Low-energy puzzles and the role of lattice QCD

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PRISMA+

HIM
Helmholtz-Institut Mainz

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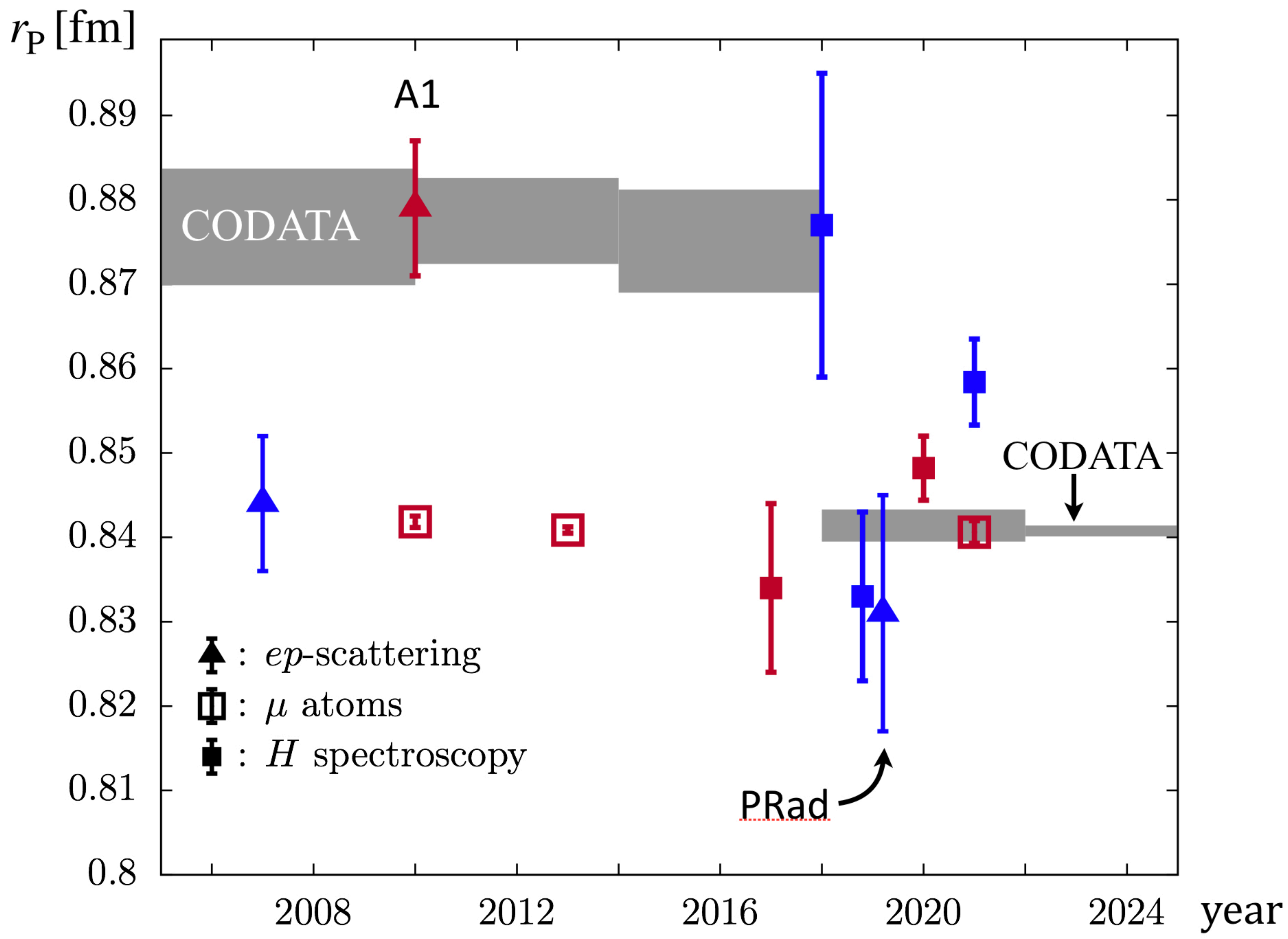
Is there a proton radius puzzle?

Discrepant measurements of r_p in muonic / electronic hydrogen and *ep* scattering



Muonic hydrogen:

$r_E = 0.84087 \pm 0.00039$ fm *[Antognini et al., 2013]*

Electron-proton scattering:

$r_E = 0.879 \pm 0.008$ fm *[A1, Bernauer et al., 2010]*

$r_E = 0.831 \pm 0.014$ fm *[PRad, Xiong et al., 2019]*

Signal for new physics or poorly understood systematic effects?

→ calls for *ab initio* calculation of the proton radius from QCD

Is there a signal for new physics in the muon $g-2$?

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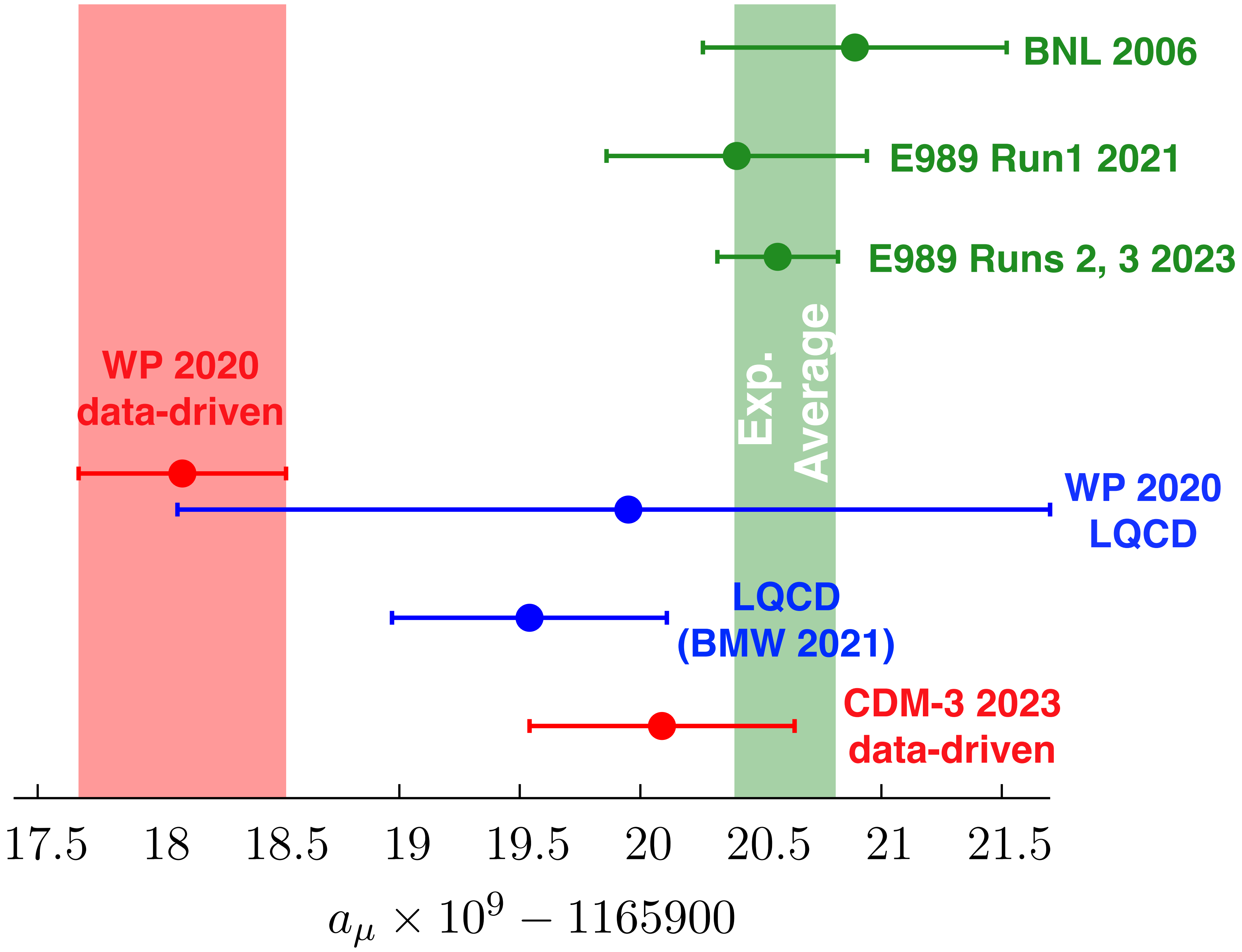
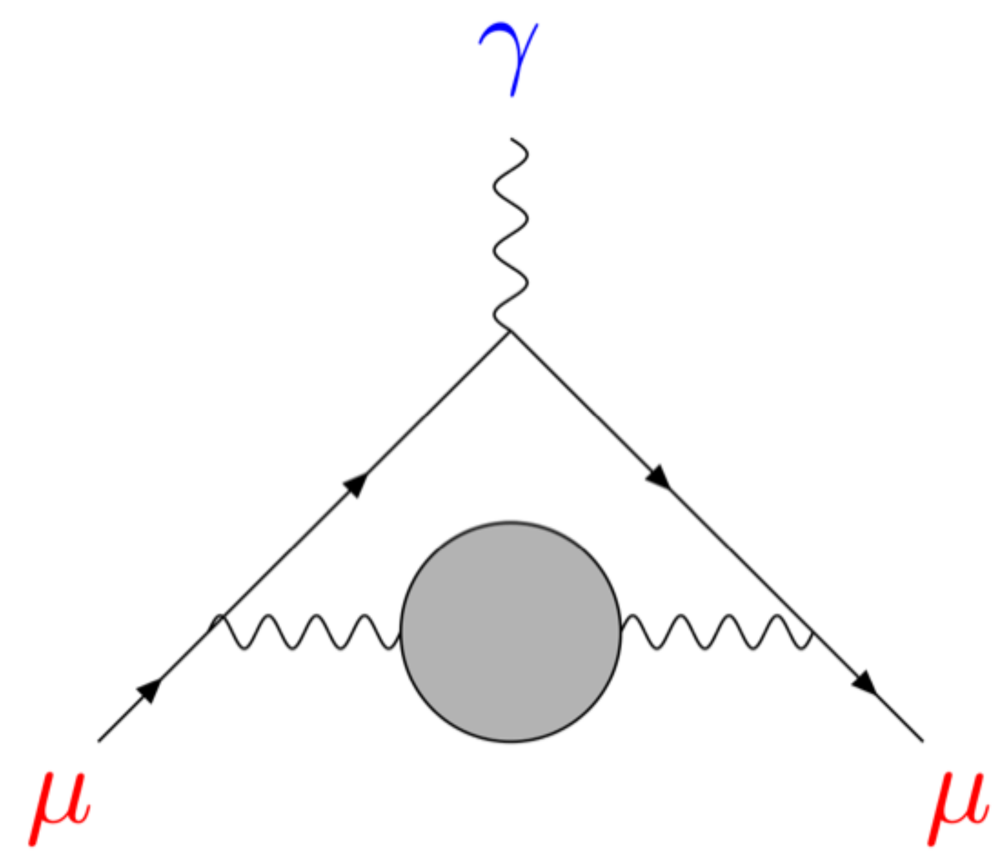
WP 2020: $a_\mu^{\rm SM} = 116\,591\,810(43)\times10^{-11}$ [0.37 ppm] $\Rightarrow$ $a_\mu^{\rm exp} - a_\mu^{\rm SM} = (249\pm48)\cdot10^{-11}$ $[5.1\sigma]$





Hadronic vacuum polarisation

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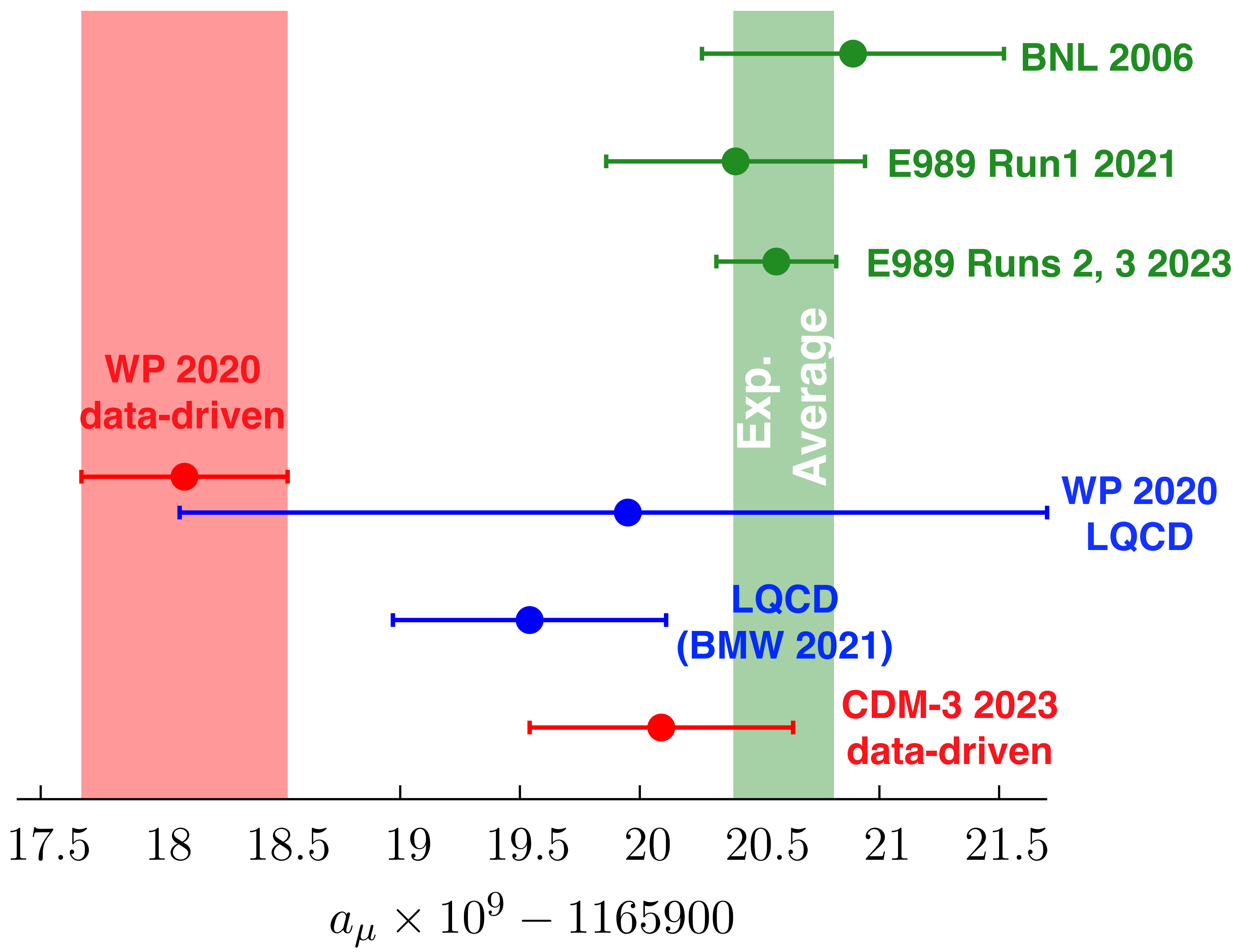
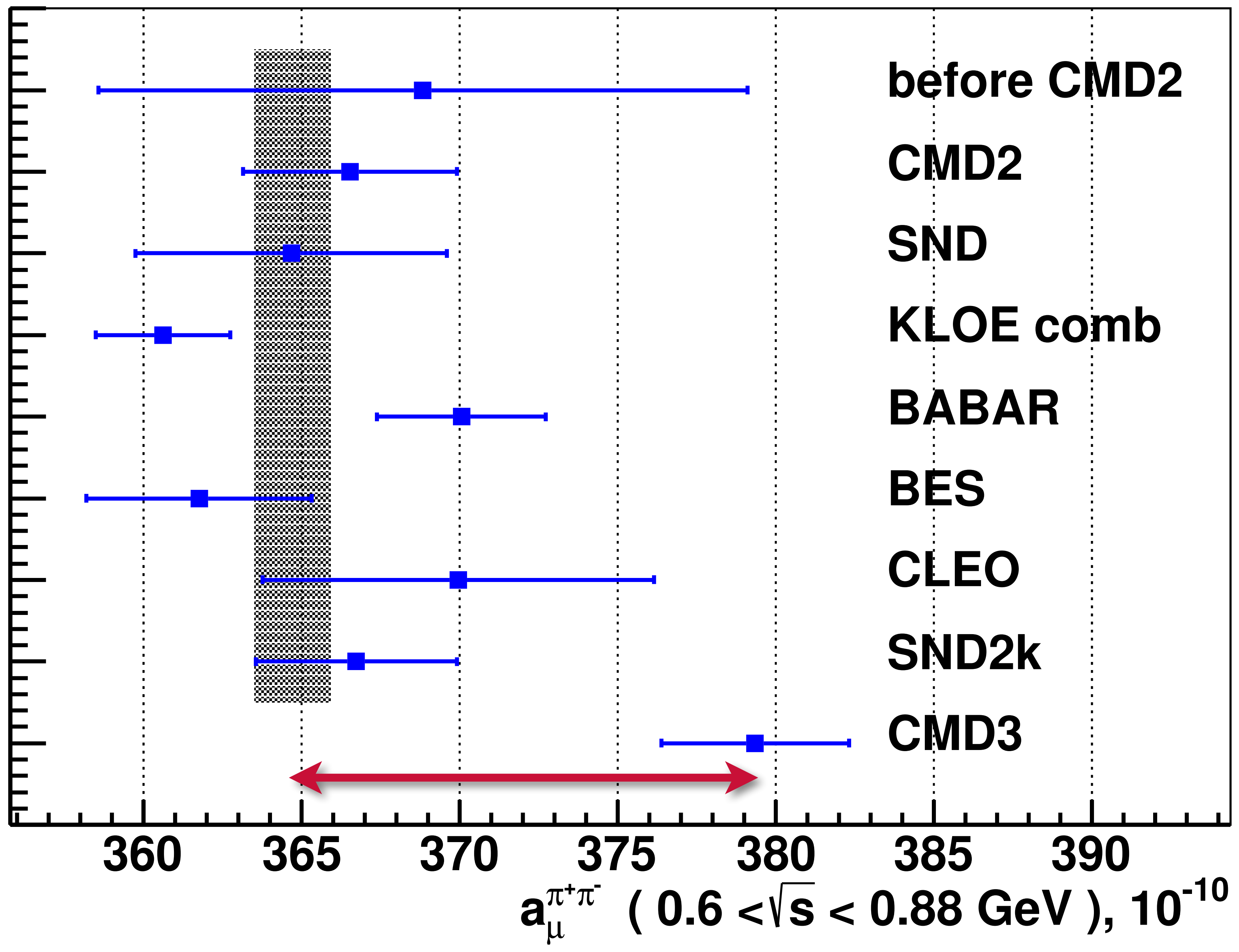




White paper estimate challenged by lattice QCD calculations [Borsányi et al., Nature 593 (2021) 7857]

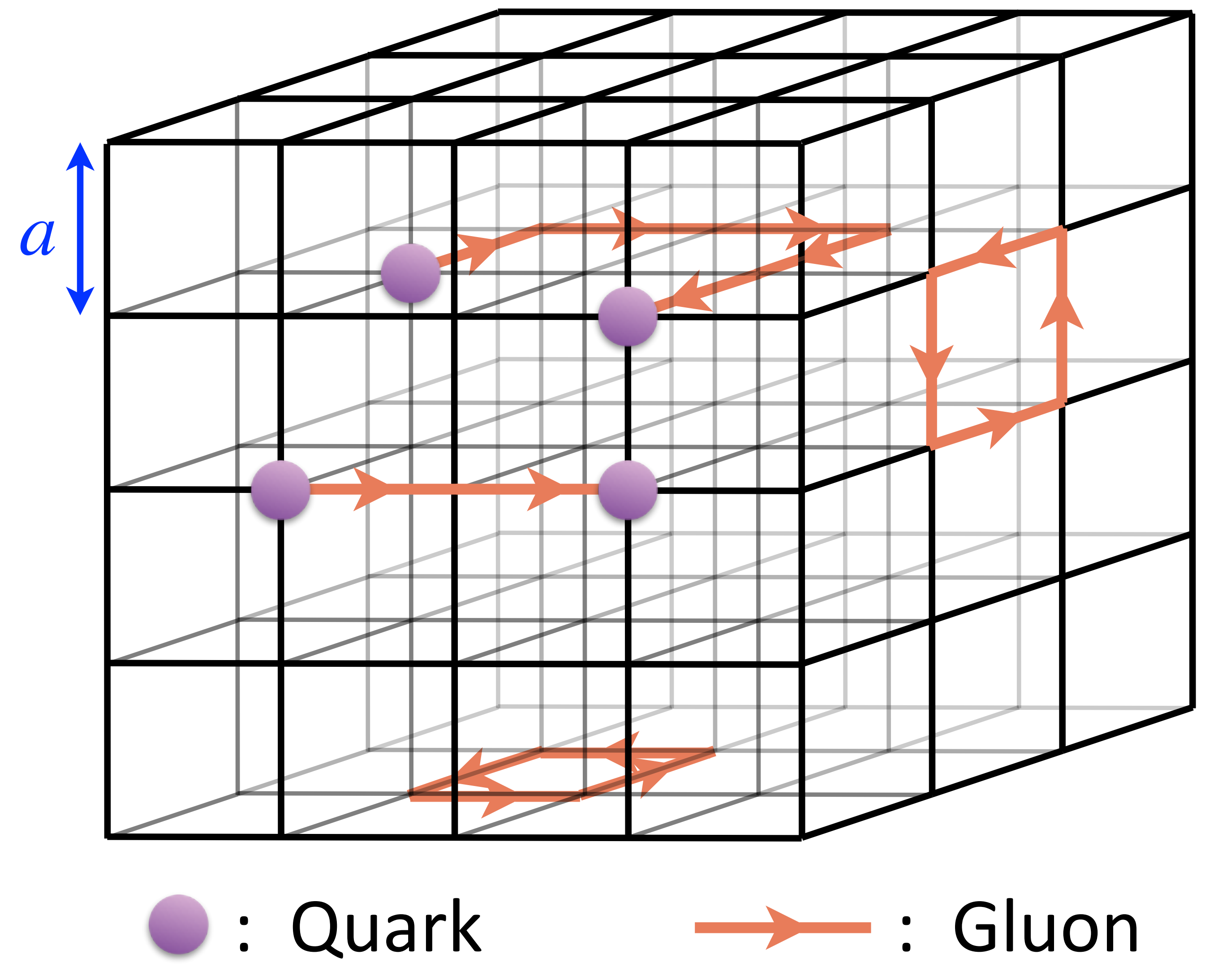
and tensions among cross section measurements for $e^+e^- \to$ hadrons [Ignatov et al., Phys Rev D109 (2024) 112002]

