EFT for boosted top production at hadron colliders

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Factorization and resummation for boosted top production

Collaboration with Andrea Ferroglia, Li Lin Yang, Simone Marzani, Alessandro Broggio

1) Factorization and resummation for large pair invariant mass distribution (arXiv:1205.3662)

2) Four Wilson-line soft function to NNLO (arXiv:1207.4798)

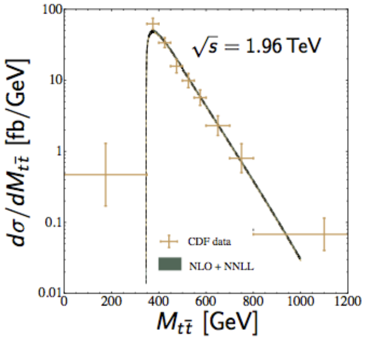
3) An NNLO soft plus virtual approximation for $d\sigma/dM_{t\bar{t}}$ in $m_t \ll M_{t\bar{t}}$ limit (arXiv:1306.1537)

4) Factorization and resummation for single-particle inclusive distributions (i.e. $d\sigma/dp_T$) (arXiv:1310.3836)

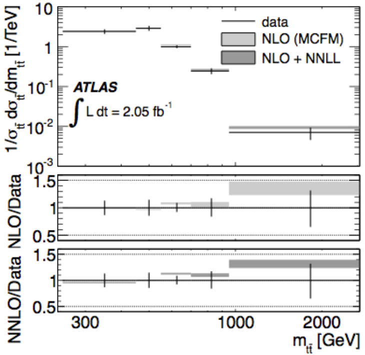
5) Massless hard function to NNLO (for all two-to-two QCD processes: arXiv:1409.5294)

Production at high top-pair invariant mass

Tevatron $\sqrt{s} \approx 2\ TeV$



LHC: $\sqrt{s} = 7\ TeV$



- LHC has data in "boosted regime" $M_{t\bar{t}} \gg m_t$
- not just "corner of phase space": important for new physics searches

QCD corrections in boosted top production

Consider very large pair invariant mass where $\tau = M_{t\bar{t}}^2/s \to 1$

$$\frac{d\sigma}{dM_{t\bar{t}}}(s, m_t, M_{t\bar{t}}) = \sum_{i,j} \int_{\tau}^{1} \frac{dz}{z}\, ff_{ij}(\tau/z, \mu_f)\, \frac{d\hat{\sigma}_{ij}}{dM_{t\bar{t}}}(z, m_t, M_{t\bar{t}}, \mu_f)$$

$$ff_{ij}(y, \mu_f) = \int_{y}^{1} \frac{dx}{x}\, f_{i/h_1}(x, \mu_f)\, f_{j/h_2}(y/x, \mu_f)$$

Two kinds of large logarithms appear:

- soft logs: $[\ln(1-z)/(1-z)]_+ \quad (z \equiv M_{t\bar{t}}^2/\hat{s})$
- small-mass (collinear) logs: $\ln m_t/M_{t\bar{t}}$

Goal: set up a framework which can factorize cross sections and resum both types of logs, i.e. understand factorization in double soft and small-mass limit

Reference point: factorization of partonic cross sections in the soft limit

1) Pair invariant mass distributions (PIM kinematics i.e. $d\sigma/dM$)

$$d\hat{\sigma}_{ij}(z, M, m_t, \cos\theta, \mu_f) = \mathrm{Tr}\left[\mathbf{H}_{ij}^{m}(M, m_t, t_1, \mu_f)\,\mathbf{S}_{ij}^{m,\mathrm{PIM}}(\sqrt{\hat{s}}(1-z), m_t, t_1, \mu_f)\right] + \mathcal{O}(1-z)$$

2) Single particle inclusive distributions (1PI kinematics i.e. $d\sigma/dp_T$)

$$d\hat{\sigma}_{ij}(s_4, \hat{s}, \hat{t}_1, \hat{u}_1, m_t, \mu_f) = \mathrm{Tr}\left[\mathbf{H}_{ij}^{m}(\hat{s}, \hat{t}_1, \hat{u}_1, m_t, \mu_f)\,\mathbf{S}_{ij}^{m,1\mathrm{PI}}(s_4, \hat{s}, \hat{t}_1, \hat{u}_1, m_t, \mu_f)\right] + O\left(\frac{s_4}{m_t^2}\right)$$

- both cases known to NNLL [Ahrens, Ferroglia, Neubert, BP, Yang; Kidonakis], enough for all soft logs and μ-dependent terms at NNLO
- In small-mass limit both $\mathbf{H}^m$ and $\mathbf{S}^m$ have logarithms in $\ln m_t^2/\hat{s}$

