EFT for Jets with Massive Quarks

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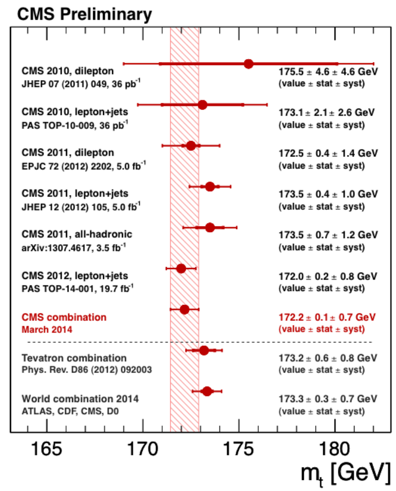
Why complete mass dependence for jets?

Aims:
- Full quark mass dependence of jet observables.
- Theory description for all kinematic regions
  ("decoupling limit" ⇔ "massless limit")
- Understanding factorization with quark masses
- Account for initial state and final state jets

Possible applications:

- Quark mass effects in precision QCD analyses
  e.g.
  - Event shapes in e^+e^- (bottom effects for low Q data)
  - Top quark mass measurements in reconstruction
- Understanding of Monte-Carlo top mass parameter
- Role of massive quark vacuum polarization effects
- Intrinsic charm ?



This talk: I will mostly talk about final state jets.
Show that SCET (+ extensions) is a good framework to address the problem of quark masses.

Outline

- Motivation and Aims
- Factorization for massless quarks
- Effective theories including massive quark effects
- Flavor number dependent renormalization
- Rapidity logs
- Running short-distance mass scheme
- Conclusions & Outlook

* In collaboration with: P. Pietrulewicz, V. Mateu, I. Jemos, S. Gritschacher
B. Dehnadi, M. Butenschön

arXiv:1302.4743 (PRD 88, 034021 (2013))
arXiv:1309.6251 (PRD 89, 014035 (2013))
arXiv:1405.4860 (PRD ..)
More to come …

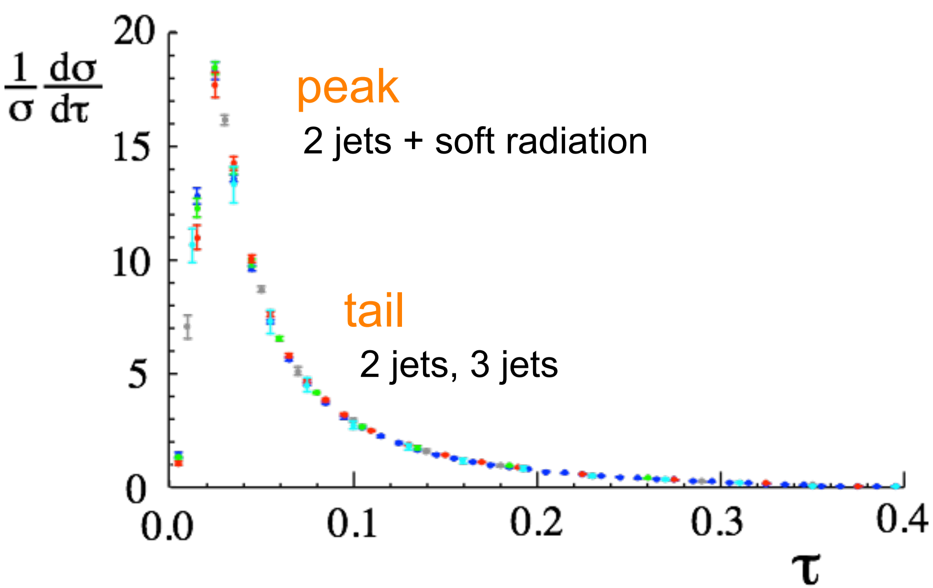
Thrust

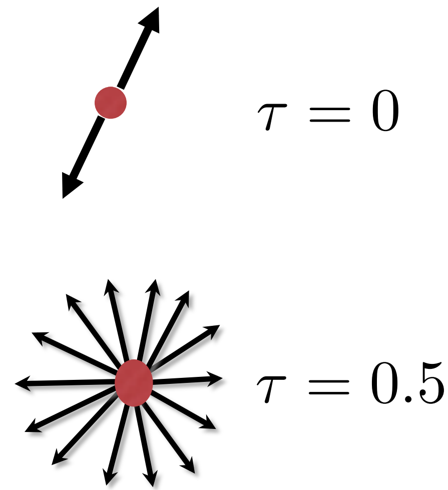
→ consider: dijet in e^+e^- annihilation

e.g. Thrust:

$$T = \max_{\hat{t}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \vec{p}_i|}{Q} \qquad \tau = 1 - T$$

ALEPH, DELPHI, L3, OPAL, SLD





→ Mass mode treatment of this talk applicable to any SCET-1-type observable

→ We use thrust to be definite and as a first important application.

Massless Quark Thrust in FO

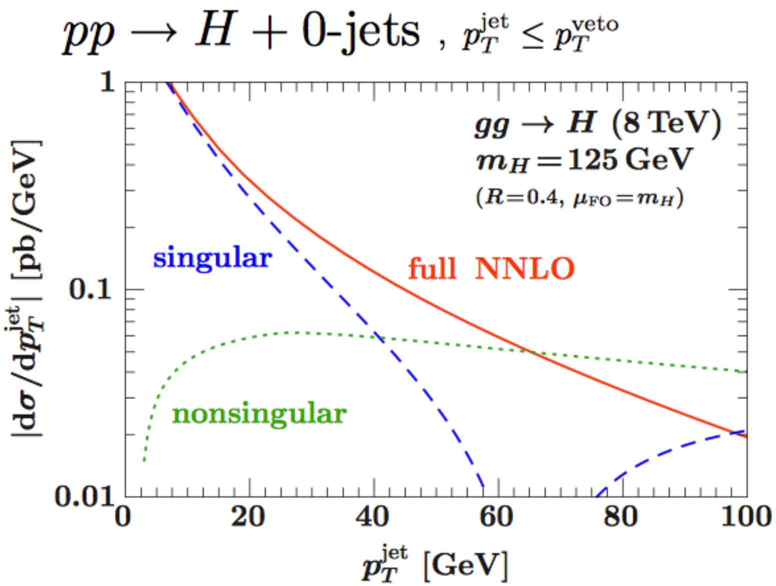
$$\frac{1}{\sigma_{\rm tot}^{\rm Born}}\frac{d\sigma}{d\tau} = \delta(\tau) + \frac{C_F\alpha_s}{\pi}\Big[\big(\frac{\pi^2}{6}-\frac{1}{2}\big)\delta(\tau) + \frac{-3+9\tau+3\tau^2-9\tau^3}{2\tau(1-\tau)} - \frac{2-3\tau+3\tau^2}{(1-\tau)}\Big(\frac{\ln(\frac{\tau}{1-2\tau})}{\tau}\Big)_+\Big]$$

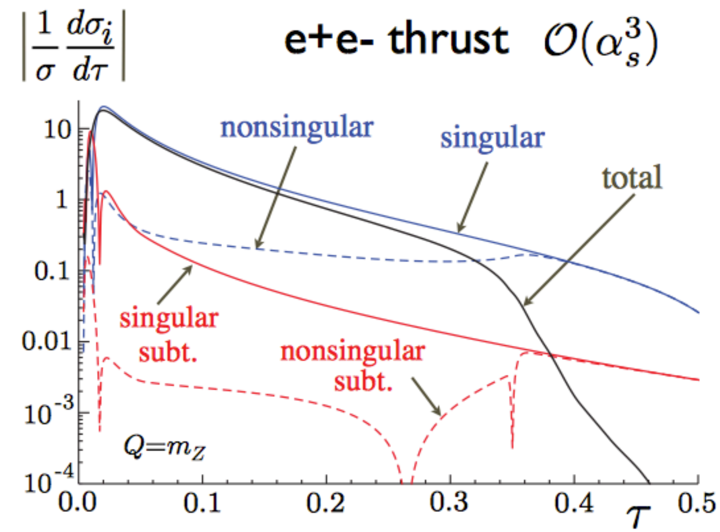
$$= \boxed{\delta(\tau) + \frac{C_F\alpha_s}{\pi}\Big[\big(\frac{\pi^2}{6}-\frac{1}{2}\big)\delta(\tau) - \frac{3}{2}\big(\frac{1}{\tau}\big)_+ - 2\big(\frac{\ln(\tau)}{\tau}\big)_+} + \ \{\text{non-sing. terms}\}\Big]$$

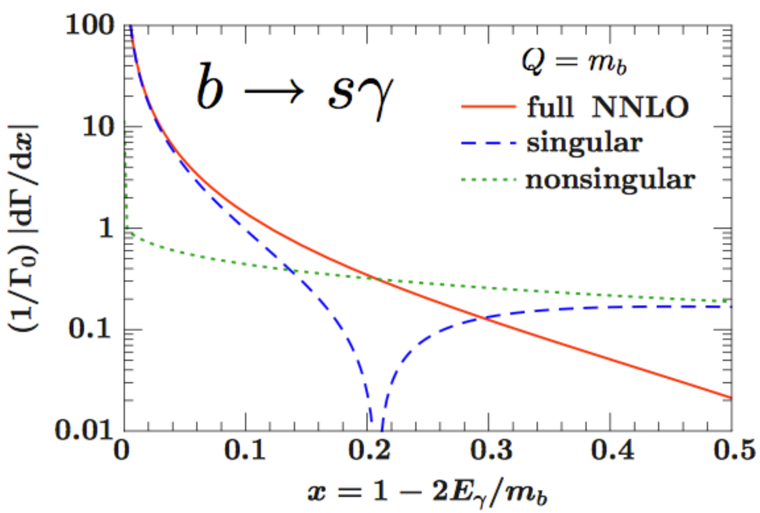
singular terms

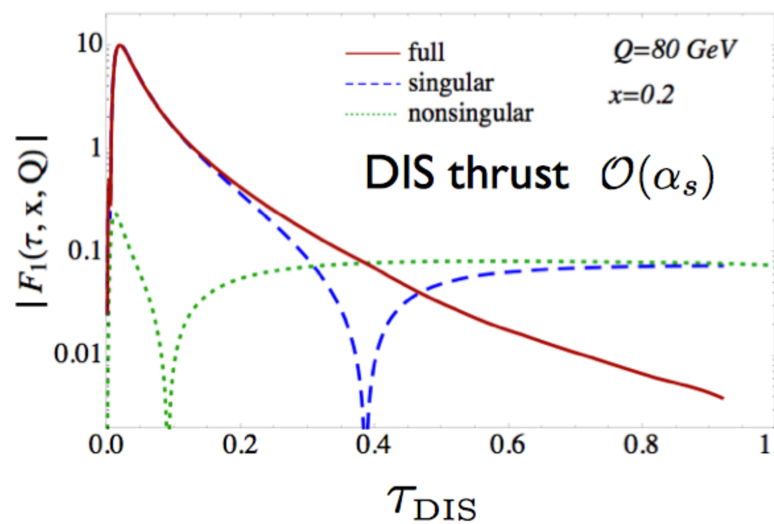
Strongly dominate in kinematic
regions where jets are produced

Singular vs. Non-singular





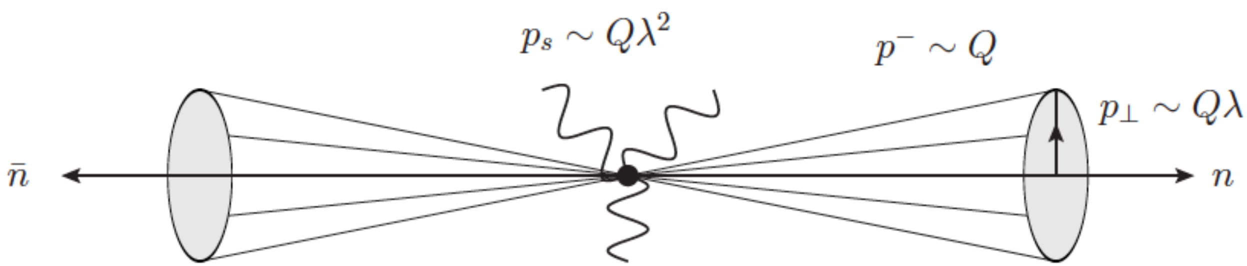




Massless Quark SCET

Bauer, Fleming, Luke
Bauer, Fleming, Pirjol, Stewart

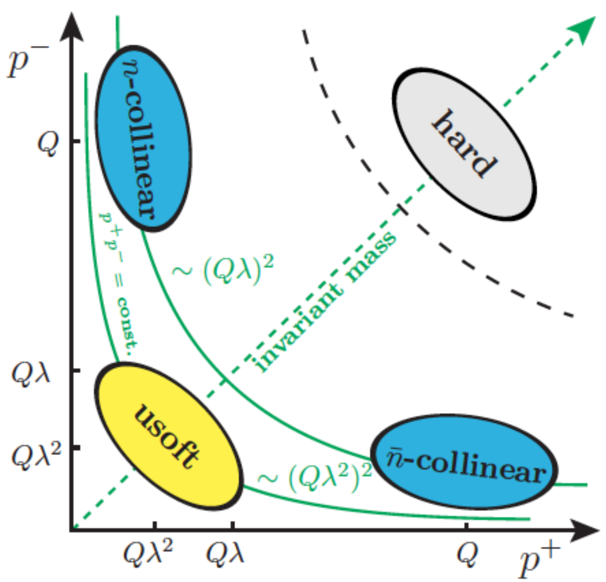
→ consider: dijet in e^+e^- annihilation, all quarks are light ($m_q < \Lambda$)



$$n^\mu = (1,0,0,1) \qquad \bar{n}^\mu = (1,0,0,-1)$$

$$p^\mu = p^- \frac{n^\mu}{2} + p^+ \frac{\bar{n}^\mu}{2} + p_\perp$$

$$p^2 = p^- p^+ + p_\perp^2$$



mode	$p^\mu = (+,-,\perp)$	p^2	fields
hard	$Q(1,1,1)$	Q^2	–
n-collinear	$Q(\lambda^2,1,\lambda)$	$Q^2\lambda^2$	ξ_n, A_n^μ
$\bar{n}$-collinear	$Q(1,\lambda^2,\lambda)$	$Q^2\lambda^2$	$\xi_{\bar{n}}$, $A_{\bar{n}}^\mu$
ultrasoft	$Q(\lambda^2,\lambda^2,\lambda^2)$	$Q^2\lambda^4$	q_{us}, A_{us}^μ

