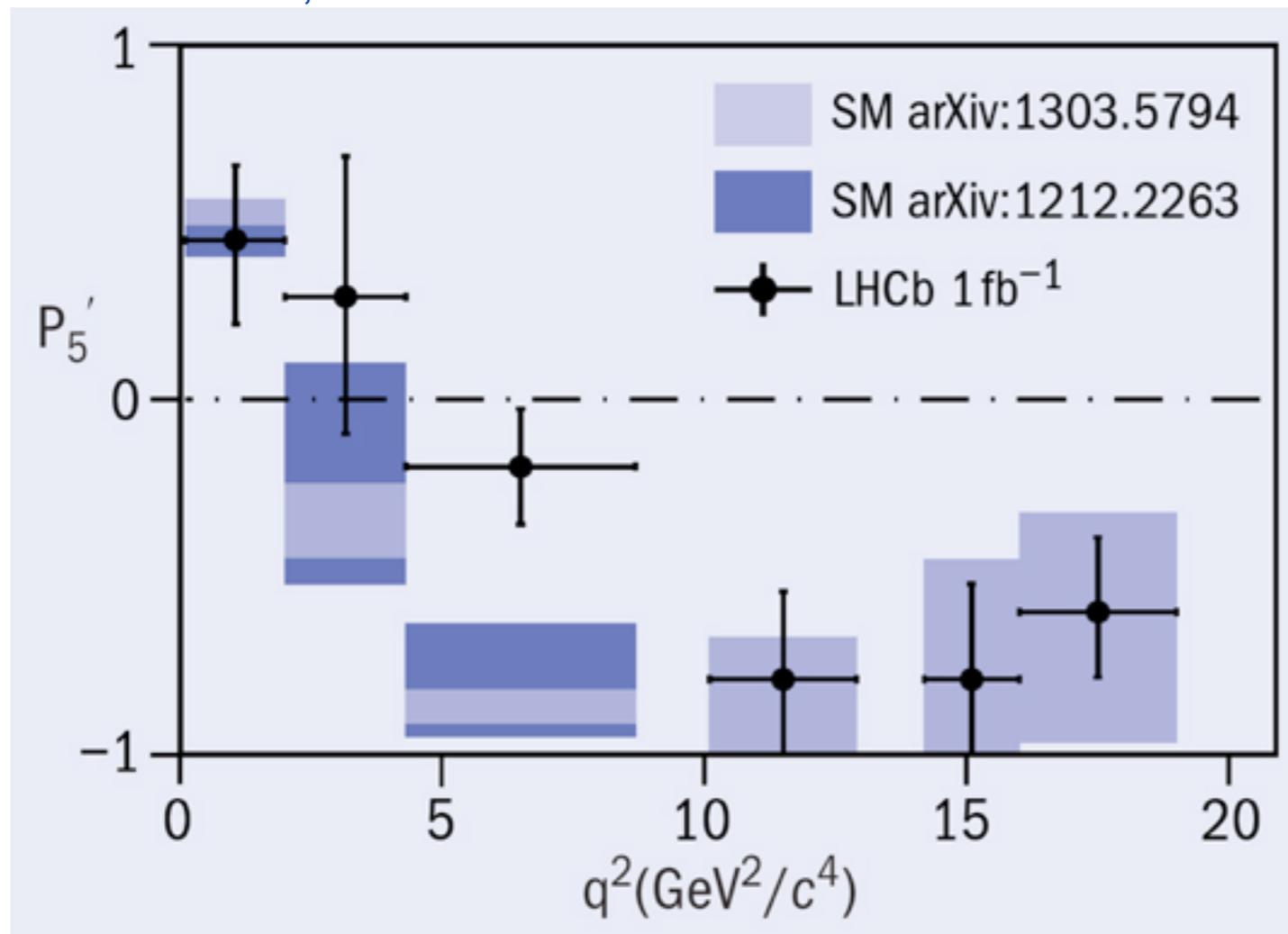
Tension with SM, driven largely but not solely  
by a single bin of a single angular coefficient ( $S_5 \sim P_5'$ )

(Essentially same form factor input as employed in LHCb  
significance estimate.)

# $P_5'$ “anomaly”

$$\langle P_5' \rangle = \frac{\langle \beta(\text{Re}[(H_V^- - H_V^+)H_A^{0*} + (H_A^- - H_A^+)H_V^{0*}]) \rangle}{\sqrt{\langle \beta^2 |H_V^0|^2 + |H_A^0|^2 \rangle \langle \beta^2 (|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2) \rangle}}$$

CERN Courier, December 2013



Descotes-Genon, Matias, Virto [DMV]

SJ, J Martin Camalich (4.3..8.68 bin is actually a private update, not stated in paper)

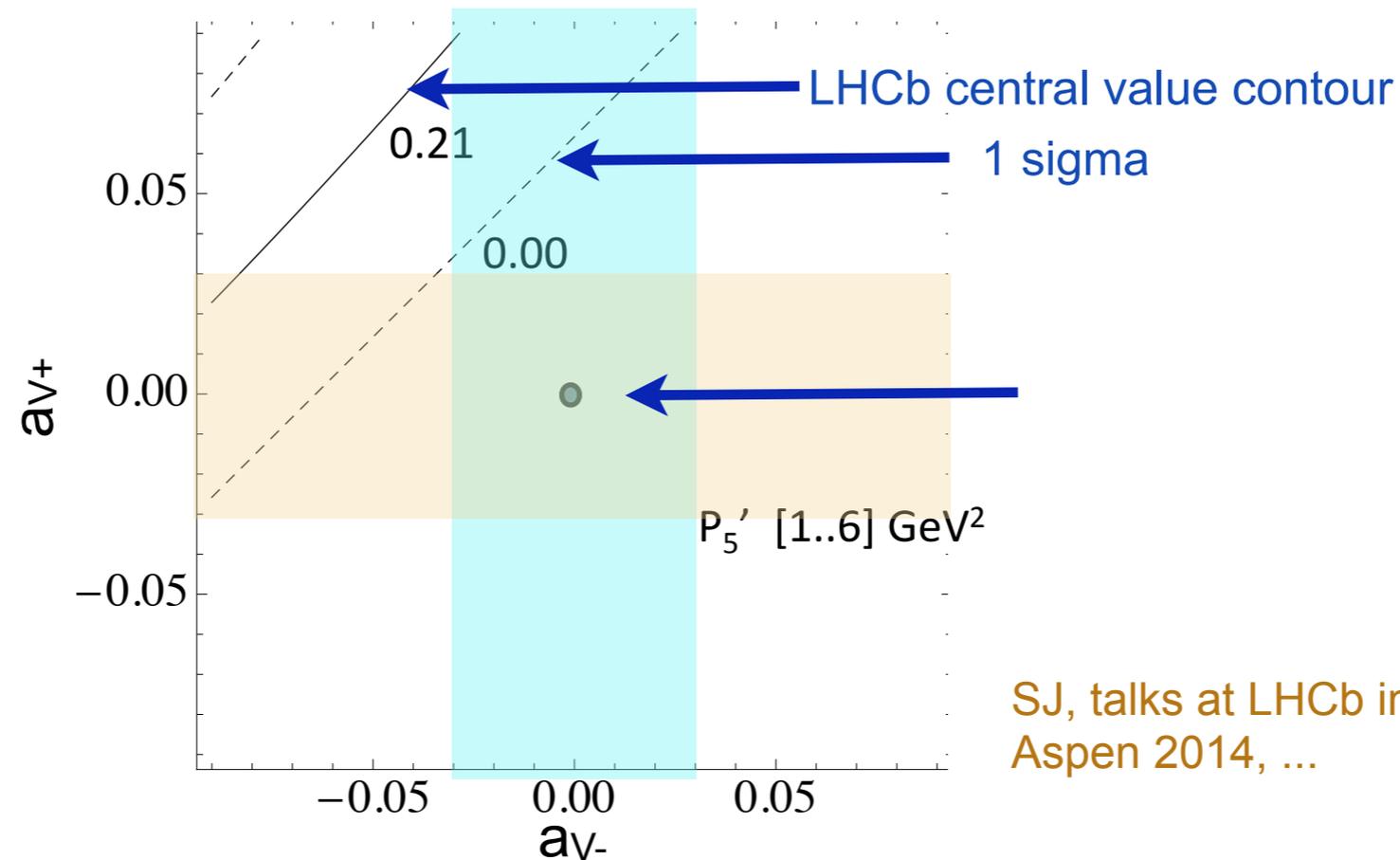
- \* Significance of the effect depends strongly on treatment of theory uncertainties!
- \* The most significant effect occurs in a bin extending well above the perturbative charm threshold, outside the range of validity of the theory framework

