ExoHad

CRC-TR 211
Strong-interaction matter under extreme conditions

JLU
NEUE WEGE. SEIT 1607.

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

HFHF
Helmholtz Forschungsakademie Hessen für FAIR

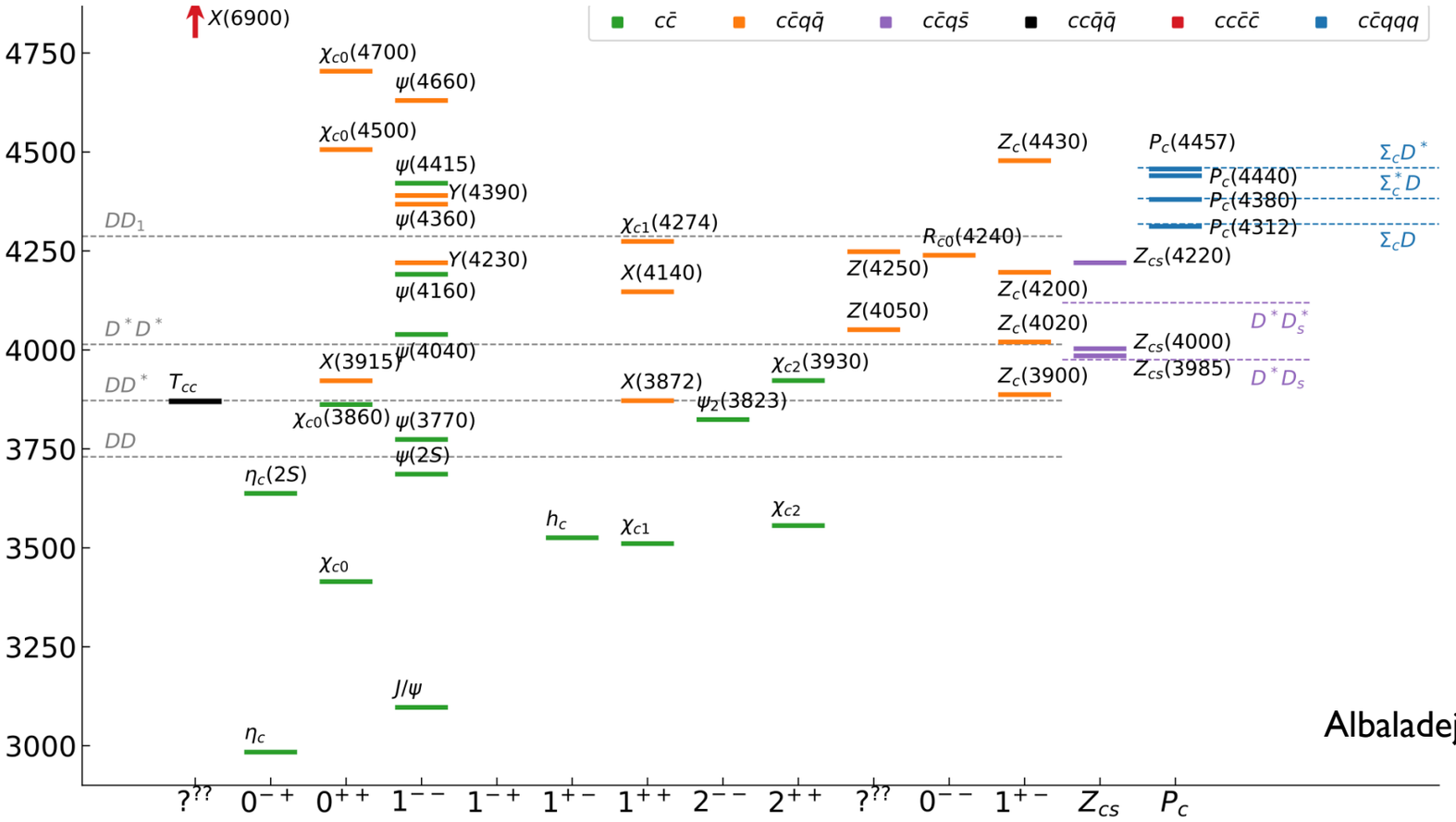
Bormio 2025

Four-quark states with functional methods

with Gernot Eichmann and Joshua Hoffer

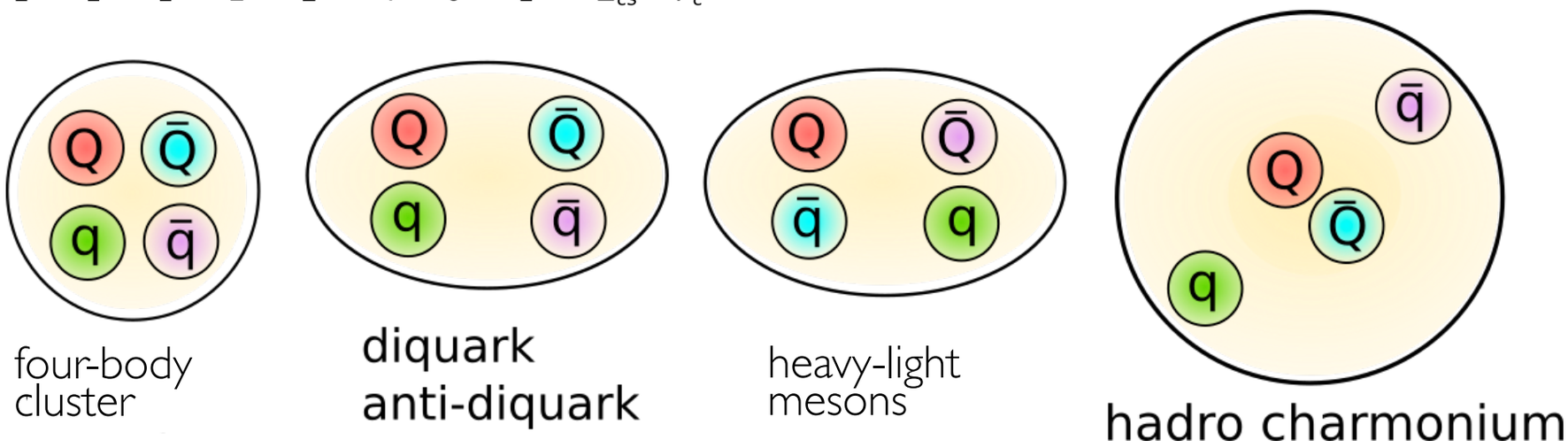
Hoffer, Eichmann, CF, PRD 109 (2024) 7 074025
Hoffer, Eichmann, CF, 2409.05779
Hoffer, Eichmann, CF, in preparation

Exotic hadrons at Belle, BABAR, BES, LHCb,…



Albaladejo et al. [JPAC], PPNP 127 (2022), 103981

Four-quark states:



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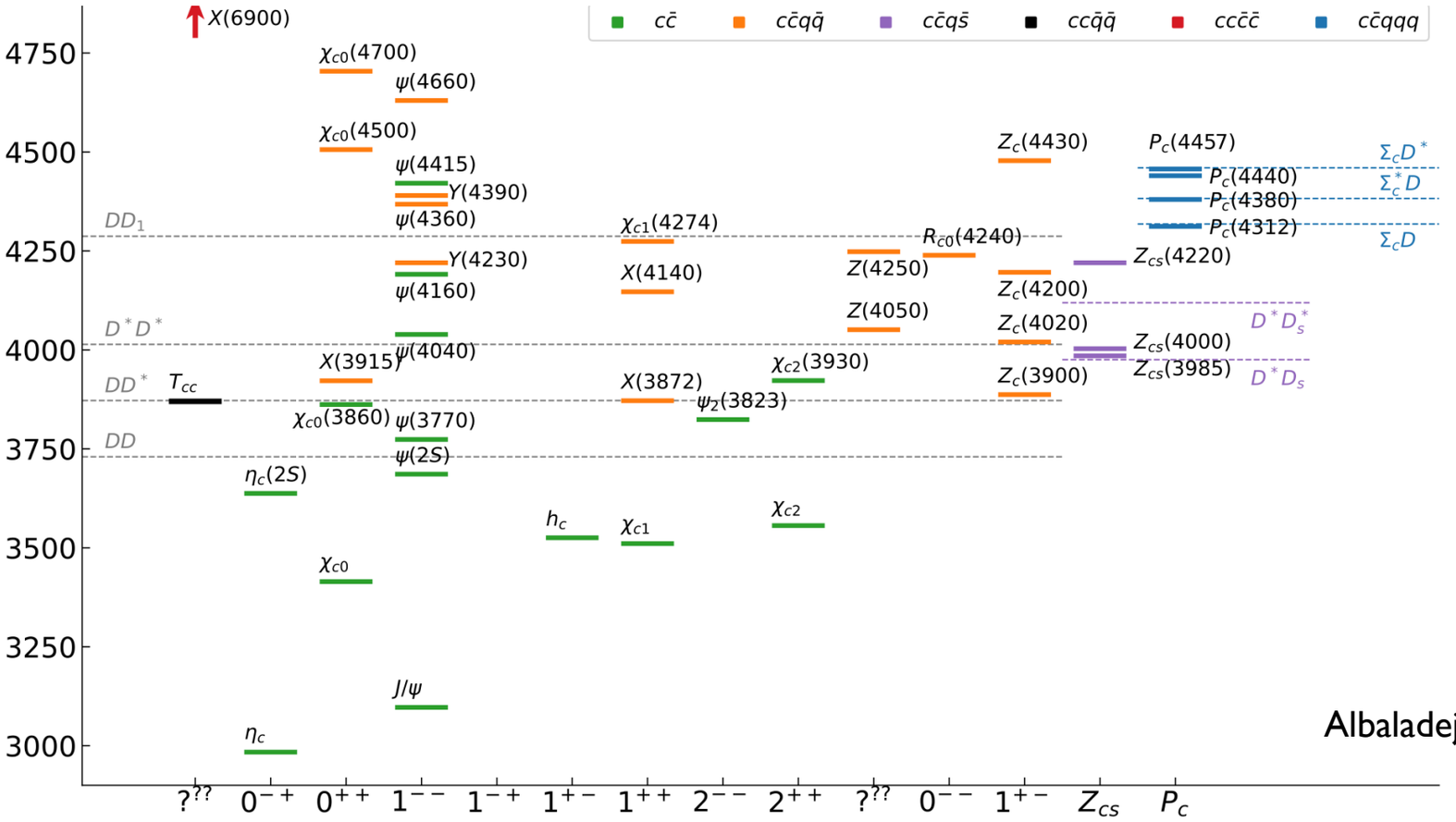

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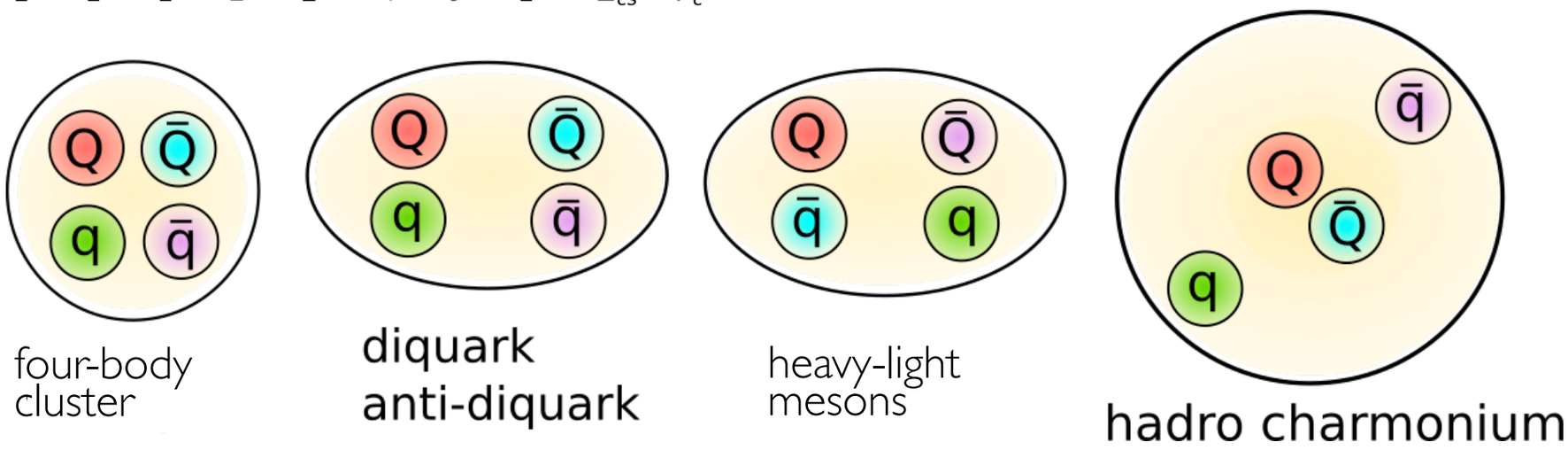
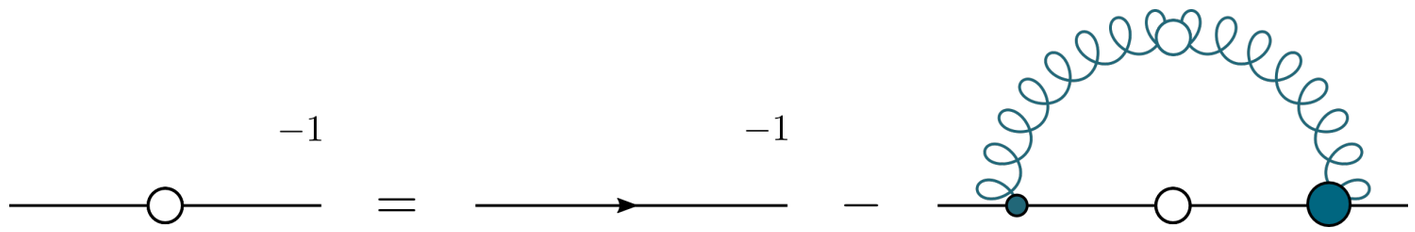


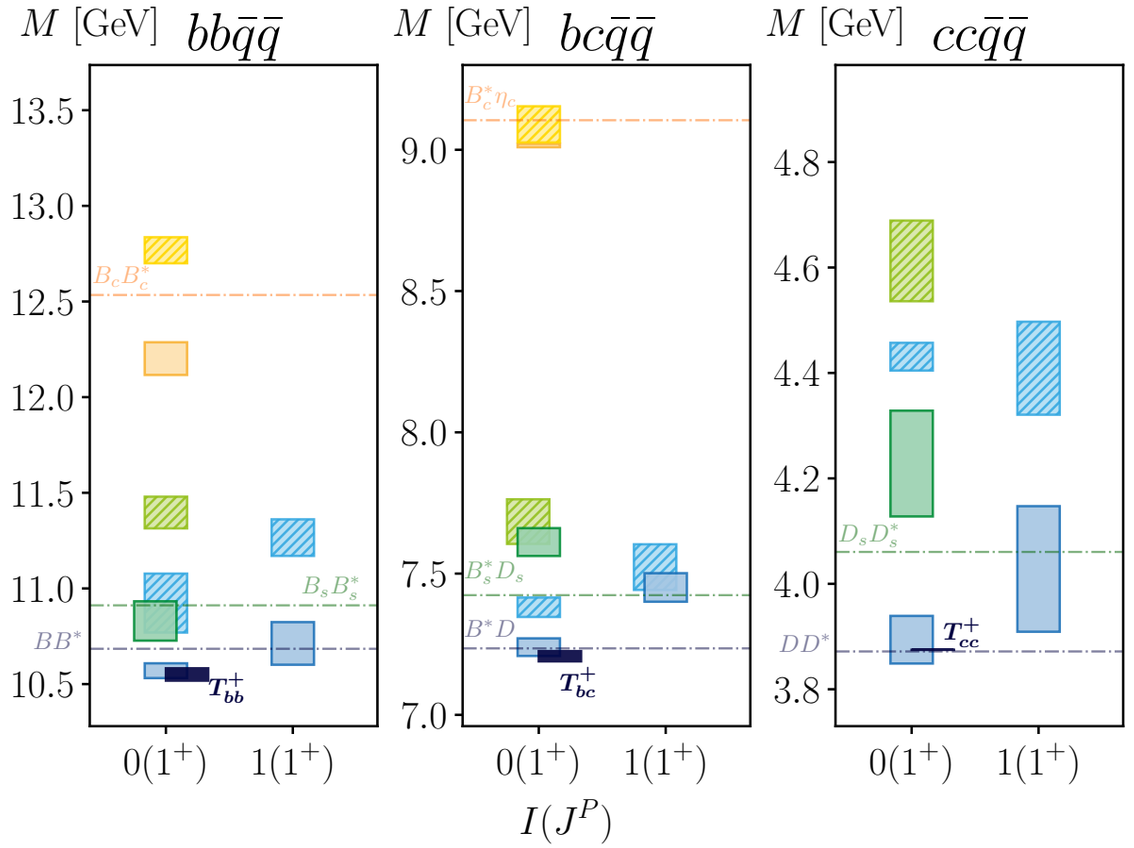
Related to details of underlying QCD forces

Overview

1. The formalism: glueballs, quarks and two-quark states



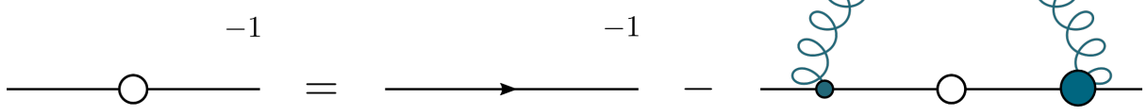
2. Four-quark states: hidden and open flavour



Dyson-Schwinger equations - "3PI vs RL"

Non-perturbative treatment of strong interaction via Euclidean correlation functions

propagators



CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

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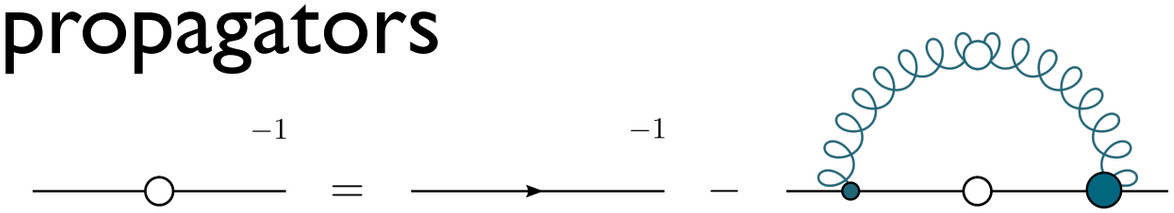

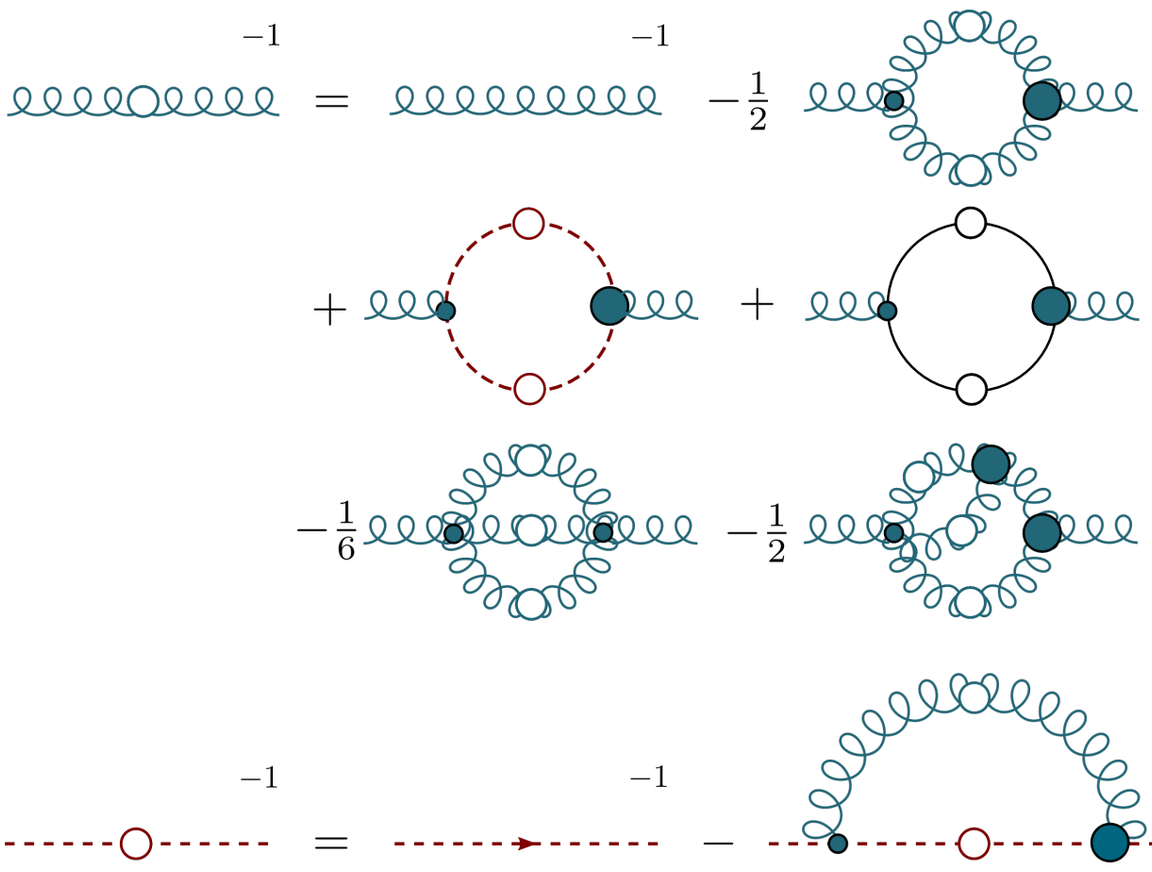

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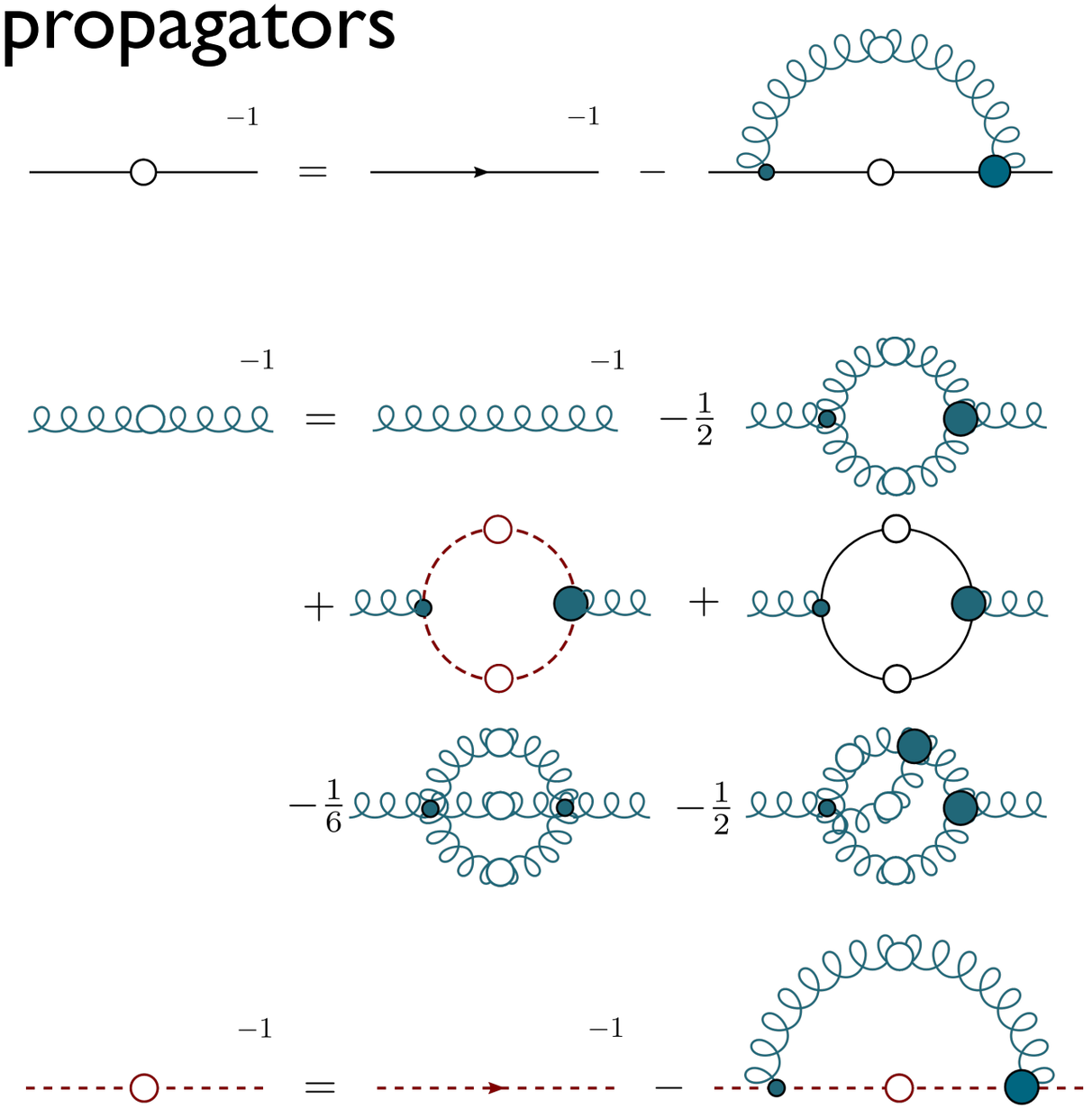

vertices

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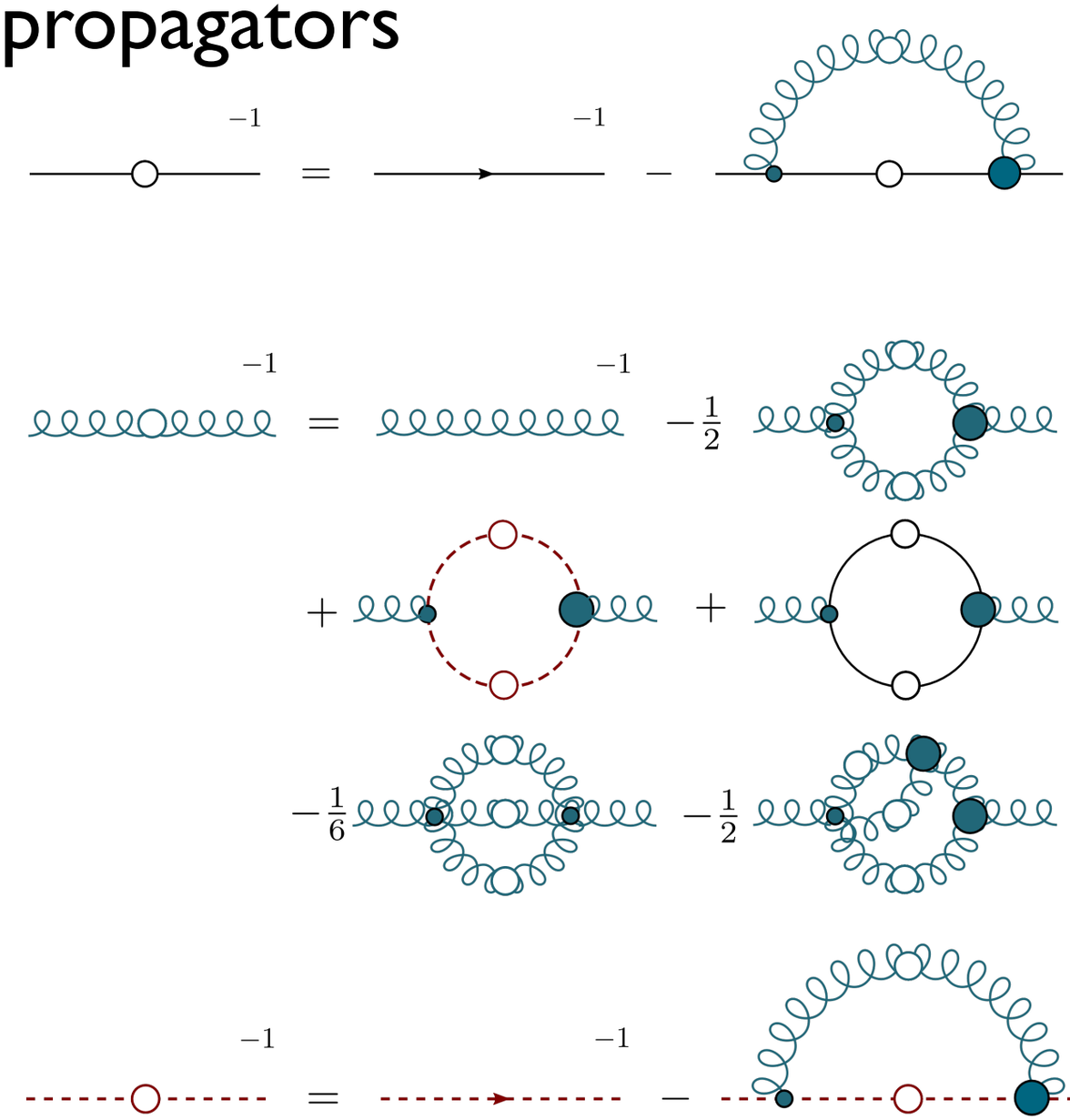

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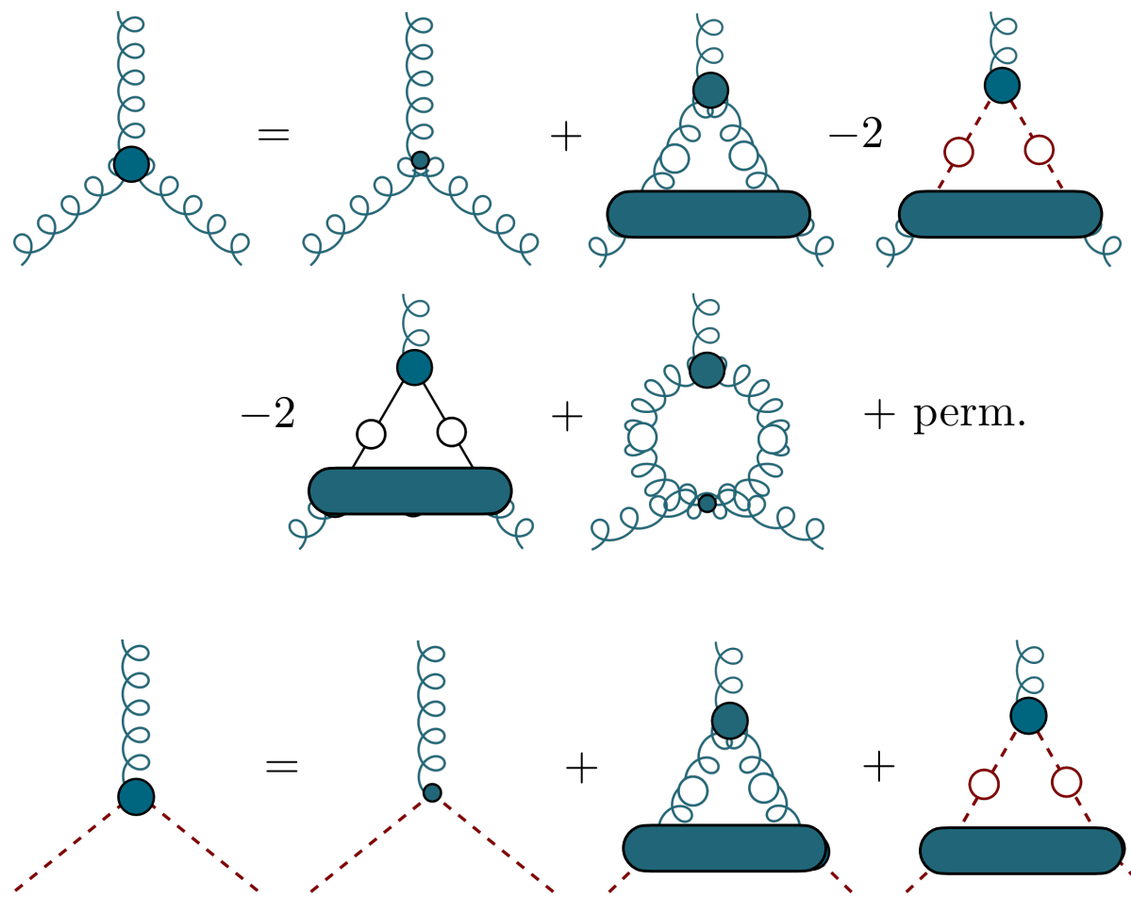

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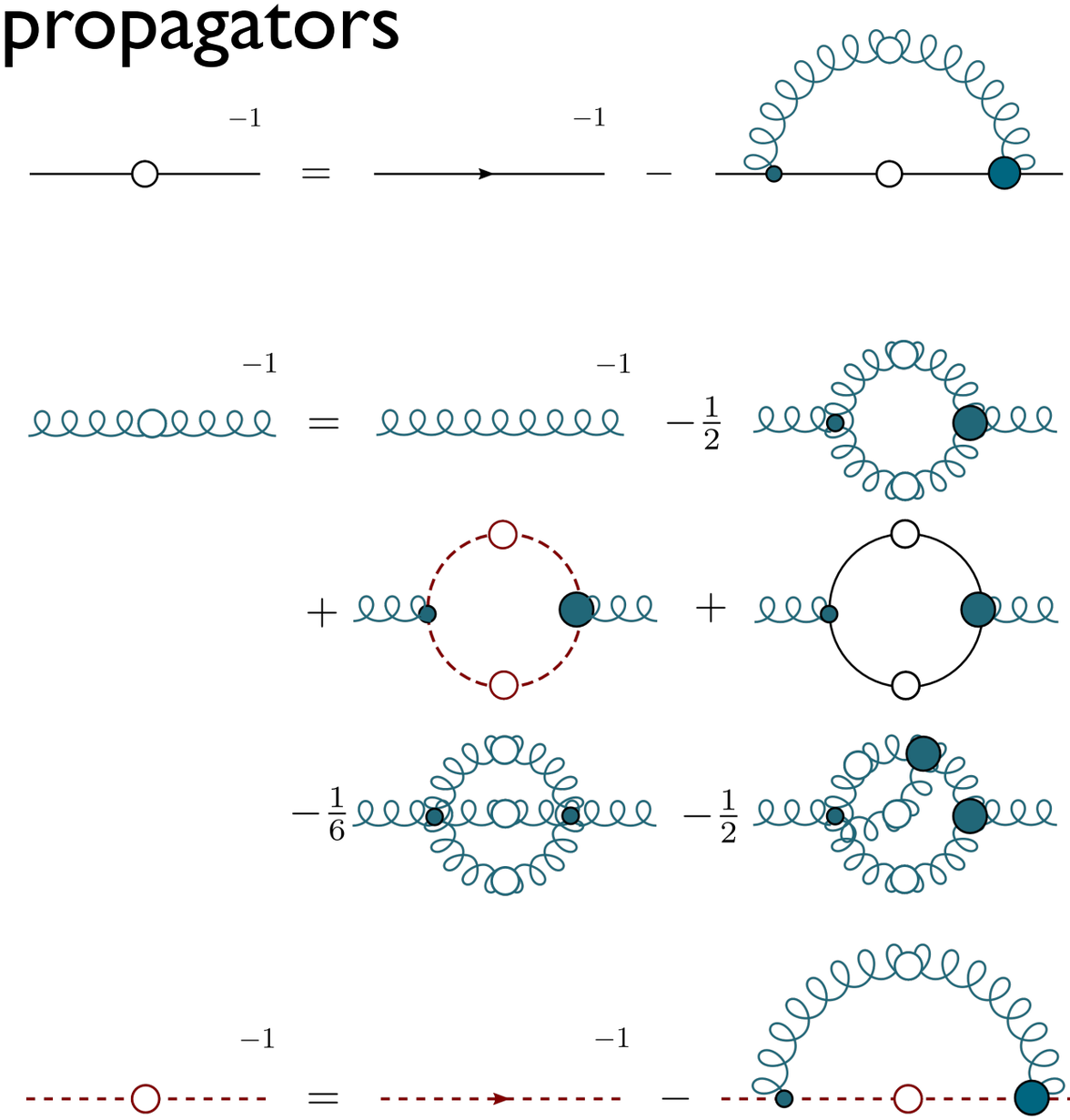

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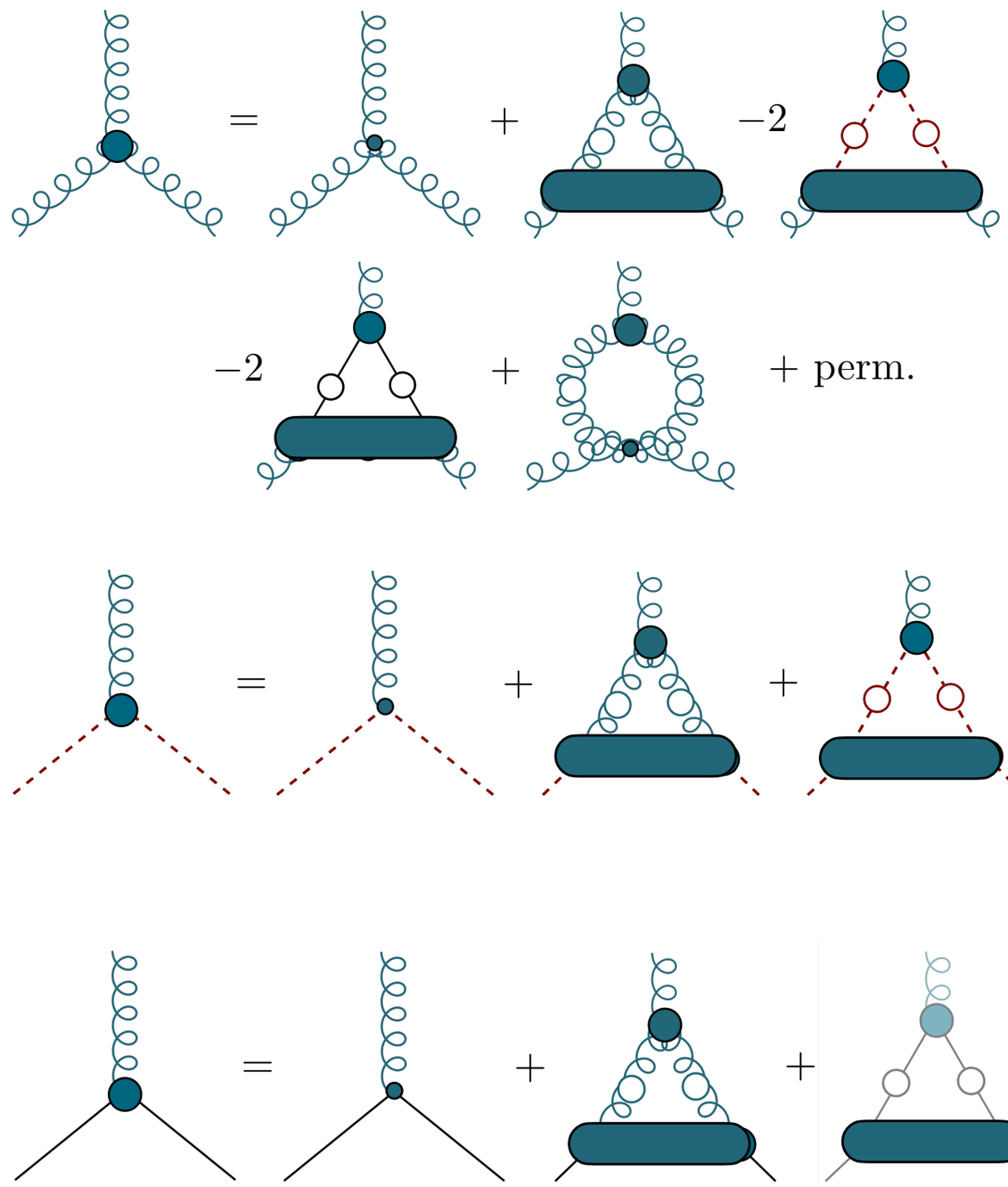

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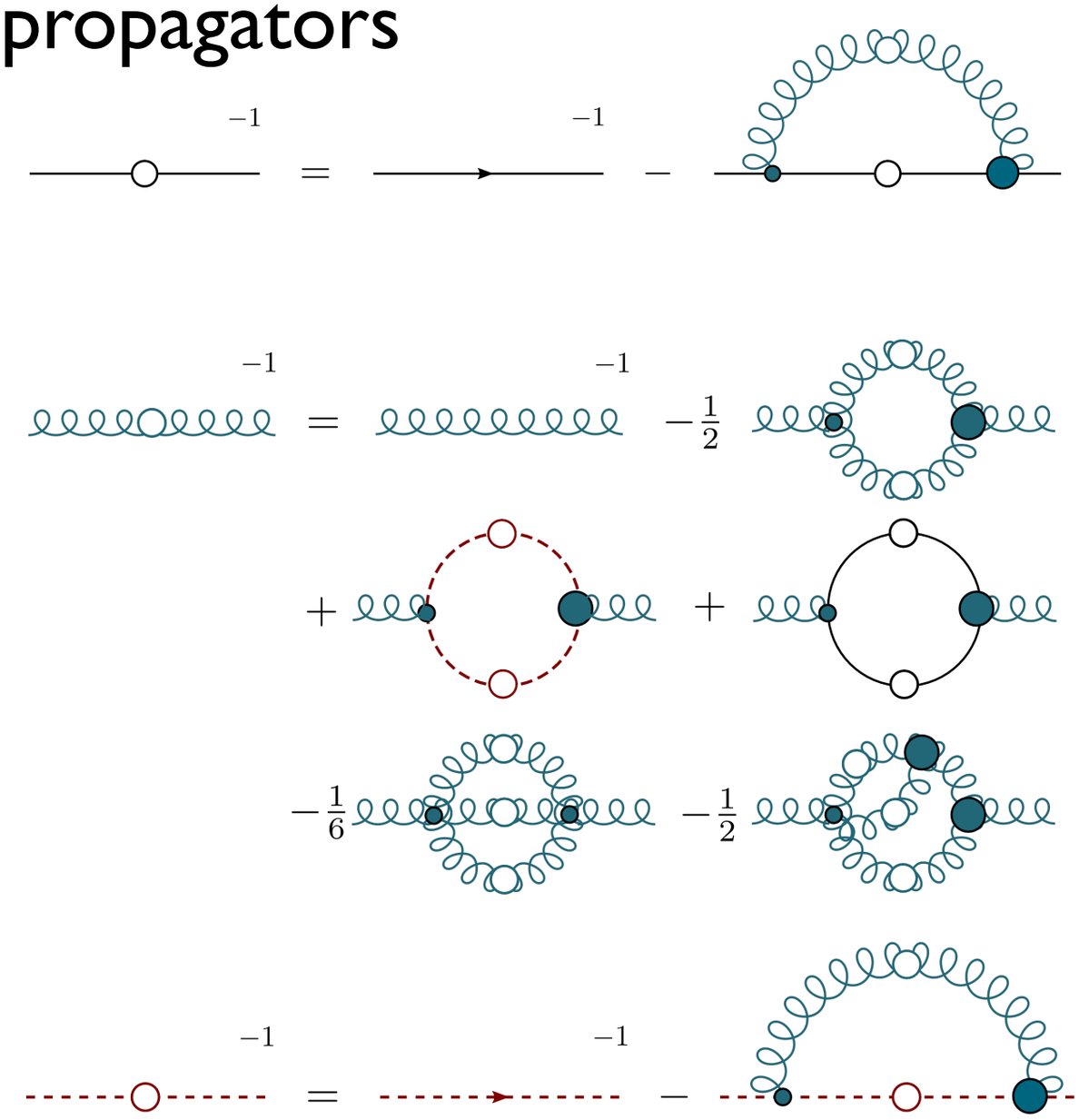

