DO LHC CROSS SECTIONS FACTORIZE ?

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erc
EFT4jets

61th International Winter Meeting on Nuclear Physics – Bormio, Italy – January 27-31, 2025

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ON THE MEANING OF "FACTORIZATION"

Most fundamental: **separation of energy/distance scales**

- Without this principle, physics would not exist
- **Effective Field Theories** (EFTs) describe phenomena using only the relevant degrees of freedom, quantum effects from shorter distances are "integrated out" and included in the couplings of the EFT
- EFT for collider physics: **Soft-Collinear Effective Theory**

[Bauer, Fleming, Pirjol, Stewart (2000);
Bauer, Pirjol, Stewart (2001);
Bauer, Fleming, Pirjol, Rothstein, Stewart (2002);
Beneke, Chapovsky, Diehl, Feldmann (2002)]

ON THE MEANING OF "FACTORIZATION"

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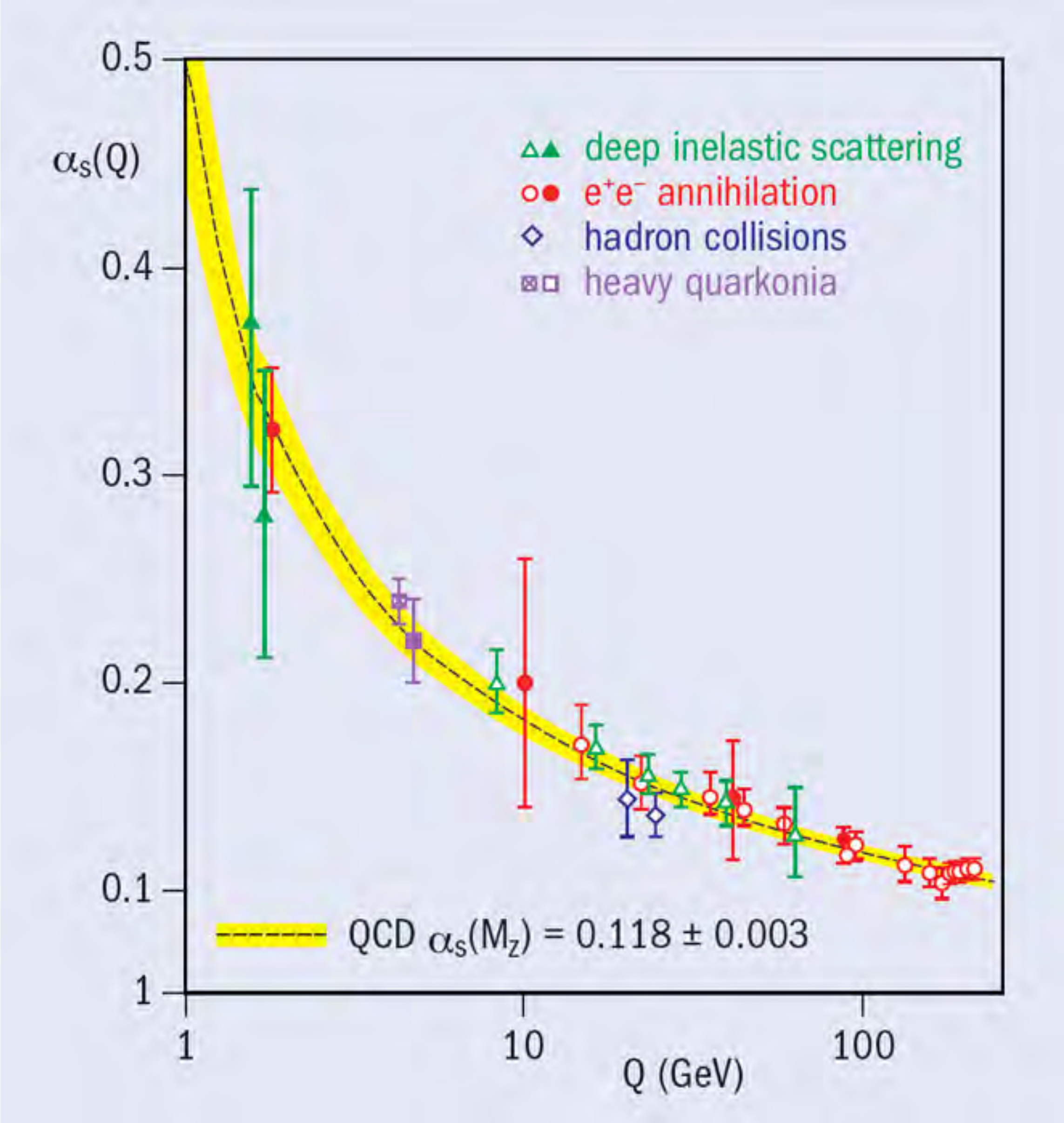
- **Effective Field Theories** (EFTs) describe phenomena using only the relevant degrees of freedom, quantum effects from shorter distances are "integrated out" and included in the couplings of the EFT
- EFT for collider physics: **Soft-Collinear Effective Theory**
- Relevant in QCD: separation of perturbative partonic (short-distance) from non-perturbative hadronic (long-distance) effects



"PDF FACTORIZATION"

Stronger assumption:

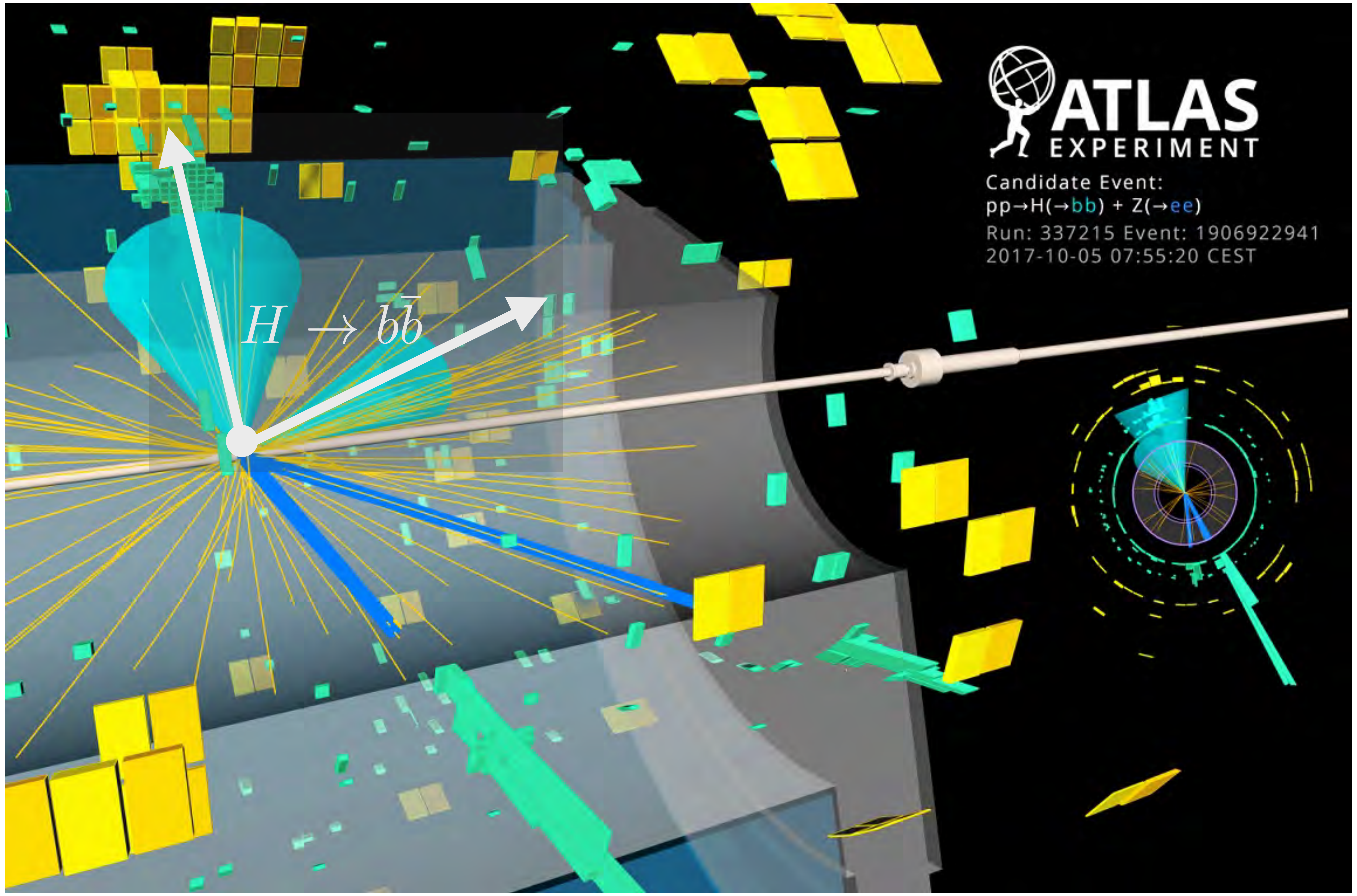
- Up to power corrections, all long-distance effects in hadron collider scattering are contained in **universal parton distribution functions** (PDFs) of the nucleon:

$$d\sigma_{pp\to f}(s) = \sum_{a,b=q,\bar{q},g} \int dx_1 dx_2\, f_{a/p}(x_1,\mu)\, f_{b/p}(x_2,\mu)\, d\sigma_{ab\to f}(\hat{s}=x_1x_2s,\mu)$$

- Used in all calculations of LHC processes, but proved only for Drell-Yan processes: $pp \to$ color-neutral state (γ^*, W, Z, H) [Collins, Soper, Sterman (1985)]

- What about more complicated **processes with colored particles (jets)** in the final state?

JET PROCESSES AT HADRON COLLIDERS



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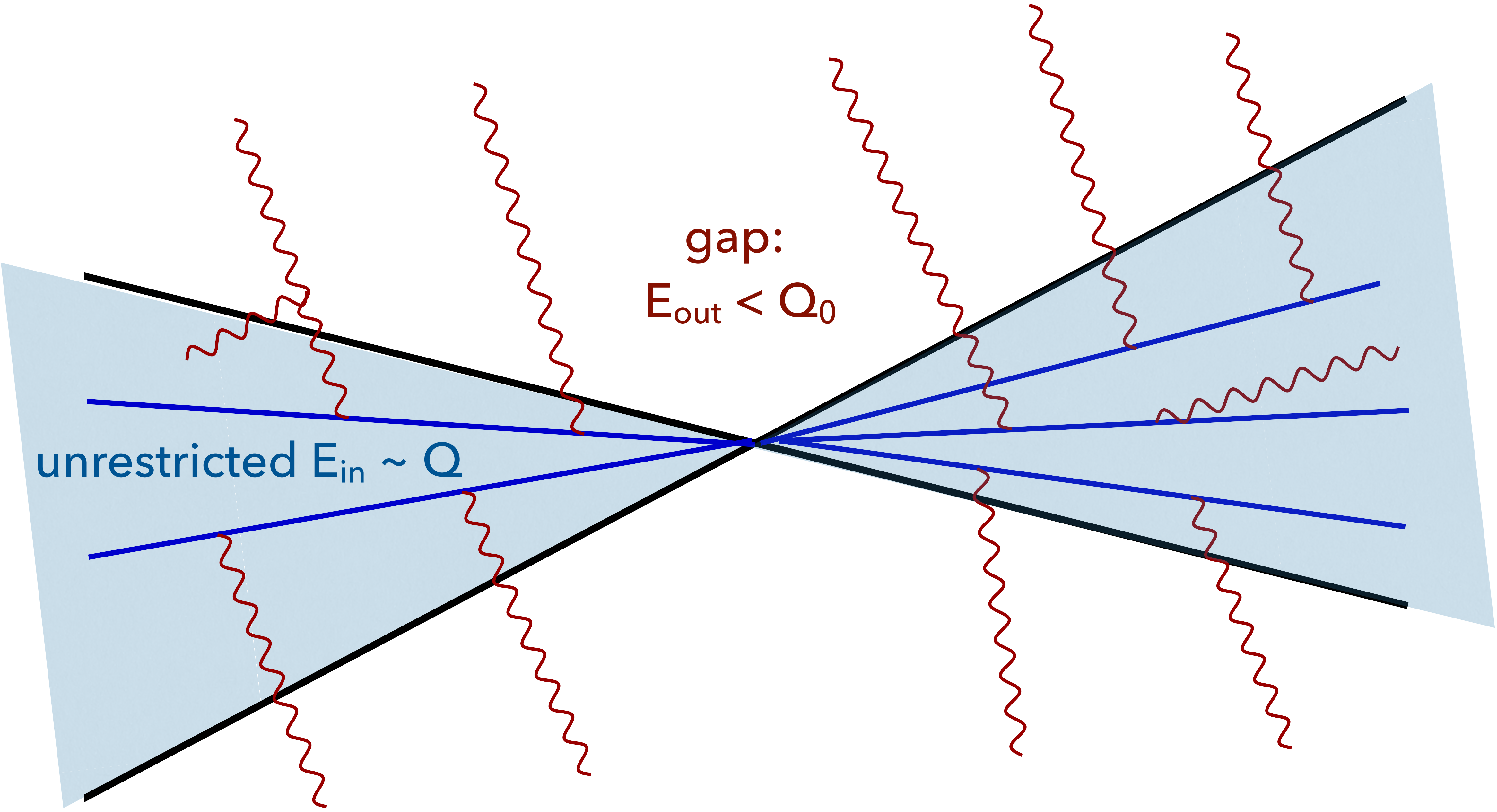
JET PROCESSES AT HADRON COLLIDERS



Perturbative expansion includes **super-leading logarithms** not present at e^+e^-:

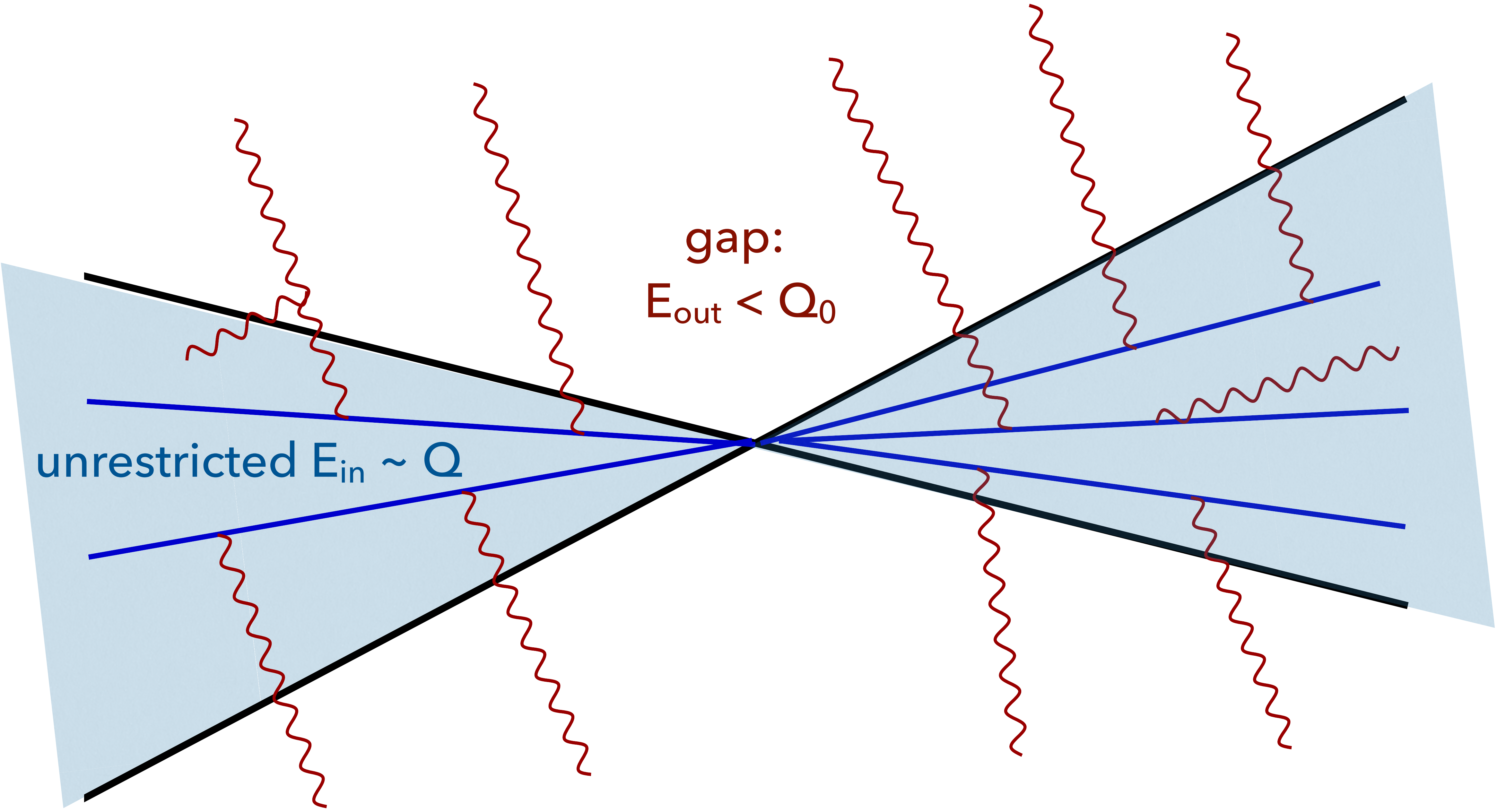
$$\sigma \sim \sigma_{\text{Born}} \times \{1 + \alpha_s L + \alpha_s^2 L^2 + \alpha_s^3 L^3 + \underbrace{\alpha_s^4 L^5 + \alpha_s^5 L^7}_{\text{formally larger than } O(1)} + \dots\}$$