Lattice QCD in a Nutshell

Non-perturbative treatment of strong interaction via regularised Euclidean path integrals

Lattice spacing: $a, \quad x_\mu = n_\mu a, \quad a^{-1} = \Lambda_{\rm UV}$

Expectation value: $\langle\Omega\rangle = \frac{1}{Z}\int \prod_{x,\mu} dU_\mu(x)\; \Omega\; \mathrm{e}^{-S_{\rm G}^{\rm eff}[U]}$



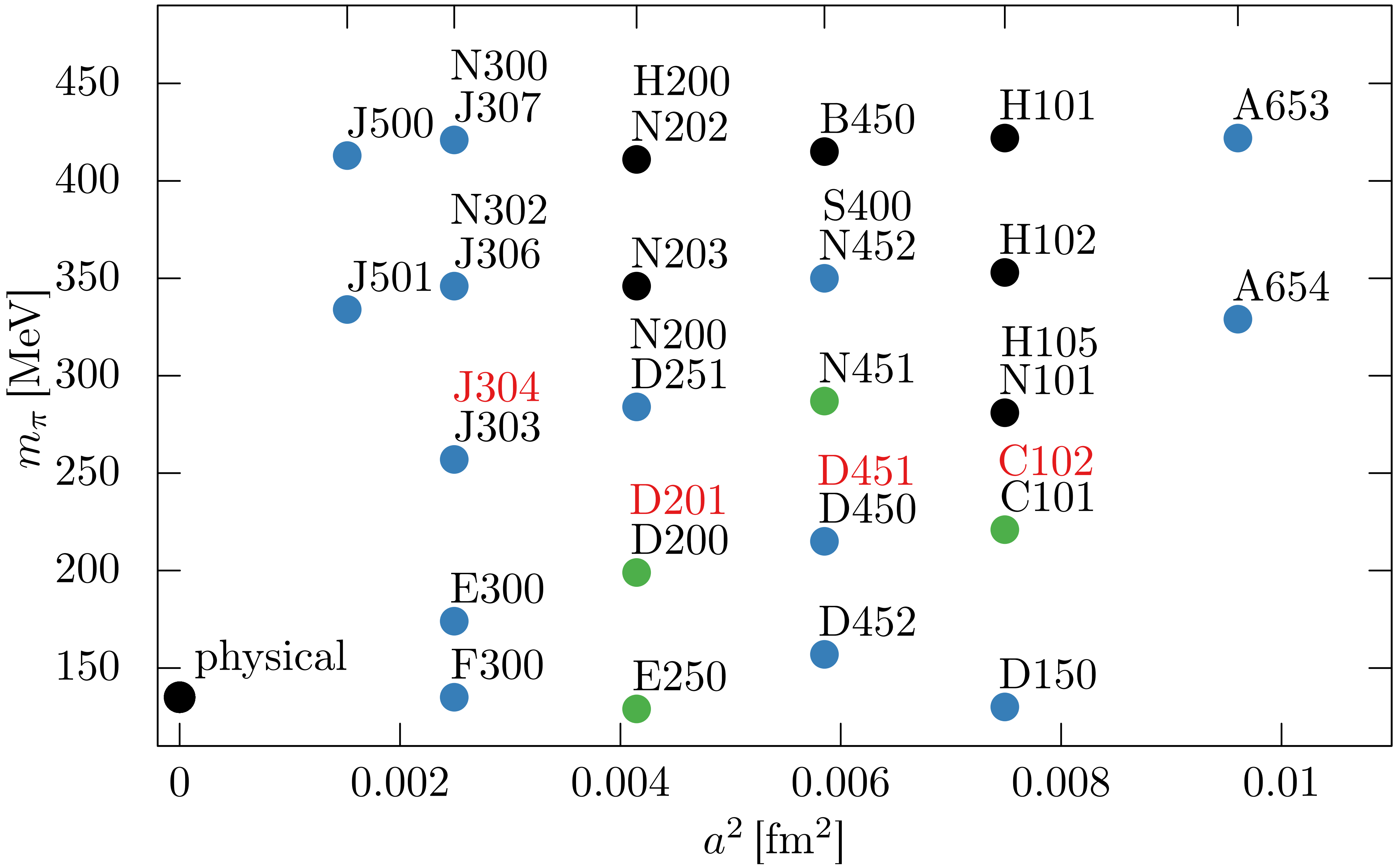
Procedure:

- Choose discretisation of QCD action
- Evaluate $\langle\Omega\rangle$ via Monte Carlo Integration:
  generate ensembles of gauge configurations via a Markov chain
- Ensemble average: $\langle\Omega\rangle \simeq \overline{\Omega}$ Statistical error: $\sqrt{\overline{\Omega^2} - \overline{\Omega}^2} \propto 1/N_{\rm cfg}^{1/2}$
- Extrapolate observables to the continuum limit: $a \to 0$ and tune quark masses to physical values

Lattice QCD at Mainz

Ensembles with $N_f = 2 + 1$ flavours of $\mathrm{O}(a)$ improved Wilson fermions generated by CLS effort

Six lattice spacings: $a = 0.099 - 0.035\,\mathrm{fm}$; Pion masses: $m_\pi = 130 - 420\,\mathrm{MeV}$

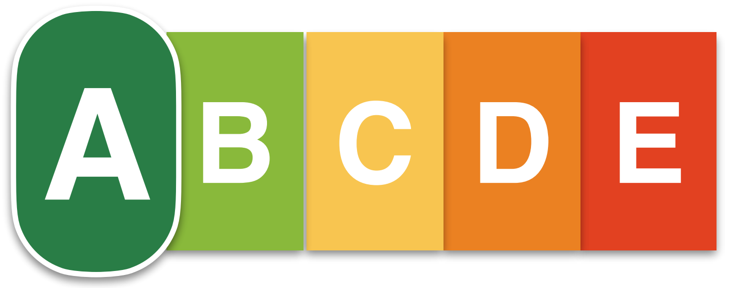


Computational cost: ≈ 400 M core-hrs p.a.

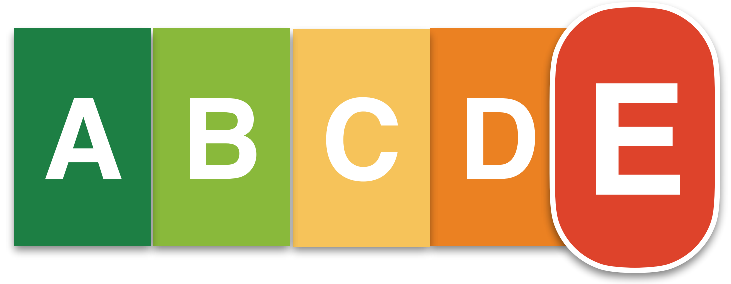
Intrinsic numerical cost

Wilson fermions

Staggered fermions



Domain-wall fermions



Ensemble "F300":

$m_\pi \approx 135\,\mathrm{MeV}\,, \quad a = 0.050\,\mathrm{fm}\,, \quad 128^3 \cdot 256$

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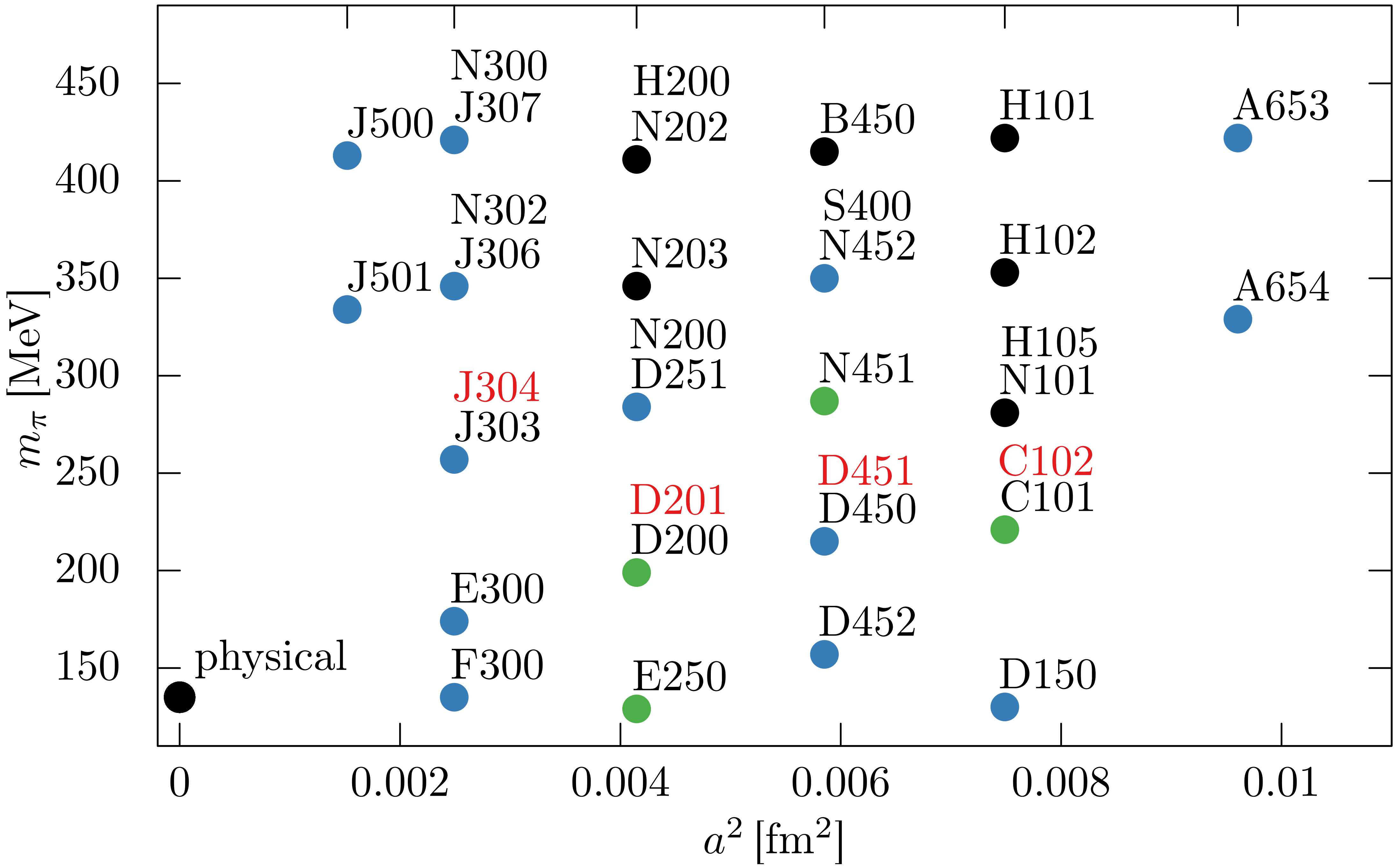

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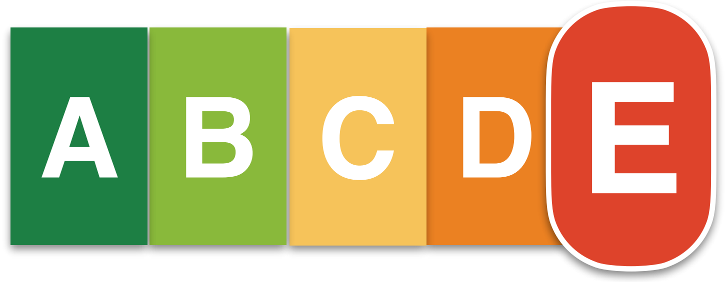
Conceptual issues / fermion doubling problem

Wilson fermions

Staggered fermions



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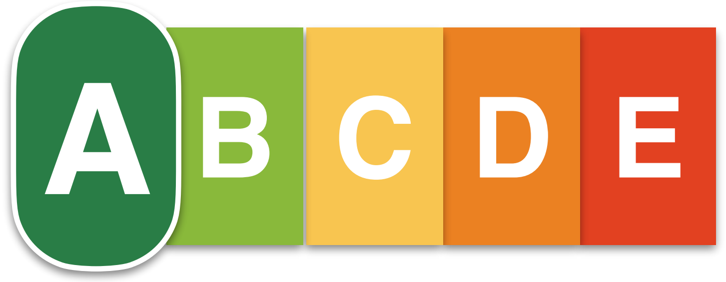


Nucleon form factors and the proton radius puzzle

Challenges for lattice QCD: The noise problem

Statistical errors grow exponentially in baryonic correlators:

$$R_{\rm NS}(t) \propto \mathrm{e}^{(m_N - \frac{3}{2}m_\pi)t}$$

→ Calculations of baryonic three-point functions limited to source-sink separations $t_s \lesssim 1.7\,\mathrm{fm}$

→ Potential bias from unsuppressed excited-state contributions

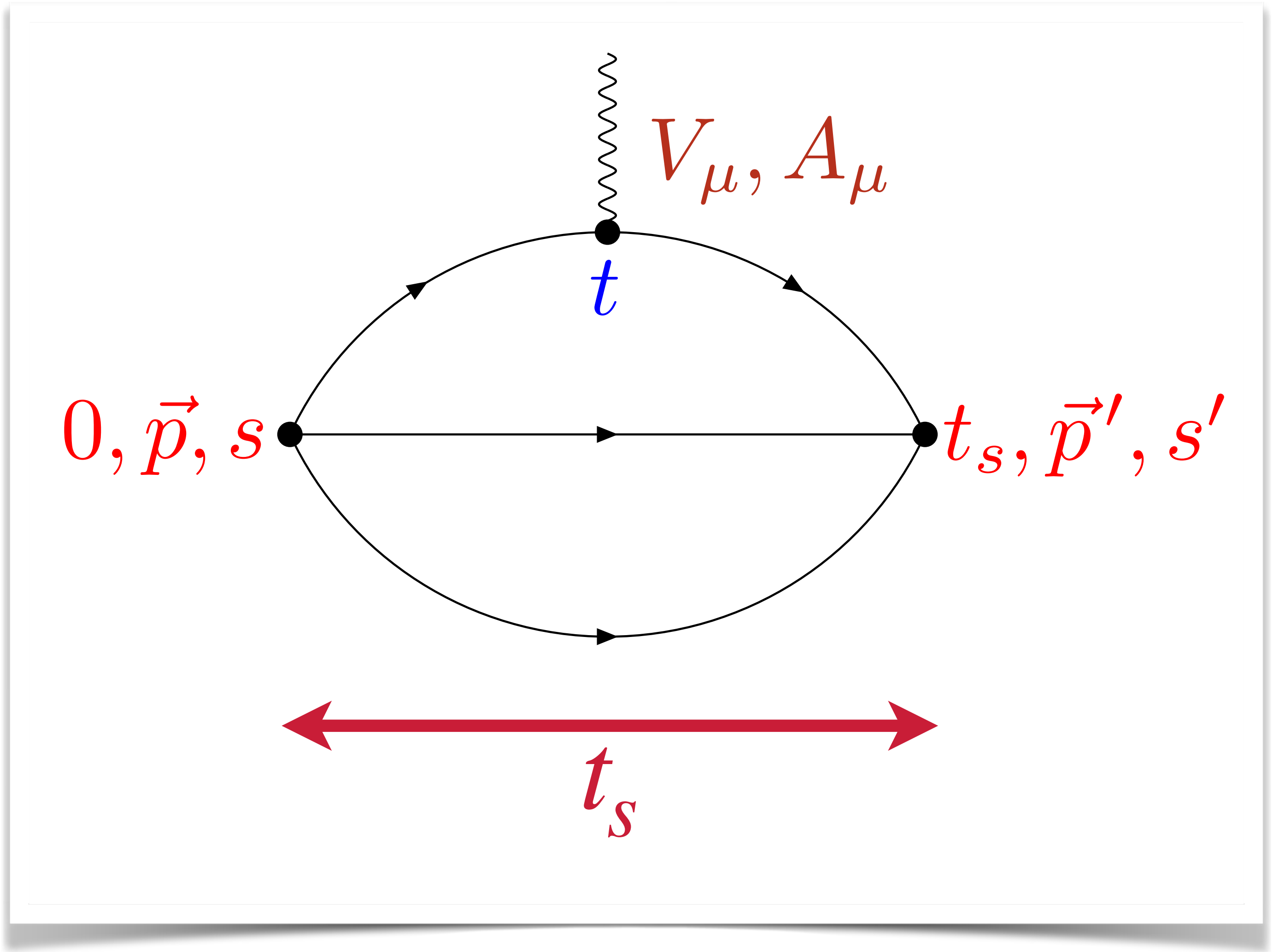
Encounter dense spectrum of $N\pi, N\pi\pi, \ldots$ states

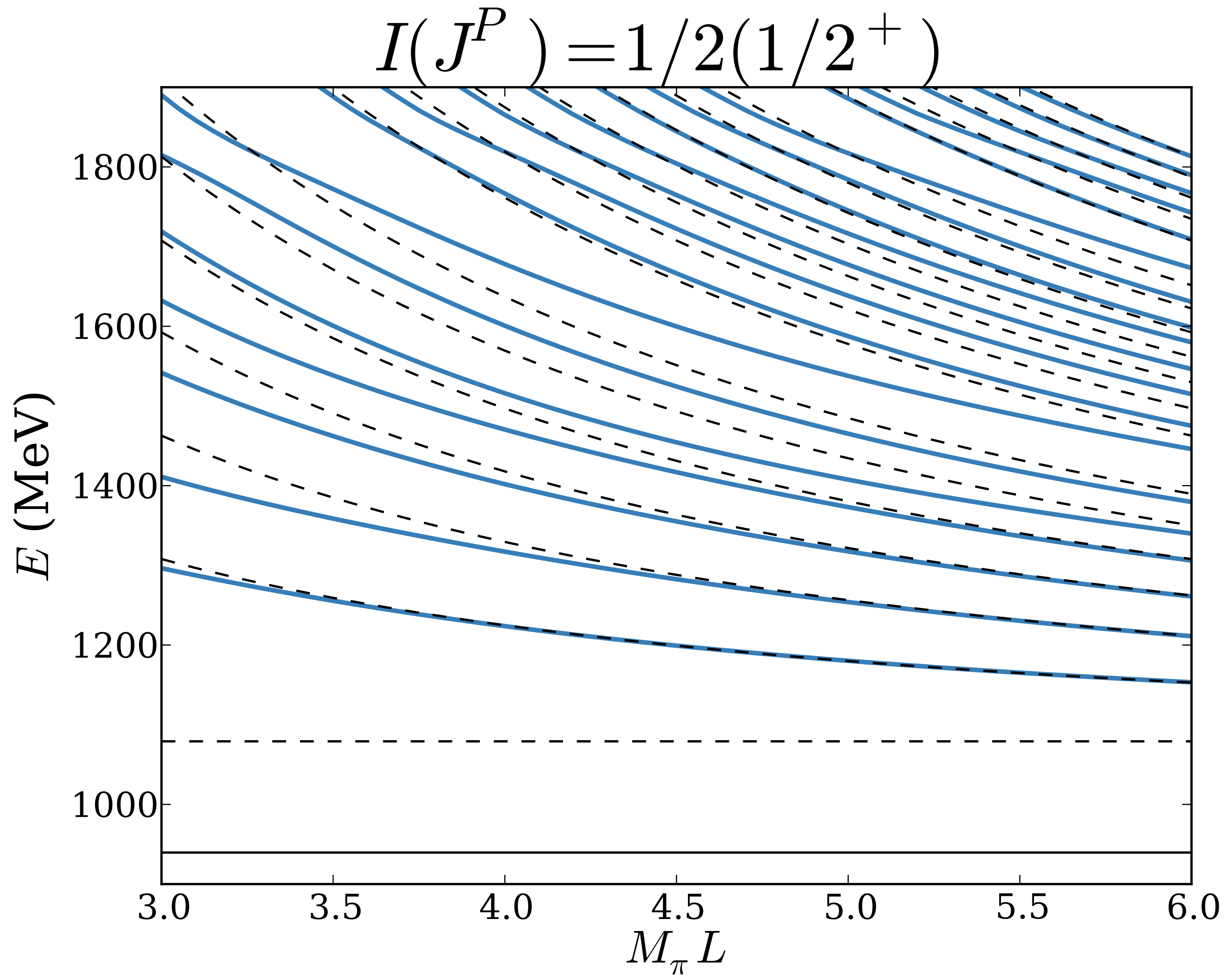
Nucleon matrix elements from ratios of 3- and 2-point functions:

$$R_\Gamma(t,\, t_s) \equiv \frac{C_3^\Gamma(\boldsymbol{q}=0;t,\,t_s)}{C_2(\boldsymbol{p}=0;\,t_s)}$$

$$= g_\Gamma + c_{01}\,\mathrm{e}^{-\Delta t} + c_{10}\,\mathrm{e}^{-\Delta(t_s-t)} + c_{11}\,\mathrm{e}^{-\Delta t_s} + \ldots$$

$$\Delta = (E_1 - E_0)\,, \qquad \Gamma = A,\, S,\, T,\, \ldots$$





[Hansen & Meyer, 1610.03843]

