Holomorphy without Supersymmetry in the SMEFT

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Talk based on:

Rodrigo Alonso, Elizabeth Jenkins, Michael Trott, AM

- C. Grojean, E.E. Jenkins, A.V. Manohar and M. Trott, *Renormalization Group Scaling of Higgs Operators and $h \to \gamma\gamma$ Decay*, JHEP 1304:016 (2013).
- E.E. Jenkins, A.V. Manohar and M. Trott, *On Gauge Invariance and Minimal Coupling*, JHEP 1309:063 (2013).
- E.E. Jenkins, A.V. Manohar and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators I: Formalism and λ Dependence*, JHEP 1310:087 (2013).
- E.E. Jenkins, A.V. Manohar and M. Trott, *Naive Dimensional Analysis Counting of Gauge Theory Amplitudes and Anomalous Dimensions*, Phys. Lett. B726 (2013) 697-702.
- E.E. Jenkins, A.V. Manohar and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators II: Yukawa Dependence*, JHEP 1401:035 (2014).
- R. Alonso, E.E. Jenkins, A.V. Manohar and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators III: Gauge Coupling Dependence and Phenomenology*, JHEP 1404:159 (2014).
- R. Alonso, E.E. Jenkins and A.V. Manohar, *Holomorphy without Supersymmetry in the Standard Model Effective Field Theory*, arXiV:1409.0868 [hep-ph].

Related Work

- J. Elias Miro, J.R. Espinosa, E. Masso, A. Pomarol
  Higgs windows to new physics through d=6 operators: constraints and one-loop anomalous dimensions
  JHEP 1311:066 (2013)

- J. Elias Miro, J.R. Espinosa, E. Masso, A. Pomarol
  Renormalization of dimension-six operators relevant for the Higgs decays $h \to \gamma\gamma, \gamma Z$
  JHEP 1308:033 (2013)

Generalize SM to SMEFT

Fields are three generations of fermions

$$\text{L} : q_r,\ l_r, \qquad \text{R} : u_r,\ d_r,\ e_r \qquad r = 1, \ldots, n_g = 3$$

the scalar doublet H, and $SU(3) \times SU(2) \times U(1)$ gauge fields.

$$L = L_{SM} + \sum \frac{1}{\Lambda^n} L^{(4+n)} = L_{SM} + \frac{1}{\Lambda^2} L^{(6)} + \ldots \quad \textit{note the dots}$$

Λ is the scale of new physics, and assume $\Lambda > v$

Power Counting

$$L^{(6)} \sim \frac{m_H^2}{\Lambda^2} \qquad L^{(8)} \sim \frac{m_H^4}{\Lambda^4} \qquad L^{(6)} \times L^{(6)} \sim \frac{m_H^4}{\Lambda^4}$$

H breaks electroweak symmetry

Baryon and Lepton Number Violation

Dimension five operator:

$$\frac{1}{\Lambda_5}(H\,l)(H\,l)$$

$\Delta L = 2$ operator which gives neutrino masses.

Not relevant for 1 TeV LHC processes.

Similarly, baryon number violating operators can be dropped.

SMEFT Operators

- Leading higher dimension operators are $d = 6$.
- Assuming B and L conservation, there are 59 independent dimension-six operators which form complete basis of $d = 6$ operators.
- 59 operators divided into eight operator classes.

1 : X^3	2 : H^6	3 : $H^4 D^2$	4 : $X^2 H^2$
5 : $\psi^2 H^3$	6 : $\psi^2 XH$	7 : $\psi^2 H^2 D$	8 : ψ^4

$$X = G^A_{\mu\nu}, W^I_{\mu\nu}, B_{\mu\nu} \qquad \psi = q, l, u, d, e$$

Buchmuller & Wyler 1986

Grzadkowski, Iskrzynski, Misiak and Rosiek 2010

Dimension Six Operators

1 : X^3	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$Q_{\widetilde{W}}$	$\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$

2 : H^6	
Q_H	$(H^\dagger H)^3$

3 : $H^4 D^2$	
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$\left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right)$

5 : $\psi^2 H^3 + \text{h.c.}$	
Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$

