Few-Nucleon Scattering in Pionless Effective Field Theory with Non-Perturbative Coulomb Interaction

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61st International Winter Meeting on Nuclear Physics 2025, Bormio
January 27-31, 2025

Motivation

Nuclear Efective Field Theories

- Systematic framework for studying nuclear interaction
- Degrees of freedom $\longrightarrow$ Lagrangian $\longrightarrow$ Interaction
- Very low energies $\longrightarrow$ **Pionless effective field theory ($\not{\pi}$EFT)**

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- Very low energies $\longrightarrow$ **Pionless effective field theory ($\not{\pi}$EFT)**

Few-body problem can be solved highly accurately $\longrightarrow$ Testing ground for nuclear interaction

Furthermore $\longrightarrow$ Low-energy regime in few-body nuclear reactions is important in astrophysics

(B. D. Fields, Big Bang Nucleosynthesis, In: Handbook of Nuclear Physics, Springer (2023).)

$$d + d \to {}^3\text{H} + p \qquad {}^3\text{H} + d \to {}^4\text{He} + n$$

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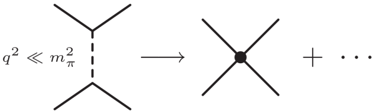
- $Q < 1.5$ MeV $\longrightarrow$ domain of $\not{\pi}$EFT

Pionless Effective Field Theory

- Low energies ($Q \ll m_\pi$) $\rightarrow$ pion fields integrated out $\rightarrow$ $\not\pi$EFT
- Only degress of freedom are nucleonic fields $\rightarrow$ expansion in contact terms and their derivatives



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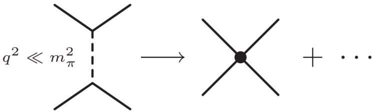

- Increasingly singular interactions $\longrightarrow$ Local Gaussian regulators

$$C(N^\dagger N)^2 \quad \longrightarrow \quad C\delta^{(3)}(\boldsymbol{r}_{ij}) \quad \longrightarrow \quad C \exp\left(-\Lambda^2 \boldsymbol{r}_{ij}^2/4\right)$$

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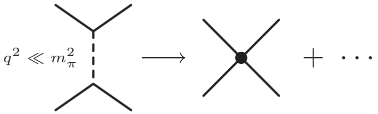

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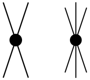
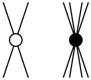
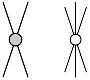
- LECs depend on the UV regulator $\Lambda \longrightarrow C \equiv C(\Lambda)$
- Renormalization = matching to exp. data; binding energies, scattering lengths, effective ranges...
- **Renormalization Group (RG) invariance** for $\Lambda \gg m_\pi$

$$\mathscr{M}(\Lambda) = \mathscr{M}(\infty) + \mathcal{O}(\Lambda^{-1})$$

Power Counting in $\not{\pi}$EFT (without Coulomb)

LO		$\delta(\mathbf{r}_{12})$, $\delta(\mathbf{r}_{12})\delta(\mathbf{r}_{23})$
NLO		$\overleftarrow{\nabla}^2_{\mathbf{r}_{12}}\delta(\mathbf{r}_{12}) + \delta(\mathbf{r}_{12})\overrightarrow{\nabla}^2_{\mathbf{r}_{12}}$, $\delta(\mathbf{r}_{12})\delta(\mathbf{r}_{23})\delta(\mathbf{r}_{34})$
N^2LO		$S-D$ tensor ($T=0$), momentum dep. 3-body
N^3LO	...	$(\nabla_{\mathbf{r}_1}\cdot\nabla_{\mathbf{r}_2})\delta(\mathbf{r}_{12})$, LS, tensor ($T=1$), more 4-body ?

