Non-Cancellation of Electroweak Logarithms in High-Energy Scattering

Sascha Turczyk

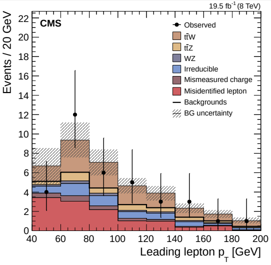
Work in collaboration with A. Manohar, B. Shotwell and Ch. Bauer

[arXiv:1409.1918]

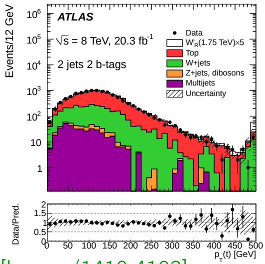
ERC workshop
Schloss Waldthausen, October 13th 2014

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Example of pp(later gg) $\to t\bar{t}$



[hep-ex/1406.7830]



[hep-ex/1410.4103]

Current Status

- Analysis extends to high energies
- $\sigma(t\bar{t})$: NNLO QCD, NNLL soft gluon resummation: top++2.0

  [Baernreuther,Cacciari,Czakon,Fiedler,Mangano,Mitov,Nason; Beneke,Falgari,Klein,Schwinn]
- No clear sign of New Physics (yet?)

$\Rightarrow$ Electroweak effects become important for precision predictions

Appearance of Sudakov Logs

Example of e^+e^-quarks ($\to$ KLN Theorem)

- Regularize individual contributions
- Virtual corrections are IR divergent $\propto \log^2 \frac{M^2}{s} + 3 \log \frac{M^2}{s}$
- Real emission is IR divergent $\propto -(\log^2 \frac{M^2}{s} + 3 \log \frac{M^2}{s})$
  - $\Rightarrow$ Fully inclusive rate is not IR sensitive: $1 + \alpha_s/\pi$
  - $\Rightarrow$ Applying phase-space cuts, Sudakov logs may survive

Sudakov Logs for Electroweak Corrections [Ciafalon et.al., Kuhn et.al., Denner et.al.]

- Presence of Sudakov double logs $\alpha_W \log^2 s/M^2$ regularized by $M_{W,Z}$
- Arise from collinear and soft infrared divergences
- Known to survive for
  1. Phase-space cuts [Bell, Kuhn, Rittinger]
  2. Non $SU(2)$-Singlet initial state [Ciafaloni, Comelli]
- Here: Restrict specific gauge boson emission

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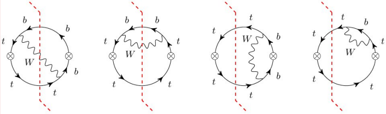
Simplifying Assumptions

- Consider gauge singlett current $J \to q\bar{q}$



$$\sigma_T = \frac{\alpha_W}{2\pi} (G_R - G_V) \widehat{\sigma}_0 \left\{ \ln^2 M_W^2/s + 3 \ln M_W^2/s + \ldots \right\}$$

1. Fully inclusive rate
2. Any fermion, no gauge boson
3. 1 fermion, and gauge bosons
4. 1 fermion, no gauge bosons
5. 2 fermions, and gauge bosons
6. 2 fermions, and no gauge bosons
7. Any fermion, specific gauge boson

Case	G_R	G_V	$G_R - G_V$
1	NC_F	NC_F	0
2	0	NC_F	$-NC_F$
3	C_F	C_F	0
4	0	C_F	$-C_F$
5	$\frac{1}{2} - \frac{1}{2N}\delta_{ij}$	$C_F\delta_{ij}$	$\frac{1}{2} - \frac{N}{2}\delta_{ij}$
6	0	$C_F\delta_{ij}$	$-C_F\delta_{ij}$
7	$\frac{1}{2}$	NC_F	$1 - \frac{N^2}{2}$

Relevant for EW Corrections

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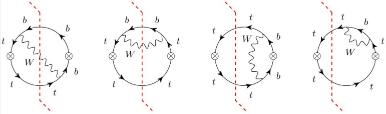