4 : $X^2 H^2$	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$
$Q_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$
Q_{HWB}	$H^\dagger \tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$
$Q_{H\widetilde{W}B}$	$H^\dagger \tau^I H\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$

6 : $\psi^2 X H + \text{h.c.}$	
Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I H W_{\mu\nu}^I$
Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$
Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r)\widetilde{H}\, G_{\mu\nu}^A$
Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\tau^I \widetilde{H}\, W_{\mu\nu}^I$
Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\widetilde{H}\, B_{\mu\nu}$
Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H\, G_{\mu\nu}^A$
Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r)\tau^I H\, W_{\mu\nu}^I$
Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$

7 : $\psi^2 H^2 D$	
$Q_{Hl}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar{l}_p \tau^I \gamma^\mu l_r)$
Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar{q}_p \tau^I \gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$

Dimension Six Operators

$8 : (\bar{L}L)(\bar{L}L)$		$8 : (\bar{R}R)(\bar{R}R)$		$8 : (\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$8 : (\bar{L}R)(\bar{R}L) + \text{h.c.}$		$8 : (\bar{L}R)(\bar{L}R) + \text{h.c.}$	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_{tj})$	$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)$
		$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)$
		$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)$
		$Q_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu} u_t)$

Buchmuller & Wyler 1986

Grzadkowski, Iskrzynski, Misiak and Rosiek 2010

$$\psi^4 \to JJ,\ (\bar{L}R)(\bar{L}R),\ (\bar{L}R)(\bar{R}L)$$

Field redefinitions (equations of motion) used to eliminate operators.

59 baryon number conserving operators, not including flavor indices.

2499 independent coefficients: 1350 *CP*-even and 1149 *CP*-odd including flavor indices

156 different irreducible flavor representations: $\otimes_{q,l,u,d,e} SU(n_g)$

$$Q_{\substack{He\\pr}} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$$

under $SU(n_g)_e$ has both singlet and adjoint pieces,

$$\left(\frac{1}{n_g} Q_{\substack{He\\ss}} \delta_{pr}\right) + \left(Q_{\substack{He\\pr}} - \frac{1}{n_g} Q_{\substack{He\\ss}} \delta_{pr}\right)$$

Four-quark operators have more complicated representations.

Equations of Motion

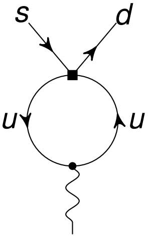
Used to eliminate operators with derivatives:

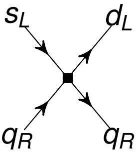
$$D^\mu F_{\mu\nu} = g\, j_\nu$$

$$g\, \overline{d}\gamma^\mu T^A P_L s\, D^\mu F^A_{\mu\nu} \to g^2\, \overline{d}\gamma^\mu T^A P_L s\;\; \overline{q}\gamma^\mu T^A q$$

Penguin operators give LL and LR operators; no 1PI diagram for LR

Gaillard and Lee; Gilman and Wise





Eliminate $v \cdot D$ terms in HQET.

SM Equations of Motion

$$D^2 H_k - \lambda v^2 H_k + 2\lambda (H^\dagger H) H_k + \overline{q}^j\, Y_u^\dagger\, u\, \epsilon_{jk} + \overline{d}\, Y_d\, q_k + \overline{e}\, Y_e\, l_k = 0\,,$$

for the Higgs field (mix H and fermions)

$$\begin{aligned} i\not{D}\, q_j &= Y_u^\dagger\, u\, \widetilde{H}_j + Y_d^\dagger\, d\, H_j\,, & i\not{D}\, d &= Y_d\, q_j\, H^{\dagger\, j}\,, & i\not{D}\, u &= Y_u\, q_j\, \widetilde{H}^{\dagger\, j}\,, \\ i\not{D}\, l_j &= Y_e^\dagger\, e H_j\,, & i\not{D}\, e &= Y_e\, l_j H^{\dagger\, j}\,, \end{aligned}$$

for the fermion fields, and

$$[D^\alpha, G_{\alpha\beta}]^A = g_3\, j_\beta^A, \qquad [D^\alpha, W_{\alpha\beta}]^I = g_2\, j_\beta^I, \qquad D^\alpha B_{\alpha\beta} = g_1\, j_\beta,$$

for the gauge fields, where $[D^\alpha, F_{\alpha\beta}]$ is the covariant derivative in the adjoint representation.