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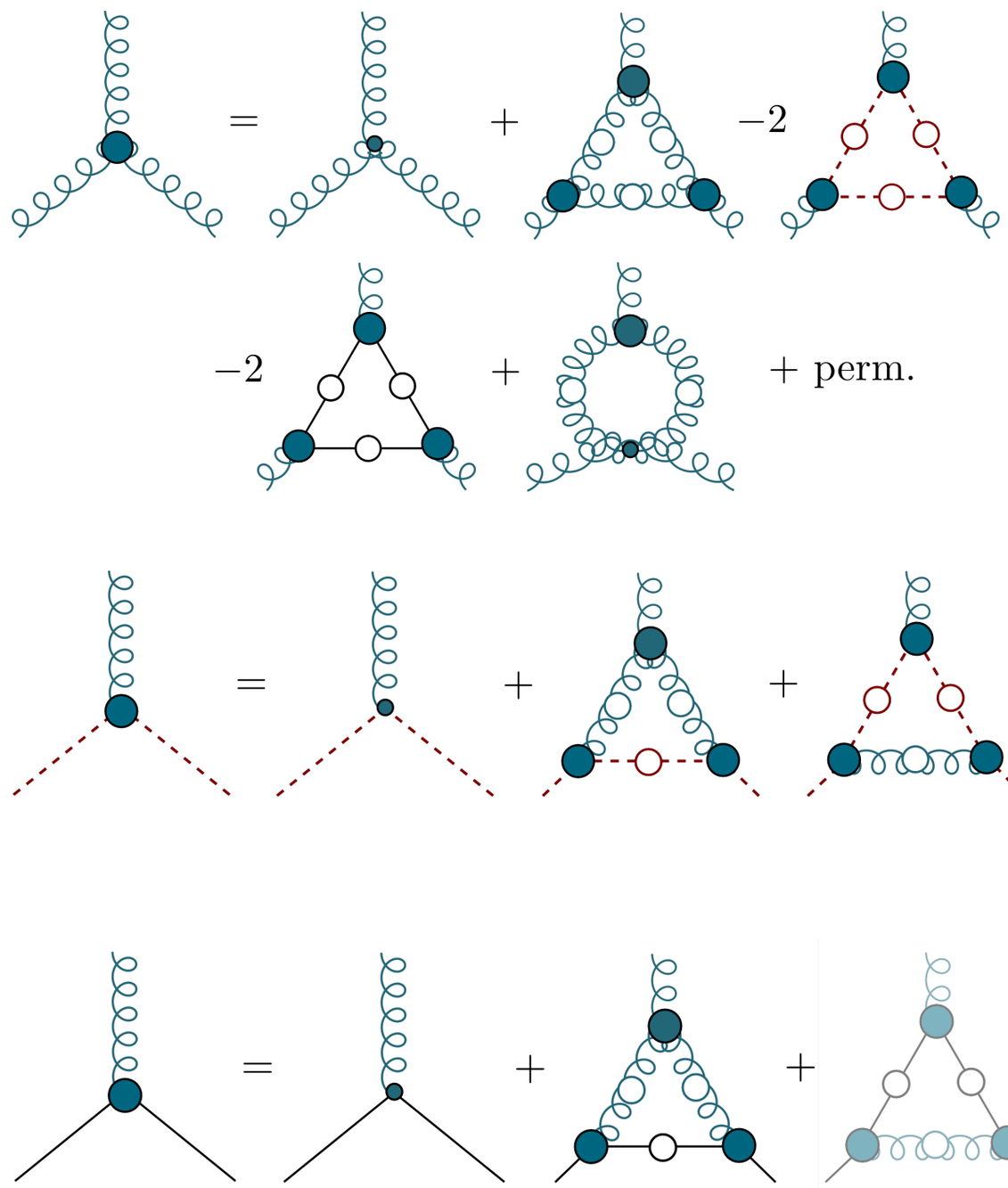

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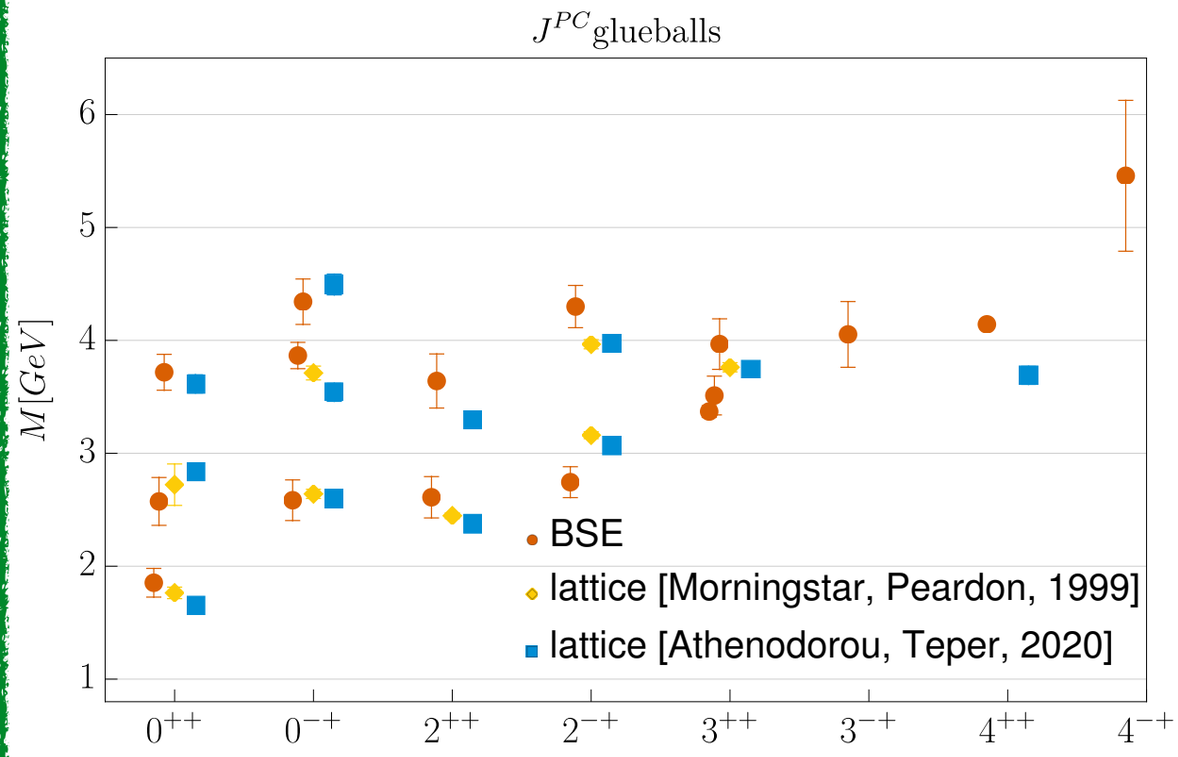

CF, Huber, Sanchis-Alepuz, EPJC 80 (2020) [arXiv:2004.00415]
Huber, CF, Sanchis-Alepuz, EPJC 81 (2021) [arXiv:2110.09180]

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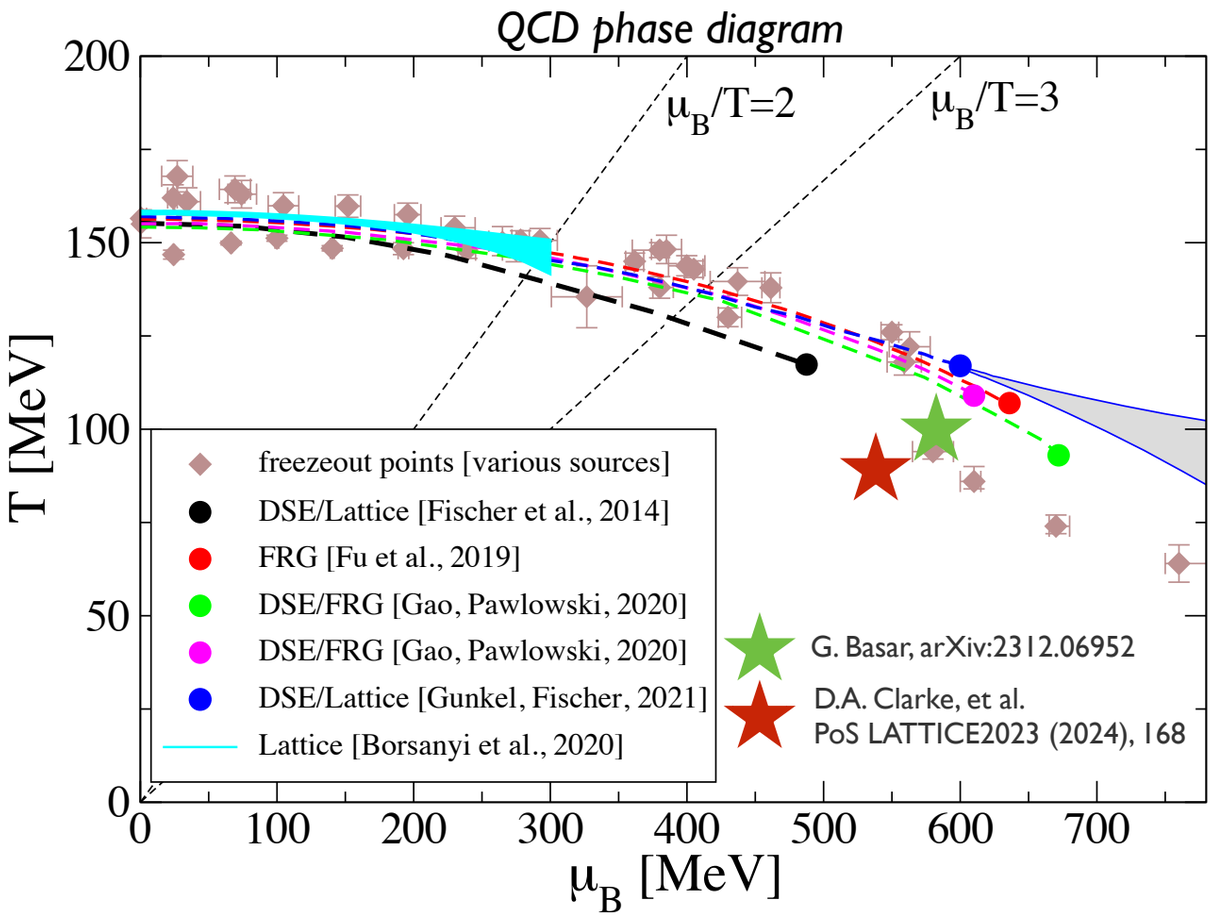
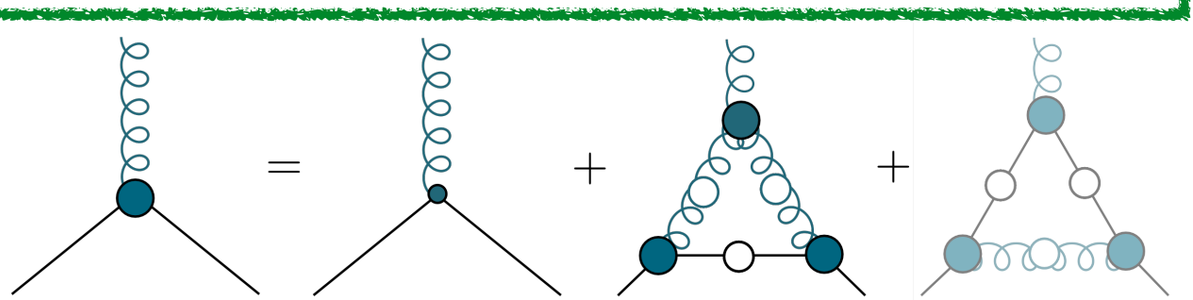


Bernhardt, CF and Isserstedt, PLB 841 (2023)



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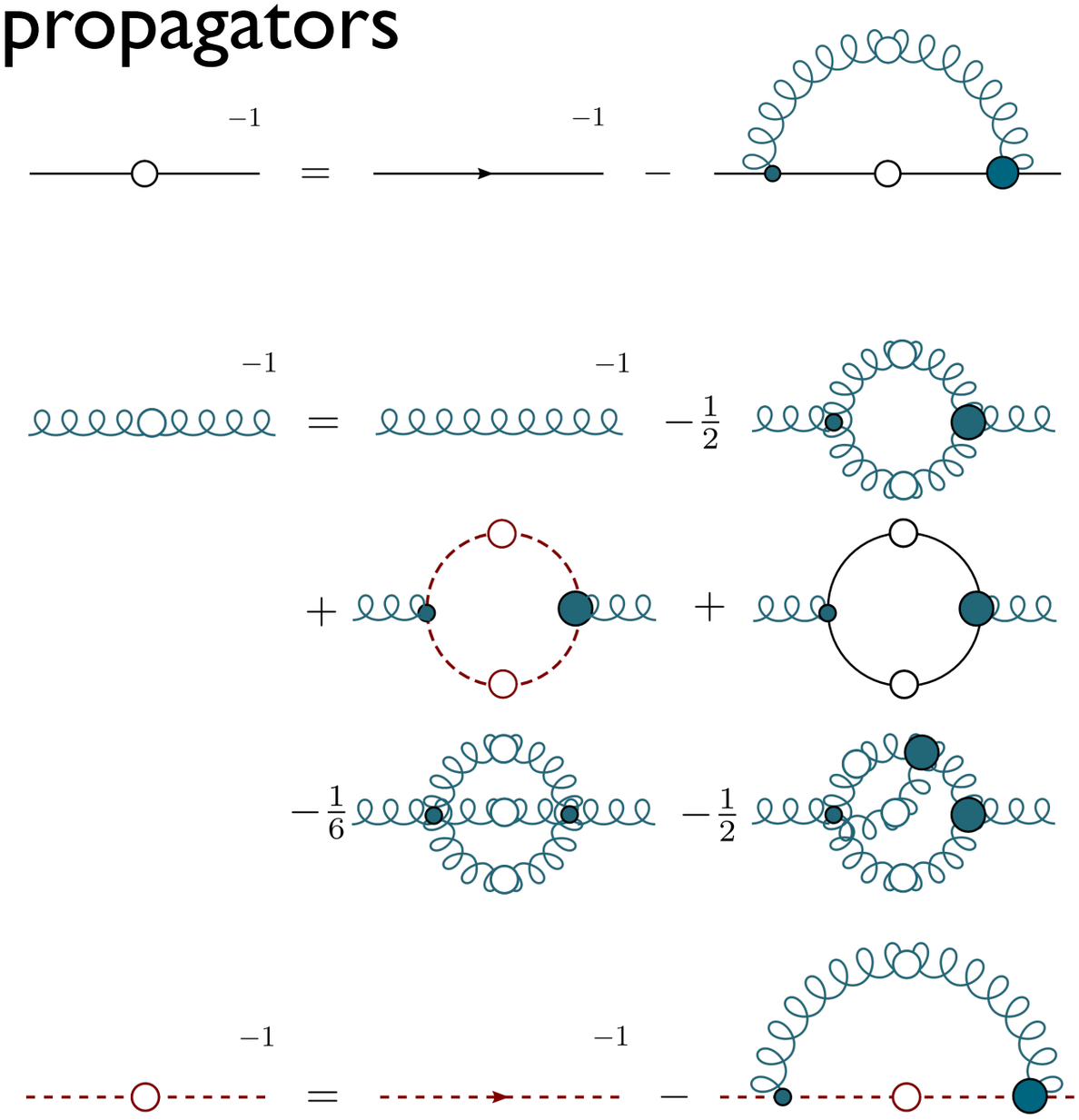

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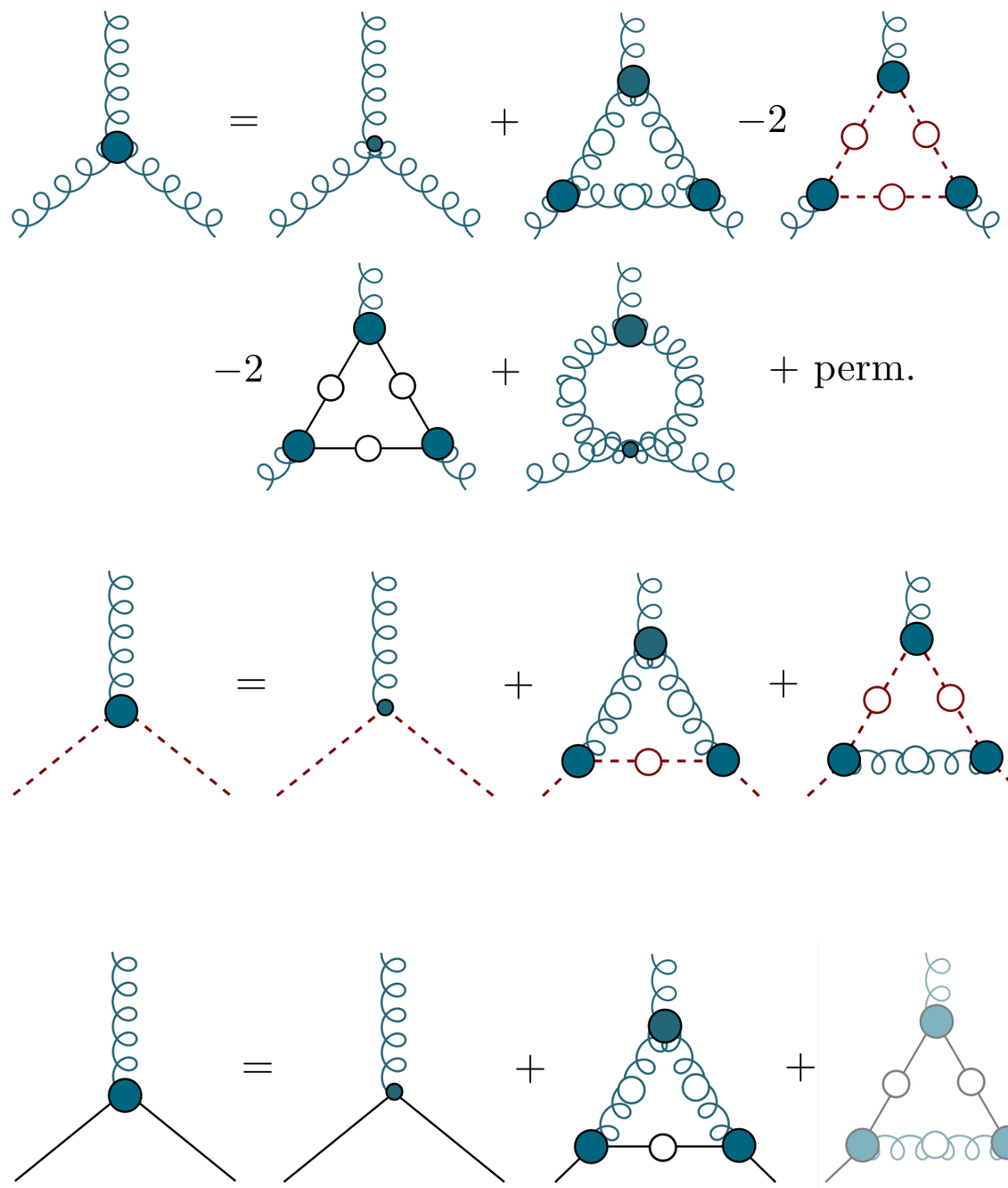

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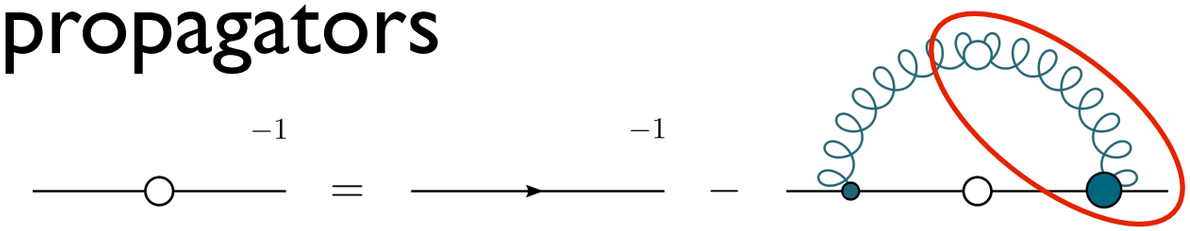

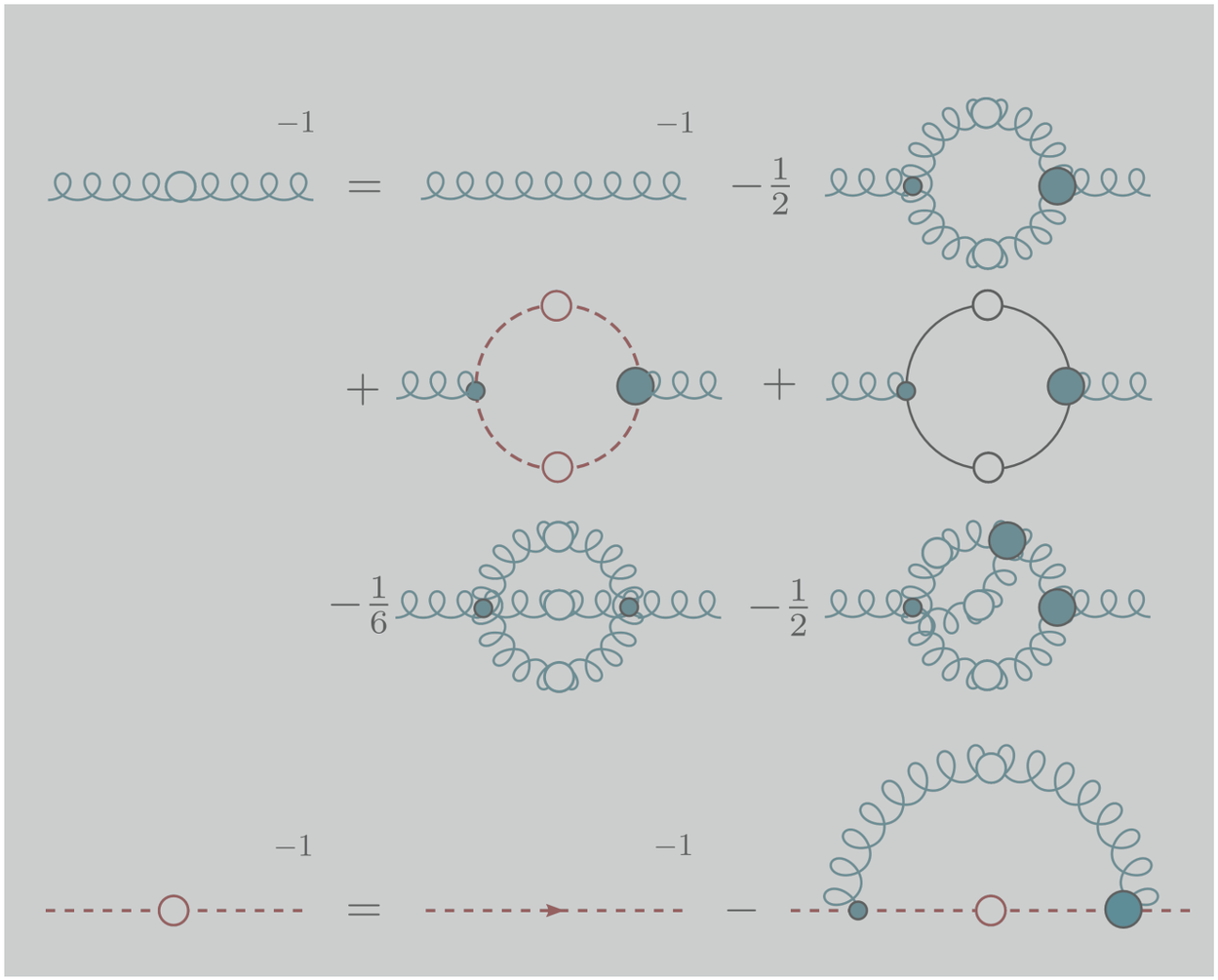

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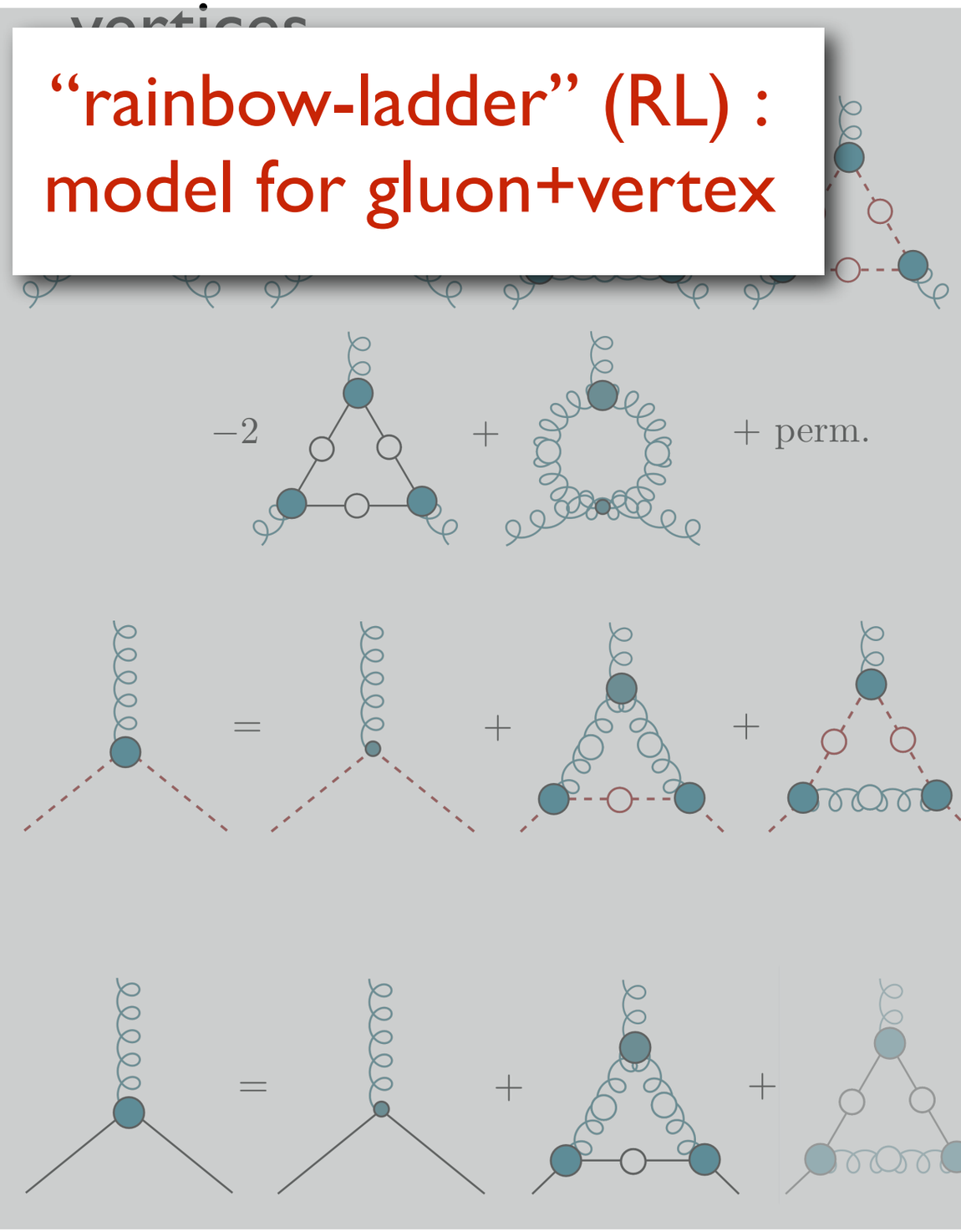
vertices

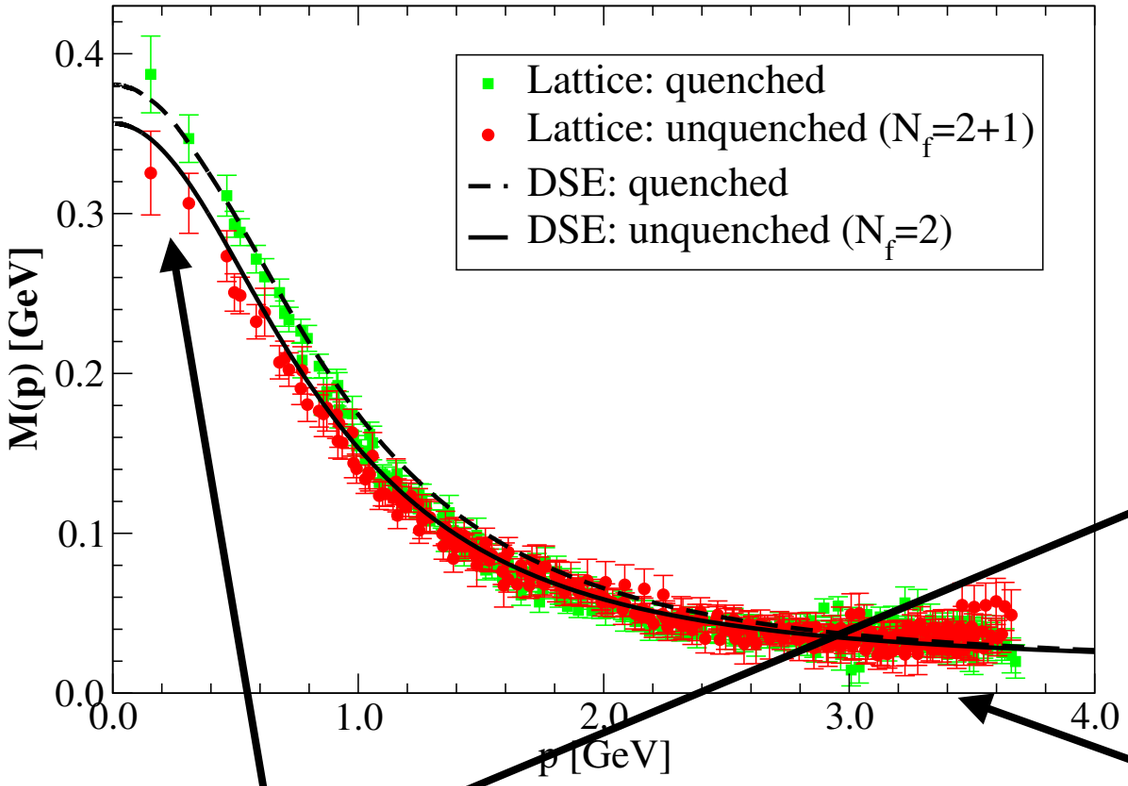
"rainbow-ladder" (RL) : model for gluon+vertex

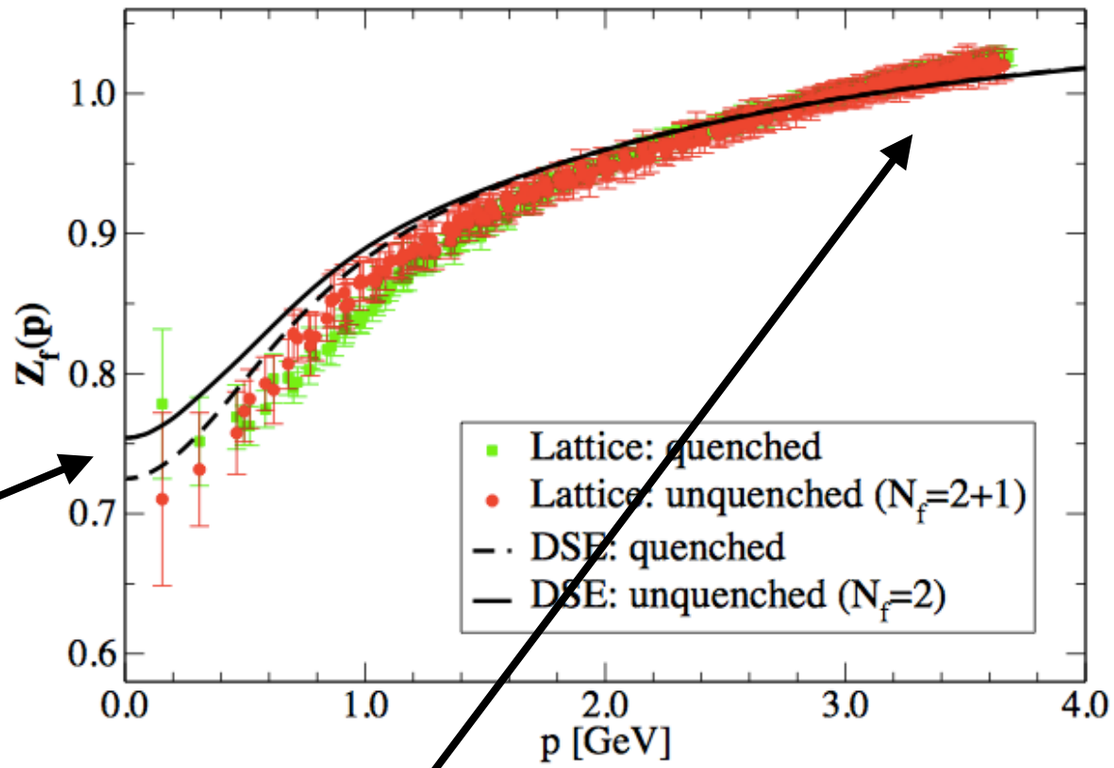


Quark dressing - comparison with lattice

$$S(p) = Z_f(p^2)\frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$

DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507

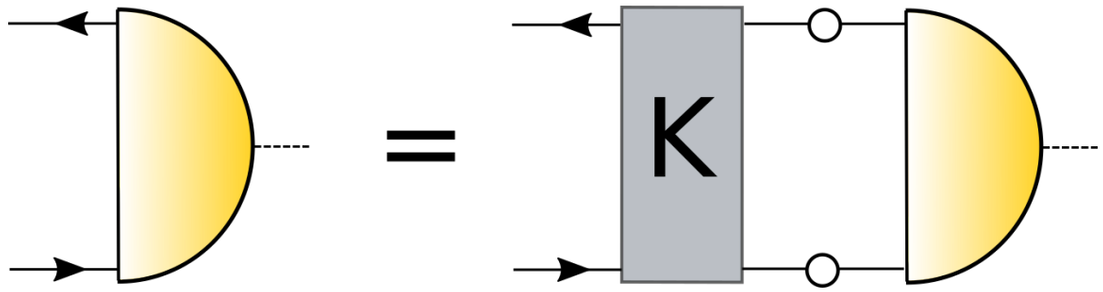




'constituent quark':
large mass; very composite

'current quark':
- small mass; non-composite

The complex P^2-plane



$\lambda(P^2) \circ BSA = kernel \circ BSA$

$\lambda(P^2) \stackrel{!}{=} 1$

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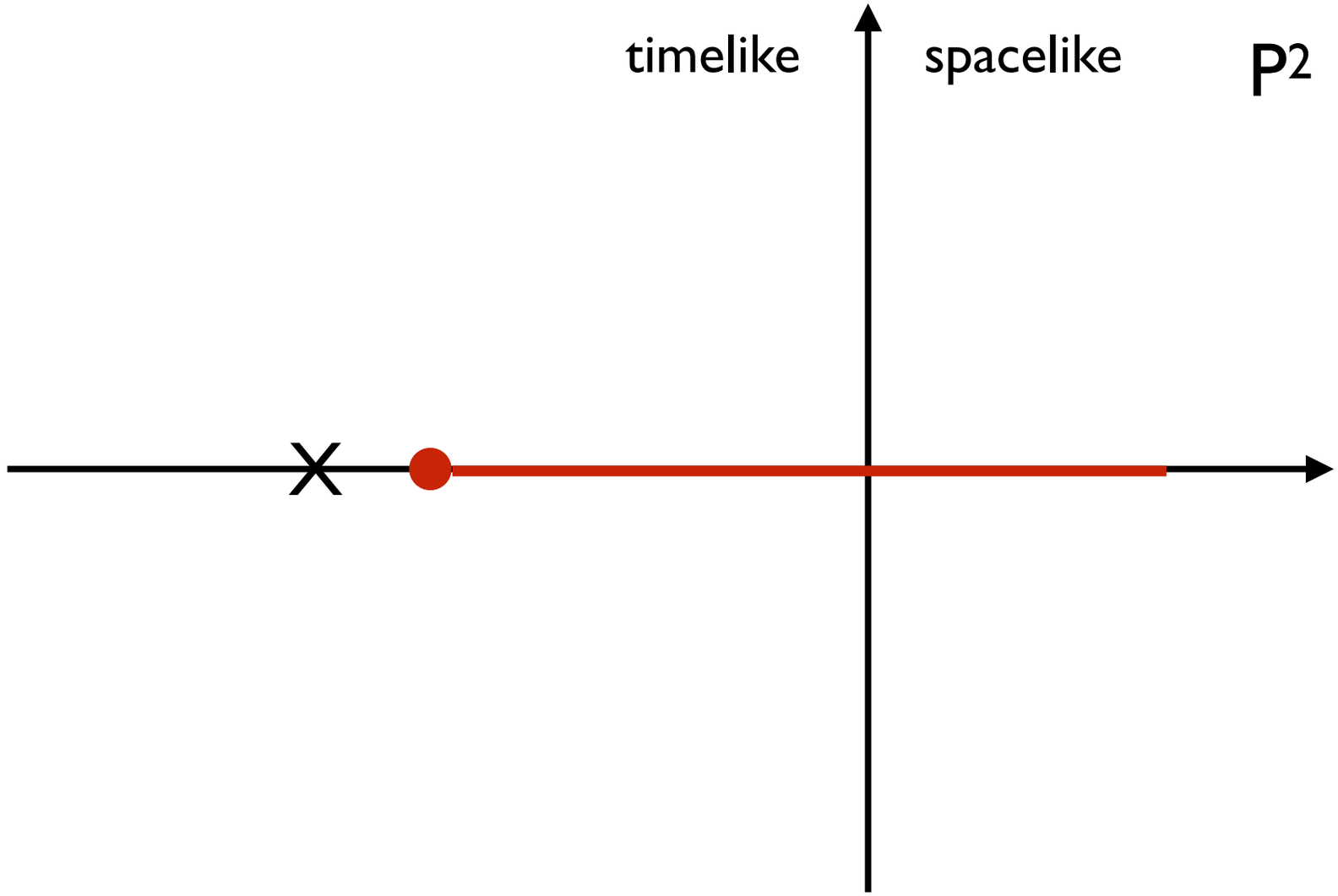
generic situation



Williams, PLB 798 (2019) 134943, [arXiv:1804.11161]

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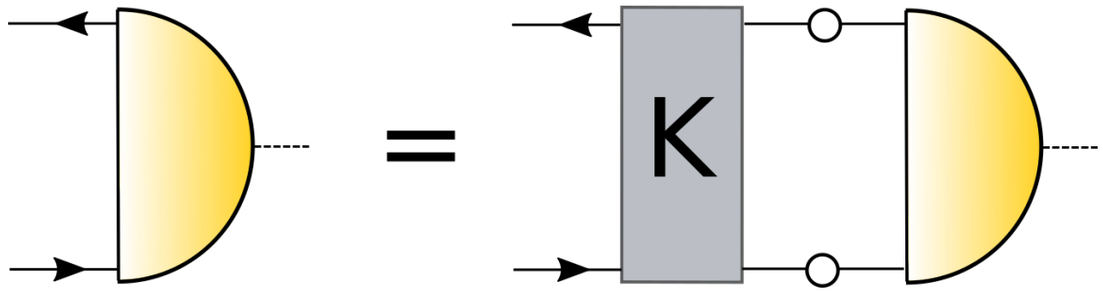

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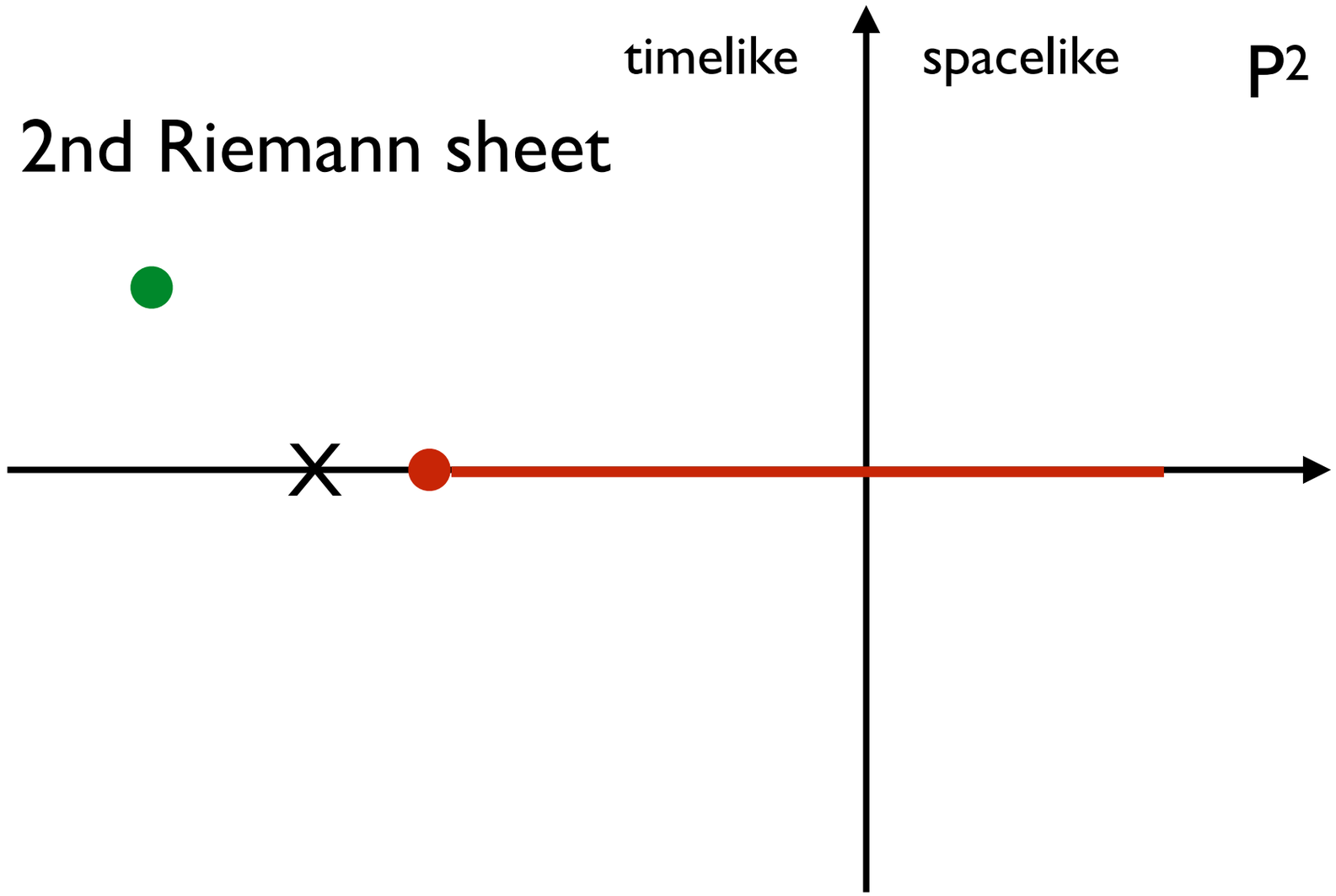
extrapolation to pole in 2nd Riemann sheet

