Some Recent Results in Precision Flavour Physics

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Contents

Introduction

Precision Bottom Physics has become possible
due to a large arsenal of model-independent methods

- Effective Theories
- ... such as Heavy Quark Expansions
- QCD sum rules

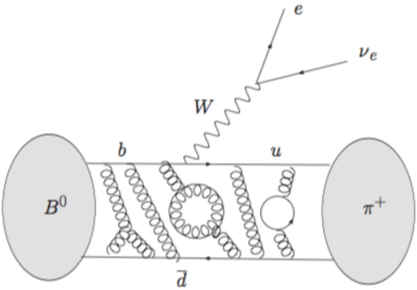
Two papers

I. S. Imsong, A. Khodjamirian, ThM, D. van Dyk, arXiv:1409.7816, *($B \to \pi$ Form Factors)*

ThM, A. Pivovarov, D. Rosenthal, arXiv:1405.5072 *($\mu_G^2 \alpha_s$ Corrections)*

$B \to \pi \ell \nu_\ell$, determination of $|V_{ub}|$

- decay amplitude parametrized by hadronic form factors



$$\langle \pi^+(p)|\bar{u}\gamma_\mu b|\bar{B}^0(p+q)\rangle = f^+_{B\pi}(q^2)\left[\ldots\right]_\mu + f^0_{B\pi}(q^2)\left[\ldots\right]_\mu$$

- $|V_{ub}|$ determination [BaBar,Belle]

$$\left(\frac{1}{\tau_B}\right)\frac{dBR(\bar{B}^0 \to \pi^+ l^- \nu)}{dq^2} = \frac{G_F^2|V_{ub}|^2}{24\pi^3} p_\pi^3 |f^+_{B\pi}(q^2)|^2 + O(m_l^2)$$

QCD Light-Cone Sum Rules (LCSR)

- the method uses full QCD with finite m_b,
  $1/m_b$ expansion possible
- involves analytic calculation, truncated α_s and power expansion
  $\sim 1/\sqrt{m_b \chi}$, $\Lambda_{QCD} \ll \chi \ll m_b$
- employs universal nonperturbative input
  light-cone distribution amplitudes of mesons; condensate densities
- "indirect access" to the form factor
  via hadronic dispersion relation $\oplus$ quark-hadron duality approximation
- form factors accessible at $0 < q^2 < 12 - 16\ \text{GeV}^2$

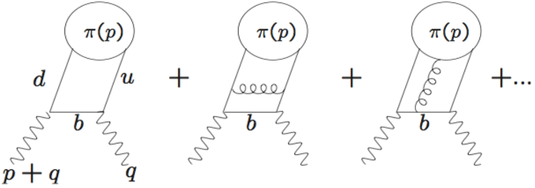
□ LCSR for $B \to \pi$ form factor: the correlation function

- an artifitially "designed" amplitude
- external currents with $(p+q)^2, q^2 \ll m_b^2 \Rightarrow b$-quark virtual,
- Operator product expansion (OPE) of the correlation function:

  $\mu \sim \sqrt{m_b \chi}$,

$$F(q^2,(p+q)^2) = \sum_{t=2,3,4,..} \int du\, T^{(t)}(q^2,(p+q)^2, m_b^2, \alpha_s, u, \mu)\, \varphi_\pi^{(t)}(u,\mu)$$

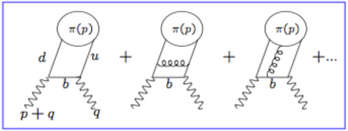
hard scattering amplitudes ⊗ pion light-cone DA's



LO NLO {power corr.}

LCSR for $B \to \pi$ form factor: the hadronic dispersion relation

{correlation function, OPE } = {sum over intermediate B states}



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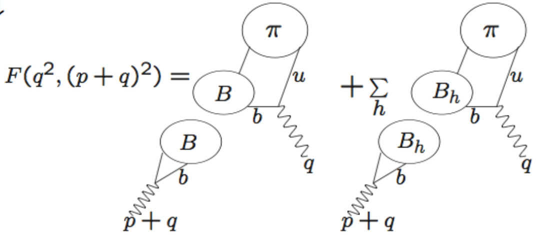


