Light-cone Distribution Amplitudes for Heavy Hadrons in HQET

Thorsten Feldmann (U Siegen)

G. Bell, TF, Y.-M. Wang, MWY Yip, JHEP 11 (2013) 191.

TF, B. O. Lange, Y.-M. Wang, Phys. Rev. D89 (2014) 114001.

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Theoretische Physik 1

UNIVERSITÄT SIEGEN

DFG FOR 1873
quark **f**lavour physics and **e**ffective field **t**heories

Brief Motivation

B-Meson Light-Cone Distribution Amplitudes:

- Essential hadronic input for QCD factorization theorems:
  - charmless non-leptonic B-decays [Beneke/Buchalla/Neubert/Sachrajda]
  - spectator corrections for heavy-to-light form factors [Beneke/TF]
  - spectator corrections for radiative/semi-leptonic decays [Beneke/TF/Seidel]
  - correlation functions for QCD sum rules [Khodjamirian/Mannel/Offen; De Fazio/TF/Hurth]
- Resummation of large logs $\leftrightarrow$ RG evolution equations:
  - Renormalization of light-cone operators in HQET [Lange/Neubert, Descotes-Genon/Knodlseder/Offen, Kawamura et al. ...]
- Non-trivial constraints from local OPE [Lee/Neubert, Braun/Ivanov/Korchemsky]
- Experimental constraints from $B \to \gamma \ell \nu$ [Beneke/Rohrwild; Braun/Khodjamirian]

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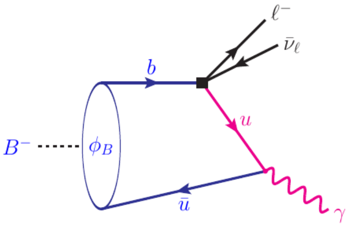
... but keep in mind (non-factorizable) power corrections ...



For large photon energy, $E_\gamma \sim m_b/2$:

- Sensitive to light-cone projection ω of light antiquark momentum in B-meson.

$$F^{B\to\gamma}(E_\gamma) \simeq \text{[kinematic factor]} \times \int_0^\infty \frac{d\omega}{\omega}\,\phi_B(\omega)$$

A first prototype of the "Buras-Counter"

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Outline

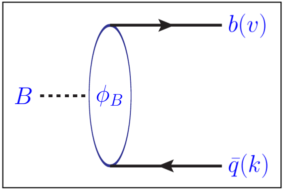
1. *B*-Meson LCDAs
   - Renormalization-group evolution
   - OPE constraints from HQET
2. LCDAs for Heavy Baryons (briefly)
   - mainly Λ_b

Summary / Main Messages

$\boxed{B\text{-Meson LCDAs}}$

("Zeptomechanics")

RGE for the LCDA of the B-Meson



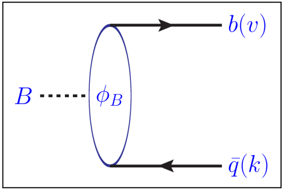
Definition of $\phi_B^+(\omega)$:

Fourier-Transform of Light-Cone Matrix Element in HQET $(n^2 = 0)$

$\omega = n \cdot k$: light-cone projection of light-quark momentum

$$m_B f_B^{(\mathrm{HQET})} \, \phi_B^+(\omega) = \int \frac{d\tau}{2\pi} \, e^{i\omega\tau} \, \langle 0 | \, \bar{q}(\tau n) \, [\tau n, 0] \, \not{n} \gamma_5 \, h_v^{(b)}(0) \, | \bar{B}(m_B v) \rangle$$

[there is another Dirac structure, leading to another LCDA, notes as $\phi_B^-(\omega)$]



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Eigenfunctions of LN Kernel

[Bell/TF/Wang/Yip '13]

RGE for $\phi_B^+(\omega,\mu)$ as Integro-Differential Equation: [Lange/Neubert]

$$\frac{d\phi_B^+(\omega,\mu)}{d\ln\mu} = -\left[\Gamma_{\rm cusp}\,\ln\frac{\mu}{\omega}+\gamma_+\right]\phi_B^+(\omega,\mu) - \omega\int\limits_0^\infty d\eta\,\Gamma(\omega,\eta)\,\phi_B^+(\eta,\mu)$$