$\rho \to \pi\pi$

$\sigma \to \pi\pi$



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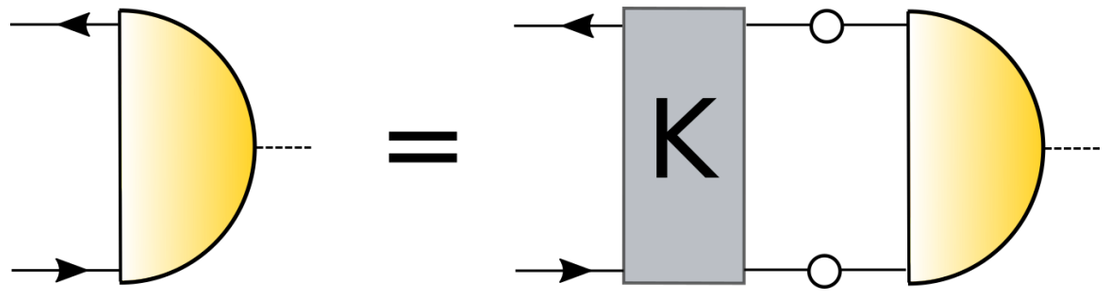

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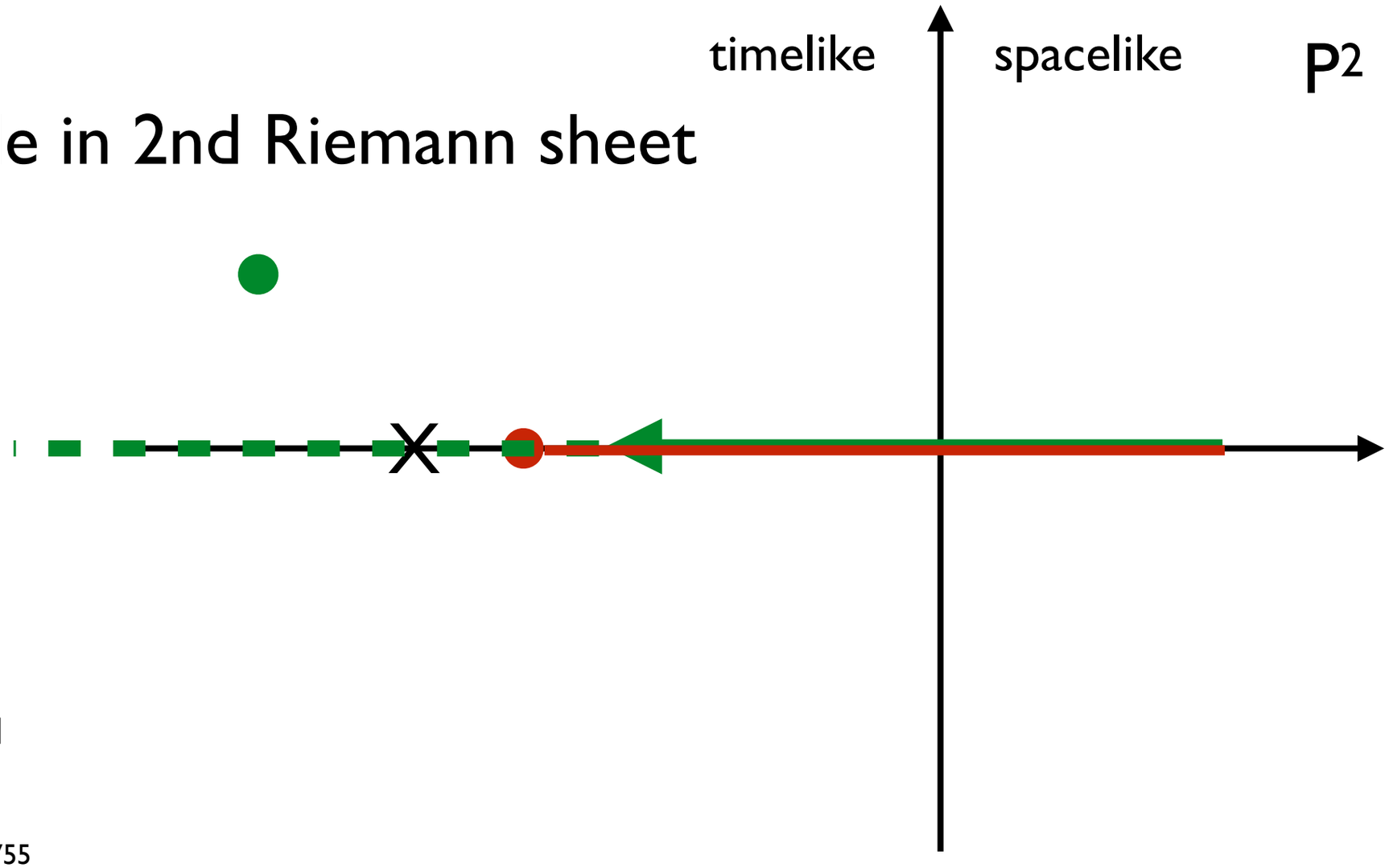

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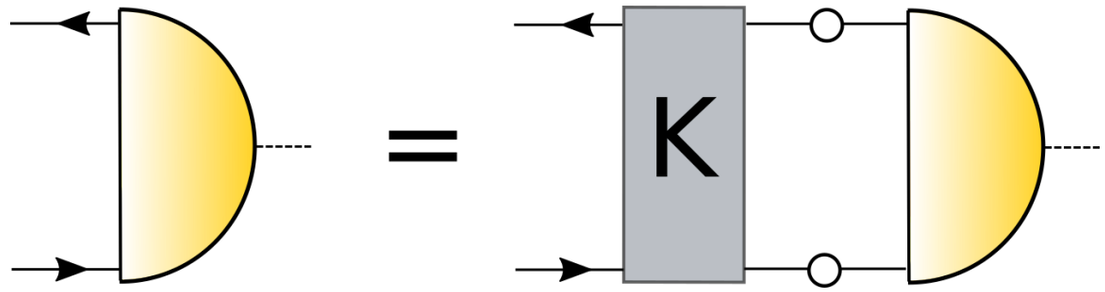

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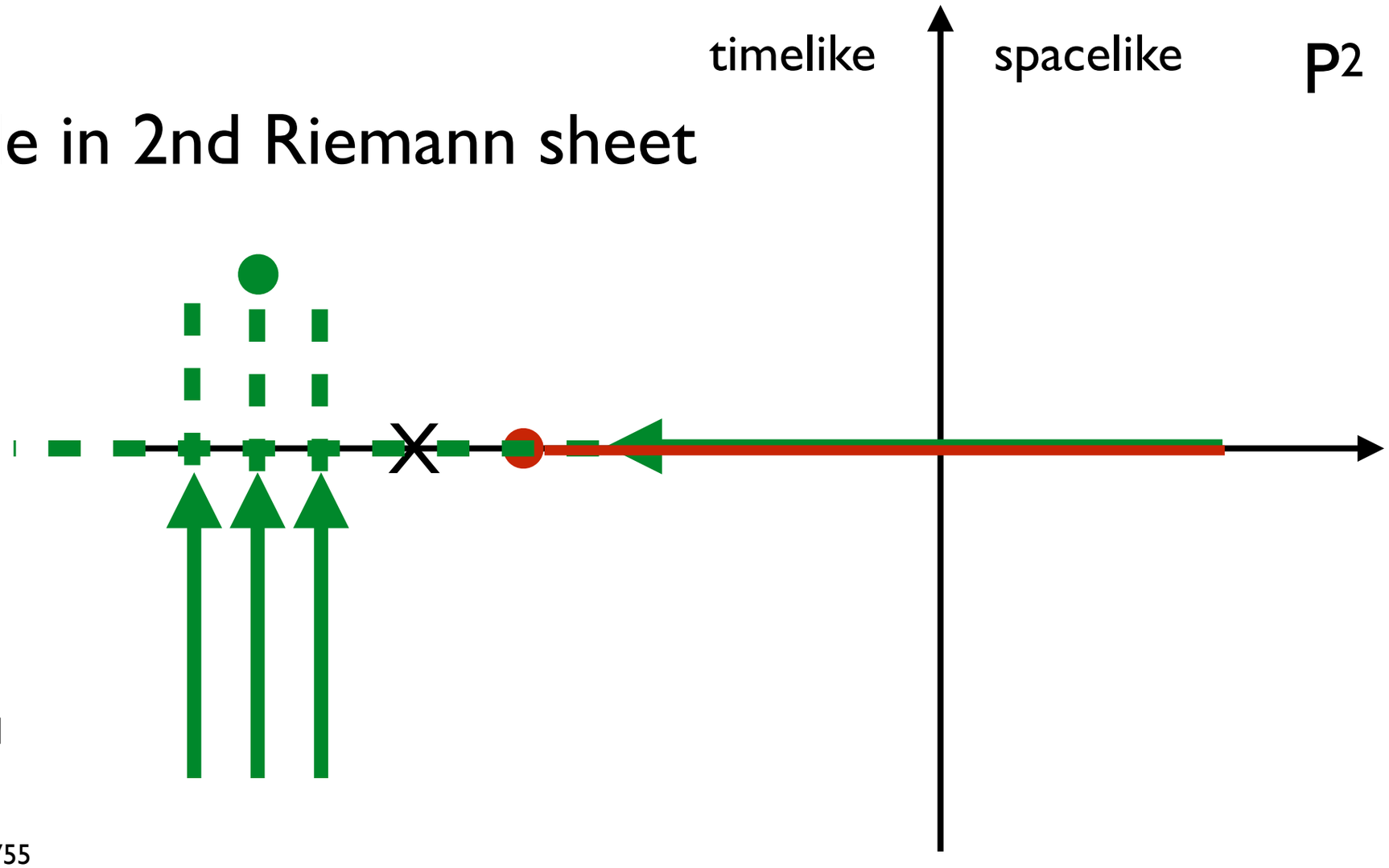

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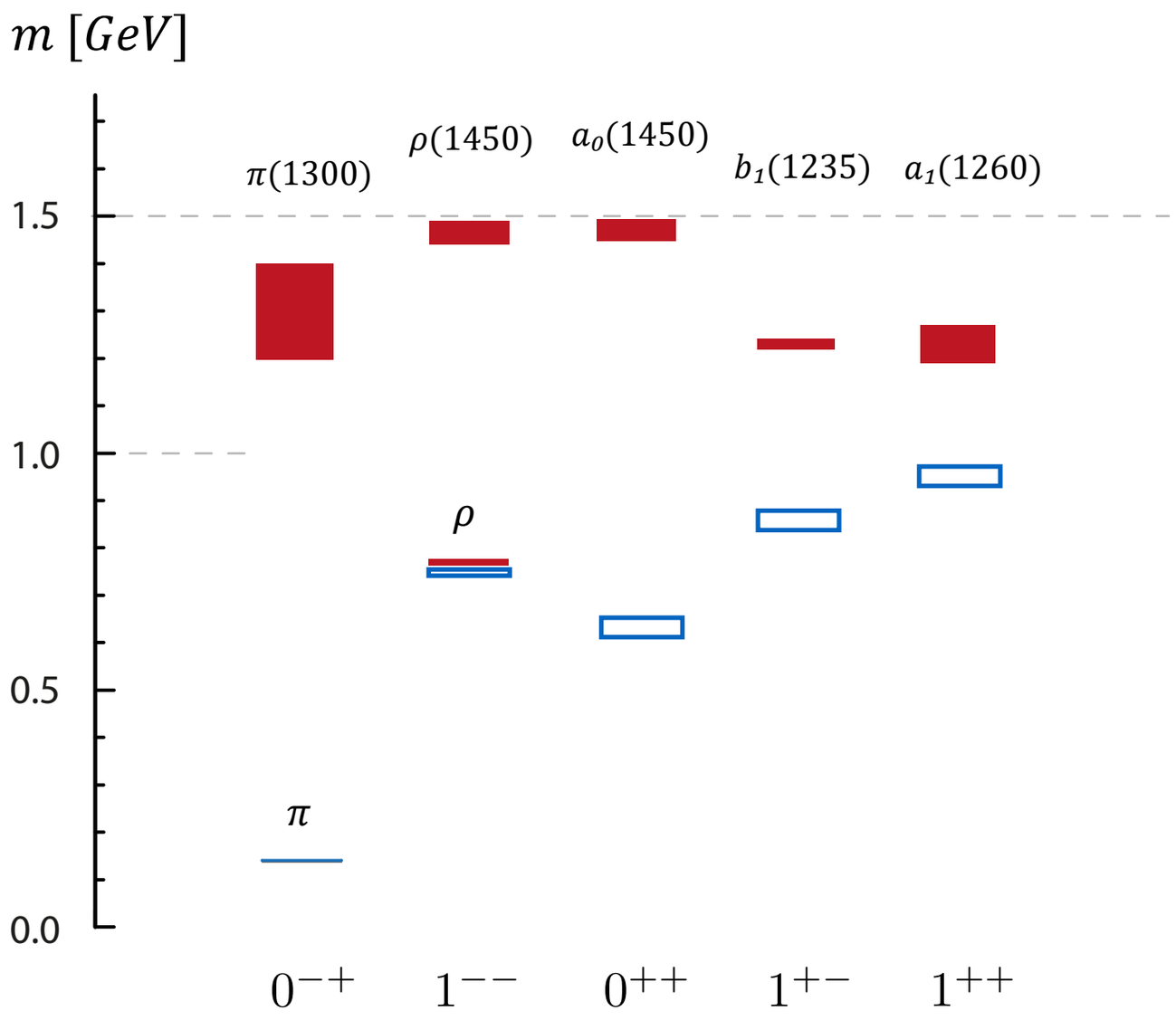
Light meson spectrum - RL vs full 3PI-calculation



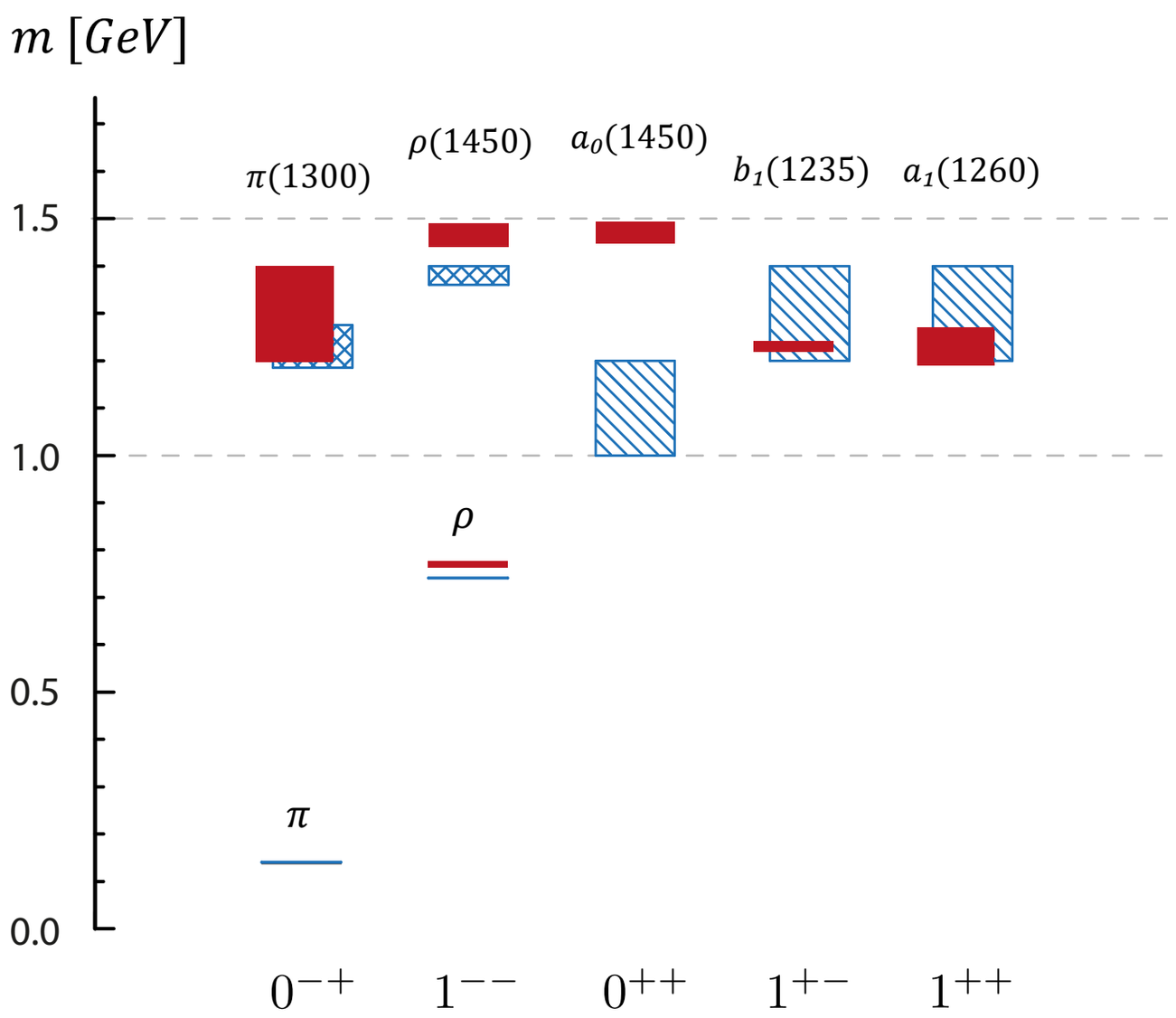
RL

CF, Kubrak, Williams, EPJA (2014) 126

PDG

Experiment

Light meson spectrum - RL vs full 3PI-calculation



Williams, CF, Heupel, PRD93 (2016) 034026

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- special channels:
  pseudoscalar 0^{-+} : (pseudo-) Goldstone bosons
  scalar 0^{++} : complicated channel…

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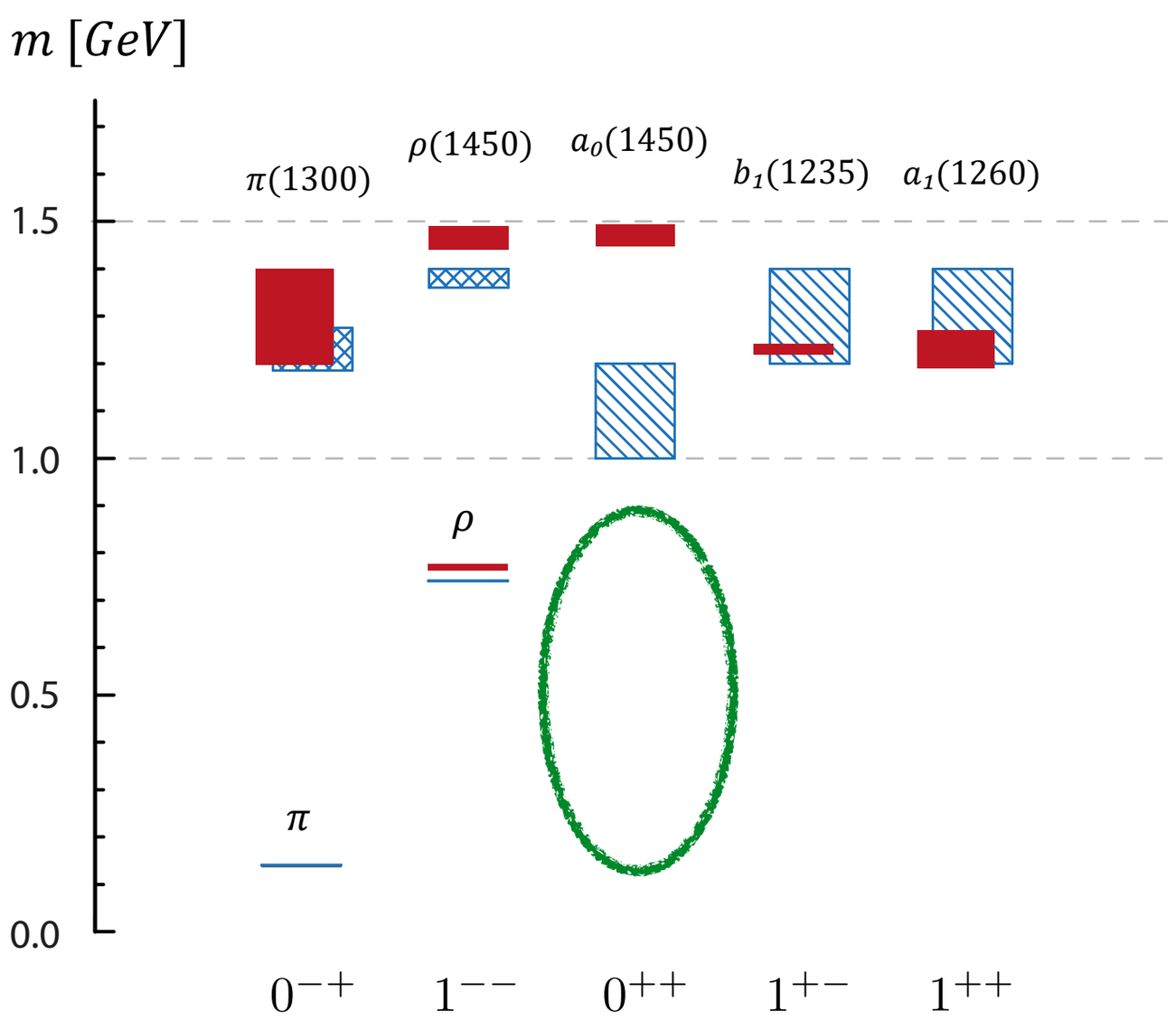

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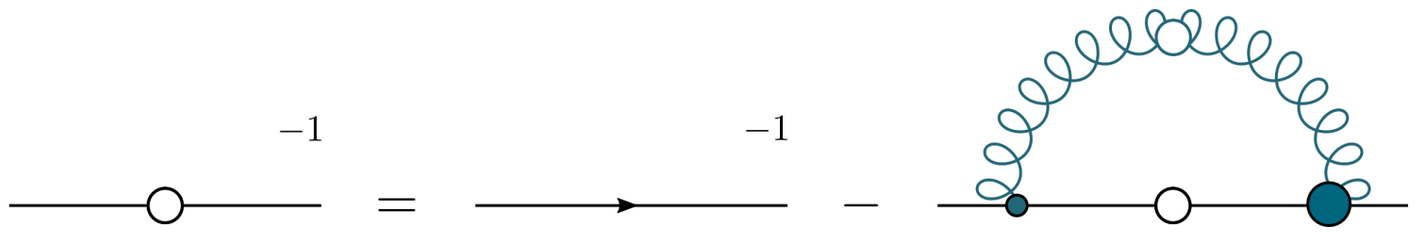

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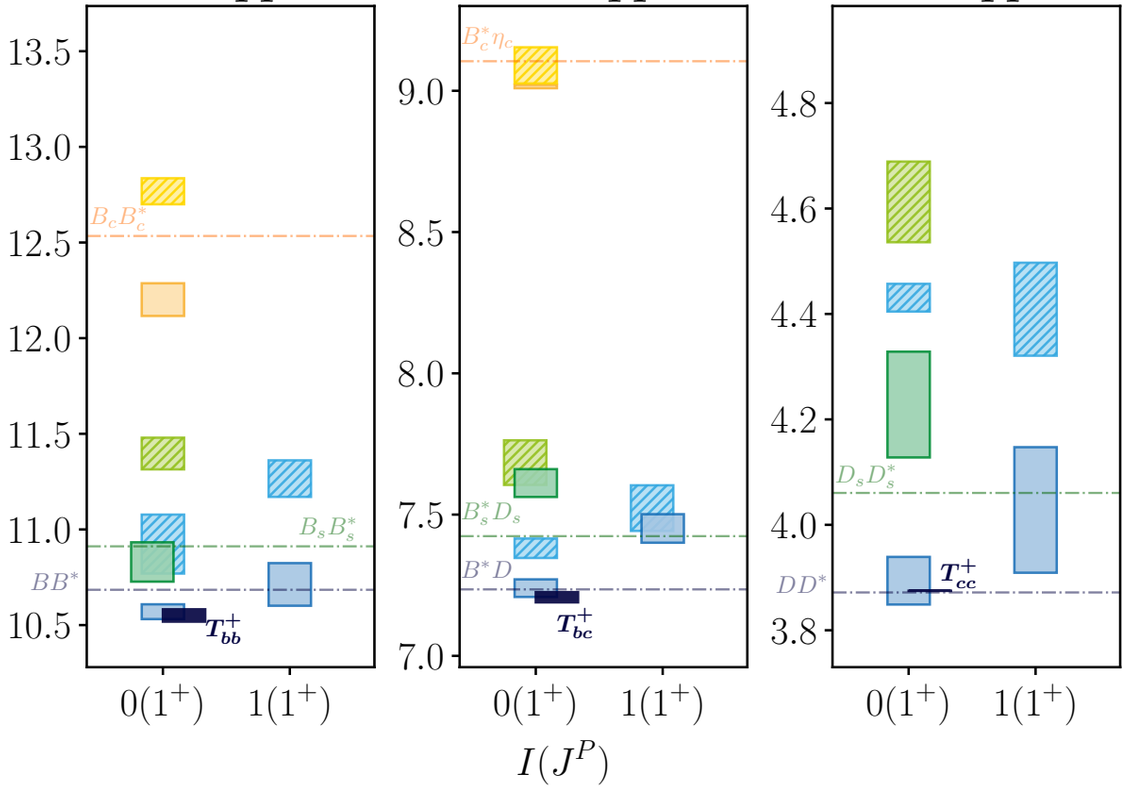
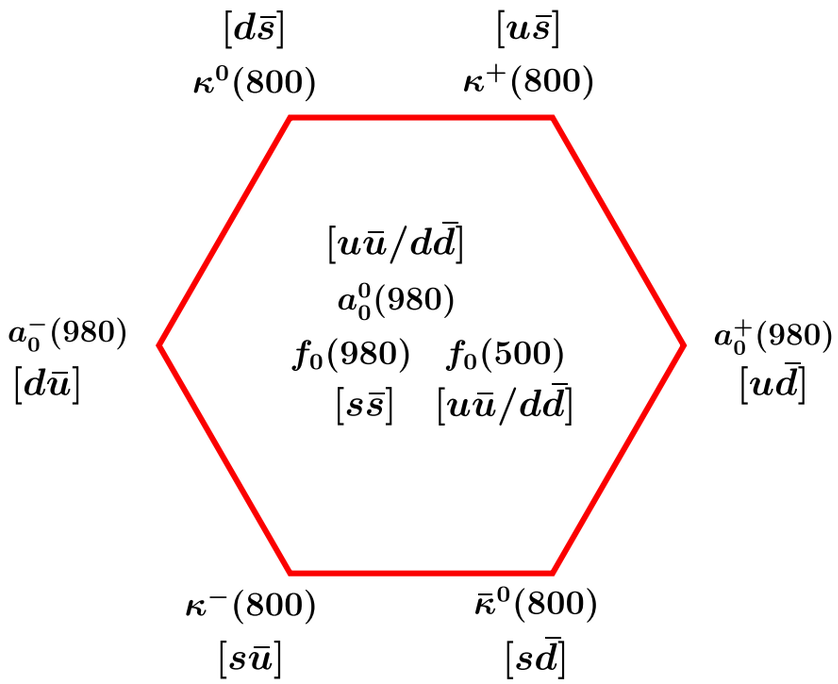


Light mesons with $\bar{q}\bar{q}qq$-content

Light scalar mesons:



wrong level ordering

R. L. Jaffe, Phys. Rev. D 15, 267 (1977)

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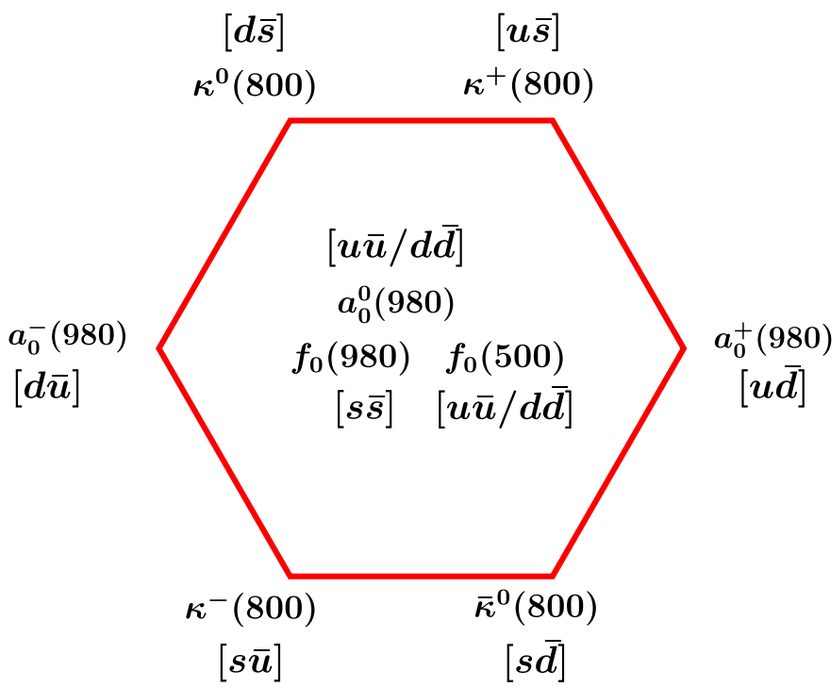

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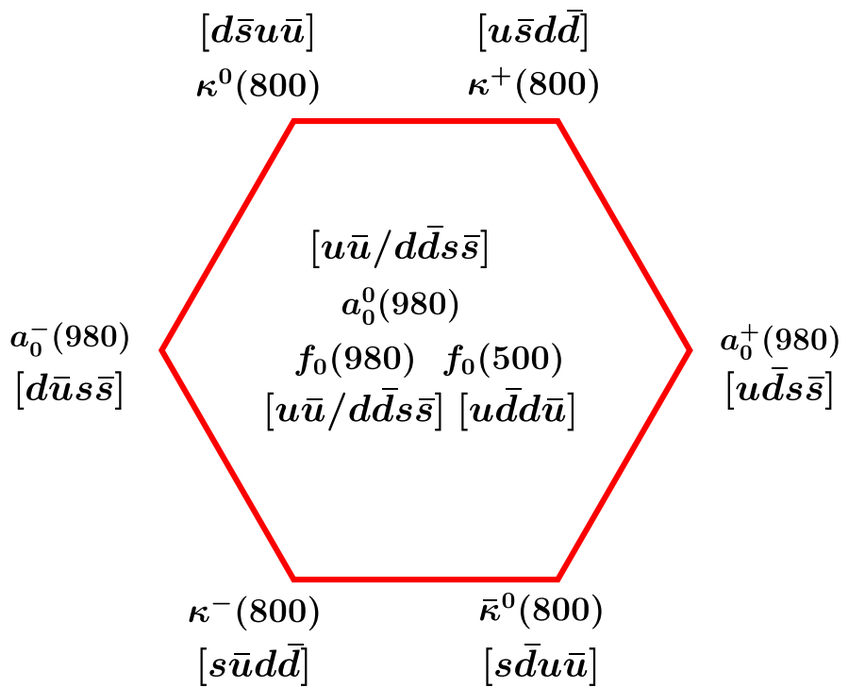

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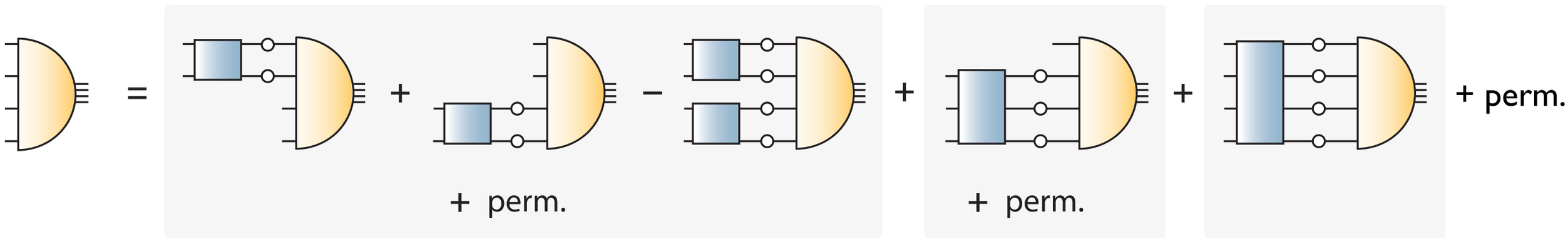
Tetraquarks from the four-body equation

Exact equation:

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)
Heupel, Eichmann, CF, PLB 718 (2012) 545-549
Eichmann, CF, Heupel, PLB 753 (2016) 282-287



Two-body interactions

Three- and four-body interactions

Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

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Heupel, Eichmann, CF, PLB 718 (2012) 545-549
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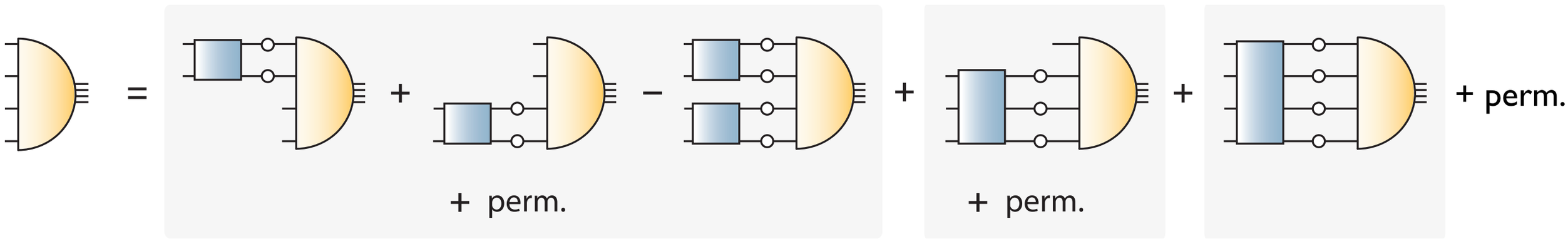
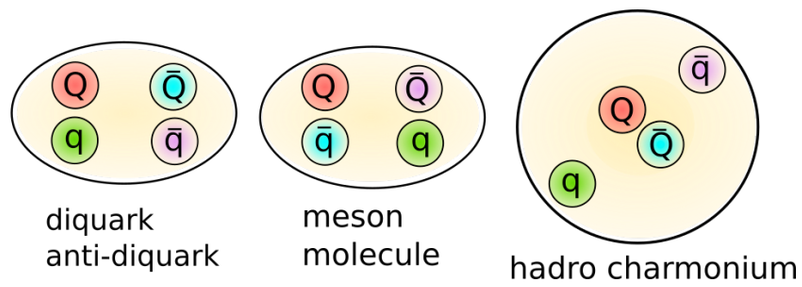
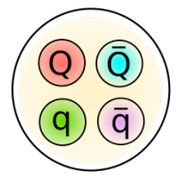


Two-body interactions

Three- and four-body interactions





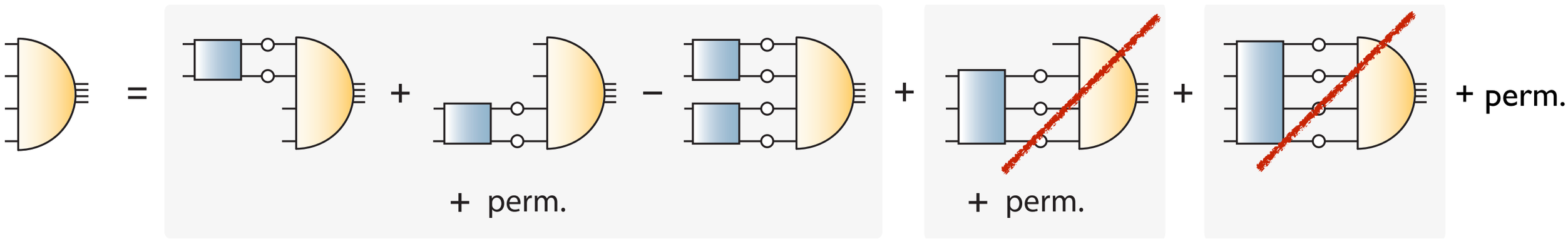
- Two-body interactions: allow for internal clustering
- use rainbow-ladder approximation…

Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

Tetraquarks from the four-body equation

Exact equation:

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)
Heupel, Eichmann, CF, PLB 718 (2012) 545-549
Eichmann, CF, Heupel, PLB 753 (2016) 282-287



Two-body interactions

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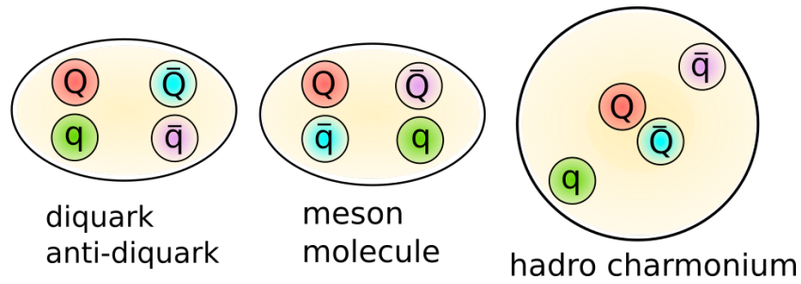

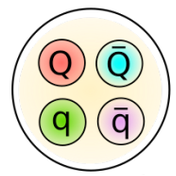

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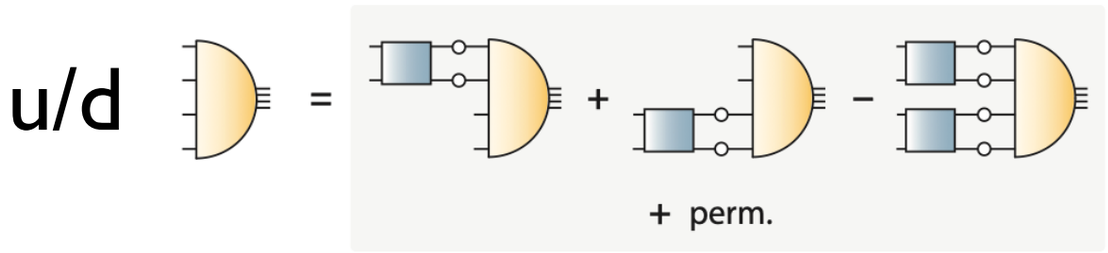
Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

Bound state vs resonance: scalar four-quark states



without twobody-clustering

0

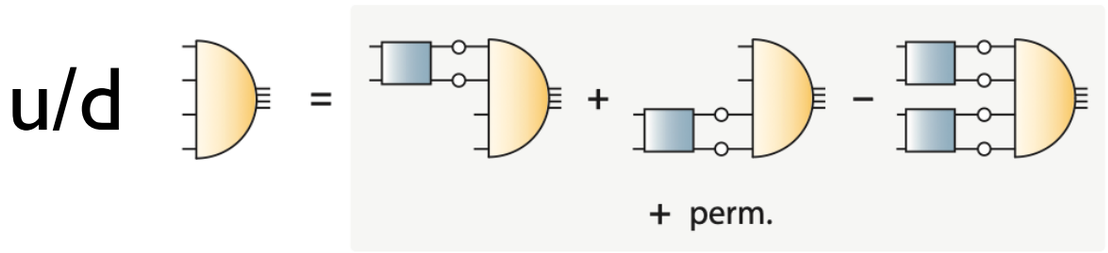
1200

$M_{\mathrm{Tetra}}[MeV]$

Bound state of four massive quarks

Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

Bound state vs resonance: scalar four-quark states



without twobody-clustering

0 300-400 1200 $M_{\mathrm{Tetra}}[MeV]$

with π-clustering

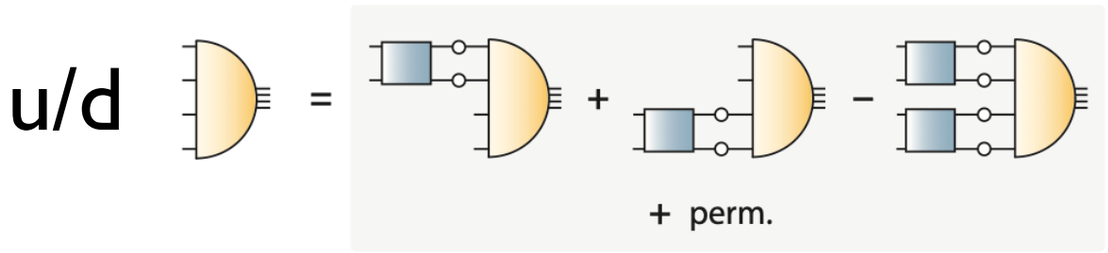
Two-pion resonance

Bound state of

four massive quarks

Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

Bound state vs resonance: scalar four-quark states



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Two-pion resonance

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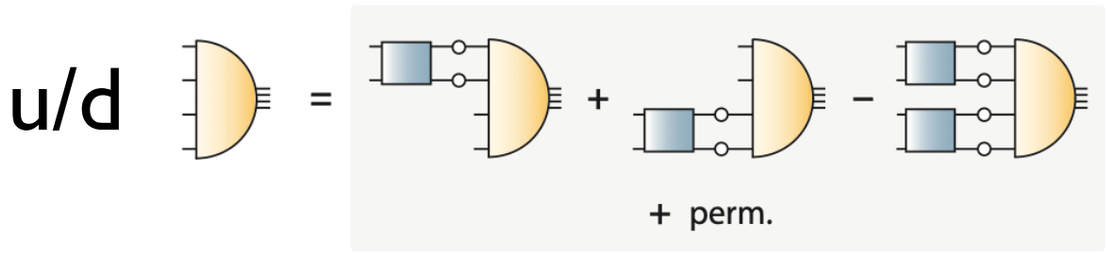
⟶ identify with $f_0(500)$ (‘σ-meson’)

Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

Bound state vs resonance: scalar four-quark states



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with strange quarks: $m(a_0, f_0) \approx 1GeV$

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Santowsky, CF, PRD 105 (2022) 4,313

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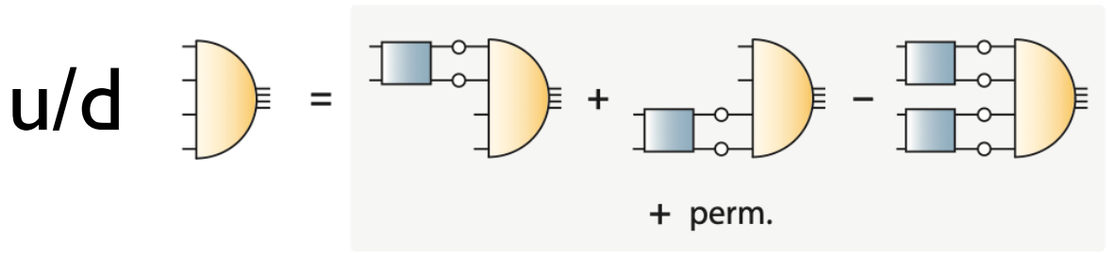

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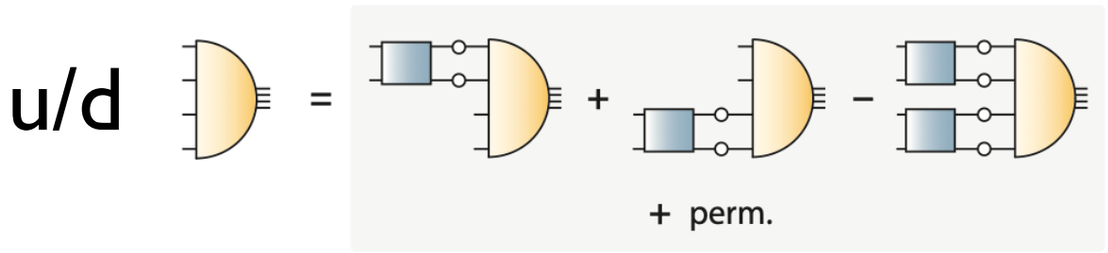
Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

Meson-meson components completely dominate over diquarks!