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Necessity for Resummation

- $\frac{\alpha_{\mathrm{EW}}}{4\pi \sin^2\theta_W} \log^2 s/M_W^2 \sim 0.15$ for $\sqrt{s} = 4$ TeV
- Power corrections are less important than logarithmic ones

$\Rightarrow$ We have advantages to use an EFT approach: SCET

- Amplitude expansion structure
- Fixed order: Row
- Resummation: Column

- Exponentiated Amplitude
- Natural EFT result

$\Rightarrow$ Non-trivial relation

$$A = \begin{pmatrix} 1 & & & & \\ \alpha L^2 & \alpha L & \alpha & & \\ \alpha^2 L^4 & \alpha^2 L^3 & \alpha^2 L^2 & \alpha^2 L & \alpha^2 \\ \alpha^3 L^6 & & \dots & & \\ \vdots & & & & \end{pmatrix}$$

$$\log A = \begin{pmatrix} \alpha L^2 & \alpha L & \alpha & & \\ \alpha^2 L^3 & \alpha^2 L^2 & \alpha^2 L & \alpha^2 & \\ \alpha^3 L^4 & \alpha^3 L^3 & \alpha^3 L^2 & \alpha^3 L & \alpha^3 \\ \alpha^4 L^5 & & \dots & & \\ \vdots & & & & \end{pmatrix}$$

- Leading Log (LL) Regime: $L \sim \frac{1}{\alpha}$
- Leading Log squared (LL^2) Regime: $L \sim \frac{1}{\sqrt{\alpha}}$

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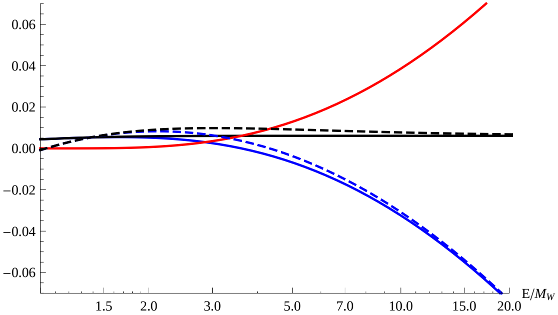
Comparing Fixed Order vs EFT

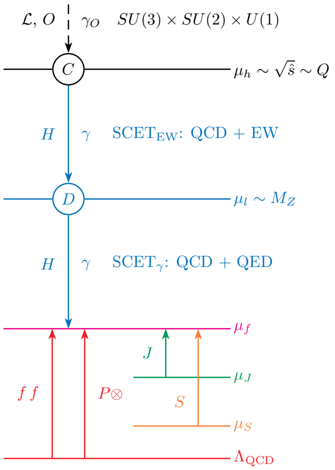


Legend

- Red: Real radiation
- Blue: Virtual corrections
- Black: Sum of contributions

- Solid: Full result
- Dashed: EFT result

Scales of the Problem



- Needed: High-Scale matching C
  - No large logs!
- Known analytic results
  1. Collinear running γ in $\mathrm{SCET}_{\mathrm{EW},\gamma}$ for all fields within the SM
  2. Soft running γ in $\mathrm{SCET}_{\mathrm{EW},\gamma}$ for all field representations within the SM
  3. Soft matching D at weak scale $\Rightarrow$ symmetry breaking effects
  4. Collinear matching D at weak scale

$\Rightarrow$ Corrections are available analytically for an arbitrary process (up to high scale matching)

[Chiu, Fuhrer, Kelley, Manohar, 0909.0012[hep-ph]]