# $P_5'$ power-correction dependence

plot in plane of two form factor power correction parameters

relating to  $V_+$  and  $V_-$ , respectively

(there are 10 power-correction parameters to order  $q^2/m_B^2$ )



SJ, talks at LHCb implications 10/2013, Aspen 2014, ...

$\sim \pm 0.03$  for either power correction parameter corresponds to a 10% power correction & is sufficient to bring data in agreement with SM theory

Drawing conclusions based on this observable requires **sufficient accuracy on the form factor calculations** (not even considering nonfactorizable effects). DMV and most other phenomenology employs one of two light-cone-sum rule calculations.

This conclusion relies solely on the functional dependence of  $P_5'$  and holds **irrespectively** of statistical treatments, assumptions on soft form factors at  $q^2=0$ , etc.

# Fitting the power-correction parameters?

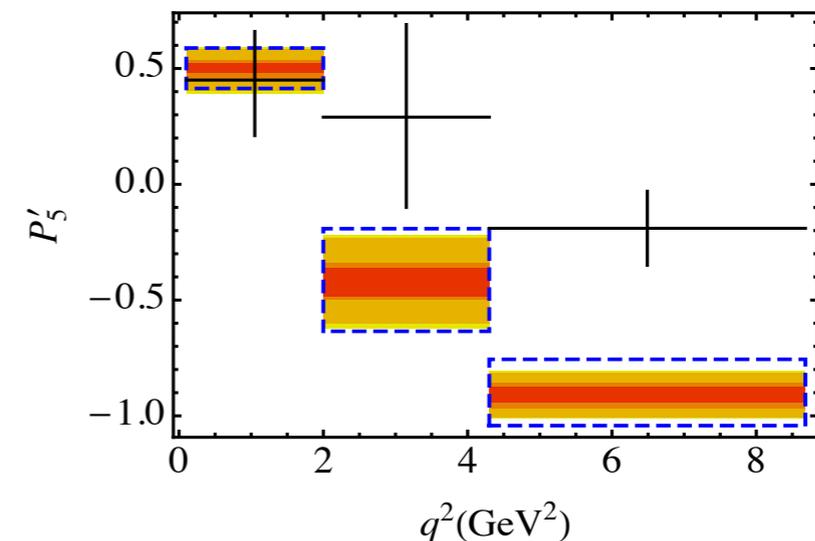
Dedicated analysis employing same power correction model

Descotes-Genon, Hofer, Matias, Virto arXiv:1407.8526

\* identify a different set of 2 QCD form factors, claiming this reduces sensitivity of  $P_5'$  to power correction parameters

\* **fit the power-correction parameters to two different light-cone sum rule calculations**