state-of-the-art

formally larger than $O(1)$

[Forshaw, Kyrieleis, Seymour (2006)]

JET PROCESSES AT HADRON COLLIDERS



Really, a **double-logarithmic series** starting at 3-loop order:

$$\sigma \sim \sigma_{\rm Born} \times \left\{ 1 + \alpha_s L + \alpha_s^2 L^2 + (\alpha_s \pi^2) \underbrace{\left[\alpha_s^2 L^3 + \alpha_s^3 L^5 + \dots \right]}_{\text{formally larger than } O(1)} \right\}$$

$(\Im\mathrm{m}\, L)^2$

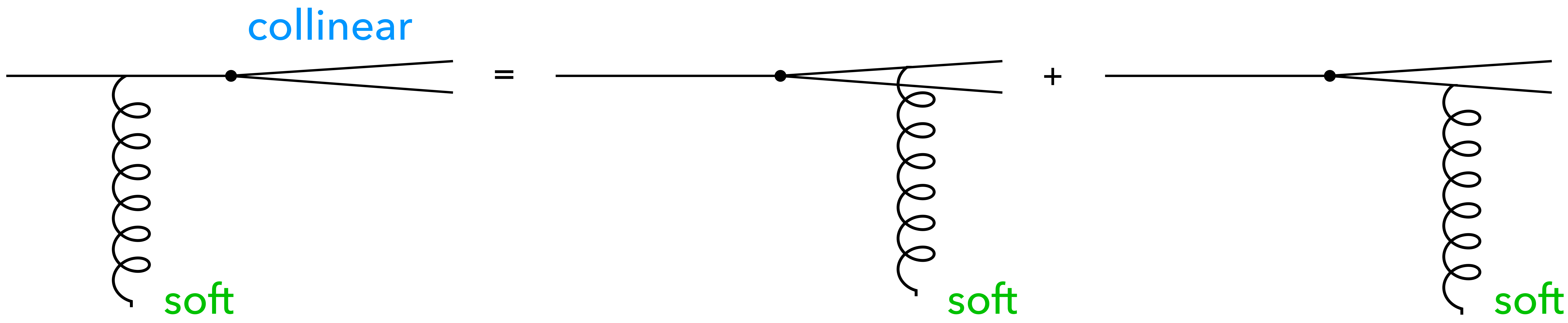
Glauber phases

[Becher, MN, Shao (2021)]

IMPORTANCE OF COLOR COHERENCE

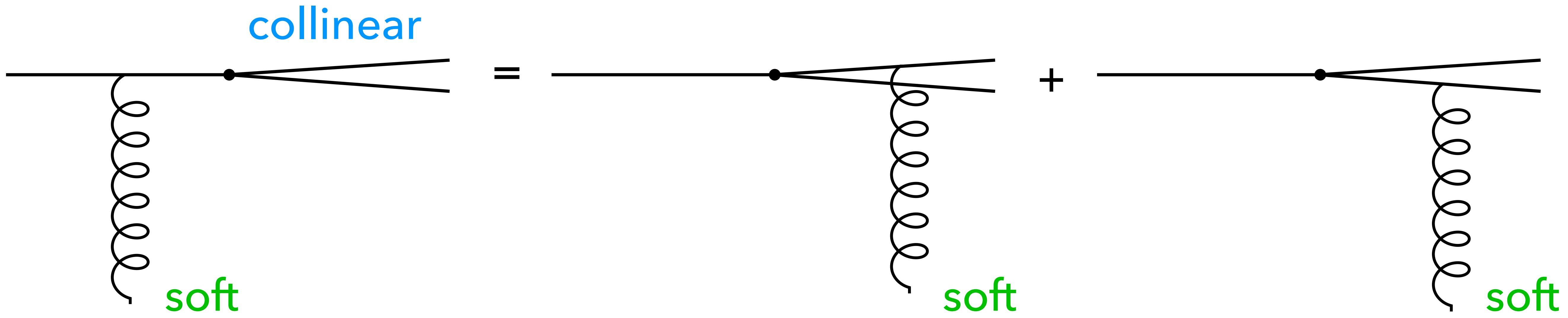
Color coherence is essential for the cancellation of collinear singularities in scattering processes

- Intuitive concept familiar from Low's **soft-photon theorem** in QED: soft photons only probe the charge and direction of energetic particles

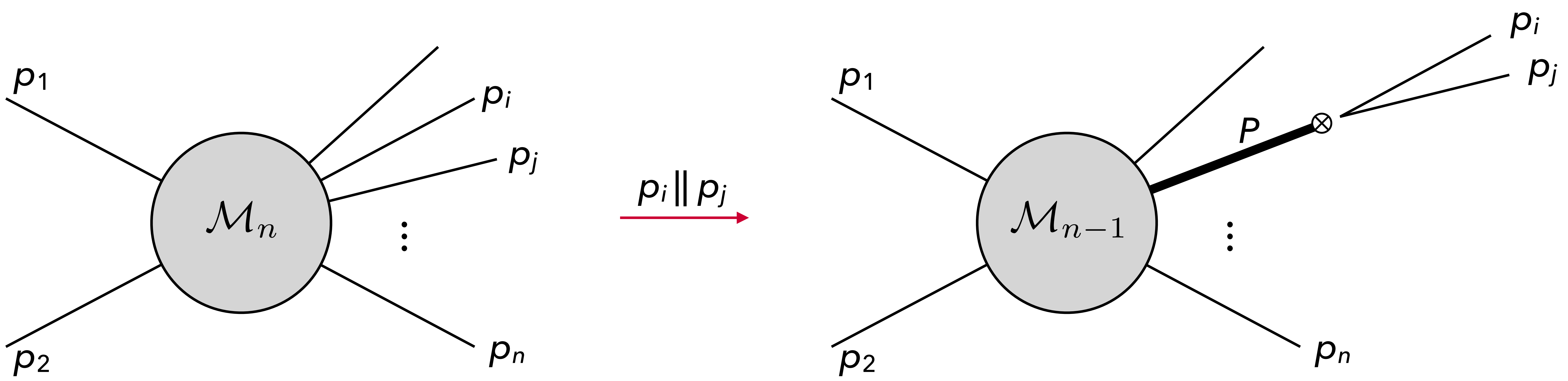


IMPORTANCE OF COLOR COHERENCE

- **Color coherence holds** if all three particles are in the final state of a scattering process (time-like splitting):