(Relatively) complicated solution as convolution integral with hypergeometric functions

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(Relatively) complicated solution as convolution integral with hypergeometric functions

Identify Continuous Set of Eigenfunctions for (1-loop) LN-Kernel:

$$f_{\omega'}(\omega) \equiv \sqrt{\frac{\omega}{\omega'}}\,J_1\left(2\sqrt{\frac{\omega}{\omega'}}\right) \qquad \text{with Eigenvalues: } \gamma_{\omega'} = -\left(\Gamma_{\rm cusp}\,\ln\frac{\mu}{\hat\omega'}+\gamma_+\right)$$

$J_1(z)$: Bessel function, $\hat\omega' \equiv \omega'\,e^{-2\gamma_E}$

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Eigenfunctions can be understood as momentum-space representation of the eigenvectors of a generator of collinear conformal transformations:

$$S_+ = \tau^2 \partial_\tau + 2j\,\tau$$

(conformal spin of light quark: $j = 1$)

[Braun/Manashov 1402.5822]

“Dual” Representation of B-meson LCDA

$$f_{\omega'}(\omega) \equiv \sqrt{\frac{\omega}{\omega'}}\, J_1\left(2\sqrt{\frac{\omega}{\omega'}}\right)$$ with Eigenvalues: $$\gamma_{\omega'} = -\left(\Gamma_{\rm cusp}\, \ln \frac{\mu}{\hat\omega'} + \gamma_+\right)$$

→ LCDA from convolution with Dual (spectral) Function

$$\phi_B^+(\omega) = \int_0^\infty \frac{d\omega'}{\omega'}\, f_{\omega'}(\omega)\, \rho_B^+(\omega') \quad \Leftrightarrow \quad \rho_B^+(\omega') = \int_0^\infty \frac{d\omega}{\omega}\, f_{\omega'}(\omega)\, \phi_B^+(\omega)$$

- Example 1: Exponential Model $(\omega_0 \sim \mathcal{O}(\Lambda_{\rm QCD}))$

$$\phi_B^+(\omega) = \frac{\omega}{\omega_0^2}\, \exp\left(-\frac{\omega}{\omega_0}\right) \quad \leftrightarrow \quad \rho_B^+(\omega') = \frac{1}{\omega'}\, \exp\left(-\frac{\omega_0}{\omega'}\right)$$

- Example 2: Free Parton Model $(\bar\Lambda = M_B - m_b)$

$$\phi_B^+(\omega) = \frac{\omega}{2\bar\Lambda^2}\, \theta(2\bar\Lambda - \omega) \quad \leftrightarrow \quad \rho_B^+(\omega') = \frac{1}{\bar\Lambda}\, J_2\left(2\sqrt{\frac{2\bar\Lambda}{\omega'}}\right)$$

Detour: “Wandzura-Wilczek Wave Functions”

[Bell/TF/Wang/Yip ’13]

General momentum-space representation for 2-particle Fock state:

$$\propto \; f_B \left[(1+\not{v})\,\not{k}\,\gamma_5\right] \cdot \psi_B(x \equiv 2\,v\cdot k = 2E_{\rm light})$$

- Heavy-Quark Limit: Two independent 2-particle LCDAs

$$\Rightarrow \quad \phi_B^+(\omega) = \omega \int_\omega^\infty dx\,\psi_B(x)\,, \qquad \phi_B^-(\omega) = \int_\omega^\infty dx\,(x-\omega)\,\psi_B(x)$$

- Wandzura-Wilczek relations, if 3-particle LCDAs are neglected

$$\phi_B^+(\omega,\mu) = -\omega\,\frac{d\phi_B^-(\omega,\mu)}{d\omega}$$

- Corrections from 3-particle LCDAs can be systematically included . . .