-> Again obtain small theory uncertainties

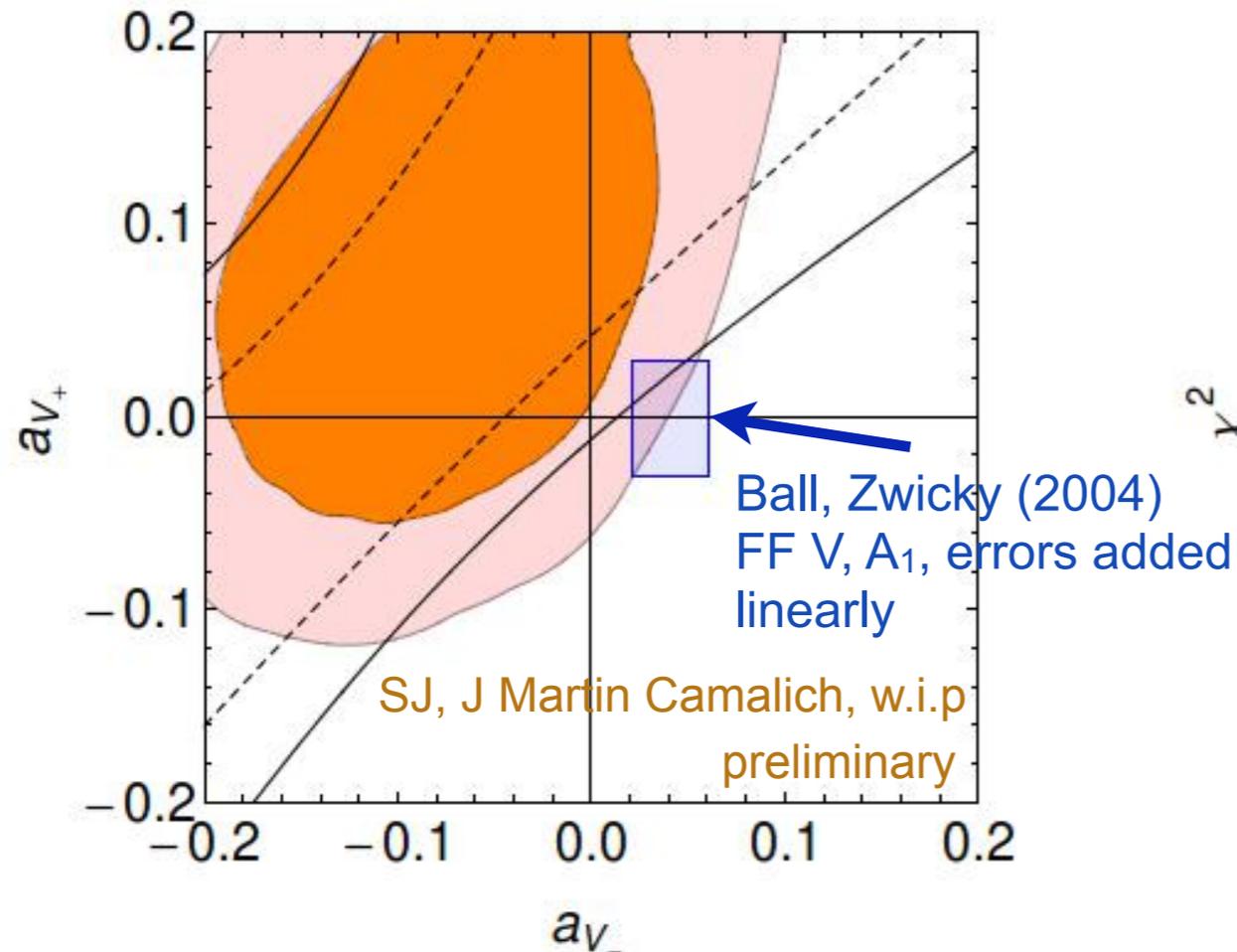


$\langle P_5' \rangle_{[1,6]}$	$-0.412^{+0.042+0.026+0.096+0.014}_{-0.070-0.045-0.089-0.017}$	$-0.416^{+0.039+0.000+0.083+0.014}_{-0.064-0.000-0.086-0.017}$
--------------------------------	--	--

- Can LCSR be trusted to the required level of <10% accuracy? Parton/hadron duality model, ...

- Reminder: bin in plot extends above pert. charm threshold

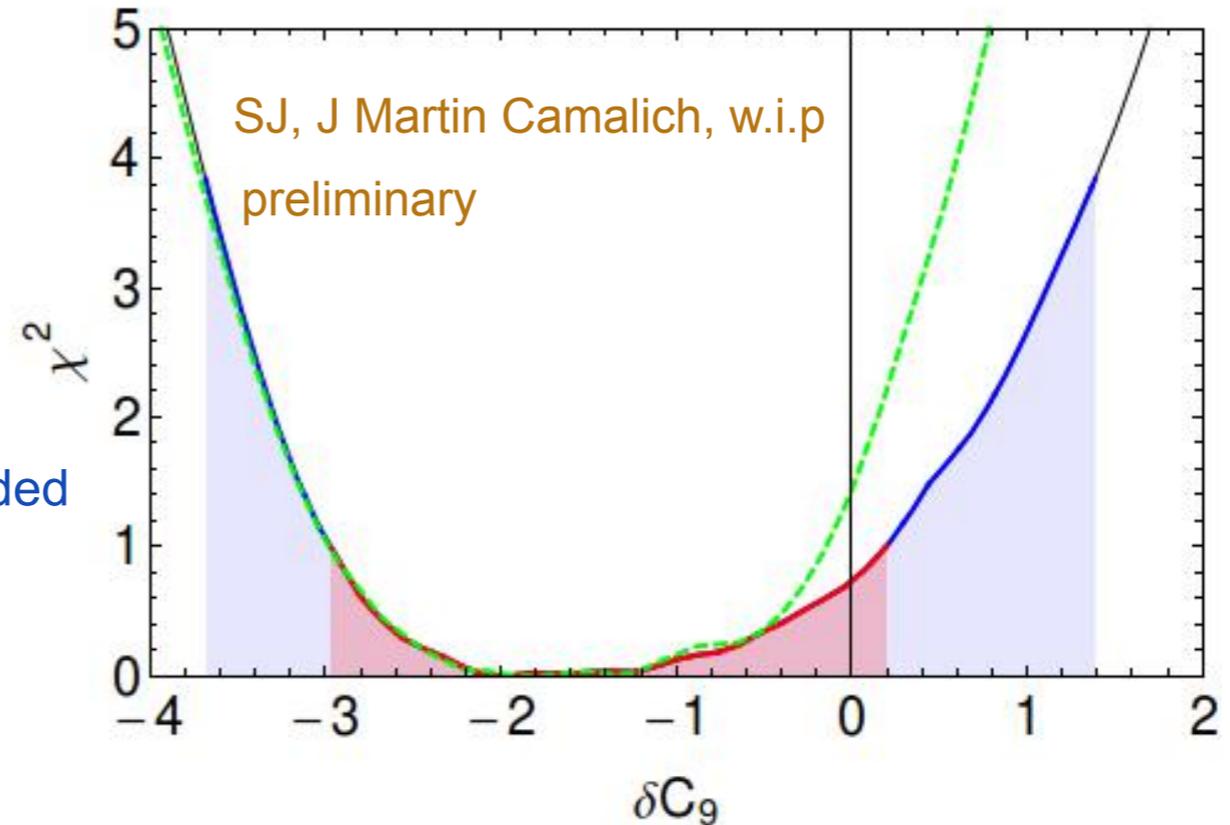
# Anomaly significance



all angular observables  
low- $q^2$  only [1..6 GeV<sup>2</sup>]

all other theory errors profiled

setting either power correction parameter  
to zero [by soft form factor redefinition]  
does not seem to help!



all angular observables  
low- $q^2$  only [1..6 GeV<sup>2</sup>]

profile likelihood ("Rfit" theory  
error treatment a la CKMfitter)

significance below 1 sigma

# Clean SM Null Tests & RH dipoles

Extending to BSM Wilson coefficients, have

neglecting strong phase differences  
[tiny; take into account in numerics]

close to  $q^2 = 0$  (photon  
pole dominance)

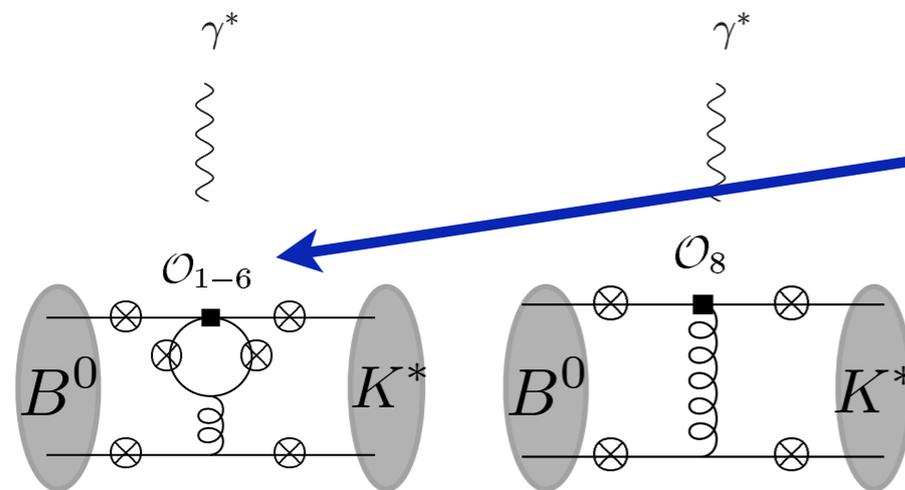
$$P_1 \equiv \frac{I_3 + \bar{I}_3}{2(I_{2s} + \bar{I}_{2s})} \stackrel{\downarrow}{=} \frac{-2 \operatorname{Re}(H_V^+ H_V^{-*} + H_A^+ H_A^{-*})}{|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2} \approx 2 \frac{\operatorname{Re}(C_7 C_7'^*)}{|C_7|^2 + |C_7'|^2}$$

$$P_3^{CP} \equiv -\frac{I_9 - \bar{I}_9}{4(I_{2s} + \bar{I}_{2s})} = -\frac{\operatorname{Im}(H_V^+ H_V^{-*} + H_A^+ H_A^{-*})}{|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2} \approx \frac{\operatorname{Im}(C_7 C_7'^*)}{|C_7|^2 + |C_7'|^2}$$