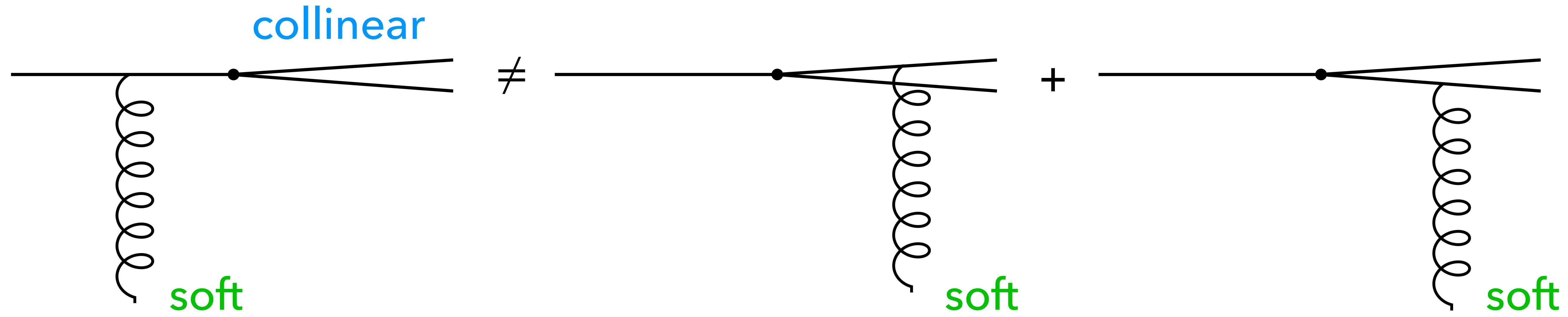


- **Then collinear factorization holds:**

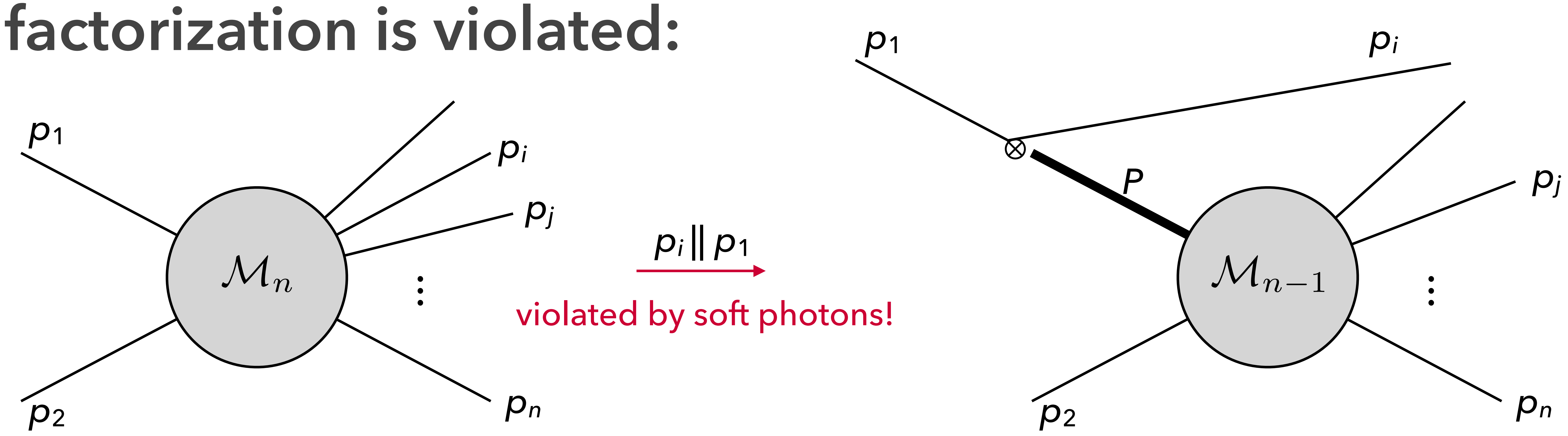


BREAKING OF COLOR COHERENCE

- **Color coherence is broken** if not all particles are outgoing (space-like splitting), since both sides receive different phase factors at higher orders:



- **Collinear factorization is violated:**



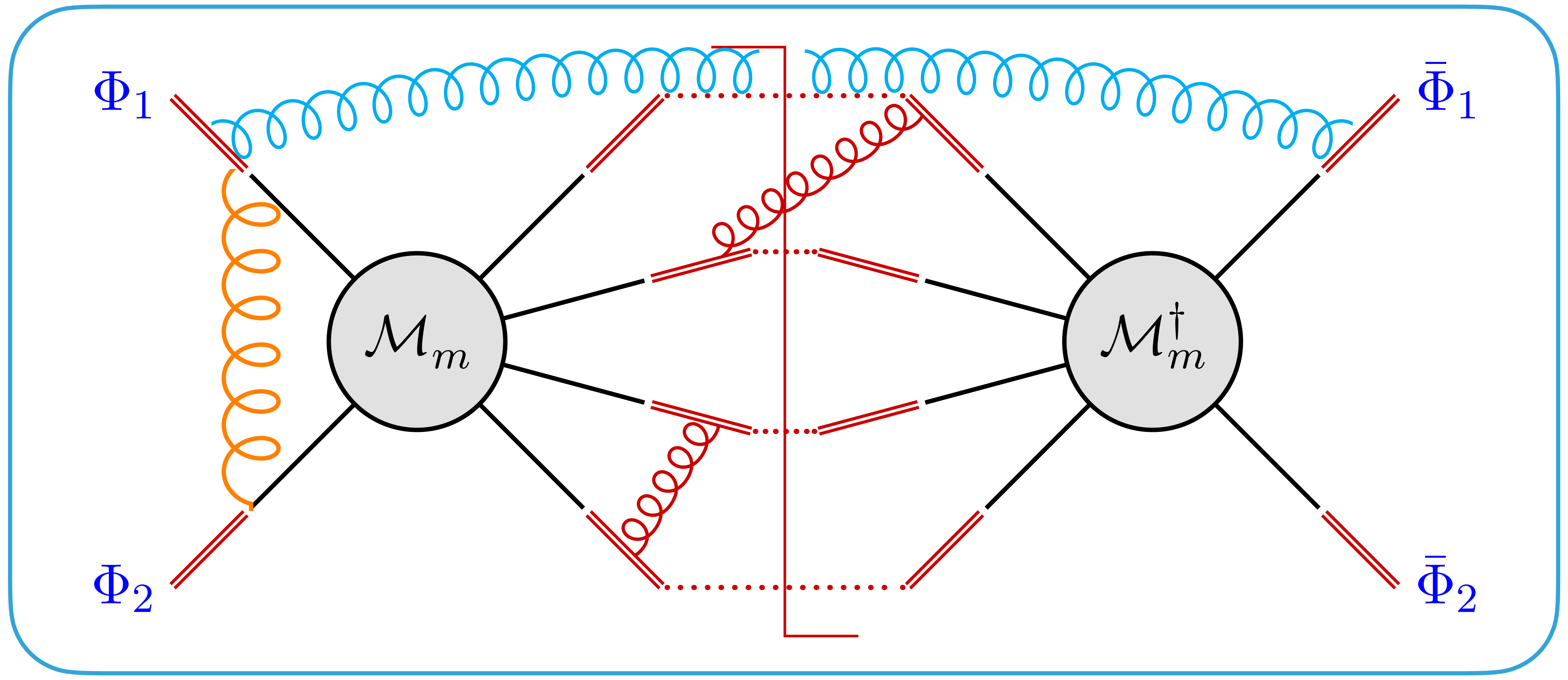
[Catani, de Florian, Rodrigo (2011); Forshaw, Seymour, Siodmok (2012)]