PIM kinematics: factorization with fragmentation functions in small-mass limit

Idea: start with well-known factorization theorem for energetic top-quark production [Mele and Nason 1990]

Generic heavy-quark production cross section (i.e. $e^+e^- \to t + X$)

$$\frac{d\sigma_t}{dz}(z, m_t, \mu) = \sum_a \int_z^1 \frac{dx}{x} \frac{d\tilde{\sigma}_a}{dx}(x, m_t = 0, \mu) D_{a/t}^{(n_l+n_h)}\left(\frac{z}{x}, m_t, \mu\right)$$

- a lot like factorization with PDFs, just apply to our case
- then weave together with soft limit of each function [arXiv:1205.3662]

caveat: closed heavy-quark loops proportional to powers of N_h complicate things, leave aside in rest of talk

First layer: the small-mass limit

1) When $m_t \ll M \equiv M_{t\bar{t}}$ $\quad\left(f(z) \otimes g(z) \equiv \int_z^1 \frac{dx}{x} f(x)\, g(z/x)\right)$

$$\frac{d\hat{\sigma}}{dM}(M, m_t, z, \mu) \sim \frac{d\hat{\sigma}}{dM}(M, z, m_t = 0, \mu) \otimes D_{t/t}^2(m_t, z, \mu) + \mathcal{O}\left(\frac{m_t^2}{M^2}\right)$$

- m_t dependence factorized into heavy-quark fragmentation function $D_{t/t}$ (collinear emissions)
- M dependence factorized into massless partonic cross section (wide-angle emissions)

- next step: factorize $d\hat{\sigma}/dM$ and $D_{t/t}$ in soft limit

Second layer: soft limit ($z \to 1$)

1) Massless partonic cross section

$$d\hat{\sigma}_{ij}(z, M, t_1, \mu_f) = \mathrm{Tr}\left[\mathbf{H}_{ij}(M, t_1, \mu_f)\,\mathbf{S}_{ij}(\sqrt{\hat{s}}(1-z), t_1, \mu_f)\right] + \mathcal{O}(1-z)$$

- $\mathbf{H}_{ij}$ related to virtual corrections to massless 2→2 scattering
- $\mathbf{S}_{ij}$ related to soft real corrections to massless 2→2 scattering

2) Fragmentation function

$$D_{t/t}^{(n_l)}(z, m_t, \mu_f) = C_D(m_t, \mu_f)\,S_D(m_t(1-z), \mu_f) + \mathcal{O}(1-z)$$

- C_D related to virtual corrections to fragmentation function
- S_D related to real corrections to fragmentation function (and proposed to be equivalent to perturbative shape function [Gardi '05 ;Neubert '07])

Factorization at NNLO

The factorized cross section is

$$\frac{d\hat{\sigma}}{dM} \sim \mathrm{Tr}[\mathbf{H}(M,\mu)\mathbf{S}(M(1-z),\mu)] \otimes C_D^2(m_t,\mu)S_D^2(m_t(1-z),\mu) + \mathcal{O}(1-z) + \mathcal{O}\left(\frac{m_t^2}{M^2}\right)$$

All component functions known at NNLO

- C_D and S_D from fragmentation function for generic z [Melnikov, Mitov '04]
- **H** from virtual corrections to $gg, q\bar{q} \to \bar{Q}Q$ scattering [Glover et. al '00-'01] after IR renormalization procedure [Ferroglia, BP, Yang '13, '14]
- **S from real emission corrections to** $gg, q\bar{q} \to \bar{q}'q'$ **in soft limit** [Ferroglia, BP, Yang '12]

If NNLO corrections are large because of logarithmic corrections, can use renormalization group to resum them to NNLL accuracy (standard SCET methods, momentum space or otherwise)

The massless PIM soft function

Basic object is Wilson loop

$$\mathbf{O}_s(x) = \left[\mathbf{S}_{n_1}\mathbf{S}_{n_2}\mathbf{S}_{n_3}\mathbf{S}_{n_4}\right](x)$$

built out of light-like Wilson lines

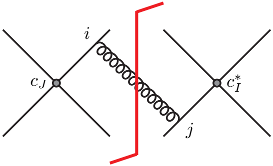
$$\mathbf{S}_i(x) = \mathcal{P}\exp\left(ig_s\int_{-\infty}^{0} ds\; n_i\cdot A^a(x+sn_i)\,\mathbf{T}_i^a\right)$$

Soft function is defined after squaring amplitude:

$$\mathbf{S}(\omega, t_1/M^2, \mu) = \frac{1}{d_R}\sum_{X_s}\langle 0|\mathbf{O}_s^\dagger(0)|X_s\rangle\,\langle X_s|\mathbf{O}_s(0)|0\rangle\;\delta(\omega-(n_1+n_2)\cdot p_{X_s})$$