The Wilson Coefficient

Matrices for EW Corrections in SCET [Chiu, Fuhrer, Kelley, Manohar, Hoang, Golf]

$$\mathcal{M}_{\text{low}} = \exp\left[\int_{\mu}^{M_z} \frac{\mathrm{d}\mu}{\mu}\, R^{\text{soft+col}}_{U(1)_{\text{EM}}}\right]$$

$$\mathcal{M}_{\text{col}} = \exp\left[M^{\text{col}}_{U(1)_Y \otimes SU(2)_W}\right] \exp\left[\int_{M_z}^{\sqrt{s}} \frac{\mathrm{d}\mu}{\mu}\, R^{\text{col}}_{U(1)_Y \otimes SU(2)_W}\right]$$

$$\mathcal{M}_{\text{soft}} = \exp\left[\int_{M_z}^{\sqrt{s}} \frac{\mathrm{d}\mu}{\mu}\, R^{\text{soft}}_{U(1)_Y}\right] M^{\text{break}}_{SU(2)} \exp\left[\int_{M_z}^{\sqrt{s}} \frac{\mathrm{d}\mu}{\mu}\, R^{\text{soft}}_{SU(2)_W}\right]$$

- R is running matrix (anomalous dimension)
- M is a matching matrix

$$\mathbf{C}_{\text{low}} = \mathcal{M}_{\text{low}}\, \mathcal{M}_{\text{col}}\, \mathcal{M}_{\text{soft}}\, \mathbf{C}_{\text{high}}\, \mathcal{M}^{T}_{SU(3)}$$

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Goals

⇒ Numerical estimate for $t\bar{t}$ production

- Effect for final state gauge bosons larger $C_F \to C_A$
- Enhancement by phase-space cuts and non-singlet initial states

- Consider three scenarios for the top
  1. $q = u, d$
  2. $q = t, b$ with m_b=100 GeV and $m_t = 173$ GeV
  3. $q = t, b$ with m_b=4.7 GeV and $m_t = 173$ GeV.

Simplifications

- Partonic level
- Unbroken $SU(2)$ theory
- Singlet initial state ⇒ $gg \to t\bar{t}$
- No Decay, shower or hadronization effects
- To avoid t-channel singularity demand
  1. $|\eta| < 1,3$ for highest transverse momentum particle
  2. $|\eta| < 5$ for second highest transverse momentum particle

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Virtual and Real Corrections

Virtual Corrections

- We use the results obtained in SCET
- Expand out to order α_W to demonstrate cancellation

$$\sigma_V(gg \to t\bar{t}) = \sigma_{0,t}\left\{v_W + 3v_t + v_b\right\}$$
$$\sigma_V(gg \to b\bar{b}) = \sigma_{0,b}\left\{v_W + v_t + 3v_b\right\}$$

$$v_W = \frac{C_F \alpha_W}{4\pi}\left[-\mathrm{L}^2 + 3\mathrm{L}\right], \quad v_t = -\frac{y_t^2}{32\pi^2}\mathrm{L}, \quad v_b = -\frac{y_b^2}{32\pi^2}\mathrm{L}$$

Real Radiation

- To our considered order: Tree Level process

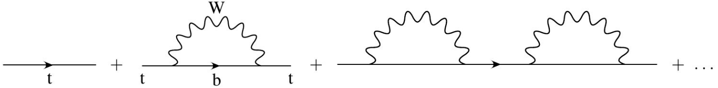
$\Rightarrow$ We use MadGraph5_aMC@NLO for calculating this numerically

Subtlety for Decaying Top

- Usually done: Narrow width approximation

$$\frac{1}{p^2 - m_t^2 + i\epsilon} \to \frac{1}{p^2 - m_t^2 + i m_t \Gamma_t}$$

- Equivalent to sum all imaginary part of



- Same as one cut of the $J \to t\bar{t}$ example

$\Rightarrow$ Cuts cannot be treated separately

$\Rightarrow$ Not gauge invariant

$\Rightarrow$ Mixes different orders in α_W: Γ_t is $\mathcal{O}(\alpha_W m_t)$

- If $t \to Wb$ is kinematically allowed: $\mathcal{O}(\alpha_W)$ piece is wrong

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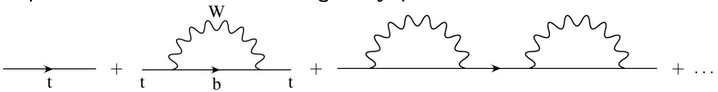