GAP-BETWEEN-JETS OBSERVABLES

SCET factorization theorem for *M*-jet production at the LHC

$$\sigma(Q_0) = \sum_{m=m_0}^{\infty} \int d\xi_1 d\xi_2 \, \langle \underbrace{\boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \xi_1, \xi_2, \mu)}_{\text{high scale}} \otimes \underbrace{\boldsymbol{\mathcal{W}}_m(\{\underline{n}\}, Q_0, \xi_1, \xi_2, \mu)}_{\text{low scales } Q_0 \text{ and } \Lambda_{\text{QCD}}} \rangle$$

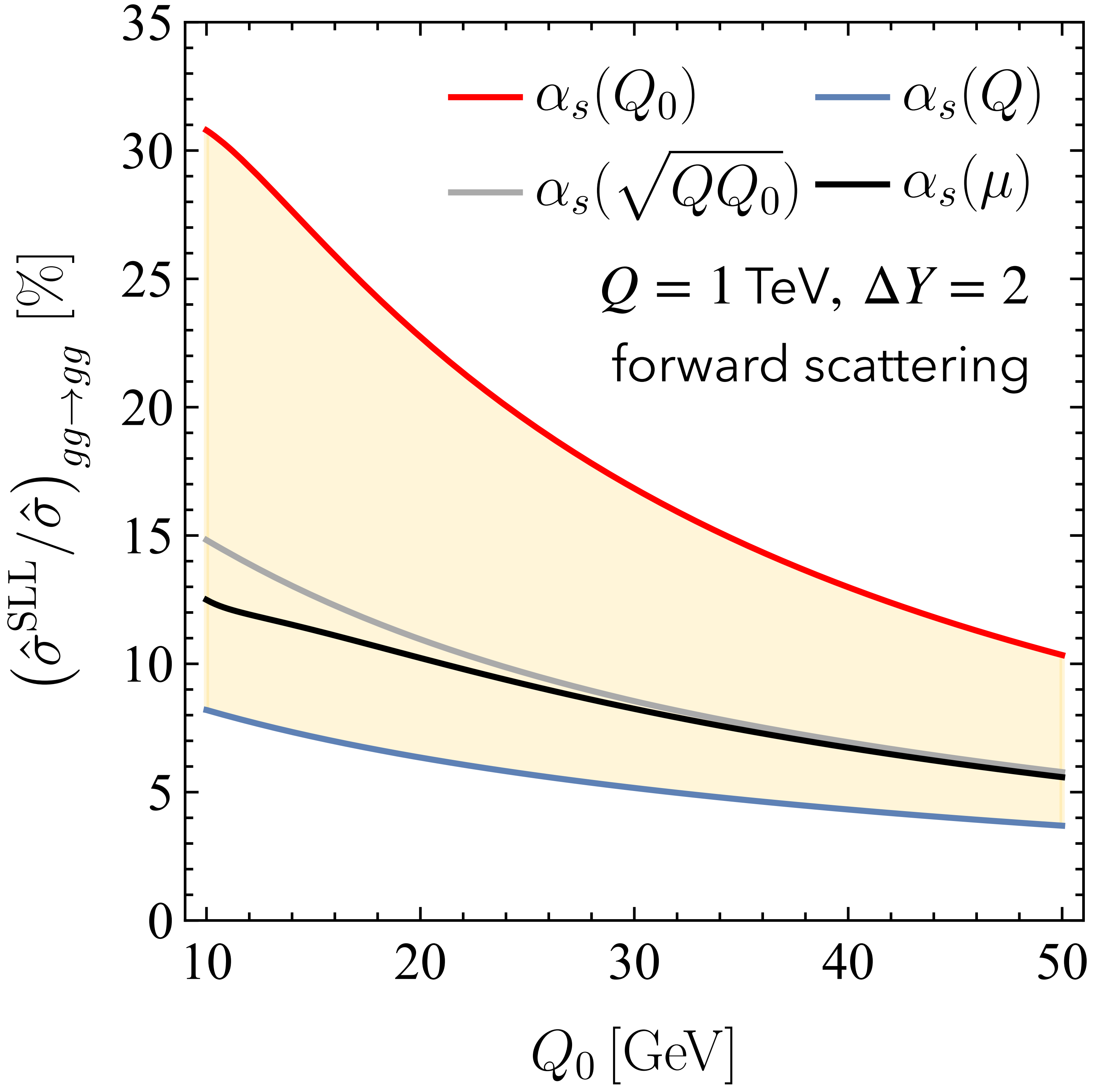
[Becher, MN, Shao (2021);
Becher, MN, Shao, Stillger (2023)]

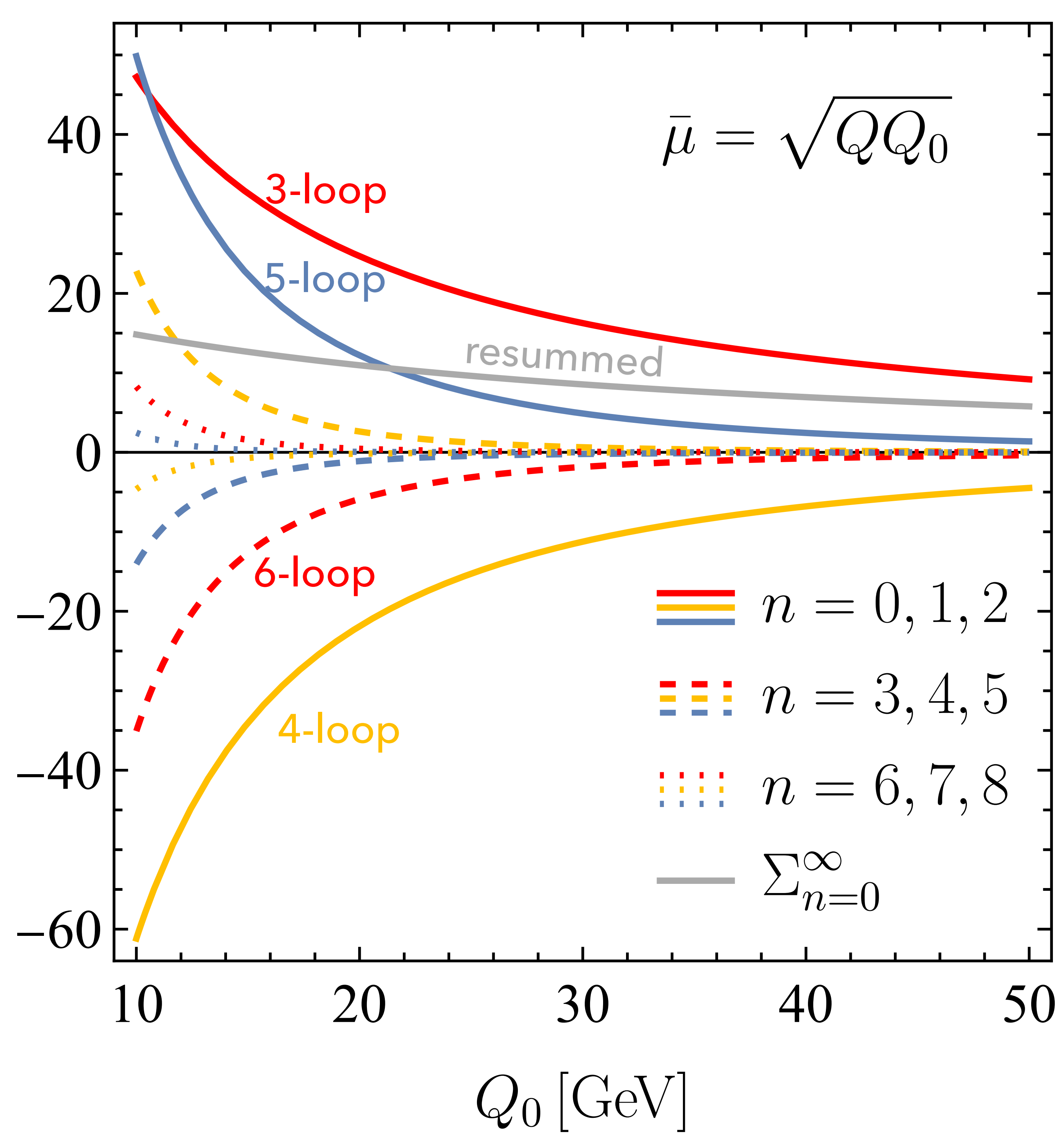


- **new perspective** to think about non-global observables
- large logs can be **resummed using RGEs**
- **all-order understanding** of super-leading logarithms for arbitrary processes