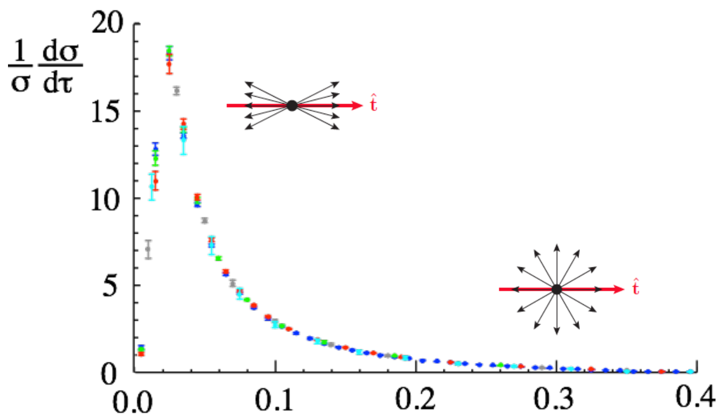
$$d\sigma = \mathcal{H} \cdot \mathcal{J} \otimes \mathcal{S}$$

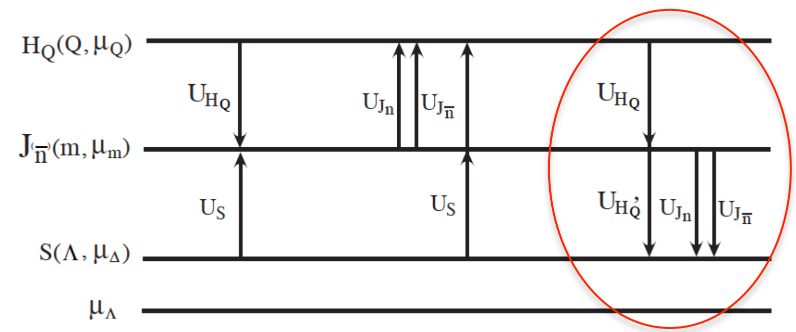
Korchemsky, Sterman

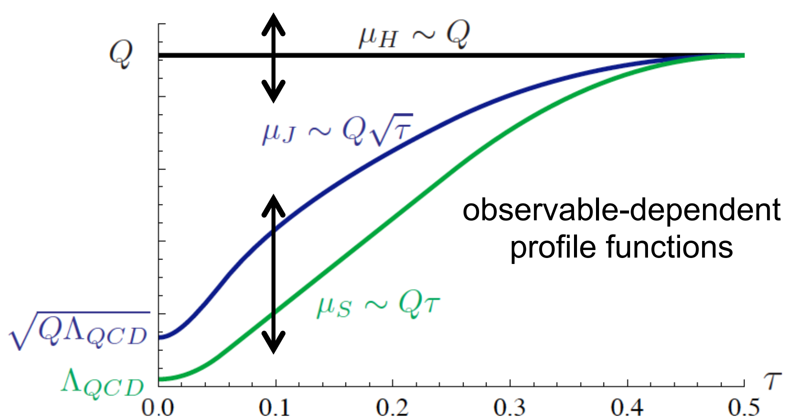
Factorization for Massless Quarks

$$\frac{d\sigma}{d\tau} = Q^2 \sigma_0 H_0(Q,\mu) \int d\ell \, J_0(Q\ell,\mu)\, S_0\,(Q\tau - \ell, \mu)$$

Schwartz
Fleming, AH, Mantry, Stewart
Bauer, Fleming, Lee, Sterman







→ evolution with n_l light quark flavors
→ consistency conditions w.r. to different evolution choices
→ top-down evolution considered in the following

$$\left(\frac{d\sigma}{d\tau}\right)^{\text{sing}}_{\text{part}} \sim \sigma_0\, H(Q,\mu_Q) U_H(Q,\mu_Q,\mu_s) \int d\ell d\ell' \, U_J(Q\tau - \ell - \ell', \mu_Q, \mu_s)\, J_T(Q\ell', \mu_j)\, S_T(\ell - \Delta, \mu_s)$$

Final State Jets with a Massive Quark

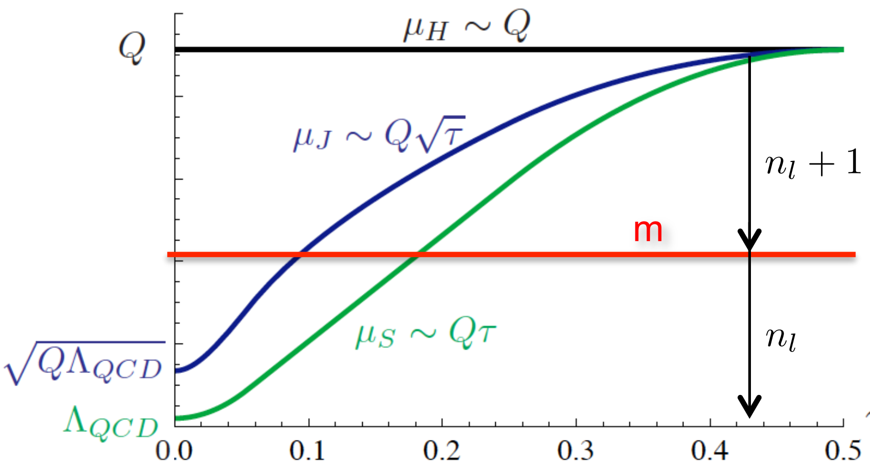
Gritschacher, AH, Jemos, Pietrulewicz

→ consider: dijet in e^+e^- annihilation, n_l light quarks ⊕ one massive quark

→ obvious: (n_l+1)-evolution for $\mu \gtrsim m$ and (n_l)-evolution for $\mu \lesssim m$

→ obvious: different EFT scenarios w.r. to mass vs. Q – J – S scales

"profile functions"



Aims:

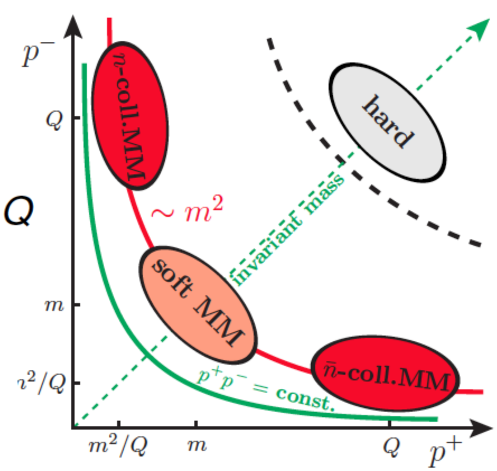
- Full mass dependence (little room for any strong hierarchies): decoupling, massless limit
- Smooth connections between different EFTs
- Determination of flavor matching for current-, jet- and soft-evolution
- Reconcile problem of SCET_2-type rapidity divergences

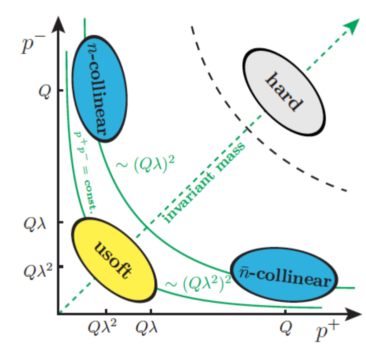
"Variable Flavor Number Scheme" (VFNS)

→ Deal with collinear and soft "mass modes"

→ Additional power counting parameter $\lambda_m = m/Q$

mode	$p^\mu = (+,-,\perp)$	p^2
n-coll MM	$Q(\lambda_m^2, 1, \lambda_m)$	m^2
soft MM	$Q(\lambda_m, \lambda_m, \lambda_m)$	m^2

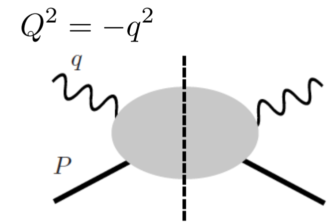




universität wien

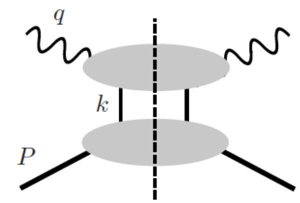
VFNS for Hadron Collisions

e.g. Deep Inelastic Scattering: $\frac{d\sigma(e^- p \to e^- + X)}{dQ\, dx}$



→ consider all quarks as as light ($m_q < \Lambda$)

→ quark number operators with an anomalous dimension between proton states → DGLAP equations



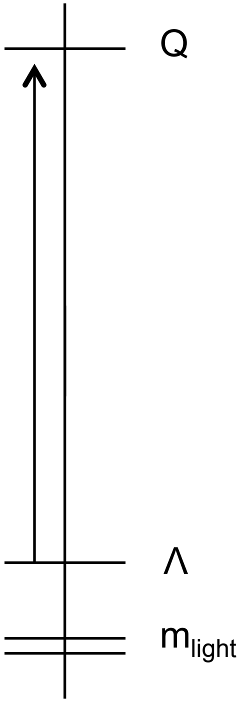
→ Hadronic tensor:

$$W_{\mu\nu}(Q,x) \sim \sum_{\text{partons a}} f_a(\mu) \otimes w_{\mu\nu}(Q,x,\mu)$$

→ μ-dependence with DGLAP equations for (light) parton distribution functions

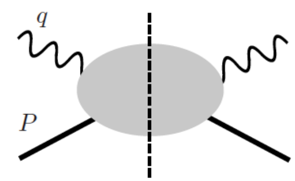
$$\frac{\partial}{\partial \ln Q^2} \begin{pmatrix} q_i(x,Q^2) \\ g(x,Q^2) \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi} \sum_j \int_x^1 \frac{d\xi}{\xi} \times \begin{pmatrix} P_{q_i q_j}\left(\frac{x}{\xi}, \alpha_s(Q^2)\right) & P_{q_i g}\left(\frac{x}{\xi}, \alpha_s(Q^2)\right) \\ P_{g q_j}\left(\frac{x}{\xi}, \alpha_s(Q^2)\right) & P_{gg}\left(\frac{x}{\xi}, \alpha_s(Q^2)\right) \end{pmatrix} \begin{pmatrix} q_j(\xi,Q^2) \\ g(\xi,Q^2) \end{pmatrix}, \quad (11)$$

$$\frac{d\alpha_s(Q)}{d\ln Q^2} = -\beta_0 \frac{\alpha_s^2(Q)}{(4\pi)} + \ldots \qquad \beta_0 = 11 - \frac{2}{3} n_{\text{light}}$$



VFNS for Hadron Collisions

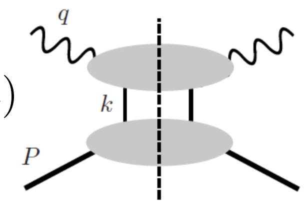
e.g. Deep Inelastic Scattering: $\dfrac{d\sigma(e^- p \to e^- + X)}{dQ\,dx}$



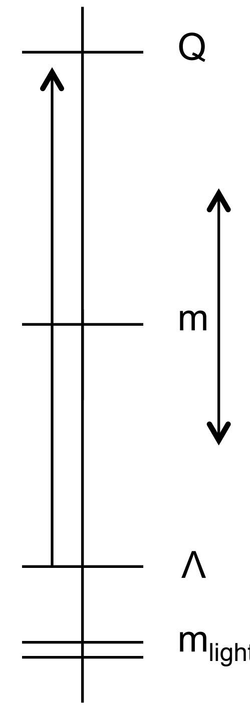
→ realistic case: massive quarks with Q > m > Λ
(charm, bottom [top])

→ Hadronic tensor:

$$W_{\mu\nu}(m,Q,x) \sim \sum_{a=q,g,Q} f_a^{(n_l+1)}(\mu) \otimes w_{\mu\nu}(m,Q,x,\mu)$$



ACOT scheme:

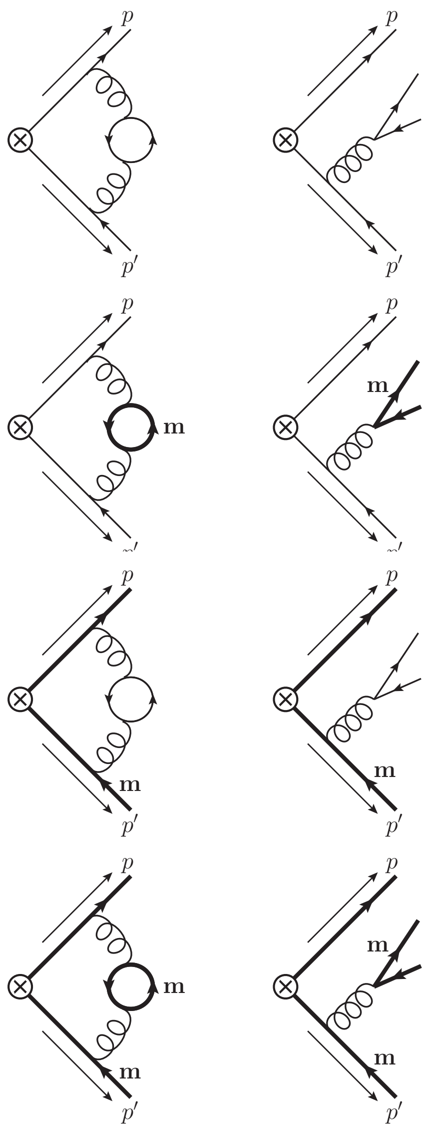


- DGLAP evolution for n_l flavors for $\mu \lesssim m$ (only light quarks)
- DGLAP evolution for n_l+1 flavors for $\mu \gtrsim m$ (light quarks + massive quark)
- Flavor matching for α_s and the pdfs at $\mu_m \sim m$

$$f_{q,g,Q}^{(n_l+1)}(\mu_m) = \sum_{a=q,g} F_{q,g,Q|a}(m,\mu_m) \otimes f_a^{(n_l)}(\mu_m)$$

→ hard coefficient $w_{\mu\nu}(m,Q,x)$ approaches massless $w_{\mu\nu}(Q,x)$ for $m \to 0$
→ calculations of $w_{\mu\nu}(m,Q,x)$ involves subtraction of pdf IR mass singularities
→ full dependence on m/Q without any large logarithms

Fully Massive Thrust



→ fully massless

- Full N^{3}LL$'$ (u.t. 4-loop cusp)+ 3-loop non-singular
- Gap scheme for soft function

Only SCET authors: Becher, Schwartz,
Fleming, AH, Mantry, Stewart
Bauer, Fleming, Lee, Sterman

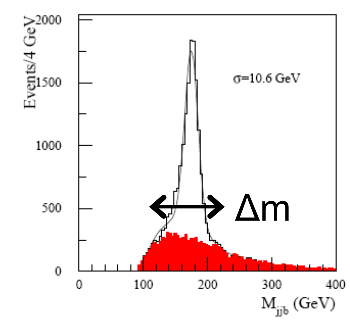
→ <u>secondary</u> massive

- Full N^{2}LL$'$/N^{3}LL

→ New: In detail in this talk.

→ <u>primary</u> massive

- bHQET: full N^{2}LL$'$/N^{3}LL
  → valid for: Δm_{jet} << m
- NLL$'$/N^{2}LL for other cases



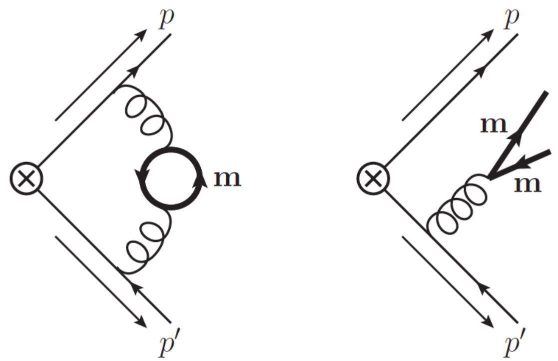
→ <u>primary</u> massive
<u>secondary</u> massive

New: complete and systematic description
→ Only briefly in this talk.

VFN Scheme: Secondary Massive Quarks

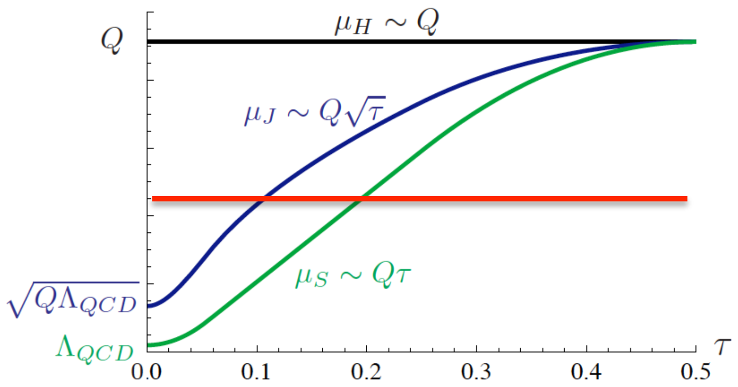
Simplest non-trivial case to study:

→ massless primary quark dijet production in e^+e^- annihilation:
n_l light quarks ⊕ one massive quark arise only through secondary production

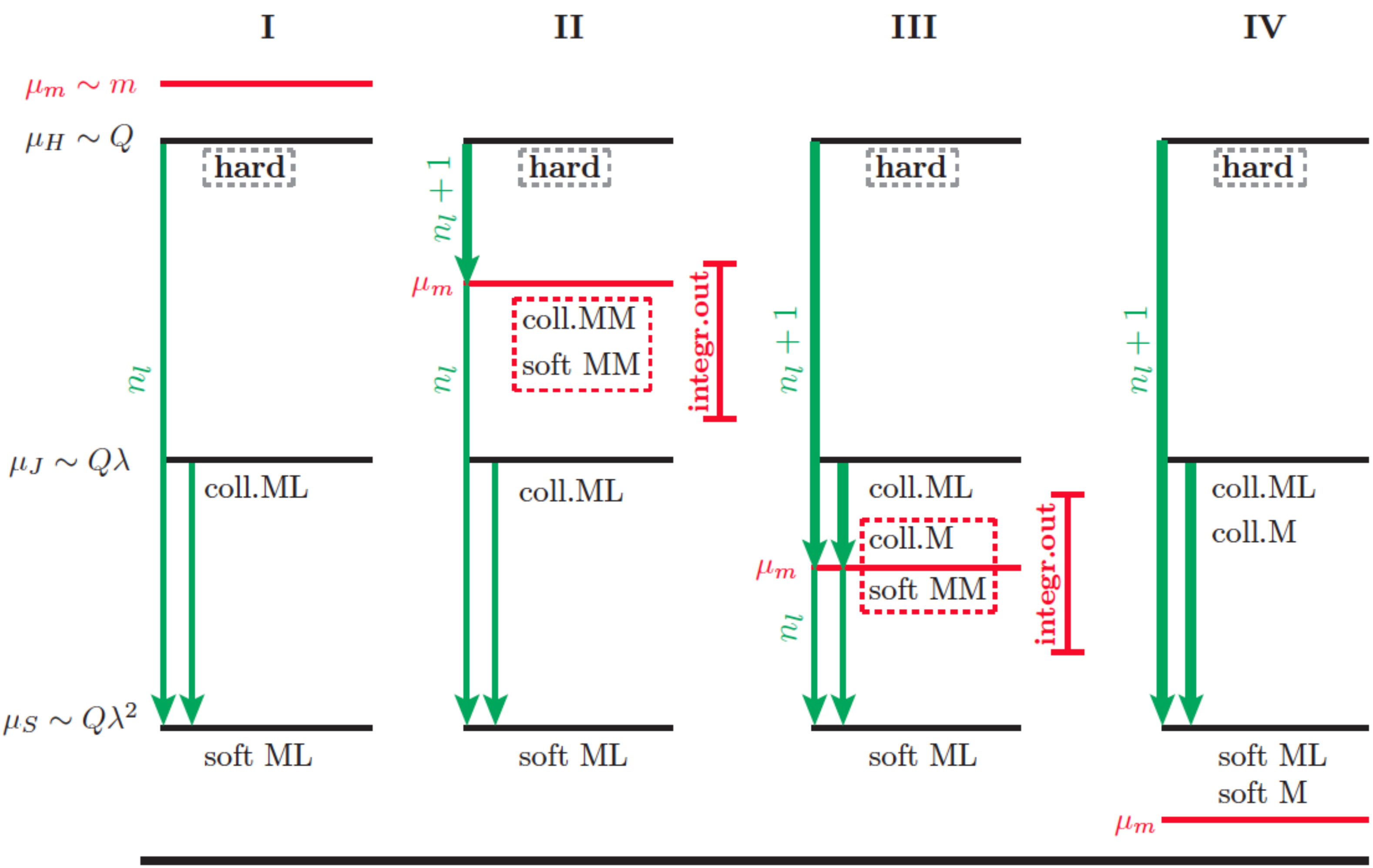


→ does not lead to bHQET-type theory when the jet scale approaches the quark mass

→ only SCET-type theories



VFN Scheme: Secondary Massive Quarks

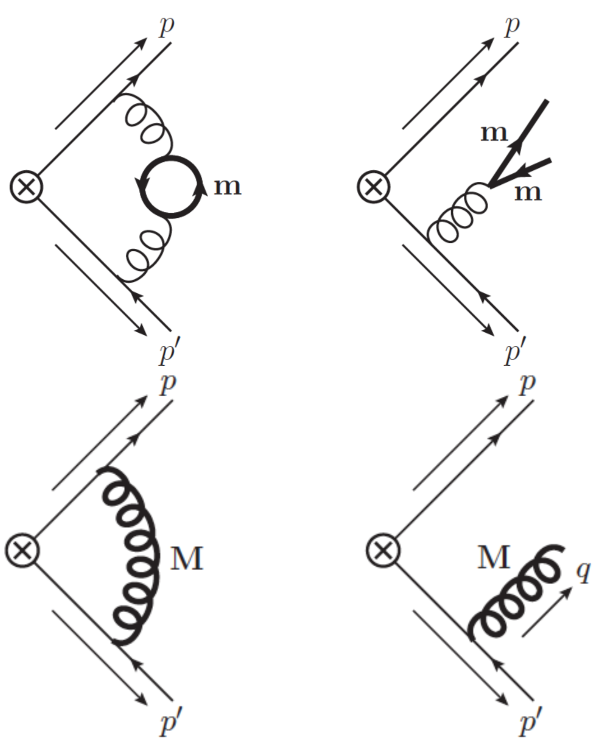


MM = mass-mode, ML = massless, M = massive

VFN Scheme: Secondary Massive Quarks

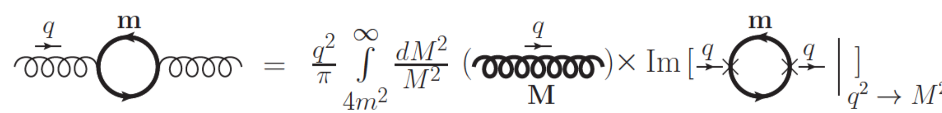
Simplest non-trivial case to study:

→ massless primary quark dijet production in e^+e^- annihilation:
n_l light quarks ⊕ one massive quark arise only through secondary production



→ field theory: close relation to the problem of massive gauge boson radiation

→ dispersion relation: massive quark results can be obtained directly from massive gluon calculations when quark pair treated inclusively (e.g. hard coefficient, jet function)



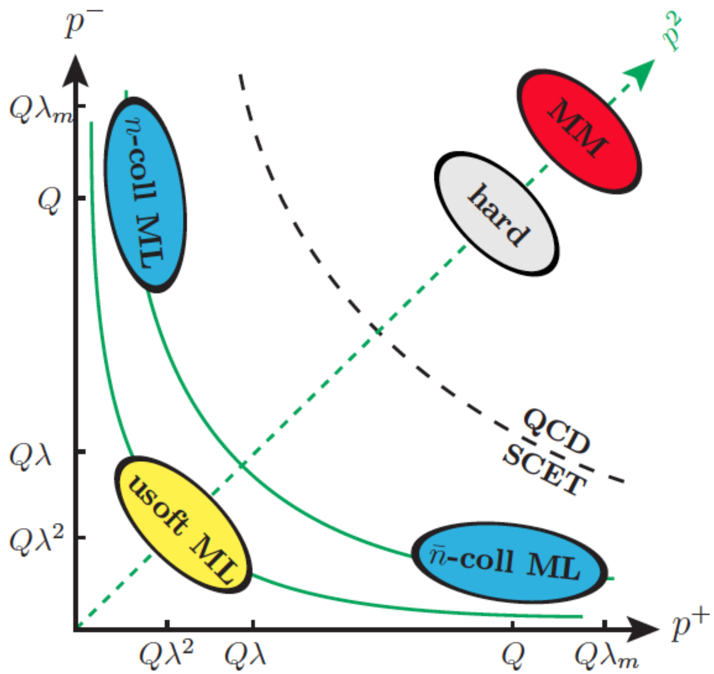
→ separation of conceptual issues to be resolved and calculations issues related to gluon splitting.