Power Counting for the RGE

Amplitudes and anomalous dimensions obey power counting:

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C^{(6)} \propto C^{(6)}$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C^{(8)} \propto C^{(8)} + \left[C^{(6)}\right]^2$$

In the SM, because of the dimension two operator $H^\dagger H$, have

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C^{(4)} \propto C^{(4)} + m_H^2 C^{(6)} + \ldots$$

equivalently $m_H^2 \to v^2$

$\star$ SM parameter RG evolution affected.

Anomalous Dimension Matrix

		g^3X^3	H^6	H^4D^2	$g^2X^2H^2$	$y\psi^2H^3$	$gy\psi^2XH$	ψ^2H^2D	ψ^4
		1	2	3	4	5	6	7	8
g^3X^3	1	g^2	0	0	1	0	0	0	0
H^6	2	$g^6\lambda$	λ, g^2, y^2	$g^4, g^2\lambda, \lambda^2$	$g^6, g^4\lambda$	$\lambda y^2, y^4$	0	$\lambda y^2, y^4$	0
H^4D^2	3	g^6	0	g^2, λ, y^2	g^4	y^2	g^2y^2	g^2, y^2	0
$g^2X^2H^2$	4	g^4	0	1	g^2, λ, y^2	0	y^2	1	0
$y\psi^2H^3$	5	g^6	0	g^2, λ, y^2	g^4	g^2, λ, y^2	$g^2\lambda, g^4, g^2y^2$	g^2, λ, y^2	λ, y^2
$gy\psi^2XH$	6	g^4	0	0	g^2	1	g^2, y^2	1	1
ψ^2H^2D	7	g^6	0	g^2, y^2	g^4	y^2	g^2y^2	g^2, λ, y^2	y^2
ψ^4	8	g^6	0	0	0	0	g^2y^2	g^2, y^2	g^2, y^2

Structure of anomalous dimension matrix. Jenkins, Trott, AM: 1309.0819

Many entries exist because of EOM

Naive Dimensional Analysis

Georgi, AM: NPB234 (1984) 189

Jenkins, Trott, AM: 1309.0819

Related recent work by Buchalla et al. arXiv:1312.5624

$$L = f^2\Lambda^2 \left(\frac{\psi}{f\sqrt{\Lambda}}\right)^a \left(\frac{H}{f}\right)^b \left(\frac{yH}{\Lambda}\right)^c \left(\frac{D}{\Lambda}\right)^d \left(\frac{gX}{\Lambda^2}\right)^e, \qquad \Lambda = 4\pi f$$

NDA weight $w \equiv$ powers of f^2 in denominator.

$$L = f^2\Lambda^2 \left(\frac{H}{f}\right)^6 = \Lambda^2 \frac{(H^\dagger H)^3}{f^4}, \qquad w = 2$$

$$\gamma_{ij} \propto \left(\frac{\lambda}{16\pi^2}\right)^{n_\lambda} \left(\frac{y^2}{16\pi^2}\right)^{n_y} \left(\frac{g^2}{16\pi^2}\right)^{n_g}, \qquad N = n_\lambda + n_y + n_g$$

$$N = 1 + w_i - w_j$$

Familiar Example: $b \to s\gamma$

Use

$$O_q = \overline{b}\gamma^\mu P_L u \, \overline{u}\gamma_\mu P_L s$$
$$O_g = \frac{g}{16\pi^2} m_b \, \overline{b}\sigma^{\mu\nu} G_{\mu\nu} P_L s$$

Then

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \begin{bmatrix} C_q \\ C_g \end{bmatrix} = \begin{bmatrix} L & L+1 \\ L-1 & L \end{bmatrix} \begin{bmatrix} C_q \\ C_g \end{bmatrix}$$

where L is the number of loops of the diagram.

To get all terms to order $g^2/(16\pi^2)$, need γ_{gq} at two loops.

Full 2499×2499 anomalous dimension matrix computed.