GAP-BETWEEN-JETS OBSERVABLES

Based on this new approach, we have performed the **first all-order resummation of super-leading logarithms** for jet processes [Becher, MN, Shao, Stillger (2023)]





GAP-BETWEEN-JETS OBSERVABLES

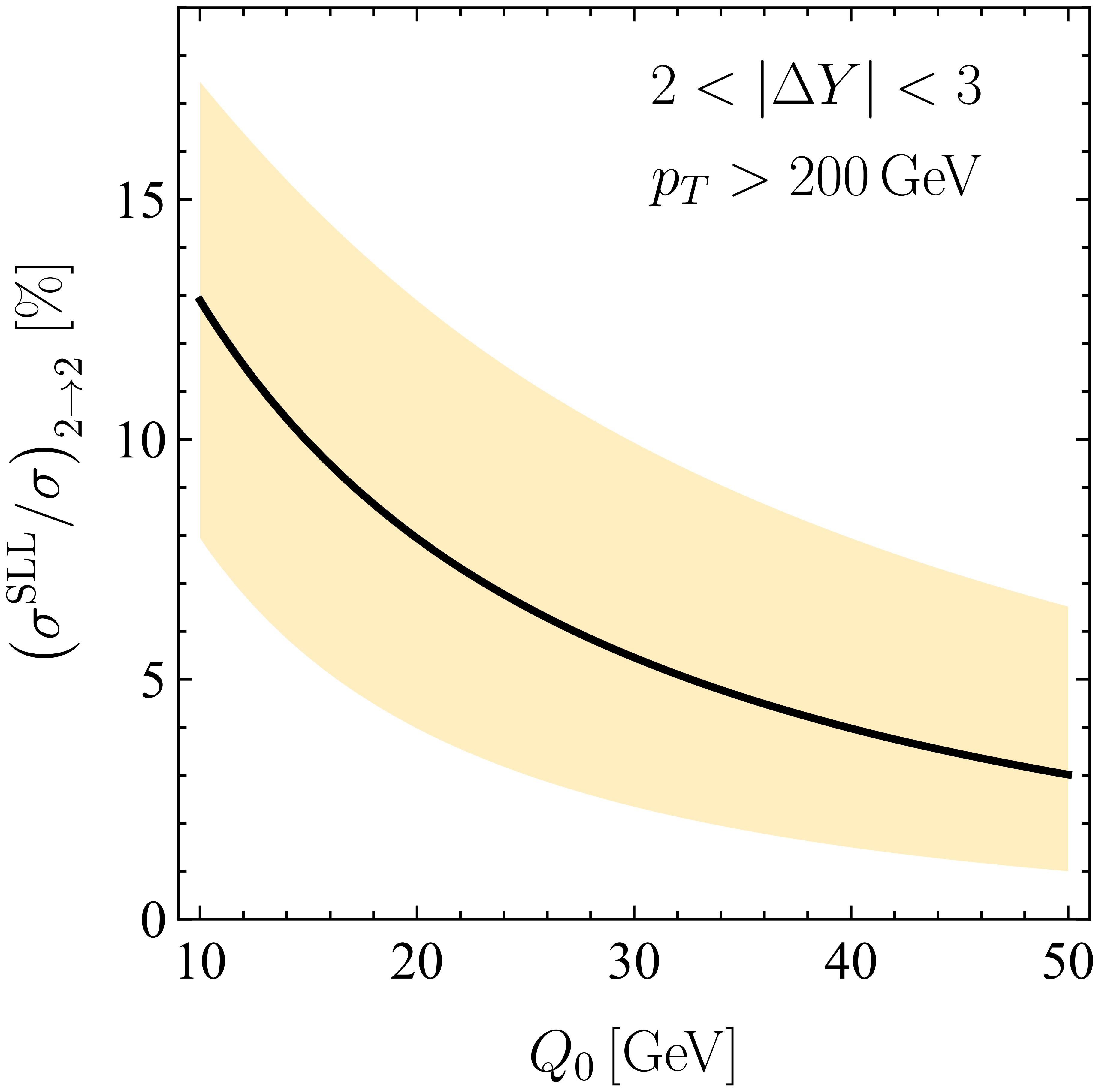


Figure 2: SLL contribution to the $pp \to 2\,\text{jets}$ cross section at the LHC as a function of the veto scale Q_0, for a center-of-mass energy $\sqrt{s} = 13\,\text{TeV}$ and jet radius $R = 0.6$.

[Becher, Hager, Martinelli, MN, Schwienbacher, Stillger (2024)]

PDF FACTORIZATION ?

It was believed by many experts that the breakdown of collinear factorization would imply a **breakdown of PDF factorization**

JHEP

Published for SISSA by Springer

Received: *January 4, 2020*
Accepted: *May 3, 2020*
Published: *May 27, 2020*

Soft gluon emission at two loops in full color

Lance J. Dixon,[a] Enrico Herrmann,[a] Kai Yan[b] and Hua Xing Zhu[c]

Abstract: The soft emission factor is a central ingredient in the factorization of generic n-particle gauge theory amplitudes with one soft gluon in the external state. We present

...

In the limit where the outgoing soft gluon is also collinear with an incoming hard parton, potentially dangerous factorization-violating terms can arise.

We speculate that at next-to-next-to-next-to-leading order (NNNLO) in QCD, integrating over the phase space of the collinear splitting can give rise to soft-collinear poles which depend on the color charge of non-collinear partons entering the process. Such poles cannot be canceled by the conventional counterterms associated with renormalization of the parton distribution functions (PDFs), which by definition are process independent.

An interesting example that can contain such factorization-violating contributions is the NNNLO QCD corrections to dijet production at hadron colliders.

STRUCTURE OF THE FACTORIZATION THEOREM ?

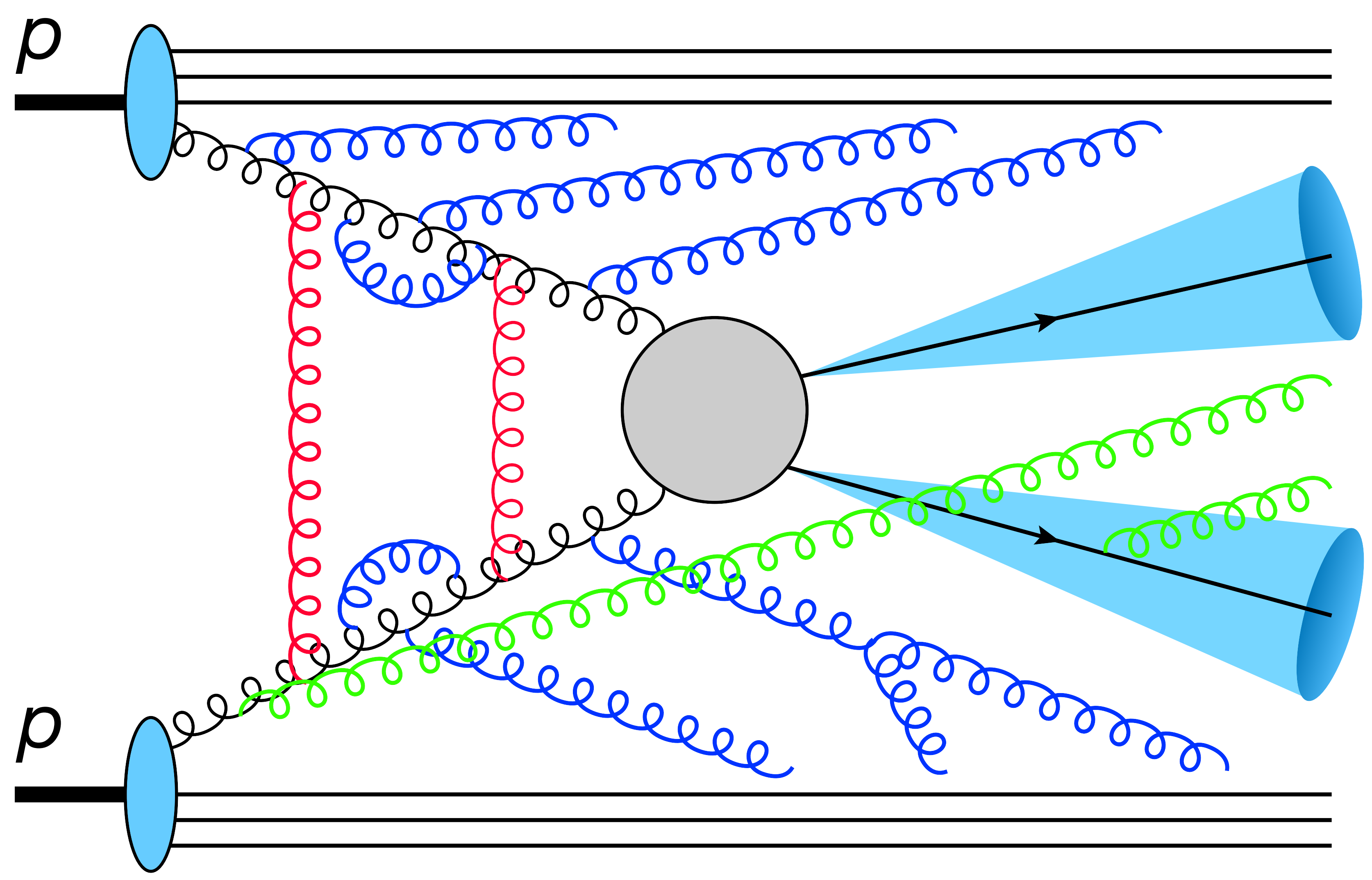
hard scale $Q \sim \sqrt{\hat{s}}$ — $\mathcal{H}_m$

