W^+W^- and ZZ production at NNLO

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PRISMA

ERC workhop

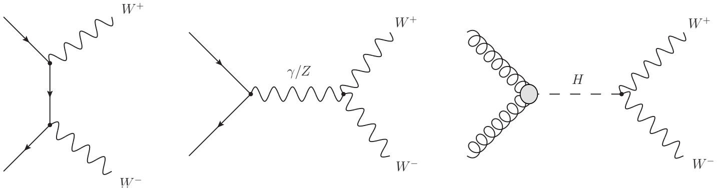
"Effective Field Theories for Collider Physics, Flavor Phenomena and Electroweak Symmetry Breaking"

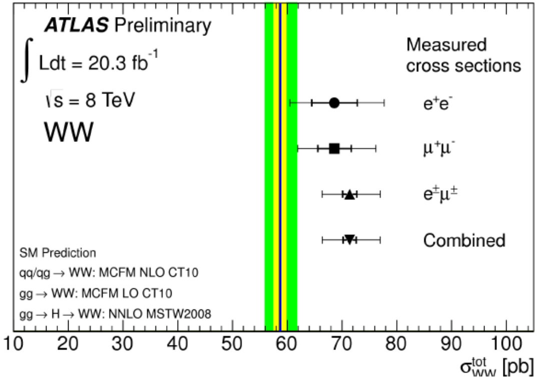
Schloß Waldthausen, 12. November 2014

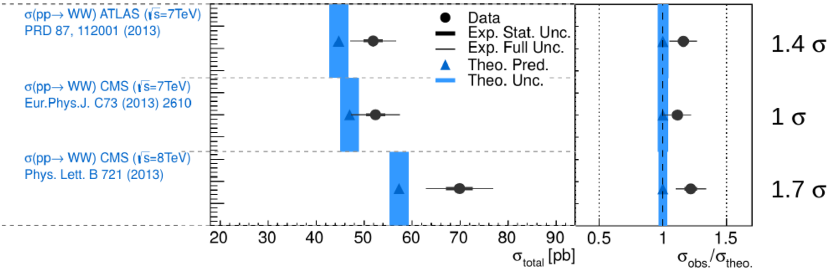
Vector boson pair production at LHC:

- sensitive to details of EWSB
- possible NP contributions at tree or loop level

e.g. W^+W^- production:




ATLAS Preliminary
∫ Ldt = 20.3 fb⁻¹
√s = 8 TeV
WW
Measured cross sections
e⁺e⁻
μ⁺μ⁻
e±μ±
Combined
SM Prediction
qq/qg → WW: MCFM NLO CT10
gg → WW: MCFM LO CT10
gg → H → WW: NNLO MSTW2008
10 20 30 40 50 60 70 80 90 100
σ tot WW [pb]


σ(pp→ WW) ATLAS (√s=7TeV)
PRD 87, 112001 (2013)
σ(pp→ WW) CMS (√s=7TeV)
Eur.Phys.J. C73 (2013) 2610
σ(pp→ WW) CMS (√s=8TeV)
Phys. Lett. B 721 (2013)
Data
Exp. Stat. Unc.
Exp. Full Unc.
Theo. Pred.
Theo. Unc.
20 30 40 50 60 70 80 90
σ_total [pb]
0.5 1 1.5
σ_obs./σ_theo.
1.4 σ
1 σ
1.7 σ

QCD approx. NNLO and electroweak NLO:

- *gg* initiated (one-loop only): Binoth et al. (2005,2008); Duhrssen et al. (2005); Ametller et al. (1985); van der Bij, Glover (1988); Adamson, de Florian, Signer (2000)
- high energy WW: Chachamis, Czakon, Eiras (2008)
- electroweak NLO: Hollik, Meier (2004); Accomando, Denner, Meier (2005); Bierweiler, Kasprzik, Kühn, Uccirati (2012); Billoni, Dittmaier, Jäger, Speckner (2013)

QCD full NNLO:

- WW, ZZ
  - master integrals: Gehrmann, Tancredi, Weihs (2013); Henn, Melnikov, Smirnov (2013); Gehrmann, AvM, Tancredi, Weihs (2014); Caola, Henn, Melnikov, Smirnov (2014)
  - New: **full NNLO for** ZZ: Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, AvM, Pozzorini, Rathlev, Tancredi, Weihs (2014)
  - New: **full NNLO for** W^+W^-: Gehrmann, Grazzini, Kallweit, Maierhöfer, AvM, Pozzorini, Rathlev, Tancredi (2014)
  - in progress: Melnikov et al, Anastasiou et al

ingredients for $VV + X$ ($V = W, Z$) production at NNLO:

- **VV**: two-loop + one-loop^2 ME for VV
  - $q\bar{q} \to VV$: two-loop + one-loop^2
  - $gg \to VV$: one-loop^2
- **RV**: one-loop ME for $VV + 1$ parton
  - $q\bar{q} \to VVg$
  - $gg \to VVg$
  - plus crossings
- **RR**: tree level ME for $VV + 2$ partons
  - $q\bar{q} \to VVgg$
  - $q\bar{q} \to VVq\bar{q}$
  - $gg \to VVgg$
  - plus crossings
- **subtraction terms**: up to 2 unresolved partons needed
  - q_T **subtraction**: Catani, Grazzini (2007); Catani, Cieri, de Florian, Ferrera, Grazzini (2013)
  - antenna subtraction: Gehrmann-De Ridder, Gehrmann, Glover (2005)
  - sector decomposition based: Czakon (2010)

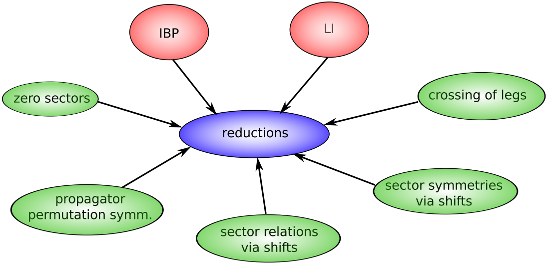
Expertise for all ingredients crucial for VV @ NNLO QCD

analytic results for two-loop corrections:

1. generate Feynman diagrams with Qgraf (Nogueira)
2. compute amplitude using projectors
3. IBP reduce integrals with Reduze 2 (AvM, Studerus)
4. compute master integrals with differential equations

Reduze 2 (AvM, C. Studerus)
arXiv:1201.4330, HepForge

uses GiNaC (Bauer, Frink, Kreckel)
and Fermat (Lewis)

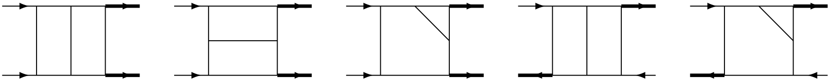


- distributed **Feynman integral reduction**
- advanced **shift finders**
- upcoming version features:
  - **bilinear propagators**
    (3-loop heavy flavour Wilson coefficients in DIS (Blümlein et al. '13-'14))
  - **phase space integrals**
    (RRV threshold contributions to N^3LO Higgs and DY (Li,AvM,Schabinger,Zhu '14))
  - **family finder**, ...

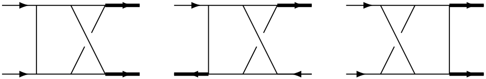
Master integrals for $q\bar{q} \to VV$

88 master integrals (61 w/ products, w/o crossings)

planar two-loop master integrals



non-planar master integrals



Differential equations

- general form, $\epsilon = (4-d)/2$:

$$\frac{\partial}{\partial x_i} f_j(x_i,\varepsilon) = A^{(i)}_{jk}(x_i,\varepsilon) f_k(x_i,\varepsilon)$$

- in certain cases proper choice of basis achieves (Henn '13):

$$A^{(i)}_{jk}(x_i,\varepsilon) = \varepsilon\,\bar{A}^{(i)}_{jk}(x_i)$$

Canonical form

$$\mathrm{d}f(x_i,\varepsilon) = \varepsilon\,\mathrm{d}A(x_i)\,f(x_i,\varepsilon)$$

with

$$A(x_i) = A^{(l)}\,\ln R_l(x_i)$$

- decoupling, clear structure, general solution:

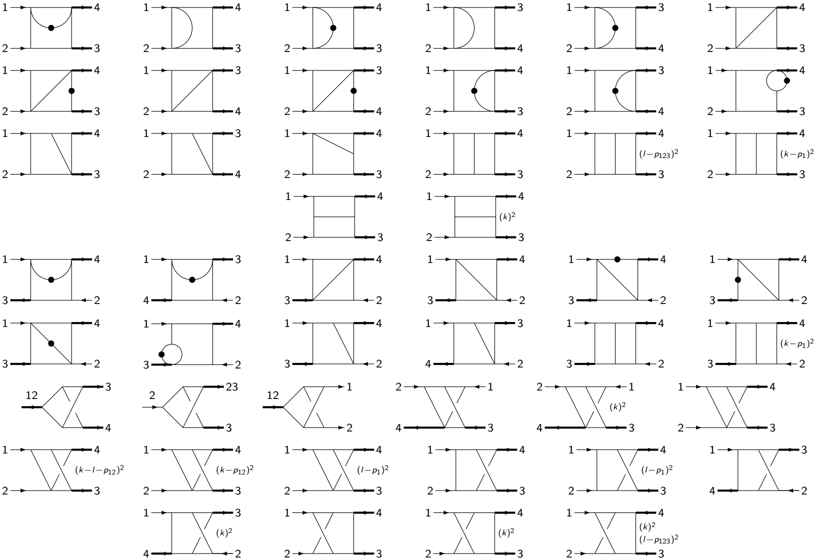
$$f = P\,e^{\varepsilon\int_{\mathcal{C}}\mathrm{d}A}\,f(\varepsilon=0)$$

(pure functions for each order in ε)

- construction of canonical form (heuristic): Gehrmann, AvM, Tancredi, Weihs '14

Master integrals for $q\bar{q} \to VV$

(planar bubbles, triangles and one-loop products not shown)



Gehrmann, AvM, Tancredi, Weihs '14

Structure of result

vector of 75 master integrals in canonical basis with alphabet:

$$x,\ 1-x,\ 1+x,\ z,\ 1+z,\ x-z,\ 1-xz,\ 1+x^2-xz,\ 1+x+x^2-xz,\ z(1+x+x^2)-x$$

where $s = m^2(1+x)^2/x,\ t = -m^2 y,\ u = -m^2 z$

integrated in terms of:

Multiple polylogarithms [Remiddi, Gehrmann; Goncharov]

$$G(a_1, a_2, \ldots, a_n; x) = \int_0^x dt\ \frac{dt}{t-a_1} G(a_2, \ldots, a_n; t),$$

integration constants:

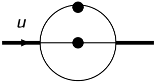
- independent input for a couple of very simple bubbles and triangles
- remaining boundary functions fixed by regularity:

$$z \to x, \qquad z \to 1/x, \qquad z \to -1, \qquad z \to (1+x+x^2)/x, \qquad x \to 1$$

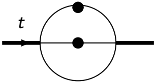
coproduct based and other algorithms:

- partly from Brown '11, Duhr '12, Duhr, Gangl, Rhodes '11, Vollinga, Weinzierl '04
- employ also generalised weights $[f(o)]$ (see AvM, Schabinger, Zhu '13)

example result: (dots are squared propagators)

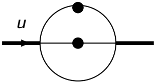
$-\epsilon^2 m^{2\epsilon} u$ 

$$= 1 - \epsilon 2G(0,z) + \epsilon^2 4G(0,0,z) + \epsilon^3(-8G(0,0,0,z) - 6\zeta(3)) + \epsilon^4(-\pi^4/10 + 16G(0,0,0,0,z) + 12G(0,z)\zeta(3)) + O(\epsilon^5)$$

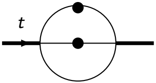
$-\epsilon^2 m^{2\epsilon} t$ 

$$= 1 + \epsilon(2G(0,x) - 2G((1+x^2)/x,z) - 2G([1+o^2],x)) + \epsilon^2(-4G(0,x)G((1+x^2)/x,z) + 4G((1+x^2)/x,z)G([1+o^2],x) + 4G(0,0,x) - 4G(0,[1+o^2],x) + 4G((1+x^2)/x,(1+x^2)/x,z) - 4G([1+o^2],0,x) + \epsilon^3(\ldots(\text{lengthy})) + \epsilon^4(\ldots(\text{lengthy})) + O(\epsilon^5)$$

optimize functional basis for numerical evaluation:

$-\epsilon^2 m^{2\epsilon} u$ 

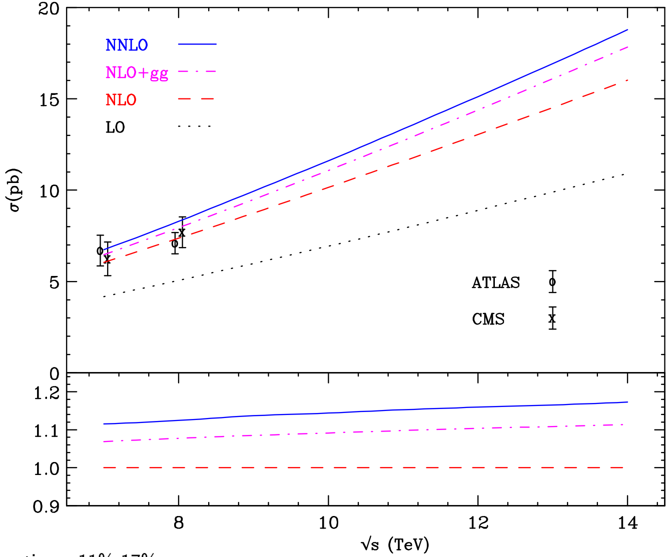
$$= 1 - \epsilon 2\ln z + \epsilon^2 2\ln^2 z + \epsilon^3(-(4/3)\ln^3 z - 6\zeta(3)) + \epsilon^4(-\pi^4/10 + (2/3)\ln^4 z + 12\ln z\zeta(3)) + O(\epsilon^5),$$

$-\epsilon^2 m^{2\epsilon} t$ 

$$= 1 - \epsilon 2\ln y + \epsilon^2 2\ln^2 y + \epsilon^3(-(4/3)\ln^3 y - 6\zeta(3)) + \epsilon^4(-\pi^4/10 + (2/3)\ln^4 y + 12\ln y\zeta(3)) + O(\epsilon^5),$$

full 2-loop amplitude fast & stable, takes $\mathcal{O}(35\text{ms})$ per point

Result: *ZZ* production at NNLO



- NNLO corrections: 11%-17%
- *gg* contributes 60% of NNLO

Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, AvM, Pozzorini, Rathlev, Tancredi, Weihs '14

ZZ PRODUCTION: SCALE UNCERTAINTIES

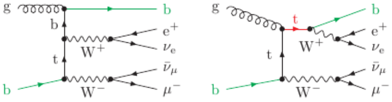
$\sqrt{s}$ (TeV)	σ_{LO} (pb)	σ_{NLO} (pb)	σ_{NNLO} (pb)
7	$4.167^{+0.7\%}_{-1.6\%}$	$6.044^{+2.8\%}_{-2.2\%}$	$6.735^{+2.9\%}_{-2.3\%}$
8	$5.060^{+1.6\%}_{-2.7\%}$	$7.369^{+2.8\%}_{-2.3\%}$	$8.284^{+3.0\%}_{-2.3\%}$
9	$5.981^{+2.4\%}_{-3.5\%}$	$8.735^{+2.9\%}_{-2.3\%}$	$9.931^{+3.1\%}_{-2.4\%}$
10	$6.927^{+3.1\%}_{-4.3\%}$	$10.14^{+2.9\%}_{-2.3\%}$	$11.60^{+3.2\%}_{-2.4\%}$
11	$7.895^{+3.8\%}_{-5.0\%}$	$11.57^{+3.0\%}_{-2.4\%}$	$13.34^{+3.2\%}_{-2.4\%}$
12	$8.882^{+4.3\%}_{-5.6\%}$	$13.03^{+3.0\%}_{-2.4\%}$	$15.10^{+3.2\%}_{-2.4\%}$
13	$9.887^{+4.9\%}_{-6.1\%}$	$14.51^{+3.0\%}_{-2.4\%}$	$16.91^{+3.2\%}_{-2.4\%}$
14	$10.91^{+5.4\%}_{-6.7\%}$	$16.01^{+3.0\%}_{-2.4\%}$	$18.77^{+3.2\%}_{-2.4\%}$