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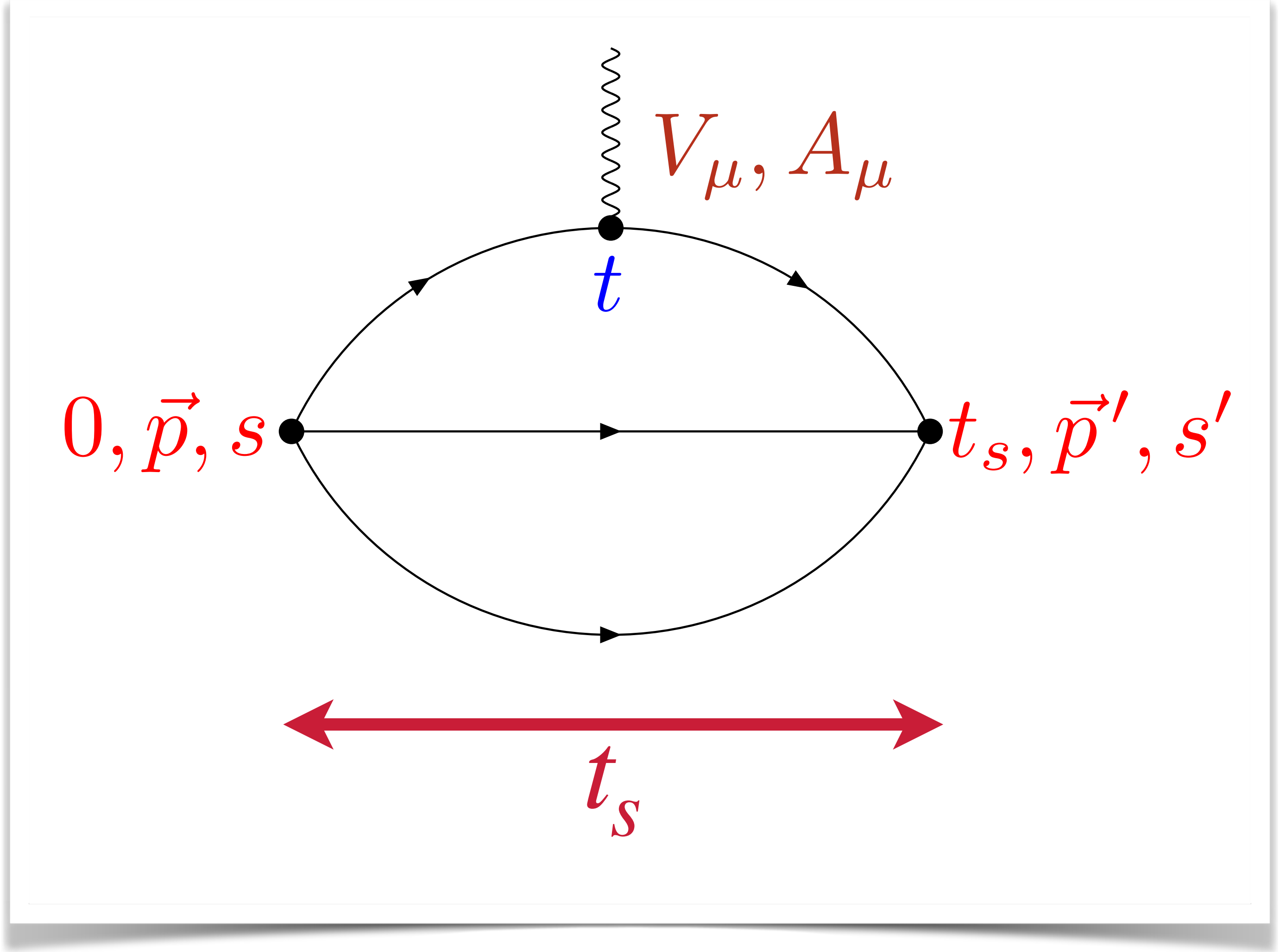
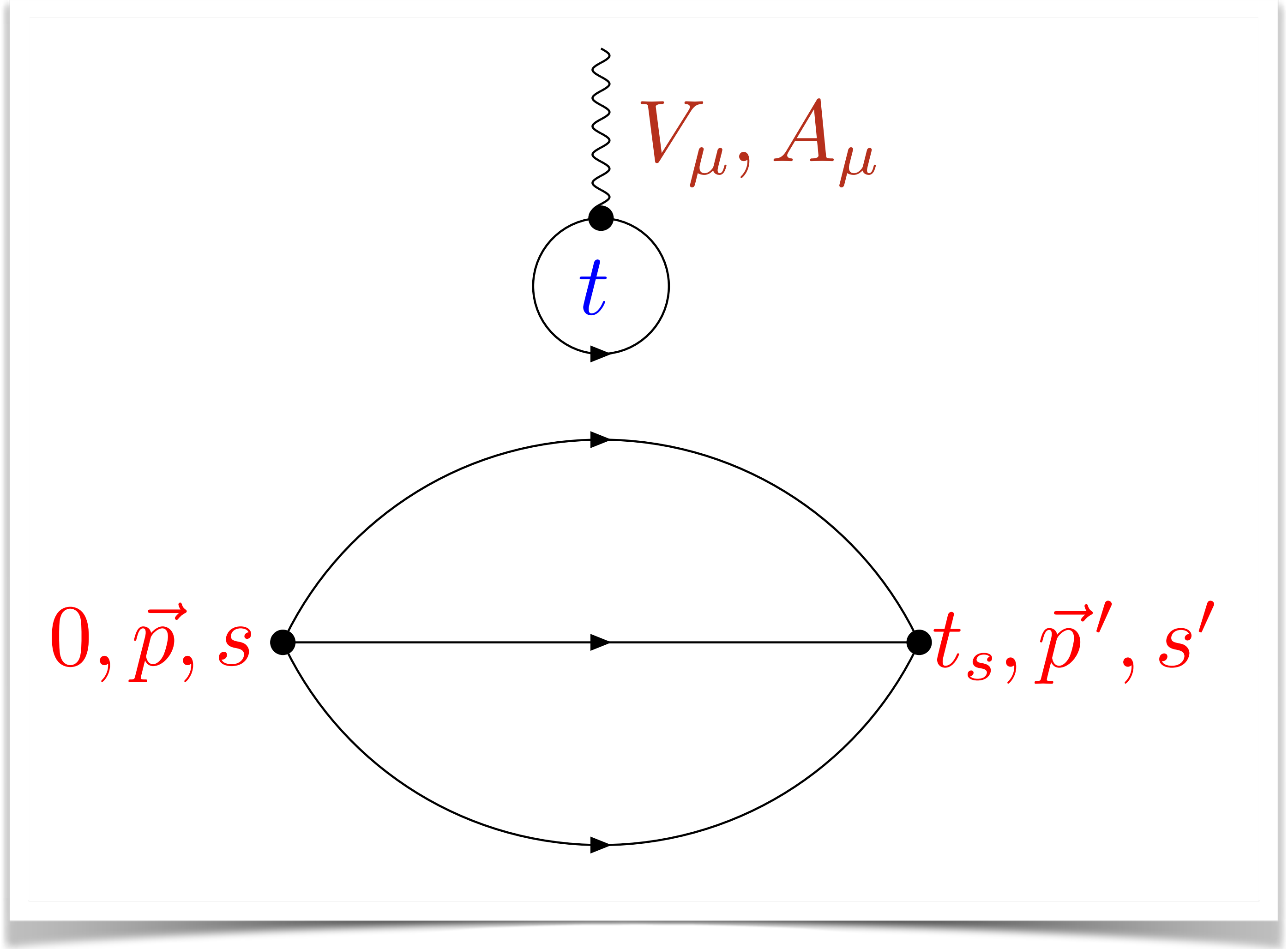


Quark-disconnected diagrams

- Large inherent statistical noise
- Contribute to isoscalar quantities and sigma-terms
- Contribute exclusively to strange form factors



Fighting the noise problem

$$R_\Gamma(t,\ t_s) = g_\Gamma + c_{01}\,\mathrm{e}^{-\Delta t} + c_{10}\,\mathrm{e}^{-\Delta(t_s-t)} + c_{11}\,\mathrm{e}^{-\Delta t_s} + \ldots\,,$$

Multi-state fits

Include sub-leading terms in $R_\Gamma(t, t_s)$ or in individual two- and three-point functions with or without priors for the excitation spectrum

Summed operator insertions ("summation method")

Excited-state contributions more strongly suppressed

$$S_\Gamma(t_s) \equiv \sum_{t=0}^{t_s-a} R_\Gamma(t,\ t_s) = K_\Gamma + (t_s - a)\,g_\Gamma + (t_s - a)\,\mathrm{e}^{-\Delta t_s}\,d_\Gamma + \mathrm{e}^{-\Delta t_s}\,f_\Gamma + \ldots$$

Variational approach

Compute correlator matrices; solve GEVP; optimise projection on ground state

The Mainz nucleon structure project

Past and present members:

A. Agadjanov, A. Barone, S. Capitani, M. Della Morte, D. Djukanovic, T. Harris, G. von Hippel, J. Hua, B. Jäger, P. Junnarkar, B. Knippschild, J. Koponen, H.B. Meyer, D. Mohler, K. Ottnad, T.D. Rae, M. Salg, T. Schulz, J. Wilhelm, H. Wittig

Recent publications based on ensembles with $N_f = 2 + 1$ flavours of $\mathrm{O}(a)$ improved Wilson quarks

Nucleon charges
[Harris et al., Phys Rev D 100 (2019) 034513; Djukanovic et al., Phys Rev D 109 (2024) 074507]

Strange form factors
[Djukanovic et al., Phys Rev Lett 123 (2019) 212001]

Axial form factors
[Djukanovic et al., Phys Rev D 106 (2022) 074503]

Nucleon sigma terms
[Agadjanov et al., Phys Rev Lett 131 (2023) 26]

Electromagnetic form factors
[Djukanovic et al., Phys Rev D 103 (2021) 094522; Phys Rev D 109 (2024) 9; Phys Rev Lett 132 (2024) 21]

Zemach and Friar radii of the proton and neutron
[Djukanovic et al., Phys Rev D 110 (2024) 1]

Nucleon form factors and charge radii: Definitions

Dirac and Pauli form factors:

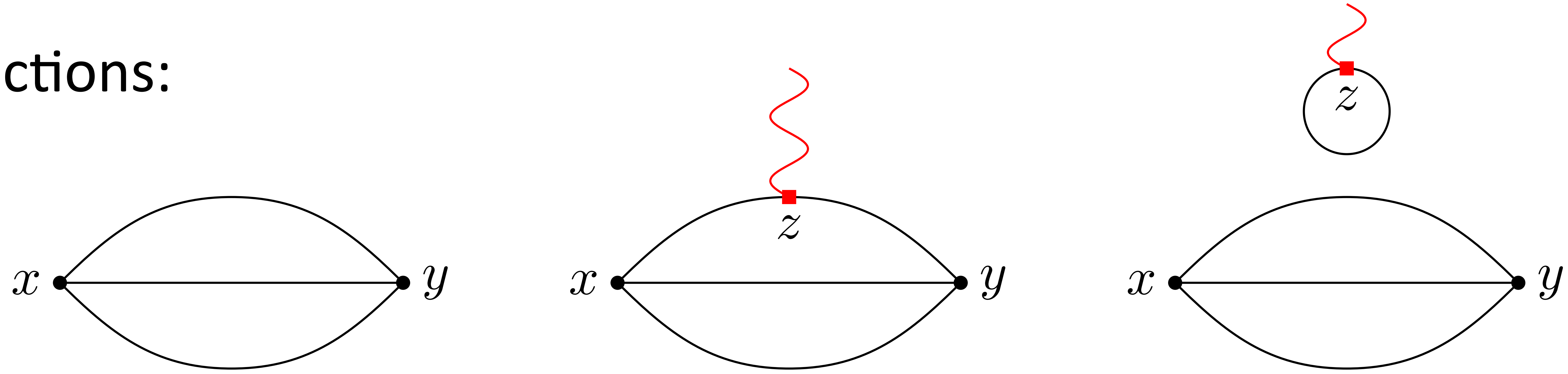
$$\langle N(p',s') | J_\mu^{\rm em}(0) | N(p,s) \rangle = \bar{u}(p',s') \left[\gamma_\mu F_1(Q^2) + \sigma_{\mu\nu} \frac{Q_\nu}{2m_{\rm N}} F_2(Q^2) \right] u(p,s)$$

$$G_{\rm E}(Q^2) = F_1(Q^2) - \frac{Q^2}{(am_{\rm N})^2} F_2(Q^2)\,, \qquad G_{\rm M}(Q^2) = F_1(Q^2) + F_2(Q^2)$$

Charge radii, magnetic moment:

$$G_{\rm E}(Q^2) = \left(1 - \tfrac{1}{6}\langle r_{\rm E}^2 \rangle Q^2 + {\rm O}(Q^4)\right)\,, \quad G_{\rm M}(Q^2) = \mu\left(1 - \tfrac{1}{6}\langle r_{\rm M}^2 \rangle Q^2 + {\rm O}(Q^4)\right)$$

Correlation functions:



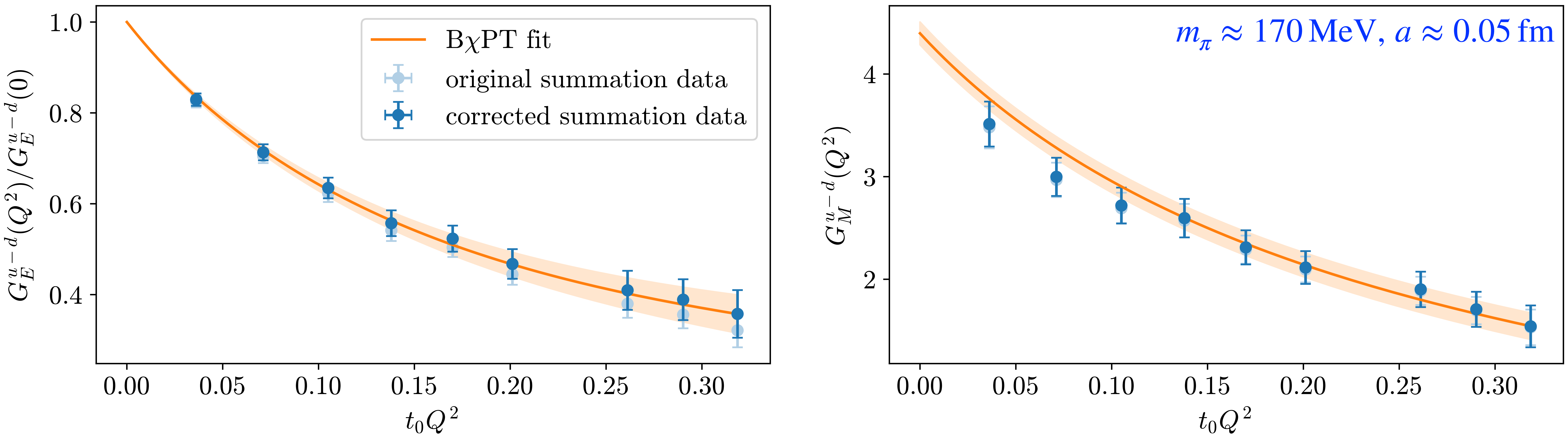
Nucleon form factors and charge radii: Procedure

Extract isovector $(u-d)$ and isoscalar $(u+d-2s)$ form factors from correlators $\rightarrow$ $G^{\rm eff}_{\rm E/M}(Q^2,\, m_\pi^2;\, a,\, L)$

11 gauge ensembles: 4 lattice spacings: $a = 0.050 - 0.087\,{\rm fm}$; Pion masses: $128 - 300\,{\rm MeV}$

Perform simultaneous fit to describe dependence on Q^2, m_π^2, lattice spacing and volume:

$$G^{\rm eff}_{\rm E/M}(Q^2,\, m_\pi^2;\, a,\, L) = G^{\rm B\chi PT}_{\rm E/M}(Q^2,\, m_\pi^2) + F_{\rm lat}(a^2;\, Q^2) + F_{\rm vol}(m_\pi L)$$



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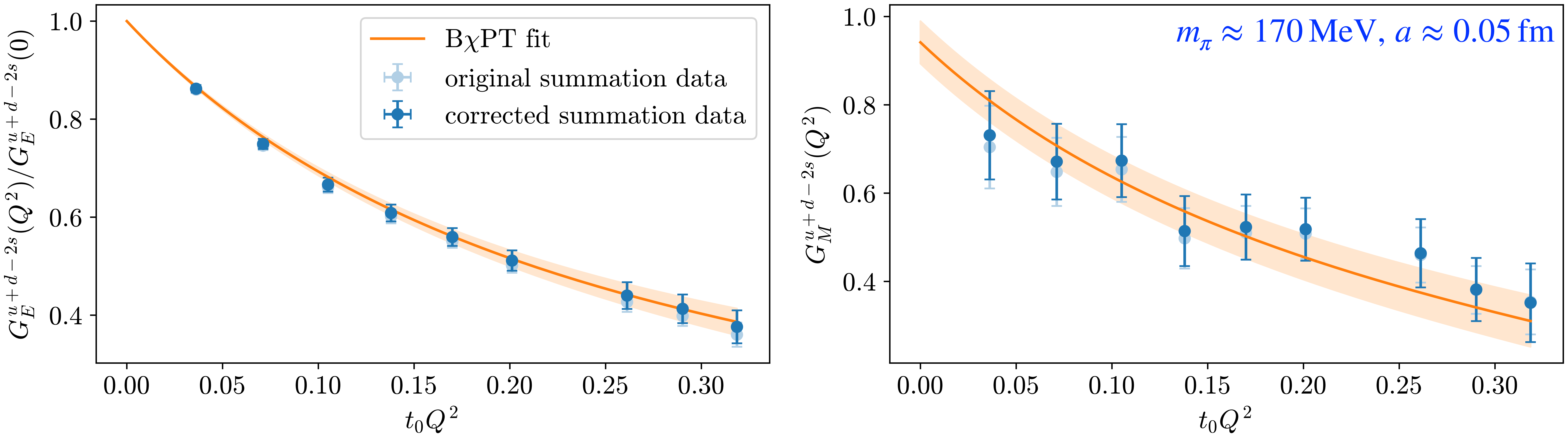

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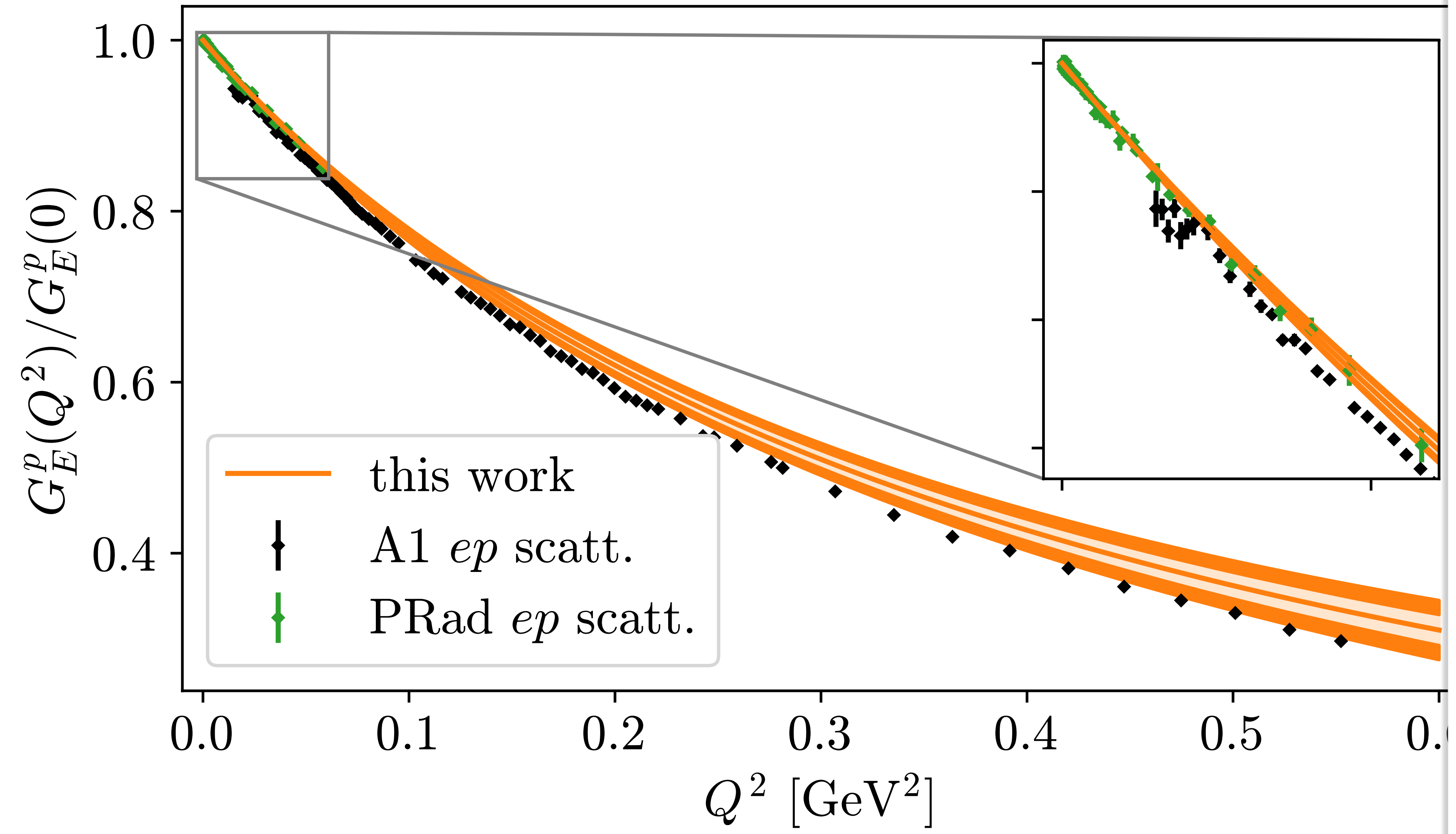
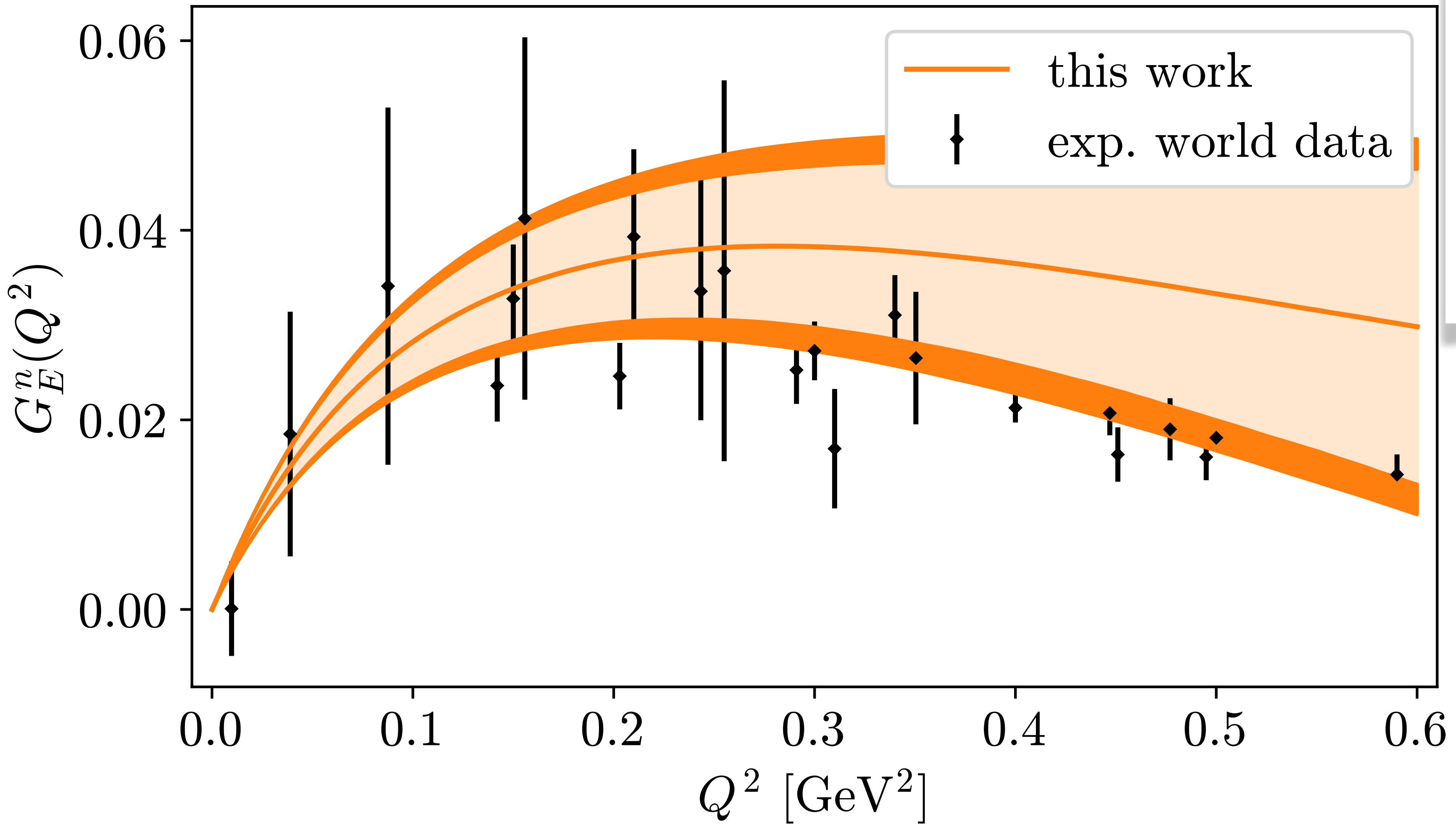
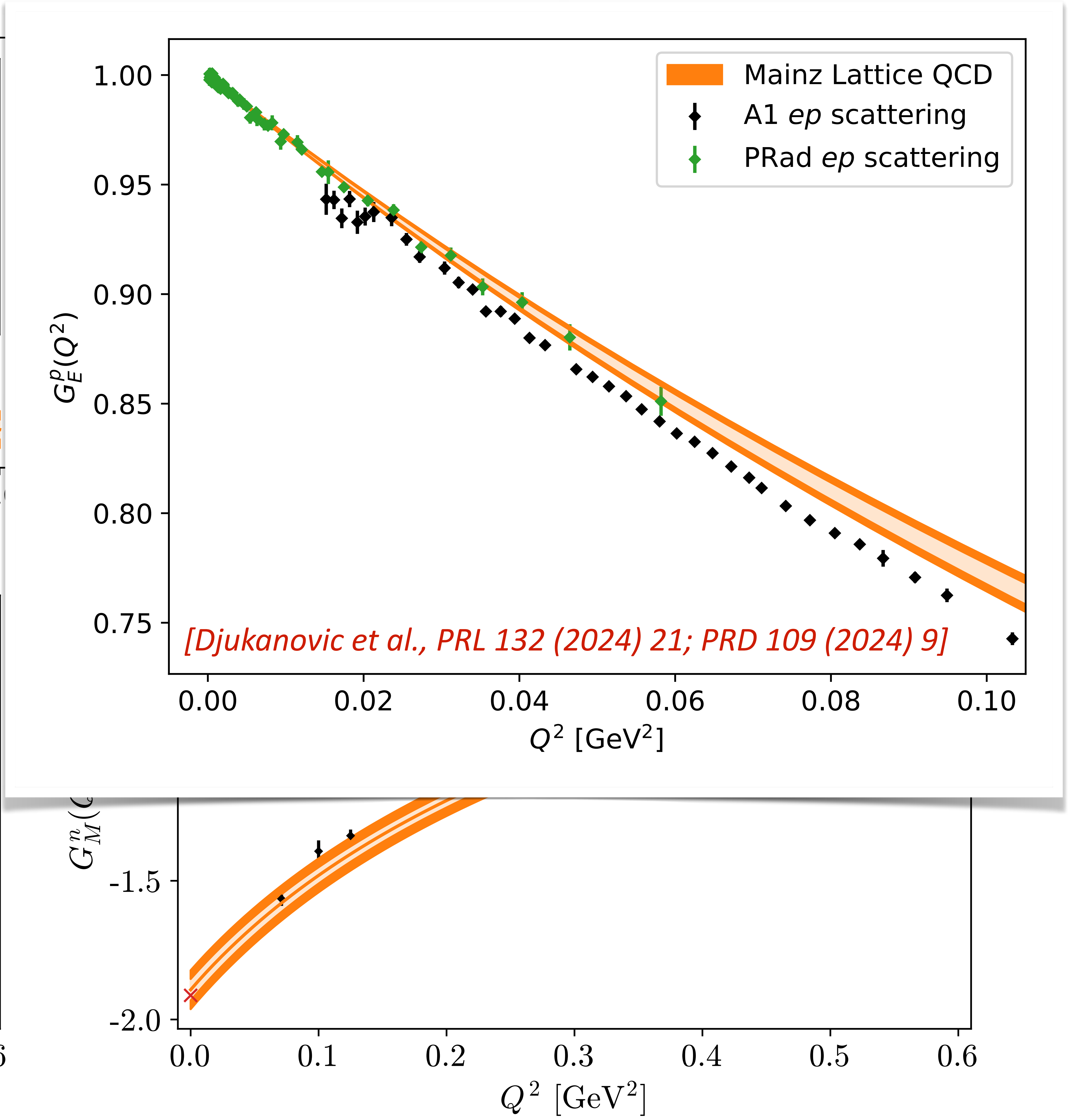
Nucleon form factors and charge radii: Results