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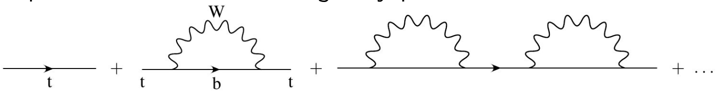

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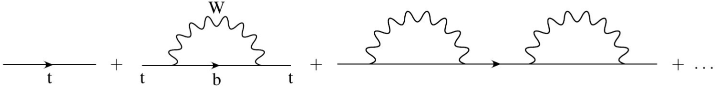

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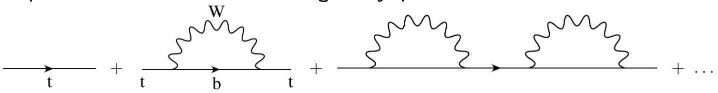

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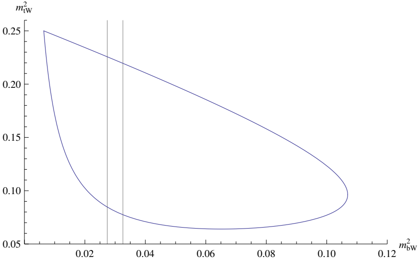

$\sqrt{s} = 0.5$ TeV

Divide into resonant (A) and non-resonant (A') region

$$I = \int_{m_t^2-\Delta}^{m_t^2+\Delta} dm_{bW}^2 \, \frac{f(m_{bW}^2)}{(m_{bW}^2 - m_t^2)^2 + \epsilon^2}$$

ϵ regulator from $i\epsilon$ prescription

- Need to subtract singular piece in region A by expanding

$$f(m_{bW}^2) = f_0 + (m_{bW}^2 - m_t^2)f_1 + (m_{bW}^2 - m_t^2)^2 f_2 + \ldots$$

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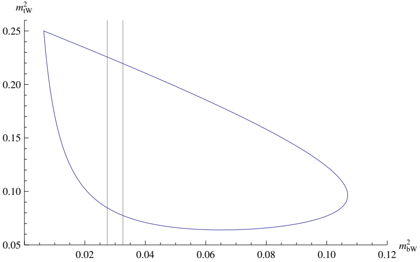

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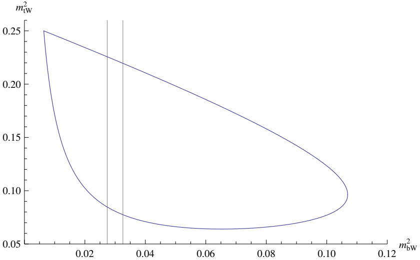

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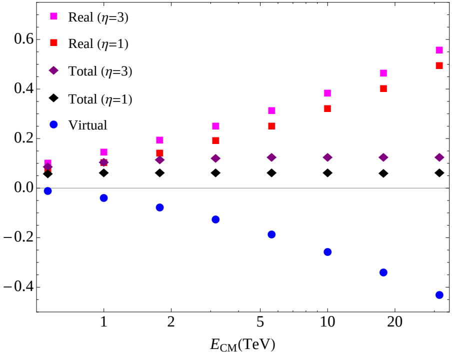
Results for Massless Quarks



- Tree level: $gg \to u\bar{u}$ and $gg \to d\bar{d}$
- Real radiation: $gg \to u\bar{u}Z$, $gg \to d\bar{d}Z$, $gg \to u\bar{d}W^-$ and $gg \to d\bar{u}W^+$
- $SU(2)$: $\sigma(u\bar{u}) = \sigma(d\bar{d})$, $\sigma(u\bar{d}W^-) = \sigma(d\bar{u}W^+) = 2\sigma(u\bar{u}Z) = 2\sigma(d\bar{d}Z)$
- Cancellation $3\sigma(u\bar{d}W) + 2v_W\sigma(u\bar{u}) \to 0$
- No full cancellation for experimental observables!