Comparison: Dual representation for $\phi_B^\pm(\omega)$ and $\psi_B(x)$

$$\phi_B^-(\omega,\mu) = \int_0^\infty \frac{d\omega'}{\omega'}\, J_0\left(2\sqrt{\frac{\omega}{\omega'}}\right)\, \rho_B^+(\omega',\mu)\,,$$

$$\phi_B^+(\omega,\mu) = \int_0^\infty \frac{d\omega'}{\omega'}\, \sqrt{\frac{\omega}{\omega'}}\, J_1\left(2\sqrt{\frac{\omega}{\omega'}}\right)\, \rho_B^+(\omega',\mu)\,,$$

$$\psi_B(x,\mu) = \frac{1}{x}\int_0^\infty \frac{d\omega'}{\omega'}\, \frac{1}{\omega'}\, J_2\left(2\sqrt{\frac{x}{\omega'}}\right)\, \rho_B^+(\omega',\mu)$$

$\rightarrow$ Wave Function Models can be directly translated to $\rho_B^+(\omega')$ $\surd$

- exponential model: $\psi_B(x) = \frac{1}{\omega_0^3}\, e^{-x/\omega_0}$
- free parton model: $\psi_B(x) = \frac{1}{2\bar\Lambda^2}\,\delta(x-2\bar\Lambda)$

RGE in "Dual Space"

Properties:

RGE local in ω':
$$\frac{d\rho_B^+(\omega',\mu)}{d\ln\mu} = \gamma_{\omega'}\,\rho_B^+(\omega',\mu)\,,$$

Simple solution:
$$\rho_B^+(\omega',\mu) = U_{\omega'}(\mu,\mu_0)\,\rho_B^+(\omega',\mu_0) \qquad \surd$$

The RG factor is known from SCET [see e.g. Neubert et al. 2004]

$$U_{\omega'}(\mu,\mu_0) \quad = \quad e^{V(\mu,\mu_0)}\left(\frac{\mu_0}{\hat\omega'}\right)^{-g(\mu,\mu_0)}$$

with

$$g(\mu,\mu_0) \quad = \int\limits_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha)}\,\Gamma_{\rm cusp}(\alpha)\,, \qquad V(\mu,\mu_0) = -\int\limits_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha)}\left[\gamma_+(\alpha) + \Gamma_{\rm cusp}(\alpha)\int\limits_{\alpha_s(\mu_0)}^{\alpha}\frac{d\alpha'}{\beta(\alpha')}\right]$$

Example: Factorization in $B \to \gamma \ell \nu$ (@large recoil)

Leading $B \to \gamma$ Form Factor factorizes in **Hard**, **Jet** and **Soft** Dynamics:

$$F^{B\to\gamma}(E_\gamma) = \left[e^{V_h(\mu,\mu_h)} \left(\frac{\mu_h}{2E_\gamma}\right)^{-g(\mu,\mu_h)} H(E_\gamma,\mu_h)\right]$$
$$\times \int_0^\infty \frac{d\omega'}{\omega'} \left[e^{-2V_{\rm hc}(\mu,\mu_{\rm hc})} \left(\frac{\mu_{\rm hc}^2}{2E_\gamma\hat\omega'}\right)^{g(\mu,\mu_{\rm hc})} j(2E_\gamma\hat\omega',\, \mu_{\rm hc})\right]$$
$$\times \left[e^{V(\mu,\mu_0)} \left(\frac{\mu_0}{\hat\omega'}\right)^{-g(\mu,\mu_0)} \rho_B^+(\omega',\mu_0)\right]$$

- In dual space, all RG factors are multiplicative!
  (i.e. jet function j in dual space obeys simple RGE, too)

Relations between Logarithmic Moments

- Jet Function has perturbative expansion in α_s and $\ln\omega'$
- Need Logarithmic Moments of $\rho_B^+(\omega', \mu_{\rm hc})$.

$$L_n(\mu) \equiv \int_0^\infty \frac{d\omega'}{\omega'} \ln^n\left(\frac{\hat{\omega}'}{\mu}\right) \rho_B^+(\omega', \mu)$$

[for $n = 0, 1, 2$ identical to logarithmic moments of $\phi_B^+(\omega)$]

RGE: $$\frac{dL_n(\mu)}{d\ln\mu} = \Gamma_{\rm cusp}(\alpha_s)\, L_{n+1}(\mu) - \gamma_+(\alpha_s) L_n(\mu) - n\, L_{n-1}(\mu)$$

Formal solution: $$L_n(\mu) = e^V \sum_{m=0}^\infty \frac{g^m}{m!} \sum_{j=0}^n \frac{n!}{(n-j)!\, j!} \ln^{n-j}\left(\frac{\mu_0}{\mu}\right) L_{m+j}(\mu_0)$$