Proton:



Neutron:



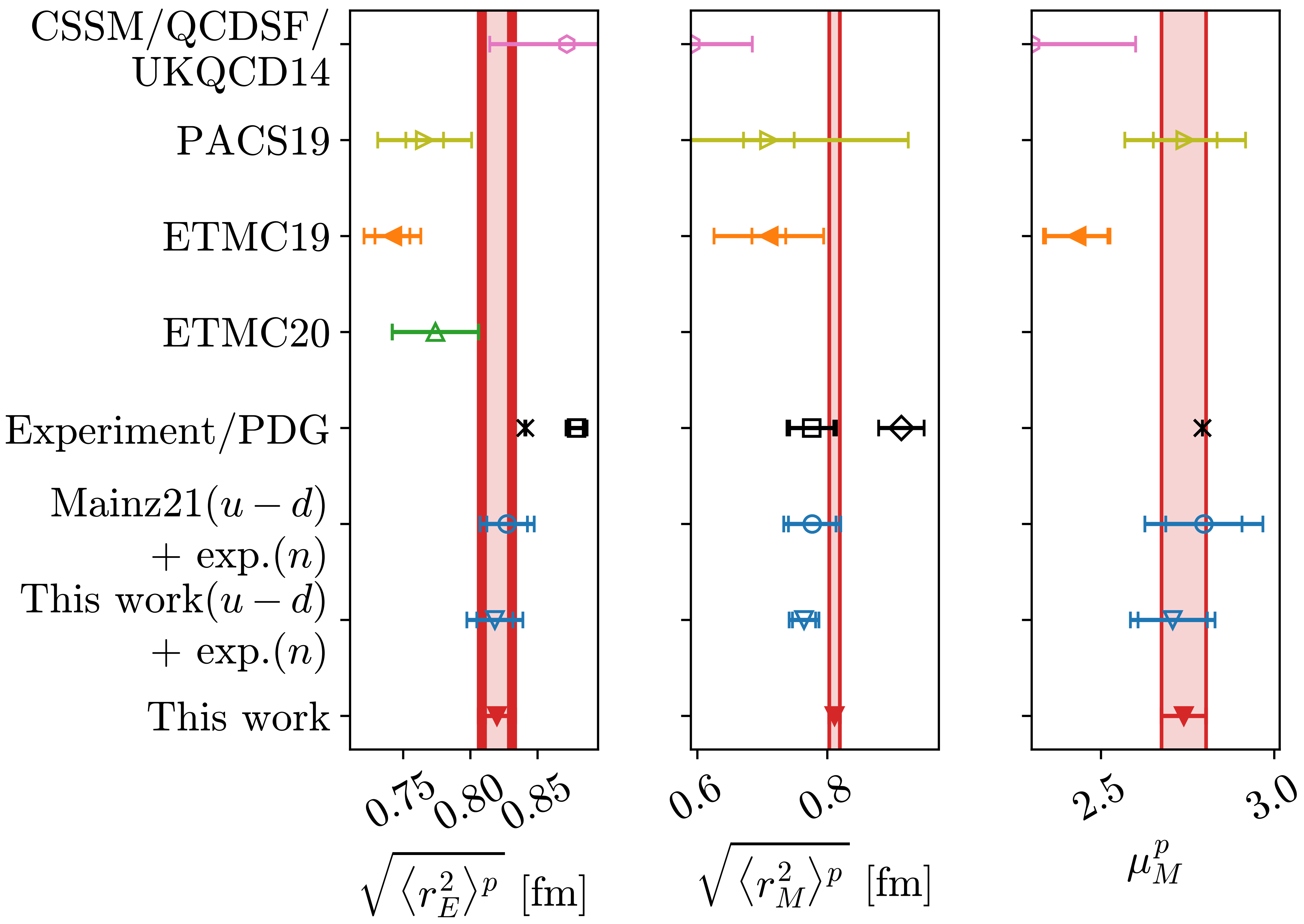


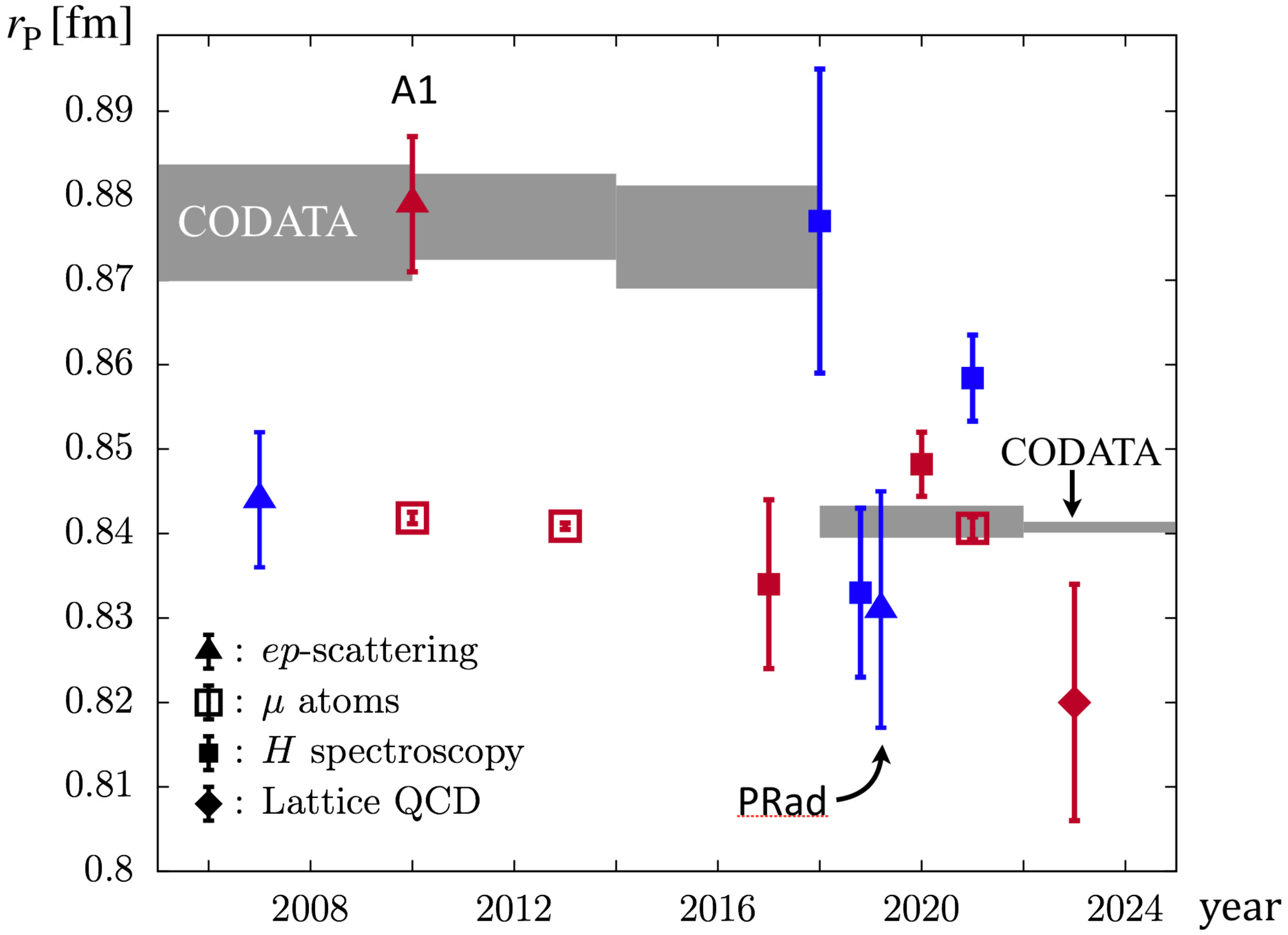
Lattice QCD estimate for the proton radius

- Complete error budget

[Djukanovic et al., PRL 132 (2024) 21; PRD 109 (2024) 9]

- Proton's electric and magnetic radii obtained with 1.7% and 1.1%, respectively
  - → Competitive with *ep*-scattering experiments





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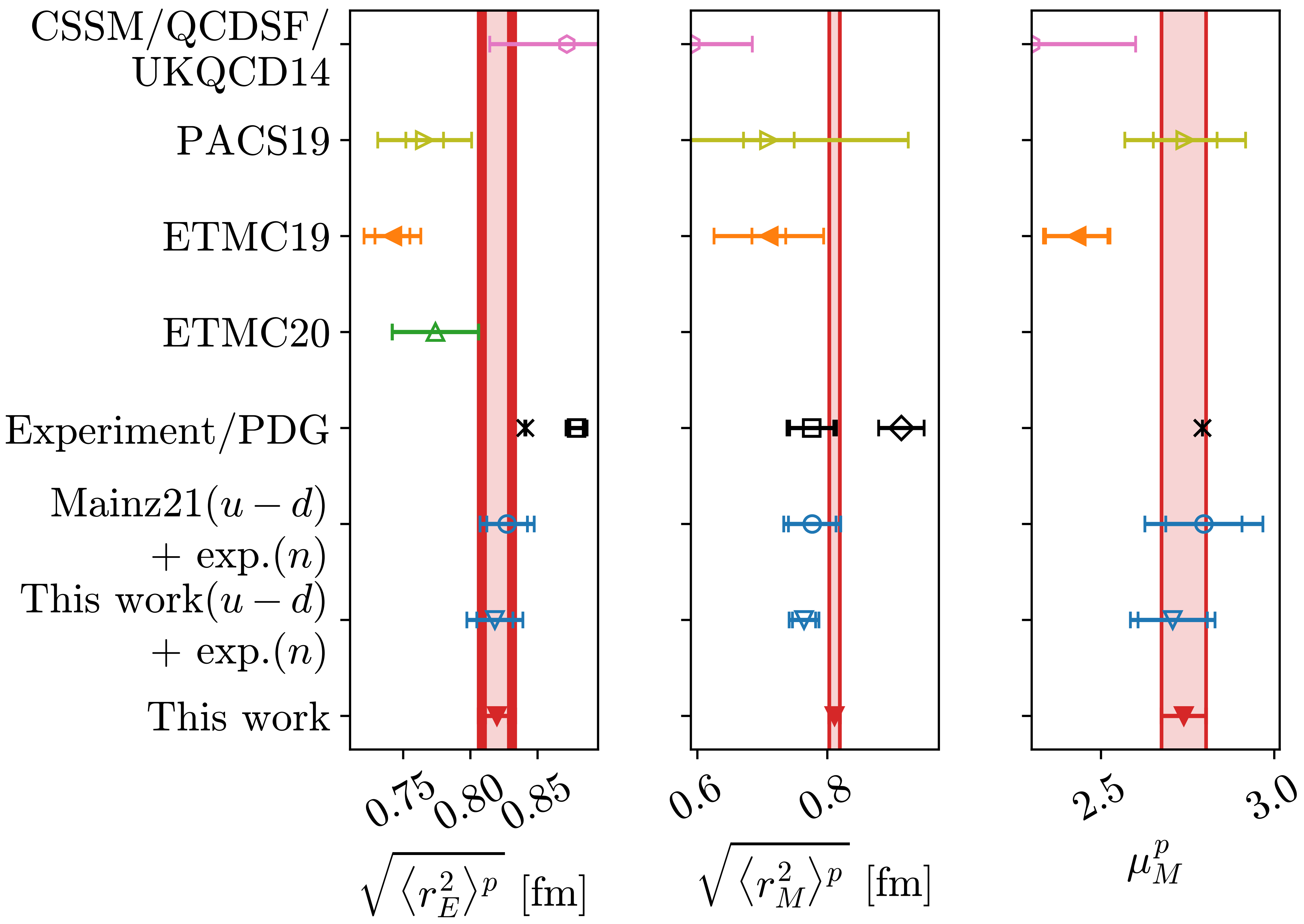

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Lattice QCD:

$r_E = 0.820 \pm 0.014$ fm [Djukanovic et al., 2023]

Hyperfine splitting and the Zemach radius

Electromagnetic structure of the proton affects the HFS of the *s*-state of hydrogen

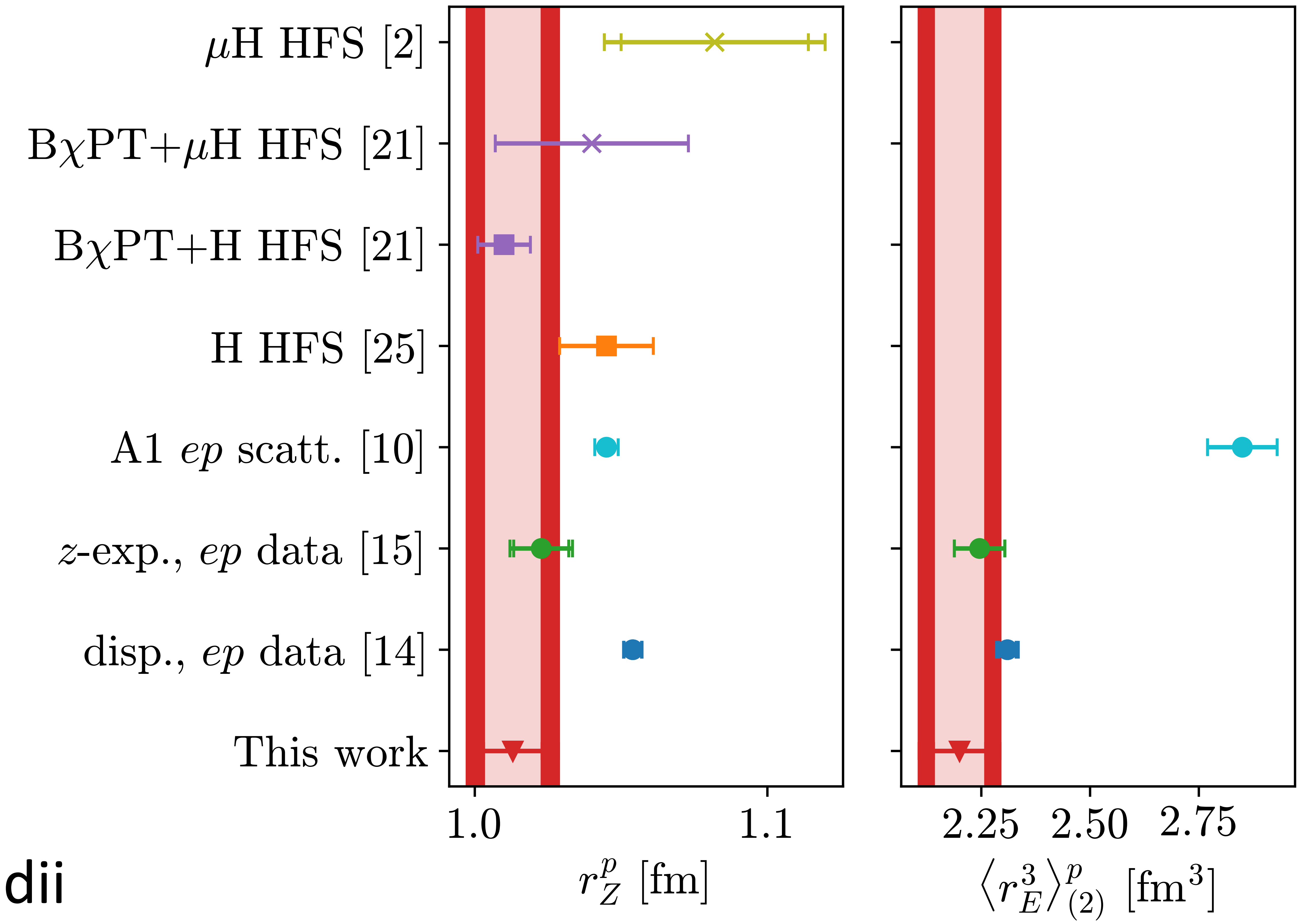
Relevant parameter: Zemach radius and third Zemach moment

$$r_Z^p = -\frac{2}{\pi}\int_0^\infty \frac{dQ^2}{(Q^2)^{3/2}}\left(\frac{G_E^p(Q^2)\,G_M^p(Q^2)}{\mu_M^p} - 1\right)$$

$$= 1.013(10)_{\rm stat}(12)_{\rm sys}\ \mathrm{fm}$$

$$\langle r_F^3\rangle^p = \frac{48}{\pi}\int_0^\infty \frac{dQ^2}{Q^4}\left([G_E^p(Q^2)]^2 - 1 + \tfrac{1}{3}Q^2\langle r_E^2\rangle^p\right)$$

$$= 2.200(60)(71)\mathrm{fm}^3$$

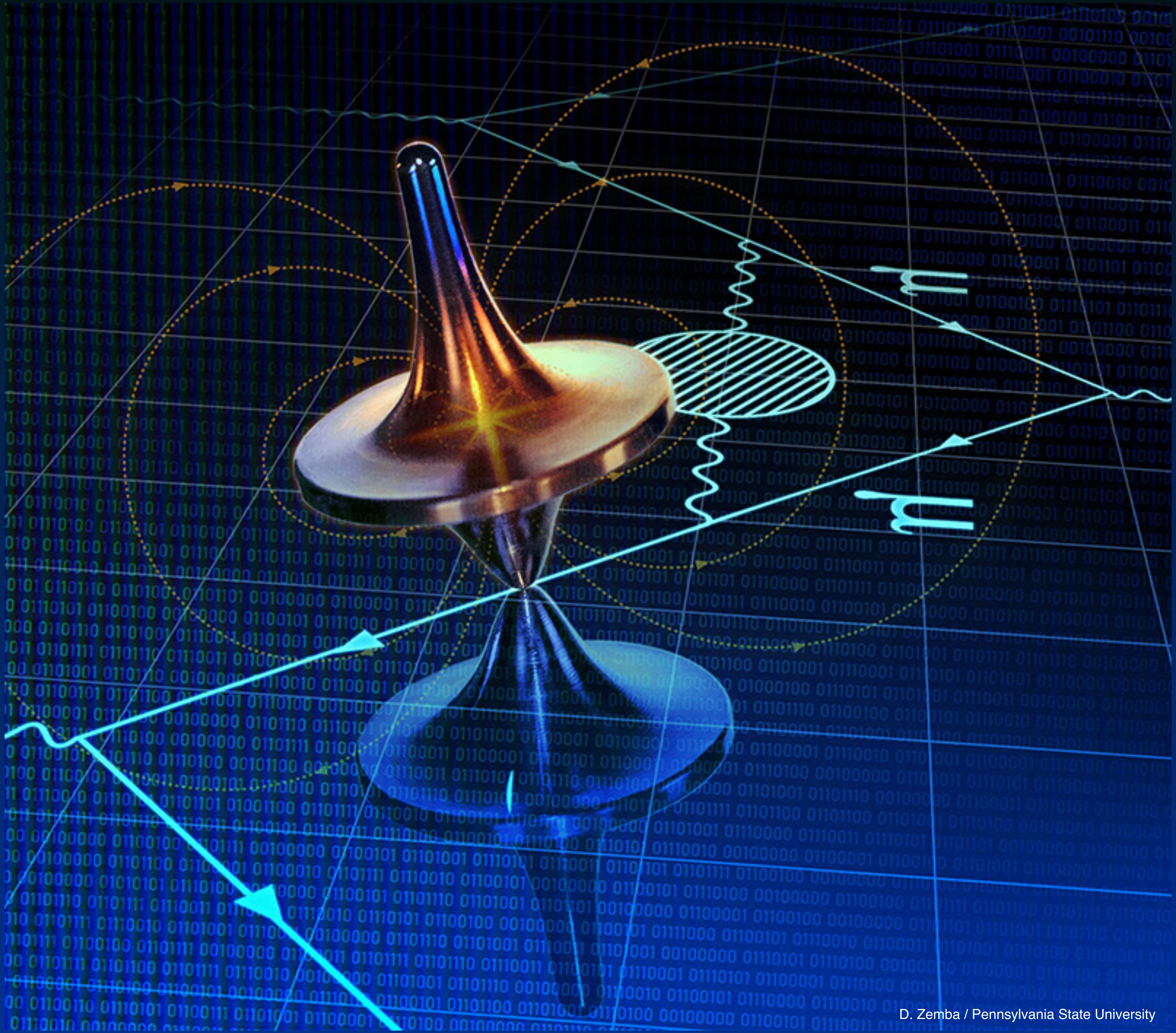


First-ever lattice calculations of Zemach and Friar radii

Results signal tension with A1 measurement and dispersion theory

[Djukanovic et al., Phys Rev D 110 (2024) 1]