Alonso, Jenkins, Trott, AM:

JHEP **1310** (2013) 087, JHEP **1401** (2014) 035, JHEP **1404** (2014) 159

Group theory can be quite complicated, have to Fierz into standard form.

There are some big numbers:

The evolution of the H^6 coefficient is

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C_H = \frac{1}{16\pi^2} \left[108\,\lambda\, C_H - 160\,\lambda^2\, C_{H\Box} + 48\,\lambda^2\, C_{HD} \right] + \ldots$$

Independent of normalization of C_H.

For $m_H \sim 126$ GeV, $108\,\lambda/(16\pi^2) \approx 0.1$.

RGE

- Computed the one-loop anomalous dimensions of the full set of operators.
- Interesting mixing betweeen different flavor sectors
- Test of MFV since MFV violation in one sector feeds into the other sectors

Magnetic Dipole Operator

$$\mathscr{C}_{\substack{e\gamma\\rs}} = \frac{1}{g_1} C_{\substack{eB\\rs}} - \frac{1}{g_2} C_{\substack{eW\\rs}} \qquad \mathcal{L} = \frac{ev}{\sqrt{2}} \mathscr{C}_{\substack{e\gamma\\rs}}\, \overline{e}_r \sigma^{\mu\nu} P_R e_s\, F_{\mu\nu} + h.c.$$

where r and s are flavor indices ($\{e_e\,,\ e_\mu\,,\ e_\tau\} \equiv \{e\,,\ \mu\,,\ \tau\}$) and

$$\begin{aligned}
\dot{\mathscr{C}}_{\substack{e\gamma\\rs}} = & \left\{ Y(s) + e^2 \left(12 - \frac{9}{4}\csc^2\theta_W + \frac{1}{4}\sec^2\theta_W \right) \right\} \mathscr{C}_{\substack{e\gamma\\rs}} \\
& + 2\mathscr{C}_{\substack{e\gamma\\rv}} [Y_e Y_e^\dagger]_{vs} + \left(\frac{1}{2} + 2\cos^2\theta_W \right) [Y_e^\dagger Y_e]_{rw} \mathscr{C}_{\substack{e\gamma\\ws}} + e^2 \left(12 \cot 2\theta_W\right) \mathscr{C}_{\substack{eZ\\rs}} \\
& - \left(2\sin\theta_W\cos\theta_W\right) [Y_e^\dagger Y_e]_{rw} \mathscr{C}_{\substack{eZ\\ws}} - \cot\theta_W [Y_e^\dagger]_{rs} \left(C_{HWB} + i C_{H\widetilde{W}B} \right) \\
& + \frac{8}{3} e^2 [Y_e^\dagger]_{rs} \left(\mathscr{C}_{\gamma\gamma} + i\widetilde{\mathscr{C}}_{\gamma\gamma} \right) + e^2 \left(\cot\theta_W - \frac{5}{3}\tan\theta_W \right) [Y_e^\dagger]_{rs} \left(\mathscr{C}_{\gamma Z} + i\widetilde{\mathscr{C}}_{\gamma Z} \right) \\
& + 16 [Y_u]_{wv} C^{(3)}_{\substack{lequ\\rsvw}}
\end{aligned}$$

as corrected by Signer and Pruna, arXiv:1408:3565

Empirical Observation

$$\dot{\mathscr{C}}_{e\gamma} \propto \left(\mathscr{C}_{\gamma\gamma} + i\widetilde{\mathscr{C}}_{\gamma\gamma}\right), \ \mathscr{C}_{e\gamma}$$

no

$$\left(\mathscr{C}_{\gamma\gamma} - i\widetilde{\mathscr{C}}_{\gamma\gamma}\right), \ \mathscr{C}^*_{e\gamma}$$

$$f(z) = z, \ \sin z, e^z, \qquad\qquad \not{z}^*, |z|^2 = zz^*$$

Find this after adding all graphs and using the EOM. Individual contributions are not holomorphic, and only the total respects holomorphy.

- Observation: 1-loop anomalous dimension matrix respects holomorphy to a large extent.
- Using EOM, so equivalent to computing (on-shell) S-matrix elements.