→ explicit two-loop calculation needed when quarks are treated exclusively (e.g. soft function → hemisphere prescription)

Gritschacher, AH, Jemos, Pietrulewicz 2013

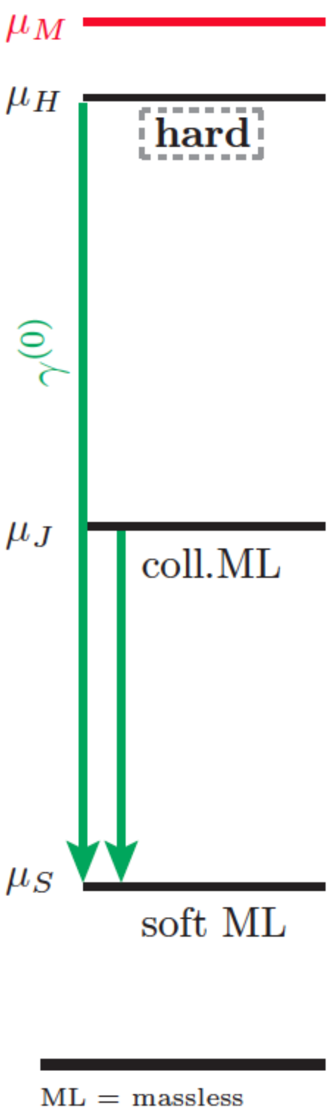
VFN Scheme: Secondary Massive Quarks

<u>Scenario 1:</u> $\lambda_m > 1 > \lambda > \lambda^2$ (m > Q > J > S)



- EFT only contains light quarks
- Massive quark only in current matching coeff.
- Decoupling for m/Q → ∞

VFN Scheme: Secondary Massive Quarks



integrate out mass modes at QCD level

$$\frac{d\sigma}{d\tau} \sim \left|C^I(\mu_H)\right|^2 U_H^{(0)}(\mu_H,\mu_S) \times \int d\ell \int ds\, J_0(s)\, U_J^{(0)}(Q\ell - s,\mu_S,\mu_J)\, S_0(Q\tau-\ell,\mu_S)$$

$U_H^{(0)}$, $U_J^{(0)}$: massless evolution factors

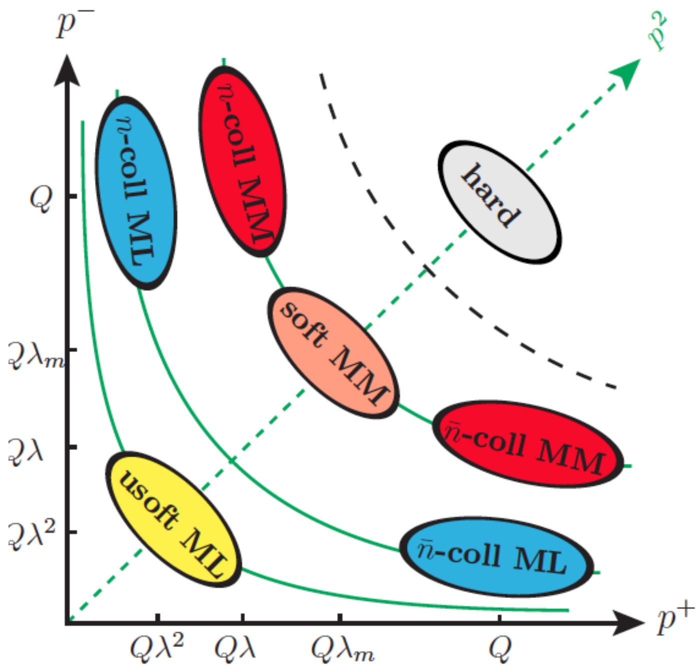
$$C^I(\mu_H) = C_0(\mu_H) + \delta F_m^{\rm QCD}$$

C_0: massless matching coefficient
$\delta F_m^{\rm QCD}$: massive full theory contribution (OS)
$\to$ decoupling for $M/Q \to \infty$
$\to$ IR-divergent expression for $M/Q \to 0$

$U_i^{(0)}$ stands for: (a) massive gluon integrated out
(b) (n_l)-evolution

VFN Scheme: Secondary Massive Quarks

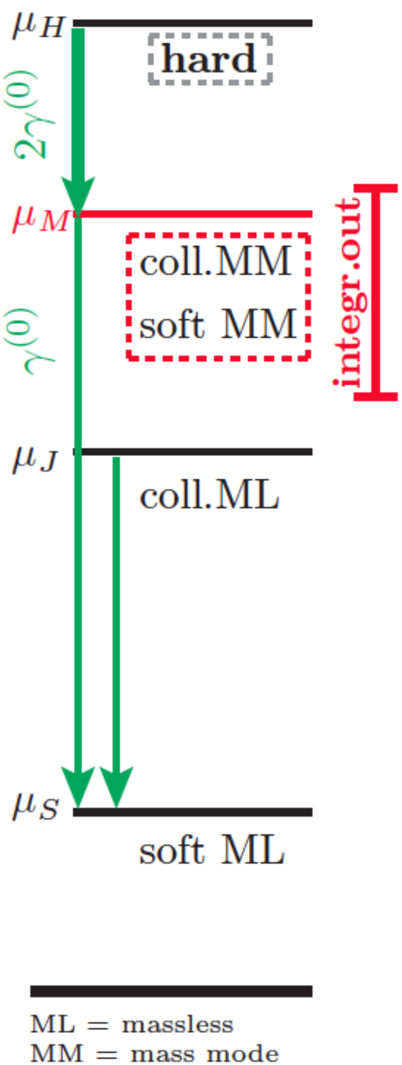
Scenario 2: 1> λ_m > λ > λ^2 (Q > m > J > S)



- Massive modes only virtual
- Jet and soft function as in massless case
- Hard coefficient must have massless limit
- Known Sudakov problem for massive gauge boson

Chiu, Golf, Kelley, Manohar
Chiu, Führer, Hoang, Kelley

VFN Scheme: Secondary Massive Quarks



mass modes enter SCET, but integrated out before the jet scale

$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^{II}(\mu_H)\right|^2 U_H^{(1)}(\mu_H,\mu_M)\left|\mathcal{M}_H(\mu_M)\right|^2 U_H^{(0)}(\mu_M,\mu_S) \times \int d\ell \int ds\, J_0(s,\mu_J)\, U_J^{(0)}(Q\ell - s,\mu_J,\mu_S)\, S_0(Q\tau-\ell,\mu_S)$$

$\mathcal{C}^{II}(\mu_H) = \mathcal{C}^{I}(\mu_H) - \delta F_m^{\rm eff}(\mu_H)$ ⟵ Contains all mass-singularities

$\mathcal{M}_H(\mu_M) = 1 + \delta F_m^{\rm eff}(\mu_M)$

$\delta F_m^{\rm eff}$: massive SCET contribution

continuity to scenario I for $\mu_M = \mu_H$:

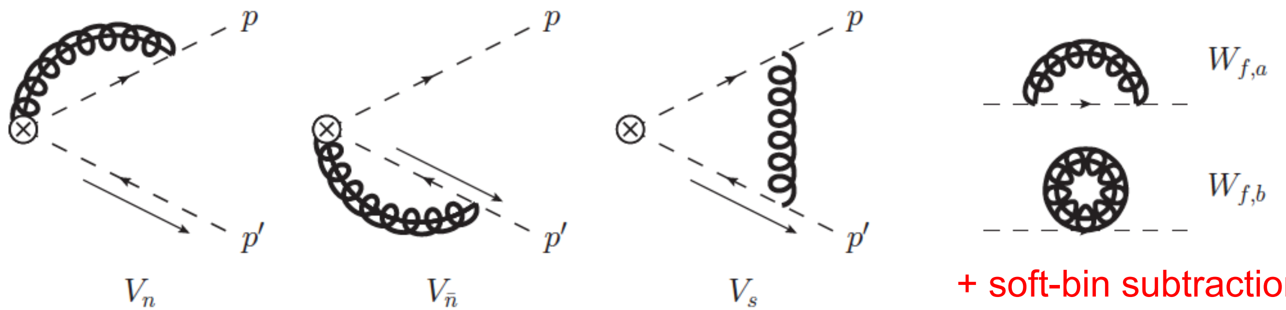
⟶ $\left|\mathcal{C}^{II}(\mu_H)\right|^2 \left|\mathcal{M}_H(\mu_H)\right|^2 = \left|\mathcal{C}^{I}(\mu_H)\right|^2$

$U_i^{(0)}$ stands for: (a) massive gluon integrated out
(b) (n_l)-evolution

$U_i^{(1)}$ stands for: (a) massive gluon dynamical
(b) (n_l+1)-evolution

VFN Scheme: Secondary Massive Quarks

Scenario 2: mass mode SCET calculation



+ soft-bin subtractions
(collinear for $k^2=M^2$)

$$\delta F_m^{\mathrm{eff}}(Q,M,\mu) = \frac{\alpha_s C_F}{4\pi}\left\{\ln\left(\frac{M^2}{\mu^2}\right)\left[2\ln\left(\frac{-Q^2}{\mu^2}\right) - \ln\left(\frac{M^2}{\mu^2}\right) - 3\right] - \frac{5\pi^2}{6} + \frac{9}{2}\right\}$$

Chiu, Golf, Kelley, Manohar (2008)

Chiu, Fuhrer, Hoang, Kelley, Manohar (2009)

rapidity logarithms

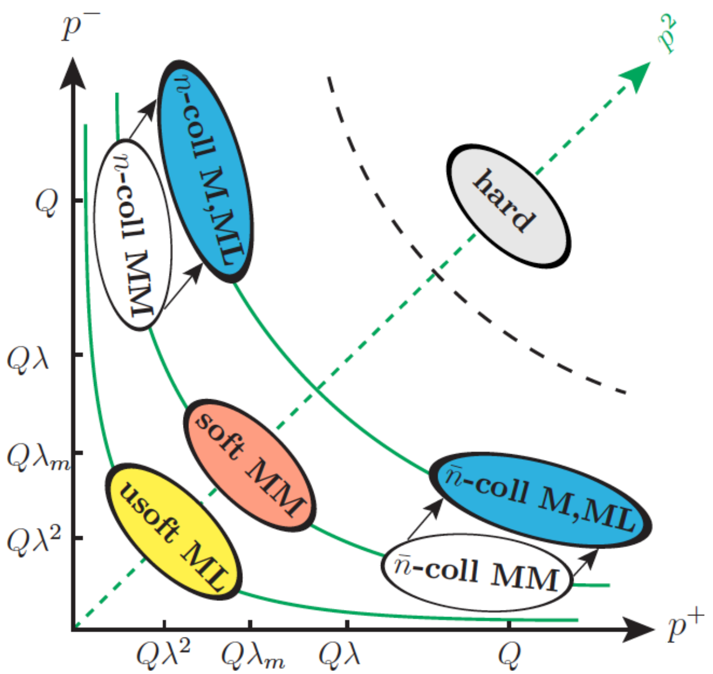
large logarithm $\ln\left(\frac{M^2}{\mu_H^2}\right)$ cancels between $\mathcal{C}^I$ and $\delta F_m^{\mathrm{eff}}$

correct massless limit for $\mathcal{C}^{II}(\mu_H)$:

$$\mathcal{C}^{II}(Q,M,\mu_H) = \mathcal{C}^I(Q,M,\mu_H) - \delta F_m^{\mathrm{eff}}(Q,M,\mu_H) \xrightarrow{M\to 0} 2C_0(Q,\mu_H)$$

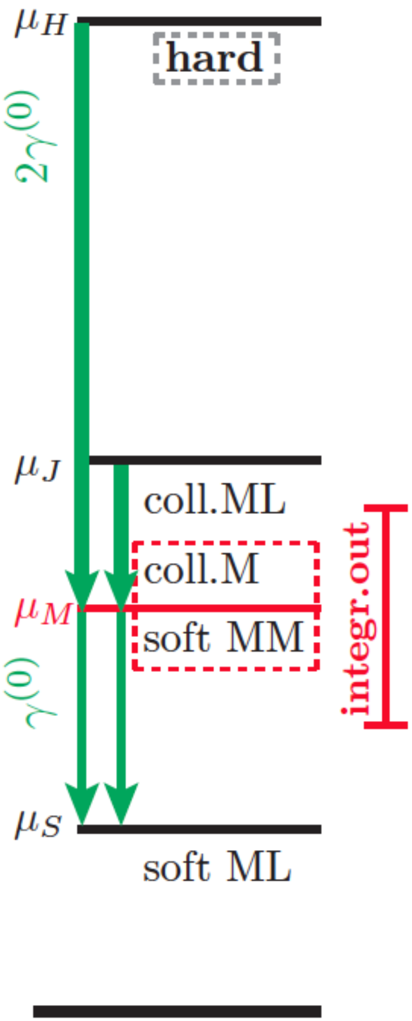
VFN Scheme: Secondary Massive Quarks

Scenario 3: 1 > λ > λ_m > λ^2 (Q > J > m > S)



- Current evolution unchanged w.r. to Scen. 2
- Hard coefficient must have massless limit
- Jet function has massless limit
- Massive and massless collinear in same sector
- Collinear mass modes integrated out at m

VFN Scheme: Secondary Massive Quarks



ML = massless
MM = mass mode
M = massive

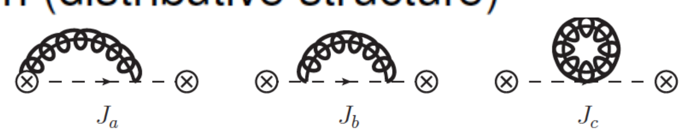
mass modes enter jet sector,
but integrated out before the soft scale