The Muon Anomalous Magnetic Moment



D. Zemba / Pennsylvania State University

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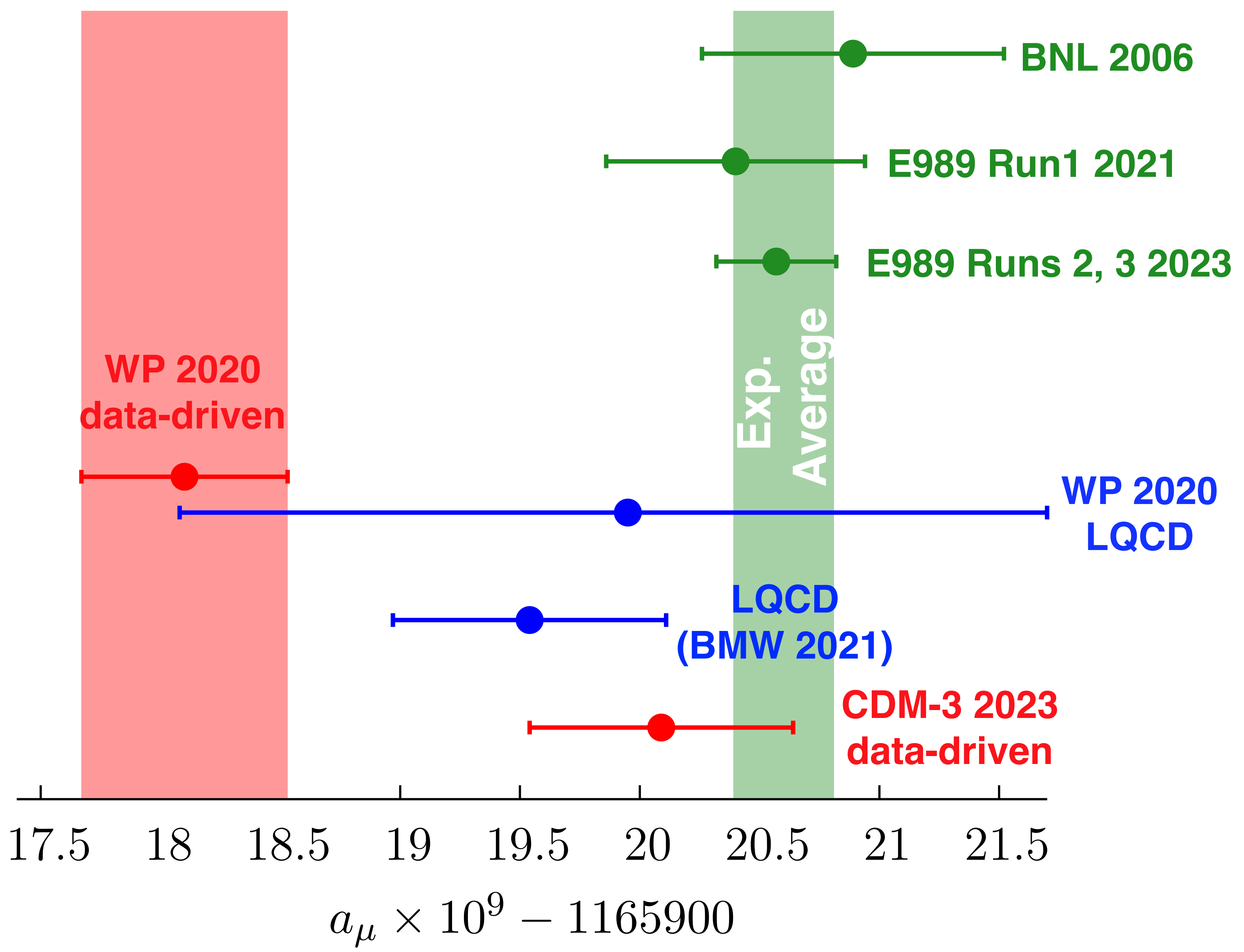

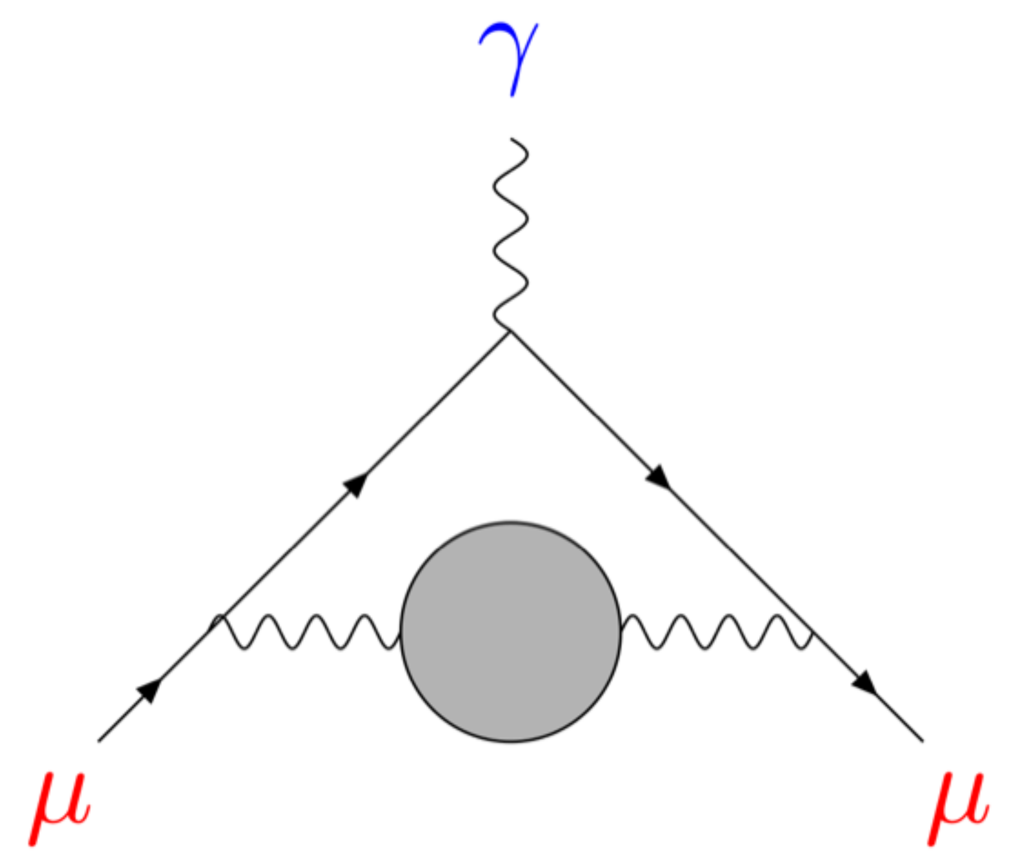

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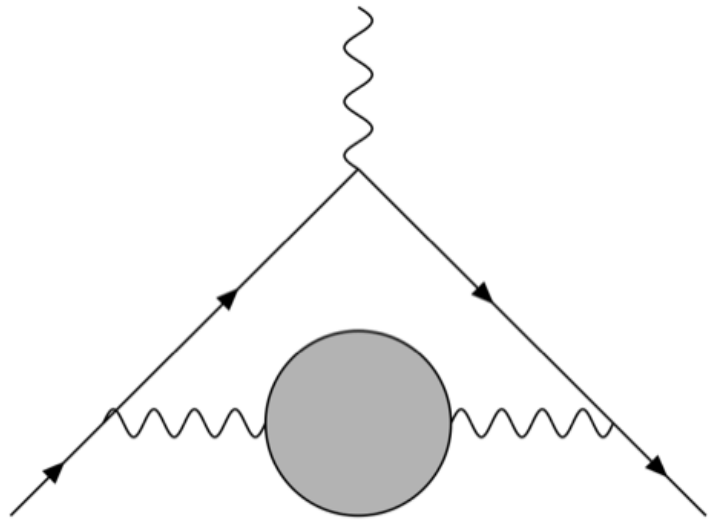
HVP contribution: Dispersion Theory vs. Lattice QCD



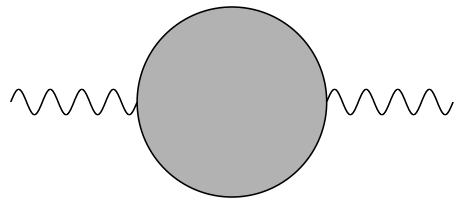
Integral representations:

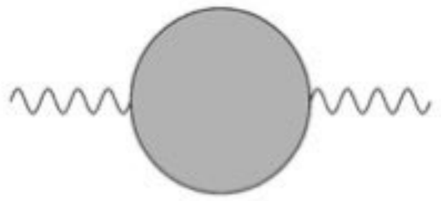
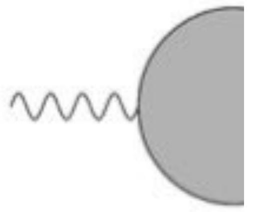
$$a_\mu^{\mathrm{LO,\,hvp}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_{\pi^0}^2}^{\infty} ds\, \frac{R_{\mathrm{had}}(s)\hat{K}(s)}{s^2}$$

$$a_\mu^{\mathrm{LO,\,hvp}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^{\infty} dt\, \tilde{K}(t)\, G(t)$$

Primary observables:

$$R_{\mathrm{had}}(s) = \frac{3s}{4\pi\,(\alpha(s))^2}\,\sigma(e^+e^- \to \mathrm{hadrons})$$

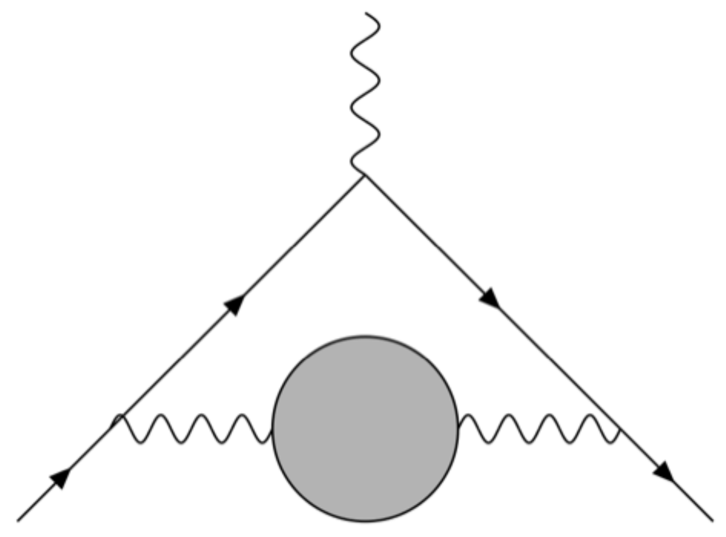
$$G(t) = -\frac{1}{3}\sum_k \int d^3x \left\langle j_k^{\mathrm{em}}(\vec{x},t)\, j_k^{\mathrm{em}}(0)\right\rangle =$$ 

 $$= \frac{1}{2}\int \frac{ds}{\pi(s-q^2)} \sum_{\mathrm{had}} \int d\Phi \,\Big|$$  $$\Big|^2$$

$$j_\mu^{\mathrm{em}}(x) = \tfrac{2}{3}\bar{u}\gamma_\mu u - \tfrac{1}{3}\bar{d}\gamma_\mu d - \tfrac{1}{3}\bar{s}\gamma_\mu s + \tfrac{2}{3}\bar{c}\gamma_\mu c + \ldots$$

$$G(t) = \frac{1}{12\pi^2}\int_{m_{\pi^0}^2}^{\infty} d(\sqrt{s})\, R_{\mathrm{had}}(s)\, s\, \mathrm{e}^{-\sqrt{s}t}$$

HVP contribution: Dispersion Theory vs. Lattice QCD



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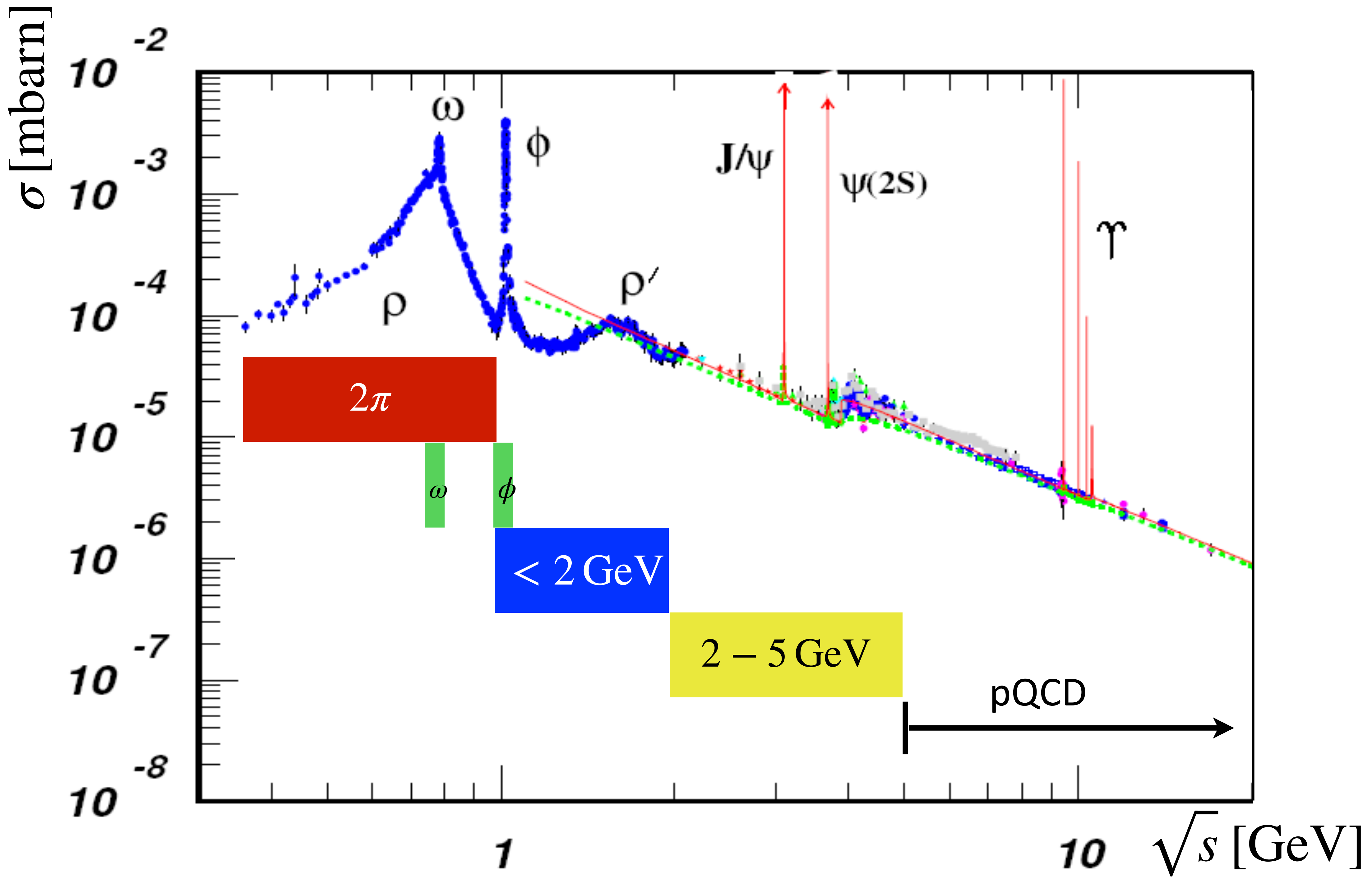
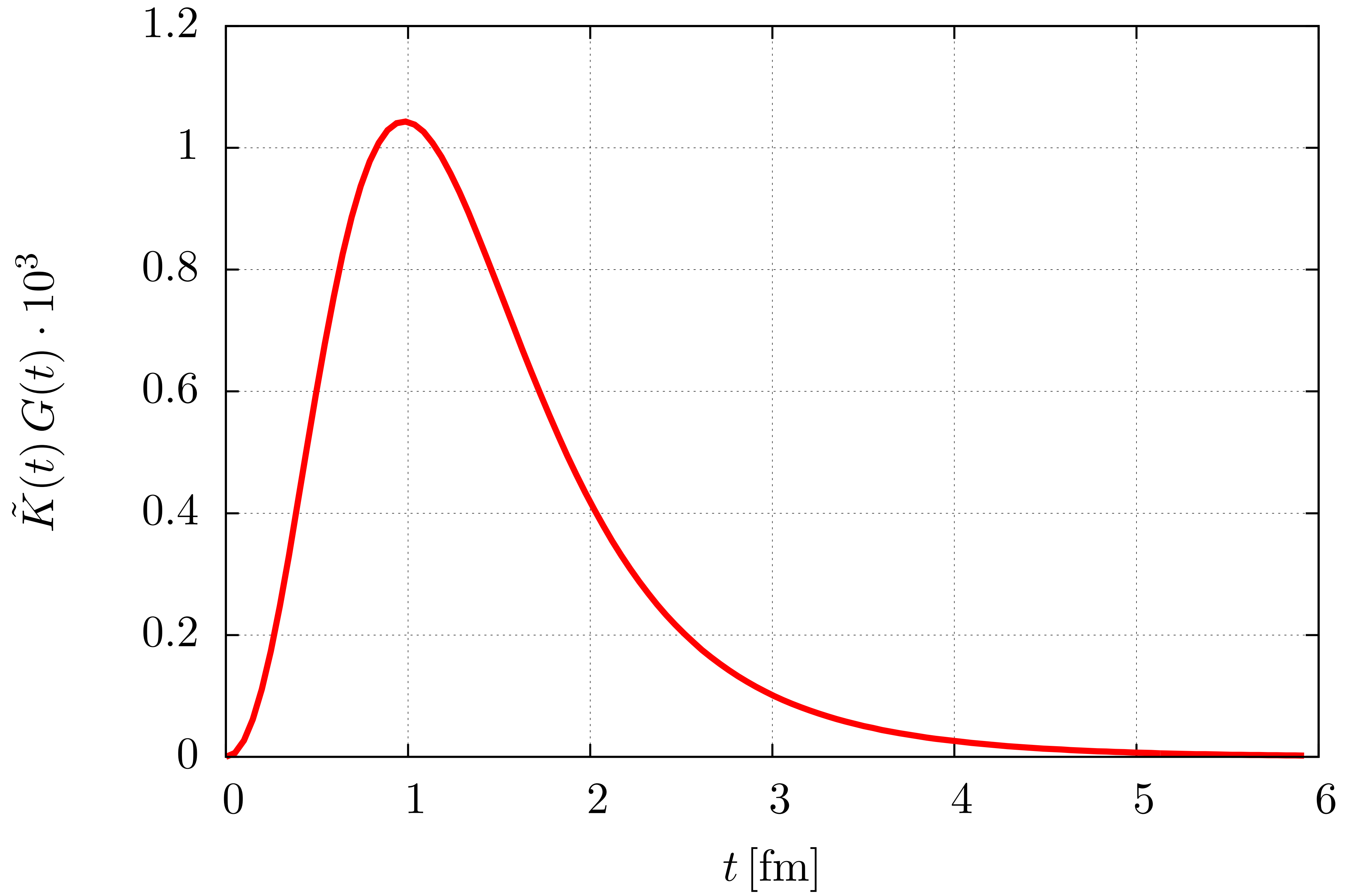
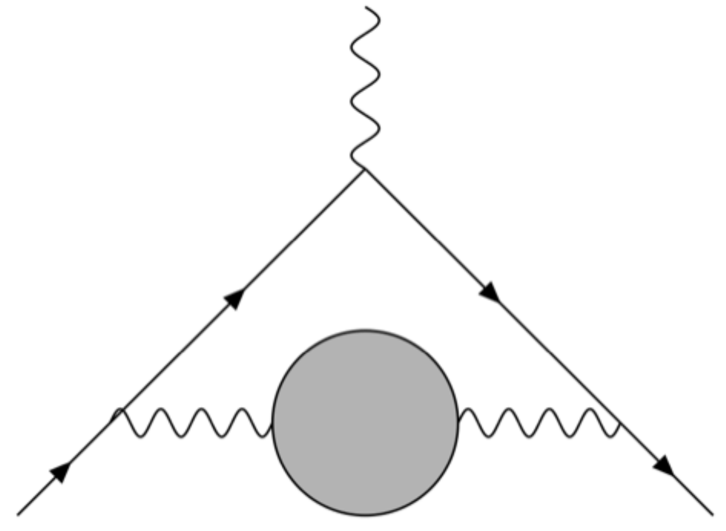
$$a_\mu^{\mathrm{LO,hvp}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^{\infty} dt\, \tilde{K}(t)\, G(t)$$