- **double** suppression  $T_+(q^2) = \mathcal{O}(q^2/m_B^2) \times \mathcal{O}(\Lambda/m_b)$

- What is the fate of the helicity hierarchy in the nonlocal term?  
**Crucial question for sensitivity to  $C_7'$**

# Nonlocal terms (hadronic hamiltonian)



includes  $Q_1^c, Q_2^c$  - large Wilson coefficients

+ “vertex corrections” + annihilation

Beneke, Feldmann, Seidel 2001

leading-power: factorises into perturbative kernels, form factors, meson light-cone distribution amplitudes (including hard/hard-collinear gluon corrections to all orders)

$$h_\lambda = \int_0^1 du \phi_K^*(u) T(u, \alpha_s) + \mathcal{O}(\Lambda/m_b)$$

$\alpha_s^0$  :  $C_7 \rightarrow C_7^{\text{eff}}$      $C_9 \rightarrow C_9^{\text{eff}}(q^2)$     + 1 annihilation diagram

$\alpha_s^1$  : (convergent) convolutions of hard- scattering kernels with meson LCDA

unambiguous (save for parametric uncertainties)  
state-of-the-art in phenomenology

at subleading powers: breakdown of factorisation

some contributions have been estimated as end-point divergent convolutions with a cut-off

Kagan&Neubert 2001, Feldmann&Matias 2002

LCSR computation finds effective shifts of transversity amplitudes as large as  $\sim 10\%$

Khodjamirian et al 2010

# Digression: relation to $B \rightarrow K^* \gamma$

$$\begin{aligned}
 \mathcal{A}(\bar{B} \rightarrow V(\lambda)\gamma(\lambda)) &= \lim_{q^2 \rightarrow 0} \frac{q^2}{e} H_V(q^2 = 0; \lambda) && \text{exact (LSZ)} \\
 &= \frac{iN m_B^2}{e} \left[ \frac{2\hat{m}_b}{m_B} (C_7 \tilde{T}_\lambda(0) - C'_7 \tilde{T}_{-\lambda}(0)) - 16\pi^2 h_\lambda(q^2 = 0) \right]
 \end{aligned}$$

(only  $\lambda = \pm 1$ )

**same** amplitudes as in  $B \rightarrow K\ell$  including all long-distance details

$$S_{K^* \gamma} = 2 \frac{\text{Im}(e^{-i\phi_d} H_V^+(0) H_V^{-*}(0))}{|H_V^+(0)|^2 + |H_V^-(0)|^2} \approx 2 \frac{\text{Im}(e^{-i\phi_d} C_7 C_7'^*)}{|C_7|^2 + |C_7'|^2}$$

earlier estimates of **long-distance** effects in  $h_\lambda(0)$

~ 10%

Grinstein, Grossman, Ligeti, Pirjol 2004 (SCET)

few percent

Ball, Jones, Zwicky 2006 (LCSR)

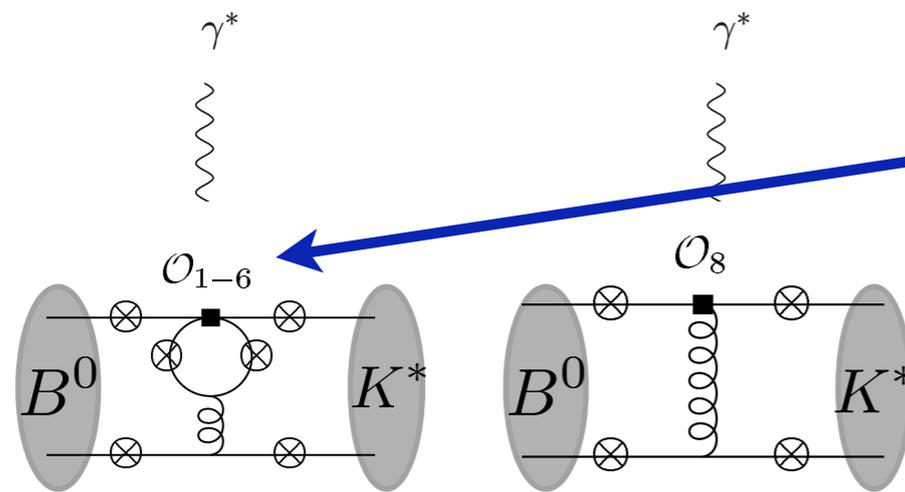
also Muheim, Xie, Zwicky 2008 (refer to unpub LCSR)

- understand differences

- clarify relation/double counting with heavy-quark limit

- important for fate of clean null tests!

# Nonlocal terms: power corrections



includes  $Q_1^c, Q_2^c$  - large Wilson coefficients

+ “vertex corrections” + annihilation

Beneke, Feldmann, Seidel 2001

subleading power: breakdown of factorisation. Schematically for  $Q_1^c, Q_2^c$ :

$$r_\lambda^c = \int_{\Lambda_h}^1 du \phi_K^*(u) T(u, \alpha_s) + r_{\lambda, \text{soft}}^c$$

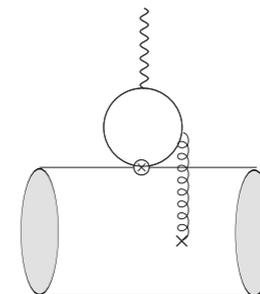
1) power corrections from: (i) higher-twist 2-particle LCDA; (ii) multi-particle LCDA, and from soft endpoint region (iii)

2) some endpoint-divergent contributions from hard-collinear gluon exchanges;

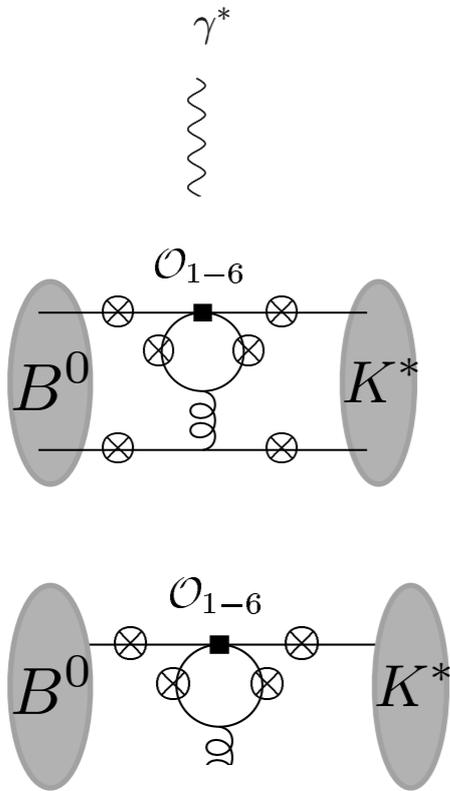
Kagan&Neubert 2001, Feldmann&Matias 2002

3) need to allow for “soft” remainder even if endpoint convergent: means only that endpoint region is power suppressed relative to “bulk” region!