$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^{II}(\mu_H)\right|^2 U_H^{(1)}(\mu_H,\mu_M)\left|\mathcal{M}_H(\mu_M)\right|^2 U_H^{(0)}(\mu_M,\mu_S)$$
$$\times \int d\ell \int ds \int ds' \int ds''\, J_{0+m}(s,\mu_J)\, U_J^{(1)}(s'-s,\mu_J,\mu_M)$$
$$\times\, \mathcal{M}_J(s''-s',\mu_M)\, U_J^{(0)}(s''-Q\ell,\mu_M,\mu_S)\, S_0\left(Q\tau-\ell,\mu_S\right)$$

$$J_{0+m}(s,\mu_J) = J_0(s,\mu_J) + \delta J_m^{\mathrm{virt}}(s,\mu_J) + \theta(s-M^2)\,\delta J_m^{\mathrm{real}}(s)$$

$\delta J_m^{\mathrm{virt}}$: virtual piece of jet function (distributive structure)

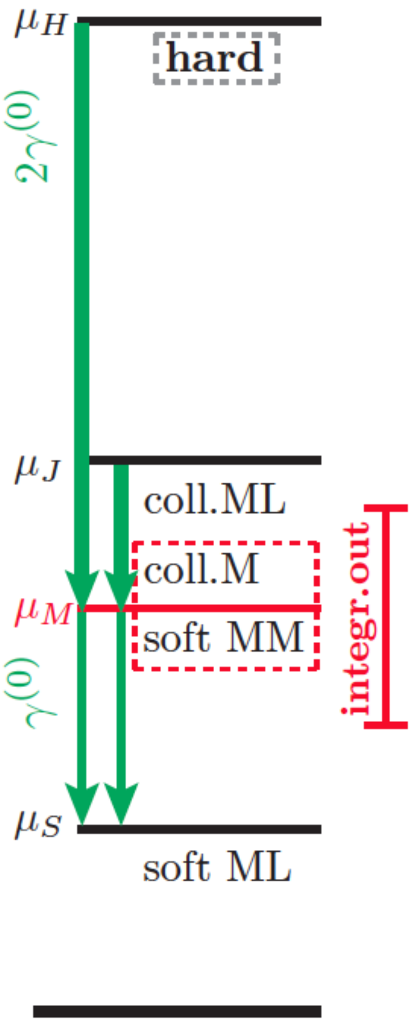
- Soft-bin subtraction
- Rapidity singularities cancel
- UV divergences agree with massless case



$\delta J_m^{\mathrm{real}}$: real radiation piece of jet function (function)

- finite
- sum of virtual and real: rapidity logs cancel
- sum of virtual and real: approaches massless jet function for m $\rightarrow$ 0

VFN Scheme: Secondary Massive Quarks



ML = massless
MM = mass mode
M = massive

mass modes enter jet sector,
but integrated out before the soft scale

$$
\begin{aligned}
\frac{d\sigma}{d\tau} \sim & \left|\mathcal{C}^{ll}(\mu_H)\right|^2 U_H^{(1)}(\mu_H,\mu_M)\left|\mathcal{M}_H(\mu_M)\right|^2 U_H^{(0)}(\mu_M,\mu_S) \\
& \times \int d\ell \int ds \int ds' \int ds''\, J_{0+m}(s,\mu_J)\, U_J^{(1)}(s'-s,\mu_J,\mu_M) \\
& \times \mathcal{M}_J(s''-s',\mu_M)\, U_J^{(0)}(s''-Q\ell,\mu_M,\mu_S)\, S_0\,(Q\tau-\ell,\mu_S)
\end{aligned}
$$

$$J_{0+m}(s,\mu_J) = J_0(s,\mu_J) + \delta J_m^{\rm virt}(s,\mu_J) + \theta(s-M^2)\,\delta J_m^{\rm real}(s)$$

$\delta J_m^{\rm virt}$: virtual piece of jet function (distributive structure)

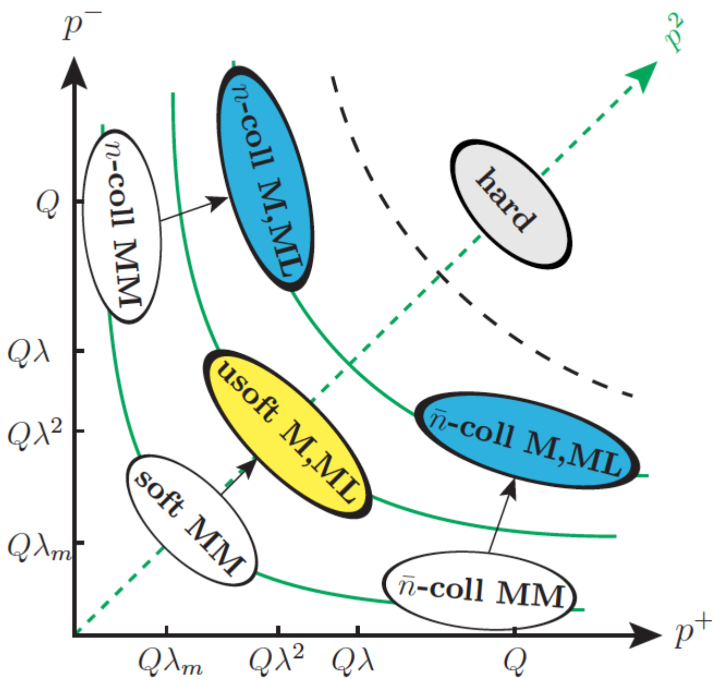
$$\mathcal{M}_J(s,\mu_M) = \delta(s) - \delta J_m^{\rm virt}(s,\mu_M)$$

continuity to scenario II for $\mu_M = \mu_J$ ($\mu_M \leq M$):

$$J_{0+m}(s,\mu_J)\,\mathcal{M}_J(s,\mu_J) = J_0(s,\mu_J)$$

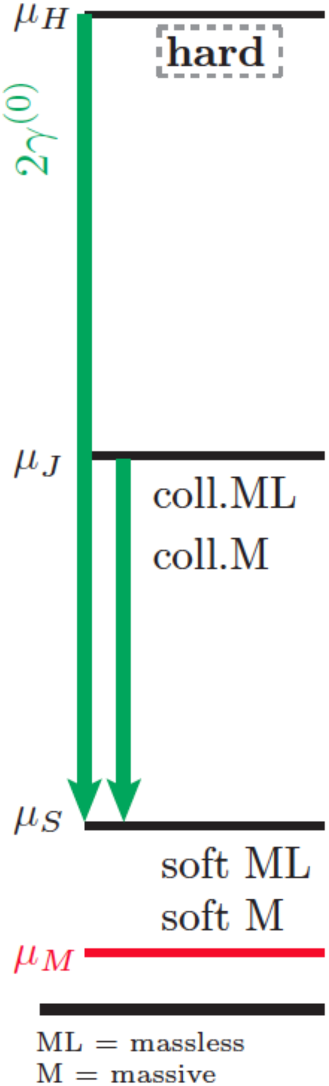
VFN Scheme: Secondary Massive Quarks

Scenario 4: $1 > \lambda > \lambda^2 > \lambda_m$ (Q > J > S > m)



- Current evolution unchanged w.r. to Scen. 2
- Jet function and evolution as in Scen. 2
- Massive and massless coll. modes same sector
- Massive and massless soft modes same sector
- Hard coefficient, jet and soft function must have massless limit
- All RG-evolution for (n_l+1) flavors

VFN Scheme: Secondary Massive Quarks



mass modes enter all sectors

$$\frac{d\sigma}{d\tau} \sim \left|C^{II}(\mu_H)\right|^2 U_H^{(1)}(\mu_H,\mu_S) \times \int d\ell \int ds\, J_{0+m}(s,\mu_J)\, U_J^{(1)}(Q\ell - s,\mu_J,\mu_S)\, S_{0+m}(Q\tau-\ell,\mu_S)$$

$$S_{0+m}(\ell,\mu_S) = S_0(\ell,\mu_S) + \delta S_m^{\rm virt}(\ell,\mu_S) + \theta(\ell - M)\,\delta S_m^{\rm real}(\ell)$$

$\delta S_m^{\rm virt}$: virtual piece of massive soft function (distributive structure)

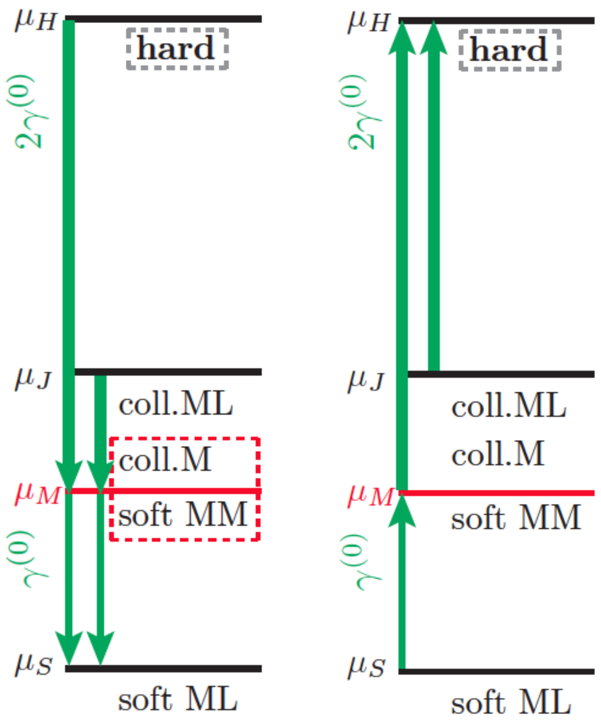
- Rapidity singularities cancel between contributions from both hemispheres (+,-)
- UV divergences agree with massless case

$\delta S_m^{\rm real}$: real radiation piece of massive soft function (function)

- finite
- sum of virtual and real: rapidity logs cancel
- sum of virtual and real: approaches massless soft function for m → 0

Consistency Conditions: Threshold Corrections

Important role of consistency relation: soft – jet – hard for scenario III



alternative description in bottom-up running ($\mu \sim \mu_H$):

$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^{II}(\mu_H)\right|^2 \int d\ell \int d\ell' \int d\ell'' \int ds \int ds'$$
$$\times\, U_J^{(1)}(s-s',\mu_J,\mu_H)\, J_0(s',\mu_J)\, U_S^{(1)}(\ell''-s/Q,\mu_M,\mu_H)$$
$$\times\, \mathcal{M}_S(\ell'-\ell'',\mu_M)\, U_S^{(0)}(\ell-\ell',\mu_S,\mu_M)\, S_0\left(Q\tau-\ell,\mu_S\right)$$

$$\mathcal{M}_S(\ell,\mu_M) = \delta(\ell) + \delta S_m^{\rm virt}(\ell,\mu_M)$$

consistency relation: $\boxed{\mathcal{M}_S(\ell,\mu_M) = Q\,\left|\mathcal{M}_H(\mu_M)\right|^2 \mathcal{M}_J(Q\ell,\mu_M)}$

similarly: $U_S^{(1)}(\ell,\mu_S,\mu_M) = Q\, U_H^{(1)}(\mu_M,\mu_S)\, U_J^{(1)}(Q\ell,\mu_M,\mu_S)$

VFN Scheme: Threshold Corrections

The calculation of the mass mode matching corrections for current, jet and soft function can be carried out by matching the factorization theorem to a full QCD calculation.

But there is a more efficient method based on the fact that current, jet and soft functions are gauge-invariant quantities that can be renormalized separately.

- Evolution with VFN and matching can be related to the use of different renormalization conditions within a single effective theory.
- Use scenario 4 effective theory where the massive quark is contained in hard, collinear and soft sectors.

Example: Jet function $\quad J^{\text{bare}} = Z_J^{OS} \otimes J^{\text{OS}} = Z_J^{\overline{\text{MS}}} \otimes J^{\overline{\text{MS}}}$

On-shell condition: decoupling for m→∞ : (n_l-flavor scheme)

$$J^{\text{OS}}(s,m,\mu) = J^{(n_l)}(s,\mu) + \theta(s-4m^2)\,\delta J_m^{\text{real}}(s,m) \xrightarrow{m \gg s} J^{(n_l)}(s,\mu)$$

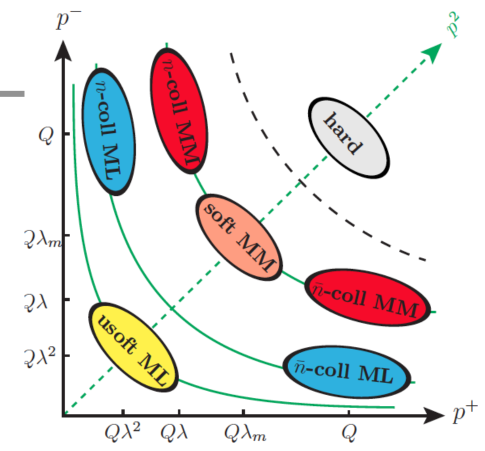
$\overline{\text{MS}}$ condition: massless limit for m→0 : ((n_l+1)-flavor scheme)

$$J^{\overline{\text{MS}}}(s,m,\mu) = J^{(n_l+1)}(s,\mu) + \delta J_m^{\text{dist}}(s,m,\mu) + \theta(s-4m^2)\,\delta J_m^{\text{real}}(s,m) \xrightarrow{m \ll s} J^{(n_l+1)}(s,\mu)$$

$$\Rightarrow \quad \boxed{\mathcal{M}_J(s,m,\mu) = J^{\overline{\text{MS}}}(s,m,\mu) \otimes \left(J^{\text{OS}}(s,m,\mu)\right)^{-1}}$$

- Renormalization approach automatically implies (perturbative) continuity of the evolution through the MM threshold $\rightarrow$ no scale hierarchies are involved/needed anywhere!