double-logarithmic evolution (super-leading logs) & non-DGLAP *) collinear evolution

jet-veto scale Q_0 — $\mathcal{W}_m^{\rm pert}$

single-logarithmic PDF evolution

hadronic scale $\Lambda_{\rm QCD}$ — $f_{a/p}(\xi_1)\, f_{b/p}(\xi_2)$



$$\sigma \overset{?}{\sim} \sum_m \mathcal{H}_m \otimes \mathcal{W}_m^{\rm pert} \otimes f_{a/p} \otimes f_{b/p}$$

*) Dokshitzer–Gribov–Lipatov–Altarelli–Parisi

STRUCTURE OF THE FACTORIZATION THEOREM ?

hard scale $Q \sim \sqrt{\hat{s}}$ — $\mathcal{H}_m$

double-logarithmic evolution (super-leading logs) & non-DGLAP *) collinear evolution

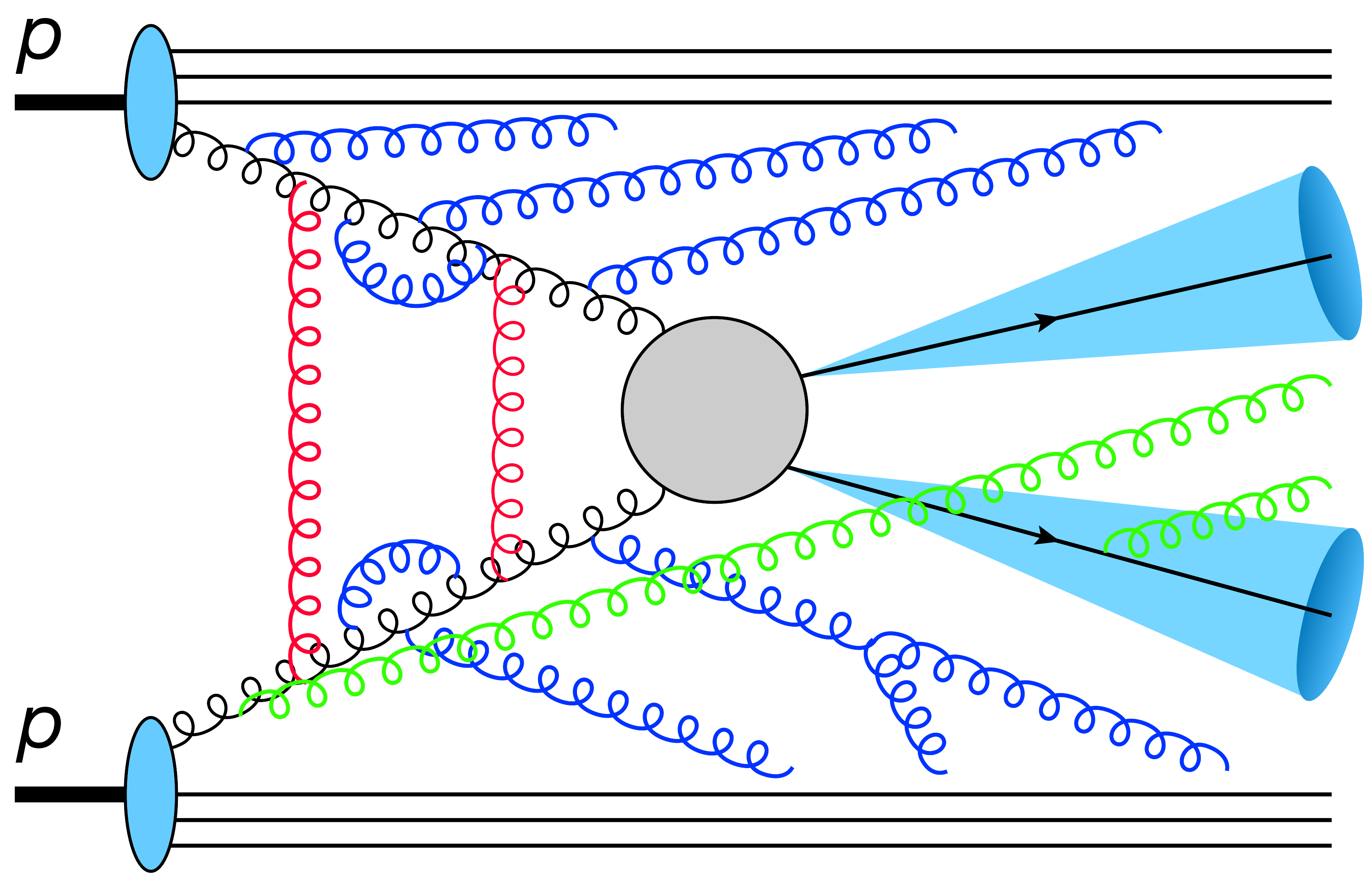
jet-veto scale Q_0 — $\mathcal{W}_m^{\mathrm{pert}}$

single-logarithmic PDF evolution

OR: *double-logarithmic evolution below Q_0 and breaking of PDF factorization?*

hadronic scale Λ_{QCD} — $f_{a/p}(\xi_1)\, f_{b/p}(\xi_2)$

OR: *something more complicated, e.g. a combined two-proton distribution?*



$$\sigma \sim \sum_m \mathcal{H}_m \otimes \mathcal{W}_m^{\mathrm{pert}} \otimes \cancel{f_{a/p} \otimes f_{b/p}}$$

*) Dokshitzer–Gribov–Lipatov–Altarelli–Parisi

STRUCTURE OF THE FACTORIZATION THEOREM ?