- for two-to-two scattering $n_i\cdot n_j$ can be expressed in terms of one angle $-t_1/M^2$
- the δ-function sets $\omega = 2E_s = M(1-z)$ in partonic center-of-mass frame

Soft function at NLO



Feynman diagram proportional to (using dim. reg. in $d = 4 - 2\epsilon$ dimensions)

$$I_1(\omega, a_{ij}) = \int d^d k\, \delta(k^2)\, \theta(k^0)\, \frac{n_i \cdot n_j\, \delta(\omega - n_0 \cdot k)}{n_i \cdot k\ n_j \cdot k} \equiv \pi^{1-\epsilon}\, e^{-\epsilon\gamma_E}\, \omega^{-1-2\epsilon}\, \bar{I}_1(a_{ij});$$

$$a_{ij} \equiv 1 - \frac{n_0^2\ n_i \cdot n_j}{2\, n_0 \cdot n_i\ n_0 \cdot n_j}; \qquad \bar{I}_1(a) = \frac{2\, e^{\epsilon\gamma_E}\, \Gamma(-\epsilon)}{\Gamma(1-2\epsilon)}\, (1-a)^{-\epsilon}\, {}_2F_1(-\epsilon, -\epsilon, 1-\epsilon, a)$$

Bare soft function matrix obtained by summing over legs and evaluating color factors

$$\left[\mathbf{S}^{(1)}_{\mathrm{bare}}\right]_{IJ} = \frac{2}{\omega}\left(\frac{\mu}{\omega}\right)^{2\epsilon} \sum_{\text{legs}} \langle c_I | \mathbf{T}_i \cdot \mathbf{T}_j | c_J \rangle\, \bar{I}_1(a_{ij})$$

Bare Soft function at NNLO

Three basic types of topologies at NNLO

- two-Wilson-line graphs (depend on one scale a_{ij})
- three-Wilson-line graphs (depend on two scales a_{ij}, a_{ik})
- four-Wilson-line graphs (depend on two scales a_{ij}, a_{kl} but factorize)

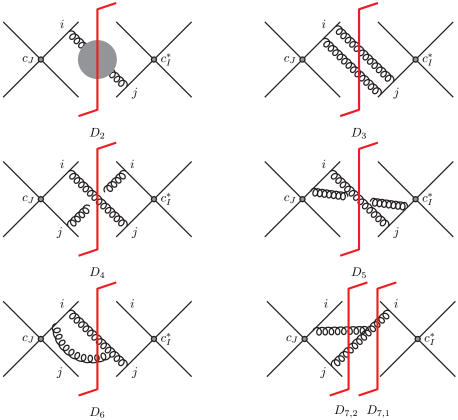
Especially the three-Wilson-line graphs are potentially very difficult, but turned out to be much simpler that we originally thought!

Result at NNLO has form

$$\mathbf{S}^{(2)}_{\text{bare}} = \frac{4}{\omega}\left(\frac{\mu}{\omega}\right)^{4\epsilon}\sum_{\text{legs}}\left(\sum_{n=2}^{7}\mathbf{w}^{(n)}_{ij}\,\bar{I}_n(a_{ij}) + \mathbf{w}^{(8)}_{ijk}\,\bar{I}_8(a_{ij},a_{ik}) + \mathbf{w}^{(9)}_{ijkl}\,\bar{I}_9(a_{ij},a_{kl})\right)$$

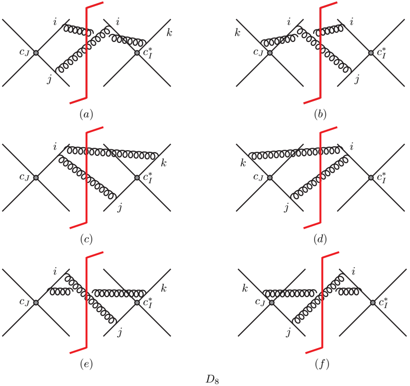
- integrals $\bar{I}_n$ solved in terms of various harmonic polylogarithms (HPLs)
- $1/\epsilon$ poles in bare function subtracted by renormalization procedure (derived from factorization so important cross check)

Two-wilson-line graphs



- parameterize phase-space in terms of light-cone coordinates
- hardest integrals evaluated by expanding in ϵ and deriving and solving differential equations w.r.t a_{ij} (checked with integrals in [Li, Mantry, Petriello '11])

Abelian three-Wilson-line graphs



Graphs (a) and (b) factorize.