Results for Massive and Stable Top: Remarks

- Use equivalence theorem for longitudinal polarization of W

$$\sigma(t\bar{b}W^-) \to \sigma(u\bar{d}W^-) + 2(y_t^2 + y_b^2)\sigma_S$$
$$\sigma(t\bar{t}Z) \to \frac{1}{2}\sigma(u\bar{d}W^-) + 2y_t^2\sigma_S$$
$$\sigma(b\bar{b}Z) \to \frac{1}{2}\sigma(u\bar{d}W^-) + 2y_b^2\sigma_S$$
$$\sigma(t\bar{t}H) \to 2y_t^2\sigma_S\,, \quad \sigma(b\bar{b}H) \to 2y_b^2\sigma_S$$
$$\sigma_V(t\bar{t}) \to (v_W + 3v_t + v_b)\,\sigma(u\bar{u})$$
$$\sigma_V(b\bar{b}) \to (v_W + v_t + 3v_b)\,\sigma(u\bar{u})$$

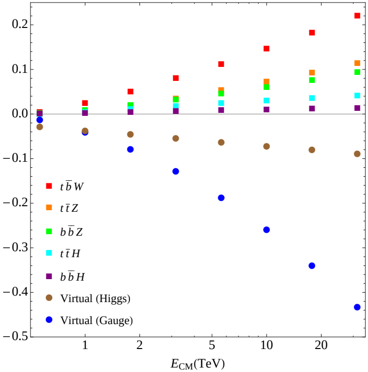
- Total real radiation

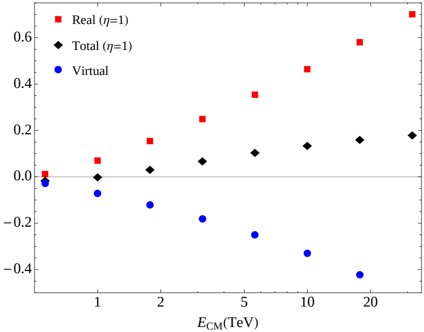
$$\sigma_R = 2\sigma(t\bar{b}W^-) + \sigma(t\bar{t}Z) + \sigma(b\bar{b}Z) + \sigma(t\bar{t}H) + \sigma(b\bar{b}H)$$
$$\to 3\sigma(u\bar{d}W^-) + 8(y_t^2 + y_b^2)\sigma_S$$

- Total virtual correction

$$\sigma_V = \sigma_V(t\bar{t}) + \sigma_V(b\bar{b}) = (2v_W + 4v_t + 4v_b)\,\sigma(u\bar{u})$$

Numerical Result for Massive and Stable Top



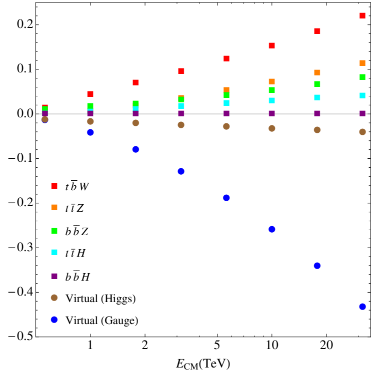


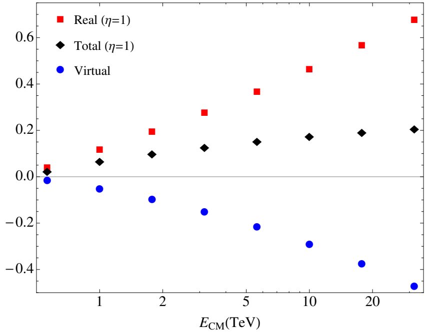
- $v_{t,b}$ linear in Log
- Gauge and Higgs part cancel separately

$$8(y_t^2 + y_b^2)\sigma_S + (4v_t + 4v_b)\,\sigma(u\bar{u}) \to 0.$$

- No full cancellation for experimental observables!