- variation: $0.5 m_Z < \mu_R, \mu_F < 2 m_Z$ with $0.5 < \mu_F/\mu_R < 2$
- scale uncertainty 3% not decreased

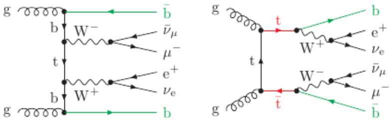
Definition of top-contamination free WW cross section in 5FNS

Definition of WW cross section beyond LO

- straightforward in 4FNS (massive b's)
- non-trivial in 5FNS (massless b's)
  - Single-top production enters at NLO.



  - Top-pair production enters at NNLO.

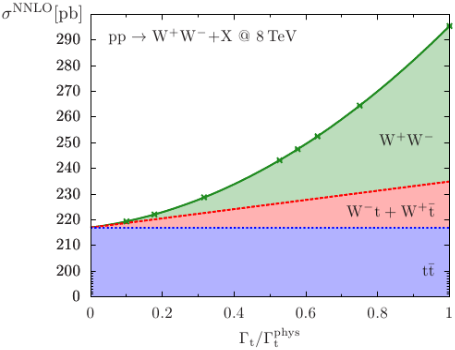


↪ Huge "higher-order corrections" from top-resonance contamination in 5FNS.

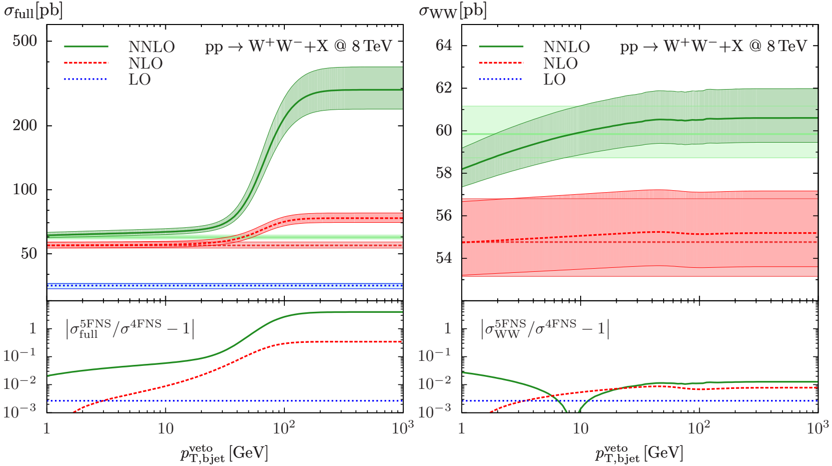
Γ_t-dependence of NNLO cross section can be used to isolate the different processes:

$\sigma_{WW} \propto 1, \quad \sigma_{tW} \propto 1/\Gamma_t, \quad \sigma_{t\bar{t}} \propto 1/\Gamma_t^2.$

↪ Parabolic fit of the $(\Gamma_t/\Gamma_t^{phys})^2$-rescaled cross section delivers σ_{WW}, σ_{tW}, $\sigma_{t\bar{t}}$.