4) In endpoint region hard-collinear gluon becomes soft



# Long-distance “charm loop”



$$r_{\lambda}^c = \int_{\Lambda_h}^1 du \phi_K^*(u) T(u, \alpha_s) + r_{\lambda, \text{soft}}^c$$

$Q_1^c, Q_2^c$  insertions with hard-collinear gluon(s):  
**cannot generate soft gluon (in Breit frame) with two-particle LCDA**

multi-particle LCDA collinear factorization viewpoint this represents the endpoint region, which is known to give a power-suppressed contribution

$Q_1^c, Q_2^c$  insertions with soft gluon: can still integrate out charm, but not the gluons

Grinstein, Grossmann, Ligeti, Pirjol 2004

for single soft gluon the two gluon attachments to the charm line give

$$r_{\lambda, \text{soft}}^c = \epsilon^{\mu*}(\lambda) \langle M(k, \lambda) | \tilde{\mathcal{O}}_{\mu} | \bar{B} \rangle$$

where the light-cone operator (in notation of [Khodjamirian 2010](#) )

$$\tilde{\mathcal{O}}_{\mu} = \int d\omega I_{\mu\rho\alpha\beta}(q, \omega) \bar{s}_L \gamma^{\rho} \delta\left(\omega - \frac{in_+ \cdot D}{2}\right) \tilde{G}^{\alpha\beta} b_L$$

(corresponds to the two photon attachments to the charm loop, treating  $\Lambda^2/(4 m_c^2) \sim \Lambda/m_b$  )

matrix element power counting:  $\Lambda^2/(4 m_c^2) \sim \Lambda/m_b$  per soft gluon [Khodjamirian 2010](#)

power suppression as expected from heavy-quark power counting!  
 no double counting! - but 4 more photon attachments

# Helicity hierarchies survive!

- LCSR helicity amplitudes

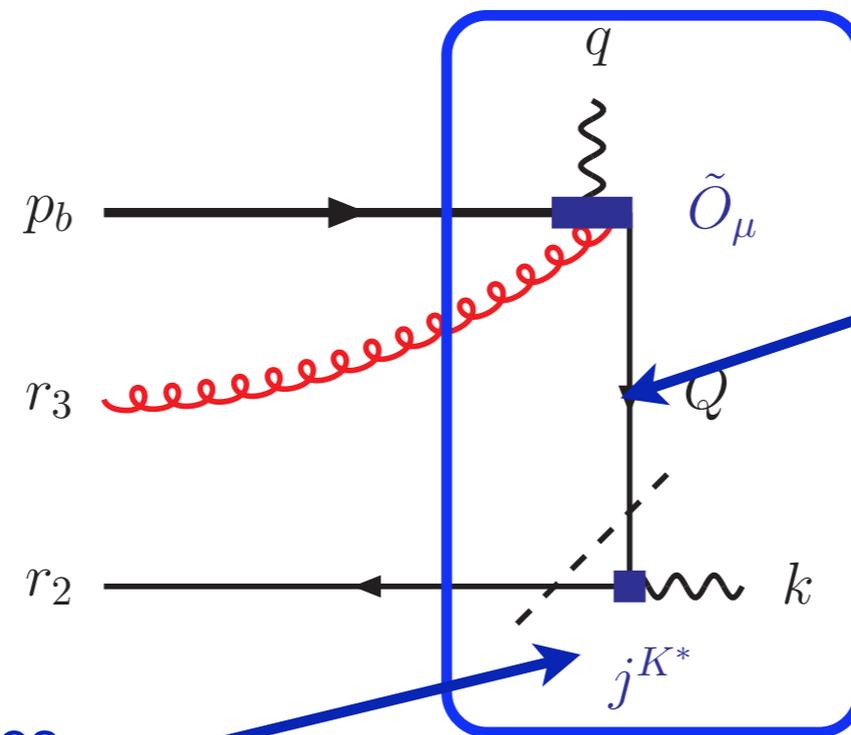
SJ, Martin Camalich 2012  
(also for helicity-+ form factors!)

$$G_{h\lambda}(q^2; k^2) = -i \int d^4y e^{iky} \langle 0 | T \{ \epsilon^{\nu*}(\hat{z}; \lambda) j_\nu^{K^*}(y) \epsilon^{\mu*}(-\hat{z}; \lambda) \tilde{O}_\mu(0) \} | B \rangle$$

This has a hadronic representation containing the desired matrix elements

$$G_{h\lambda}(q^2; k^2) = \frac{f_{K^*} m_{K^*}}{m_{K^*}^2 - k^2} \langle K^*(\tilde{k}; \lambda) | \epsilon^{\mu*}(-\hat{z}; \lambda) \tilde{O}_\mu(0) | B \rangle + \text{continuum contributions.}$$

based on Khodjamirian et al 2010



for  $k^2 \sim -1 \text{ GeV}^2$   
this line is hard-collinear  
(numerically only - no heavy-quark expansion!)

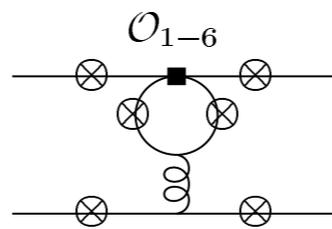
integrate out  
(standard  
LCSR step)

key: project out helicities  
through interpolating current

operator defining 3-particle  
B-meson LCDA

vanishes for + helicity, up to higher power of  $\Lambda/m_b$

SJ, Martin Camalich 2012



1) further photon attachments:

attachments to b or s quark quite local operator; simpler argument; again helicity hierarchy

attachments to spectator lines should give nonlocal operator product of [s G b] operator and light-quark part of em current.

However as photon always hard, soft-gluon exchange appears kinematically impossible (more detailed investigation desirable)

2) earlier estimates of **long-distance** effects in  $h_\lambda(0)$

SCET-based [Grinstein, Grossman, Ligeti, Pirjol 2004](#)

identify SCET<sub>I</sub> operator  $\sim \tilde{O}_\mu$

only power counting estimate of matrix element, misses helicity hierarchy (cannot match onto SCET<sub>II</sub> b/c endpoint divergences)

LCSR-based [Ball, Jones, Zwicky 2006](#) [\(also Muheim, Xie, Zwicky 2008\)](#)

derive sum rule with external  $K^*$  external (instead of B)

- does not single out the soft (endpoint) configuration

- moreover expand a light-cone operator in local operators; but the neglected higher-dimensional matrix elements scale like  $m_B^2/(4 m_c^2)$ : not justified!