Bound state vs resonance: scalar four-quark states



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Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

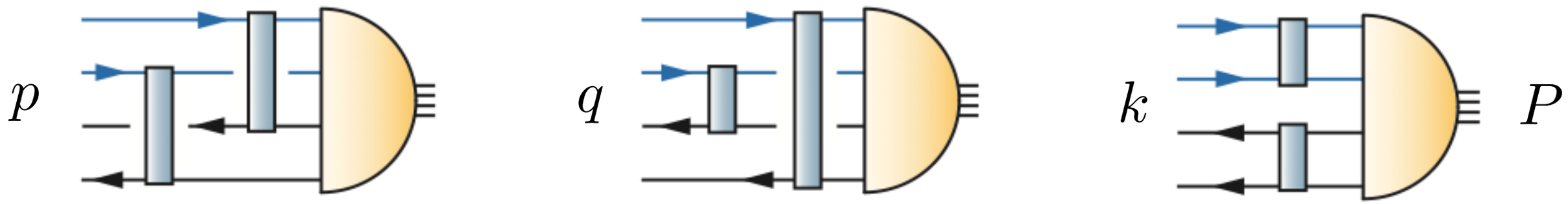
Meson-meson components completely dominate over diquarks!

Mixing with $q\bar{q}$: small effect

Santowsky, Eichmann, CF, Wallbott and Williams, PRD 102 (2020) no.5, 056014
Santowsky, CF, PRD 105 (2022) 4,313

Structure of the amplitude: open flavor

Vector tetraquark:



$$\Gamma(P,p,q,k) = \sum_i f_i(s_1,\cdots,s_9) \times \tau_i(P,p,q,k) \times color \times flavor$$

9 Lorentz scalars (built from P,p,q,k)

768 tensor structures (vector)

$3 \otimes \bar{3}, 6 \otimes \bar{6}$ or $1 \otimes 1, 8 \otimes 8$

- reduce # tensor structures guided by physics

Structure of the amplitude: open flavor

Vector

p

$\Gamma(P, p$

P

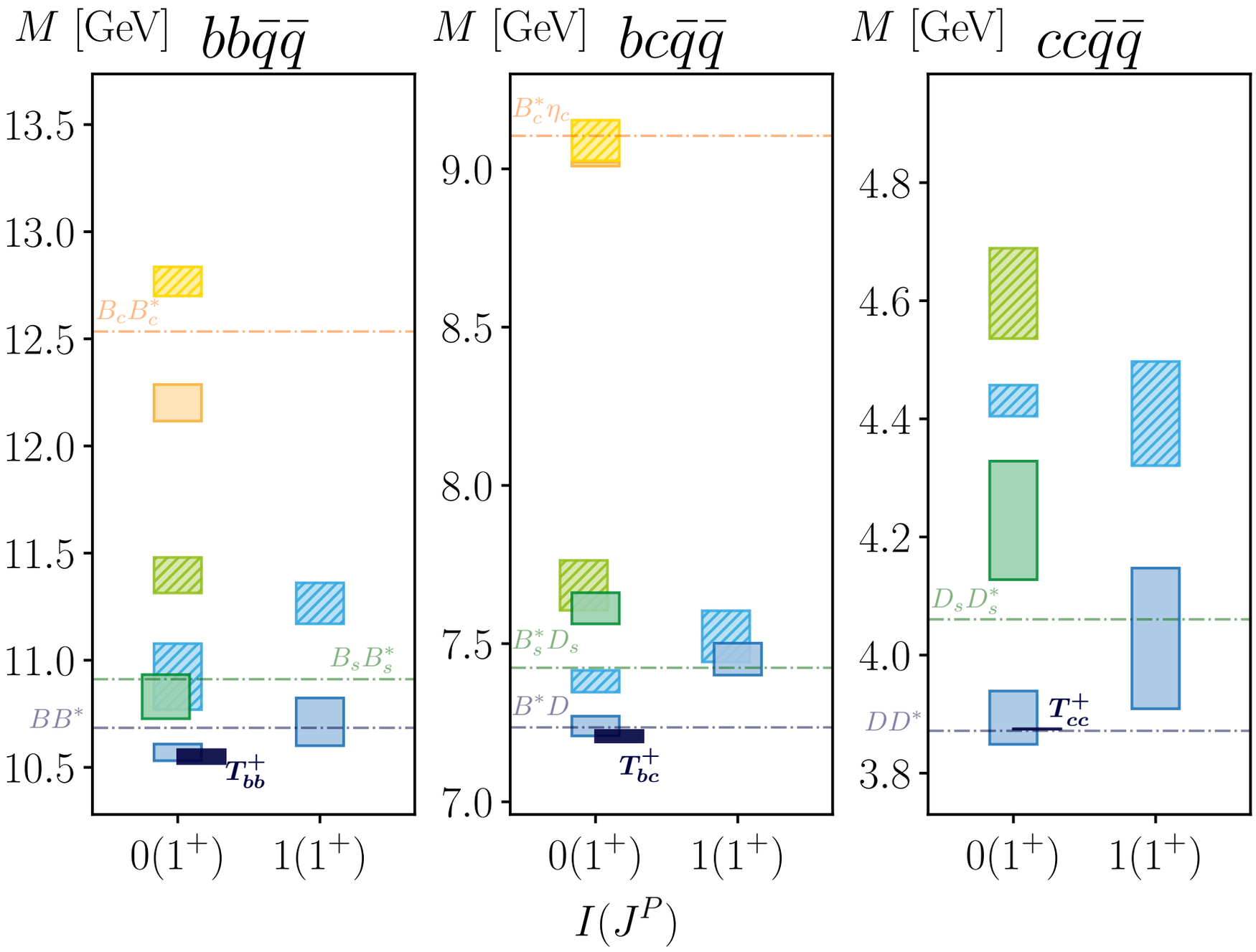
$avor$

$n \in \{u, d\}$

$I(J^P)$		Physical components					
		$\mathbf{1}\otimes\mathbf{1}$		$\bar{\mathbf{3}}\otimes\mathbf{3}$	$\mathbf{8}\otimes\mathbf{8}$		$\mathbf{6}\otimes\bar{\mathbf{6}}$
		f_0	f_1	f_2	f_3	f_4	f_5
$0(1^+)$	$bb\bar{n}\bar{n}$	BB^*	B^*B^*	$A_{bb}S$	BB^*	B^*B^*	$S_{bb}A$
	$bc\bar{n}\bar{n}$	BD^*	B^*D	$A_{bc}S$	BD^*	B^*D	$S_{bc}A$
	$cc\bar{n}\bar{n}$	DD^*	D^*D^*	$A_{cc}S$	DD^*	D^*D^*	$S_{cc}A$
	$bb\bar{s}\bar{s}$	$B_sB_s^*$	–	$A_{bb}A_{ss}$	$B_sB_s^*$	–	–
	$bc\bar{s}\bar{s}$	$B_sD_s^*$	$B_s^*D_s$	$S_{bc}A_{ss}$	$B_sD_s^*$	$B_s^*D_s^*$	$A_{bc}S_{ss}$
	$cc\bar{s}\bar{s}$	$D_sD_s^*$	–	$A_{cc}A_{ss}$	$D_sD_s^*$	–	–
$1(1^+)$	$bb\bar{q}\bar{q}$	BB^*	–	$A_{bb}A$	BB^*	–	–
	$bc\bar{q}\bar{q}$	BD^*	B^*D	$S_{bc}A$	BD^*	B^*D^*	$A_{bc}S$
	$cc\bar{q}\bar{q}$	DD^*	–	$A_{cc}A$	DD^*	–	–

- red

bb, bc and cc four-quark-states



$n \in \{u, d\}$

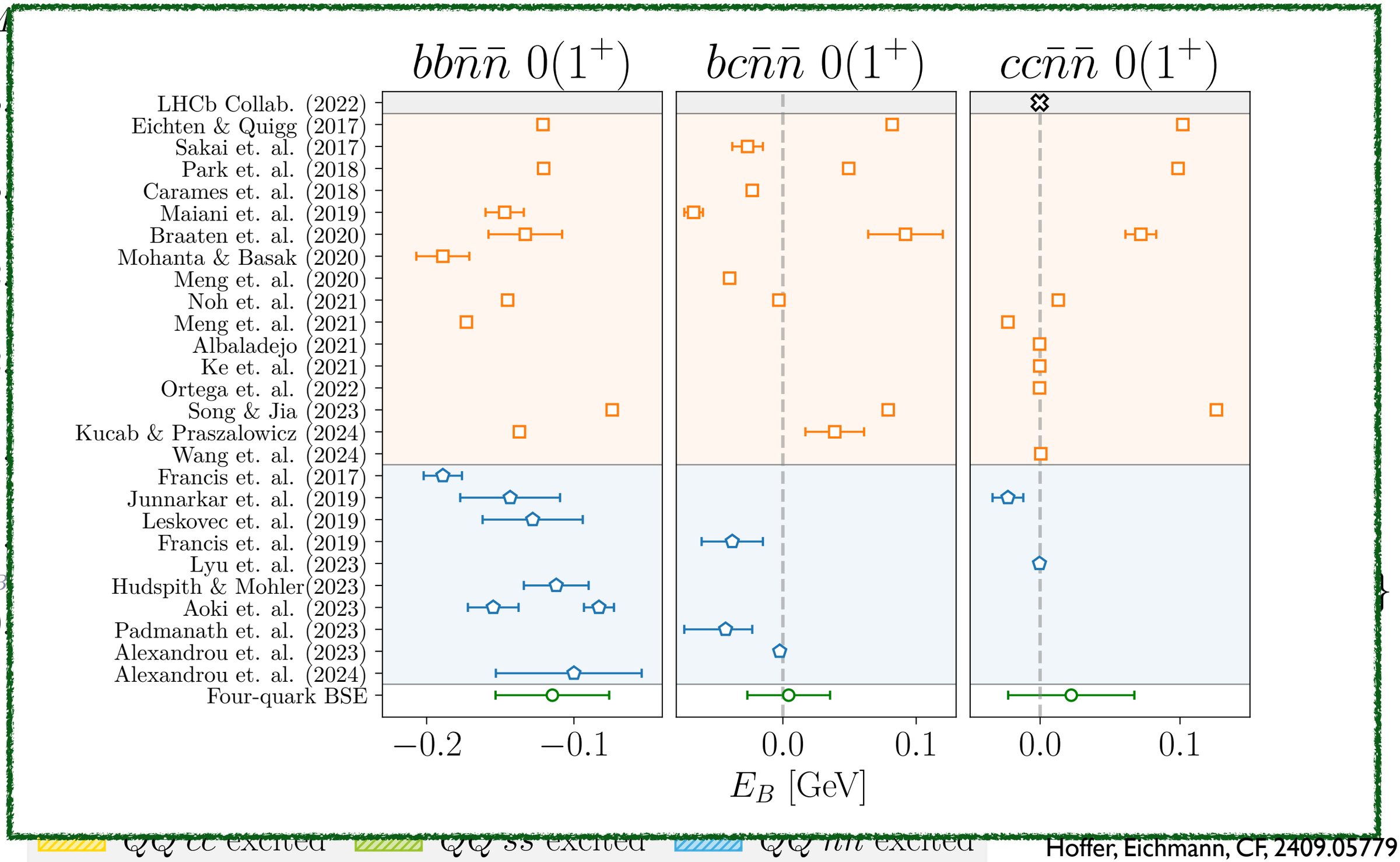
$QQ'\bar{c}\bar{c}$ excited $QQ'\bar{s}\bar{s}$ excited $QQ'\bar{n}\bar{n}$ excited

$QQ'\bar{c}\bar{c}$ ground $QQ'\bar{s}\bar{s}$ ground $QQ'\bar{n}\bar{n}$ ground

Hoffer, Eichmann, CF, 2409.05779

Lattice:
Francis, PPNP 140 104143 (2025)

bb, bc and cc four-quark-states



Hoffer, Eichmann, CF, 2409.05779

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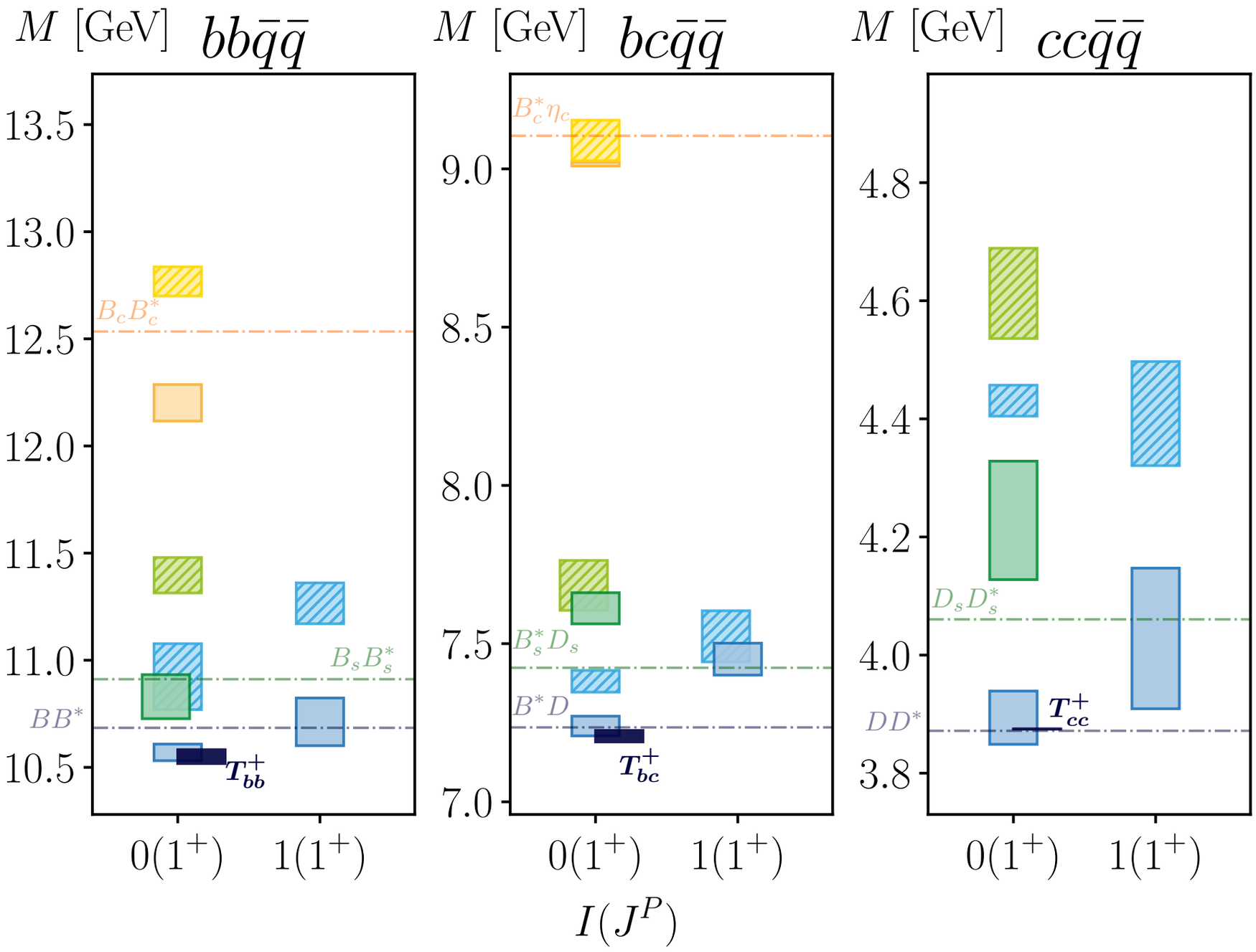

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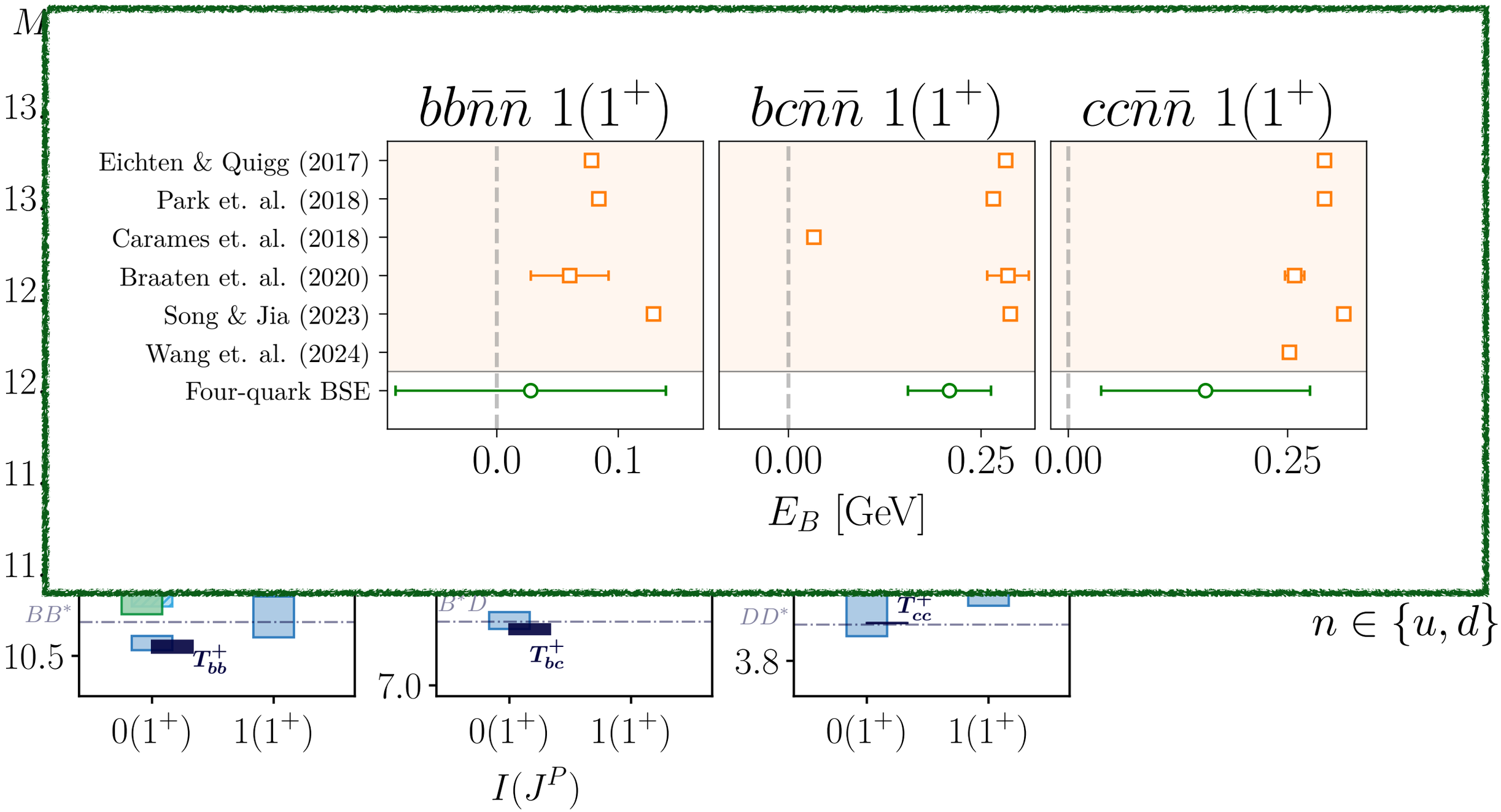

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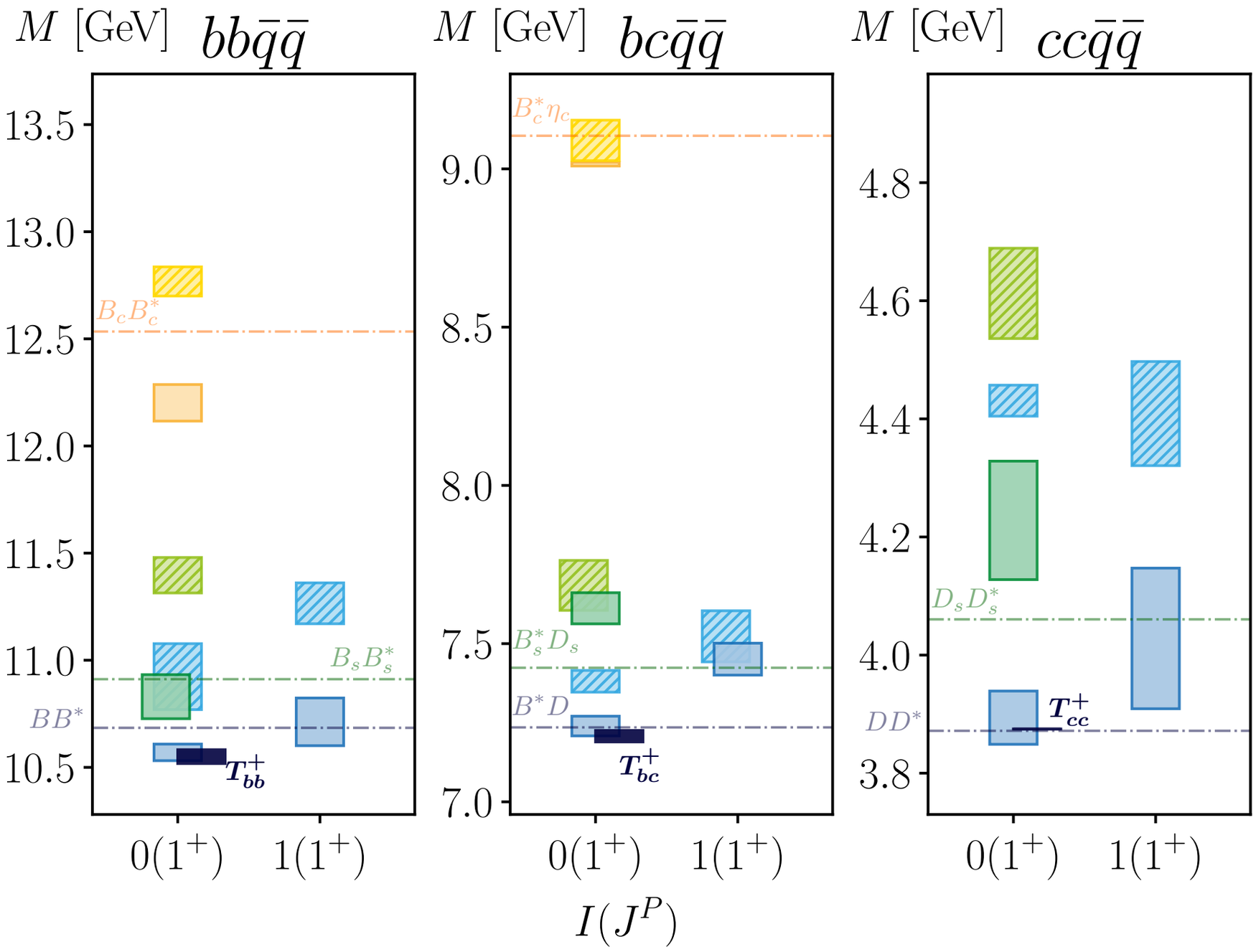

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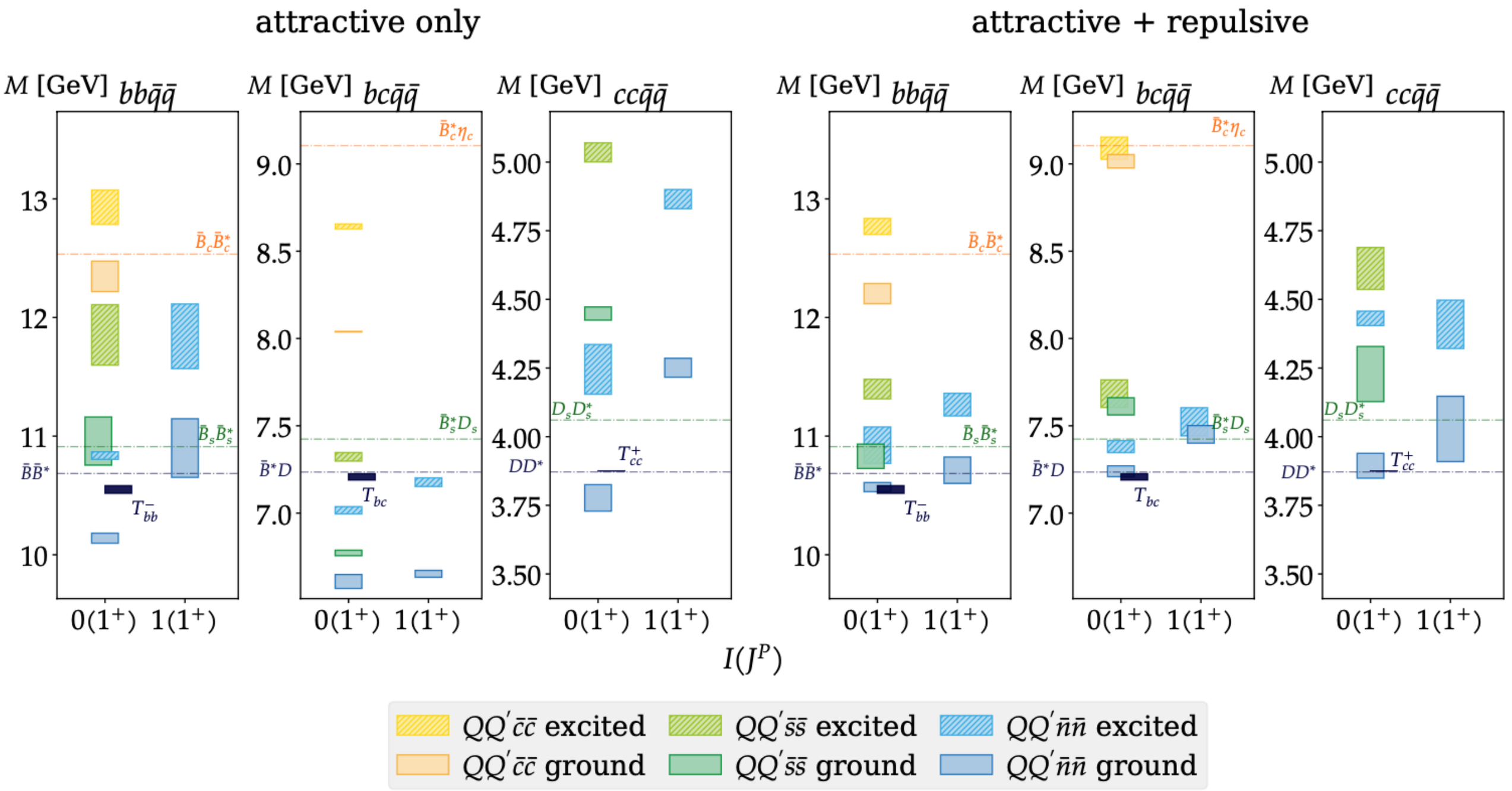
Hoffer, Eichmann, CF, 2409.05779

Lattice:
Francis, PPNP 140 104143 (2025)

Effect of repulsive forces



Hoffer, Eichmann, CF, 2409.05779

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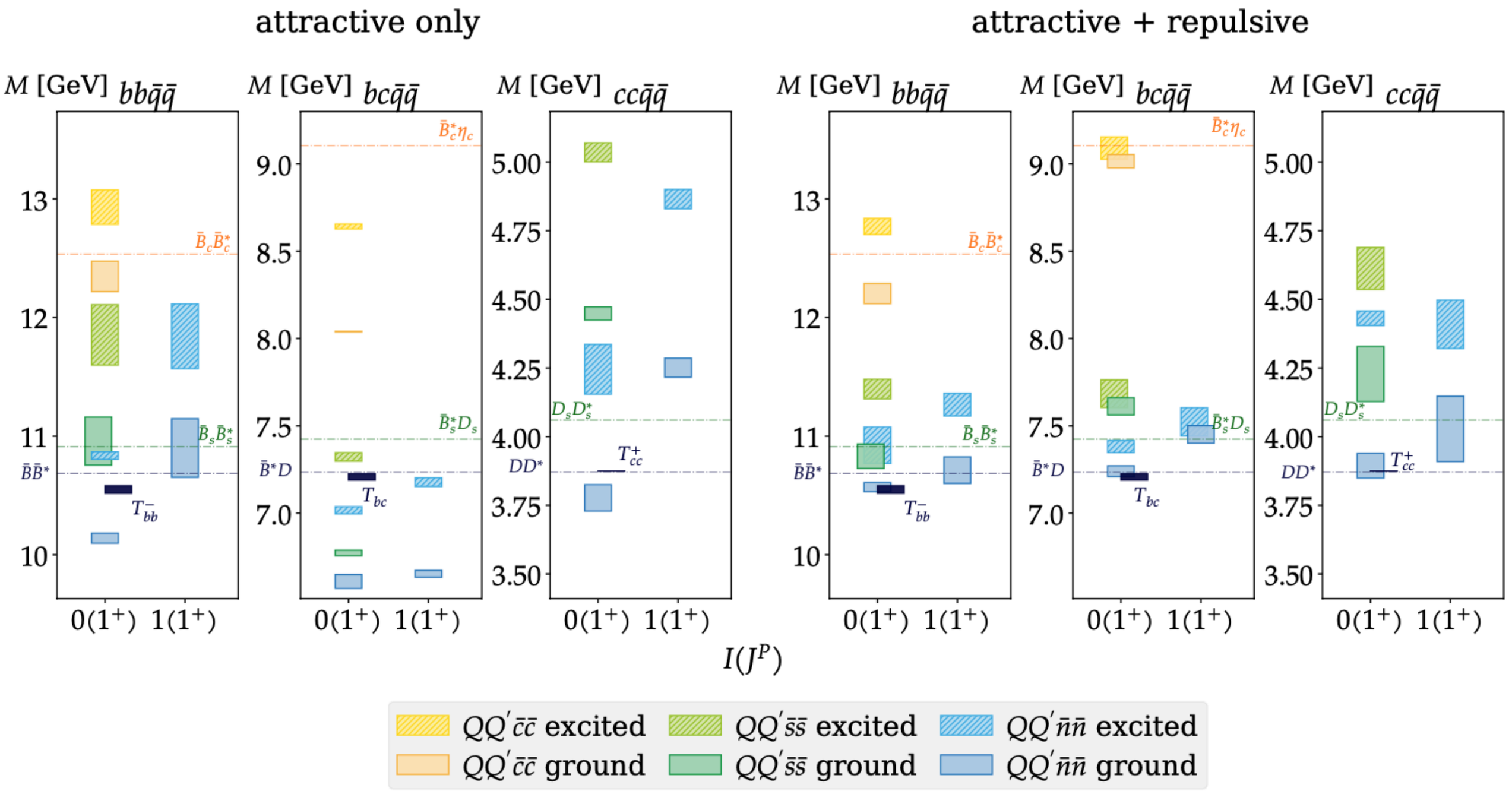
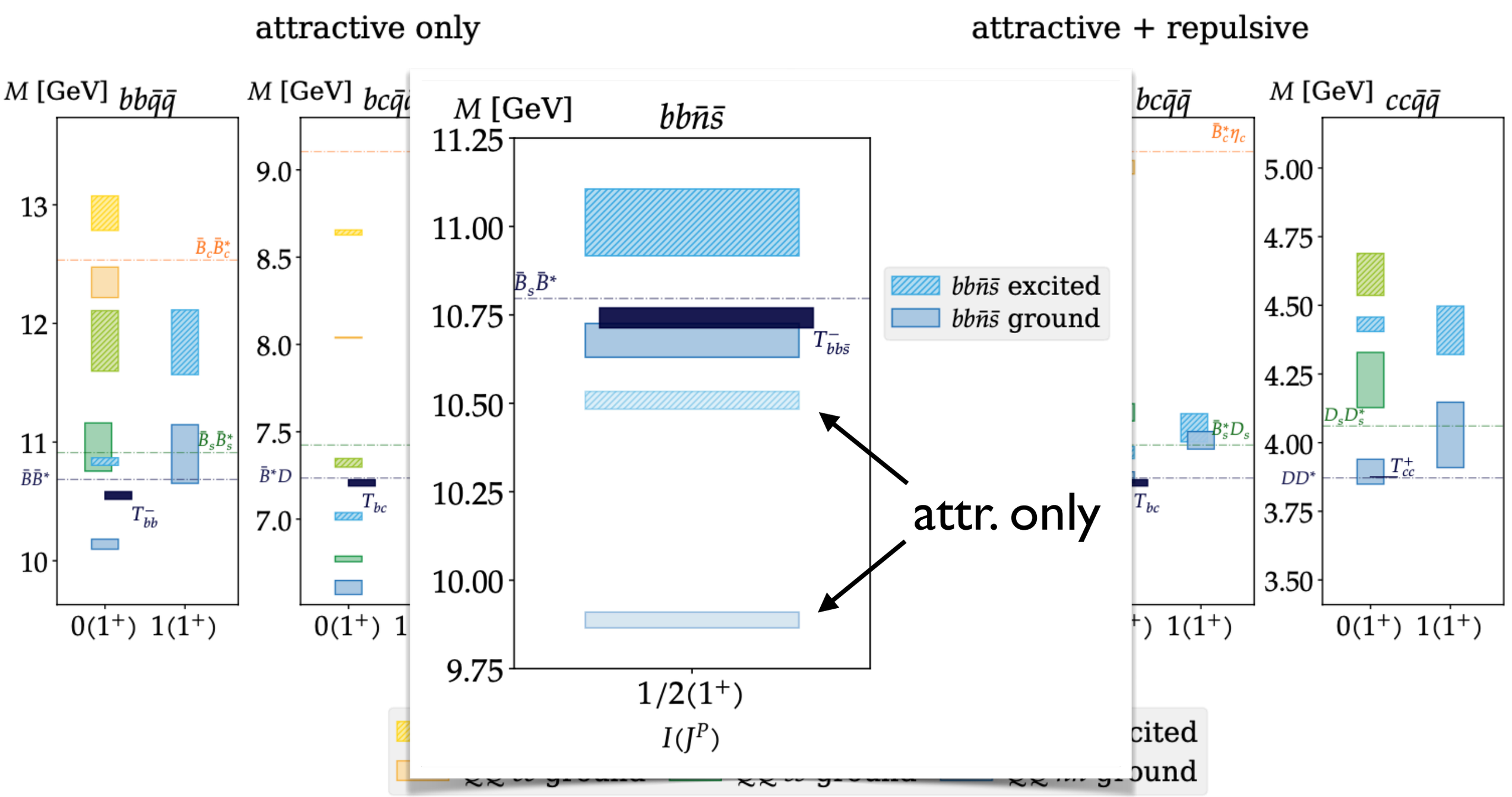


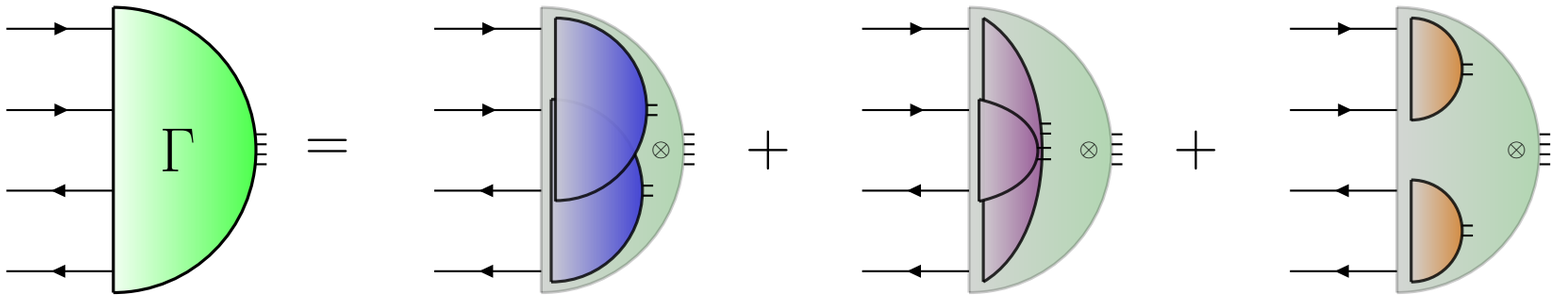
Hoffer, Eichmann, CF, 2409.05779
Hoffer, Eichmann, CF, in preparation

Effect of repulsive forces



Hoffer, Eichmann, CF, 2409.05779
Hoffer, Eichmann, CF, in preparation

Identifying leading structures…

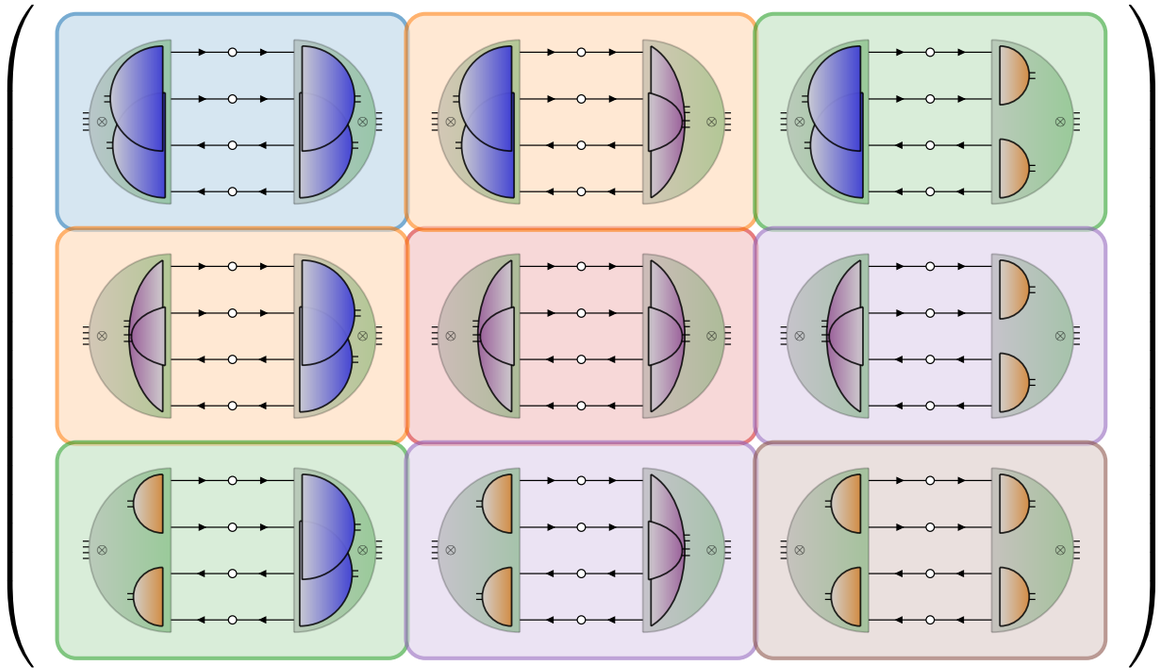


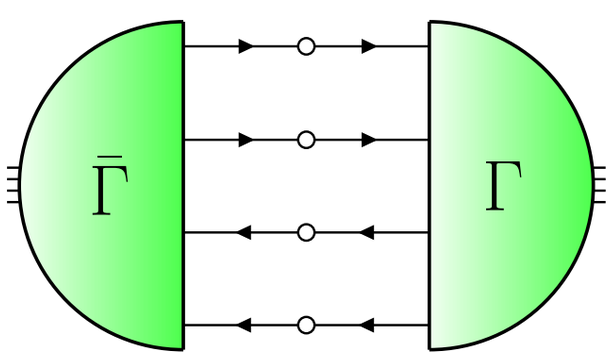
meson-meson

hadro-onium

diquark-antidiquark

- norm contributions

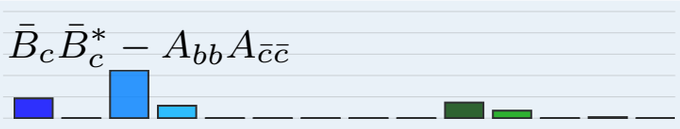




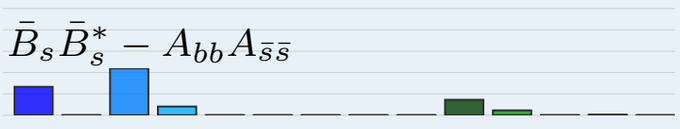
Internal structure

$n \in \{u, d\}$

$bb\bar{c}\bar{c}$

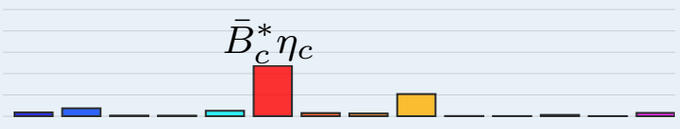


$bb\bar{s}\bar{s}$



$bb\bar{n}\bar{n}$

$bc\bar{c}\bar{c}$



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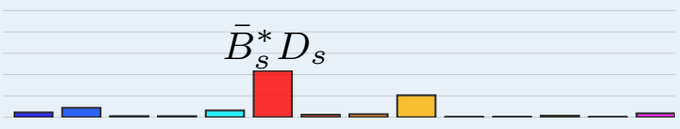

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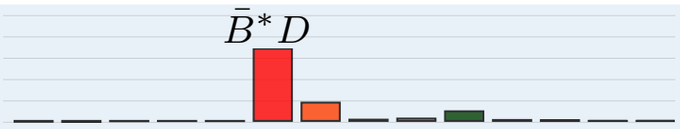