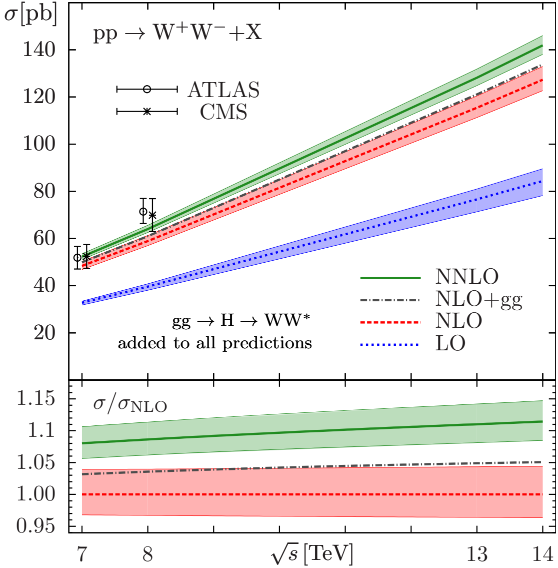
New definition of W^+W^- cross section



- top-substracted *WW* cross section: robust & precise

Gehrmann, Grazzini, Kallweit, Maierhöfer, AvM, Pozzorini, Rathlev, Tancredi '14

Result: W^+W^- production at NNLO



Gehrmann, Grazzini, Kallweit, Maierhöfer, AvM, Pozzorini, Rathlev, Tancredi '14

Conclusions & outlook

- technical progress
  - algorithms for multiple polylogarithms
  - procedure to put DEQ into normal form
- $q\bar{q} \to ZZ,\ W^+W^-$
  - complete set of two-loop master integrals
  - analytic two-loop amplitudes: fast & precise
  - first NNLO prediction for ZZ and W^+W^- production at LHC
  - new definition of W^+W^- cross section: top-subtraction
- outlook
  - helicity amplitudes for VV off-shell and VV' $(V, V' = \gamma, W, Z)$
  - fiducial cross section (cmp. Monni, Zanderighi '14)
  - differential observables . . .

W^+W^- production at NNLO: scale uncertainties

$\frac{\sqrt{s}}{\mathrm{TeV}}$	σ_{LO}	σ_{NLO}	σ_{NNLO}	$\sigma_{gg\to H\to WW^*}$
7	$29.52^{+1.6\%}_{-2.5\%}$	$45.16^{+3.7\%}_{-2.9\%}$	$49.04^{+2.1\%}_{-1.8\%}$	$3.25^{+7.1\%}_{-7.8\%}$
8	$35.50^{+2.4\%}_{-3.5\%}$	$54.77^{+3.7\%}_{-2.9\%}$	$59.84^{+2.2\%}_{-1.9\%}$	$4.14^{+7.2\%}_{-7.8\%}$
13	$67.16^{+5.5\%}_{-6.7\%}$	$106.0^{+4.1\%}_{-3.2\%}$	$118.7^{+2.5\%}_{-2.2\%}$	$9.44^{+7.4\%}_{-7.9\%}$
14	$73.74^{+5.9\%}_{-7.2\%}$	$116.7^{+4.1\%}_{-3.3\%}$	$131.3^{+2.6\%}_{-2.2\%}$	$10.64^{+7.5\%}_{-8.0\%}$

Rotating to canonical basis

how to find **canonical basis** ?

- some heuristics in Smirnov, Smirnov, Henn '13: cuts, explicite bubble insertions

our proposal:

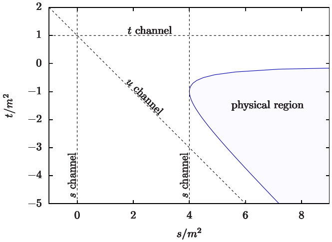
PROCEDURE

construct canonical basis starting from rough first guess

1. **bottom up** strategy, assume subtopos in canonical form
2. for given sector, guess basis: **triangular for** $\epsilon = 0$, diff. eq. **linear in** ϵ (top level only)
3. integrate out **homogeneous part for** $\epsilon = 0$ (top level only)
4. remove unwanted terms $1/(u - v\epsilon)^n$, 1, ϵ^n iteratively
   simplifying assumption: restrict to minimal shifts

result: canonical basis

Kinematics and alphabet



- Landau variable x to rationalize roots

$$s = m^2(1+x)^2/x, \qquad t = -m^2 y, \qquad u = -m^2 z$$

- non-linear relations for crossed kinematics (non-planar !)