Graphs (c)-(f) are complicated, but their sum factorizes

Final result for sum of graphs

$$D_8 \sim \{\mathbf{T}_i^a, \mathbf{T}_i^b\}\, \mathbf{T}_j^a\, \mathbf{T}_k^b \times (1-a_{ij})^{-\epsilon}\, (1-a_{ik})^{-\epsilon} \times {}_2F_1(-\epsilon,-\epsilon,1-\epsilon,a_{ij})\, {}_2F_1(-\epsilon,-\epsilon,1-\epsilon,a_{ik})$$

- factorized form follows from non-abelian exponentiation theorem, which states that coefficient of symmetric color structure is proportional to NLO graph squared

Cancellations

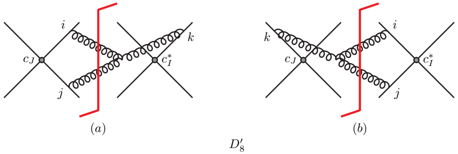


Figure : Examples of non-abelian three-Wilson-line integrals which add to zero.

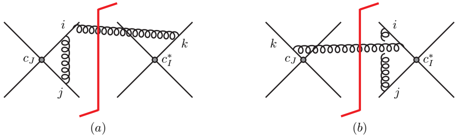


Figure : Example of a pair of mixed virtual-real one-particle cuts which adds up to a scaleless integral.

A STEP BACK

The factorized cross section is

$$\frac{d\hat{\sigma}}{dM} \sim \mathrm{Tr}[\mathbf{H}(M,\mu)\mathbf{S}(M(1-z),\mu)] \otimes C_D^2(m_t,\mu) S_D^2(m_t(1-z),\mu) + \mathcal{O}(1-z) + \mathcal{O}\left(\frac{m_t^2}{M^2}\right)$$

- we derived this by taking $m_t \to 0$ limit then soft limit
- natural question: how to do it the other way around?
- in other words, would like to start with

$$d\hat{\sigma}_{ij}(z, M, m_t, \cos\theta, \mu_f) = \mathrm{Tr}\left[\mathbf{H}_{ij}^m(M, m_t, t_1, \mu_f)\, \mathbf{S}_{ij}^{m,\mathrm{PIM}}(\sqrt{\hat{s}}(1-z), m_t, t_1, \mu_f)\right] + \mathcal{O}(1-z)$$

and subfactorize $\mathbf{H}_{ij}^m$ and $\mathbf{S}_{ij}^m$ in $m_t^2/\hat{s}$ limit...

FACTORIZED HARD FUNCTION IN $m_t^2/\hat{s} \ll 1$ LIMIT

Three pieces

$$|\mathcal{M}(\epsilon, M, m_t, t_1)\rangle = Z_{[q]}(\epsilon, m_t, \mu)\,|\mathcal{M}(\epsilon, M, t_1)\rangle \quad \text{[Mitov/Moch]}$$

$$\lim_{\epsilon\to 0} \mathbf{Z}_m^{-1}(\epsilon, M, m_t, t_1, \mu)\,|\mathcal{M}(\epsilon, M, m_t, t_1)\rangle = |\mathcal{M}_{\text{ren}}(M, m_t, t_1, \mu)\rangle \quad \text{[Neubert/Becher]}$$

$$\lim_{\epsilon\to 0} \mathbf{Z}^{-1}(\epsilon, M, t_1, \mu)\,|\mathcal{M}(\epsilon, M, t_1)\rangle = |\mathcal{M}_{\text{ren}}(M, t_1, \mu)\rangle \quad \text{[Catani; Neubert/Becher]}$$

Combine to get a relation between Z factors and finite matching function f

$$Z_{[q]}(\epsilon, m_t, \mu)\mathbf{Z}_m^{-1}(\epsilon, M, m_t, t_1, \mu) = f(m_t, \mu)\mathbf{Z}^{-1}(\epsilon, M, t_1, \mu)\,.$$

It follows that

$$\mathbf{H}_{ij}^m(M, m_t, t_1, \mu) = f^2(m_t, \mu)\mathbf{H}_{ij}(M, t_1, \mu) + \mathcal{O}\left(\frac{m_t^2}{\hat{s}}\right)\,.$$

Have factorized hard function in $m_t^2 \ll \hat{s}$ limit. Have checked this works to NLO, and to NNLO for μ dependent terms.

Factorization for the soft function

- Work backwards, and use that all m_t dependence should be in fragmentation function $D_{t/t} = C_D S_D$

$$\mathbf{S}^m_{ij}(\sqrt{\hat{s}}(1-z), m_t, t_1, \mu) =$$
$$\mathbf{S}_{ij}(\sqrt{\hat{s}}(1-z), t_1, \mu) \otimes \frac{C_D(m_t,\mu) S_D(m_t(1-z),\mu)}{f(m_t,\mu)} \otimes \frac{C_D(m_t,\mu) S_D(m_t(1-z),\mu)}{f(m_t,\mu)} .$$