Rapidity Logarithms



- Secondary mass effects start at $O(\alpha_s^2)$
- Counting for rapidity logs: α_s Log ~ 1
- At $O(\alpha_s^2)$:
  - No resummation to all orders needed
  - Need terms at $O(\alpha_s^3$ Log) and $O(\alpha_s^4$ Log$^2)$

$O(\alpha_s)$

$O(\alpha_s^2)$

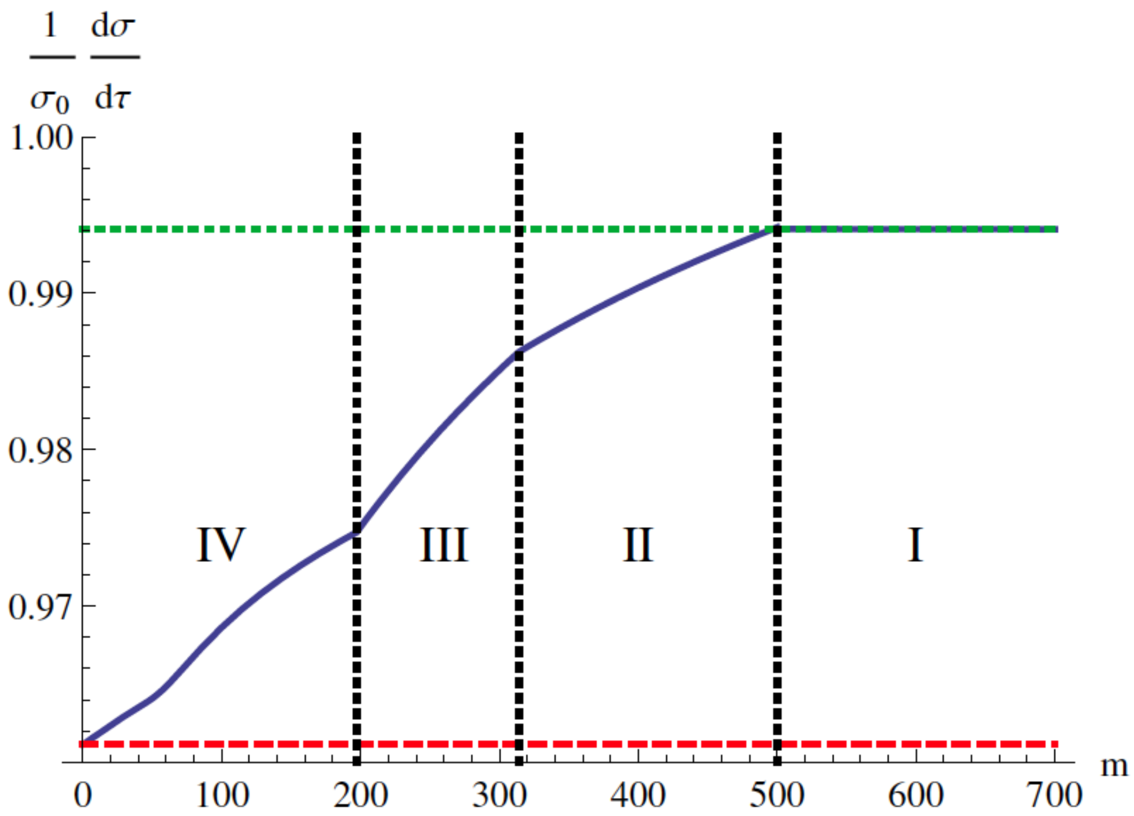
$$
\begin{aligned}
\mathcal{M}_H(Q,m,\mu_m) = 1 &+ \boxed{\frac{\left(\alpha_s^{(n_l+1)}\right)^2 C_F T_F}{(4\pi)^2}\ln\left(-\frac{\mu_m^2}{Q^2}\right)\left\{\frac{4}{3}L_m^2+\frac{40}{9}L_m+\frac{112}{27}\right\}} \\
&+ \Bigg[\frac{\left(\alpha_s^{(n_l+1)}\right)^2 C_F T_F}{(4\pi)^2}\left\{\frac{4}{9}L_m^3+\frac{38}{9}L_m^2+\left(\frac{242}{27}+\frac{2\pi^2}{3}\right)L_m-\frac{52}{9}\zeta(3)+\frac{875}{54}+\frac{5\pi^2}{9}\right\} \\
&\quad + \frac{\left(\alpha_s^{(n_l+1)}\right)^3 C_F T_F}{(4\pi)^3}\ln\left(-\frac{\mu_m^2}{Q^2}\right)\left\{\mathcal{M}_H^{(3)}+\sum_{n=0}^{3}a_n\,L_m^n\right\} \\
&+ \frac{\left(\alpha_s^{(n_l+1)}\right)^4 C_F^2 T_F^2}{(4\pi)^4}\ln^2\left(-\frac{m^2}{Q^2}\right)\left\{\frac{8}{9}L_m^4+\frac{160}{27}L_m^3+\frac{416}{27}L_m^2+\frac{4480}{243}L_m+\frac{6272}{729}\right\}\Bigg]
\end{aligned}
$$

$$L_M = \ln\left(\frac{m^2}{\mu_m^2}\right)$$

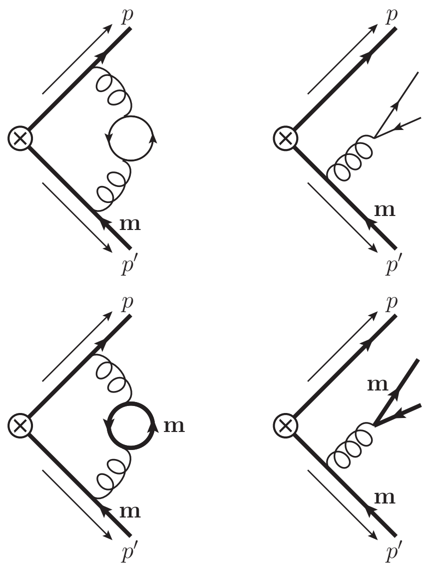
VFN Scheme for Final State Jets

Consistency check: continuous transition and correct limiting behaviour

Thrust distribution: $Q = 500$ GeV, $\tau = 0.15$ fixed, vary mass
massless limit (6 flavors): dashed
decoupling limit (5 flavors): dotted

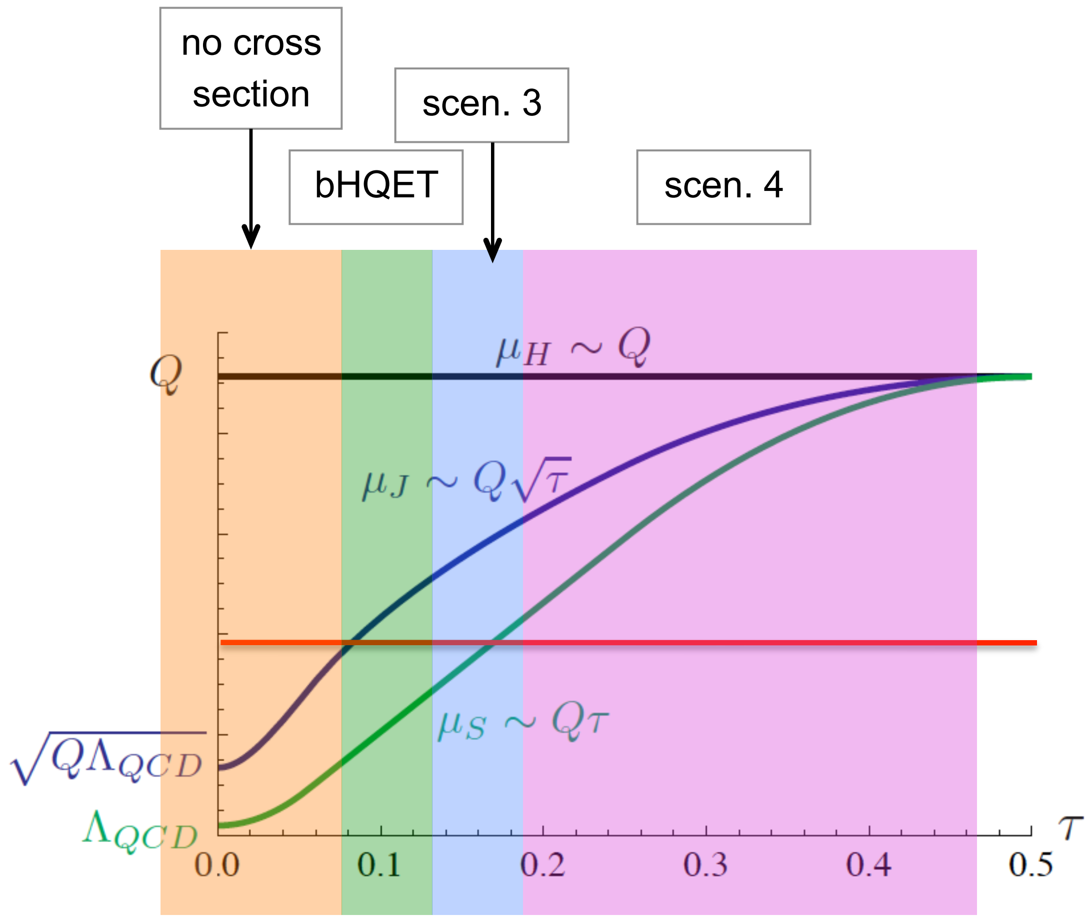


VFN Scheme: Primary Massive Quarks



→ bHQET-type theory when
the jet scale approaches the quark mass
Fleming, AH, Mantry, Stewart

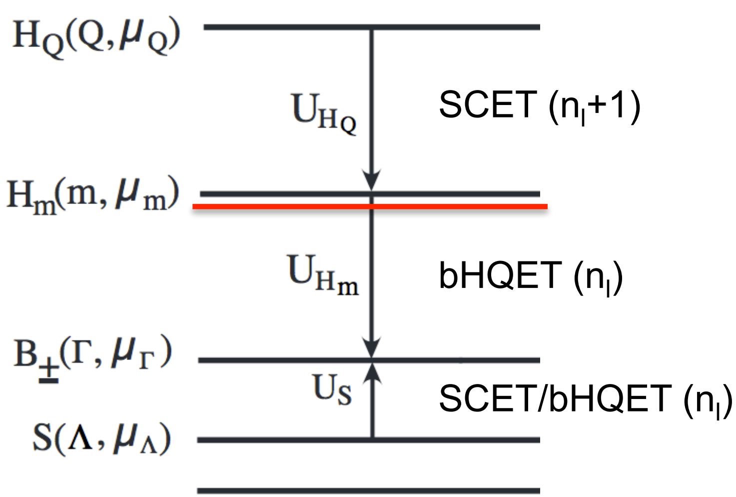
→ two SCET-type theories



VFN Scheme: Primary Massive Quarks

Fleming, AH, Mantry, Stewart (2007)

SCET/bHQET: Q >> J ~ m > Δm > m/Q Δm



- Small components of massive quark integrated out at μ_m~m
- bHQET current evolution for μ < m
- SCET current evolution for μ > m
- Soft function identical to primary massless case (boosted massive quarks)

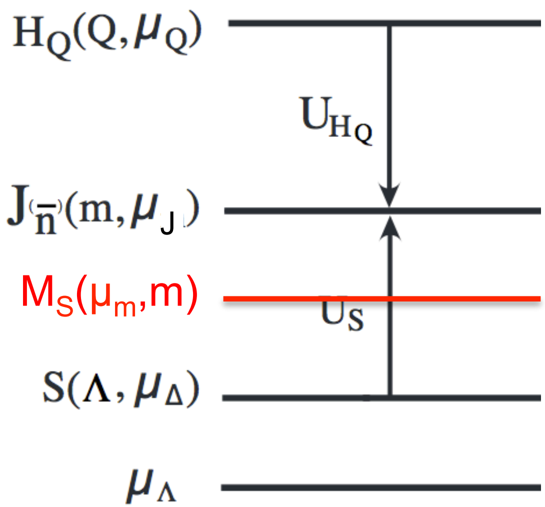
All two-loop FO input now known!
N²LL'/N³LL

$$\left|\frac{1}{\sigma_0}\frac{d\hat\sigma(\tau)}{d\tau}\right|^{\rm bHQET} = Q\, H_Q^{(n_f)}(Q,\mu_Q)\, U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_m)\, H_m^{(n_f)}(\overline{m}^{(n_f)},\mu_m)\, U_{H_m}^{(n_l)}\left(\frac{Q}{\overline{m}^{(n_l)}},\mu_m,\mu_B\right)$$

$$\int ds \int dk\, B^{(n_l)}\left(\frac{s}{m_J^{(n_l)}},\mu_B, m_J^{(n_l)}\right) U_S^{(n_l)}(k,\mu_B,\mu_S)\, S_{\rm part}^{(n_l)}\left(Q\,\tau - Q\,\tau_{\rm MIN} - \frac{s}{Q} - k,\mu_S\right)$$