Numerical Results for Physical Top Quark Case





- Same as before, Yukawa coupling for bottom much smaller
- Used $t tag for MadGraph5_aMC@NLO

$\Rightarrow$ Excludes a region of width $15\Gamma_t$ around the on-shell t-quark

- No full cancellation for experimental observables!

Summary

- Electroweak corrections for $gg \to t\bar{t}$
- Virtual corrections are around -10% for $E_{\text{cm}} \sim 2$ GeV
- If part of real radiation can be excluded, there are large electroweak radiative corrections
- Importance grows with energy and become measurable at LHC energies

Comments

- Corrections for $q\bar{q} \to t\bar{t}$ expected to be twice as large
- Explored minimal effect
- Total corrections to NLL can be written as a product $R_{\text{QCD}} R_{\text{EW}}$

$\Rightarrow$ EW can be included by reweighting QCD result

$\Rightarrow$ Inclusion of EW corrections possible with SCET approach

$\Rightarrow$ Full study with proton PDF, shower, exp. cuts, ... necessary

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Backup Slides

SCET Operators

- $g(p_1) + g(p_2) \to q(p_3) + \overline{q}(p_4)$

$$
\begin{aligned}
O_1 &= \bar{q}_4 q_3 A_2^A A_1^A \\
O_2 &= d^{ABC} \bar{q}_4 T^C q_3 A_2^A A_1^B \\
O_3 &= i f^{ABC} \bar{q}_4 T^C q_3 A_2^A A_1^B \,.
\end{aligned}
$$

- High Scale Matching contains EW and QCD corrections

Running in SCET

- First run down to Electroweak scale
- Then integrate out gauge bosons $\Rightarrow$ low scale matching

$\Rightarrow$ Breaks up $SU(2)$ doubletts into individual fields

$$
\begin{aligned}
\mathcal{M} &= \exp\left[D_C(\mu_l, \mathrm{L_M}, \bar{n}\cdot p)\right] d_S(\mu_l, \mathrm{L_M}) \\
&\times P \exp\left[\int_{\mu_h}^{\mu_l} \frac{\mathrm{d}\mu}{\mu} \gamma(\mu, \bar{n}\cdot p)\right] C(\mu_h, \mathrm{L_Q})
\end{aligned}
$$

Anomalous Dimension Matrices

$$\gamma(\mu, \bar{n} \cdot p) = \gamma_C(\mu, \bar{n} \cdot p) + \gamma_S(\mu)$$

- Soft matrix: $\hat{=}$ interactions between particles

$$\gamma_S(\mu) = -\sum_{\langle rs \rangle, i} \frac{\alpha_i(\mu)}{\pi} T_r^{(i)} \cdot T_s^{(i)} \ln \frac{-n_r \cdot n_s + i0^+}{2}$$

- Colinear Part: Knows about species (Diagonal!)

$$\gamma_C(\mu, \bar{n} \cdot p) = \mathbb{1} \sum_r \left[A_r(\mu) \ln \frac{2E_r}{\mu} + B_r(\mu) \right]$$

- $A_r(\mu)$ and $B_r(\mu)$ have a perturbative expansion in $\alpha_i(\mu)$

Idea of SCET

Ansatz

- Describing **energetic** particles with multiple scales
  1. Scattering process $\mathcal{O}(Q^2)$
  2. Collinear component
  3. Soft component
  4. Systematic power-counting $\lambda \ll 1$
- Propagating around light-cone: $p^2 \ll Q^2$

$\Rightarrow$ Factorization of perturbative and non-perturbative effects

Kinematics

- Light-cone vectors $\mathbf{n}^\mu = (1, \mathbf{n})$ and $\bar{\mathbf{n}}^\mu = (1, -\mathbf{n})$