□ LCSR for $B \to \pi$ form factor: the result

quark-hadron duality

Borel transf. $(p+q)^2 \to M^2 \sim m_b \chi$

$$f^+_{B\pi}(q^2;\vec{\theta}) = \left(\frac{e^{m_B^2/M^2}}{2m_B^2[f_B]_{2ptSR}}\right)\int_{m_b^2}^{s_0^B} ds\, \frac{1}{\pi}\mathrm{Im}F(s,q^2,\alpha_s,\mu,m_b,\vec{\theta}^{(2,3,4)}_{DA})\, e^{-s/}$$

$\Uparrow$ QCD SR for f_B

$\Uparrow$ calculated from light-cone OPE

$$\left[f_B^2\right]_{2ptSR} = \left(\frac{e^{m_B^2/\overline{M}^2}}{m_B^4}\right)\overline{\mathcal{F}}(\overline{M}^2,\bar{s}_0^B,\alpha_s,\mu,m_b,\vec{\theta}_{\mathrm{cond}})$$

- set of inputs

$$\vec{\theta} \equiv \left(\alpha_s(M_Z)\,,\overline{m}_b(m_b)\,,\vec{\theta}^{(2,3,4)}_{DA}\,,\vec{\theta}_{\mathrm{cond}}\,,M^2,\,s_0^B\,,\overline{M}^2\,,\bar{s}_0^B\right)\,.$$

□ Previous use of $f^+_{B\pi}(q^2)$ from LCSR

- $|V_{ub}|$ determinations:
  integrating $|f^+_{B\pi}(q^2)|^2$ over q^2 or using $f^+_{B\pi}(0)$
  [AK, T.Mannel, N.Offen, Y-M.Wang (2011)] [A. Bharucha (2012)]
- one-by-one variation of input parameters, added in quadrature
  an overestimate?
- correlation between normalization and shape not studied
  additional constraints on theory ?
- z-parametrization used for extrapolation to large q^2
  how reliable ?

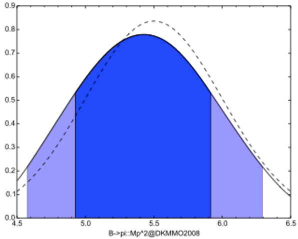
□ New statistical analysis of LCSR for $f^+_{B\pi}(q^2)$

- calculate the form factor $f^+_{B\pi}(q^2, \vec{\theta})$ from LCSR;
  use 2pt SR for f_B
- input parameters $\vec{\theta}$ include:
  - α_s, b-quark mass
  - quark condensate densities
  - coefficients of pion DA's
  - Borel parameters
  - effective thresholds
- statistical (Bayesian) analysis:
  inputs (assumed uncorrelated) taken as priors,
  constructing theoretical likelihood by imposing $[m_B]_{SR}$ within 1% of m_B

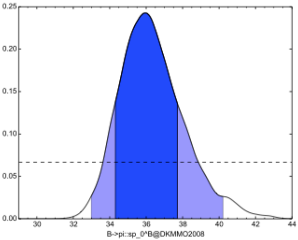
Some results

- Posterior of parameter space: one-dimension marginal PDF's

$M^2(\mathrm{GeV}^2)$



$s_0^B(\mathrm{GeV}^2)$



(prior: dashed lines, blue: 68%, light-blue: 95%)

- 6 quantities obtained from LCSR:
  $f^+_{B\pi}(q^2)$ + first + second derivative (value, slope, curvature) at $q^2 = 0, 10\ \mathrm{GeV}^2$,
  output approximately gaussian with large correlations

LCSR results fitted to BCL parameterization

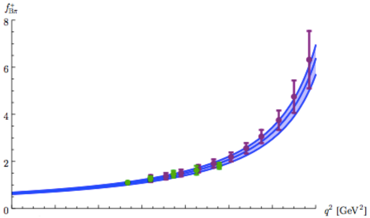
- "standard" z-series parameterization, $q^2 \to z(q^2, t_0)$,
  mapping SL region to small z, *the BCL-version [Bourrely, Caprini, Lellouch, (2008)]*

$$f^+_{B\pi}(q^2) = \frac{f^+_{B\pi}(0)}{1 - q^2/m^2_{B*}} \Big\{ 1 + b_1^+ \left[z(q^2, t_0) - z(0, t_0) - \frac{1}{3}\left(z(q^2, t_0)^3 - z(0, t_0)^3\right) \right] + b_2^+ \left[z(q^2, t_0)^2 - z(0, t_0)^2 + \frac{2}{3}\left(z(q^2, t_0)^3 - z(0, t_0)^3\right) \right] \Big\} ,$$

$$f_{B\pi}(0) = 0.307 \pm 0.02$$
$$b_1^+ = -1.31 \pm 0.42$$
$$b_2^+ = -0.904 \pm 0.444$$