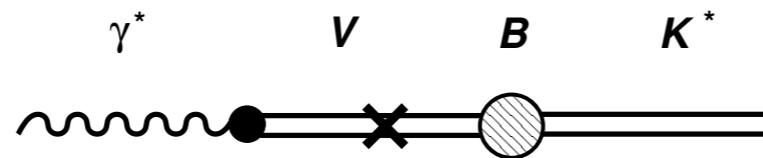
(different from somewhat analogous  $B \rightarrow X_s \gamma$  case)

# Light-quark contributions

Operators without charm have strong charm or CKM suppression; power corrections should be negligible.

However, they generate (mild) resonance structure even below the charm threshold, presumably “duality violation”

Presumably  $\rho, \omega, \phi$  most important; use vector meson dominance supplemented by heavy-quark limit  $B \rightarrow VK^*$  amplitudes

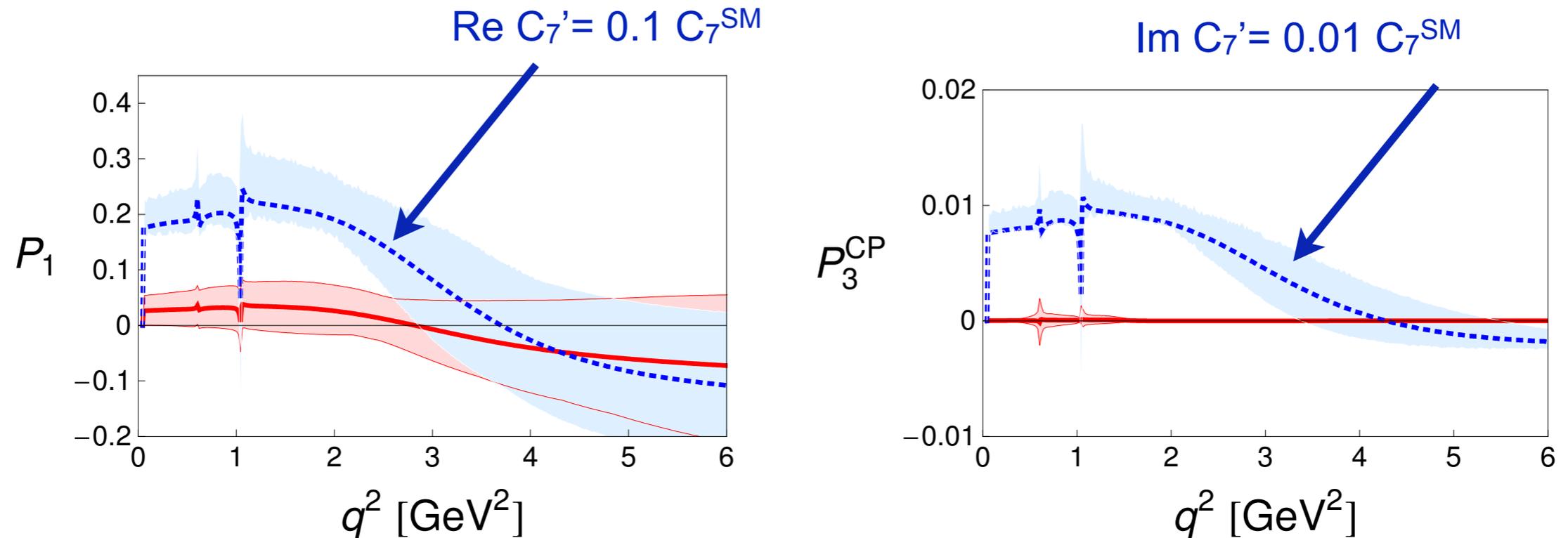


$$\tilde{a}_\mu^{\text{had, lq}} = \int d^4x e^{-iq \cdot x} \sum_{P, P'} \langle 0 | j_\mu^{\text{em, lq}}(x) | P' \rangle \langle P'(x) | P(0) \rangle \langle \bar{K}^* P | \mathcal{H}_{\text{eff}}^{\text{had}}(0) | \bar{B} \rangle$$

estimate **uncertainty** from difference between VMD model and the subset of heavy-quark limit diagrams corresponding to intermediate  $V$  states.

Helicity hierarchies in **hadronic**  $B$  decays prevent large uncertainties in  $H_V^+$  from this source, too.

# Sensitivity to $C_7'$ (muonic mode)



SJ, Martin Camalich 2012

- Two angular observables remain clean null tests of the SM in the presence of long-distance corrections
- (theoretical limit on) sensitivity to Re  $C_7'$  at  $<10\%$  ( $C_7^{\text{SM}}$ ) level, to Im  $C_7'$  at  $<1\%$
- sensitivity stems from  $q^2 \in [0.1, 2]$  GeV<sup>2</sup>

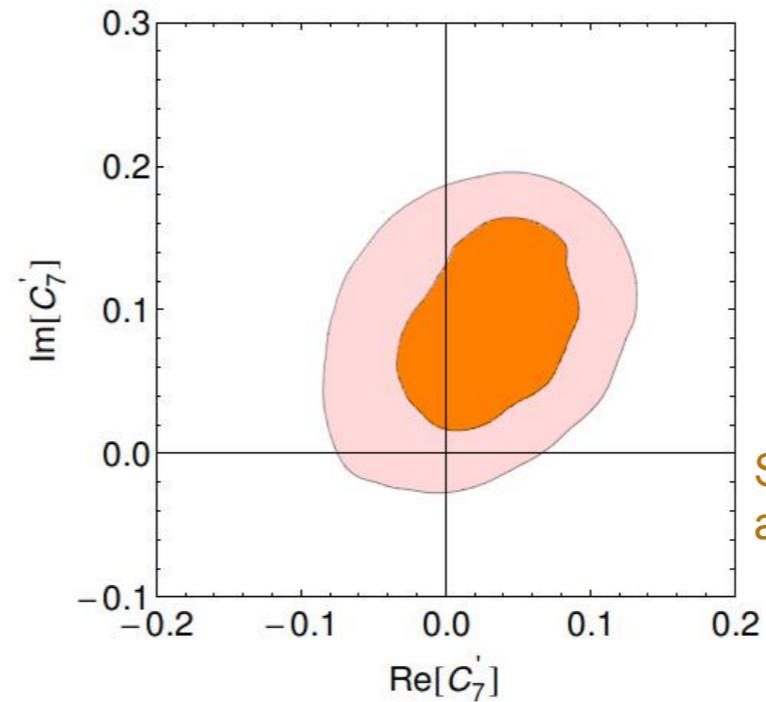
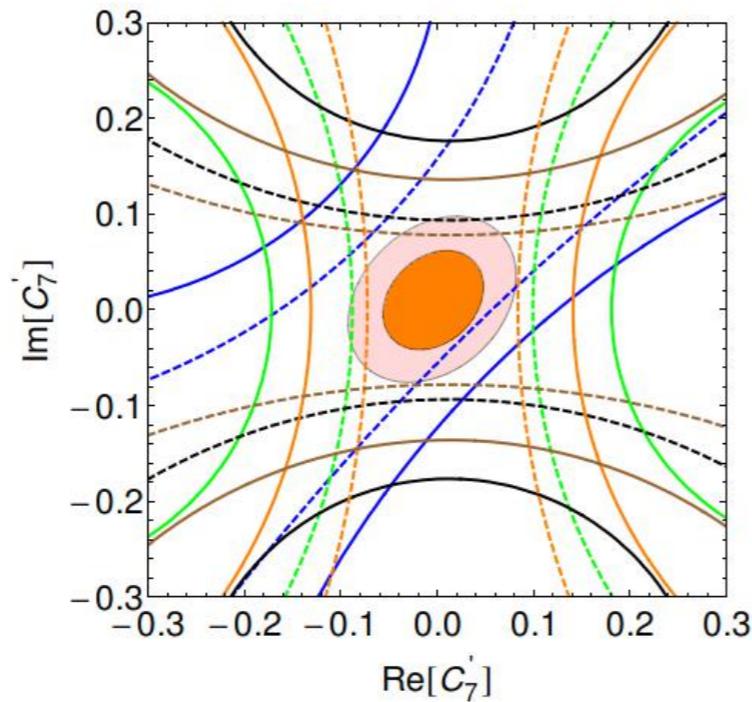
# Predictions for electronic mode