$$(1+x^2)/x - y - z = 0$$

- involved functional identities, explicite imaginary parts

Result

vector of 75 master integrals in canonical basis with alphabet:

$$x,\ 1-x,\ 1+x,\ z,\ 1+z,\ x-z,\ 1-xz,\ 1+x^2-xz,\ 1+x+x^2-xz,\ z(1+x+x^2)-x$$

Multiple polylogarithms

[Remiddi, Gehrmann; Goncharov]

$$G(a_1, a_2, \ldots, a_n; x) = \int_0^x dt \, \frac{dt}{t - a_1} G(a_2, \ldots, a_n; t),$$

with $G(; x) = 1$, complex weights a_i and complex argument x.

we employ also **generalised weights** $[f(o)]$:

$$G([f(o)], w_2, \ldots, w_n; x) = \int_0^x \mathrm{d}t \frac{f'(t)}{f(t)} G(w_2, \ldots, w_n; t)$$

example:

$$G([o^2 + 1]; x) = \int_0^x \mathrm{d}t \frac{2t}{t^2 + 1} = \int_0^x \mathrm{d}t \frac{1}{t - i} + \int_0^x \mathrm{d}t \frac{1}{t + i} = G(i; x) + G(-i; x)$$

see AvM, Schabinger, Zhu '13, related: Ablinger, Blümlein, Schneider '11 (cyclotomic polylogs)

we apply coproduct based and other algorithms

- partly from Brown '11, Duhr '12, Duhr, Gangl, Rhodes '11
- numerical routines from Vollinga, Weinzierl '04

integration and boundary terms:

- integrate partial diff. eq. in z if possible, otherwise in x
- uniform setup with same set of "physical" variables + functions
- **independent input** for a couple of very simple bubbles and triangles
- remaining boundary functions fixed by **regularity**:

$$z \to x, \qquad z \to 1/x, \qquad z \to -1, \qquad z \to (1+x+x^2)/x, \qquad x \to 1$$

- result in terms of G-functions with arguments z, x

checks:

- consistency of boundary conditions
- diff. eq. in all variables
- known planar masters
- real in Eulicidean region
- SecDec 2.1 by Borowka, Heinrich '13

main algorithms:

1. **normal form for specific arguments**
   - independent of symbol calculus
   - uses Vollinga, Weinzierl '04 for numerical evaluation, fits constants
2. **coproduct based normal form** for general choice of basis
   - based on Goncharov '02, Brown '11, Duhr '12, Duhr, Gangl, Rhodes '11
   - handles generalised weights
   - identifies products (e.g. $G(0,1;x) + G(1,0;x) \to G(0;x)G(1;x)$)
   - matches irreducible factors *at symbol level*
   - uses Vollinga, Weinzierl '04 for numerical evaluation, fits constants
3. **construct new basis** with desired properties
   - based on Duhr, Gangl, Rhodes '11, apply to generalised weights

PRIMARY RESULT

$$G(\ldots;y), \quad \text{weights} \in \left\{-1, 0, \frac{1}{x}, x, (1+x^2)/x, (1+x+x^2)/x\right\}$$

$$G(\ldots;x), \quad \text{weights} \in \left\{-1, 0, 1, [1+o^2], [1+o+o^2]\right\}$$

OPTIMISED FUNCTIONAL BASIS

choose real valued ln, $\mathrm{Li}_n(R_1)$, $\mathrm{Li}_{2,2}(R_1, R_2)$ with

$$|R_1| < 1, \qquad |R_1 R_2| < 1$$

such that Li functions have convergent power series

$$\mathrm{Li}_n(R_1) = -\sum_{j_1=1}^{\infty} \frac{R_1^{j_1}}{j_1^n}, \qquad \mathrm{Li}_{2,2}(R_1, R_2) = \sum_{j_1=1}^{\infty}\sum_{j_2=1}^{\infty} \frac{R_1^{j_1}}{(j_1+j_2)^2} \frac{(R_1 R_2)^{j_2}}{j_2^2}$$

features:

- $R_i \in \left\{\pm x,\ x^2,\ \frac{1}{y+1},\ xy,\ \frac{(1+x)y}{y+1},\ \frac{(1+x)z(x,y)}{z(x,y)+1}, \ldots\right\}$
- no spurious letters, no generalised weights
- **very fast and stable** numerical evaluation: $O(35ms)$ per generic point (full amplitude)

RESULT

MC friendly analytic expressions, ready to be used for VV at NNLO