Holomorphy

Recall $d = 6$ operator classes

1 : X^3	2 : H^6	3 : $H^4 D^2$	4 : $X^2 H^2$
5 : $\psi^2 H^3$	6 : $\psi^2 X H$	7 : $\psi^2 H^2 D$	8 : ψ^4

Divide $d = 6$ Operators into Holomorphic, Antiholomorphic and Non-Holomorphic Operators

$$X^{\pm}_{\mu\nu} = \frac{1}{2}\left(X_{\mu\nu} \mp i\widetilde{X}_{\mu\nu}\right), \qquad \widetilde{X}^{\pm}_{\mu\nu} = \pm i X^{\pm}_{\mu\nu},$$

$$\widetilde{X}_{\mu\nu} \equiv \epsilon_{\mu\nu\alpha\beta} X^{\alpha\beta}/2 \qquad \widetilde{\widetilde{X}}_{\mu\nu} = -X_{\mu\nu}$$

$$X \to X^{\pm}$$

$$\psi \to L, R$$

Complex self-duality condition in Minkowski space.

Holomorphy

Definition

The holomorphic part of the Lagrangian, $\mathcal{L}_{\mathfrak{h}}$, is the Lagrangian constructed from the fields X^+, R, $\overline{L}$, but none of their hermitian conjugates.

These transform as $(0, \frac{1}{2})$ or $(0, 1)$ under the Lorentz group, i.e. only under the $SU(2)_R$ part of $SU(2)_L \times SU(2)_R$.

Holomorphy: Class 6

Magnetic moment operators:

$$6 : \psi^2 XH + \text{h.c.}$$

The $\sigma_{\mu\nu}$ matrices are self-dual:

$$\frac{i}{2}\epsilon^{\alpha\beta\mu\nu}\sigma_{\mu\nu}P_R = -\sigma^{\alpha\beta}P_R$$

$$Q_{RX} = \left(\overline{L}\sigma^{\mu\nu}R\right) X^{\mu\nu}\ H = \left(\overline{L}\sigma^{\mu\nu}R\right) X^{+\mu\nu}\ H,$$
$$Q_{RX}^\dagger = \left(\overline{R}\sigma^{\mu\nu}L\right) X^{\mu\nu}\ H = \left(\overline{R}\sigma^{\mu\nu}L\right) X^{-\mu\nu}\ H,$$

$$\mathcal{L}^{d=6} \supset C_{RX} \underbrace{Q_{RX}}_{\text{holomorphic}} + C_{RX}^* \underbrace{Q_{RX}^\dagger}_{\text{antiholomorphic}}$$

Holomorphy: Class 1

$$1 : X^3$$

$$Q_X = f^{ABC} X_\mu^{A\nu} X_\nu^{B\rho} X_\rho^{C\mu},$$
$$Q_{\widetilde{X}} = f^{ABC} \widetilde{X}_\mu^{A\nu} X_\nu^{B\rho} X_\rho^{C\mu},$$

$$Q_{X,+} \equiv \frac{1}{2}\left(Q_X - iQ_{\widetilde{X}}\right) = f^{ABC} X_\mu^{+A\nu} X_\nu^{+B\rho} X_\rho^{+C\mu}$$
$$Q_{X,-} \equiv \frac{1}{2}\left(Q_X + iQ_{\widetilde{X}}\right) = f^{ABC} X_\mu^{-A\nu} X_\nu^{-B\rho} X_\rho^{-C\mu}$$

$$\mathcal{L}^{d=6} \supset C_X Q_X + C_{\widetilde{X}} Q_{\widetilde{X}} = C_{X,+} \underbrace{Q_{X,+}}_{\text{holomorphic}} + C_{X,-} \underbrace{Q_{X,-}}_{\text{antiholomorphic}}$$

$$C_{X,\pm} \equiv \left(C_X \pm iC_{\widetilde{X}}\right)$$

Holomorphy: Class 4

$$4 : X^2 H^2$$

$$Q_{HX} = X_{\mu\nu} X^{\mu\nu} \; H^\dagger H,$$
$$Q_{H\widetilde{X}} = X_{\mu\nu} \widetilde{X}^{\mu\nu} \; H^\dagger H,$$