- Given that all real emission should be in soft functions S_i, expect $C_D = f$. Instead we found difference at $\mathrm{N}^3\mathrm{LL}$

$$f(m_t,\mu) = C_D(m_t,\mu) - 4\pi^2 C_A C_F \left(\frac{\alpha_s}{4\pi}\right)^2$$
$$C_D^{(2)}(m_t, \mu = m_t) = \frac{21553}{162} + \frac{107\pi^2}{3} - \frac{749\pi^4}{405} + \frac{260\zeta_3}{9} + \frac{16\pi^2}{9}\ln 2 - \left(\frac{1541}{243} + \frac{74\pi^2}{81} + \frac{104\zeta_3}{27}\right) n_l$$

- exists (annoying) mismatch f and C_D (or S_D and shape function)! Only know $D_{t/t} = C_D S_D$ to NNLO from [Melnikov, Mitov '04] so hard to pinpoint problem....

FACTORIZATION FOR 1PI OBSERVABLES (I.E. $d\sigma/dp_T$)

1PI more involved than PIM case, just putting in fragmentation function doesn't work. We instead started from cross section in soft limit and tried to derive small-mass limit from scratch

$$d\hat{\sigma}_{ij}(s_4, \hat{s}, \hat{t}_1, \hat{u}_1, m_t, \mu_f) = \mathrm{Tr}\left[\mathbf{H}^m_{ij}(\hat{s}, \hat{t}_1, \hat{u}_1, m_t, \mu_f)\,\mathbf{S}^{m,\mathrm{1PI}}_{ij}(s_4, \hat{s}, \hat{t}_1, \hat{u}_1, m_t, \mu_f)\right] + \ O\left(\frac{s_4}{m_t^2}\right)$$

- know from PIM case that $\mathbf{H}^m_{ij} = f^2(m_t, \mu)\mathbf{H}_{ij}$
- problem is now: how to factorize multiscale soft function into component parts?
- Dealt with in [arXiv:1310.3836]. We used method of regions to figure out structure then worked with operators to make more formal

NLO MOMENTUM REGIONS IN SMALL-MASS LIMIT

- Basic NLO integral for massive 1PI soft function

$$I_{ij}^m = \pi^{-1+\epsilon} e^{\epsilon\gamma_E} \mu^{2\epsilon} \int d^d k \delta^+(k^2)\, \delta^+(s_4 - 2p_4 \cdot k)\, \frac{p_i \cdot p_j}{p_i \cdot k\, p_j \cdot k}$$
$$\equiv \int [dk]\, \delta^+(s_4 - 2p_4 \cdot k)\, \frac{p_i \cdot p_j}{p_i \cdot k\, p_j \cdot k}$$

- To identify momentum regions use light-cone coordinates:

$$k^\mu = k_{+ij} \frac{n_i^\mu}{\sqrt{2\, n_i \cdot n_j}} + k_{-ij} \frac{n_j^\mu}{\sqrt{2\, n_i \cdot n_j}} + k_{\perp ij}^\mu$$

- Using $i = 3$ and $j = 4$ have in partonic c.m. frame ($\lambda = m_t/\sqrt{\hat{s}}$) have scalings of $p_i^\mu = (p_{4+}, p_{4k-}, p_{4k\perp})$

$$p_3^\mu \sim \sqrt{\hat{s}}(1, \lambda^2, \lambda), \quad p_4^\mu \sim \sqrt{\hat{s}}(\lambda^2, 1, \lambda); \quad p_3^2 = p_4^2 = m_t^2$$

$$p_1^\mu \sim \sqrt{\hat{s}}(1, 1, 1), \quad p_2^\mu \sim \sqrt{\hat{s}}(1, 1, 1); \quad p_1^2 = p_2^2 = 0$$

Momentum regions and factorization

- Regions analysis shows that three momentum regions contribute to massive soft function in limit $m_t^2 \ll \hat{s}$

$$k_s^\mu \sim \frac{s_4}{\sqrt{\hat{s}}} \sim \sqrt{\hat{s}}\,\frac{s_4}{m_t^2}\,(\lambda^2,\lambda^2,\lambda^2) \qquad \text{(soft, wide angle)}$$

$$k_{sc}^\mu \sim \frac{s_4\,p_3^\mu}{\hat{s}} \sim \sqrt{\hat{s}}\,\frac{s_4}{m_t^2}\,(\lambda^2,\lambda^4,\lambda^3) \qquad \text{(soft, collinear to the top)}$$

$$k_{sc'}^\mu \sim \frac{s_4\,p_4^\mu}{m_t^2} \sim \sqrt{\hat{s}}\,\frac{s_4}{m_t^2}\,(\lambda^2,1,\lambda) \qquad \text{(soft, collinear to the anti-top)}$$