$F_L$	$P_1$	$P_2$	$P_3^{\text{CP}} [10^{-4}]$
$0.07^{+0.09}_{-0.04}$	$0.030^{+0.045}_{-0.041}$	$-0.054^{+0.015}_{-0.012}$	$0.1^{+0.7}_{-0.6}$
$P'_4$	$P'_5$	$P'_6$	$P'_8$
$0.18^{+0.06}_{-0.07}$	$0.52^{+0.10}_{-0.11}$	$0.05^{+0.07}_{-0.06}$	$0.01^{+0.08}_{-0.08}$

SJ, Martin Camalich to appear (preliminary)

- Theoretically even cleaner than muonic mode at very low  $q^2$  as tensor form factor / photon pole dominates more
- Boost in BR:  $BR_{[0.0004, 1]}^e = 31^{+15}_{-11} 10^{-8}$ ,  $BR_{[0.1, 0.98]}^\mu = 9.5^{+5.2}_{-3.5} 10^{-8}$
- Angular analysis in progress at LHCb
- Belle 2 well positioned to study this mode

# Prospects



SJ, Martin Camalich to appear (preliminary)

$$S \simeq \frac{2\text{Im}(e^{-2i\beta} C_7 C_7')}{|C_7|^2 + |C_7'|^2},$$

$$P_1 \simeq \frac{2\text{Re}(C_7 C_7')}{|C_7|^2 + |C_7'|^2},$$

$$P_3^{\text{CP}} \simeq \frac{2\text{Im}(C_7 C_7')}{|C_7|^2 + |C_7'|^2}$$

- Left: assuming  $\sigma_{P_i} = 0.25$  for muons and electrons, no theory errors
- Right: Profile likelihood for current data (1sigma and 95% CL)
- excellent sensitivity to right-handed currents remains with conservative treatment of QCD uncertainties

# Summary

Rich and growing set of measurements on  $b \rightarrow s$  II and related FCNC decays

Various anomalies (LHCb anomaly in  $P_5'$ , but also hints of lepton flavour nonuniversality and in branching fraction data)

No single observable appears to be significant, and long-distance QCD effects may be an explanation. However, a consistent picture may be emerging.

However, helicity hierarchies remain intact for robust treatment of long-distance/power corrections, preserving two null-test observables in  $B \rightarrow K^* \mu^+ \mu^-$

Precision probes of right-handed dipole transitions (and other coefficients) possible - good LHCb, and presumably Belle 2, prospects

かたじけない

**BACKUP**

# Exclusive decays at the LHC

Experimental landscape dominated by LHCb, for exam[ple in semileptonic modes alone:

channel	$\mathcal{L}^{int}$ ( $fb^{-1}$ )	Publication
$d\mathcal{B}/dq^2 B \rightarrow K^{*+} \mu^+ \mu^-$	3	[1403.8044]
$d\mathcal{B}/dq^2 B \rightarrow K^0 \mu^+ \mu^-$	3	[1403.8044]
$d\mathcal{B}/dq^2 B \rightarrow K^+ \mu^+ \mu^-$	3	[1403.8044]
$d\mathcal{B}/dq^2 B^0 \rightarrow K^{*0} \mu^+ \mu^-$	1	[JHEP08(2013)131]
$d\mathcal{B}/dq^2 B_s^0 \rightarrow \phi \mu^+ \mu^-$	1	[JHEP07(2013)084]
$d\mathcal{B}/dq^2 \Lambda_b \rightarrow \Lambda \mu^+ \mu^-$	1	[PLB725(2013)25]
$\mathcal{B} B^0 \rightarrow K^{*0} e^+ e^-$	1	[JHEP05(2013)159]
$\mathcal{B} B^+ \rightarrow \pi^+ \mu^+ \mu^-$	1	[JHEP12(2012)125]
$A_I B \rightarrow K^{(*)} \mu^+ \mu^-$	3	[1403.8044]
$A_{CP} B^+ \rightarrow K^+ \mu^+ \mu^-$	1	[PRL111,151801(2013)]
$A_{CP} B^0 \rightarrow K^{*0} \mu^+ \mu^-$	1	[PRL110,031801(2013)]
Angular $B^+ \rightarrow K^+ \mu^+ \mu^-$	3	[JHEP05(2014)082],[PRL111,112003(2013)]
Angular $B^0 \rightarrow K^0 \mu^+ \mu^-$	3	[JHEP05(2014)082]
Angular $B^0 \rightarrow K^{*0} \mu^+ \mu^-$	1	[JHEP08(2013)131],[PRL111,191801(2013)]
Angular $B_s^0 \rightarrow \phi \mu^+ \mu^-$	1	[JHEP07(2013)084]

K Petridis (LHCb), b->s workshop, Paris, June 2014

Updates and new analyses expected based on full  $3 fb^{-1}$  data set  
 $5-7 fb^{-1}$  expected during run II