$$Q_{HX,+} \equiv X^{+2} \; H^\dagger H = \frac{1}{4} \left(X - i\widetilde{X} \right)^2 \; H^\dagger H$$
$$Q_{HX,-} \equiv X^{-2} \; H^\dagger H = \frac{1}{4} \left(X + i\widetilde{X} \right)^2 \; H^\dagger H$$

$$\mathcal{L}^{d=6} \supset C_{HX} Q_{HX} + C_{H\widetilde{X}} Q_{H\widetilde{X}} = C_{HX,+} \underbrace{Q_{HX,+}}_{\text{holomorphic}} + C_{HX,-} \underbrace{Q_{HX,-}}_{\text{antiholomorphic}}$$

$$C_{HX,\pm} \equiv \left(C_{HX} \pm i C_{H\widetilde{X}} \right)$$

Holomorphy: Class 8

$$\color{red}{8} : \psi^4 = (\bar{L}R)\,(\bar{L}R) + \text{h.c.}$$

$$Q^{(1)}_{quqd} = (\bar{q}^j_p u_r)\epsilon_{jk}(\bar{q}^k_s d_t)$$

$$\mathcal{L}^{d=6} \supset C_{LRLR} \underbrace{Q_{LRLR}}_{\text{holomorphic}} + C^*_{LRLR} \underbrace{Q^\dagger_{LRLR}}_{\text{antiholomorphic}}$$

The remaining operators are nonholomorphic, e.g. $(H^\dagger H)^3$.

Holomorphy

$$\mathcal{L}^{d=6} = \mathcal{L}_{\mathfrak{h}} + \mathcal{L}_{\overline{\mathfrak{h}}} + \mathcal{L}_{\mathfrak{n}} = C_{\mathfrak{h}} Q_{\mathfrak{h}} + C_{\overline{\mathfrak{h}}} Q_{\overline{\mathfrak{h}}} + C_{\mathfrak{n}} Q_{\mathfrak{n}}$$

$$Q_{\mathfrak{h}} \subset \left\{ X^{+3},\ X^{+2} H^2,\ (\overline{L} \sigma^{\mu\nu} R)\, X^+ H,\ (\overline{L} R)(\overline{L} R) \right\}$$

$$Q_{\overline{\mathfrak{h}}} \subset \left\{ X^{-3},\ X^{-2} H^2,\ (\overline{R} \sigma^{\mu\nu} L)\, X^- H,\ (\overline{R} L)(\overline{R} L) \right\}$$

$$Q_{\mathfrak{n}} \subset \left\{ H^6,\ H^4 D^2,\ \psi^2 H^3,\ \psi^2 H^2 D,\ (\overline{L} R)(\overline{R} L),\ JJ \right\}$$

Some of the $\mathfrak{n}$ operators are complex.

Holomorphy

$$\dot{C}_i \equiv 16\pi^2 \mu \frac{\mathrm{d}}{\mathrm{d}\mu} C_i = \sum_{j=\mathfrak{h},\bar{\mathfrak{h}},\mathfrak{n}} \gamma_{ij} C_j \ , \qquad i = \mathfrak{h}, \bar{\mathfrak{h}}, \mathfrak{n}$$

$$\begin{pmatrix} \gamma_{\mathfrak{h}\mathfrak{h}} & \gamma_{\mathfrak{h}\bar{\mathfrak{h}}} & \gamma_{\mathfrak{h}\mathfrak{n}} \\ \gamma_{\bar{\mathfrak{h}}\mathfrak{h}} & \gamma_{\overline{\mathfrak{h}\mathfrak{h}}} & \gamma_{\bar{\mathfrak{h}}\mathfrak{n}} \\ \gamma_{\mathfrak{n}\mathfrak{h}} & \gamma_{\mathfrak{n}\bar{\mathfrak{h}}} & \gamma_{\mathfrak{n}\mathfrak{n}} \end{pmatrix}$$