(requires truncation — or consider $\rho_B^+(\omega')$ as generating function)

OPE Constraints and Modelling

Small and Large Values of ω' now **clearly separated** in Dual Function **!**

Small values of ω':

- Simple models / parametrizations
- Non-perturbative methods (QCD sum rules, [Lattice??])
- Phenomenological constraints using Factorization

Large values of ω':

- Perturbative dynamics ($\rightarrow$ parton picture)
- Constraints from local OPE (HQET parameters)

OPE Constraints at Large Values of ω'

Consider Positive Moments of LCDA: [Lee/Neubert '05]

$$M_n(\Lambda_{\rm UV},\mu) := \int_0^{\Lambda_{\rm UV}} d\omega\, \omega^n\, \phi_B^+(\omega,\mu)$$

- UV cut-off required $\leftrightarrow$ no naive relation to HQET parameters ($\bar\Lambda, \ldots$) **!**
- For $\boxed{\mu \sim \Lambda_{\rm UV} \gg \Lambda_{\rm hadr.}}$ perturbatively calculable (incl. power corr.)

In terms of dual function:

$$M_0(\Lambda_{\rm UV},\mu) = \Lambda_{\rm UV} \int_0^\infty \frac{d\omega'}{\omega'}\, J_2\left(2\sqrt{\frac{\Lambda_{\rm UV}}{\omega'}}\right) \rho_B^+(\omega')\,,$$

$$M_1(\Lambda_{\rm UV},\mu) = \frac{2\Lambda_{\rm UV}}{3}\, M_0(\Lambda_{\rm UV},\mu) - \frac{\Lambda_{\rm UV}^2}{3} \int_0^\infty \frac{d\omega'}{\omega'}\, J_4\left(2\sqrt{\frac{\Lambda_{\rm UV}}{\omega'}}\right) \rho_B^+(\omega')\,.$$

Fixed-order Matching for $\rho_B^+(\omega')$:

$\rightarrow$ Perturbative Expansion for $\rho_B^+(\omega', \mu)$ if $\boxed{\omega' \sim \mu \gg \bar\Lambda \equiv m_B - m_b}$

$$\rho_B^+(\omega')_{\text{pert.}} = C_0 \, \frac{1}{\bar\Lambda} \, J_2\left(2\sqrt{\frac{2\bar\Lambda}{\omega'}}\right) + (C_0 - C_1)\,\frac{4}{\bar\Lambda}\,J_4\left(2\sqrt{\frac{2\bar\Lambda}{\omega'}}\right) + \ldots$$

Model-independent Prediction: $(L = \ln \mu/\hat\omega')$

$$C_0 = 1 + \frac{\alpha_s C_F}{4\pi}\left(-2L^2 + 2L - 2 - \frac{\pi^2}{12}\right) + \mathcal{O}(\alpha_s^2)\,,$$

$$C_1 = 1 + \frac{\alpha_s C_F}{4\pi}\left(-2L^2 + 2L + \frac{5}{4} - \frac{\pi^2}{12}\right) + \mathcal{O}(\alpha_s^2)$$

$\rightarrow$ reduces to the free parton result for $\alpha_s \to 0$ $\surd$

... further power corrections in $\bar\Lambda/\Lambda_{\rm UV}$, $\lambda_{1,2}/\Lambda_{\rm UV}^2$ etc.

“(Local) RG Improvement”

- Coefficients $C_i(L)$ fulfill the same RGE as $\rho_B^+(\omega')$.
- If ω' is large, RGE can be used to resum large logs $|L| \gg 1$

Technically, this can be achieved by defining auxiliary scale

$$\mu_{\omega'} \equiv \mu_{\omega'}(\mu) := \sqrt{(k\hat{\omega}')^2 + \mu^2}\,, \qquad k \sim 1$$

with the RG-improved perturbative result for the dual function:

$$\rho_B^+(\omega', \mu)_{\rm RG} = U_{\omega'}(\mu, \mu_{\omega'})\, \rho_B^+(\omega', \mu_{\omega'})_{\rm pert} \\ \stackrel{\hat{\omega}' \gtrsim \mu}{\longrightarrow} \; e^{V(\mu,\hat{\omega}')} \rho_B^+(\omega', \mu = \hat{\omega}')_{\rm pert.}\,.$$