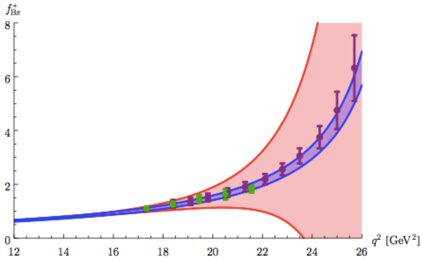
$$\rho^{BCL} = \begin{pmatrix} 1.000 & 0.503 & -0.391 \\ 0.503 & 1.000 & -0.824 \\ -0.391 & -0.824 & 1.000 \end{pmatrix}$$

- extrapolation
  beyond the LCSR region
  lattice results:
  • -HPQCD , • -Fermilab-MILC



Bounds for $B \to \pi$ form factor

- Parameterization-independent bounds following from the analytical properties of the form factor and from the unitarity of two-point correlation function [....., L.Lellouch (1996),...]
- form factor value, slope and curvature at one point yield the best constraints: [Th. Mannel, B.Postler (1998)]
- we use our results of statistical analysis at $q^2 = 10$ GeV^2



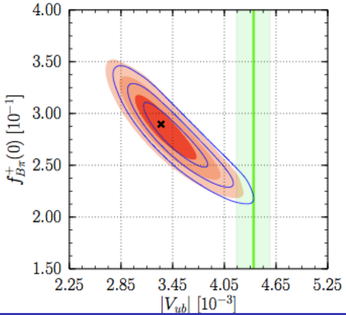
- bounds critically constraining lattice results up to $q^2 = 20$ GeV^2.

Determination of $|V_{ub}|$

fit of LCSR with the combined BaBar/Belle data at $0 < q^2 < 12$ GeV^2

$$(2010): |V_{ub}| = \left(3.43^{+0.27}_{-0.23}\right) \cdot 10^{-3}$$

$$(2013): |V_{ub}| = \left(3.32^{+0.26}_{-0.22}\right) \cdot 10^{-3}$$



blue lines: 68%, 95%, 99% prob. contours for 2010 data
red area: 68%, 95%, 99% prob. contours for 2013 data

green line/area - inclusive determination:
central value / 68% CL interval for GGOU/HFAG

Summary on exclusive V_{ub}

- statistical analysis of LCSR for $f^+_{B\pi}(q^2)$ improves error estimate
  (within fixed theoretical approximation !)
- correlations between shape and normalization of $f^+_{B\pi}(q^2)$
  to be confronted with Belle-2 future data on the shape
- unitarity bounds used to constrain extrapolations to large q^2
- $f^0_{B\pi}$, $B_{(s)} \to K$ form factors follow
- $B \to \pi\pi\ell\nu_\ell$ form factors:
  - partial wave expansion & resonances: ρ (P-wave) , f_0 (S-wave)
  - defining regions of Dalitz plot with specific QCD dynamics

  [S. Faller, T. Feldmann, A. Khodjamirian, T. Mannel and D. van Dyk, (2013)]
- calculating $B \to \pi\pi\ell\nu_\ell$ at low 2-pion mass and small q^2
  from LCSR with 2-pion DAs *(in progress)*

News on the inclusive determination of V_{cb}

- Standard tool: Heavy Quark Expansion
- Structure of the expansion (@ tree):

$$d\Gamma = d\Gamma_0 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2 d\Gamma_2 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 d\Gamma_3 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^4 d\Gamma_4$$
$$+ d\Gamma_5 \left(a_0 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^5 + a_2 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^2 \right)$$
$$+ \ldots + d\Gamma_7 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^4$$

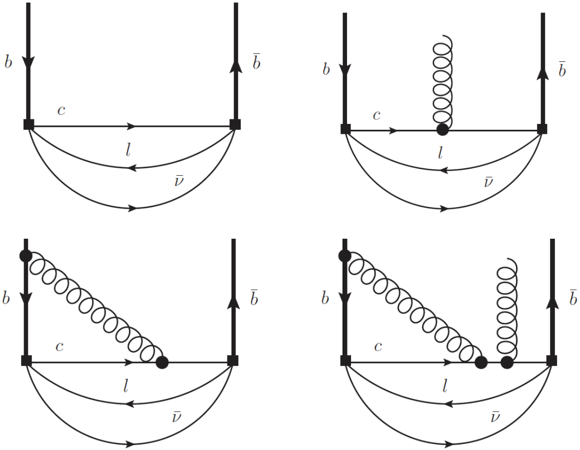
- Power counting $m_c^2 \sim \Lambda_{\rm QCD} m_b$