- This leads to factorized form (checked explicitly to NNLO)

$$\begin{aligned}\mathbf{S}_{ij}^m(s_4,\hat{s},\hat{t}_1,\hat{u}_1,m_t,\mu) = &\int d\omega_s\,d\omega_d\,d\omega_b\,\delta(s_4-\omega_s-\omega_d-\omega_b)\\ &\times \mathbf{S}_{ij}\left(\omega_s,\frac{\omega_s}{\sqrt{\hat{s}}},x_t,\mu\right) S_D\left(\omega_d,\frac{\omega_d\,m_t}{\hat{s}},\mu\right) S_B\left(\omega_b,\frac{\omega_b}{m_t},\mu\right)\\ &+\mathcal{O}(s_4/m_t^2)+\mathcal{O}(m_t^2/\hat{s})\end{aligned}$$

- Can see structure through regions calculation (next), but helps to have operator definitions in SCET (later) to connect with literature (and do all orders 'proof')

Regions I: Wide angle soft

Can approximate $p_i^\mu \sim E_i n_i^\mu$, $n_i^2 = 0$. The soft integral is then

$$
\begin{aligned}
I_{ij}^s &= \int [dk_s]\, \delta^+(s_4 - \sqrt{\hat{s}} n_4 \cdot k_s)\, \frac{n_i \cdot n_j}{n_i \cdot k_s \; n_j \cdot k_s} \\
&= \frac{1}{s_4} \left(\frac{s_4}{\sqrt{\hat{s}}\mu} \right)^{-2\epsilon} \left(\frac{2 n_i \cdot n_j}{n_4 \cdot n_i \; n_4 \cdot n_j} \right)^{-\epsilon} \left(-\frac{2}{\epsilon} + \frac{\pi^2}{6}\epsilon \right)
\end{aligned}
$$

- characteristic scale is $\mu_s \sim s_4/\sqrt{\hat{s}}$
- like an eikonal factor for "massless" partons
- total contribution from region after summing all over all legs:

$$
I^s = 2\mathbf{T}_1 \cdot \mathbf{T}_2 \, I_{12}^s + 2\mathbf{T}_1 \cdot \mathbf{T}_3 \, I_{13}^s + 2\mathbf{T}_2 \cdot \mathbf{T}_3 \, I_{12}^s
$$

- matrix in color space, and emissions from parton 4 vanish

Regions II: soft and collinear to top quark

write $p_3^\mu = \sqrt{\hat{s}} n_3^\mu/2 + n_4^\mu m_t^2/2\sqrt{\hat{s}}$, so $v_3^2 = p_3^2/m_t^2 = 1$. Then

$$I^{sc}_{(i\neq 3)3} = \int [dk_{sc}]\, \delta^+(s_4 - \sqrt{\hat{s}} k_{sc}^+)\, \frac{2v_3^+}{(v_3^+ k_{sc}^- + v_3^- k_{sc}^+)\; k_{sc}^+} = \frac{1}{s_4}\left(\frac{s_4 m_t}{\hat{s}\mu}\right)^{-2\epsilon}\left(\frac{1}{\epsilon} + \frac{\pi^2}{12}\epsilon\right)$$

$$I^{sc}_{33} = \int [dk_{sc}]\, \delta^+(s_4 - \sqrt{\hat{s}} k_{sc}^+)\, \frac{1}{v_3\cdot k_{sc}\; v_3 \cdot k_{sc}} = \frac{2}{s_4}\left(\frac{s_4 m_t}{\hat{s}\mu}\right)^{-2\epsilon}$$

- characteristic scale is $\mu_{sc} \sim m_t s_4/\hat{s}$
- integrals for $i \neq 3$ do not depend on n_i
- total contribution from region

$$I^{sc} = \mathbf{T}_3 \cdot \mathbf{T}_3\, I^{sc}_{33} + 2 I^{sc}_{13} \sum_{i\neq 3} \mathbf{T}_i \cdot \mathbf{T}_3 = C_F (I^{sc}_{33} - 2 I^{sc}_{13})$$

- color diagonal! can show this is one loop contribution to soft part of fragmentation function, S_D

Regions III: Soft and collinear to antitop quark

Write $p_4^\mu = \sqrt{\hat{s}} n_4^\mu/2 + n_3^\mu m_t^2/2\sqrt{\hat{s}}$. Then

$$I^{sc'}_{(i\neq 4)4} = \int [dk_{sc'}]\, \delta^+(s_4 - 2m_t v_4\cdot k_{sc'})\, \frac{v_4^-}{v_4\cdot k_{sc'}\;\, k^-_{sc'}} = \frac{1}{s_4}\left(\frac{s_4}{m_t\mu}\right)^{-2\epsilon}\left(-\frac{1}{\epsilon}+\frac{\pi^2}{4}\epsilon\right)$$

$$I^{sc'}_{44} = \int [dk_{sc'}]\, \delta^+(s_4 - 2m_t v_4\cdot k_{sc'})\, \frac{1}{v_4\cdot k_{sc'}\;\, v_4\cdot k_{sc'}} = \frac{1}{s_4}\left(\frac{s_4}{m_t\mu}\right)^{-2\epsilon}(2+4\epsilon)$$