VFN Scheme: Primary Massive Quarks

SCET scen. 3: Q >> J > m > S



- Same as scenario 3 for primary massless, but with massive jet function

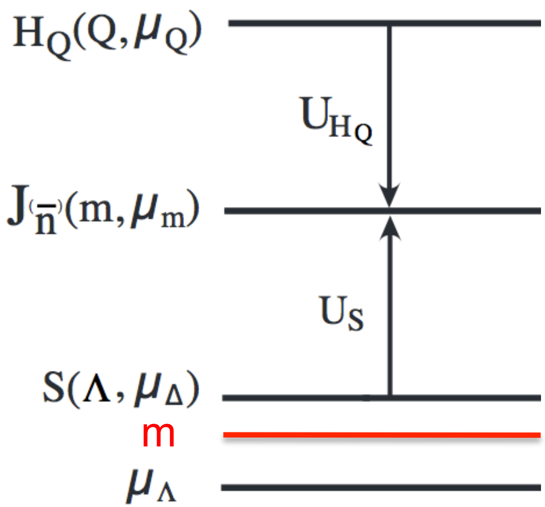
N^2LL'/N^3LL up to two-loop massive SCET jet function.

$$\left|\frac{1}{\sigma_0}\frac{d\hat{\sigma}(\tau)}{d\tau}\right|^{\mathrm{SCET-III}} = Q\, H_Q^{(n_f)}(Q,\mu_Q)\, U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_J) \int \mathrm{d}s \int \mathrm{d}k\, \mathrm{d}k'\, \mathrm{d}k''\, J^{(n_f)}(s,\mu_J,\overline{m}^{(n_f)}(\mu_J))\, U_S^{(n_f)}(k,\mu_J,\mu_m)$$
$$\mathcal{M}_S^{(n_f)}(k'-k,\overline{m}^{(n_f)}(\mu_m),\mu_m,\mu_s) U_S^{(n_l)}(k''-k',\mu_m,\mu_S)\, S_{\mathrm{part}}^{(n_l)}\left(Q\tau - Q\tau_{\min} - \frac{s}{Q} - k'',\mu_S\right)$$

$$n_f = n_\ell + 1$$

VFN Scheme: Primary Massive Quarks

SCET scen. 4: Q >> J > S > m



- Same as scenario 4 for primary massless, but with massive jet function

N^2LL'/N^3LL up to two-loop massive SCET jet function.

✓ Consistency relations: Evolution factors and mass mode threshold corrections

✓ Perturbative continuity

$$\left|\frac{1}{\sigma_0}\frac{\mathrm{d}\hat\sigma(\tau)}{\mathrm{d}\tau}\right|^{\mathrm{SCET-IV}} = Q\,H_Q^{(n_f)}(Q,\mu_Q)\,U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_J)\int\mathrm{d}s\int\mathrm{d}k\,J^{(n_f)}(s,\mu_J,\overline{m}^{(n_f)}(\mu_J))$$
$$U_S^{(n_f)}(k,\mu_J,\mu_S)\,S_{\mathrm{part}}^{(n_f)}\Big(Q\,\tau-Q\,\tau_{\min}-\frac{s}{Q}-k,\mu_S\Big)$$

$$n_f = n_\ell + 1$$

Short-Distance Masses

- Mass dependence in all FO components of all factorization theorems
- Most relevant quark mass dependence contains in the jet functions (SCET & bHQET)
- Mass definition must be close with the scale of the respective functions (→profile functions)

μ ≥ m: MSbar mass (n_l+1)

$$\bar{m}(\mu) = m_{\text{pole}} - \bar{m}(\mu) \sum_{n=1}^{\infty} \sum_{k=0}^{n} a_{nk} \left(\frac{\alpha_s(\mu)}{4\pi}\right)^n \ln^k \frac{\mu}{\bar{m}}$$

→ usual MSbar RG-evolution

μ < m: R-scale short-distance mass (n_l)

- Jet mass: from bHQET jet function
- MSR mass: derived from MSbar mass coefficients
- Many others possible

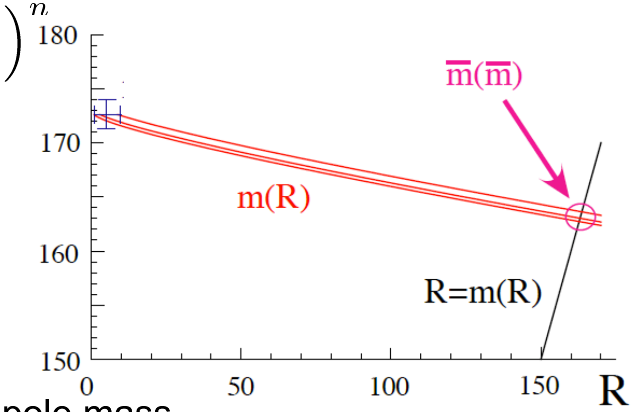
Jain, Scimemi, Stewart 08

Jain, Scimemi, Stewart, AH 08

$$m(R) = m_{\text{pole}} - \delta m(R) \qquad \delta m(R) = R \sum_{n=1}^{\infty} \left(\frac{\alpha_s(R)}{4\pi}\right)^n$$

$$R\frac{d}{dR}m(R) = -\frac{d}{d\ln R}\delta m(R) = R \sum_{n=0}^{\infty} \gamma_n^R \left[\frac{\alpha_s(R)}{4\pi}\right]^{n+1}$$

$$m(R_1) - m(R_0) = \int_{R_0}^{R_1} \frac{dR}{R} R\,\gamma^R[\alpha_s(R)]$$



μ_m~ m: matching: → pert. renormalons-free relation through pole mass

MC vs. SCET: Primary Bottom Production

Preliminary !!

Denahdi, AHH, Mateu

Compare MC with SCET (pQCD, summation, hadronization effects) @ NNLL for Thrust

- Take central values for α_s and Ω_1 from our earlier NNLL thrust analysis for data on all-flavor production (=massless quarks)

$$\alpha_s(M_Z) = 0.1192 \pm 0.006$$
$$\Omega_1 = 0.276 \pm 0.155$$

- Compare with Pythia (m_b^{Pythia}=4.8 GeV) for consistency and mass sensitivity

- Which mass does m_b^{Pythia}=4.8 GeV correspond to for a field theoretic bottom mass?

Abbate,Fickinger, AHH, Mateu, Stewart 2010

order	$\bar{\Omega}_1$ ($\overline{\text{MS}}$)	Ω_1 (R-gap)
NLL$'$	0.264 ± 0.213	0.293 ± 0.203
NNLL	0.256 ± 0.197	$\boxed{0.276 \pm 0.155}$
NNLL$'$	0.283 ± 0.097	0.316 ± 0.072
N^{3}LL	0.274 ± 0.098	0.313 ± 0.071
N^{3}LL$'$ (full)	0.252 ± 0.069	$\mathbf{0.323 \pm 0.045}$
N^{3}LL$'_{(\text{QCD}+m_b)}$	0.238 ± 0.070	0.310 ± 0.049
N^{3}LL$'_{(\text{pure QCD})}$	0.254 ± 0.070	0.332 ± 0.045

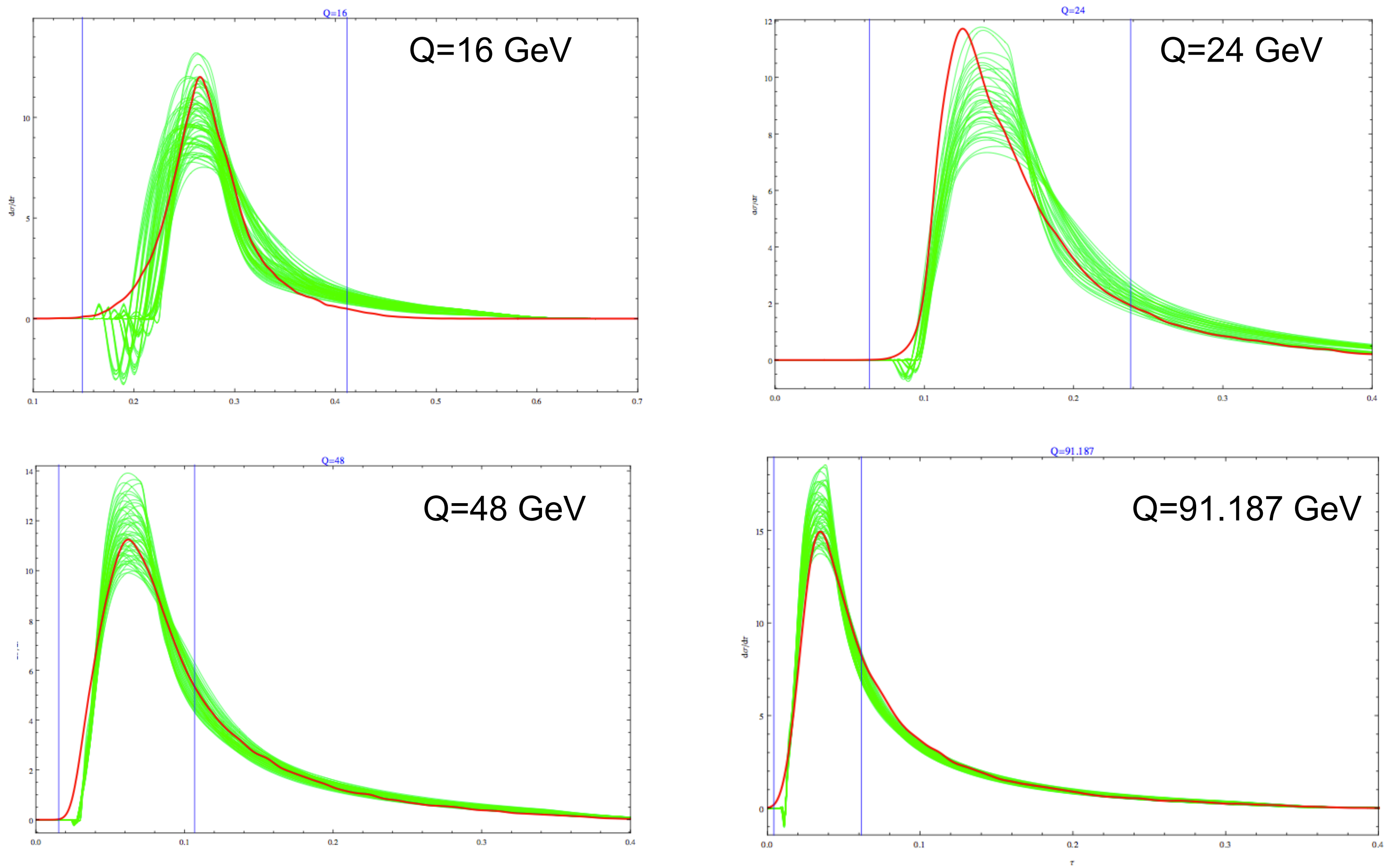
order	$\alpha_s(m_Z)$ (with $\bar{\Omega}_1^{\overline{\text{MS}}}$)	$\alpha_s(m_Z)$ (with Ω_1^{Rgap})
NLL$'$	0.1203 ± 0.0079	0.1191 ± 0.0089
NNLL	0.1222 ± 0.0097	$\boxed{0.1192 \pm 0.0060}$
NNLL$'$	0.1161 ± 0.0038	0.1143 ± 0.0022
N^{3}LL	0.1165 ± 0.0046	0.1143 ± 0.0022
N^{3}LL$'$ (full)	0.1146 ± 0.0021	$\mathbf{0.1135 \pm 0.0009}$
N^{3}LL$'_{(\text{QCD}+m_b)}$	0.1153 ± 0.0022	0.1141 ± 0.0009
N^{3}LL$'_{(\text{pure QCD})}$	0.1152 ± 0.0021	0.1140 ± 0.0008

MC vs. SCET: Primary Bottom Production

Preliminary !! (no fit yet) all NNLL+NLO

Pythia: $m_b^{\text{Pythia}} = 4.8 \text{ GeV}$

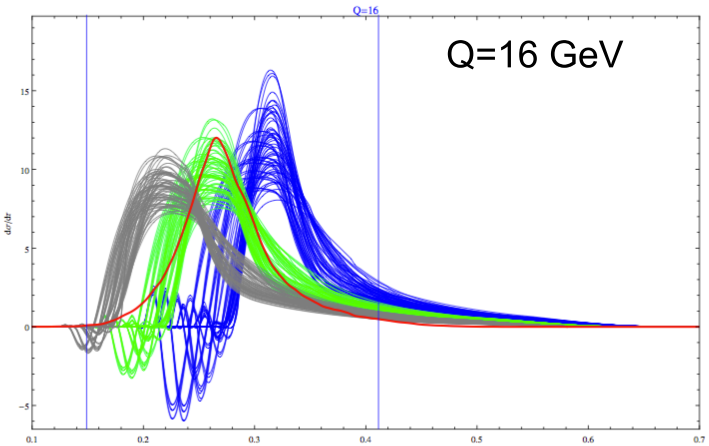
QCD calc.: $\overline{m}_b(\overline{m}_b) = 4.2 \text{ GeV}$ $\alpha_s(M_Z) = 0.1192$ $\Omega_1 = 0.276 \, \text{GeV}$