(H.-W. Hammer, Sebastian König, and U. van Kolck, Rev. Mod. Phys. 92 (2020) 025004)

The NLO $\not\pi$EFT Potential (without Coulomb)

- Leading order (LO) terms iterated to reproduce the large *NN* scattering length (bound state)
- NLO terms are treated as perturbations - necessary for renormalization

$$
\begin{aligned}
V_{2\mathrm{B}} = & \sum_{i<j} \left(C_0^{(0)} \hat{\mathcal{P}}_{ij}^{(0,1)} + C_1^{(0)} \hat{\mathcal{P}}_{ij}^{(1,0)} \right) e^{-\Lambda^2 \mathbf{r}_{ij}^2/4} + && \longrightarrow \text{LO - Iterated in Schrödinger eq.} \\
& \sum_{i<j} \left(C_0^{(1)} \hat{\mathcal{P}}_{ij}^{(0,1)} + C_1^{(1)} \hat{\mathcal{P}}_{ij}^{(1,0)} \right) e^{-\Lambda^2 \mathbf{r}_{ij}^2/4} + && \longrightarrow \text{NLO Counterterm} \\
& \sum_{i<j} \left(C_2^{(1)} \hat{\mathcal{P}}_{ij}^{(0,1)} + C_3^{(1)} \hat{\mathcal{P}}_{ij}^{(1,0)} \right) \times \left(e^{-\Lambda^2 \mathbf{r}_{ij}^2/4} \vec{\nabla}^2 + \overleftarrow{\nabla}^2 e^{-\Lambda^2 \mathbf{r}_{ij}^2/4} \right) && \longrightarrow \text{New NLO term}
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$$\sum_{i<j} (C_2^{(1)} \hat{\mathcal{P}}_{ij}^{(0,1)} + C_3^{(1)} \hat{\mathcal{P}}_{ij}^{(1,0)}) \times (e^{-\Lambda^2 \mathbf{r}_{ij}^2/4} \vec{\nabla}^2 + \overleftarrow{\nabla}^2 e^{-\Lambda^2 \mathbf{r}_{ij}^2/4}) \longrightarrow \text{New NLO term}$$

- Similarly $3N$ and $4N$

$$V_{3B} = D_0^{(0)} \sum_{i<j<k} \sum_{cyc} \hat{\mathcal{P}}_{ijk}^{(1/2,1/2)} e^{-\Lambda^2 (\mathbf{r}_{ij}^2 + \mathbf{r}_{ik}^2)/4} + D_0^{(1)} \sum_{i<j<k} \sum_{cyc} \hat{\mathcal{P}}_{ijk}^{(1/2,1/2)} e^{-\Lambda^2 (\mathbf{r}_{ij}^2 + \mathbf{r}_{ik}^2)/4}$$

$$V_{4B} = E_0^{(1)} \sum_{i<j<k<l} \hat{\mathcal{P}}^{(0,0)} e^{-\Lambda^2 (\mathbf{r}_{ij}^2 + \mathbf{r}_{ik}^2 + \mathbf{r}_{il}^2 + \mathbf{r}_{jk}^2 + \mathbf{r}_{jl}^2 + \mathbf{r}_{kl}^2)/4}$$

- 3 LO LECs and 6 NLO LECs

(Schafer M., Bazak B., Phys. Rev. C 107, 064001, 2023)

$\not{\pi}$EFT with Non-Perturbative Coulomb Interaction

- Coulomb interaction - new scale at low energies

$$V_c(Q) \sim \frac{1}{Q^2} \qquad \longrightarrow \qquad \text{Dominates over } V_{NN} \text{ for } Q \to 0$$

- Effects of V_c and V_{NN} cannot be seperated (Kong, X., Ravndal F., Nuc. Phys. A 665.1-2 (2000): 137-163.)
  $\longrightarrow$ New 2-body *pp* terms in the potential
- New 3-body *ppn* term needed $\longrightarrow$ 12 LECs total (at NLO)
  (Vanasse, J., et al. PRC 89.6 (2014): 064003)

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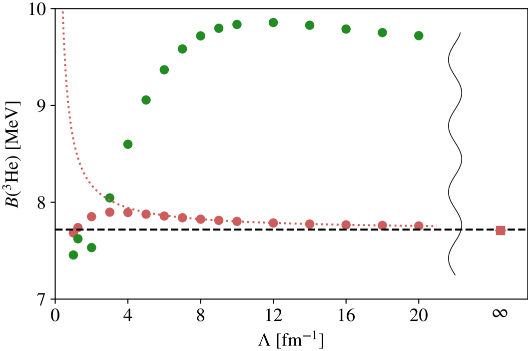

- Renormalization of ^{3}He
- Red - LO with Coulomb
- Green - NLO, no *ppn* term
- - - - : Experiment

Experiment: J. Purcell et al., Nucl. Phys. A 848, 1 (2010).