HVP contribution: Dispersion Theory vs. Lattice QCD



Integral representations:

$$a_\mu^{\mathrm{LO,hvp}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_{\pi^0}^2}^{\infty} ds\, \frac{R_{\mathrm{had}}(s)\hat{K}(s)}{s^2}$$

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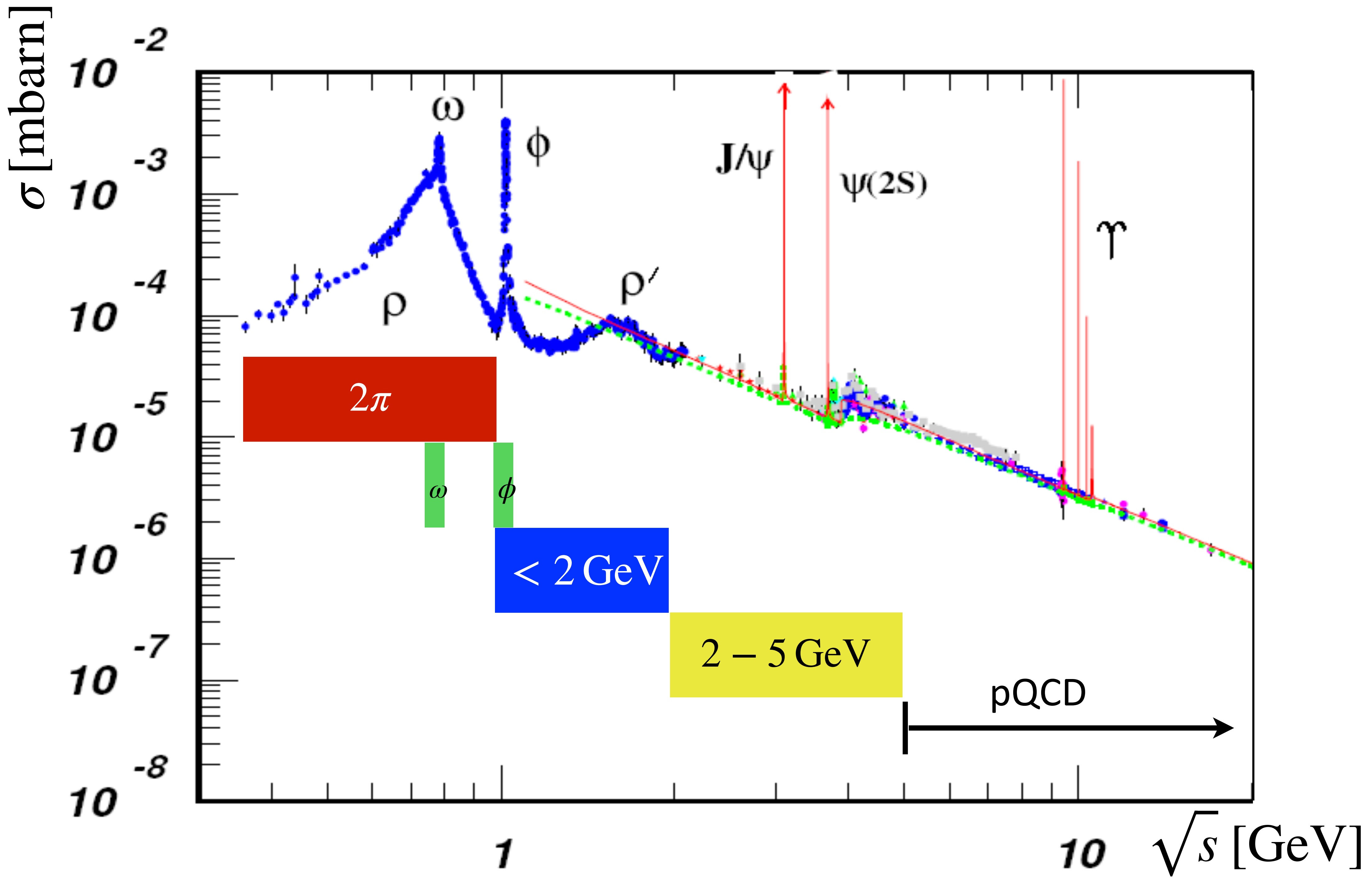
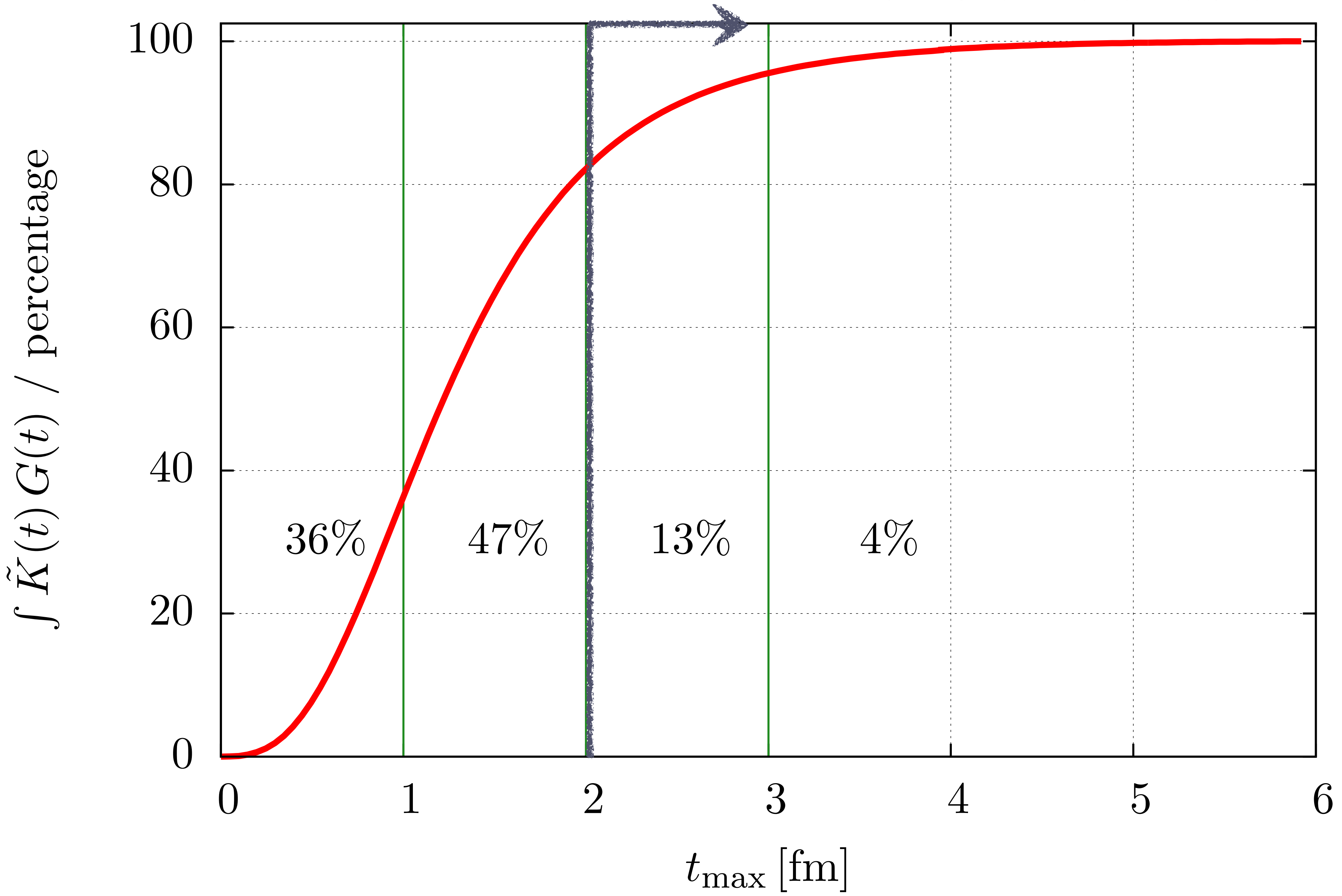


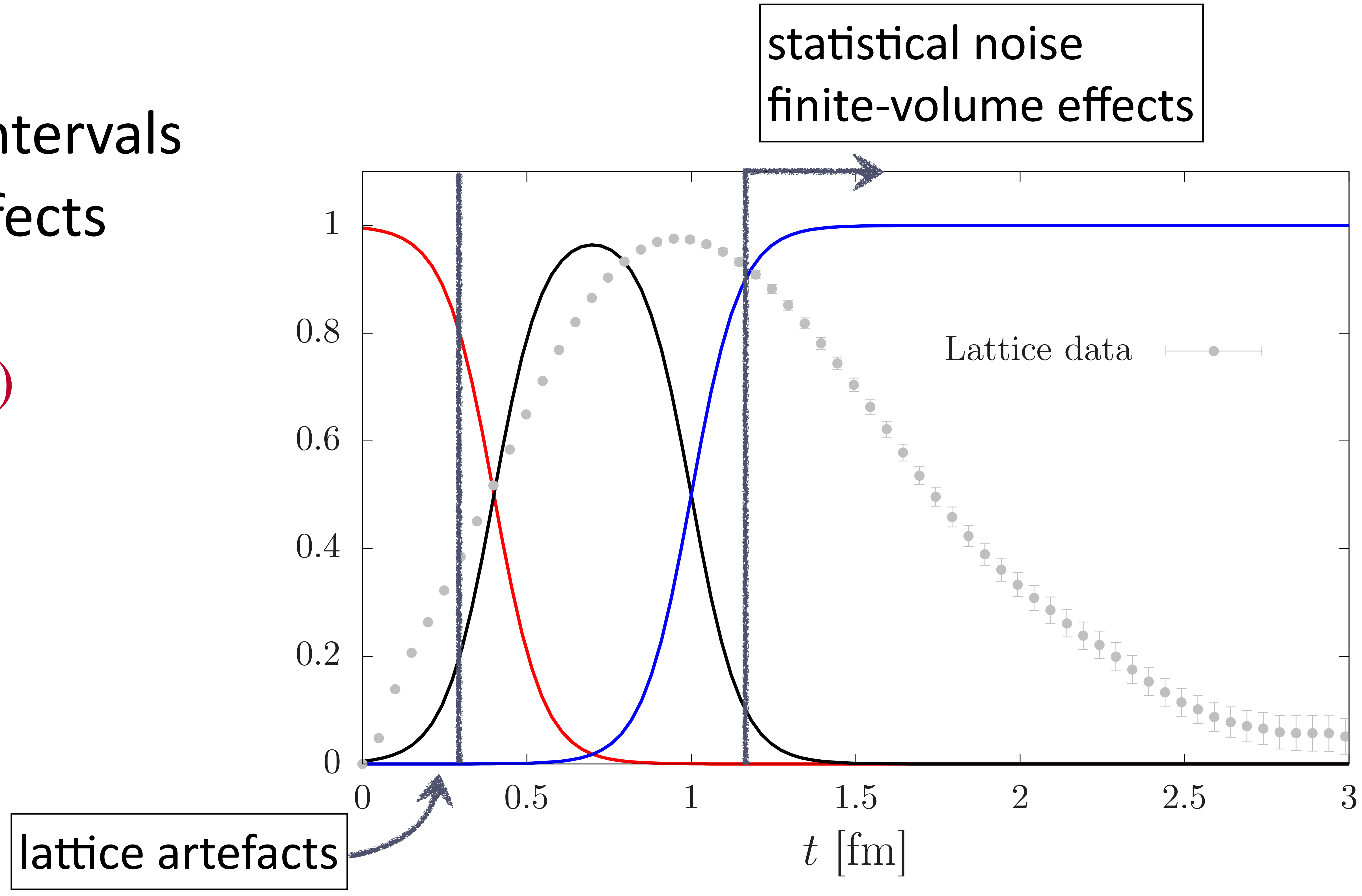
statistical noise
finite-volume effects



Window observables

Restrict integration over Euclidean time to sub-intervals
→ reduce/enhance sensitivity to systematic effects

$$a_\mu^{\mathrm{hvp,\,win}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\,\tilde{K}(t)\,G(t)\,W(t;\,t_0,\,t_1)$$



Intermediate window:

- Finite-volume correction reduced to 0.25%
- Uncertainty dominated by statistics

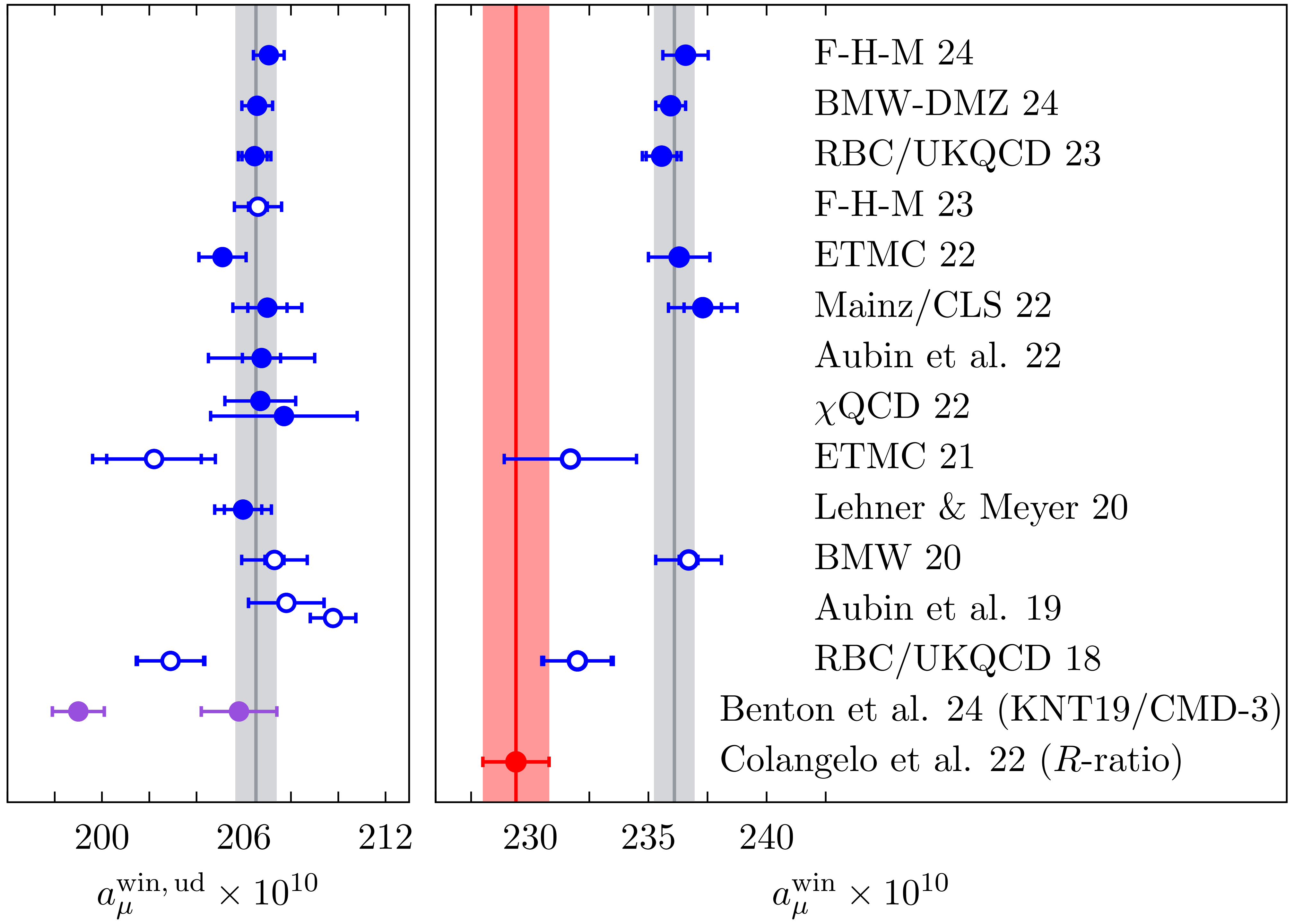
Short-distance window:

- Uncertainty dominated by ability to control continuum extrapolation

Long-distance window:

- Statistical errors in the long-distance tail dominate total error

Intermediate window observable: Lattice QCD vs. R-ratio



Left: dominant light-quark contribution to a_μ^{win}

Right: including sub-leading contributions

- Dominant light-quark contribution confirmed for wide range of discretisation with sub-percent precision
- Significant tension with results based on the R-ratio*

R-ratio estimate: $a_\mu^{\text{win}} = (229.4 \pm 1.4) \cdot 10^{-10}$

Lattice average: $a_\mu^{\text{win}} = (236.10 \pm 0.86) \cdot 10^{-10}$

(assuming 100% correlation)

- Tension of 4.1σ in the window observable evaluated from e^+e^- data* and five lattice calculations

$$a_\mu^{\text{win}}\Big|_{\langle\text{lat}\rangle} - a_\mu^{\text{win}}\Big|_{e^+e^-} = (6.70 \pm 1.64) \cdot 10^{-10} \quad [4.1\,\sigma]$$

*excluding the CMD-3 result

What can we learn from $a_\mu^{\rm win}$?

$$G(t) \equiv -\frac{a^3}{3}\sum_k\sum_{\vec{x}}\left\langle j_k^{\rm em}(\vec{x},t)\,j_k^{\rm em}(0)\right\rangle = \frac{1}{12\pi^2}\int_{m_{\pi^0}^2}^{\infty} d(\sqrt{s})\,R(s)^{\rm lat}\,s\,{\rm e}^{-\sqrt{s}t}$$

$$a_\mu^{\rm win}\big|_{\rm lat} > a_\mu^{\rm win}\big|_{e^+e^-} \quad\Rightarrow\quad R(s)^{\rm lat} > R(s)^{e^+e^-}$$ in some interval of $\sqrt{s}$

Phenomenological model for R-ratio predicts *[Mainz/CLS, Cè et al., Phys Rev D 106 (2022) 114502]*

$$\sqrt{s} = 600 - 900\,{\rm MeV}: \qquad \frac{R(s)^{\rm lat}}{R(s)^{e^+e^-}} = 1+\epsilon \quad\Rightarrow\quad \frac{(a_\mu^{\rm hvp})^{\rm lat}}{(a_\mu^{\rm hvp})^{e^+e^-}} \approx \frac{(a_\mu^{\rm win})^{\rm lat}}{(a_\mu^{\rm win})^{e^+e^-}} = 1+0.6\epsilon$$

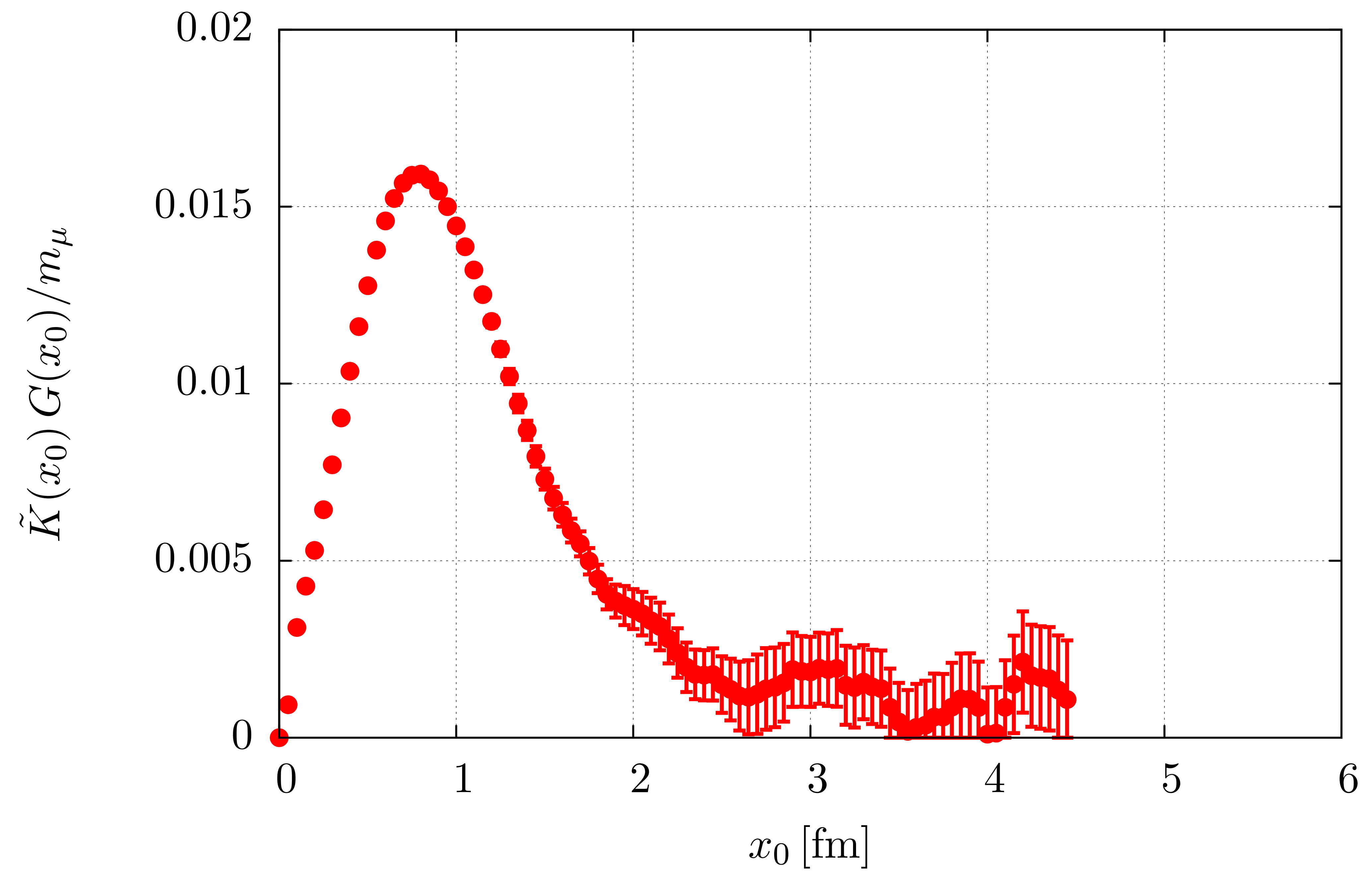
Lattice average vs. R-ratio: $(a_\mu^{\rm win})^{\rm lat}/(a_\mu^{\rm win})^{e^+e^-} = 1.029(7)$