$\Rightarrow$ for large ω' dual function falls of *faster* than $1/\omega'$

$$\rho_B^+(\omega', \mu) \sim (\omega')^{-1-g(\mu_{\omega'},\mu)} \qquad \text{with } \; g(\mu'_\omega, \mu) > 0 \; \text{ for } \; \hat{\omega}' \gg \mu$$

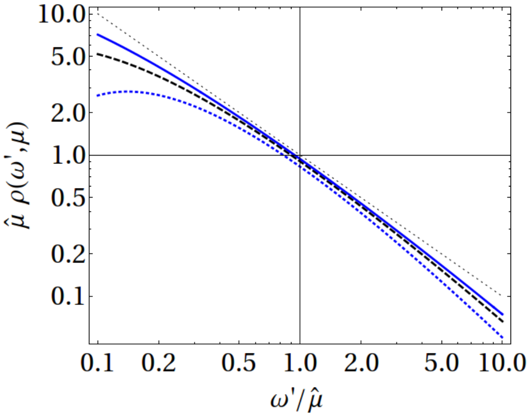
Interpolating between large and small values of ω'

Phenomenological procedure:

- Start from a given model for ϕ_B or ρ_B defined at reference scale μ_0,
- Consider $1/\omega'$ expansion and adjust to perturbative result,
  (requires to define an auxiliary parameter Ω where transition occurs)
- Implement local RG improvement.

(details for a concrete realization can be found in our paper)

Interpolating between large and small values of ω'



log-log plot:

.... asymptotic $\mu/\hat{\omega}'$ behaviour

— $\mu = 10$ GeV

- - $\mu = 3$ GeV

- - $\mu = 1$ GeV

for exp. model at $\mu_0 = 1$ GeV

$\Omega := e^{\gamma_E}\,\mu_0$

$\longleftarrow$ non-perturbative $\longleftarrow$ $\longrightarrow$ perturbative $\longrightarrow$

Consequences for Logarithmic Moments

Contributions from large ω' $(\hat{\omega}' \geq \mu)$

Completely determined perturbatively:

$$
\begin{aligned}
L_k^+(\mu) &\equiv \int_\mu^\infty \frac{d\hat{\omega}'}{\hat{\omega}'} \ln^k\left(\frac{\hat{\omega}'}{\mu}\right) \rho_B^+(\omega',\mu) \\
&\simeq \int_\mu^\infty \frac{d\hat{\omega}'}{\hat{\omega}'} \ln^k\left(\frac{\hat{\omega}'}{\mu}\right) e^{V(\mu,\hat{\omega}')} \rho_B^+(\omega',\mu=\hat{\omega}')_{\rm pert.}
\end{aligned}
$$

i.e. L_k^+ contain the information on the HQET parameters $\bar{\Lambda}$ etc.

Consequences for Logarithmic Moments

Contributions from small ω' $(\hat{\omega}' < \mu$ — model-dependent)

- Substitute $z \equiv -\ln \frac{\hat{\omega}'}{\mu}$

$$L_k^-(\mu) \equiv \int_0^\mu \frac{d\hat{\omega}'}{\hat{\omega}'} \ln^k\left(\frac{\hat{\omega}'}{\mu}\right) \rho_B^+(\omega', \mu) = \int_0^\infty dz\, (-z)^k\, \rho_B^+(\hat{\mu}\, e^{-z}, \mu)$$

- expand in terms of Laguerre polynomials $\mathrm{L}_n(z)$,

$$\rho_B^+(\hat{\mu}\, e^{-z}, \mu) := \sum_{n=0}^\infty a_n(\mu)\, e^{-z}\, \mathrm{L}_n(z)\,.$$

$$L_0^-(\mu) = a_0(\mu)\,,$$
$$L_1^-(\mu) = a_1(\mu) - a_0(\mu)\,,$$
$$L_2^-(\mu) = 2a_2(\mu) - 4a_1(\mu) + 2a_0(\mu)\,, \qquad \text{etc.}$$

Consequences for Logarithmic Moments

Numerical illustration for $\mu = 3$ GeV:

L_n	total	from $\hat{\omega}' < \mu$	from $\hat{\omega}' \geq \mu$
L_0 (model 1)	1.67	1.58	0.086
L_0 (model 2)	1.65	1.57	0.086
L_1 (model 1)	-3.85	-3.93	0.074
L_1 (model 2)	-3.46	-3.54	0.074
L_2 (model 1)	11.6	11.4	0.121
L_2 (model 2)	9.03	8.91	0.121

- Model 1: exponential with $\omega_0 = 438$ MeV at $\mu_0 = 1$ GeV
- Model 2: parton model with $\bar{\Lambda} = 465$ MeV at $\mu_0 = 1$ GeV

Log. moments dominated by small values of ω'
$\Rightarrow$ L_k are practically independent of HQET parameters

LCDAs for Heavy Baryons

. . . some general remarks . . .

- Λ_b decays less well studied experimentally . . .
  . . . but LHC will contribute . . .
- 2 light quarks coupled to static b-quark:
  – more independent Dirac structures
  – more complicated RG-evolution: LN + ERBL kernel [Ball/Braun/Gardi]
- QCD factorization in Λ_b decays more involved [W. Wang, TF/Yip]
- . . .

- **Dual representation** for "twist-2" LCDA: ($\omega_{1,2} =$ l.c.momenta of light quarks)

$$\phi_2(\omega_1, \omega_2) = \int_0^\infty \frac{d\omega_1'}{\omega_1'} \int_0^\infty \frac{d\omega_2'}{\omega_2'} \sqrt{\frac{\omega_1 \omega_2}{\omega_1' \omega_2'}} J_1\left(2\sqrt{\frac{\omega_1}{\omega_1'}}\right) J_1\left(2\sqrt{\frac{\omega_2}{\omega_2'}}\right) \rho_2(\omega_1', \omega_2')$$

- **Reduced variables:** $\omega_r' \equiv \frac{\omega_1'\omega_2'}{\omega_1'+\omega_2'}$ and $u' = 1 - \bar{u}' = \frac{\omega_1'}{\omega_1'+\omega_2'}$
- Expanding in **Gegenbauer polynomials**,

$$\hat{\rho}_2(\omega_r', u') := \sum_{n=0,2,4,\ldots}^{\infty} u' \bar{u}' \, f_n(\omega_r') \, C_n^{(3/2)}(2u' - 1) \,,$$

the coefficients satisfy a simple RGE of the form:

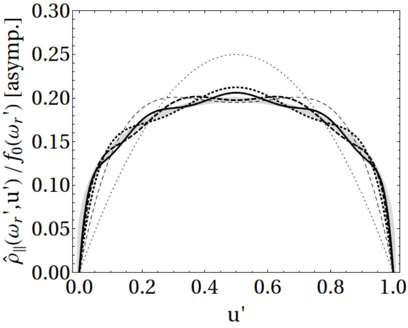
$$\frac{df_n(\omega_r')}{d \ln \mu} = -\left[\Gamma_{\rm cusp}(\alpha_s) \ln \frac{\mu}{\hat{\omega}_r'} \delta_{nm} + \gamma_{nm}(\alpha_s)\right] f_m(\omega_r')$$

- Can be diagonalized by truncating Gegenbauer expansion $\surd$

- Asymptotically, for $\mu \gg \mu_0$, the dependence on dual momentum fraction u' (approximately) approaches functional form

$$\hat{\rho}_2(\omega_r', u') \overset{\mu\to\infty}{\propto} f_0(\omega_r', \mu)\, (u'\bar{u}')^{\sim 1/3}$$

- Dependence on reduced momentum ω_r' factorizes in that limit.



different levels of Gegenbauer truncation:

- $n = 0$ (thin dotted)
- $n = 2$ (thin dashed)
- $n = 4$ (thick dotted)
- $n = 6$ (thick dashed)
- $n = 8$ (solid)

gray band $\propto (u'(1-u'))^{1/3}$.