$\Rightarrow$ Collinear field: $p^- = \bar{n} \cdot p \sim Q\,, \quad p^+ = n \cdot p \sim \lambda^2 Q\,, \quad p_\perp \sim \lambda Q$

- Ultrasoft field: All components scale as $\lambda^2 Q$

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High-Scale Operator in Full SM

Full Standard Model

- Respect full $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetry
- All masses set to zero

$\Rightarrow$ Infrared divergencies

Matching onto SCET$_{\text{ew}}$

- Describing fields with collinear Wilson lines

$$\bar{Q}\Gamma Q \to \exp C(\mu)[\bar{\xi}_{n,p_1} W_n]\Gamma[W_{\bar{n}}^\dagger \xi_{\bar{n},p_2}]$$

- Ultraviolett match onto infrared ones from full theory

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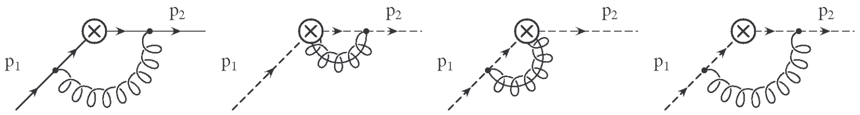

Sudakov Logarithm

Simplified Problem

- $1 \to 1$ process problem

⇒ Simple coordinates

⇒ n and $\bar{n}$ are orthogonal

$$\log F_E(Q^2, \mu) = C(\mu{=}Q) + \int_Q^{M_Z} \frac{\mathrm{d}\mu}{\mu} \gamma_1(\mu) + D_{Z,W}(\mu{=}M_Z) + \int_{M_Z}^{\mu} \frac{\mathrm{d}\mu}{\mu} \gamma_2(\mu)$$

Extension to More Particles

- r particles ⇒ Set of n_i $i = 1, \dots, r$

⇒ $n_i \cdot n_j \neq 0$ if $i \neq j$

⇒ Two types of corrections

1. Field dependent corrections
2. Corrections between different fields

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Factorization into Soft and Collinear

Comment about Regulator for divergencies

- Analytic regulator: $\frac{1}{p_i^2 - m^2} \to \frac{1}{(p_i^2 - m^2)^{1+\delta_i}}$

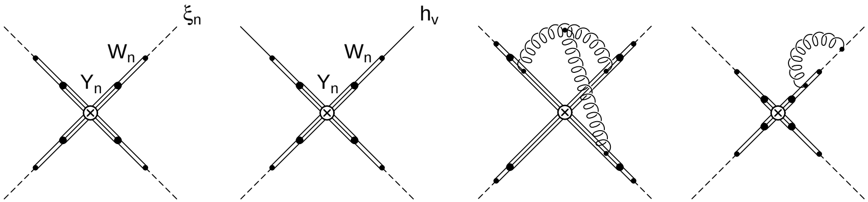
$\Rightarrow$ Breaks color Ward identity

- Use Δ regulator: $\frac{1}{p_i^2 - m^2} \to \frac{1}{p_i^2 - m^2 - \Delta_i}$

$\Rightarrow$ Preserves gauge-invariant Wilson line

- Soft function obeys Casimir scaling and linear in $n_i \cdot n_j$

$\Rightarrow$ Write soft-function as sum over all two-particle pairs



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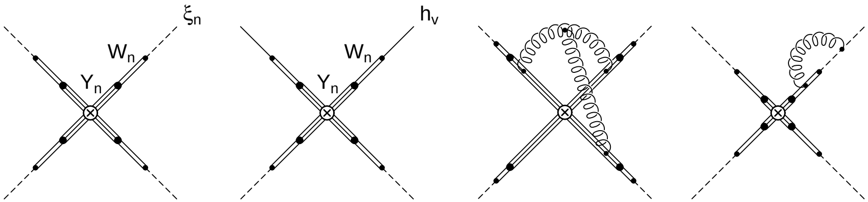