- characteristic scale is $\mu_{sc'} \sim s_4/m_t$
- integrals for $i \neq 4$ do not depend on n_i
- total contribution from region

$$I^{sc'} = \mathbf{T}_4\cdot\mathbf{T}_4\, I^{sc'}_{44} + 2 I^{sc'}_{14}\sum_{i\neq 4}\mathbf{T}_i\cdot\mathbf{T}_4 = C_F(I^{sc'}_{44} - 2 I^{sc'}_{14})$$

- color diagonal! can show this is one loop contribution to the heavy-quark jet function, S_B [Fleming, Mantry, Hoang, Stewart '08]

Factorization of Wilson loop operator I

- full theory operator is

$$\mathbf{S}^m(\omega, \hat{s}, \hat{t}_1, \hat{u}_1, m_t, \mu) = \frac{1}{d_R} \sum_X \langle 0|\mathbf{O}_s^{m\dagger}(0)|X\rangle \langle X|\mathbf{O}_s^m(0)|0\rangle \; \delta(\omega - 2p_4 \cdot p_X)$$

$$\mathbf{O}_s^m(x) = \left[\mathbf{S}_{v_1}^m \mathbf{S}_{v_2}^m \mathbf{S}_{v_3}^m \mathbf{S}_{v_4}^m\right](x); \quad \mathbf{S}_{v_i}^m(x) = \mathcal{P} \exp\left(ig_s \int_0^\infty ds \; v_i \cdot A^a(x + sv_i) \; \mathbf{T}_i^a \right)$$

- to factorize it, first decompose gluon field as

$$A^a \to A_s^a + A_{sc}^a + A_{sc'}^a$$

- then derive and use the following Wilson line identity

$$\mathcal{P} \exp\left[\int_a^b dx \, (A(x) + B(x))\right] = \mathcal{P} \exp\left[\int_a^b dx \, A(x)\right] \; \mathcal{P} \exp\left[\int_a^b dx \left(\mathcal{P} e^{\int_a^x dx' A(x')}\right)^{-1} B(x) \left(\mathcal{P} e^{\int_a^x dx' A(x')}\right)\right]$$

Factorization of Wilson loop operator II

- first define Wilson lines depending on a single mode

$$\mathbf{S}_{v_i}(x) = \mathcal{P}\exp\left(ig_s\int_0^\infty ds\ v_i\cdot A_s^a(x+sv_i)\,\mathbf{T}_i^a\right),$$
$$\mathbf{Y}_{v_i}(x) = \mathcal{P}\exp\left(ig_s\int_0^\infty ds\ v_i\cdot A_{sc}^a(x+sv_i)\,\mathbf{T}_i^a\right),$$
$$\mathbf{Y}'_{v_i}(x) = \mathcal{P}\exp\left(ig_s\int_0^\infty ds\ v_i\cdot A_{sc'}^a(x+sv_i)\,\mathbf{T}_i^a\right)$$

- can then show

$$\mathbf{S}_{v_i}^m(x) = \mathcal{P}\exp\left(ig_s\int_0^\infty ds\ v_i\cdot[A_s^a + A_{sc}^a + A_{sc'}^a](x+sv_i)\ \mathbf{T}_i^a\right) = \mathbf{Y}_{v_i}(x)\,\tilde{\mathbf{S}}_{v_i}(x)\,\tilde{\mathbf{Y}}'_{v_i}(x)$$

with

$$\tilde{\mathbf{S}}_{v_i}(x) = \mathcal{P}\exp\left(ig_s\int_0^\infty ds\left[v_i\cdot A_s^a\,\mathbf{Y}_{v_i}^\dagger\,\mathbf{T}_i^a\mathbf{Y}_{v_i}\right](x+sv_i)\right)$$
$$\tilde{\mathbf{Y}}'_{v_i}(x) = \mathcal{P}\exp\left(ig_s\int_0^\infty ds\left[v_i\cdot A_{sc'}^a\,\tilde{\mathbf{S}}_{v_i}^\dagger\mathbf{Y}_{v_i}^\dagger\,\mathbf{T}_i^a\mathbf{Y}_{v_i}\tilde{\mathbf{S}}_{v_i}\right](x+sv_i)\right)$$