Present state of the $b \to c$ semileptonic Calculations

- Tree level terms up to and including $1/m_b^5$ known
- $\mathcal{O}(\alpha_s)$ and full $\mathcal{O}(\alpha_s^2)$ for the partonic rate known
- $\mathcal{O}(\alpha_s)$ for the μ_π^2/m_b^2 is known
- QCD insprired modelling for the HQE matrix elements
- New: Complete α_s/m_b^2, including the μ_G terms
  Alberti, Gambino, Nandi arXiv:1311.7381
  ThM, Pivovarov, Rosenthal arXiv:1405.5072
- This was the remaining parametrically largest uncertainty

- Alberti et al.:
  - Calculation of the differential rate including the charm mass
  - partially numerical calculation
- ThM, Pivovarov, Rosenthal:
  - Fully analytic calculation
  - limit $m_c \to 0$
  - Possibility to include m_c in a Taylor series
- Results (seem to) agree,
  some more checks in progress

Perform the OPE for $T = i \int d^4x \, T\, [H_{\rm eff}(x) H_{\rm eff}(0)]$



- Structure of the result:

$$(\mathrm{Im}\, T)/R_0 = C_0 \left\{ \bar{b} \not{v} b - \frac{\mathcal{O}_\pi}{2m_b^2} \right\} + C_{fin} C_m \frac{\mathcal{O}_G}{2m_b^2}$$

- chromomagnetic moment operator

$$\frac{1}{2M_B} C_m(\mu) \langle B(p_B) | \mathcal{O}_G | B(p_B) \rangle = \Delta m_B^2$$

- Total rate

$$\Gamma(B \to X_c \nu \ell) = \Gamma_b |V_{cb}|^2 \left\{ C_0 \left(1 + \frac{\mu_\pi^2}{2m_b^2} \right) + C_{fin} \frac{3\Delta m_B^2}{8m_b^2} \right\}$$

- We get ($\rho = m_c^2/m_b^2$)

$$C_{fin} = -3 + \Delta_G^{(0)}(\rho) + \frac{\alpha_s}{\pi}\Delta_G^{(1)}(\rho) + \frac{\alpha_s}{\pi}\left\{ C_A \left(\frac{31}{18} - \frac{\pi^2}{9} \right) + C_F \left(\frac{43}{144} - \frac{19\pi^2}{36} \right) \right\}$$

- $\Delta_G^{(0)}(\rho)$ is known

$$\Delta_G^{(0)}(\rho) = 8\rho - 24\rho^2 + 24\rho^3 - 5\rho^4 - 12\rho^2 \ln\rho$$

- $\Delta_G^{(1)}(\rho)$ is such that $\Delta_G^{(1)}(0) = 0$
- Numerically known from Alberti et al.

- Numerically (in the $m_c \to 0$ case:

$$C_{fin} = -3(1 + 1.56\frac{\alpha_s}{\pi})$$

This has a “normal” size

- The corresponding shift in V_{cb} is

$$\frac{\Delta|V_{cb}|}{|V_{cb}|} = 4.67\frac{\alpha_s}{\pi}\frac{3\Delta m_B^2}{8m_b^2}\frac{1}{2(1+\Delta_0^{(0)}(\rho))} \sim +0.3\%$$

Conclusions on V_{cb}

- Next goals in inclusive V_{cb}:
  - Use reparametrization invariance to obtain α_s corrections to even higher orders in $1/m$
  - Study the (numerous) matrix elements appearing in $1/m^n$ with $n > 3$
  - Consider partial resummations of the $1/m$ series
- Consistency with exclusive determinations
  - Lattice values
  - Zero Recoil Sum Rules
  - "BPS" limit

Overall Conclusions

- Semileptonic B decays have provided precise values of V_{xb}
- V_{cb} looks consistent between inclusive and exclusive determinations
- V_{ub} still exhibits a tension between inclusive and exclusive determinations
- $B \to \pi \ell \bar{\nu}$ is well studied by Lattice and QCDSR
- $B \to X_u \ell \bar{\nu}$ is studied in HQE by different groups (BLNP, GGOU)
- On the way: Study of other exclusive modes, such as $B \to \rho \ell \bar{\nu} \to \pi\pi \ell \bar{\nu}$