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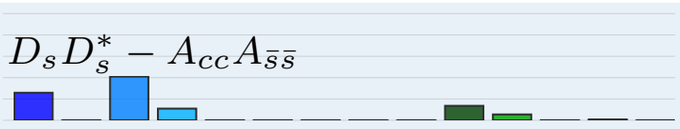

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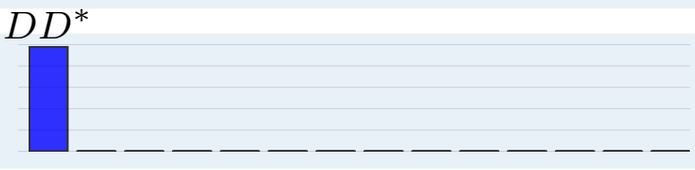

$0(1^+)$

$1(1^+)$

- decided dynamically !
- flavour and spin dependent…prediction for bc

Hoffer, Eichmann, CF, 2409.05779

Internal structure

$n \in \{u, d\}$

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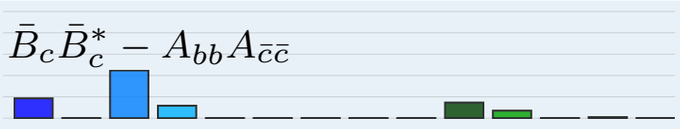

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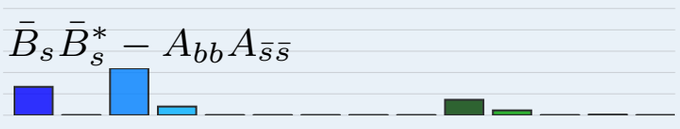

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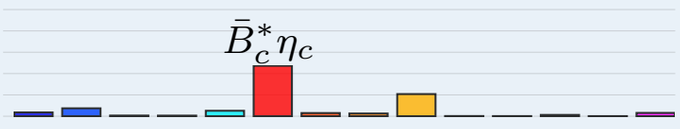

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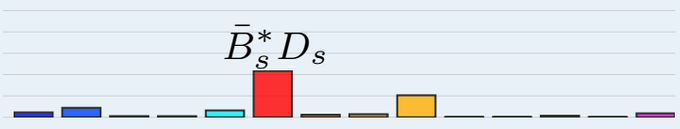

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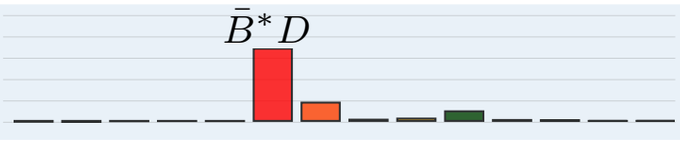
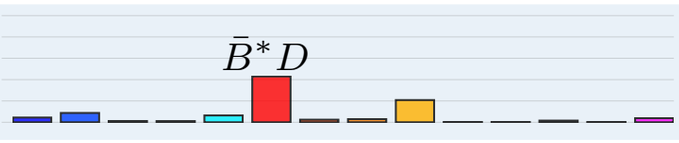
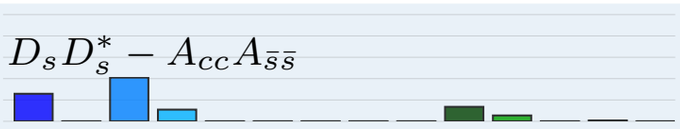
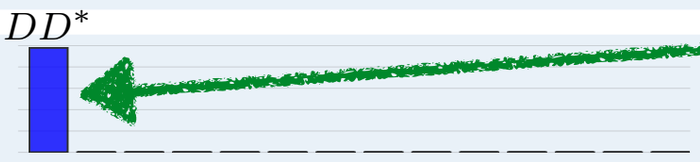


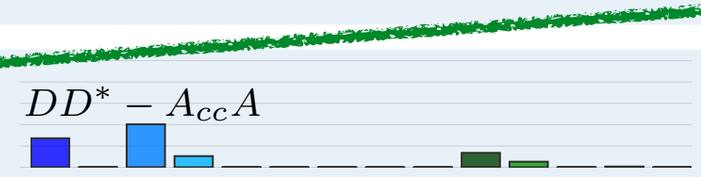


$cc\bar{s}\bar{s}$



$cc\bar{n}\bar{n}$





T_{cc}^+

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Hoffer, Eichmann, CF, 2409.05779

Internal structure

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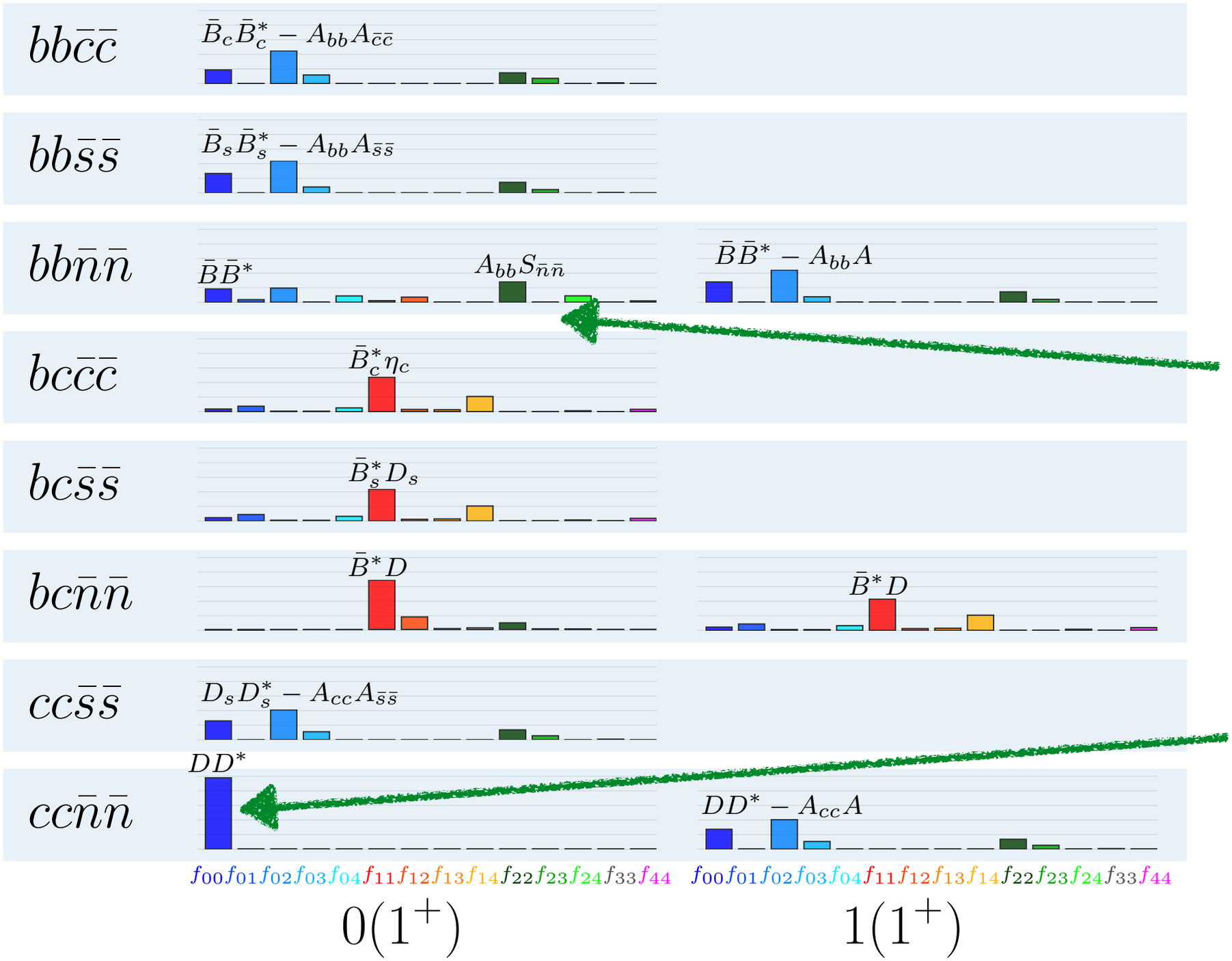


also seen on the lattice

Bicudo et al, PRD D103 (2021)

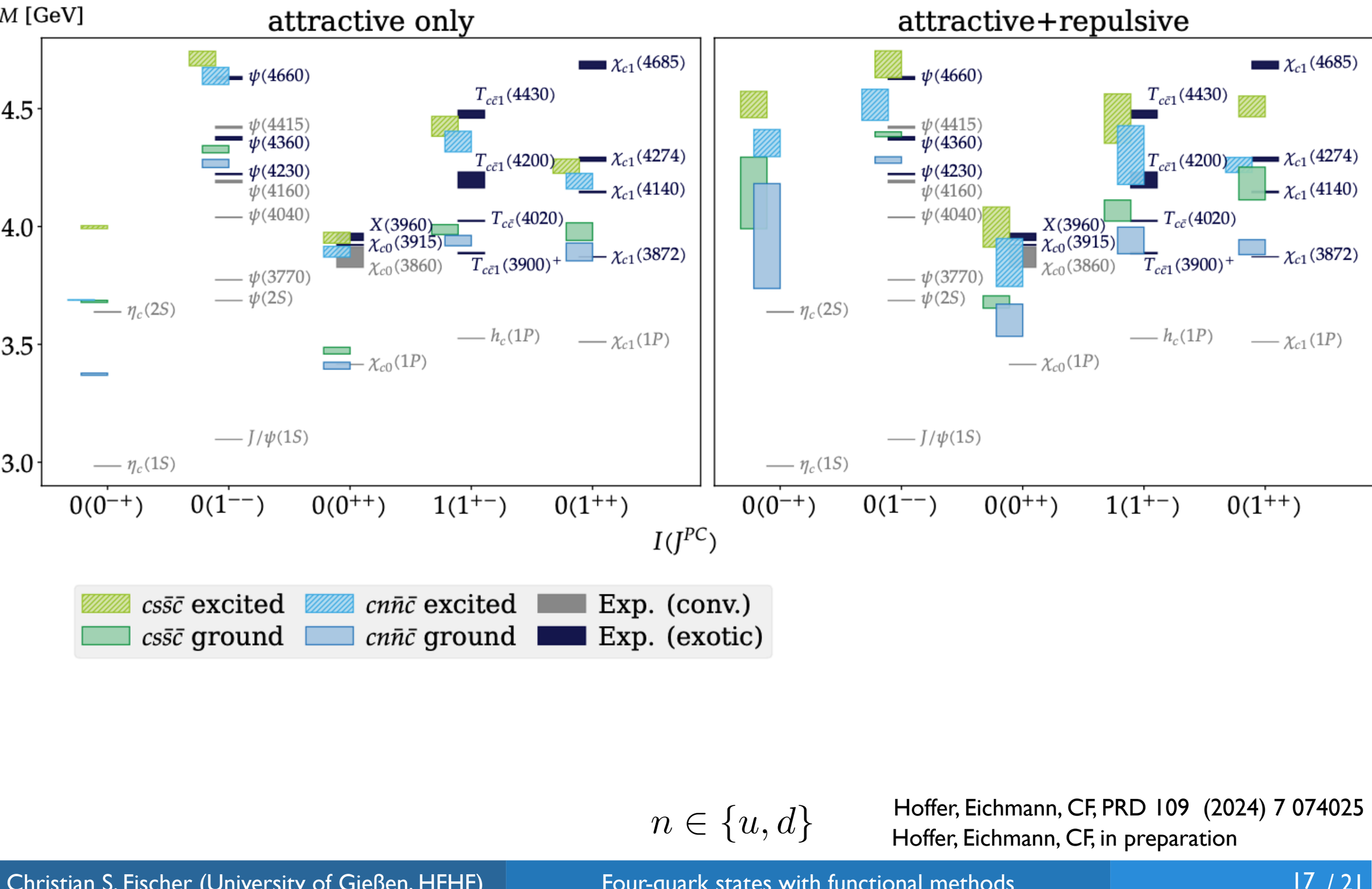
Review:
Francis, PPNP 140 (2025) 104143

T_{cc}^+

- decided dynamically !
- flavour and spin dependent…prediction for bc

Hoffer, Eichmann, CF, 2409.05779

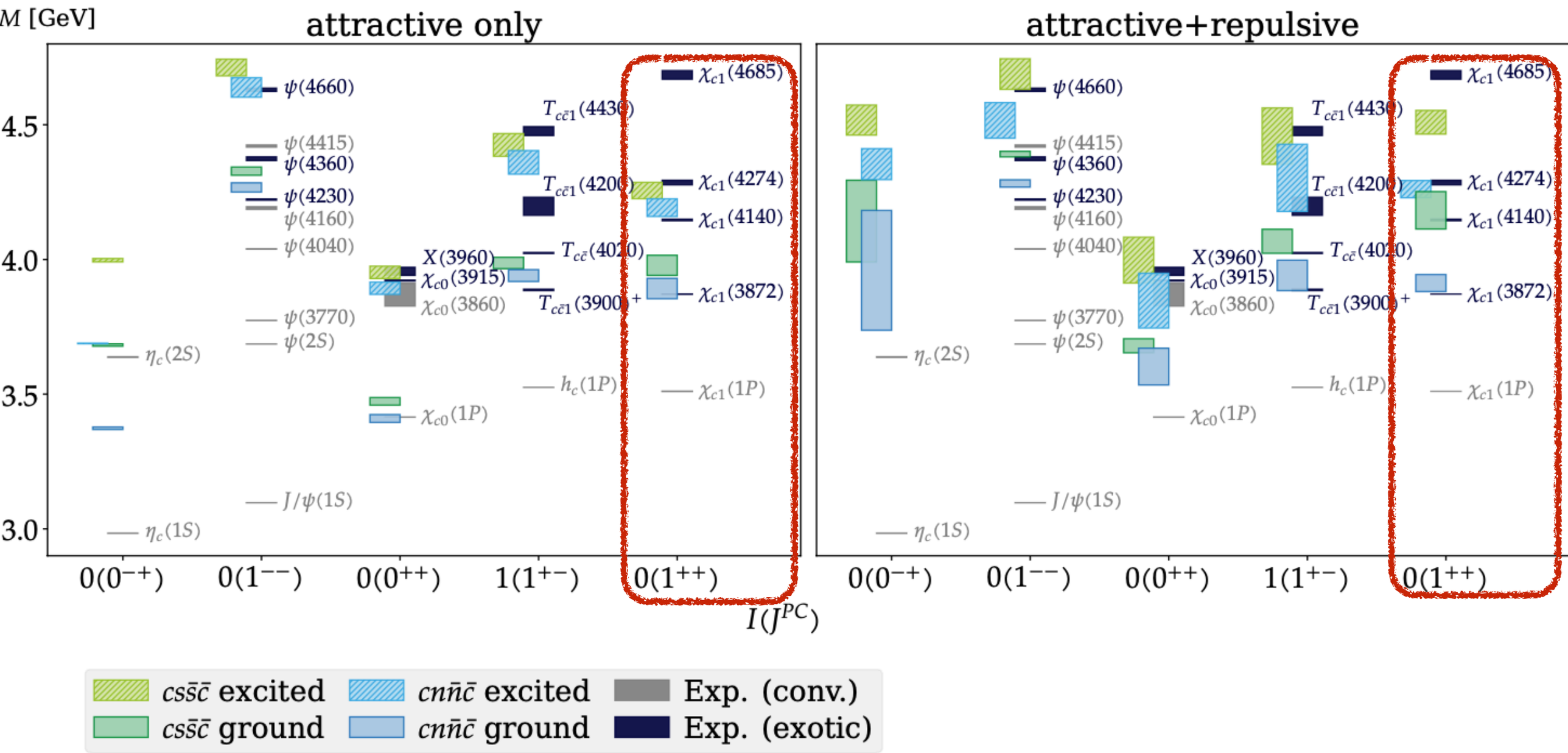
Hidden flavour four-quark states with charm



$n \in \{u, d\}$

Hoffer, Eichmann, CF, PRD 109 (2024) 7 074025
Hoffer, Eichmann, CF, in preparation

Hidden flavour four-quark states with charm



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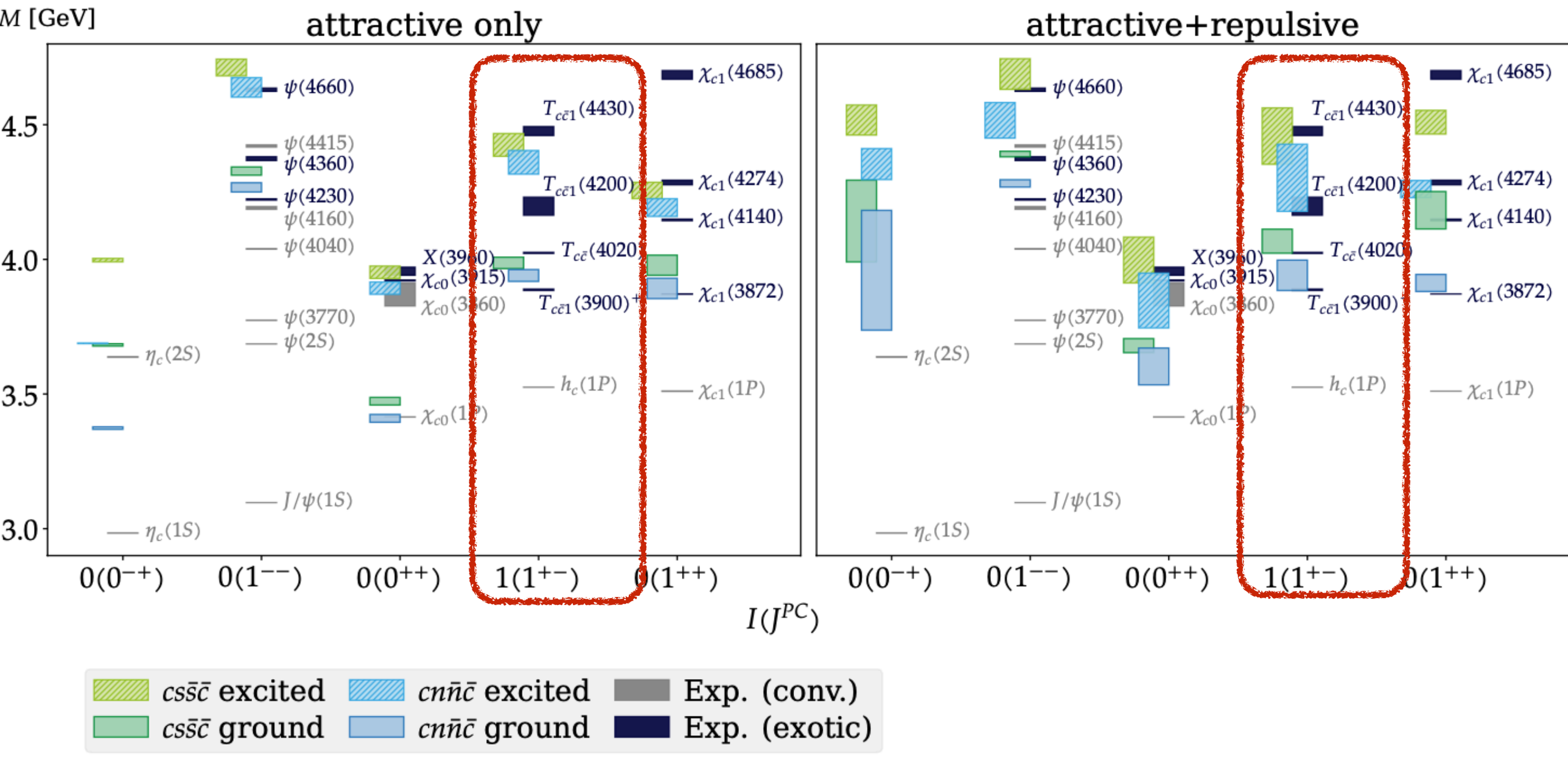

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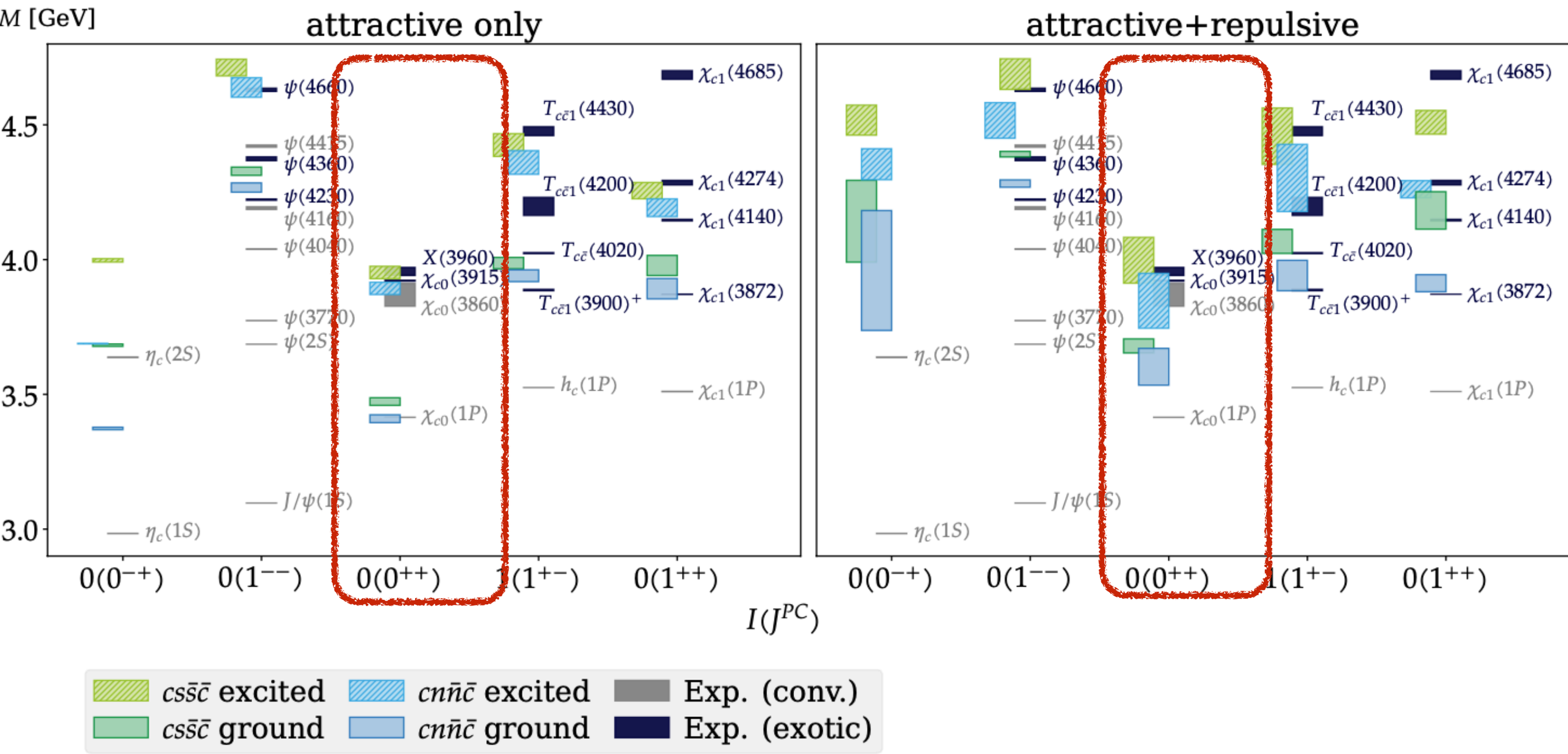

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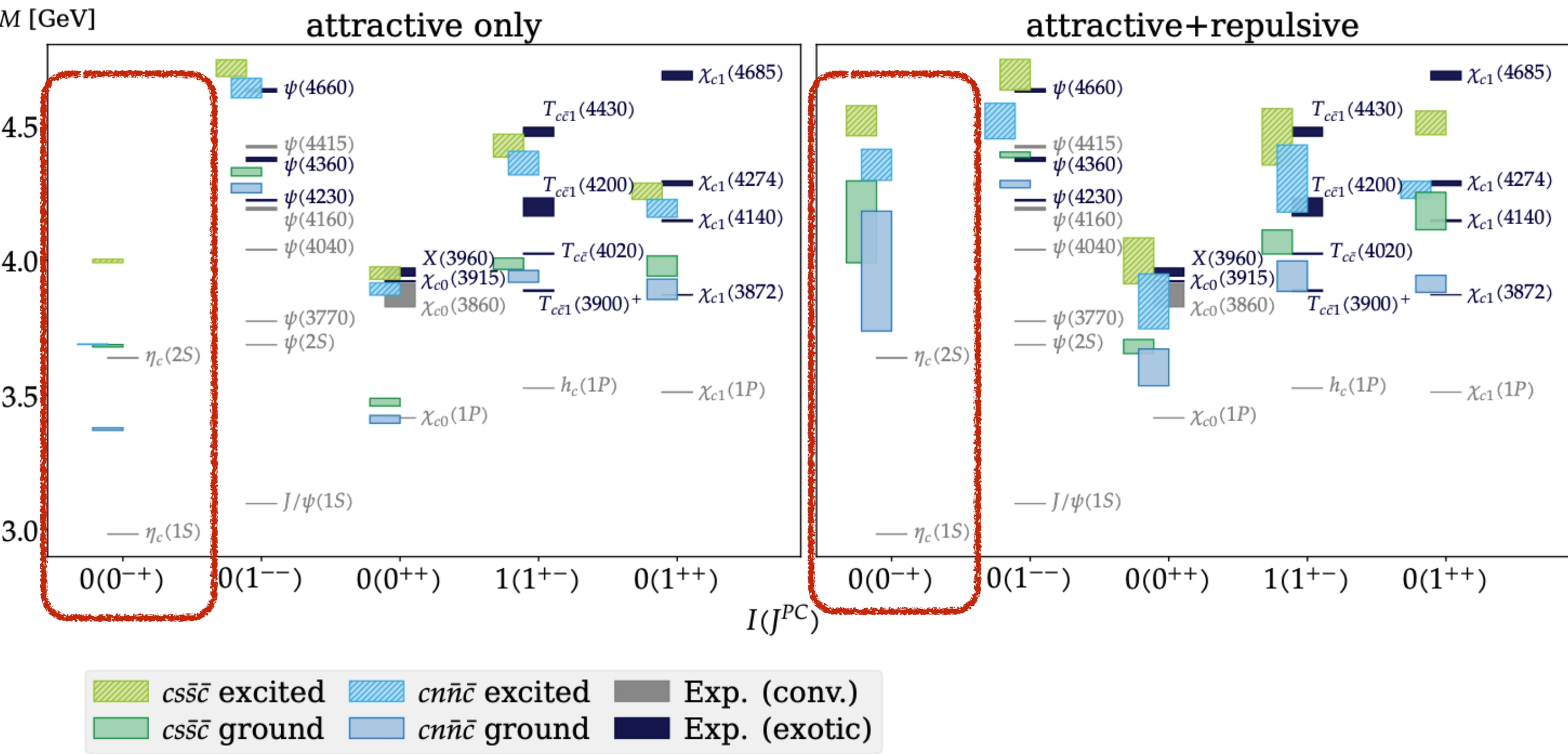

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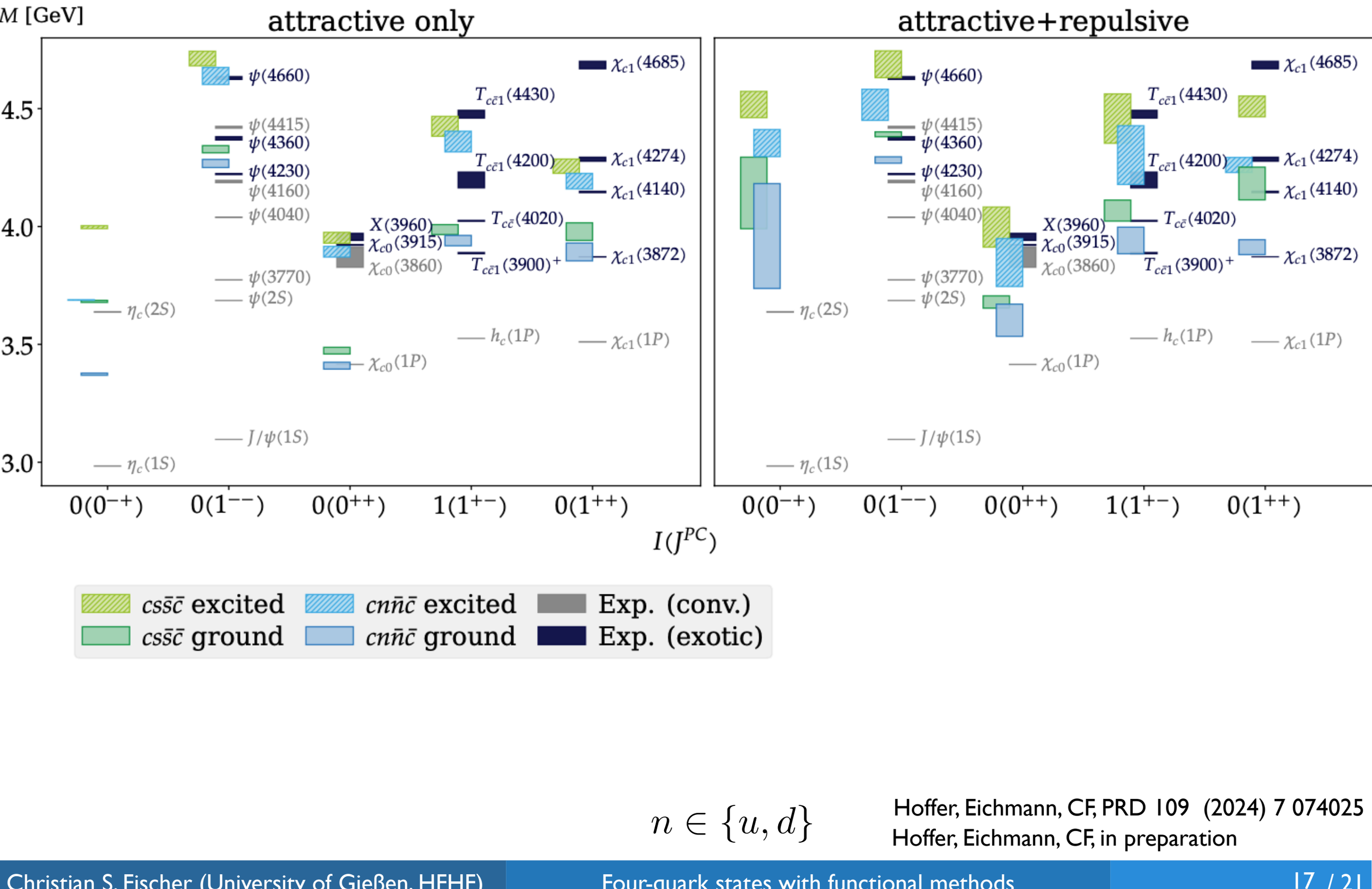

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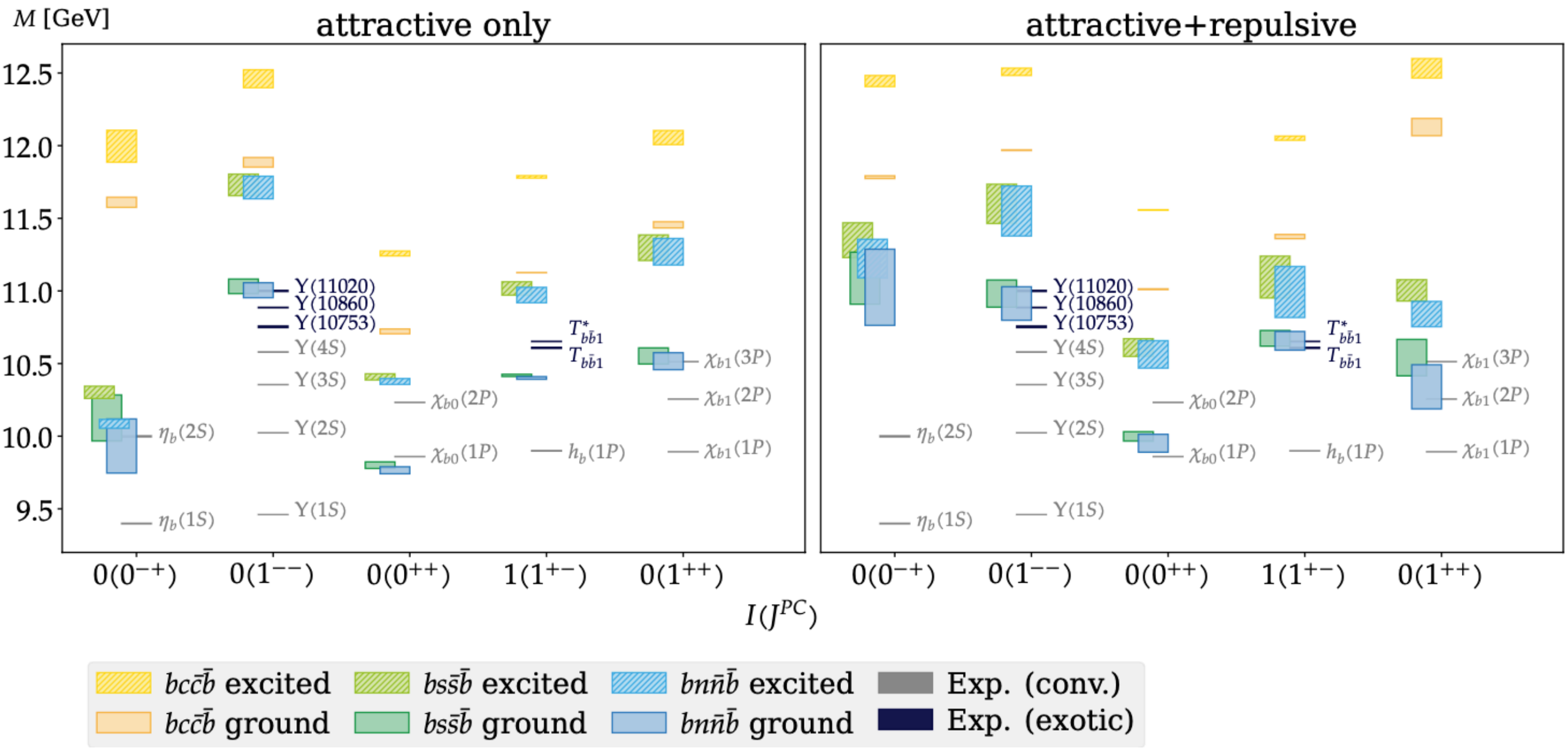
Hoffer, Eichmann, CF, PRD 109 (2024) 7 074025
Hoffer, Eichmann, CF, in preparation

Hidden flavour four-quark states with bottom



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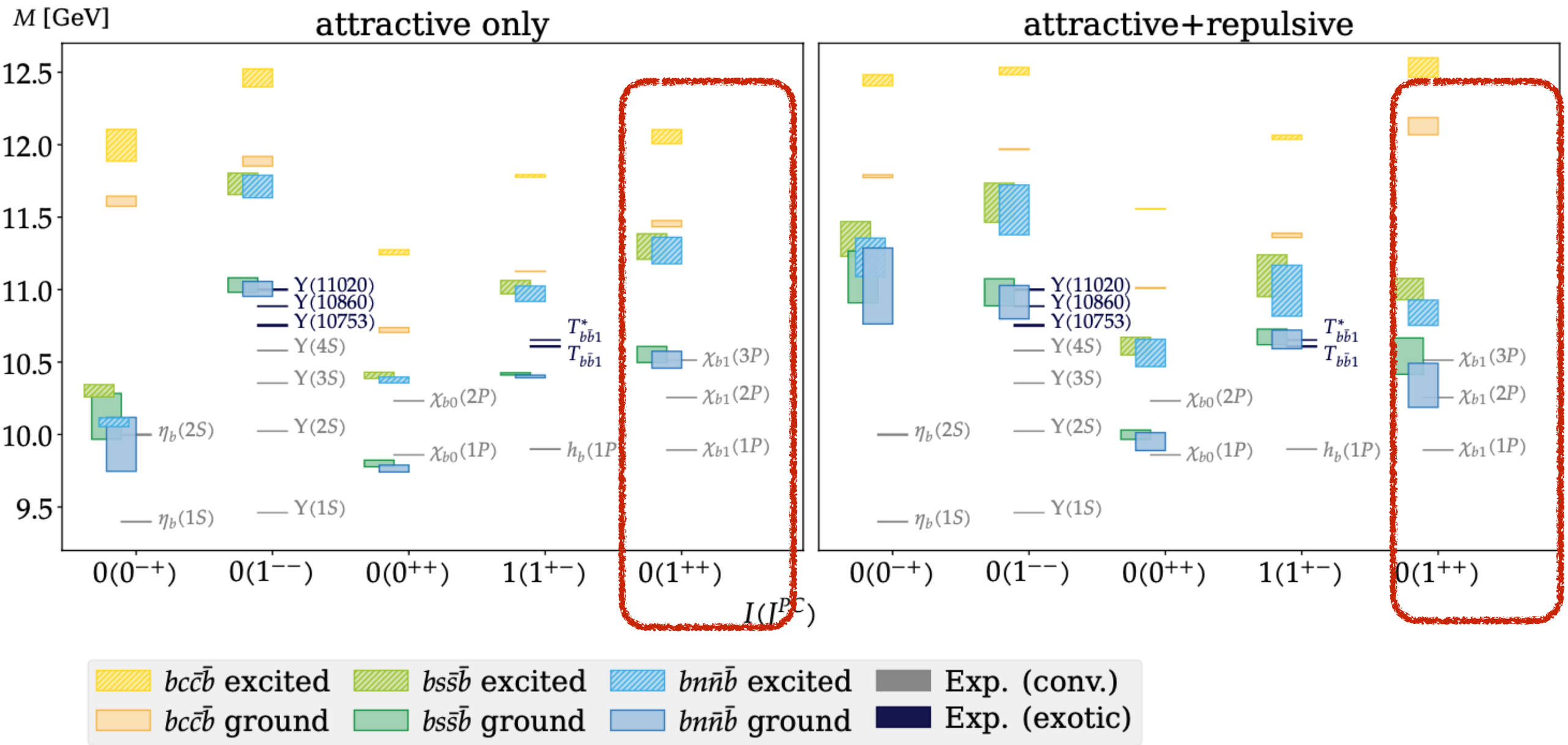

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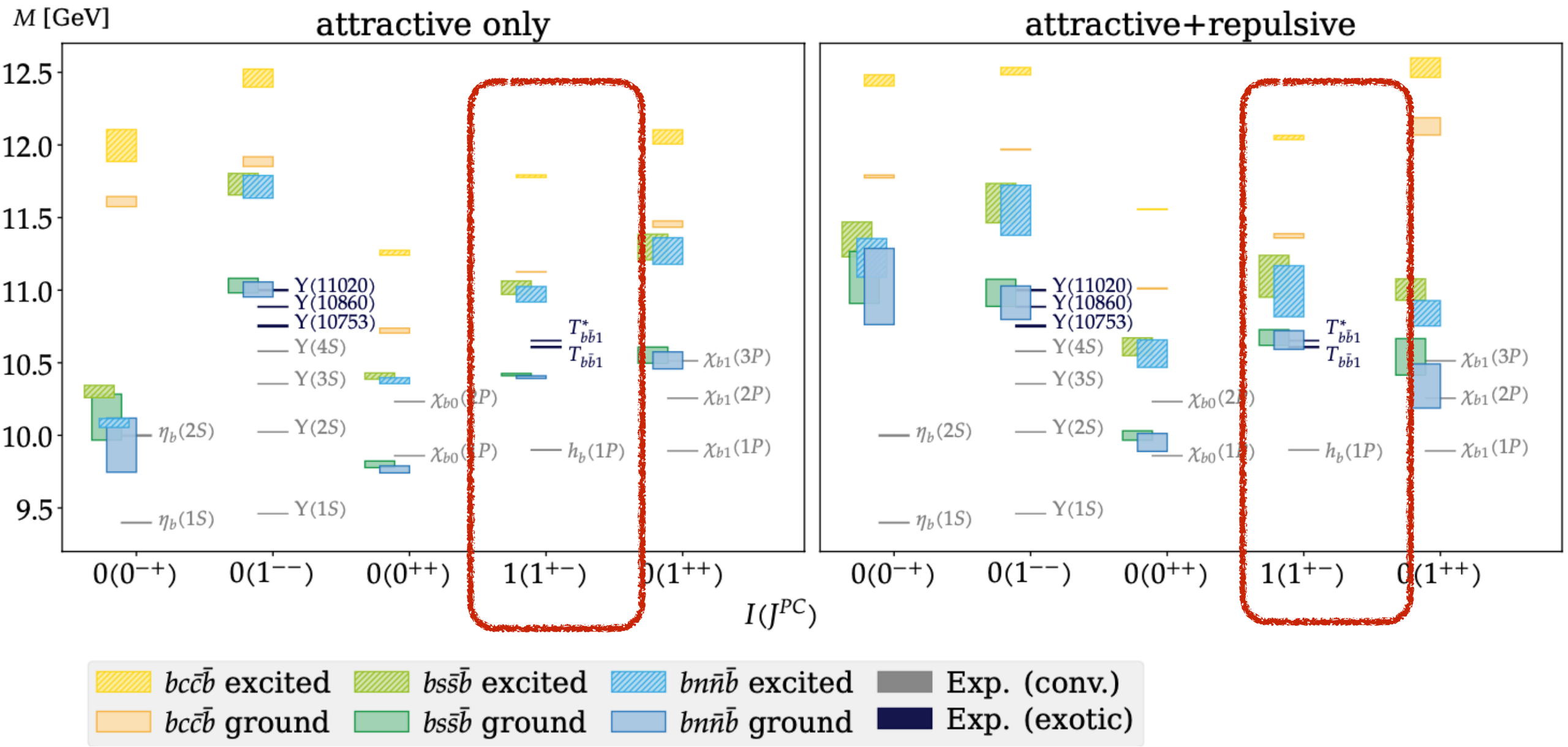