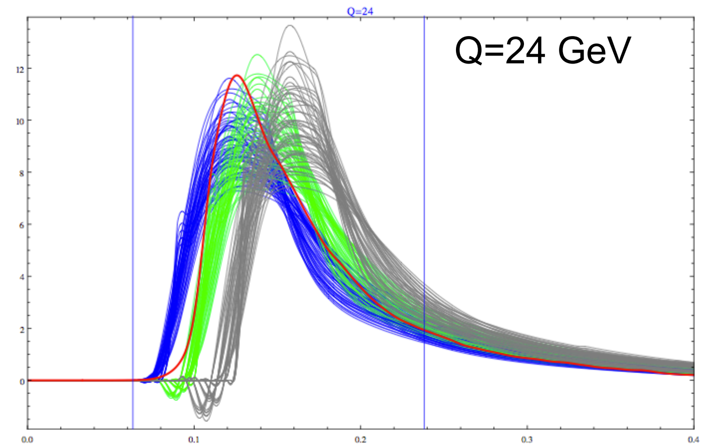


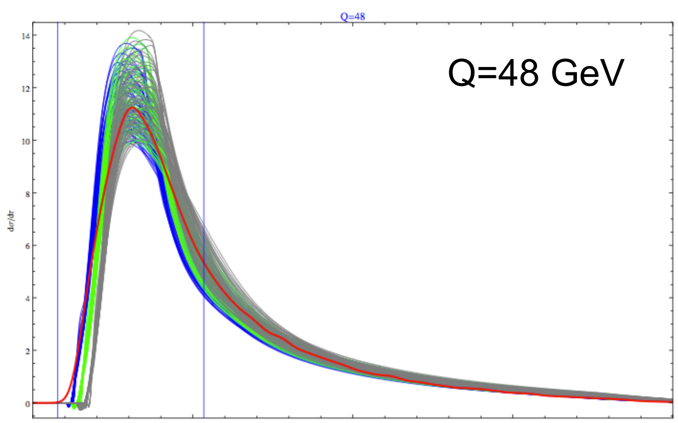
MC vs. SCET: Primary Bottom Production

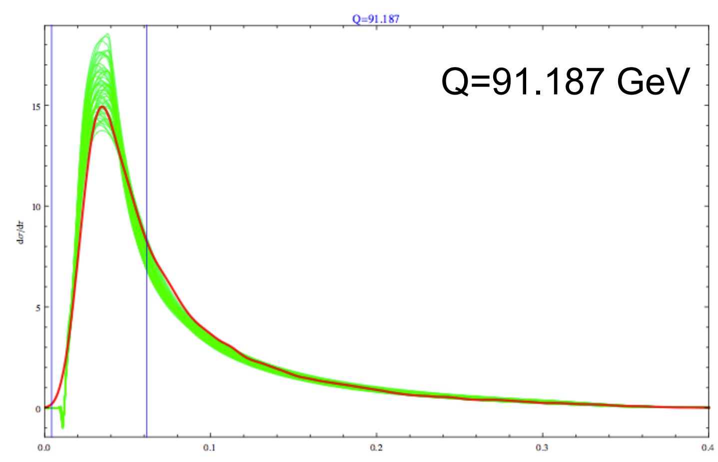
Preliminary !! (No fit yet)

$$\overline{m}_b(\overline{m}_b) = 3.7, 4.2, 4.7\ \mathrm{GeV} \qquad \alpha_s(M_Z) = 0.1192 \qquad \Omega_1 = 0.276\,\mathrm{GeV}$$



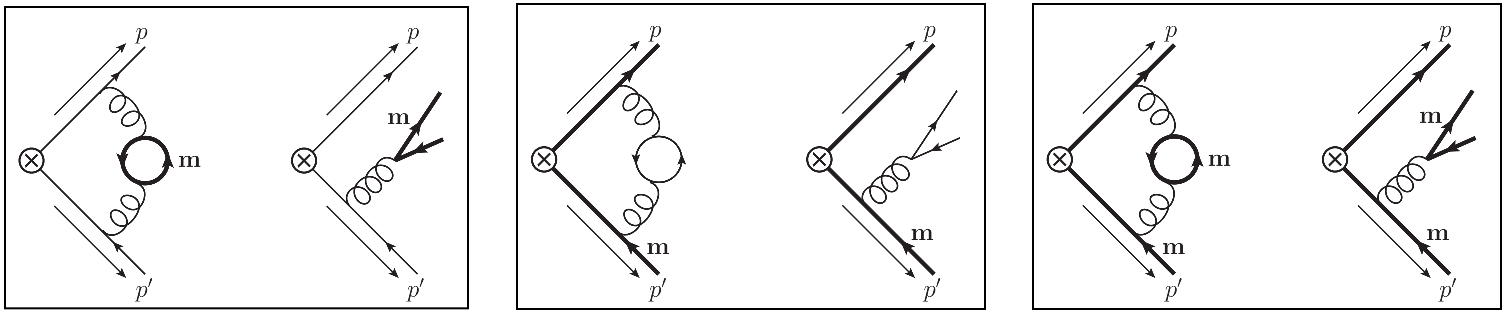






Outlook & Conclusion

$\longrightarrow$ VFN Scheme for final state jets with massive quarks



$\longrightarrow$ Sums all large logarithms including those involving m

$\longrightarrow$ Accounts for full mass dependence

$$\boxed{\begin{array}{c} Q \gg J \gg S \\ \longleftarrow\longleftarrow \quad m \quad \longrightarrow\longrightarrow \end{array}}$$

$\longrightarrow$ Fully consistent with VFNS scheme for PDFs, beam fcts, …

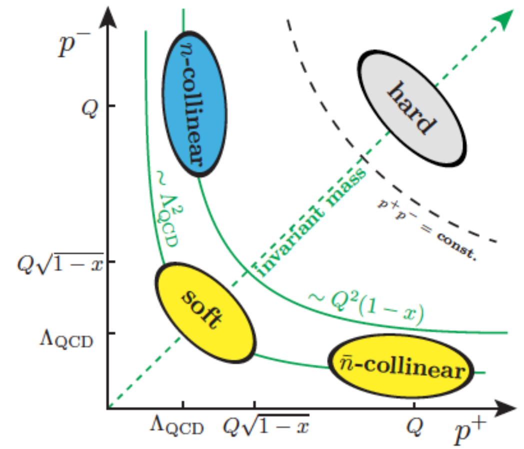
$\longrightarrow$ Allows simplified VFNS implementation in special cases.

$\longrightarrow$ Needs non-trivial mass-dependent ME calculations if mass is of order of another scale

$\longrightarrow$ Interesting applications are coming up.

Consistency with VFNS in DIS (x→1)

- $x \to 1$: experimentally barely accessible (small pdfs!)
  but: nontrivial factorization setup $\to$ interesting as a showcase for concepts
- quite a lot of SCET literature
  `Manohar (2003), Becher, Neubert, Pecjak (2006),`
  `Chay, Kim (2006, 2010, 2013), Fleming, Zhang (2013), ...`
- here: $1 - x \sim \Lambda_{\mathrm{QCD}}/Q$, conveniently: Breit frame



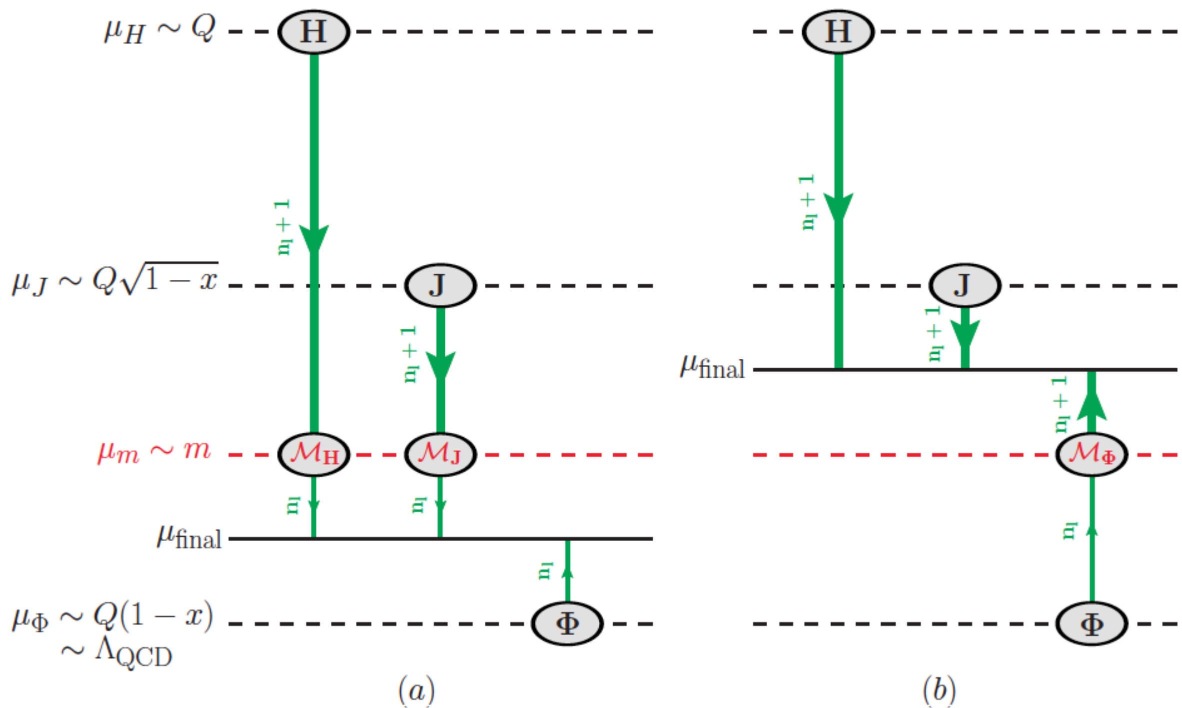
Factorization theorem:

$$F_1 \sim \sum_{i=q} H_{\mathrm{DIS}}(\mu_H) J_{\mathrm{DIS}}(\mu_J) \otimes \underbrace{S_{\mathrm{DIS}}(\mu_\Phi) \otimes f_{i/P}(\mu_\Phi)}_{=\Phi_{i/P}(\mu_\Phi)}$$

Ingredients:

- at $\mu_H \sim Q$: hard function $H_{\mathrm{DIS}}(\mu_H) = |C(\mu_H)|^2$
- at $\mu_J \sim Q\sqrt{1-x}$: final state jet function $J_{\mathrm{DIS}}(\mu_J)$
- at $\mu_\Phi \sim \Lambda_{\mathrm{QCD}}$: pdf $\Phi_{q/P}(\mu_\Phi)$
  $\leftrightarrow$ in SCET II: collinear initial state function $f_{q/P}(\mu_\Phi) \otimes$ soft function $S_{\mathrm{DIS}}(\mu_\Phi)$

Consistency with VFNS in DIS (x→1)



physical cross section independent of $\mu_{\rm final}$ $\rightarrow$ (a) and (b) equivalent

$\rightarrow$ relation between evolution factors

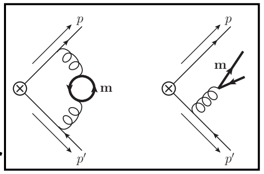
$$U_H^{(n_f)} \times U_J^{(n_f)} = \left(U_\Phi^{(n_f)}\right)^{-1} \quad \text{for } n_f = n_l, n_l+1$$

$\rightarrow$ relation between matching conditions

$$\mathcal{M}_H \times \mathcal{M}_J = \mathcal{M}_\Phi$$

Gap Parameter

- Remove O(Λ) renormalon in partonic soft function
- Gap matching in R-evolution at mass scale
- Subtraction for finite mass not strictly needed, but included to have smooth behavior for massless limit
- R-evolution mass dependent at O(α_s^2)



contains renormalon

$$S(\ell,\mu) \;=\; \int d\ell'\, S_{\rm part}(\ell-\ell',\mu)\, S_{\rm model}(\ell-\Delta)$$

$$\Delta = \bar{\Delta}(R,\mu) + \delta(R,\alpha_s,\mu)$$

renormalon-free

$$S(\ell,\mu) \;=\; \int d\ell'\, S_{\rm part}(\ell-\ell'+\delta,\mu)\, S_{\rm model}(\ell-\bar{\Delta})$$

$$\delta(R,\mu) = \frac{Re^{\gamma_E}}{2}\frac{d}{d\ln(ix)}\Big[\ln \tilde{S}_{\tau,\rm part}(x,\mu)\Big]\Big|_{x=(iRe^{\gamma_E})^{-1}}$$

Kluth, AH 10

μ_m~ m: matching:

Gritschacher, AH, Jemos, Pietrulewicz 2013

$$\bar{\Delta}^{(n_\ell)}(R,\mu) - \bar{\Delta}^{(n_\ell+1)}(R,m,\mu) = e^{\gamma_E}\, R\Big[\Big(\frac{\alpha_s(\mu)}{4\pi}\Big)^2(\delta_{2,m}(R,m,\mu) + \frac{4}{3}T_F\,\delta_1\,\ln\frac{\mu^2}{m^2})\Big]$$