Nuclear Reactions in an Oscillator Trap

- Description of few-body continuum is a challenging problem $\longrightarrow$ AGS equations, Faddeev-Yakubovsky, Hyperspherical harmonics w/ KVP, NCSM/RGM...

- **We trap the system in a Harmonic Oscillator (HO) potential**
- Phase shifts are extracted from the **quantization condition**:

  (Guo, P. PRC 103.6 (2021): 064611)

$$\boxed{-2\mu C_0^2(\eta)k\mathrm{cotg}\delta = \lim_{r,r'\to 0}\left\{\mathrm{Re}\left[G_0^{C,\infty}(r,r';E)\right] - G_0^{C,\omega}(r,r';E)\right\}}$$

- Green's functions solved for numerically

  (Bagnarol, M., Barnea, N., Rojik, M., Schäfer, M. PLB (2024), 139230)

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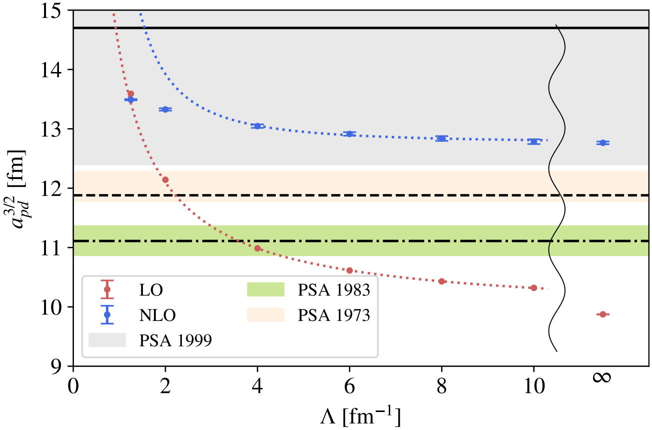
- Spectrum $\{E\} \rightarrow$ **stochastic variational method (SVM) in a correlated Gaussian basis**

$$\Psi = \sum_i c_i \hat{A} \exp\left(-\frac{1}{2} \boldsymbol{x}^T \boldsymbol{A}_i \boldsymbol{x} \right) \chi^i_{SM_S} \xi^i_{TM_T}$$

$\boldsymbol{A}_i \quad \longleftarrow \quad$ Matrix with randomly chosen basis-state parameters

Proton-Deuteron Scattering

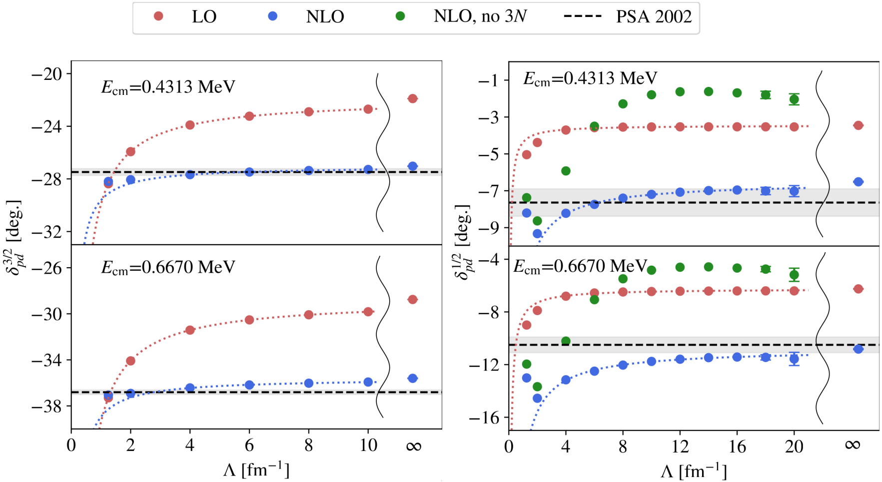
- Prediction for the *pd* scattering length in the $S = 3/2$ channel: $a_{pd}^{3/2} = 12.76(73)$ fm