[see also Braun/Derkachov/Manashov '14]

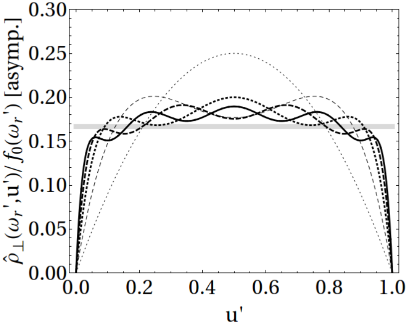
Asymptotic behaviour (Ω_b)

[work in progress]

- Asymptotically, for $\mu \gg \mu_0$, the dependence on dual momentum fraction u' approaches functional form

$$\hat{\rho}_\perp(\omega_r', u') \overset{\mu\to\infty}{\propto} f_0(\omega_r', \mu)\,\theta(u')\,\theta(1-u')$$

- Dependence on reduced momentum ω_r' factorizes in that limit.



different levels of Gegenbauer truncation:

- $n = 0$ (thin dotted)
- $n = 2$ (thin dashed)
- $n = 4$ (thick dotted)
- $n = 6$ (thick dashed)
- $n = 8$ (solid)

gray band $\propto \theta(u')\theta(1-u')$.

[see also Braun/Derkachov/Manashov '14]

Λ_b LCDAs from "WW Wave Functions" [Bell/TF/Wang/Yip]

Chiral-even and chiral-odd LCDAs can be constructed from

$$M^{(1)}(v,k_1,k_2) \;\propto\; \psi_s(x_1,x_2,K^2)\,\left[\not k_2\,\not k_1\right] h_v$$
$$M^{(2)}(v,k_1,k_2) \;\propto\; \psi_v(x_1,x_2,K^2)\,\left[\not k_2\,\not v\,\not k_1\right] h_v$$

with $x_i = 2\,v\cdot k_i$ and $K^2 = (k_1+k_2)^2$.

$\Rightarrow$ All Λ_b LCDAs from double integrals: [notation as in Ball/Braun/Gardi]

e.g. $$\phi_2(\omega_1,\omega_2) = \int_{\omega_1}^{\infty} dx_1 \int_{\omega_2}^{\infty} dx_2\,\omega_1\omega_2\,\psi_v(x_1,x_2)\,,$$
$$\phi_4(\omega_1,\omega_2) = \int_{\omega_1}^{\infty} dx_1 \int_{\omega_2}^{\infty} dx_2\,(x_1-\omega_1)(x_2-\omega_2)\,\psi_v(x_1,x_2) \quad \text{etc.}$$

(K^2-dependence neglected, for simplicity)

- Wandzura-Wilczek relations for Λ_b LCDAs $\surd$

Summary / Main Messages

- Conceptually:
  RG evolution / factorization theorems simplify, when expressed in terms of "dual" LCDAs (i.e. eigenfunctions of LN kernel)

- Numerically:
  Inverse (log) moments essentially independent of HQET par's;
  – crucial for spectator effects in exclusive B-decays;
  – λ_B ultimately to be determined by experiment (!)

- Approximately:
  Wandzura-Wilczek relations from wave function picture;
  (particularly useful for modelling heavy baryon LCDAs).

Direct estimates for dual function / log moments from lattice or sum rules **?**

Backup Slides

Lange-Neubert Solution of RGE

Standard procedure: ("continous Mellin moments" / "logarithmic F.T.")

introduce: $$\varphi_B^+(\theta,\mu) = \int_0^\infty \frac{d\omega}{\omega} \left(\frac{\omega}{\mu}\right)^{-i\theta} \phi_B^+(\omega,\mu)$$

Explicit solution in θ-space: [Lange/Neubert]

$$\varphi_B^+(\theta,\mu) = e^{V-2\gamma_E g} \left(\frac{\mu}{\mu_0}\right)^{i\theta} \frac{\Gamma(1-i\theta)\Gamma(1+i\theta-g)}{\Gamma(1+i\theta)\Gamma(1-i\theta+g)} \varphi_B^+(\theta+ig,\mu_0)$$

with RG functions $V = V(\mu,\mu_0)$ and $g = g(\mu,\mu_0)$ given in pert. theory.

...staring at the solution for $\varphi_B^+(\theta,\mu)$ ...

Def. $$\varphi_B^+(\theta,\mu) := \frac{\Gamma(1-i\theta)}{\Gamma(1+i\theta)} \int_0^\infty \frac{d\omega'}{\omega'} \rho_B^+(\omega',\mu) \left(\frac{\mu}{\omega'}\right)^{i\theta}$$