$\Rightarrow$ $R(s)^{\rm lat}$ is enhanced by 5% relative to $R(s)^{e^+e^-}$ for $\sqrt{s} = 600 - 900\,{\rm MeV}$

$\Rightarrow$ scaled estimate for HVP contribution: $(a_\mu^{\rm hvp})^{e^+e^-}_{\rm WP2020} \times 1.029 \approx 713\cdot 10^{-10}$

Long-distance window — new result by the Mainz group

Problem: exponential growth of signal-to-noise ratio in $G(t)$ for large t



"Low-mode averaging" (LMA):

- Express quark propagator in terms of eigenmodes of the Wilson-Dirac operator

$$S(y,x) = S_{\rm eigen}(y,x) + S_{\rm rest}(y,x)$$

- Low modes responsible for statistical fluctuations
- LMA leads to better sampling of the correlator

[Giusti, Hernández, Laine, Weisz, HW 2004; DeGrand & Schaefer 2004]

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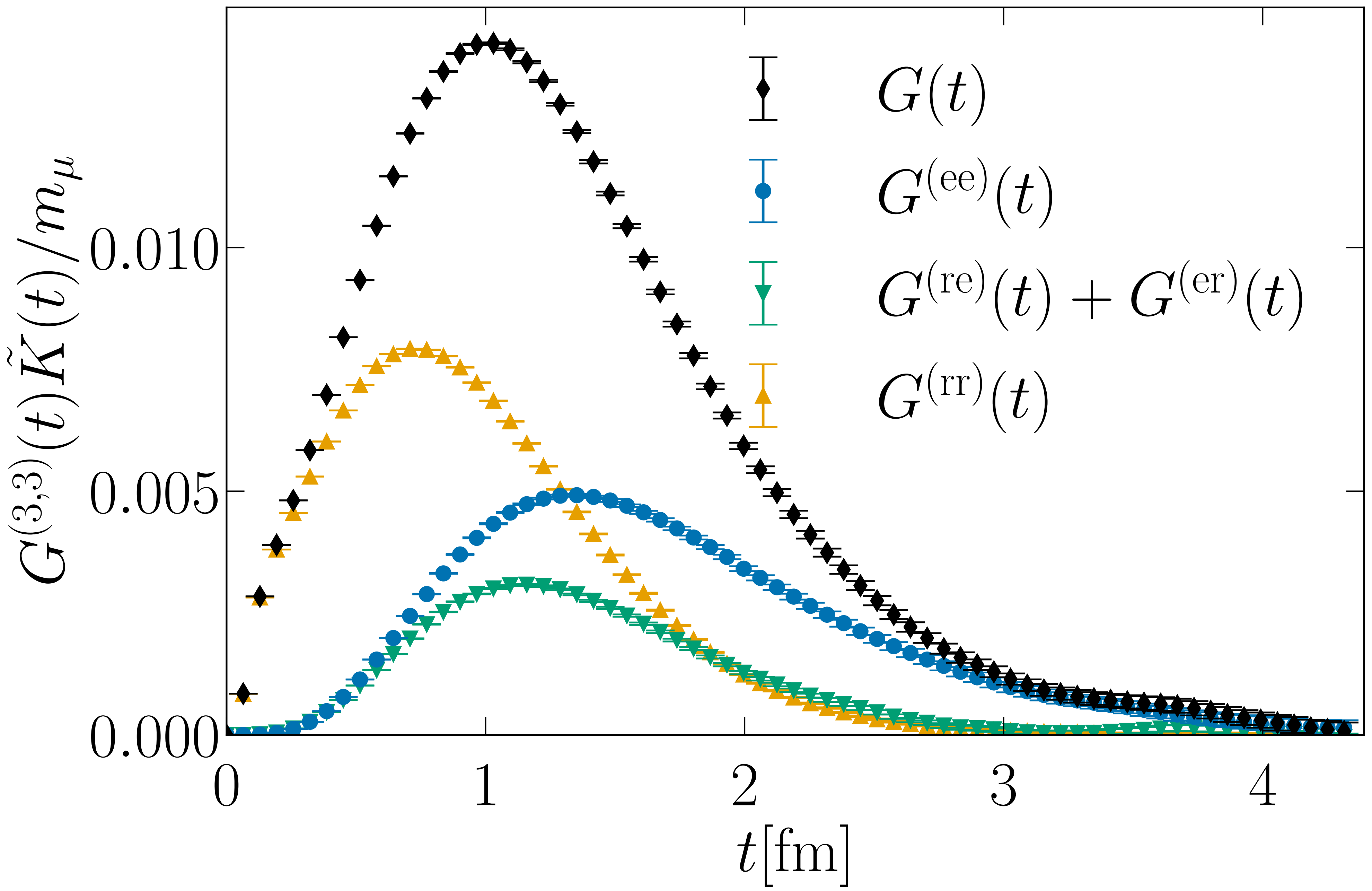

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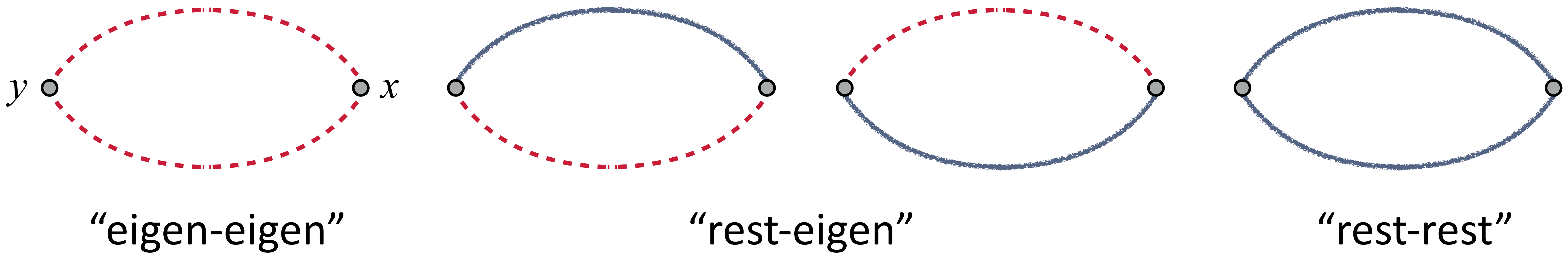

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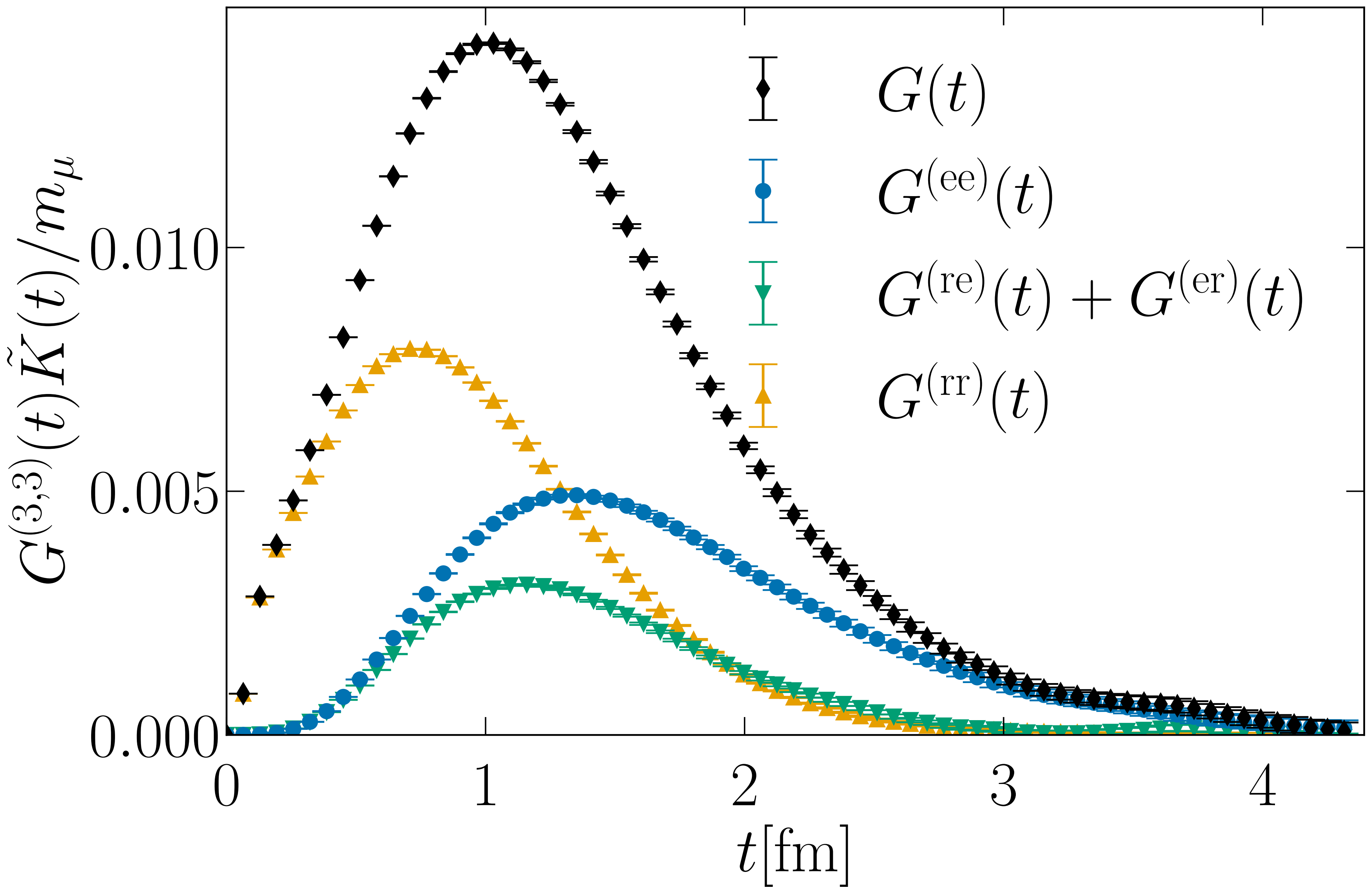

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Long-distance tail of $G(t)$ dominated by two-pion states with isospin one:

- Additional gain via spectral reconstruction: $G(t) \stackrel{t\to\infty}{=} \sum_n |A_n|^2\, e^{-\omega_n t}$

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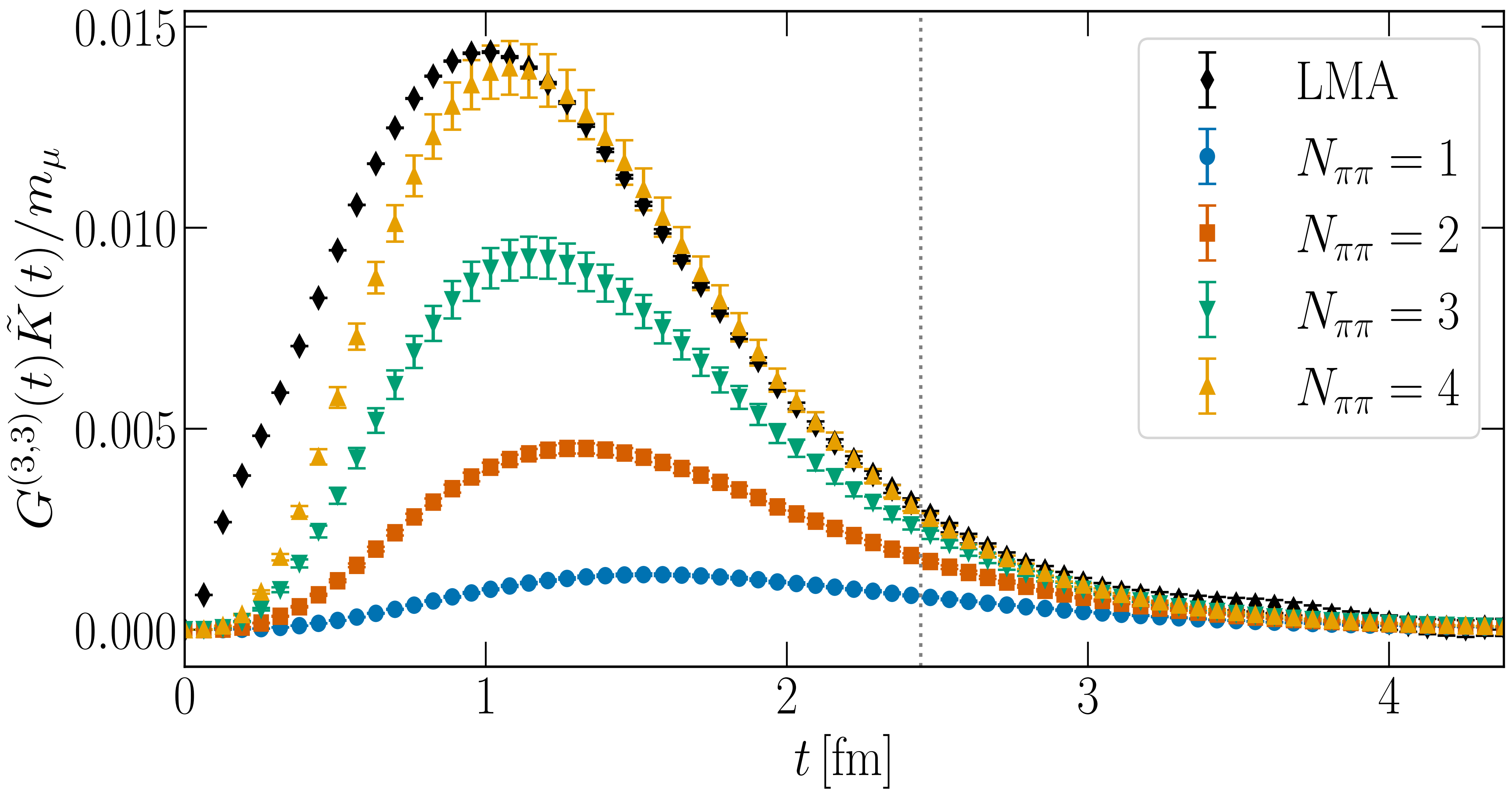

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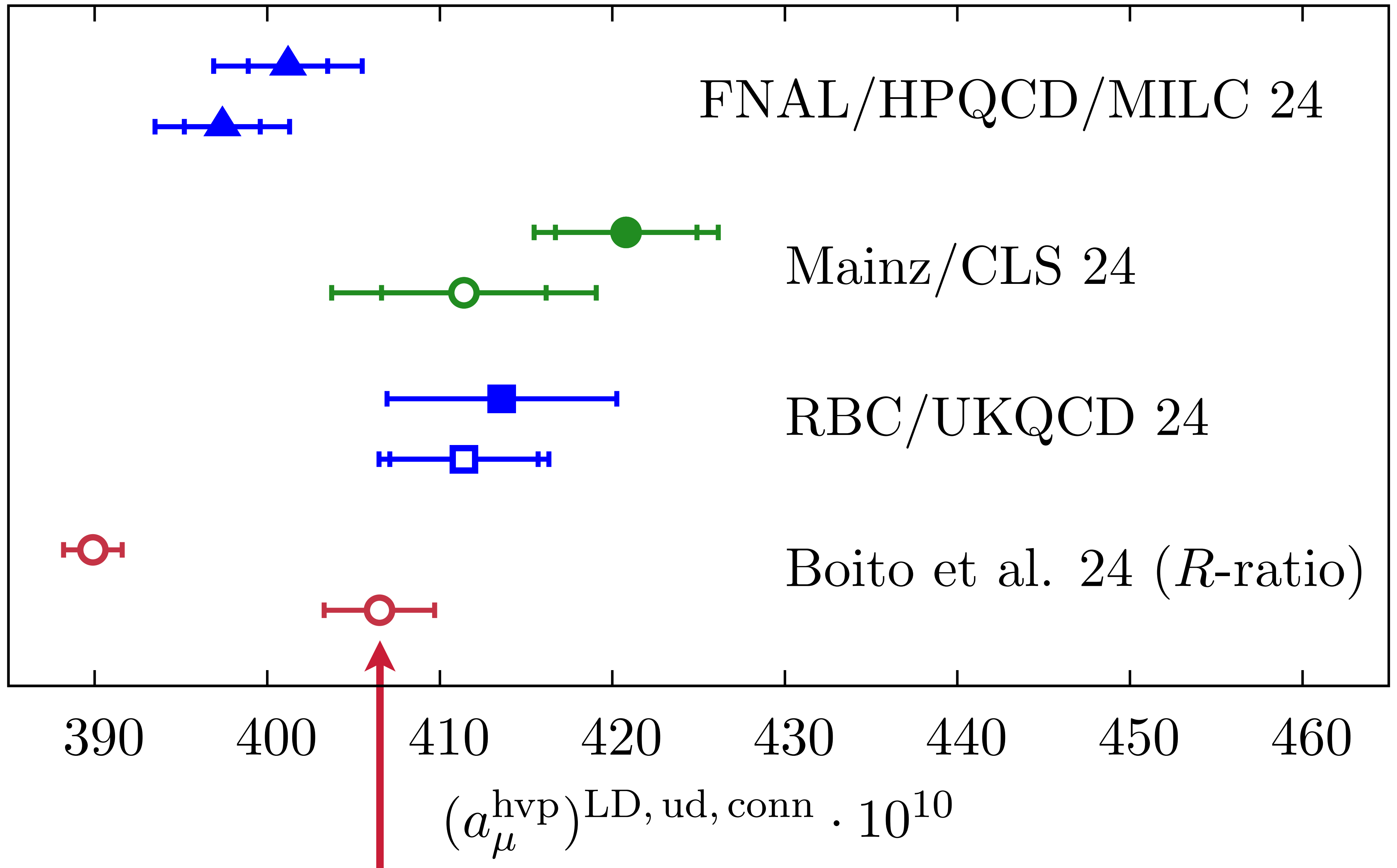
Results: long-distance window and total light-quark contribution

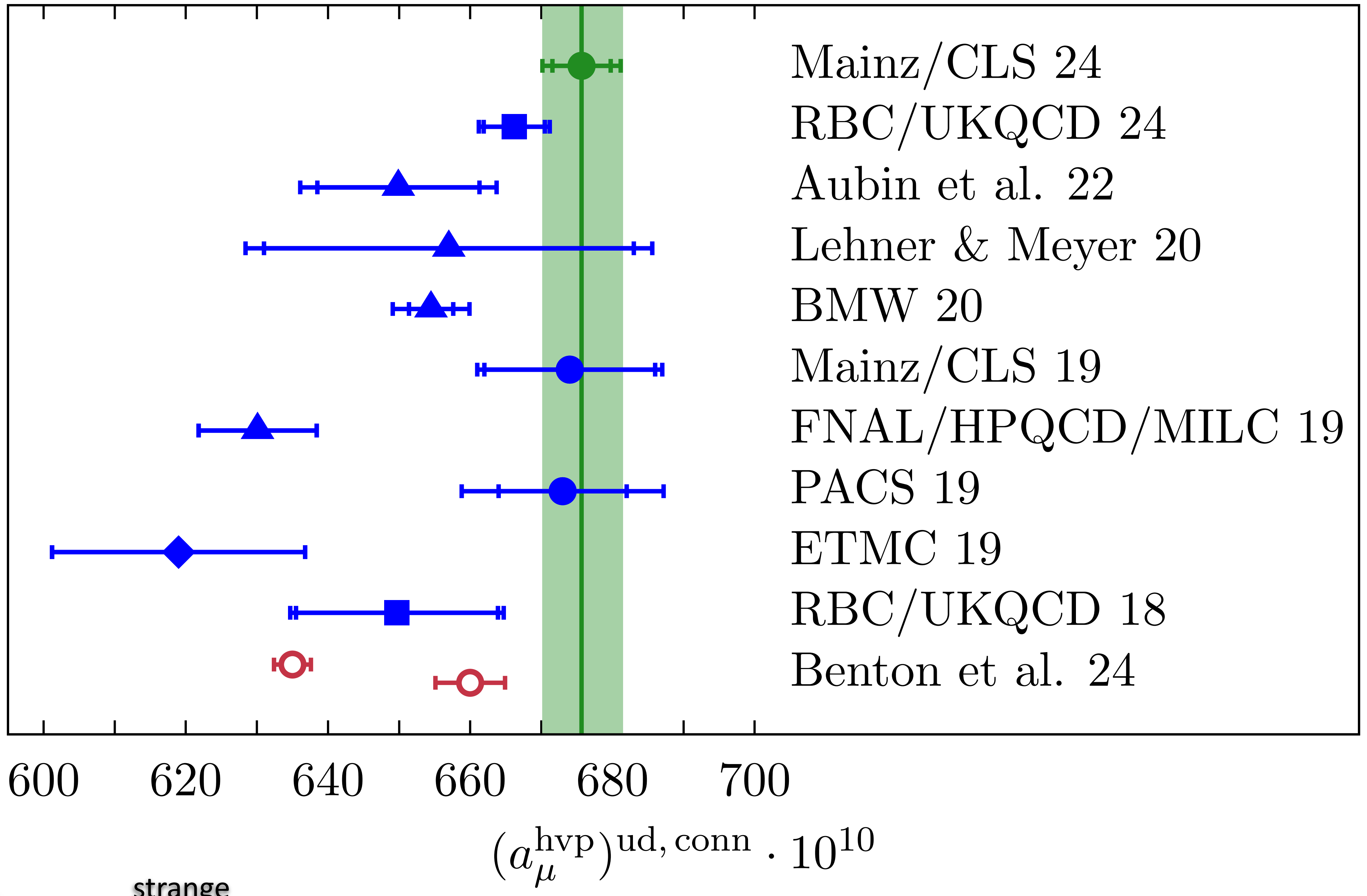
Long-distance window observable

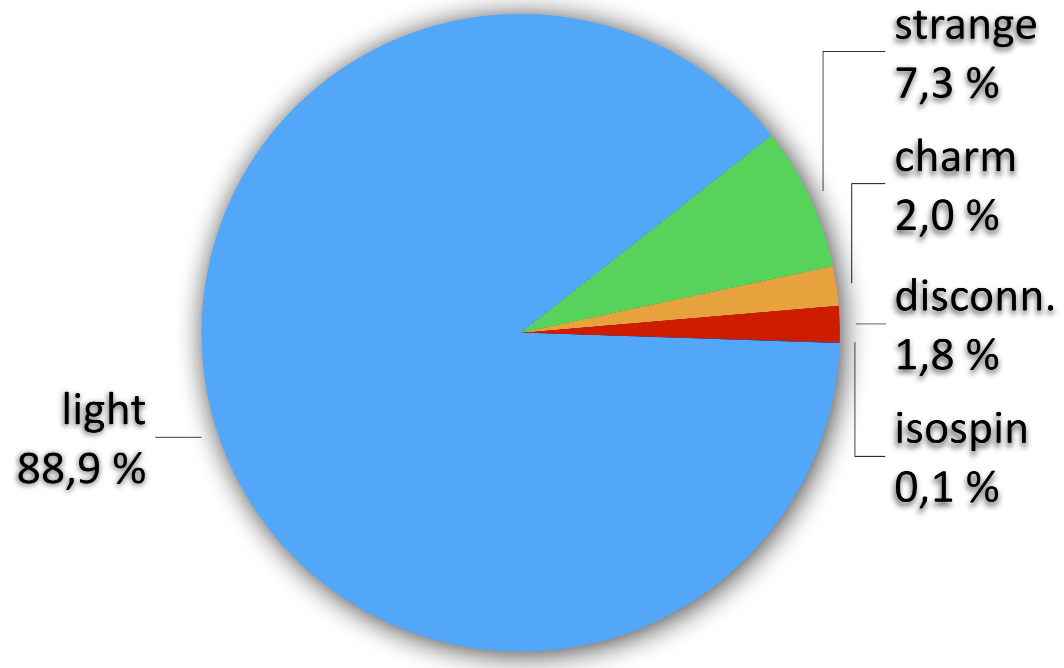
(open symbols refer to BMW20 scheme)