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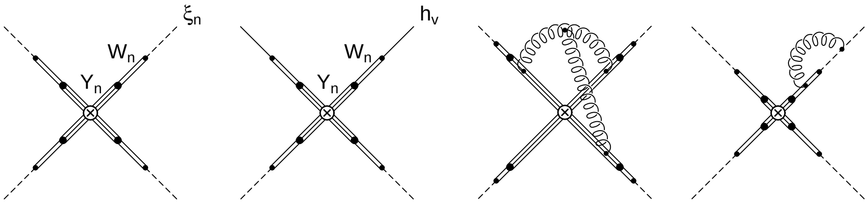

Soft Corrections

Definition

- Universal soft function $U_S(n_i, n_j) = \log \frac{-n_i \cdot n_j - i0^+}{2}$
- Anomalous dimension $\gamma_s = \Gamma(\alpha(\mu)) \left[-\sum_{\langle ij \rangle} \mathbf{T}_i \mathbf{T}_j U_S(n_i, n_j) \right]$
- Low-scale matching $D_s = J(\alpha(\mu), L_m) \left[-\sum_{\langle ij \rangle} \mathbf{T}_i \mathbf{T}_j U_S(n_i, n_j) \right]$
  1. Cusp anomalous dimension $\Gamma(\alpha(\mu)) = \frac{\alpha(\mu)}{4\pi} 4$
  2. Matching $J(\alpha(\mu), L_m) = \frac{\alpha(\mu)}{4\pi} 2 \log \frac{M^2}{\mu^2}$

Comments

- Contains all information about kinematics of process
- Assumes Casimir scaling (gauge singlett operator)

$\Rightarrow$ 3-Loop Cusp anomalous dimension is used

Collinear Corrections

Definition

- Regulator choice: n_i Wilson line interactions only with i particle

$\Rightarrow$ Particle dependent corrections

$\Rightarrow$ Sum over all particles

Comments

- Anomalous dimension contains cusp and non-cusp part
    - Cusp: 3-loop K factor
    - Non-cusp: 2 loop K factor
- Matching contains wave-function renormalization
- Matching contains $Z - \gamma$ mixing
- Additional matching in $SU(3)_C$, because top is integrated out
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Breaking of the Electroweak Symmetry

Standard Model

- Scalar field (Higgs) obtains vacuum expectation value (VEV)
- Couples to other fields

$\Rightarrow$ VEV breaks symmetry spontaneously

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_C \otimes U(1)_{\text{em}}$$

Consequences

- Mixing of fields:

$$Z = \cos\theta_W W^3 - \sin\theta_W B$$
$$A = \sin\theta_W W^3 + \cos\theta_W B$$

- Mass splitting $M_W \neq M_Z$
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Effect of Symmetry Breaking

- Operator basis is extended: $(M^{\text{break}}_{SU(2)})_{ij}$ non-square matching matrix
- Gauge structure is changed $SU(2)_L \otimes U(1)_Y \to U(1)_{\text{em}}$

$\Rightarrow$ EW symmetry breaking described by soft matrix

Matching at 1-Loop

$$R^{(1)}_{S,W} = \frac{\alpha_W}{4\pi} 2\log M_W^2 \mu^2 \left[R^{(0)} \mathcal{O}_{SU(2)} + \sum_{\langle ij \rangle} \mathbf{T}_{3,i} \mathbf{T}_{3,j} U_S(n_i, n_j) \right]$$

$$R^{(1)}_{S,Z} = \frac{\alpha_Z}{4\pi} 2\log M_W^2 \mu^2 \left[-\sum_{\langle ij \rangle} \mathbf{T}_{Z,i} \mathbf{T}_{Z,j} U_S(n_i, n_j) \right]$$

$$M^{\text{break}}_{SU(2)} = R^{(0)} + R^{(1)}_{S,W} + R^{(1)}_{S,Z}$$

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