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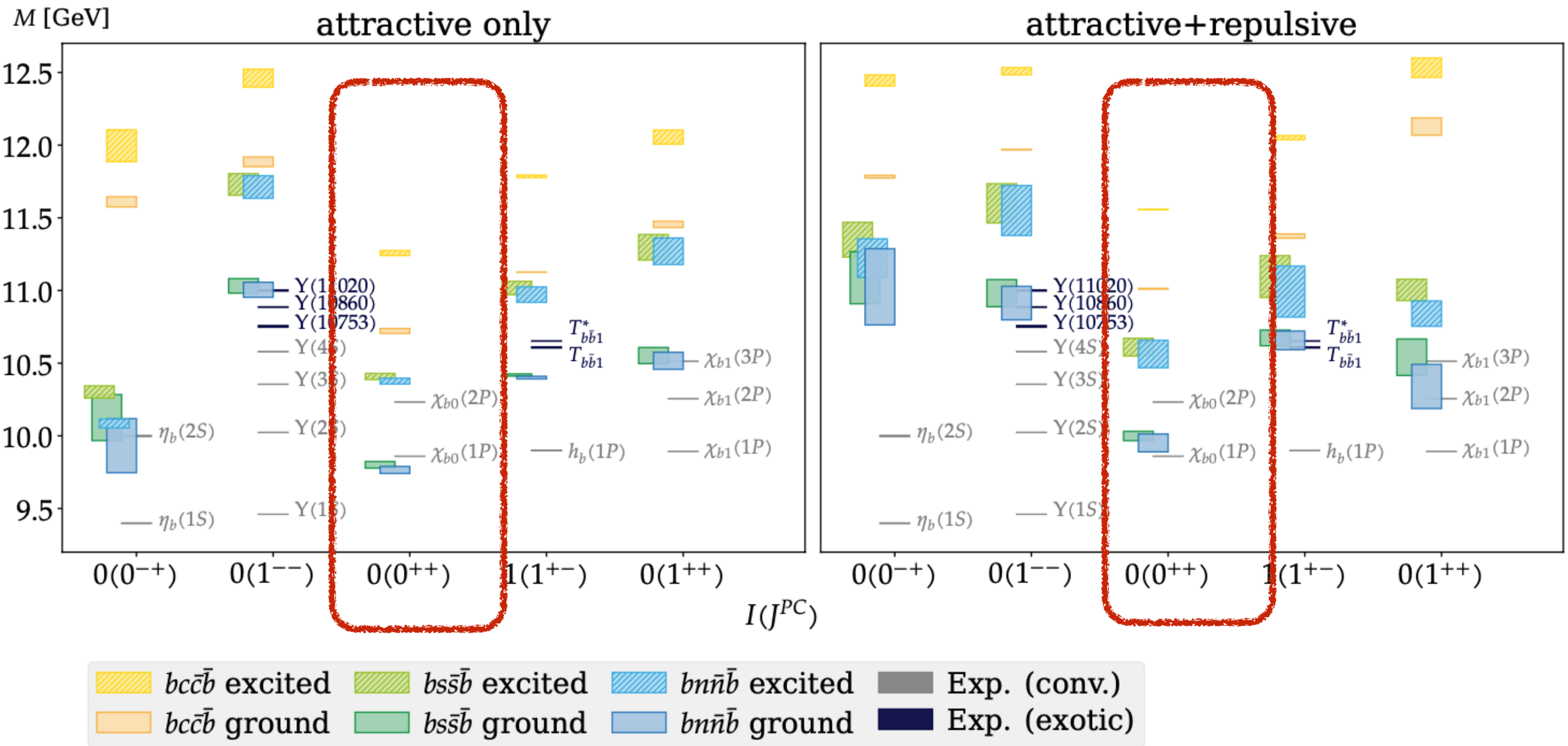

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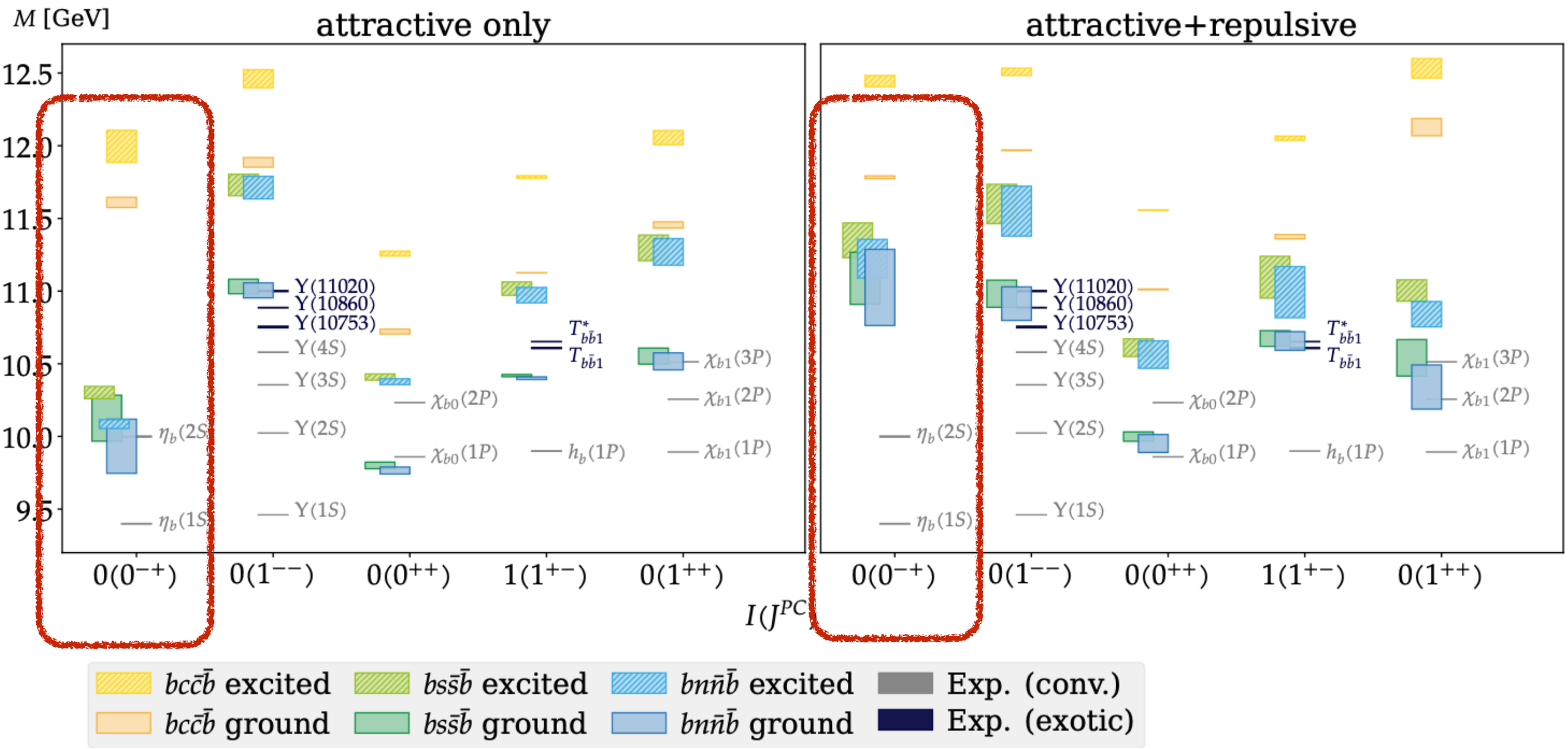

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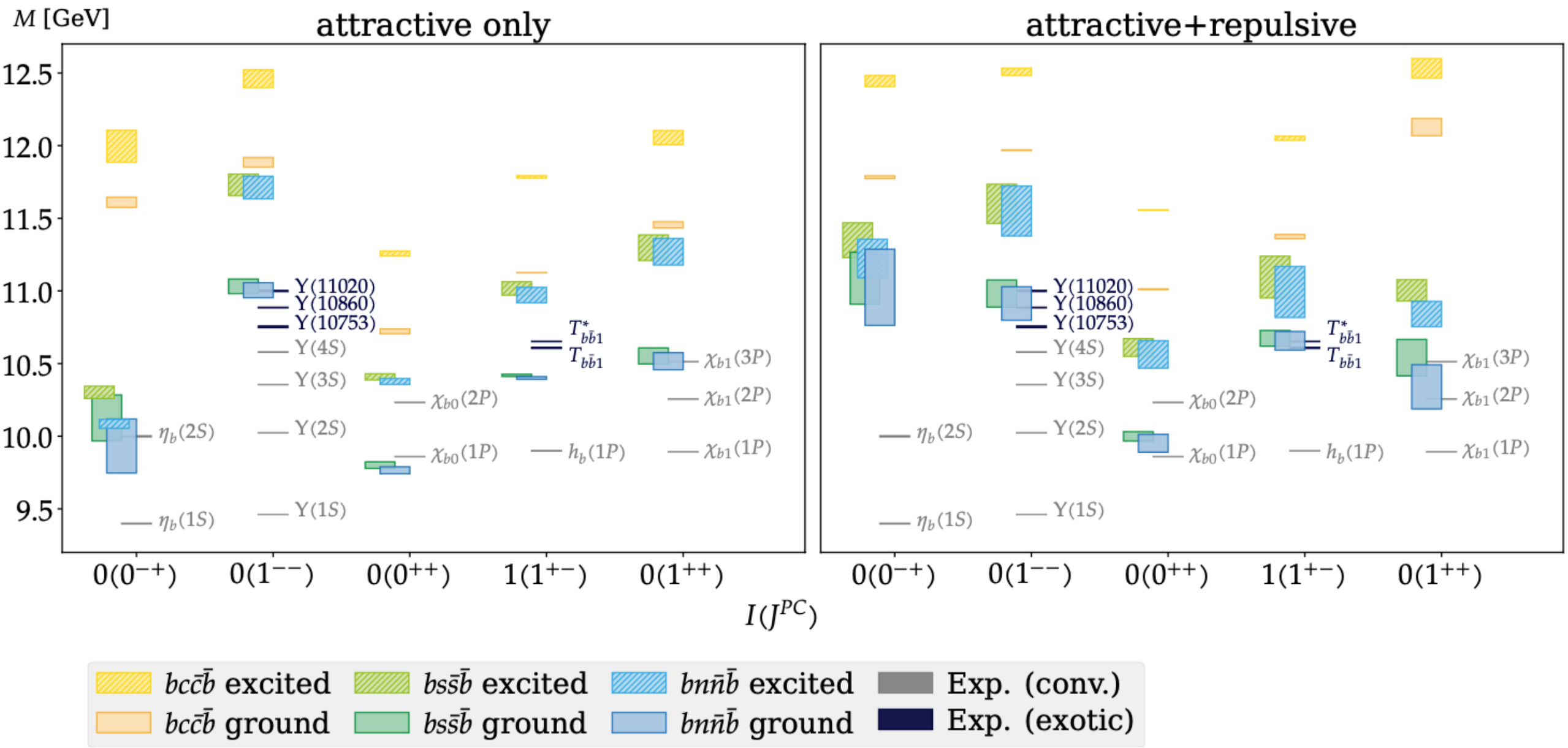
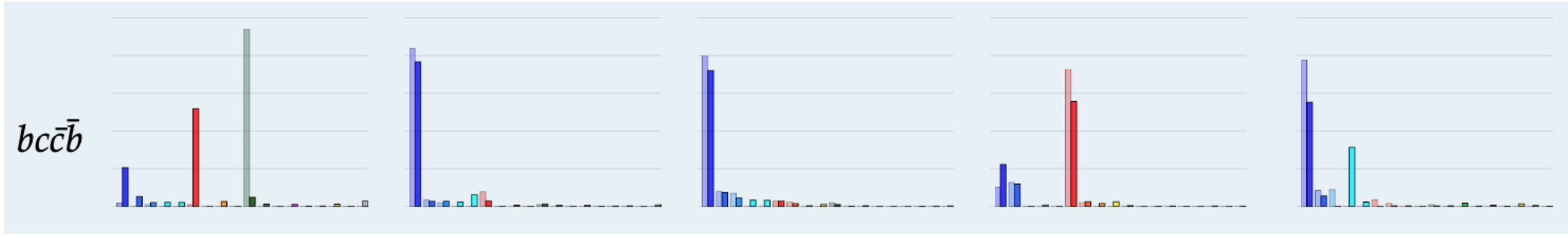
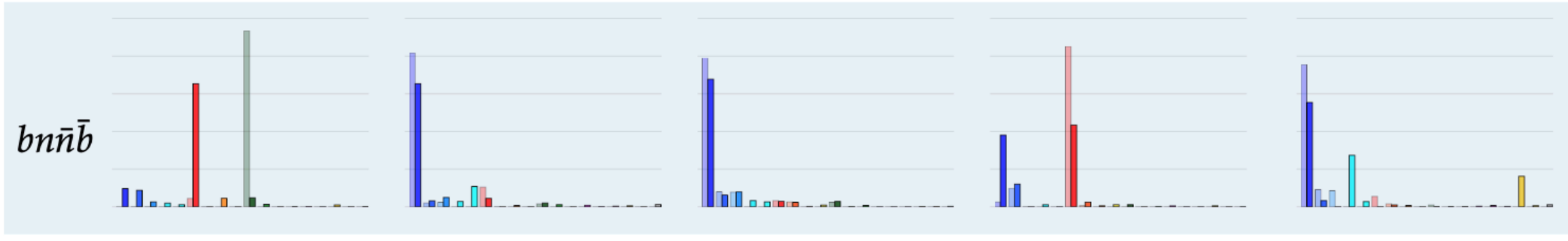


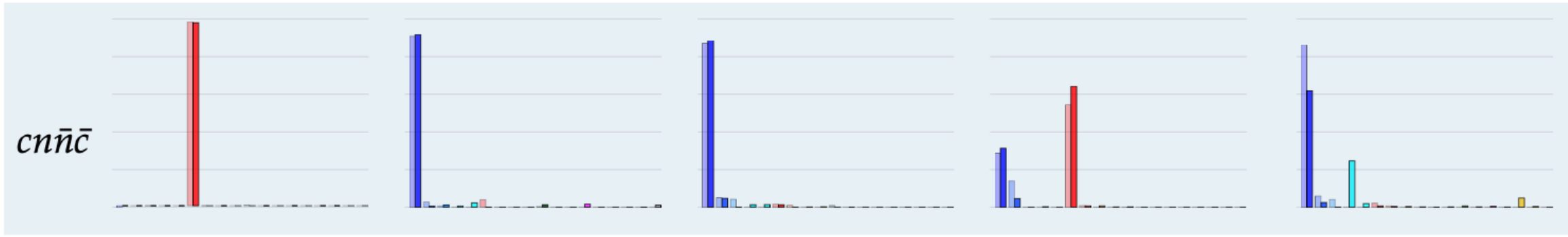
$n \in \{u, d\}$

Hoffer, Eichmann, CF, PRD 109 (2024) 7 074025
Hoffer, Eichmann, CF, in preparation

Internal structure





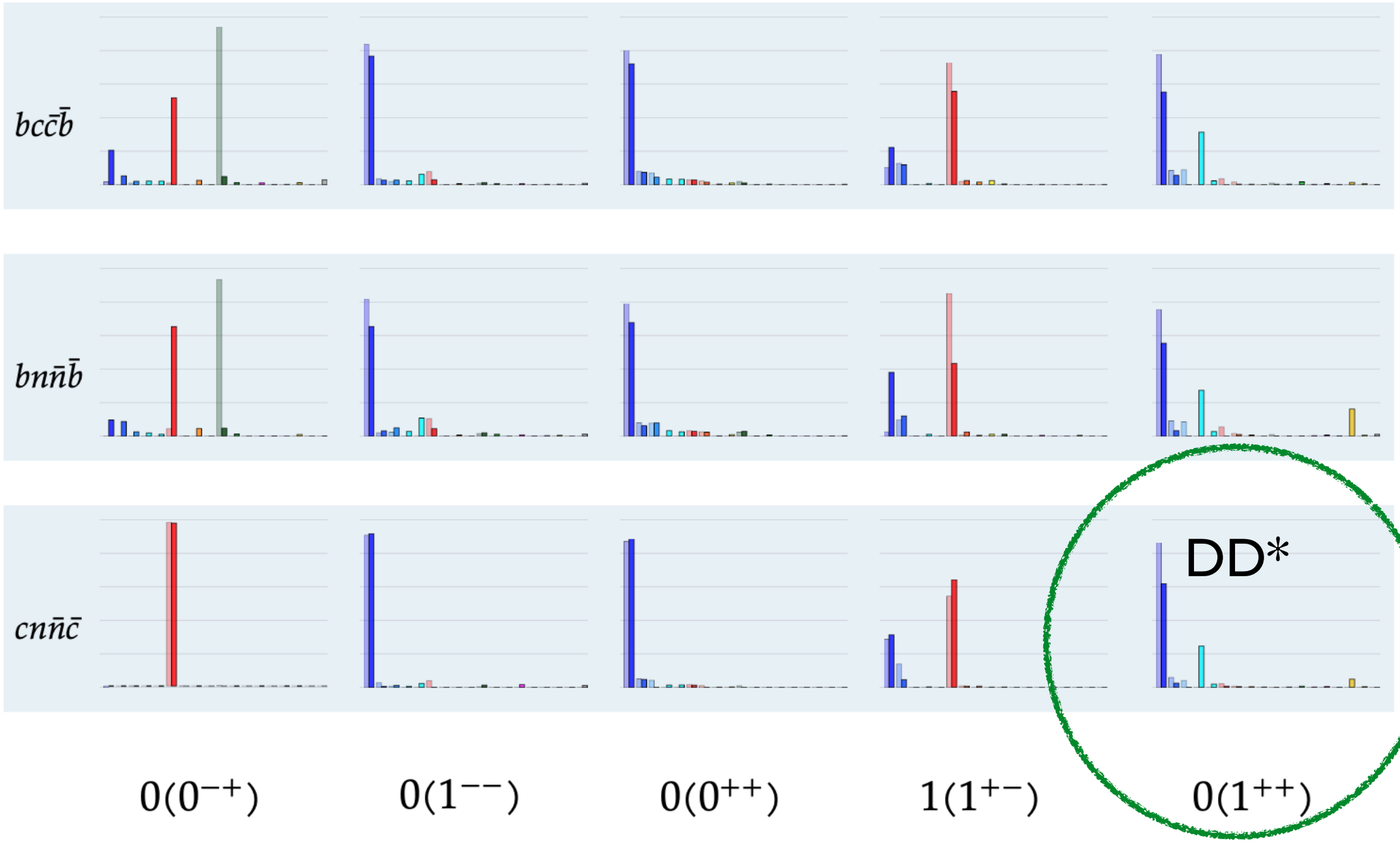


$0(0^{-+})$ $0(1^{--})$ $0(0^{++})$ $1(1^{+-})$ $0(1^{++})$

- decided dynamically !
- flavour and spin dependent…

Hoffer, Eichmann, CF, PRD 109 (2024) 7 074025
Hoffer, Eichmann, CF, in preparation

Internal structure



X(3872)

- decided dynamically !
- flavour and spin dependent…

Hoffer, Eichmann, CF, PRD 109 (2024) 7 074025
Hoffer, Eichmann, CF, in preparation

Summary

Charm and bottom energy region:

- Spectrum for hidden and open flavor
  four-quark states
- Interesting interplay: attractive and repulsive forces

- Internal structure is flavour and spin dependent!
  - meson-meson is dominating/important

Hoffer, Eichmann, CF, PRD 109 (2024) 074025

Hoffer, Eichmann, CF, 2409.05779

Hoffer, Eichmann, CF, in preparation

To do:
- improve two-body forces
- include three and four-body forces
- visit second Riemann sheet
- include mixing with two-quark states

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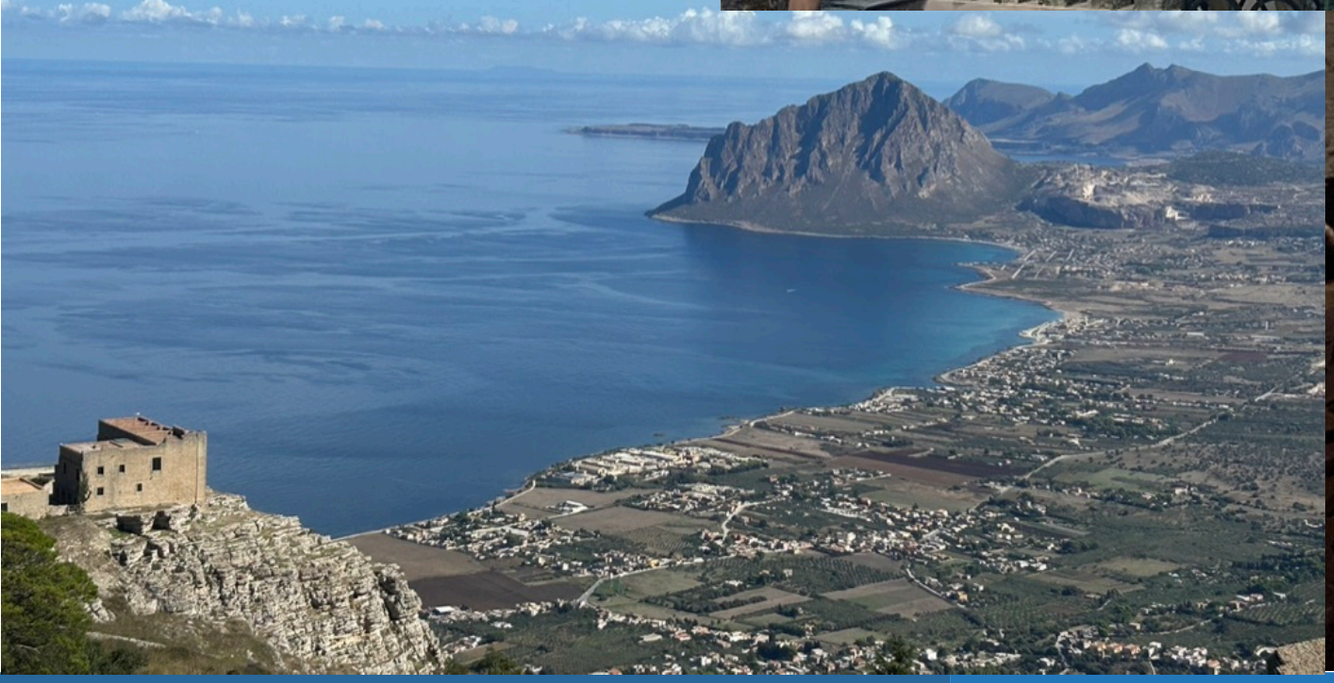

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INTERNATIONAL SCHOOL OF NUCLEAR PHYSICS

46th **Course**

QCD under extreme conditions

- from heavy-ion collisions to the phase diagram

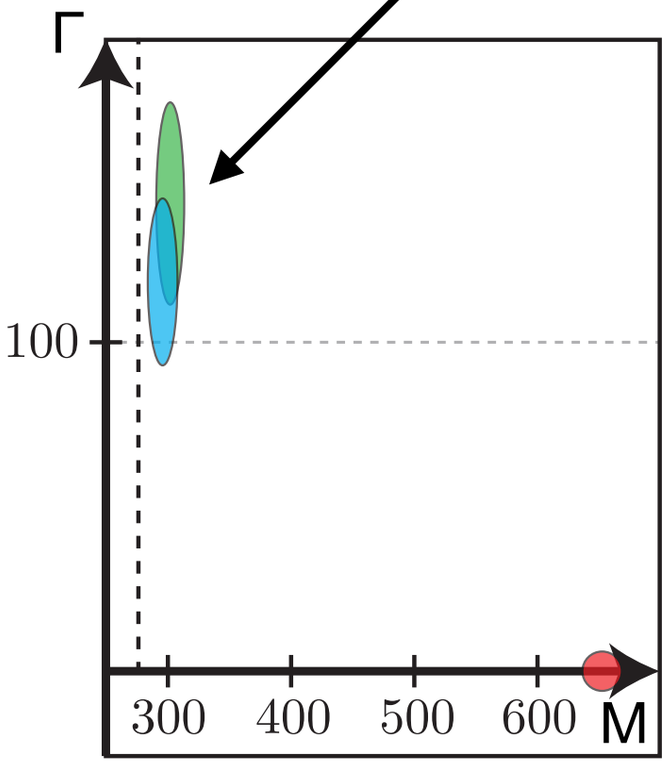
Erice-Sicily: September 16-22, 2025

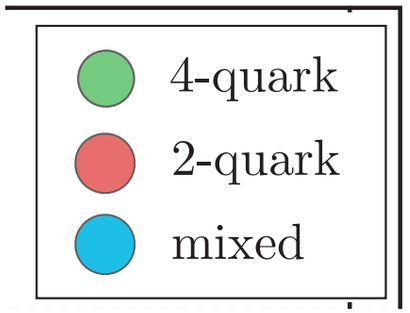
Directors of the Course:
Christian Fischer and Tetyana Galatyuk

Backup Slides

Mass evolution of four-quark state: 0++

$$f_0(500) : \pi\pi - \text{component dominates!}$$





$m_q = 3.8\,\mathrm{MeV}$

Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

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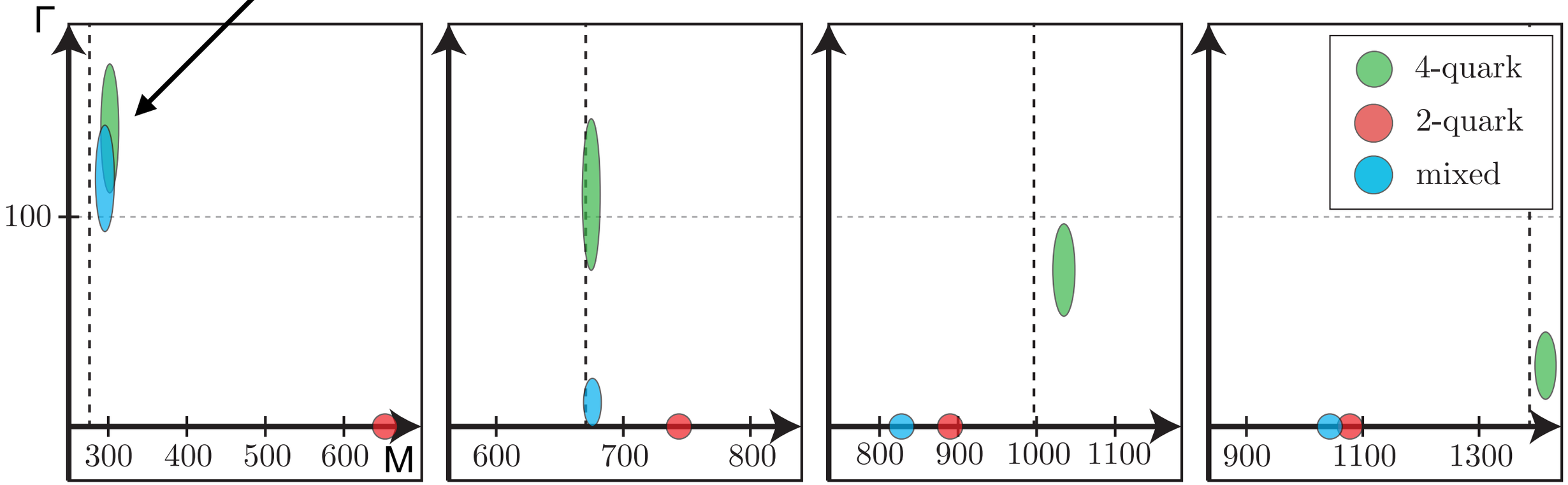

$m_q = 3.8\,\text{MeV}$ $m_q = 20.0\,\text{MeV}$ $m_q = 50.0\,\text{MeV}$ $m_q = 85.5\,\text{MeV}$

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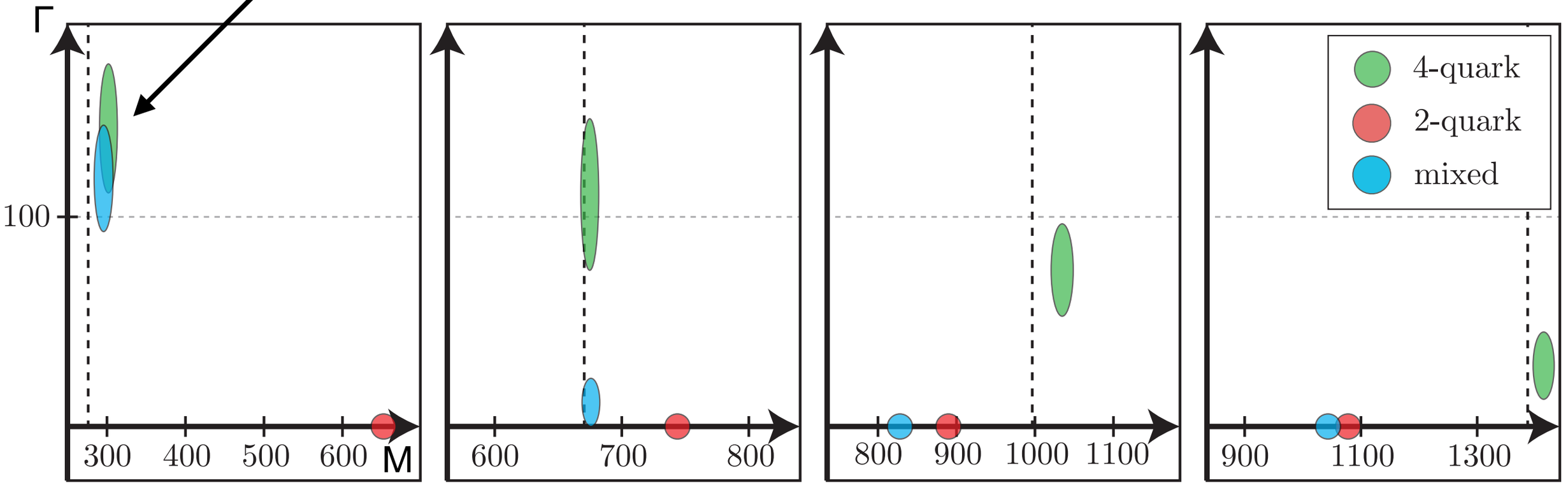

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- mixed state becomes qq-dominated for large m_q
- dynamical decision !

Mass evolution of four-quark state: 0++

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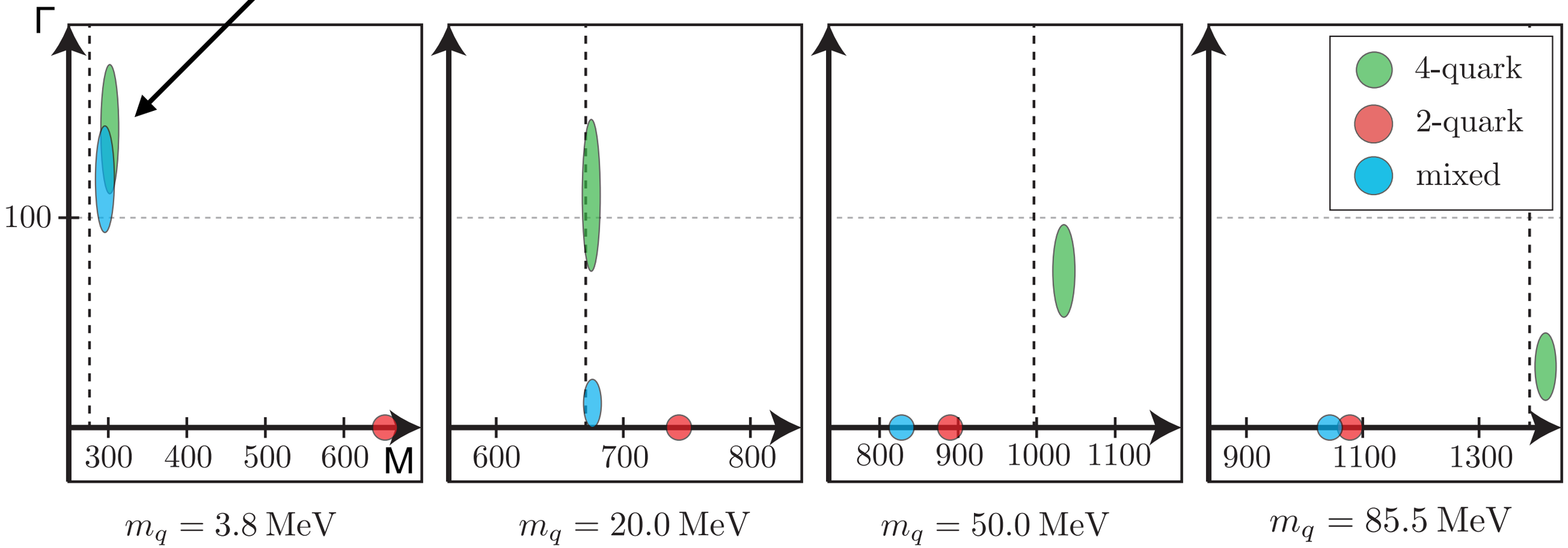

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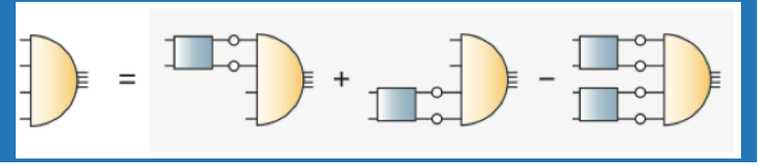
- mixed state becomes qq-dominated for large m_q
- dynamical decision !

⟶ consequences for ccqq, ccss, bbq, bbss, bbcc ?

work to be done!

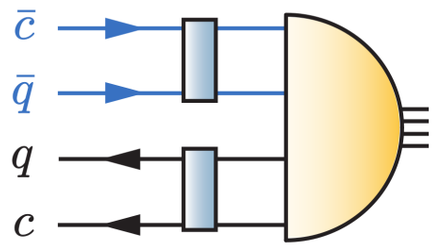
Four-body equation: permutations



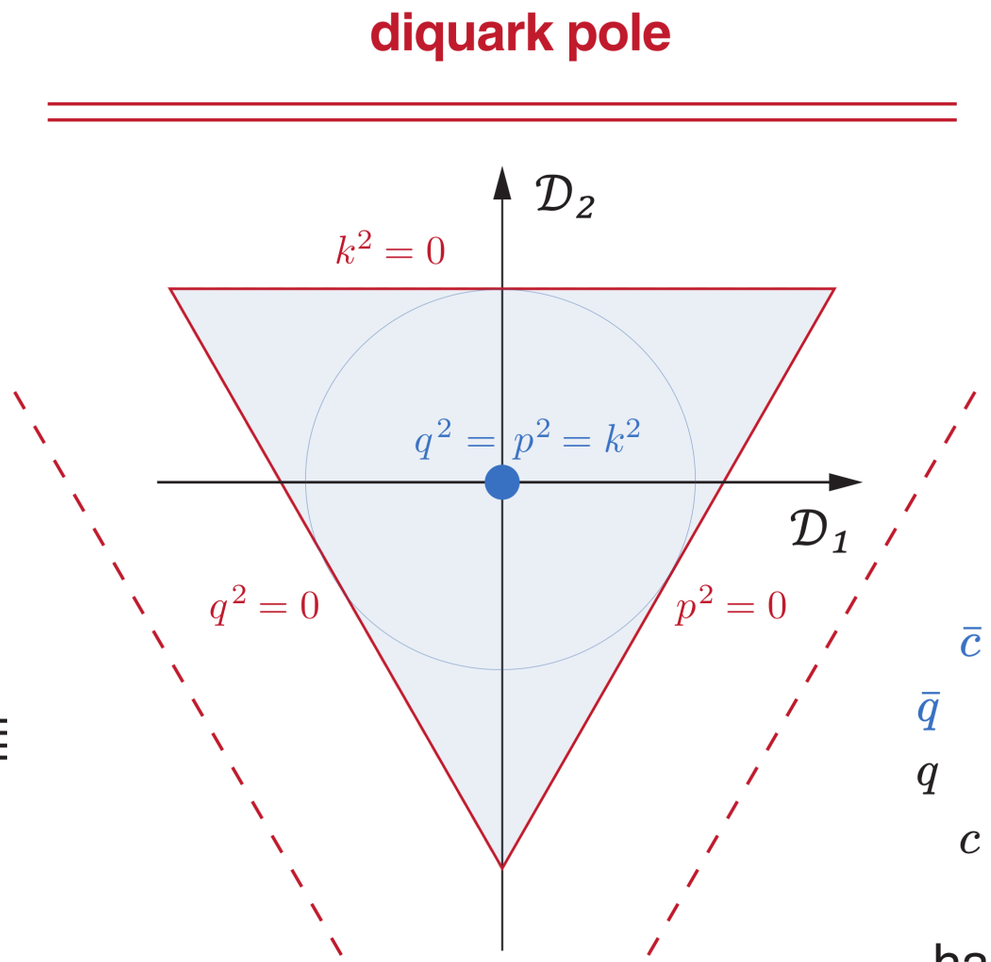
- **Singlet:** $S_0 = (p^2 + q^2 + k^2)/4$
- **Doublet:** $\mathcal{D}_1 \sim p^2 + q^2 - 2k^2$

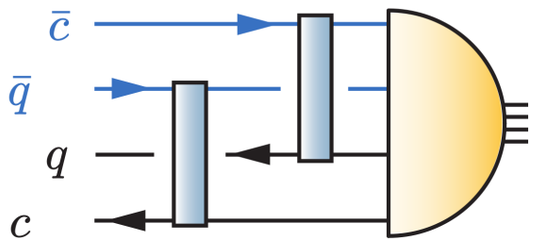
  $\mathcal{D}_2 \sim q^2 - p^2$

p, q, k : relative momenta

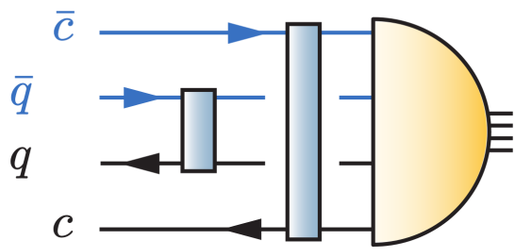


diquark-antidiquark



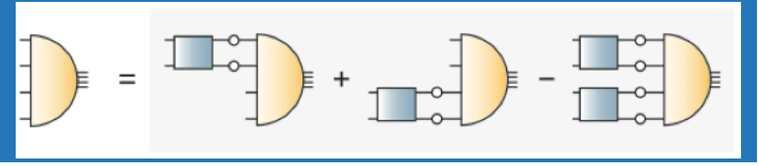


heavy-light mesons



hadrocharmonium

Four-body equation: permutations



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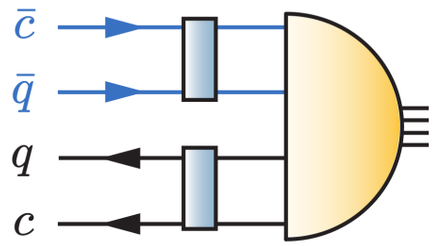
- **Doublet:** $\mathcal{D}_1 \sim p^2 + q^2 - 2k^2$

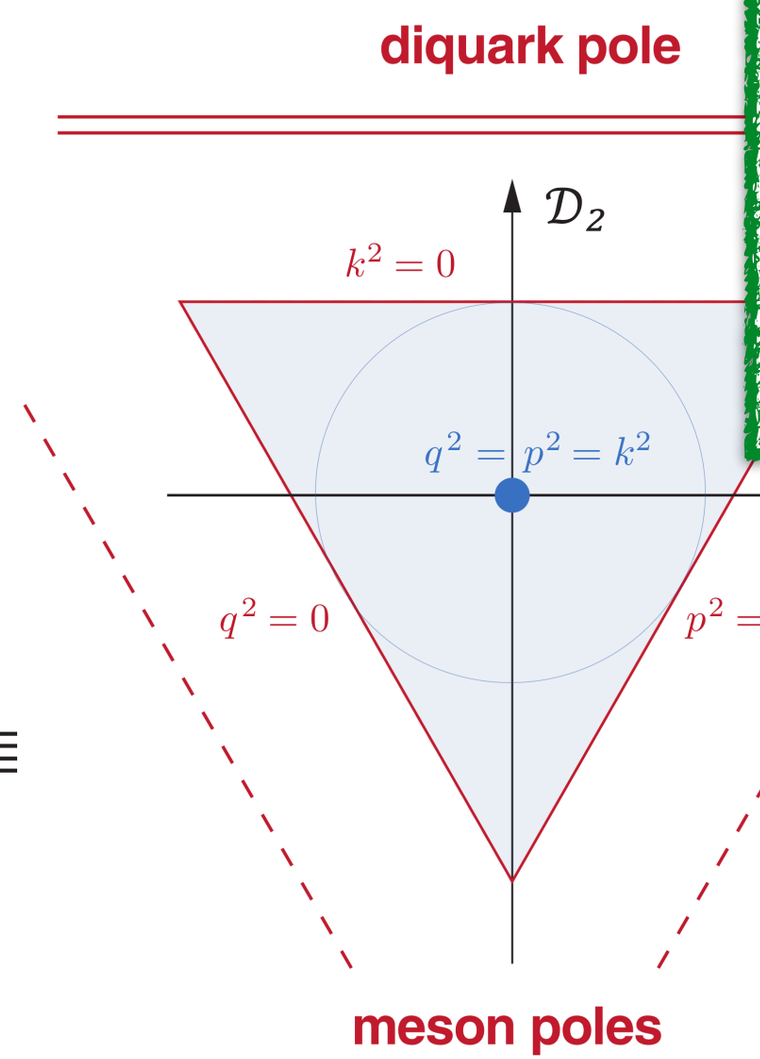
  $\mathcal{D}_2 \sim q^2 - p^2$

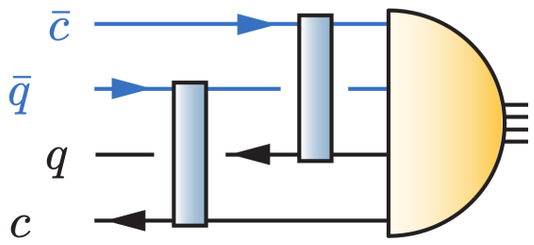
p, q, k : relative momenta

- model independent: heavy-light meson poles more important than diquark poles (color factor !)



diquark-antidiquark





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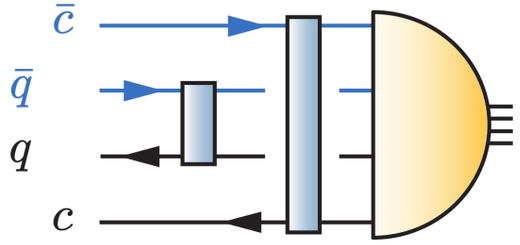
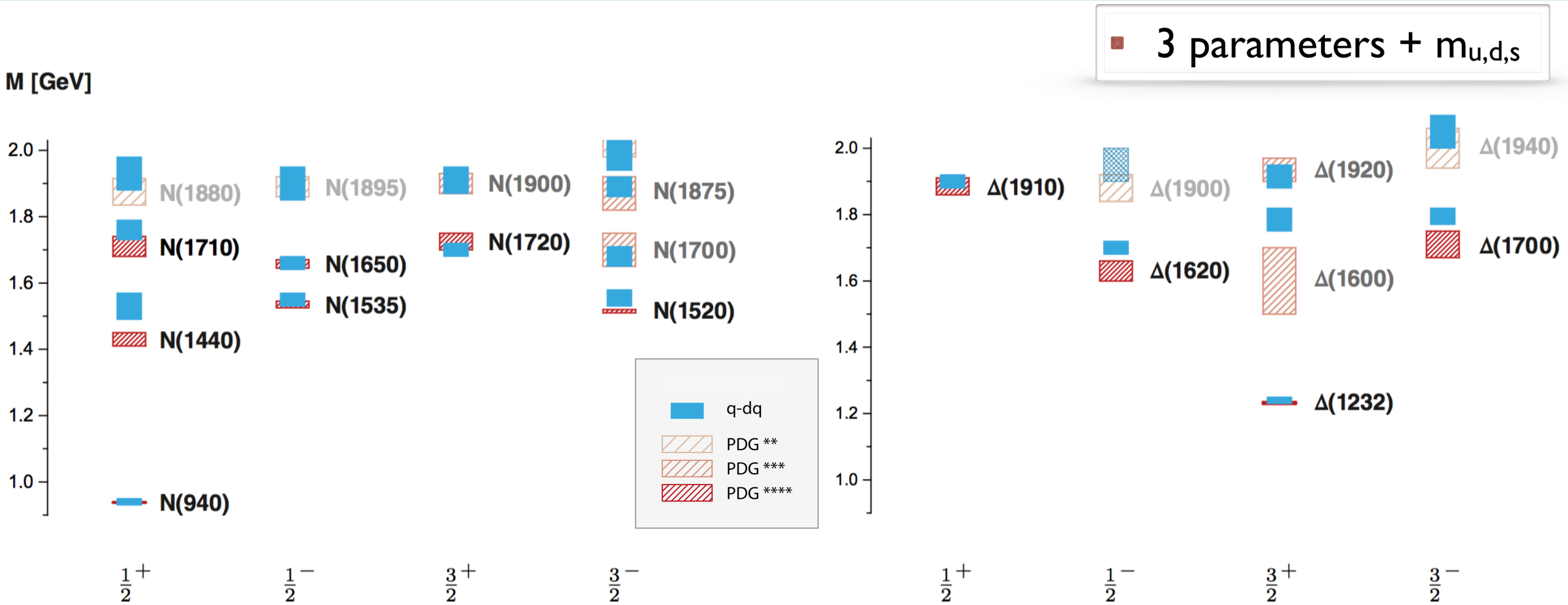


hadrocharmonium

Light baryon spectrum: diquark-picture

- 3 parameters + $m_{u,d,s}$



Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2

- spectrum in one to one agreement with experiment
- correct level ordering (without coupled channel effects…)
- strange baryons
- heavy baryons

Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
CF, Eichmann PoS Hadron 2017 (2018) 007
Sanchis-Alepuz, CF, PRD 90 (2014) 096001

Qin, Roberts, Schmidt, Few Body Syst. 60 (2019) no.2, 26
Torcato, Arriaga, Eichmann and Pena, FBS 64 (2023) 45

Review on diquarks: Barabanov et al, PPNP 116 (2021), 103835

Light baryon spectrum: diquark-picture

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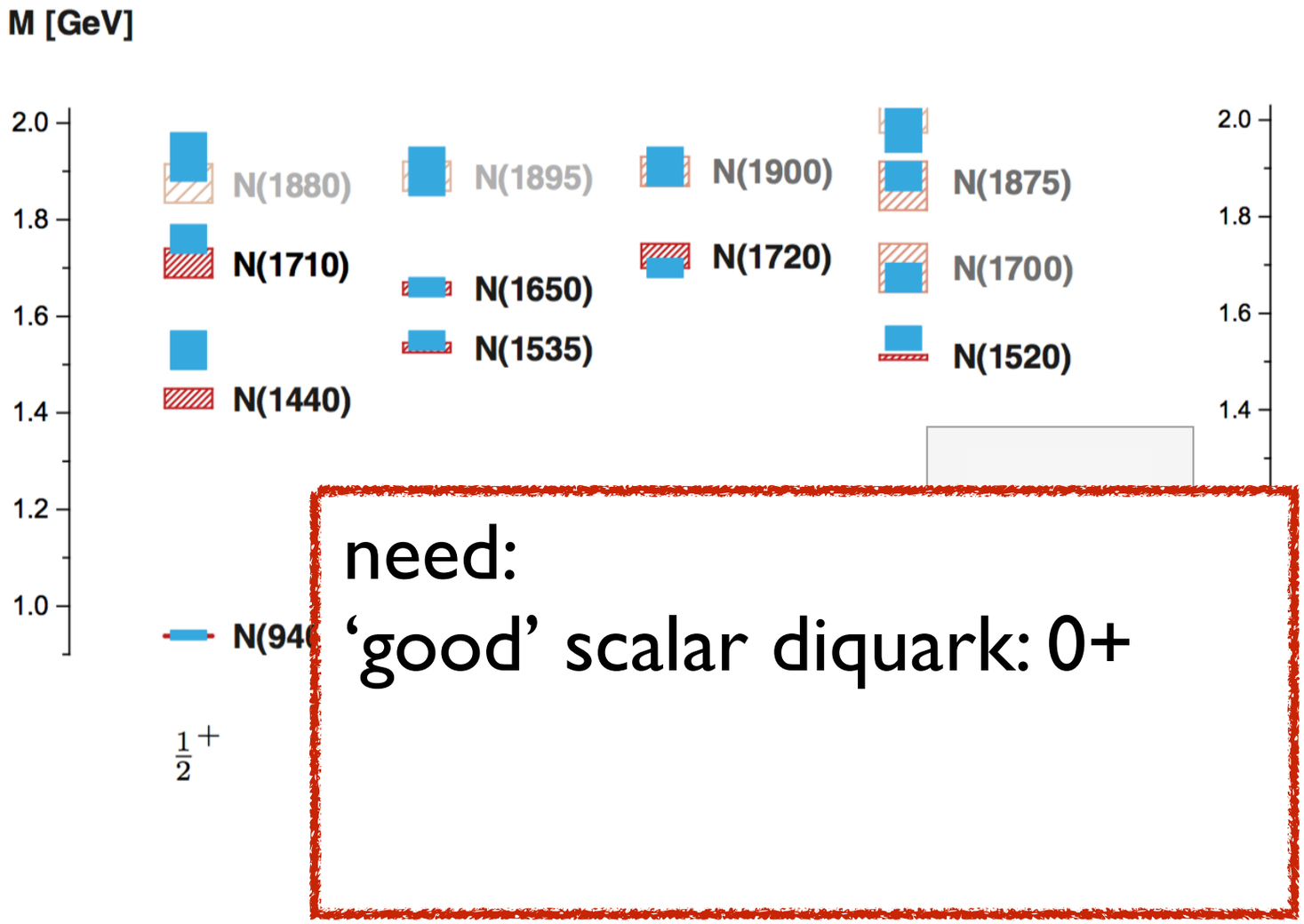
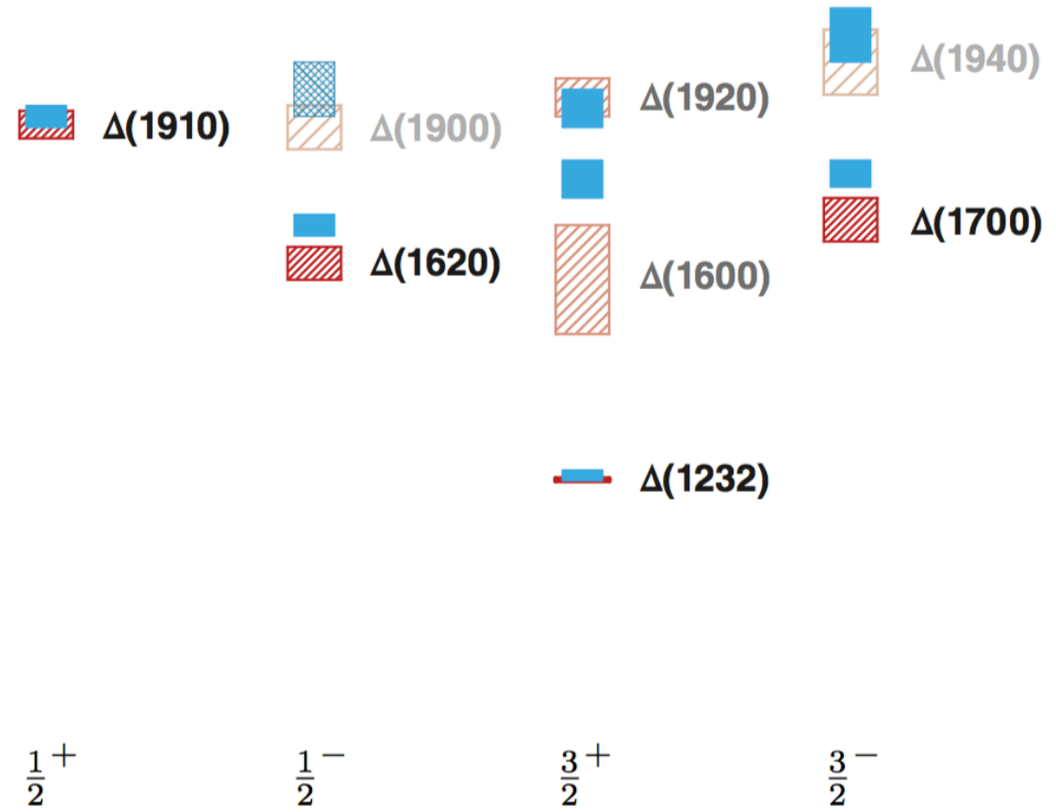


need:
‘good’ scalar diquark: 0+



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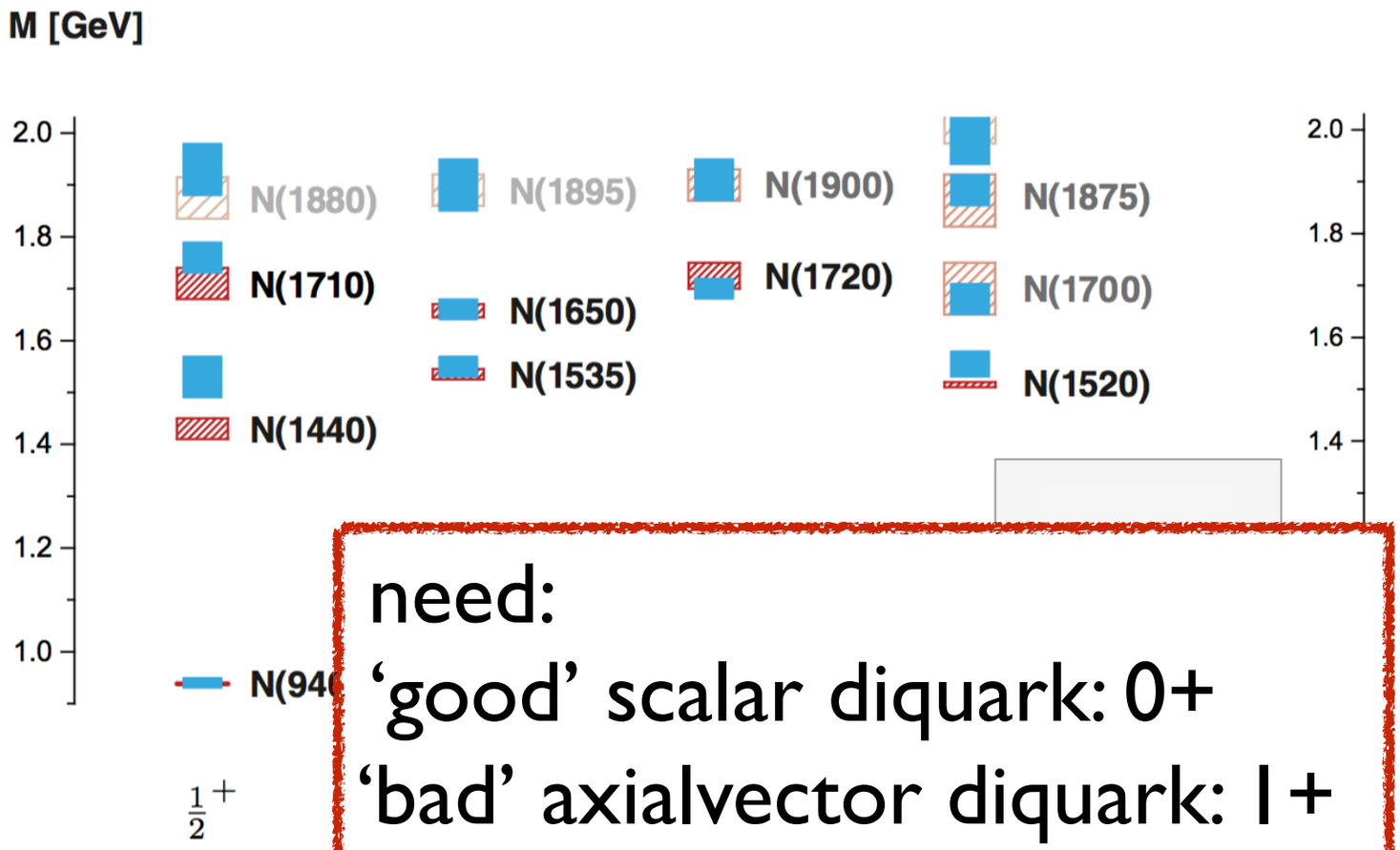

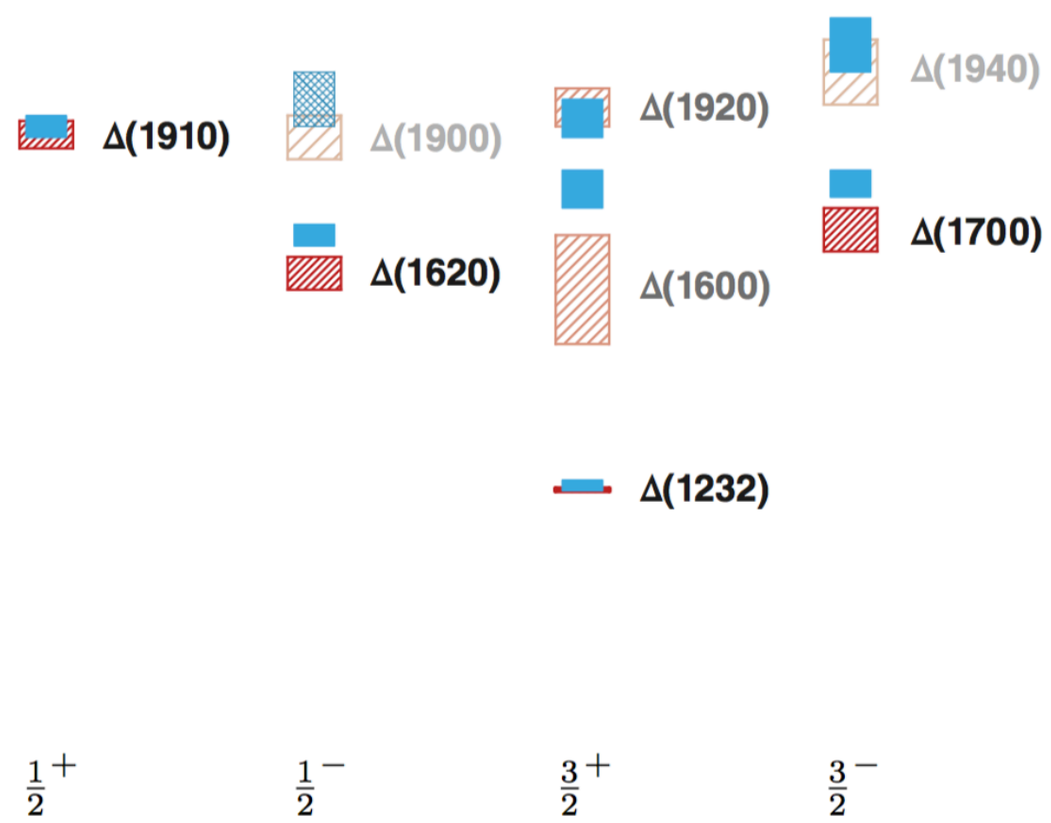

need:
'good' scalar diquark: 0+
'bad' axialvector diquark: 1+

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Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2

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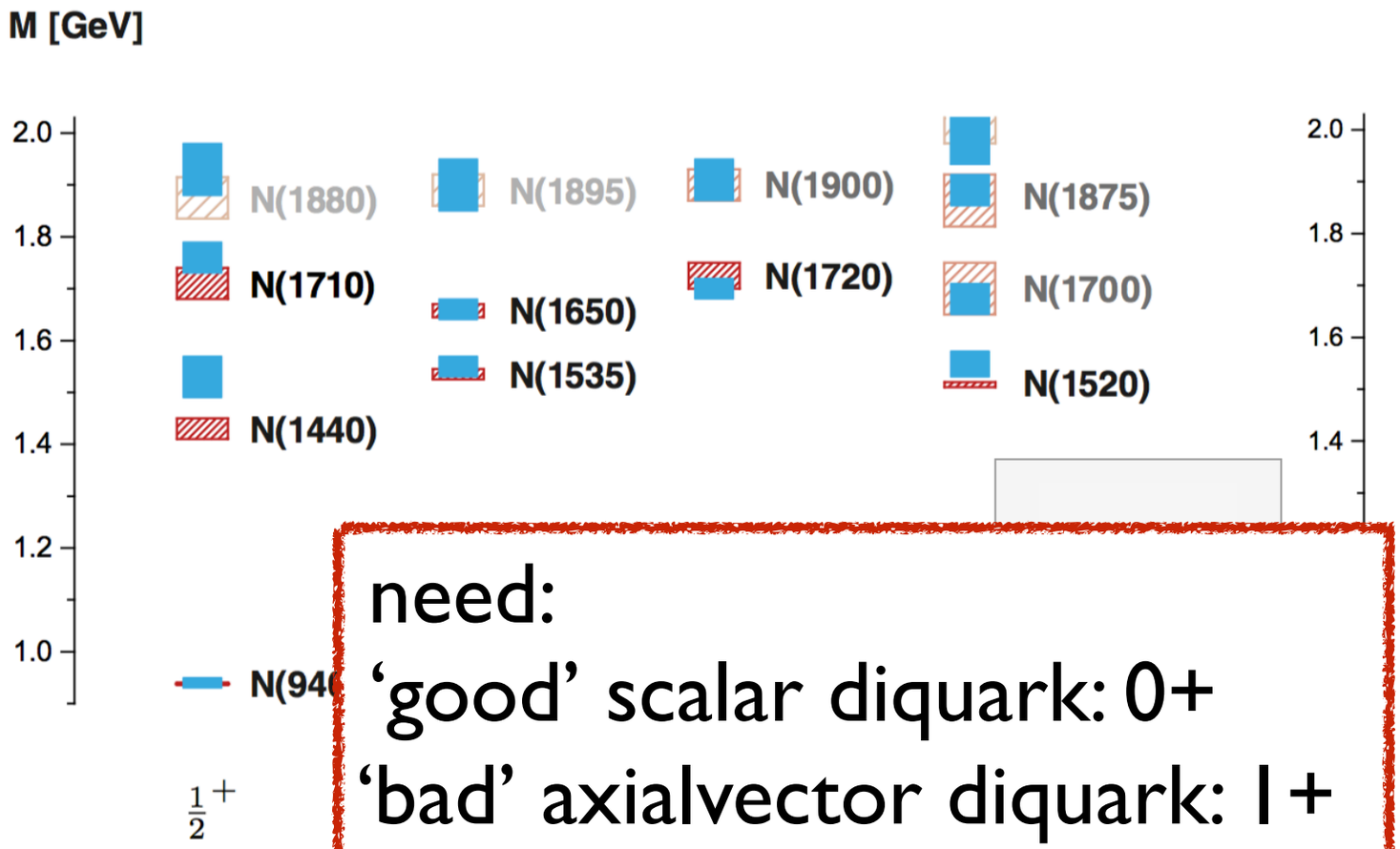
Torcato, Arriaga, Eichmann and Pena, FBS 64 (2023) 45

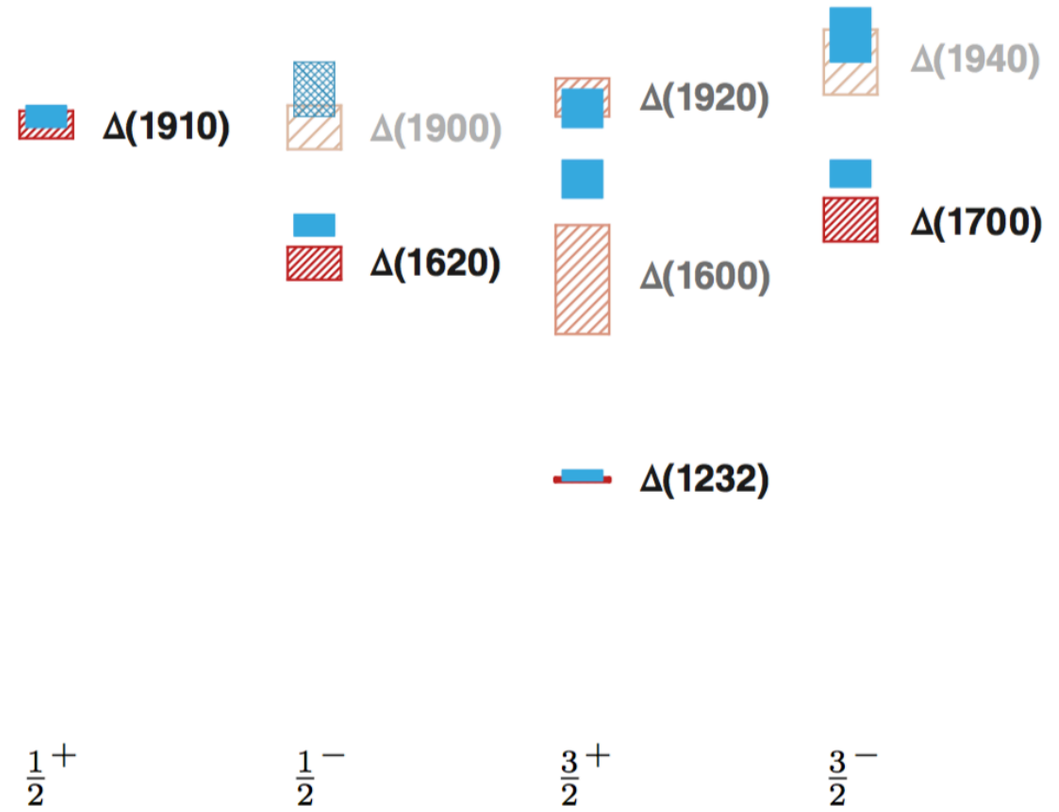
Review on diquarks: Barabanov et al, PPNP 116 (2021), 103835

Light baryon spectrum: diquark-picture

- 3 parameters + $m_{u,d,s}$



need:
‘good’ scalar diquark: 0+
‘bad’ axialvector diquark: 1+
‘ugly’ ps/v diquark



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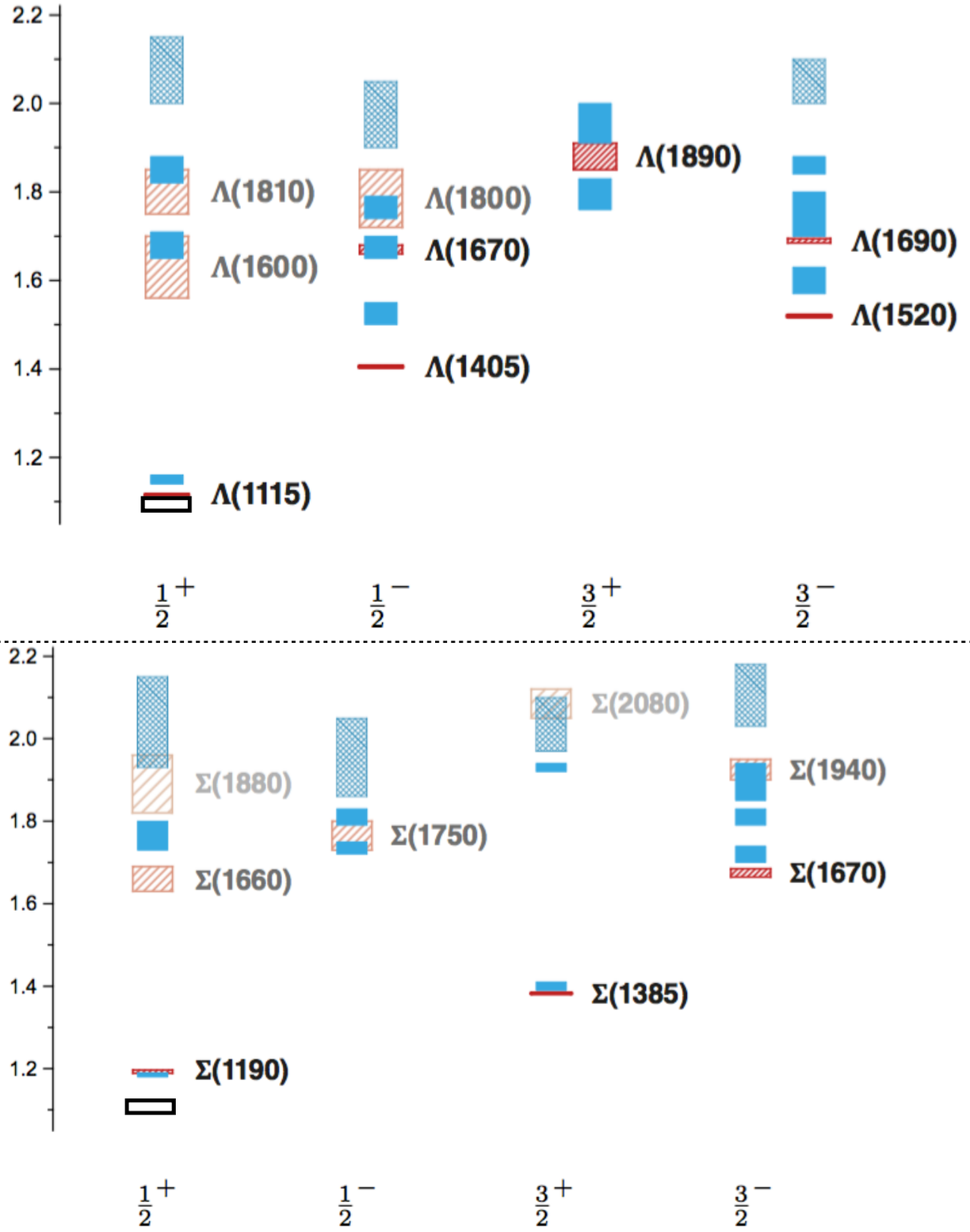
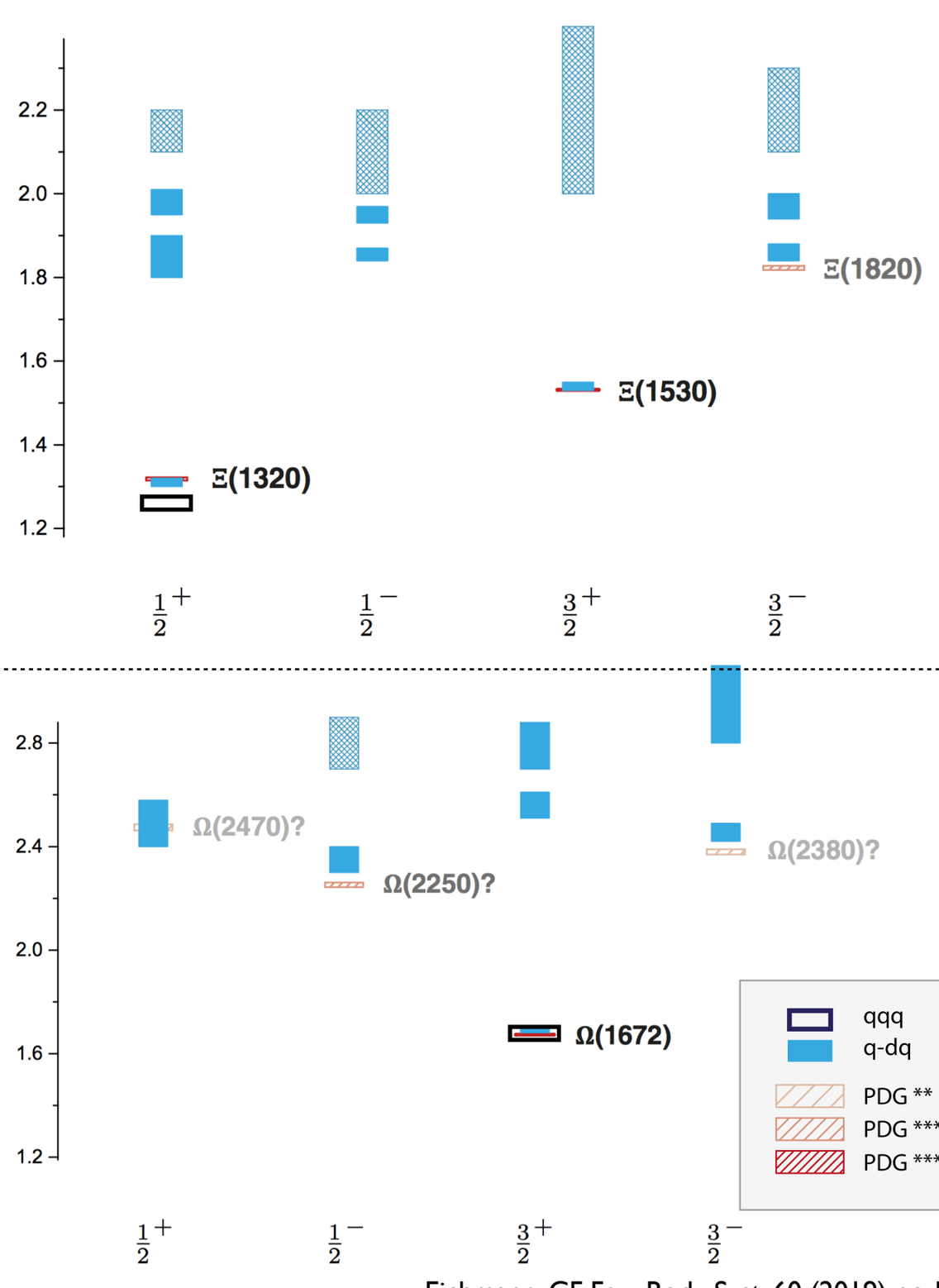
Torcato, Arriaga, Eichmann and Pena, FBS 64 (2023) 45

Review on diquarks: Barabanov et al, PPNP 116 (2021), 103835

Strange baryon spectrum: DSE-RL (preliminary !)





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Sanchis-Alepuz, CF, PRD 90 (2014) 096001

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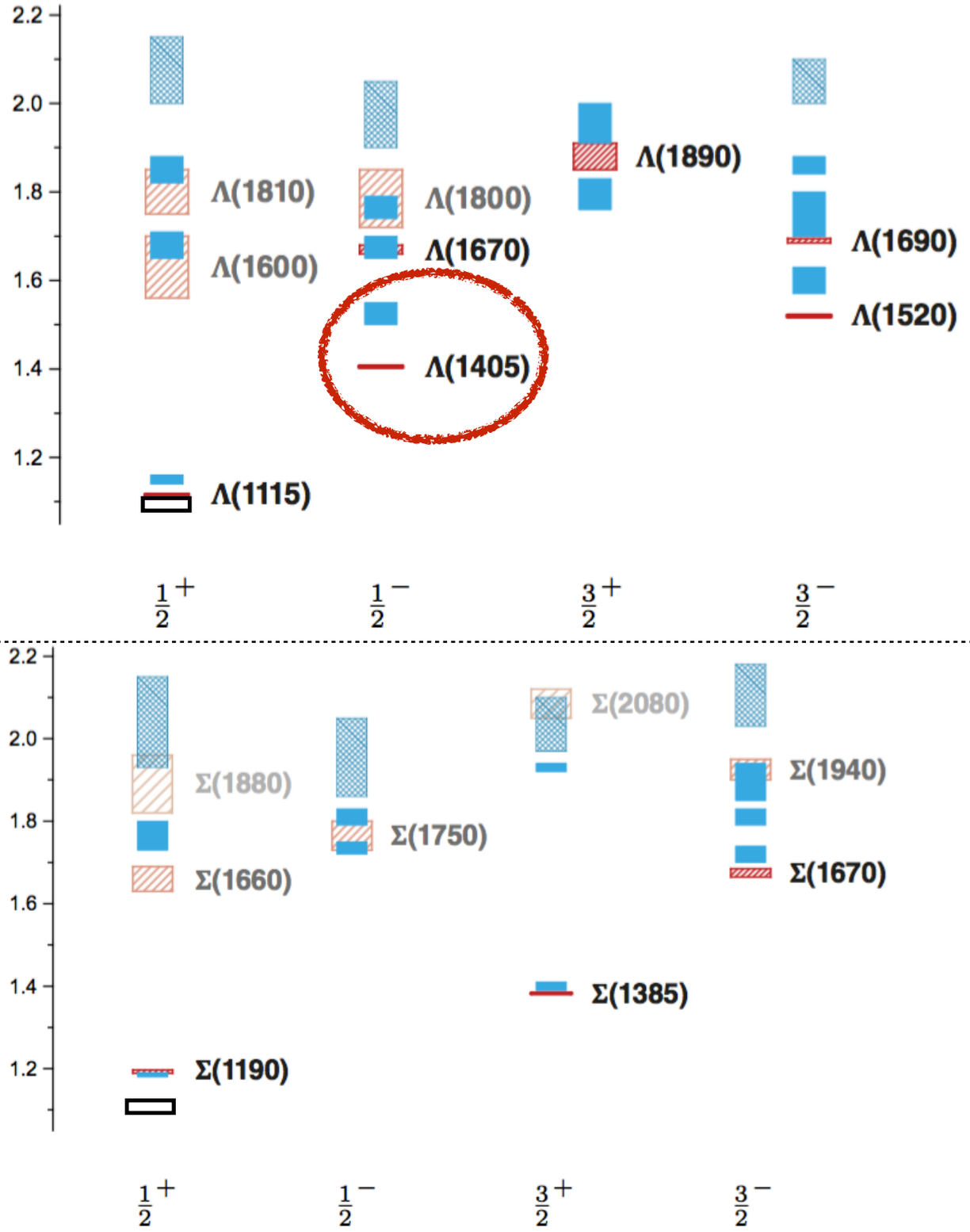

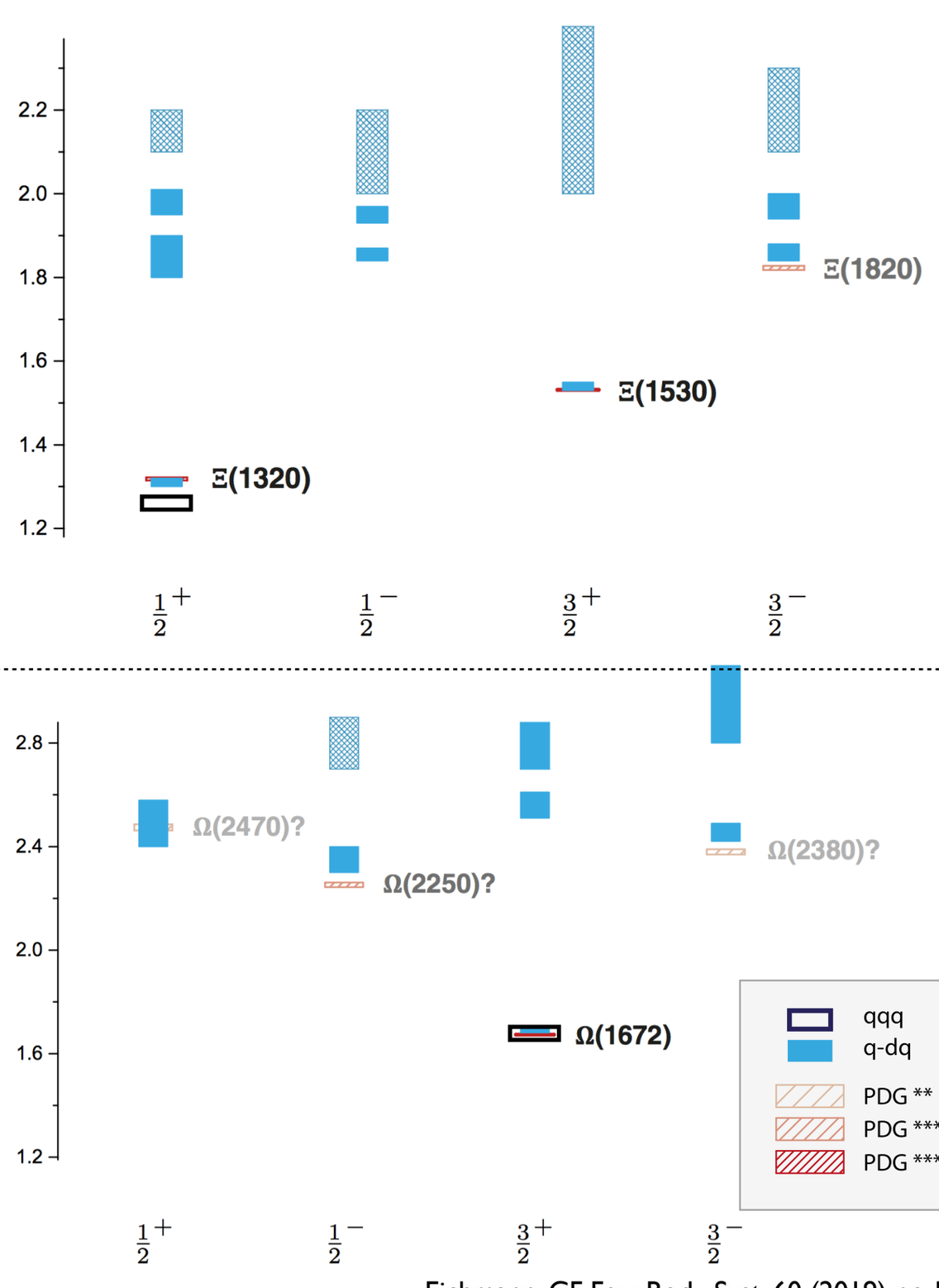

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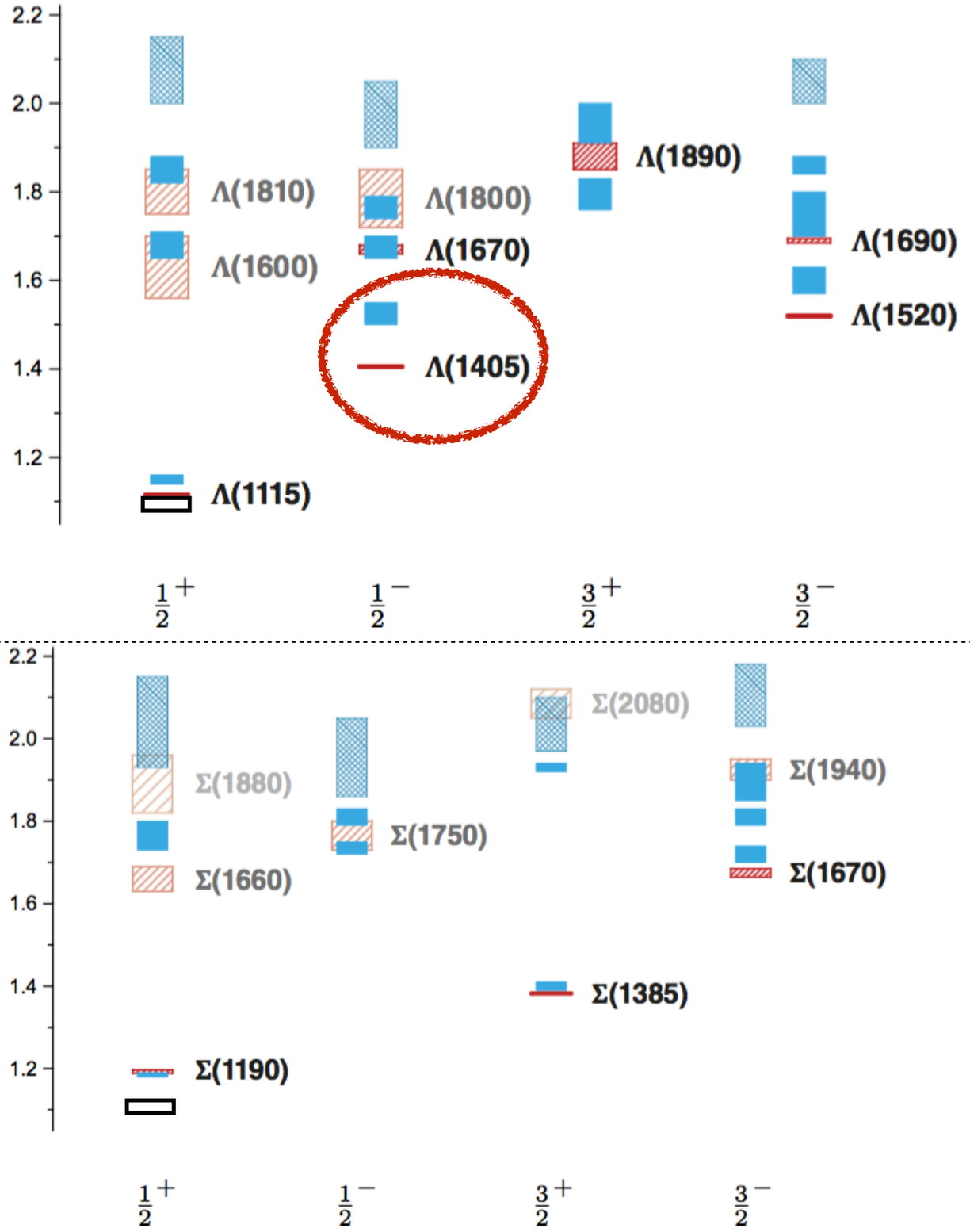