To settle the question, we calculate the **perturbative μ dependence** of $\mathcal{W}_m^{\text{pert}}$, associated with the veto scale Q_0 , and check whether the remaining scale dependence is that of the PDFs

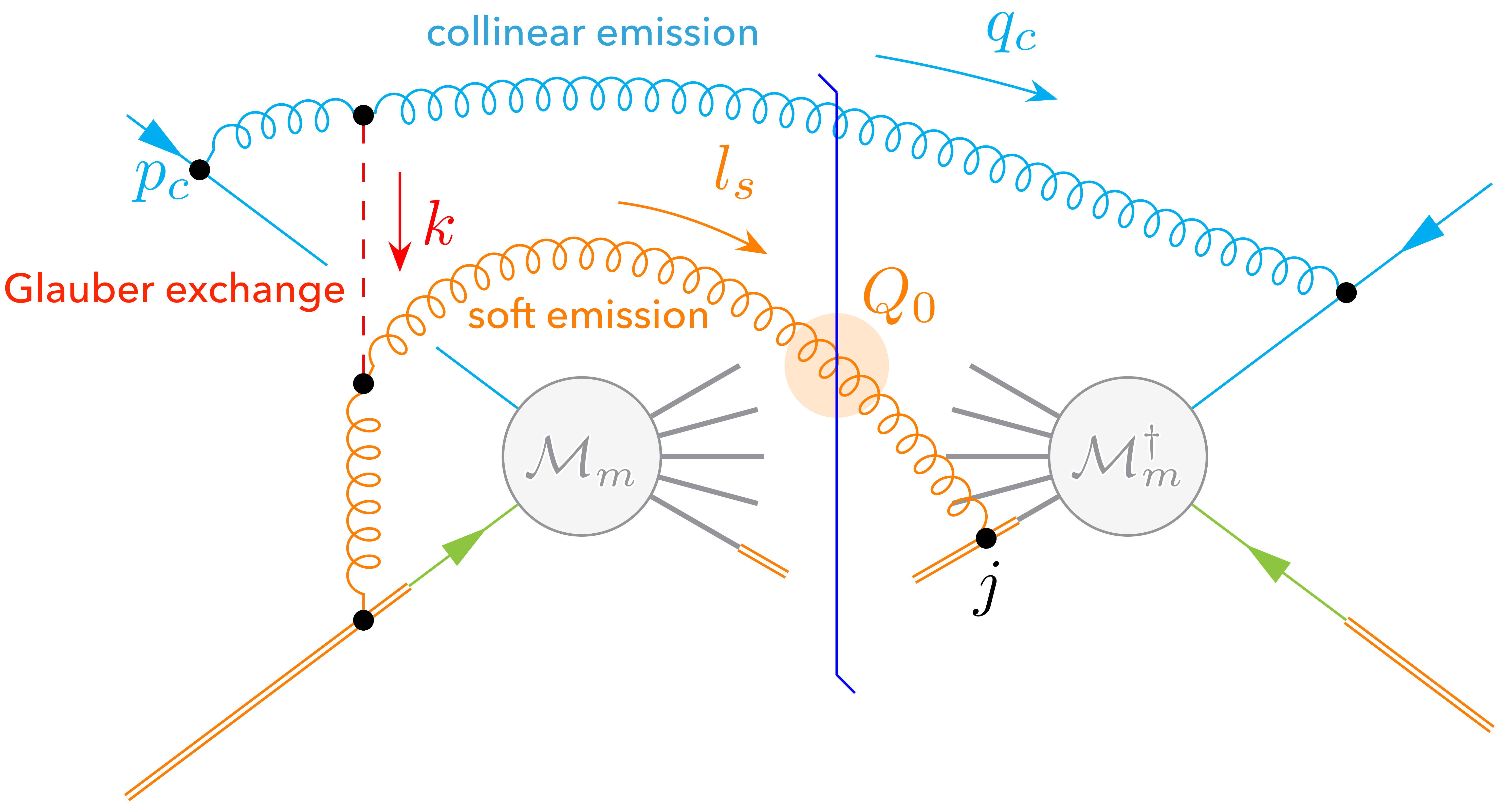
- Scale dependence of a quantity $\leftrightarrow$ $1/\varepsilon^n$ poles in dimensional regularization
- **Assuming PDF factorization,** we predict:

$$\mathcal{W}_m^{\text{bare}} = \mathbf{1} + \frac{\alpha_s}{4\pi}\,\frac{\overline{\mathbf{\Gamma}}}{2\varepsilon}\,\checkmark + \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{\boldsymbol{V}^G\,\overline{\mathbf{\Gamma}}}{2\varepsilon^2}\,\checkmark + \dots\right)$$
$$+ \left(\frac{\alpha_s}{4\pi}\right)^3 \left[\frac{\mathbf{\Gamma}^c\,\boldsymbol{V}^G\,\overline{\mathbf{\Gamma}}}{3\varepsilon^3}\left(\frac{11}{6\varepsilon} + \ln\frac{\mu_s^2}{Q^2} + \frac{9}{2}\,\ln\frac{\mu_s^2}{Q_0^2}\right) + \frac{1}{12\varepsilon^3}\,[\mathbf{\Gamma}^C, [\boldsymbol{V}^G, \overline{\mathbf{\Gamma}}]] + \frac{\boldsymbol{V}^G\,\boldsymbol{V}^G\,\overline{\mathbf{\Gamma}}}{3\varepsilon^3}\,\checkmark + \dots\right] + \mathcal{O}(\alpha_s^4)$$

STRUCTURE OF THE FACTORIZATION THEOREM ?

- Relevant graphs at 3-loop order:

[Becher, Hager, Jaskiewicz, MN, Schwienbacher (2024) → PRL]



STRUCTURE OF THE FACTORIZATION THEOREM ?

- **Assuming PDF factorization,** we predict: [Becher, Hager, Jaskiewicz, MN, Schwienbacher (2024)]

$$\mathcal{W}_m^{\text{bare}} = \mathbf{1} + \frac{\alpha_s}{4\pi}\frac{\overline{\mathbf{\Gamma}}}{2\varepsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{\boldsymbol{V}^G\,\overline{\mathbf{\Gamma}}}{2\varepsilon^2} + \dots\right)$$
$$+ \left(\frac{\alpha_s}{4\pi}\right)^3 \left[\frac{\mathbf{\Gamma}^c\,\boldsymbol{V}^G\,\overline{\mathbf{\Gamma}}}{3\varepsilon^3}\checkmark\left(\frac{11}{6\varepsilon} + \ln\frac{\mu_s^2}{Q^2} + \frac{9}{2}\ln\frac{\mu_s^2}{Q_0^2}\right) + \frac{1}{12\varepsilon^3}\left[\mathbf{\Gamma}^C, \left[\boldsymbol{V}^G, \overline{\mathbf{\Gamma}}\right]\right](\checkmark) + \frac{\boldsymbol{V}^G\,\boldsymbol{V}^G\,\overline{\mathbf{\Gamma}}}{3\varepsilon^3} + \dots\right] + \mathcal{O}(\alpha_s^4)$$

double-logarithmic evolution above Q_0

non-DGLAP collinear evolution above Q_0
(work in progress)

where:

$\overline{\mathbf{\Gamma}}$: soft emission operator

$\boldsymbol{V}^G$: Glauber operator

$\mathbf{\Gamma}^c$: soft-collinear emission operator

$\mathbf{\Gamma}^C$: collinear emission operator

STRUCTURE OF THE FACTORIZATION THEOREM

hard scale $Q \sim \sqrt{\hat{s}}$ — $\mathcal{H}_m$

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single-logarithmic PDF evolution

hadronic scale Λ_{QCD} — $f_{a/p}(\xi_1)\, f_{b/p}(\xi_2)$

$$\sigma \overset{\checkmark}{\sim} \sum_m \mathcal{H}_m \otimes \mathcal{W}_m^{\text{pert}} \otimes f_{a/p} \otimes f_{b/p}$$

[Becher, Hager, Jaskiewicz, MN, Schwienbacher (2024)]

- despite all odds, PDF factorization is restored!
- we have proved this to 3-loop order
- **conjecture** that it holds to all orders

SUMMARY

- We have uncovered a new mechanism that reconciles the breaking of collinear factorization with unbroken PDF factorization
- In an interplay of space-like collinear splittings and soft emissions, perturbative Glauber gluons restore the factorization of the cross section by converting double-logarithmic into single-logarithmic evolution below the veto scale Q_0, and turning non-DGLAP into DGLAP evolution
- It will be important to understand the all-order structure of these effects, paving the way for a proof of PDF factorization for a much wider class of observables!