PSA 1999: T. C. Black et al., PLB 471, 103 (1999).
PSA 1983: E. Huttel et al., Nucl. Phys. A 406, 443 (1983).
PSA 1973: J. Arvieux, Nucl. Phys. A 221, 253 (1974)

Proton-Deuteron Scattering

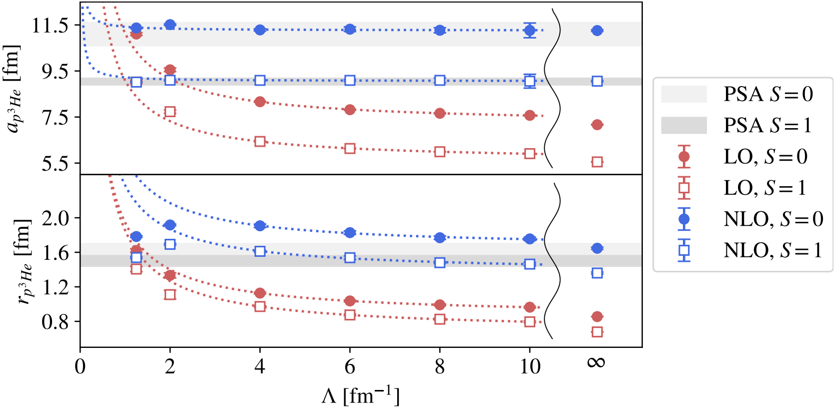
- Comparison of $\not{\pi}$EFT *pd* $S = 3/2$ and $S = 1/2$ phase shifts with experiment
- The 3-body *ppn* force $D_1^{(1)}$ must be included for renormalization



PSA: M. H. Wood et al., PRC, 65, 034002

Proton-^{3}He Scattering

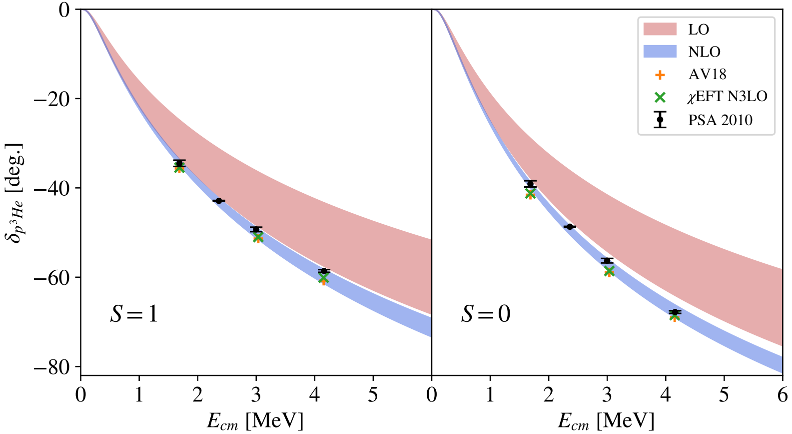
- Predictions of $\not\pi$EFT for p^3He scattering lengths and effective ranges for $S = 0$ and $S = 1$



PSA: Daniels, T., et al. PRC (2010): 034002.

Proton-^{3}He Scattering

- Comparison of S=1 (left) and S=0 (right) p^3He phase shifts with experiment and different potential models



PSA: Daniels, T., et al. PRC (2010): 034002.
AV18 and χEFT: Viviani et al. PRC 84, 054010 (2011)

Summary and Conclusions

- We have presented a detailed analysis of various bound and continuum $A \leq 4$ s-wave nuclear systems in $\not\pi$EFT up to NLO
- Nuclear interactions from $\not\pi$EFT are power-counting renormalizable
- As a consequence, Coulomb interaction and $3N$, $4N$ forces are included systematically
- Low-energy observables are in excellent agreement with experiment
- Only 9 experimental input data
- Exciting future prospects:
  - Higher orders
  - Predictions for astrophysical reactions with theoretical errors
  - Predictions for $A > 4$ systems