Field redefinitions and factorization

Now multipole expand and redefine fields

$$A_s^a(x)\,\mathbf{Y}^\dagger_{n_4,i}(x_-)\mathbf{T}_i^a\mathbf{Y}_{n_4,i}(x_-) \to A_s^a(x)\,\mathbf{T}_i^a\,,$$
$$A_{sc'}^a(x)\,\tilde{\mathbf{S}}^\dagger_{n_4,i}(x_-)\mathbf{Y}^\dagger_{n_4,i}(x_-)\mathbf{T}_i^a\mathbf{Y}_{n_4,i}(x_-)\tilde{\mathbf{S}}_{n_4,i}(x_-) \to A_{sc'}^a(x)\,\mathbf{T}_i^a$$

with

$$\mathbf{Y}_{n_4,i}(x_-) = \mathcal{P}\exp\left(ig_s\int_0^\infty ds\; n_4\cdot A_{sc}^a(x_- + sn_4)\,\mathbf{T}_i^a\right)$$

Might seem to depend on color representation of parton i but can show

$$\mathbf{Y}_{n_4,i}\mathbf{T}_i^a\mathbf{Y}^\dagger_{n_4,i} = Y^{ba}_{n_4,\mathrm{adj}}\,\mathbf{T}_i^b$$

So that the redefinitions don't depend on generator and are just the BPS ones:

$$\left[Y^\dagger_{n_4,\mathrm{adj}}(x_-)\right]^{ab} A_s^b(x) \to A_s^a(x)$$

The fields no longer interact, so can factorize total matrix element into products of them

The factorized matrix elements

- "massless soft function"

$$\mathbf{S}\left(\omega_s, \frac{\omega_s}{\sqrt{\hat{s}}}, x_t, \mu\right) = \frac{1}{d_R}\sum_{X_s} \langle 0|\mathbf{O}_s^\dagger(0)|X_s\rangle \langle X_s|\mathbf{O}_s(0)|0\rangle \; \delta(\omega_s - \sqrt{\hat{s}}\, n_4 \cdot p_{X_s})$$

$$\mathbf{O}_s(x) = \left[\mathbf{S}_{n_1}\mathbf{S}_{n_2}\mathbf{S}_{n_3}\mathbf{S}_{n_4}\right](x)$$

- "soft fragmention function" (need to use color conservation to derive)

$$S_D\left(\omega_{sc}, \frac{\omega_{sc} m_t}{\hat{s}}, \mu\right) = \sum_{X_{sc}} \langle 0|O_{sc}^\dagger(0)|X_{sc}\rangle \langle X_{sc}|O_{sc}(0)|0\rangle \; \delta(\omega_{sc} - \sqrt{\hat{s}}\, \bar{n}_3 \cdot p_{X_{sc}})$$

$$O_{sc}(x) = Y_{v_3}^\dagger(x)\, Y_{\bar{n}_3}(x)$$

- "heavy quark jet function" (need to use color conservation to derive)

$$S_B\left(\omega_{sc'}, \frac{\omega_{sc'}}{m_t}, \mu\right) = \sum_{X_{sc'}} \langle 0|O_{sc'}^\dagger(0)|X_{sc'}\rangle \langle X_{sc'}|O_{sc'}(0)|0\rangle \; \delta(\omega_{sc'} - 2 m_t v_4 \cdot p_{X_{sc'}})$$

$$O_{sc'}(x) = Y_{\bar{n}_4}'^\dagger(x)\, Y_{v_4}'(x)$$

The final result for 1PI cross sections in soft and small-mass limit

In Laplace space:

$$\tilde{c}_{ij}(N, \hat{s}, \hat{t}_1, \hat{u}_1, m_t, \mu) = C_D^2\left(\ln\frac{m_t^2}{\mu^2}, \mu\right) \mathrm{Tr}\left[H_{ij}\left(\ln\frac{\hat{s}}{\mu^2}, x_t, \mu\right) \tilde{s}_{ij}\left(\ln\frac{\hat{s}}{\bar{N}^2\mu^2}, x_t, \mu\right)\right]$$
$$\times \tilde{s}_D\left(\ln\frac{m_t}{\bar{N}\mu}, \mu\right) \tilde{s}_B\left(\ln\frac{\hat{s}}{\bar{N}m_t\mu}, \mu\right)$$
$$+ \mathcal{O}\left(\frac{\hat{s}}{Nm_t^2}\right) + \mathcal{O}\left(\frac{m_t^2}{\hat{s}}\right)$$

- cross section factorized into five one-scale functions, all known to NNLO
- can solve RG equations as usual to do double resummation of soft and small-mass logarithms

Summary and outlook

Summary:

- used SCET-based factorization formulas to derive double soft and small mass limit of differential cross sections
- components known to accuracy needed for NNLO virtual plus soft approximations plus NNLL resummation of soft and small-mass logs

To do:

- implement double small-mass and soft-gluon resummation numerically
- include electroweak corrections (important at high M)
- compare in detail with NLO parton shower programs and match to NNLO calculations once available
- deal with heavy-quark loops