$$\gamma_{\mathfrak{h}\mathfrak{h}} = \gamma_{\overline{\mathfrak{h}\mathfrak{h}}} \qquad \gamma^*_{\mathfrak{h}\bar{\mathfrak{h}}} = \gamma_{\bar{\mathfrak{h}}\mathfrak{h}} \qquad \gamma^*_{\mathfrak{h}\mathfrak{n}} = \gamma_{\bar{\mathfrak{h}}\mathfrak{n}}$$

only need to look at:

$$\begin{pmatrix} \gamma_{\mathfrak{h}\mathfrak{h}}, \gamma_{\mathfrak{h}\bar{\mathfrak{h}}} & \gamma_{\mathfrak{h}\mathfrak{n}} \\ \gamma_{\mathfrak{n}\mathfrak{h}}, \gamma_{\mathfrak{n}\bar{\mathfrak{h}}} & \gamma_{\mathfrak{n}\mathfrak{n}} \end{pmatrix}$$

($\gamma_{\mathfrak{n}\bar{\mathfrak{h}}}$ included because some $\mathfrak{n}$ operators are complex)

- 0: Vanishes by NDA, i.e. NDA gives a negative loop order
- $\nexists$: There is no one-loop diagram (including from EOM)
- $\mathfrak{h}_F$: Holomorphic. Nonholomorphic terms forbidden by NDA and flavor symmetry
- $\to 0$: Vanishes by explicit computation, after adding all contributions. Individual graphs need not vanish.
- $\mathfrak{h}$: Holomorphic, by explicit computation
- $*$: Non-zero

$$Q_{eW} = (\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I H W^I_{\mu\nu} \qquad Q^{(1)}_{lequ} = (\bar{l}^j_p e_r) \epsilon_{jk} (\bar{q}^k_s u_t)$$

$$\dot{C}_{eW} \propto Y_e Y_e Y_u^\dagger C^{(1)*}_{lequ}$$

but NDA says only order y^2.

Holomorphy

	$(X^+)^3$	$(X^+)^2H^2$	ψ^2X^+H	$(\bar{L}R)(\bar{L}R)$	$(\bar{L}R)(\bar{R}L)$	JJ	ψ^2H^3	H^6	H^4D^2	ψ^2H^2D
$(X^+)^3$	$\mathfrak{h}$	$\to 0$	0	0	0	0	0	0	0	0
$(X^+)^2H^2$	$\mathfrak{h}$	$\mathfrak{h}$	$\mathfrak{h}$	0	0	$\nexists$	0	0	$\to 0$	$\to 0$
ψ^2X^+H	$\mathfrak{h}$	$\mathfrak{h}$	$\mathfrak{h}$	$\mathfrak{h}_F$	$\to 0$	$\to 0$	$\to 0$	0	$\nexists$	$\to 0$
$(\bar{L}R)(\bar{L}R)$	$\to 0$	$\nexists$	$\mathfrak{h}_F$	$\mathfrak{h}_F$	$Y_u^\dagger Y_{e,d}^\dagger$	$Y_u^\dagger Y_{e,d}^\dagger$	$\nexists$	$\nexists$	$\nexists$	$\to 0$
$(\bar{L}R)(\bar{R}L)$	$\to 0$	$\nexists$	$\to 0$	$Y_u Y_d,\ Y_u^\dagger Y_e^\dagger$	$\mathfrak{h}_F$	$\ast$	$\nexists$	$\nexists$	$\nexists$	$\to 0$
JJ	$\to 0$	$\nexists$	$\to 0$	$Y_u Y_{e,d}$	$\ast$	$\ast$	$\nexists$	$\nexists$	$\nexists$	$\ast$
ψ^2H^3	$\to 0$	$Y_{u,d,e}^\dagger$	$\mathfrak{h}$	$\mathfrak{h}$	$\ast$	$\ast$	$\ast$	$\nexists$	$\ast$	$\ast$
H^6	$\to 0$	$\ast$	$\nexists$	$\nexists$	$\nexists$	$\nexists$	$\ast$	$\ast$	$\ast$	$\ast$
H^4D^2	$\to 0$	$\to 0$	$\to 0$	$\nexists$	$\nexists$	$\nexists$	$\to 0$	$\nexists$	$\ast$	$\ast$
ψ^2H^2D	$\to 0$	$\to 0$	$\to 0$	$\to 0$	$\to 0$	$\ast$	$\to 0$	$\nexists$	$\ast$	$\ast$