$\pi^+\pi^-$ channel from CMD-3 measurement

Total light-quark connected contribution





[Mainz/CLS: Djukanovic et al., arXiv:2411.07969]

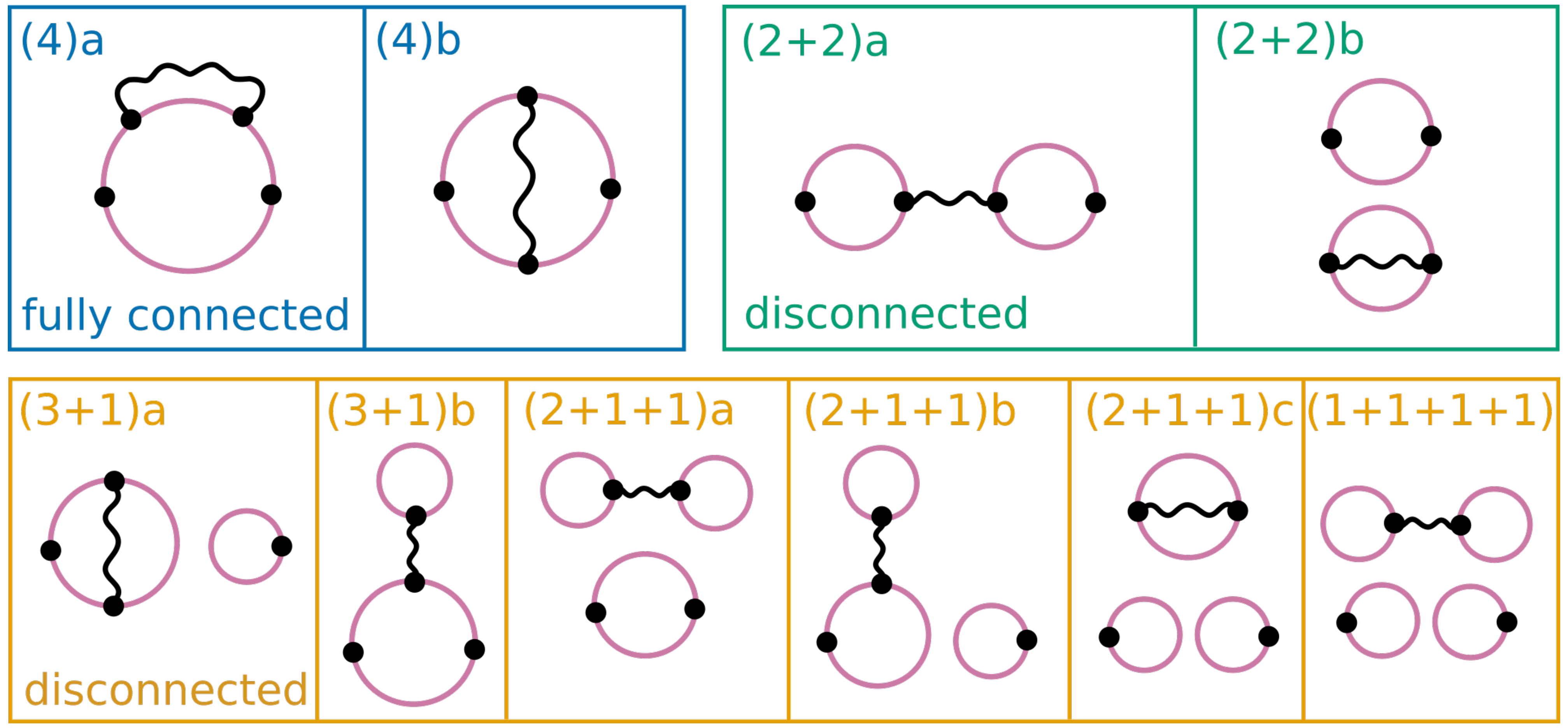
Results: full hadronic vacuum polarisation contribution

Add sub-leading contributions: isoscalar and charm; unblind analysis

Iso-symmetric QCD: $a_\mu^{\rm hvp}\Big|_{\rm isoQCD} = (728.6 \pm 4.3 \pm 3.7) \cdot 10^{-10} \quad [0.8\%]$

Definition of iso-symmetric QCD ambiguous — depends on chosen hadronic scheme

Scheme ambiguities must vanish after inclusion of strong and e.m. isospin-breaking corrections



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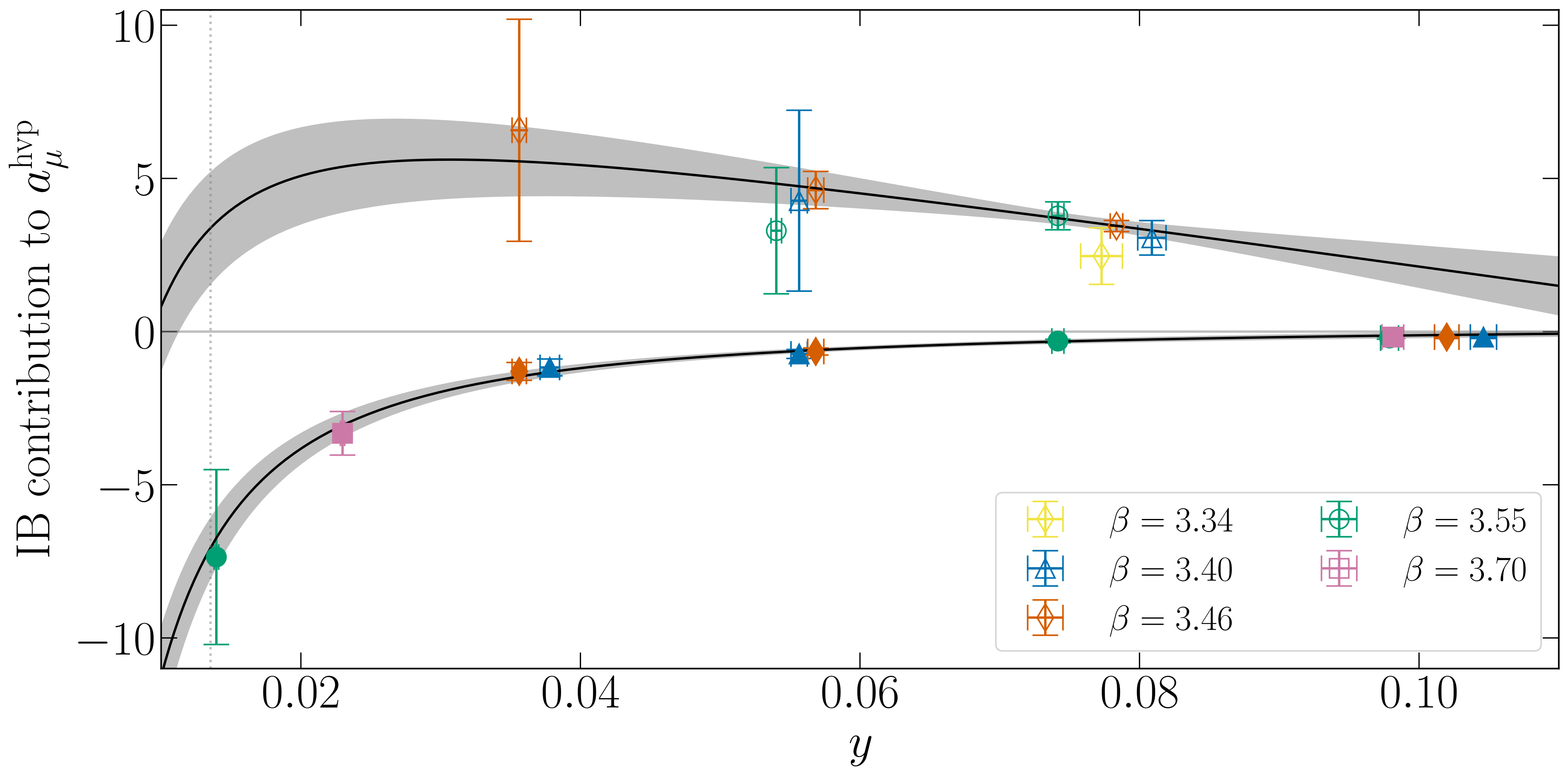
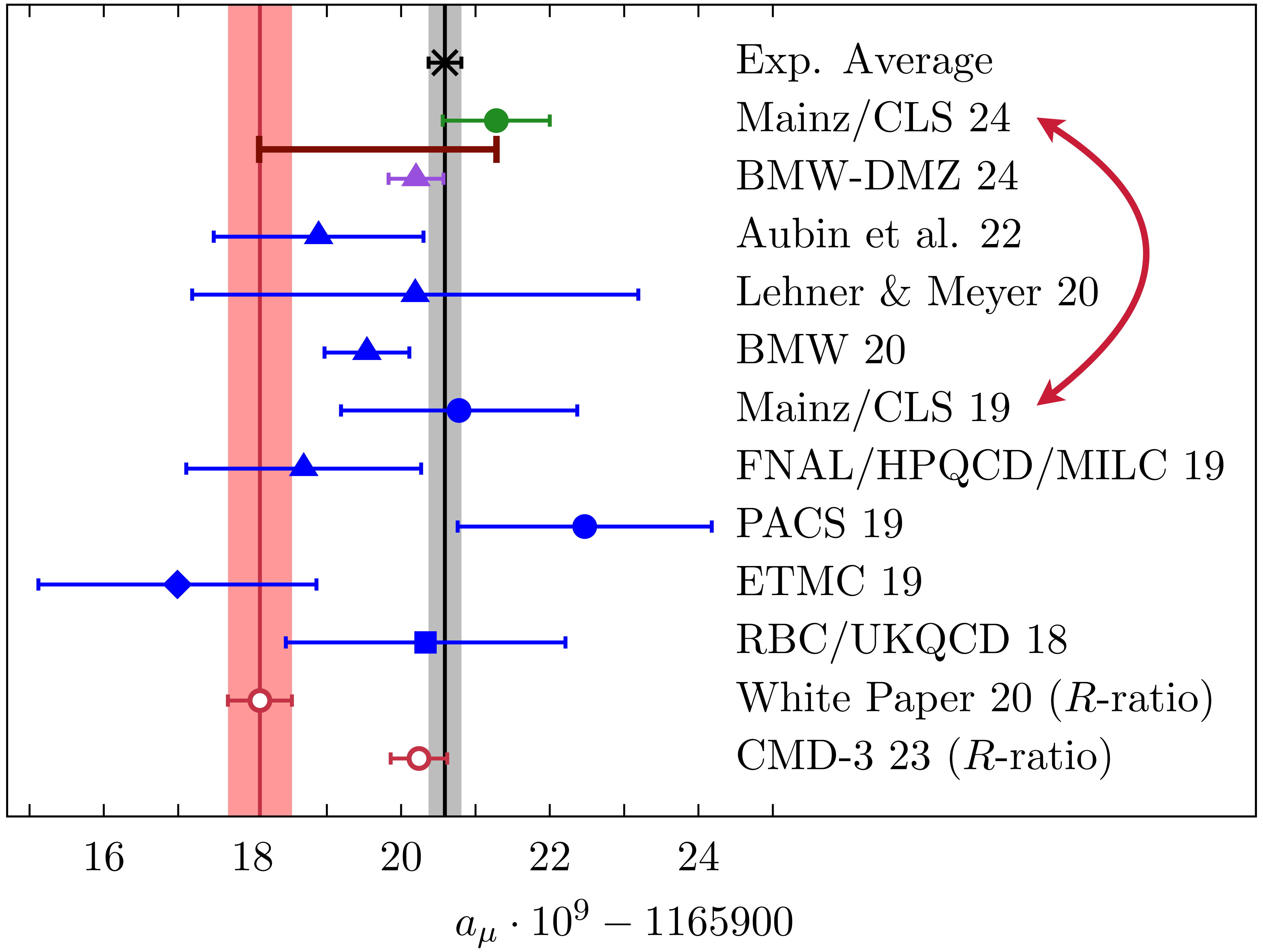


$a_\mu^{\rm hvp}\Big|_{\rm IB} = (-3.6 \pm 2.6 \pm 3.4) \cdot 10^{-10}$

Results: full hadronic vacuum polarisation contribution

Final result (Mainz/CLS):

$$a_\mu^{\rm hvp} = (724.9 \pm 5.0 \pm 4.9) \cdot 10^{-10} \quad [0.97\%]$$



- Improvement by more than a factor two
- Tension of $3.9\,\sigma$ with data-driven estimate (2020 White Paper)
- Agreement with experimental average
- Result agrees with BMW-DMZ 24 within $1.4\,\sigma$

[Mainz/CLS: Djukanovic et al., arXiv:2411.07969]

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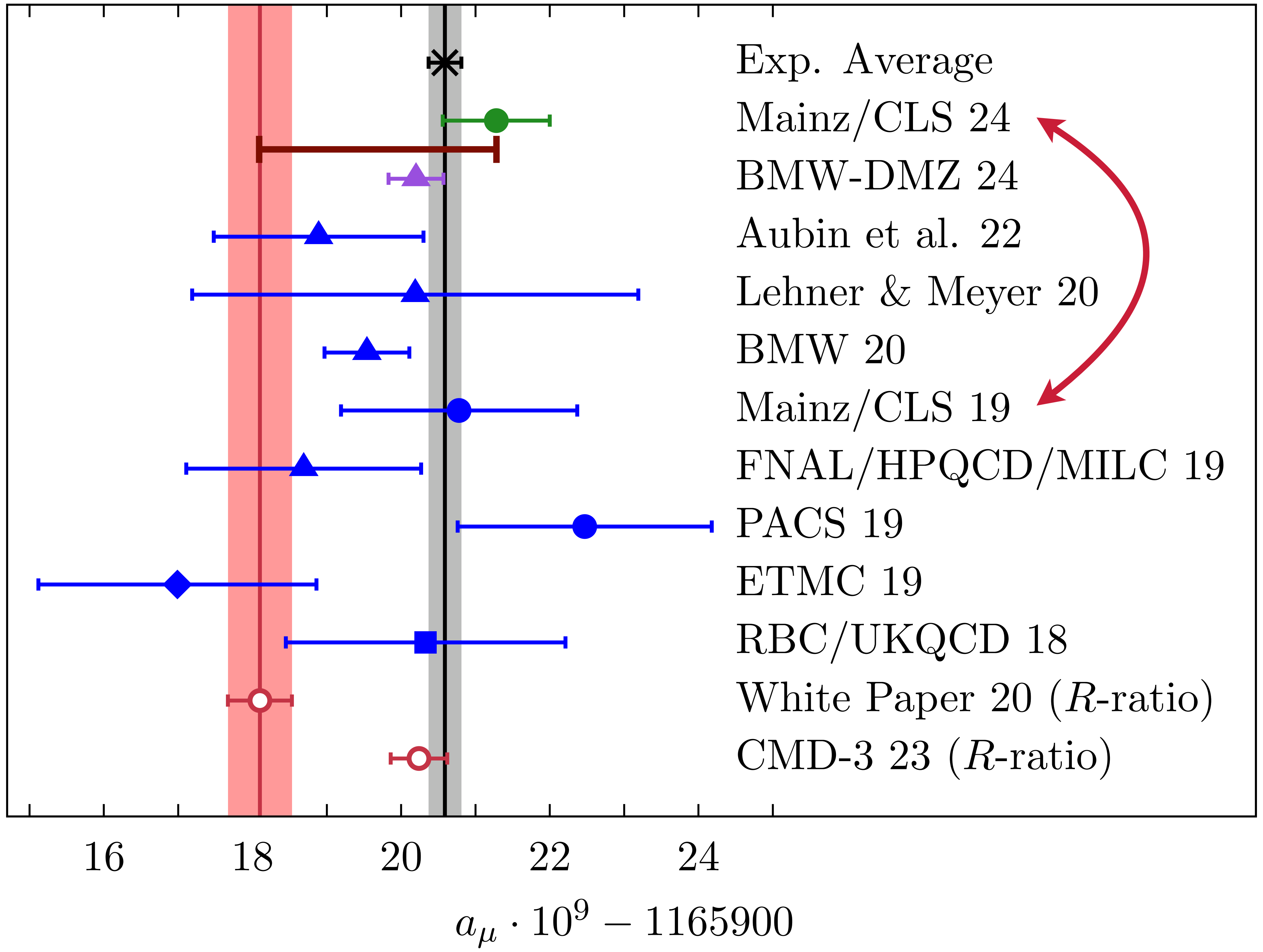
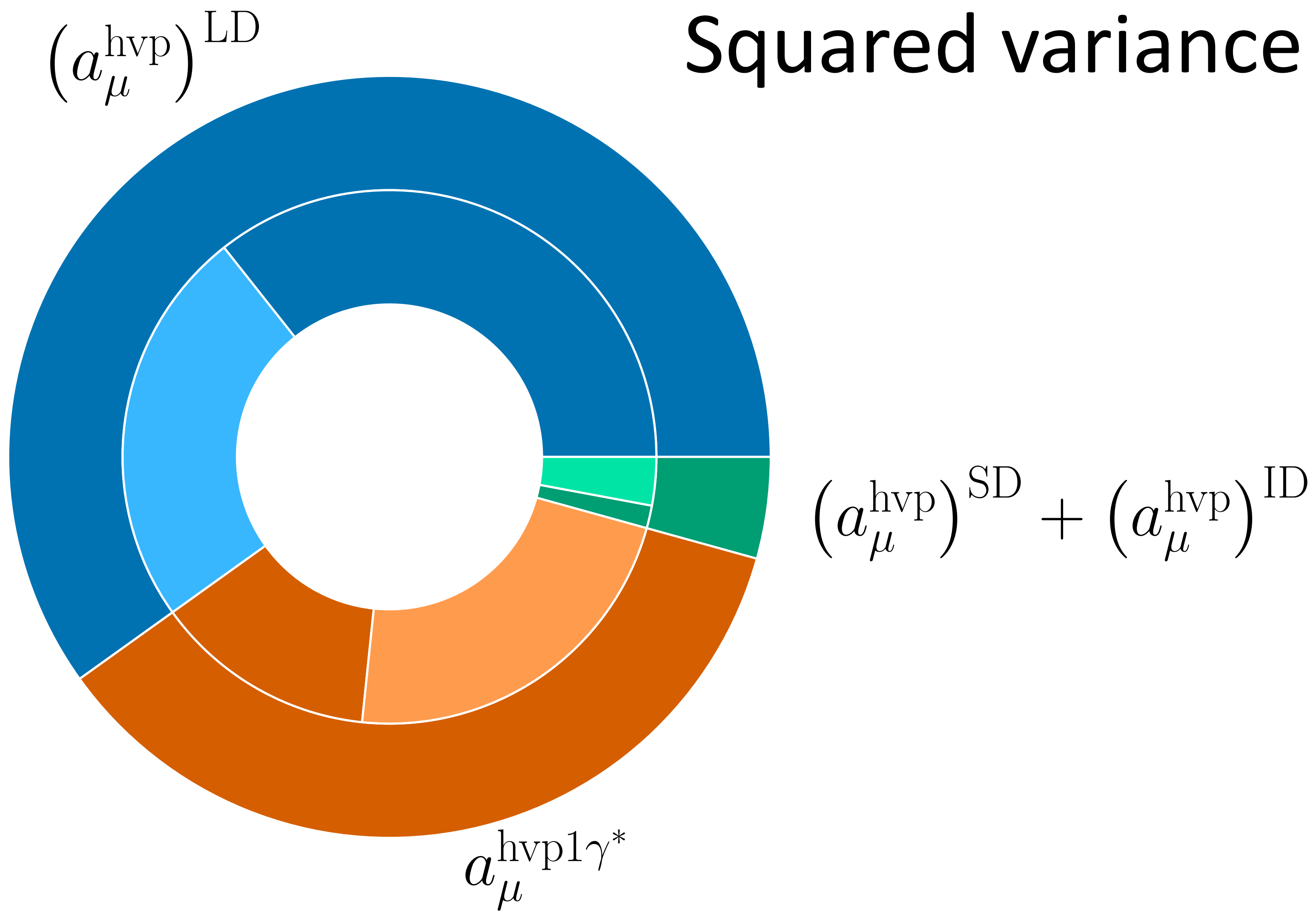


Room for improvement:



[Mainz/CLS: Djukanovic et al., arXiv:2411.07969]

Summary and outlook

"Agreement is not a useful scientific concept"

Guido Martinelli, ca. 1994

Summary and outlook

Lattice QCD calculations have entered the precision era:

- Results are precise enough to be confronted with experiment and/or phenomenology
- Lattice QCD able to resolve / confirm / create tensions

Puzzle surrounding the proton's electric charge radius largely resolved

A new puzzle has emerged in the case of the magnetic charge radius

Muon $g-2$ puzzle has shifted: explain the tension between e^+e^- data and Lattice QCD

Fermilab E989 prepares to release result including data from Runs 4–6

Second White Paper in preparation

Case for New Physics is weakening….