Some results also from ATLAS, CMS

# Angular coefficients

$$I_1^c = F \left\{ \frac{1}{2} (|H_V^0|^2 + |H_A^0|^2) + 4|H_P|^2 + \frac{2m_\ell^2}{q^2} (|H_V^0|^2 - |H_A^0|^2) + 4\beta^2 |H_S|^2 \right\},$$

$$I_1^s = F \left\{ \frac{\beta^2 + 2}{8} (|H_V^+|^2 + |H_V^-|^2 + (V \rightarrow A)) + \frac{m_\ell^2}{q^2} (|H_V^+|^2 + |H_V^-|^2 - (V \rightarrow A)) \right\}$$

$$I_2^c = -F \frac{\beta^2}{2} (|H_V^0|^2 + |H_A^0|^2),$$

$$I_2^s = F \frac{\beta^2}{8} (|H_V^+|^2 + |H_V^-|^2) + (V \rightarrow A),$$

$$I_3 = -\frac{F}{2} \text{Re} [H_V^+ (H_V^-)^*] + (V \rightarrow A),$$

$$I_4 = F \frac{\beta^2}{4} \text{Re} [(H_V^- + H_V^+) (H_V^0)^*] + (V \rightarrow A),$$

$$I_5 = F \left\{ \frac{\beta}{2} \text{Re} [(H_V^- - H_V^+) (H_A^0)^*] + (V \leftrightarrow A) - \frac{2\beta m_\ell}{\sqrt{q^2}} \text{Re} [H_S^* (H_V^+ + H_V^-)] \right\}$$

$$I_6^s = F\beta \text{Re} [H_V^- (H_A^-)^* - H_V^+ (H_A^+)^*],$$

$$I_6^c = 8F \frac{\beta m_\ell}{\sqrt{q^2}} \text{Re} [H_S^* H_V^0],$$

$$I_7 = F \left\{ \frac{\beta}{2} \text{Im} [(H_A^+ + H_A^-) (H_V^0)^*] + (V \leftrightarrow A) - \frac{2\beta m_\ell}{\sqrt{q^2}} \text{Im} [H_S^* (H_V^- - H_V^+)] \right\},$$

$$I_8 = F \frac{\beta^2}{4} \text{Im} [(H_V^- - H_V^+) (H_V^0)^*] + (V \rightarrow A),$$

$$I_9 = F \frac{\beta^2}{2} \text{Im} [H_V^+ (H_V^-)^*] + (V \rightarrow A),$$

strongly suppressed in SM  
good sensitivity to NP with  
different chirality structure  
("right-handed currents")

suppression of  $I_3, I_9$  due to  
suppression of +-amplitudes  
**must quantify corrections**

SJ, J Martin Camalich 2012

# Helicity amplitudes

decompose amplitude in lepton currents & “dilepton helicity”

$$A = - \sum_{\lambda=\pm 1,0} \mathcal{L}_V(\lambda) H_V(\lambda) - \sum_{\lambda=\pm 1,0} \mathcal{L}_A(\lambda) H_A(\lambda) + L_S H_S + L_P H_P \\ - \sum_{\lambda=\pm 1,0} \mathcal{L}_{TL}(\lambda) H_{TL}(\lambda) - \sum_{\lambda=\pm 1,0} \mathcal{L}_{TR}(\lambda) H_{TR}(\lambda),$$

polarisation vectors for dilepton

$$\mathcal{L}_V(\lambda) = \epsilon_\mu(\lambda) L_V^\mu,$$

$$\mathcal{L}_A(\lambda) = \epsilon_\mu(\lambda) L_A^\mu,$$

$$\mathcal{L}_{TL}(\lambda) = \epsilon_\mu(\lambda) L_{TL}^\mu,$$

$$\mathcal{L}_{TR}(\lambda) = \epsilon_\mu(\lambda) L_{TR}^\mu,$$

$$\mathcal{L}_S = L_S$$

$$\mathcal{L}_P = L_P$$

$$L_V^\mu = \langle \ell^+ \ell^- | \bar{l} \gamma^\mu l | 0 \rangle,$$

$$L_S = \langle \ell^+ \ell^- | \bar{l} l | 0 \rangle,$$

$$L_{TL}^\mu = \frac{i}{\sqrt{q^2}} \langle \ell^+ \ell^- | q_\nu \bar{l} \sigma^{\mu\nu} P_L l | 0 \rangle,$$

$$L_A^\mu = \langle \ell^+ \ell^- | \bar{l} \gamma^\mu \gamma^5 l | 0 \rangle,$$

$$L_P = \langle \ell^+ \ell^- | \bar{l} \gamma^5 l | 0 \rangle,$$

$$L_{TR}^\mu = \frac{i}{\sqrt{q^2}} \langle \ell^+ \ell^- | q_\nu \bar{l} \sigma^{\mu\nu} P_R l | 0 \rangle$$

most of the literature employs transversity amplitudes

$$A_{\parallel L(R)} = \frac{1}{\sqrt{2}} (H_{+1,L(R)} + H_{-1,L(R)}), \quad A_{\perp L(R)} = \frac{1}{\sqrt{2}} (H_{+1,L(R)} - H_{-1,L(R)})$$

$$H_{\lambda L/R} = i \sqrt{f} \frac{1}{2} (H_V(\lambda) \mp H_A(\lambda)), \quad A_t = i \frac{\sqrt{q^2}}{2m_\ell} \sqrt{f} H_P, \quad A_S = -i \sqrt{f} H_S$$

# Helicity amplitude definitions

Helicity amplitudes are expressed in terms of Wilson coefficients, form-factors and a nonlocal operator product

$$H_A(\lambda) = N(C_{10A}\tilde{V}_{L\lambda} + C'_{10A}\tilde{V}_{R\lambda}),$$

$$H_{TR}(\lambda) = N\frac{4\hat{m}_b m_B}{m_W \sqrt{q^2}} C_T \tilde{T}_{L\lambda},$$

$$H_{TL}(\lambda) = N\frac{4\hat{m}_b m_B}{m_W \sqrt{q^2}} C'_T \tilde{T}_{R\lambda},$$

$$H_S = -N\frac{\hat{m}_b}{m_W} (C_S \tilde{S}_L + C'_S \tilde{S}_R),$$

$$H_P = -N\left\{ \frac{\hat{m}_b}{m_W} (C_P \tilde{S}_L + C'_P \tilde{S}_R) + \frac{2m_l \hat{m}_b}{q^2} \left[ C_{10A} \left( \tilde{S}_L - \frac{m_s}{m_b} \tilde{S}_R \right) + C'_{10A} \left( \tilde{S}_R - \frac{m_s}{m_b} \tilde{S}_L \right) \right] \right\}$$

11 helicity amplitudes factorize naively (into form factors and Wilson coefficients)

$$H_V(\lambda) = N\left\{ C_{9V}\tilde{V}_{L\lambda} + C'_{9V}\tilde{V}_{R\lambda} - \frac{m_B^2}{q^2} \left[ \frac{2\hat{m}_b}{m_B} (C_{7\gamma}\tilde{T}_{L\lambda} + C'_{7\gamma}\tilde{T}_{R\lambda}) - 16\pi^2 h_\lambda \right] \right\}$$

$$h_\lambda \equiv \frac{i}{m_B^2} \square^{\mu*}(\lambda) a_\mu^{\text{had}}$$

(only) 3 helicity amplitudes are sensitive to non-(naively-)factorizing long-distance physics

$$\frac{e^2}{q^2} L_V^\mu a_\mu^{\text{had}} = -i \frac{e^2}{q^2} \int d^4x e^{-iq \cdot x} \langle \ell^+ \ell^- | j_\mu^{\text{em,lept}}(x) | 0 \rangle \int d^4y e^{iq \cdot y} \langle M | j^{\text{em, had}, \mu}(y) \mathcal{H}_{\text{eff}}^{\text{had}}(0) | \bar{B} \rangle$$

**Form factors and non-factorizable contributions control theory errors**