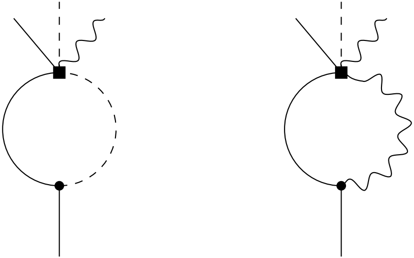
- The 1 1 block is holomorphic

$$\gamma_{\mathfrak{h}\bar{\mathfrak{h}}} = 0$$

- The 1 2 block vanishes except for the red terms proportional to $Y_u Y_e$ or $Y_u Y_d$.

$$\mathcal{L}_Y = -\overline{q}^j \, Y_d^\dagger \, d \, H_j - \overline{q}^j \, Y_u^\dagger \, u \, \widetilde{H}_j - \overline{l}^j \, Y_e^\dagger \, e \, H_j + \text{h.c.}$$
$$\widetilde{H}_j = \epsilon_{ij} H^{\dagger j}$$

- $\psi^2 H^3$ behaves to some extent like a holomorphic operator.
- one entry $*$ present even if Yukawa couplings set to zero.



In the left diagram, can treat $X^+_{\mu\nu}$ as a background field, and so the graph ends up being proportional to $X^+_{\mu\nu}$

In the right diagram, pick out the $[A_\mu, A_\nu]$ part of $X^+_{\mu\nu}$, so not obvious that the final result is proportional to $X^+_{\mu\nu}$

RGE of SM parameters

Recall that

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C^{(4)} \propto m_H^2 C^{(6)}$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \tau = \mu \frac{\mathrm{d}}{\mathrm{d}\mu} \left(\frac{4\pi}{g_X^2} - i \frac{\theta_X}{2\pi} \right) = \frac{2 m_H^2}{\pi g_X^2} C_{HX,+}$$

where θ-terms are normalized as $\mathcal{L} \supset (\theta_X g_X^2 / 32\pi^2) X \widetilde{X}$ and $X \in \{SU(3), SU(2), U(1)\}$.

τ is the SUSY holmorphic gauge coupling

∗ Entry: Some Numerology

$$\begin{aligned}\dot{C}_H = &-3g_2^2\left(g_1^2 + 3g_2^2 - 12\lambda\right)\mathrm{Re}(C_{HW,+}) \\ &-3g_1^2\left(g_1^2 + g_2^2 - 4\lambda\right)\mathrm{Re}(C_{HB,+}) \\ &-3g_1g_2\left(g_1^2 + g_2^2 - 4\lambda\right)\mathrm{Re}(C_{HWB,+}) + \ldots\end{aligned}$$

The $C_{HB,+}$ and $C_{HWB,+}$ terms vanish if $g_1^2 + g_2^2 = 4\lambda$:

$$m_H^2 = 2m_Z^2 = (129\,\mathrm{GeV})^2\,,$$

and the $C_{HW,+}$ term vanishes if $g_1^2 + 3g_2^2 = 12\lambda$:

$$m_H^2 = \frac{2}{3}m_Z^2 + \frac{4}{3}m_W^2 = (119\,\mathrm{GeV})^2\,,$$

If $g_1^2 + g_2^2 = 4\lambda$:

$$\dot{C}_H = 6g_1^2g_2^2\,\mathrm{Re}(C_{HW,+}) + \ldots$$

Summary

- Complete RGE of dimension-six operators of SM EFT computed for first time.
- Contribution of dimension-six operators to running of SM parameters calculated for first time.
- RG evolution of dimension-six operators important for Higgs boson production $gg \to h$ and decay $h \to \gamma\gamma$ and $h \to Z\gamma$, which occur at one loop in SM.
- Significant constraints on flavor structure of SM EFT. Test of MFV hypothesis.
- Approximate holomorphy of 1-loop anomalous dimension matrix of dimension-six operators.
- Does it hold in a more general gauge theory?
- Does any of it extend beyond one loop?