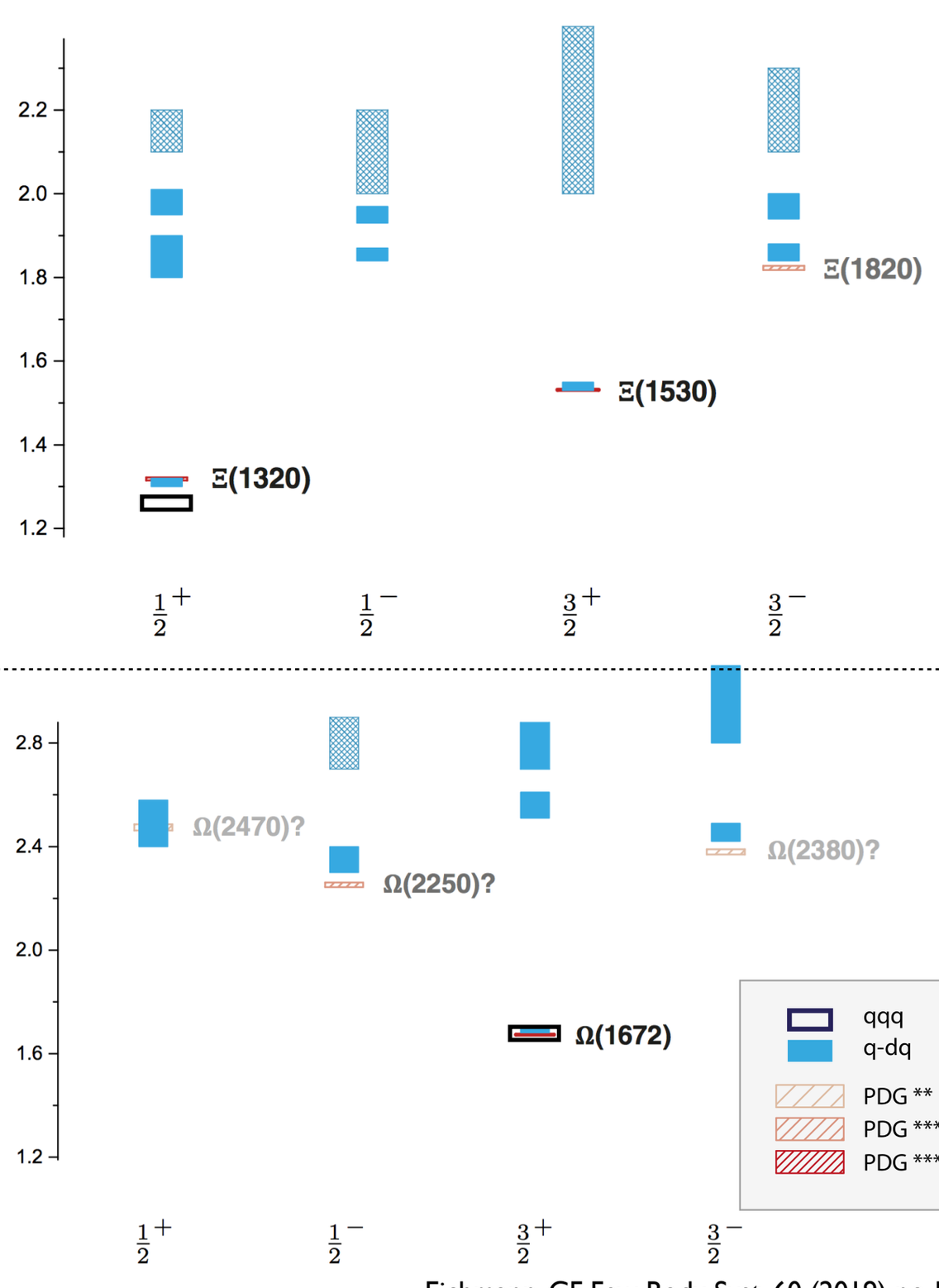

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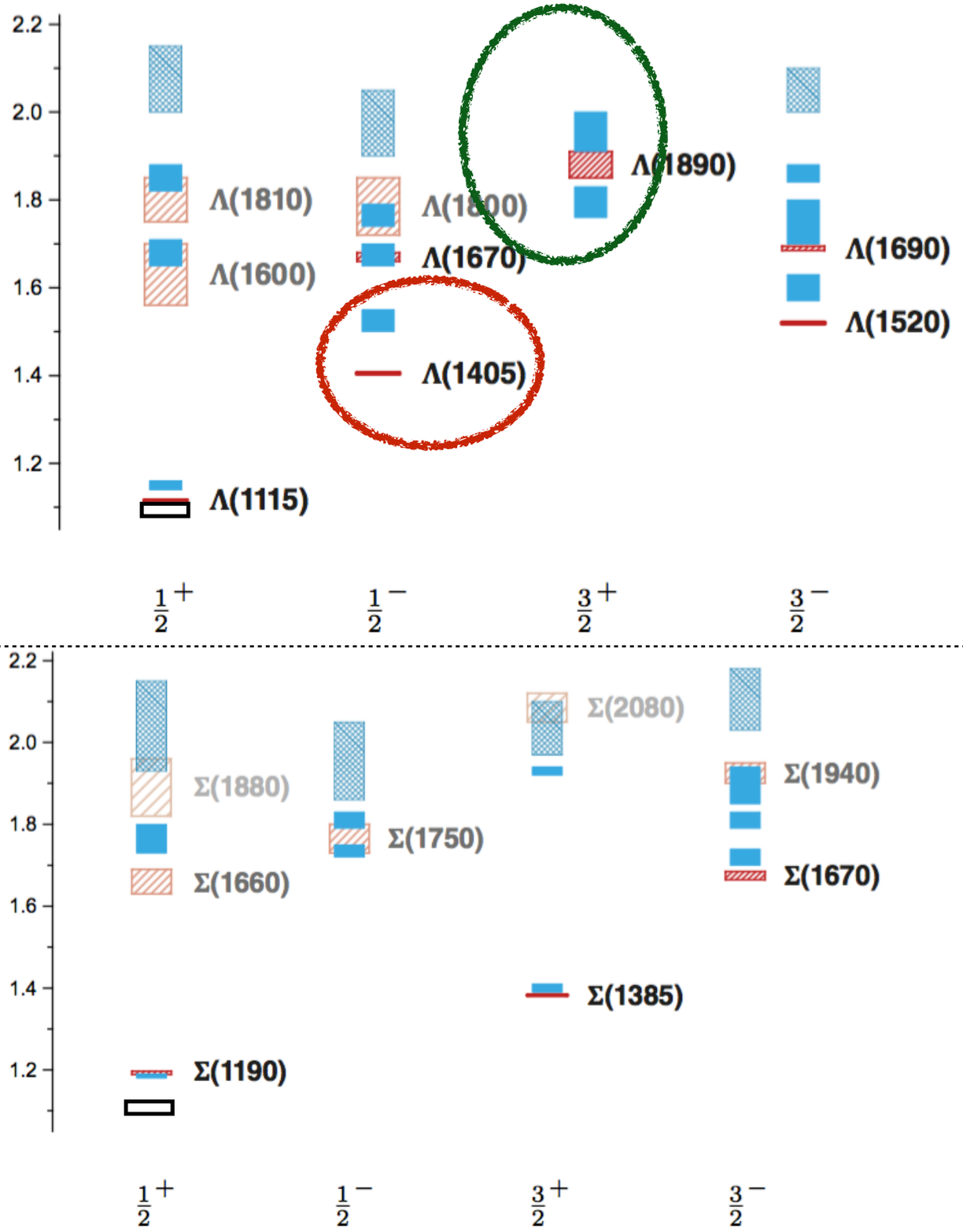

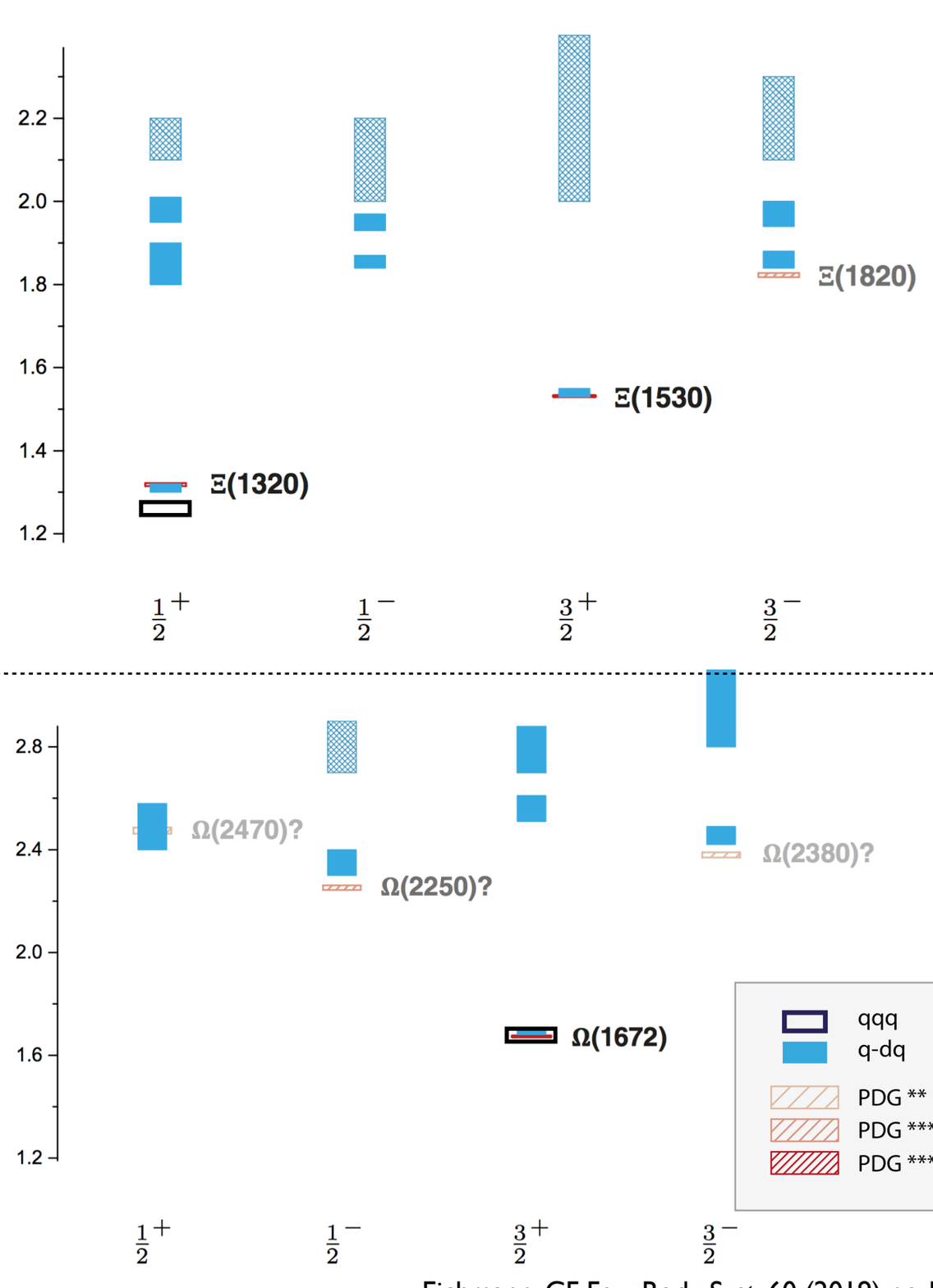

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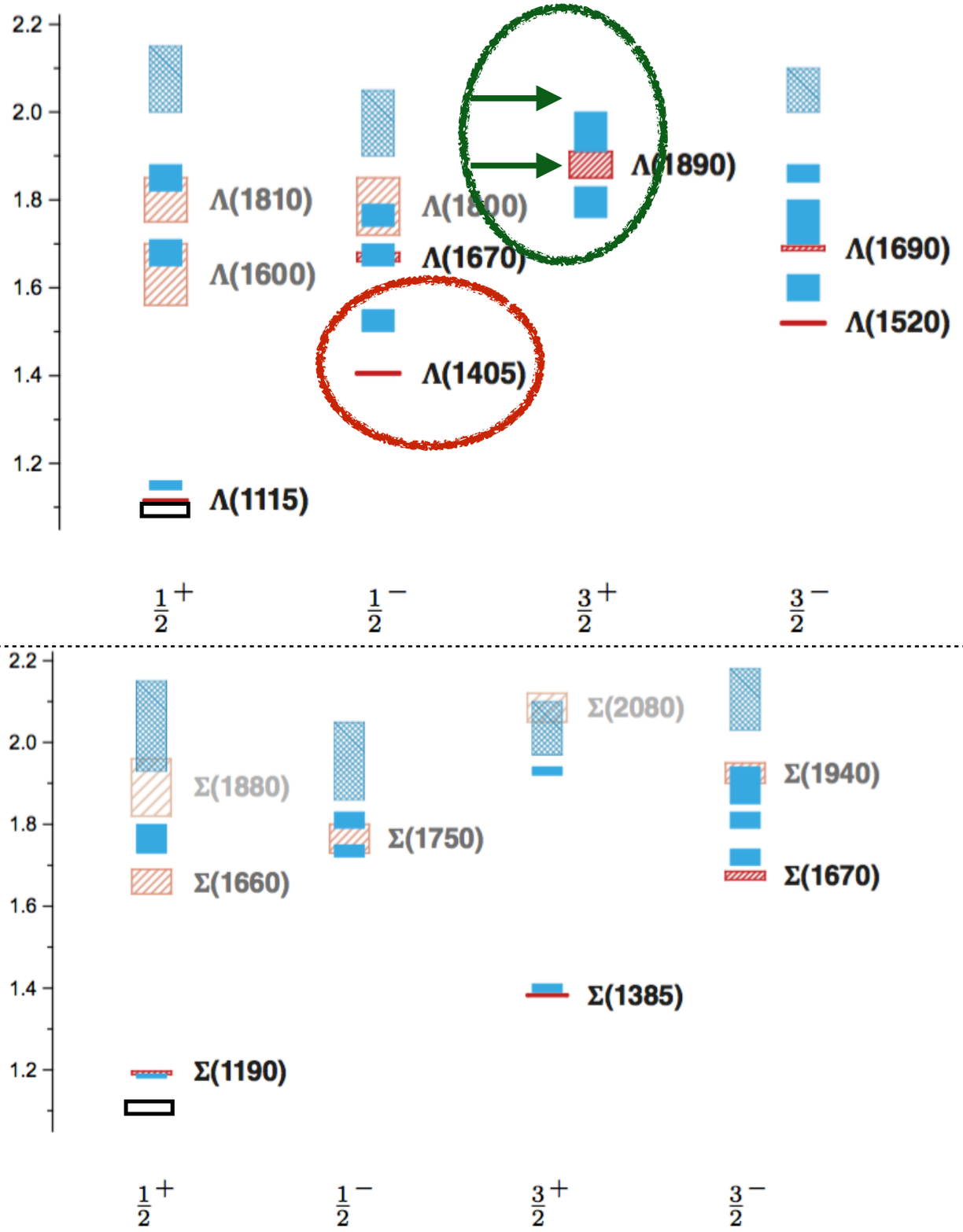

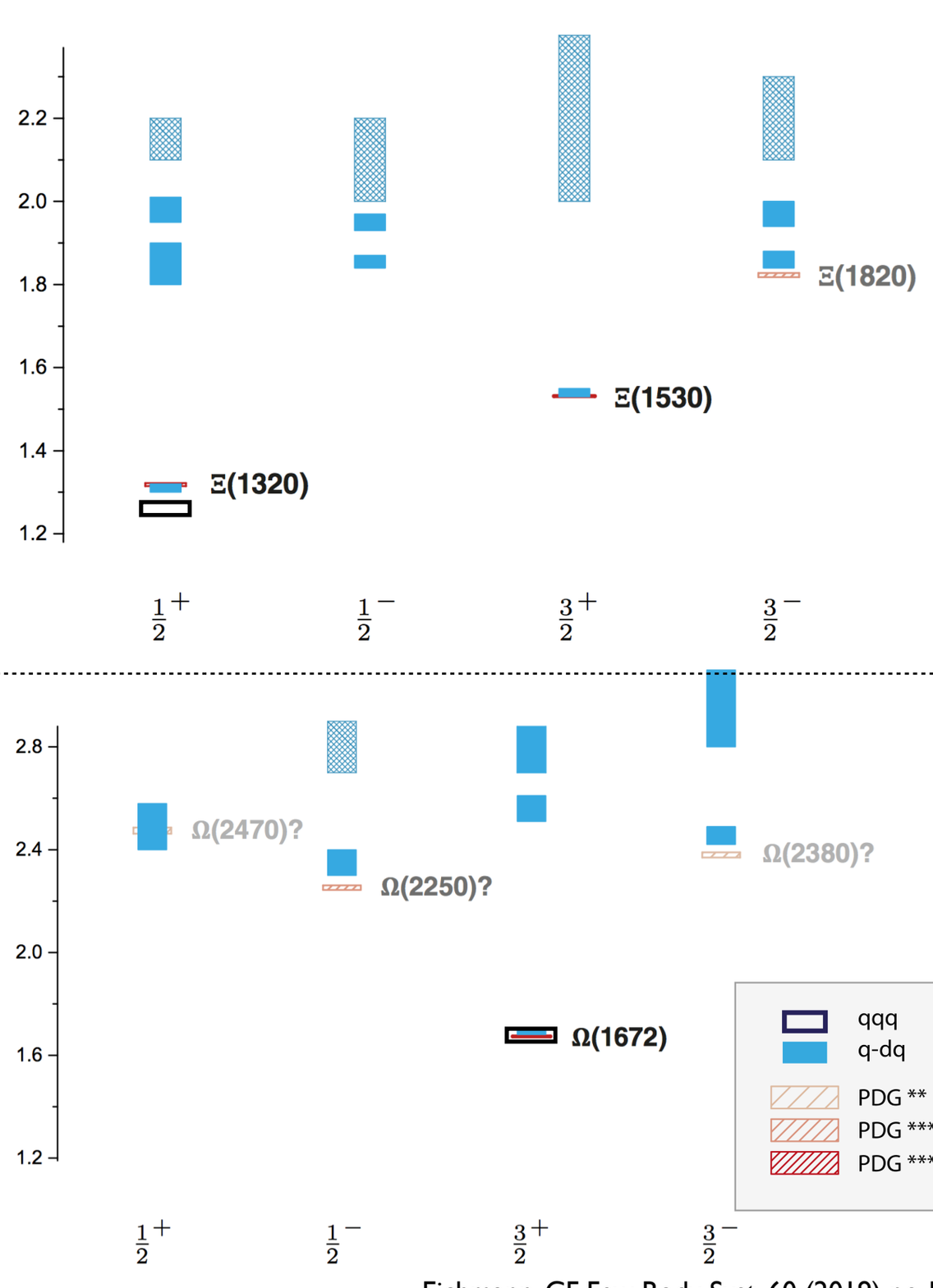

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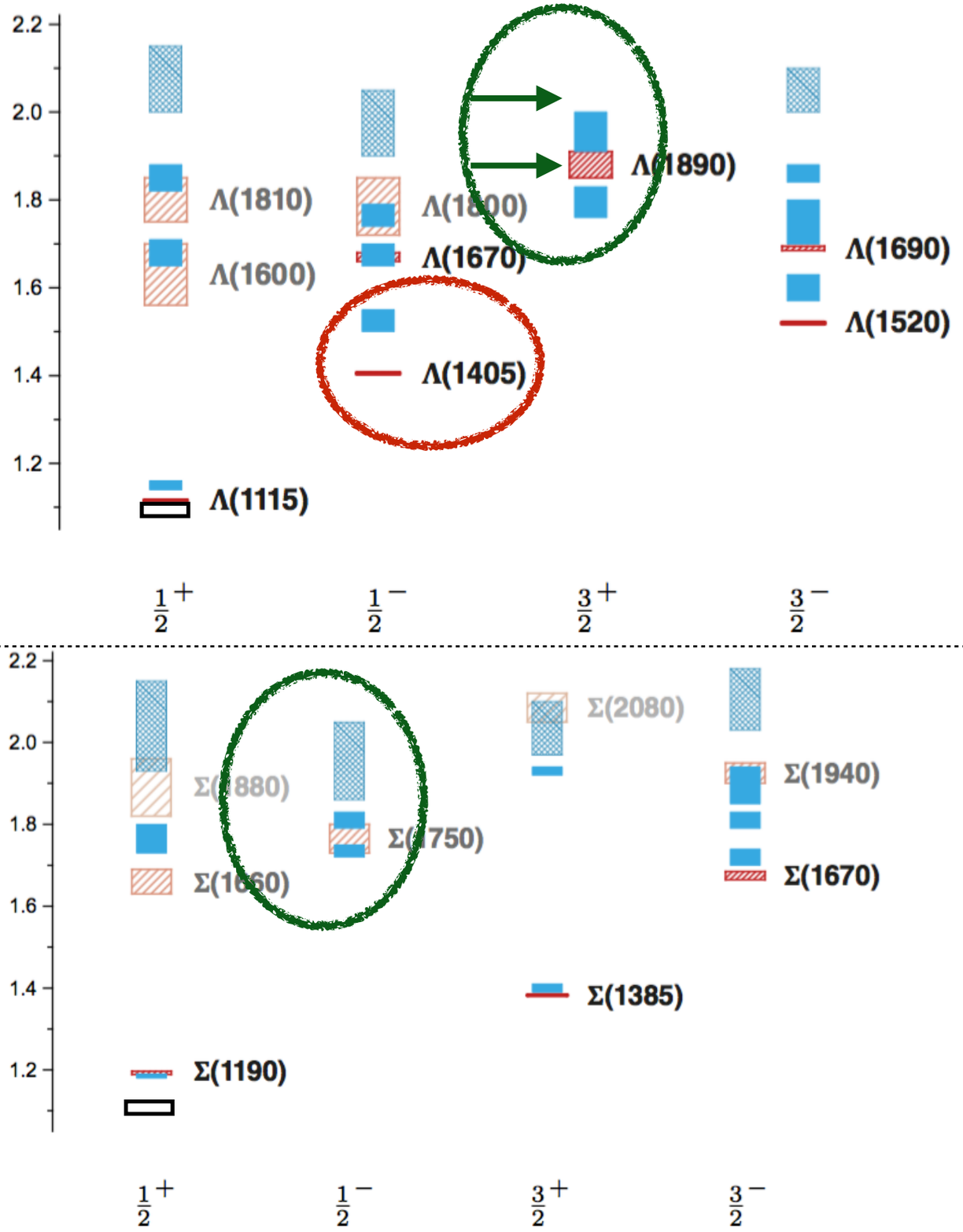

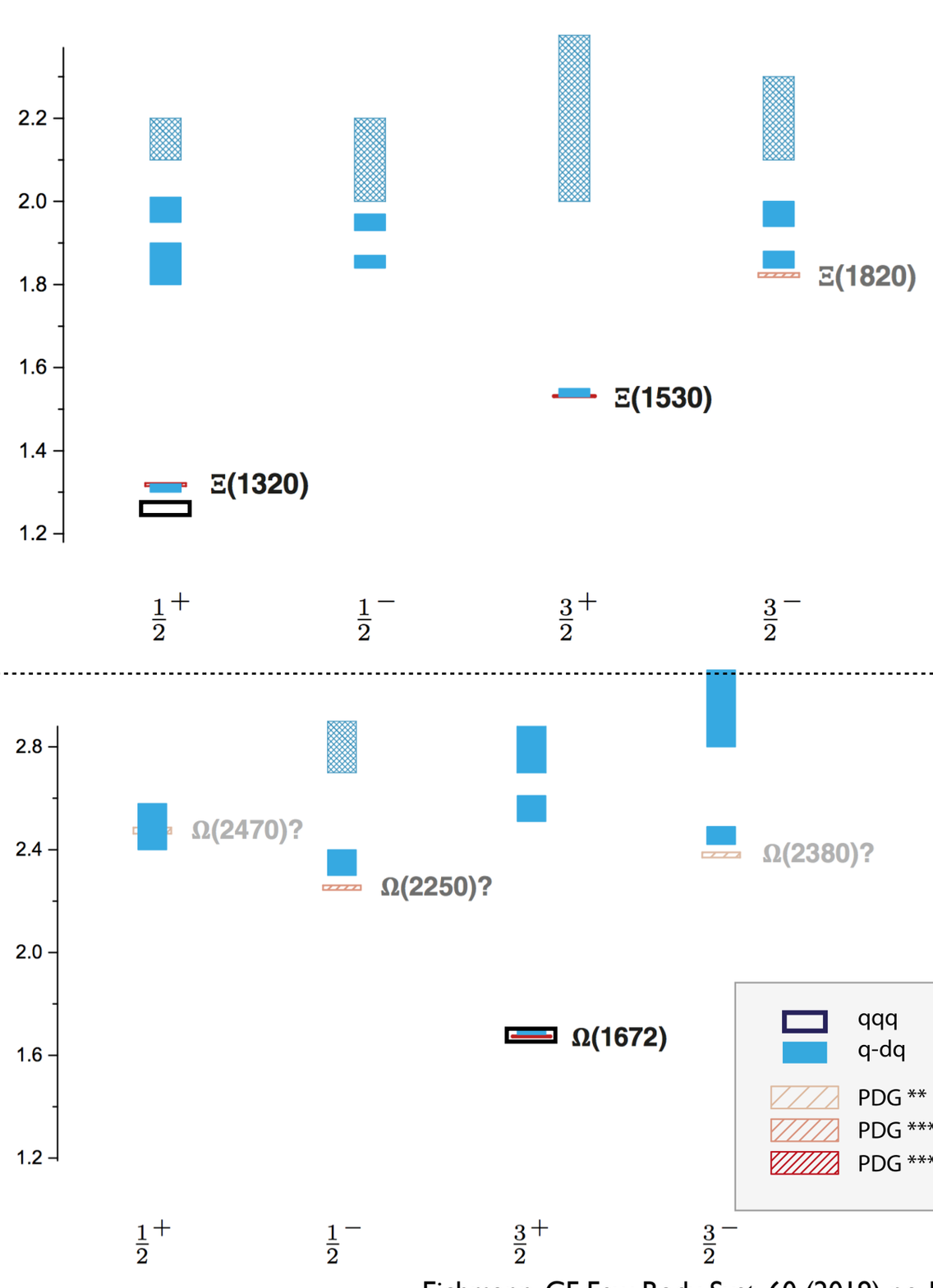

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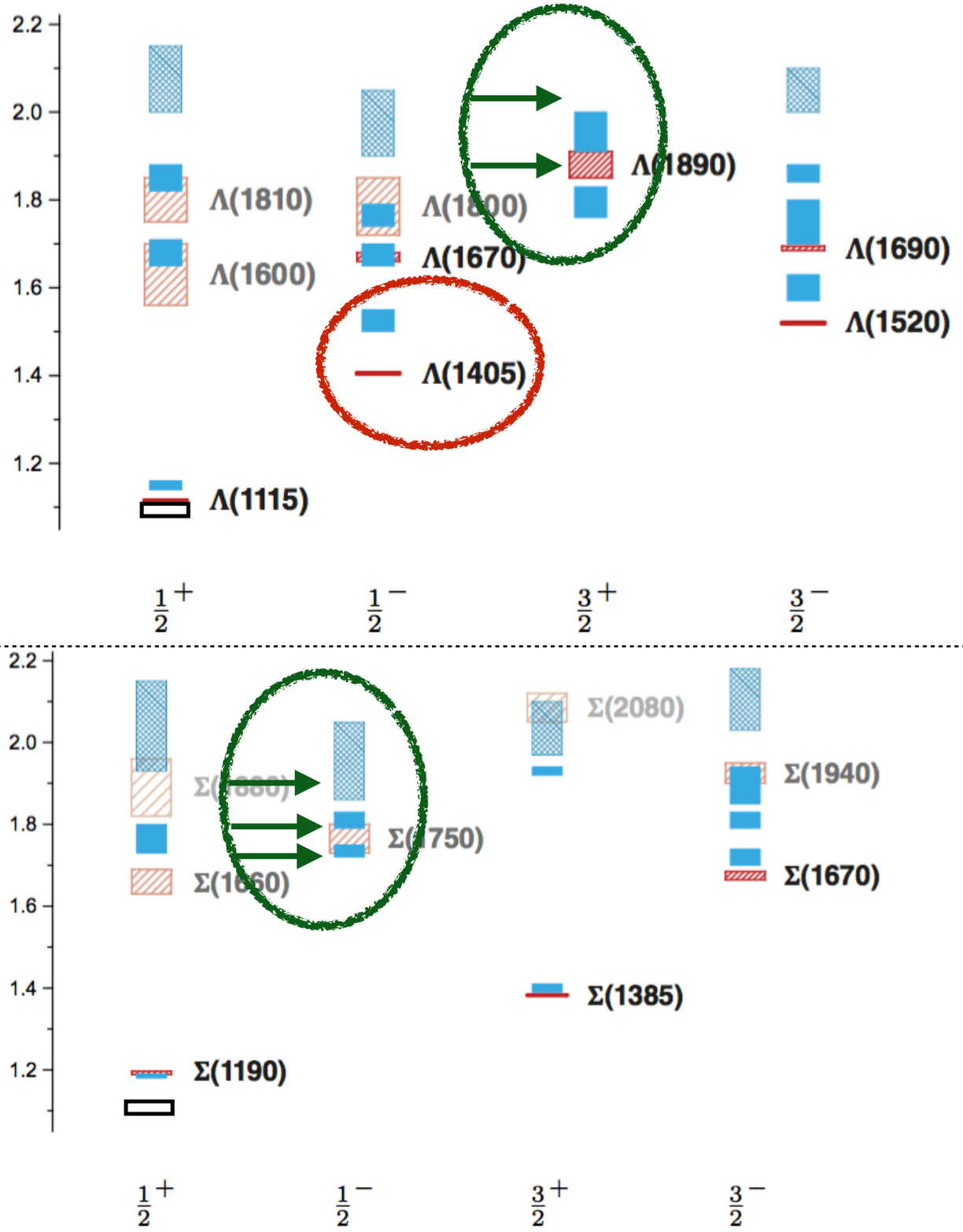

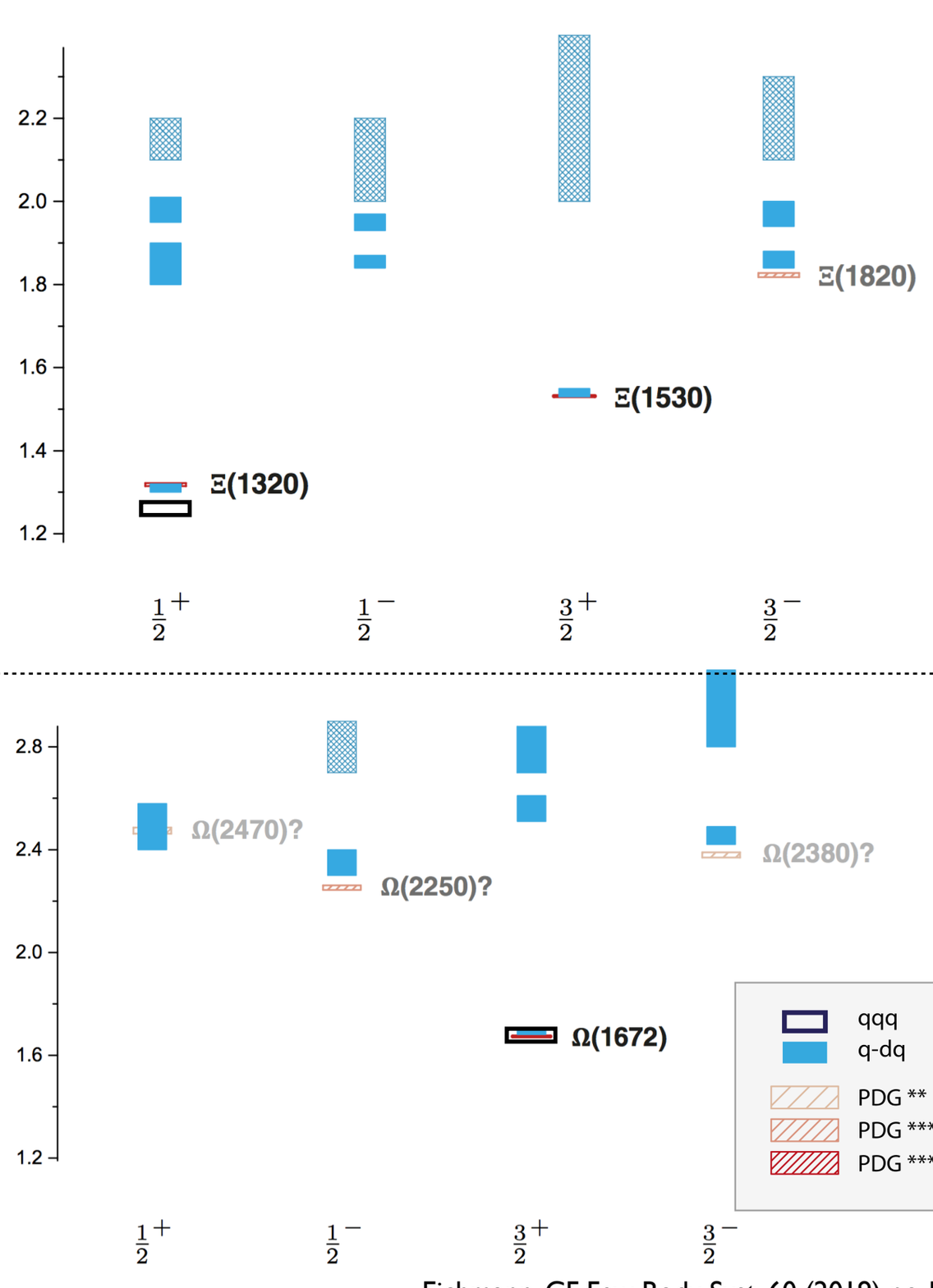

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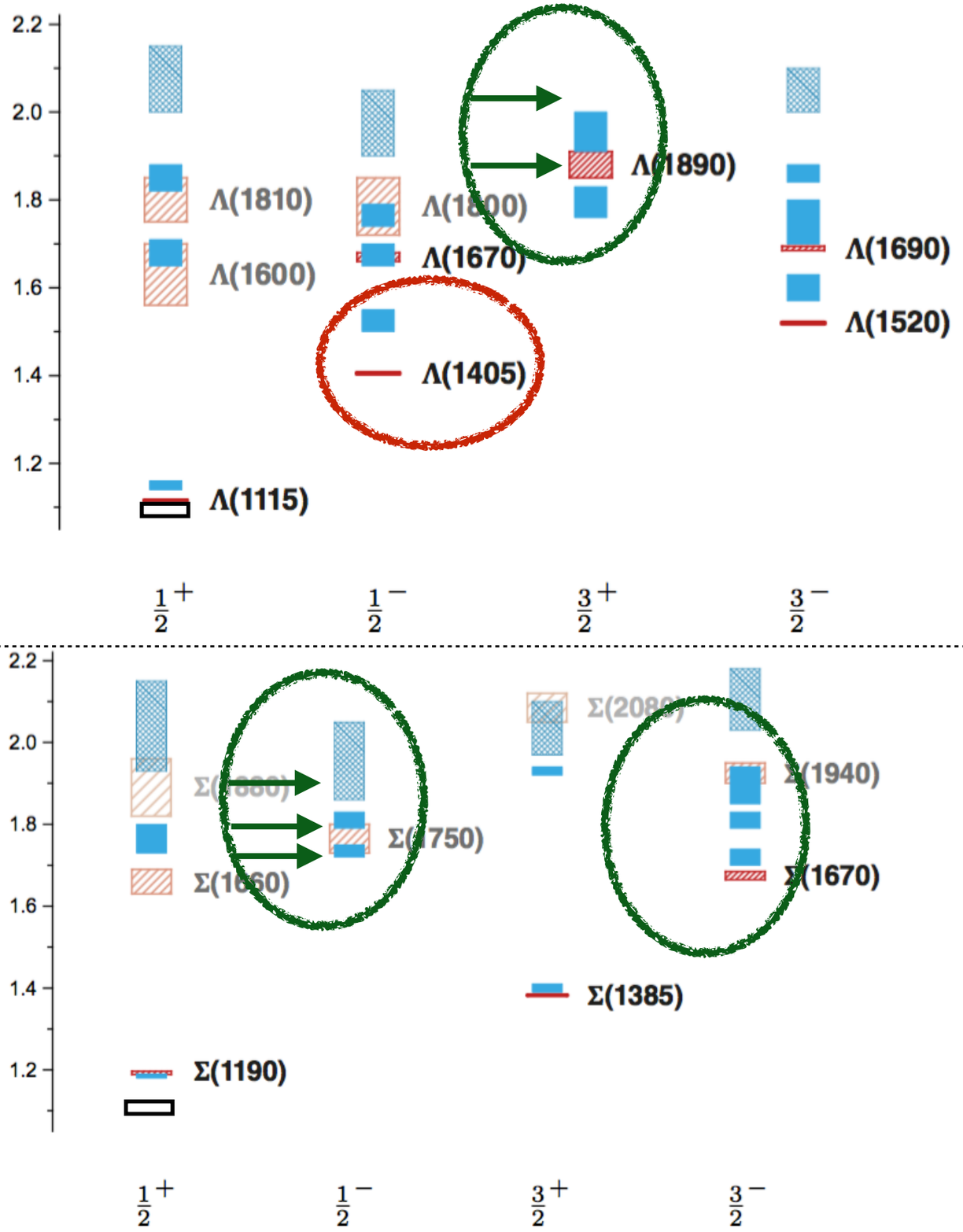

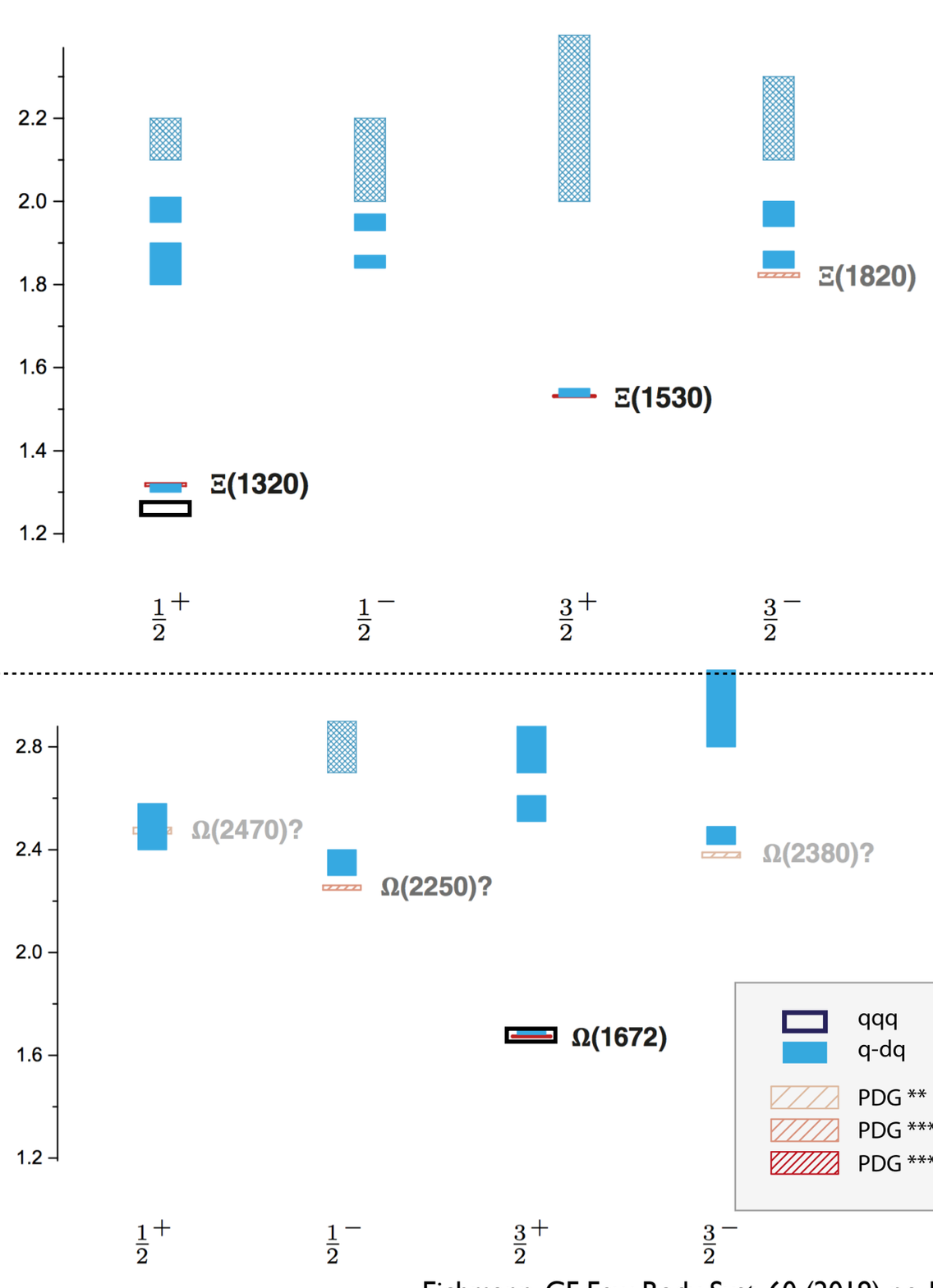

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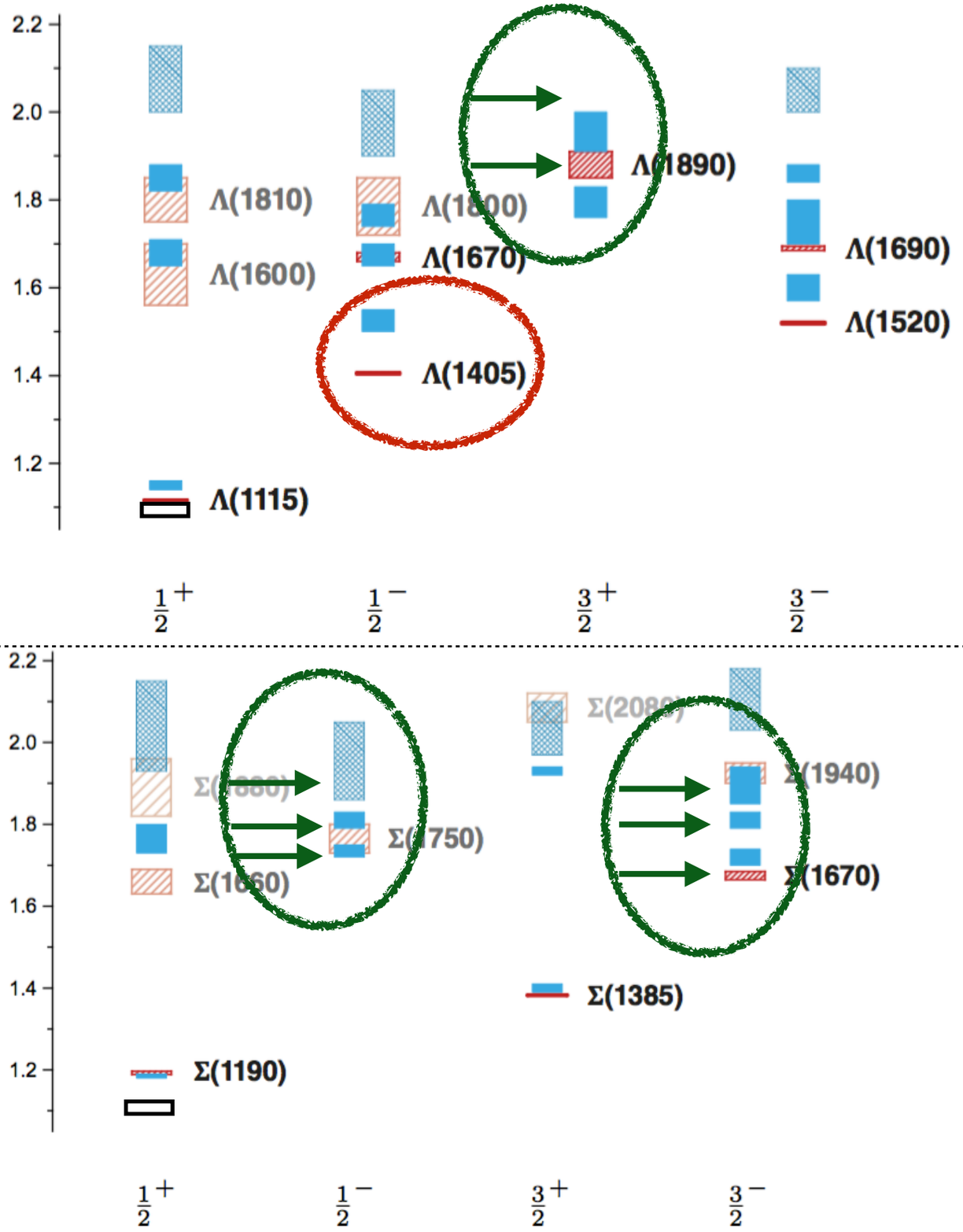

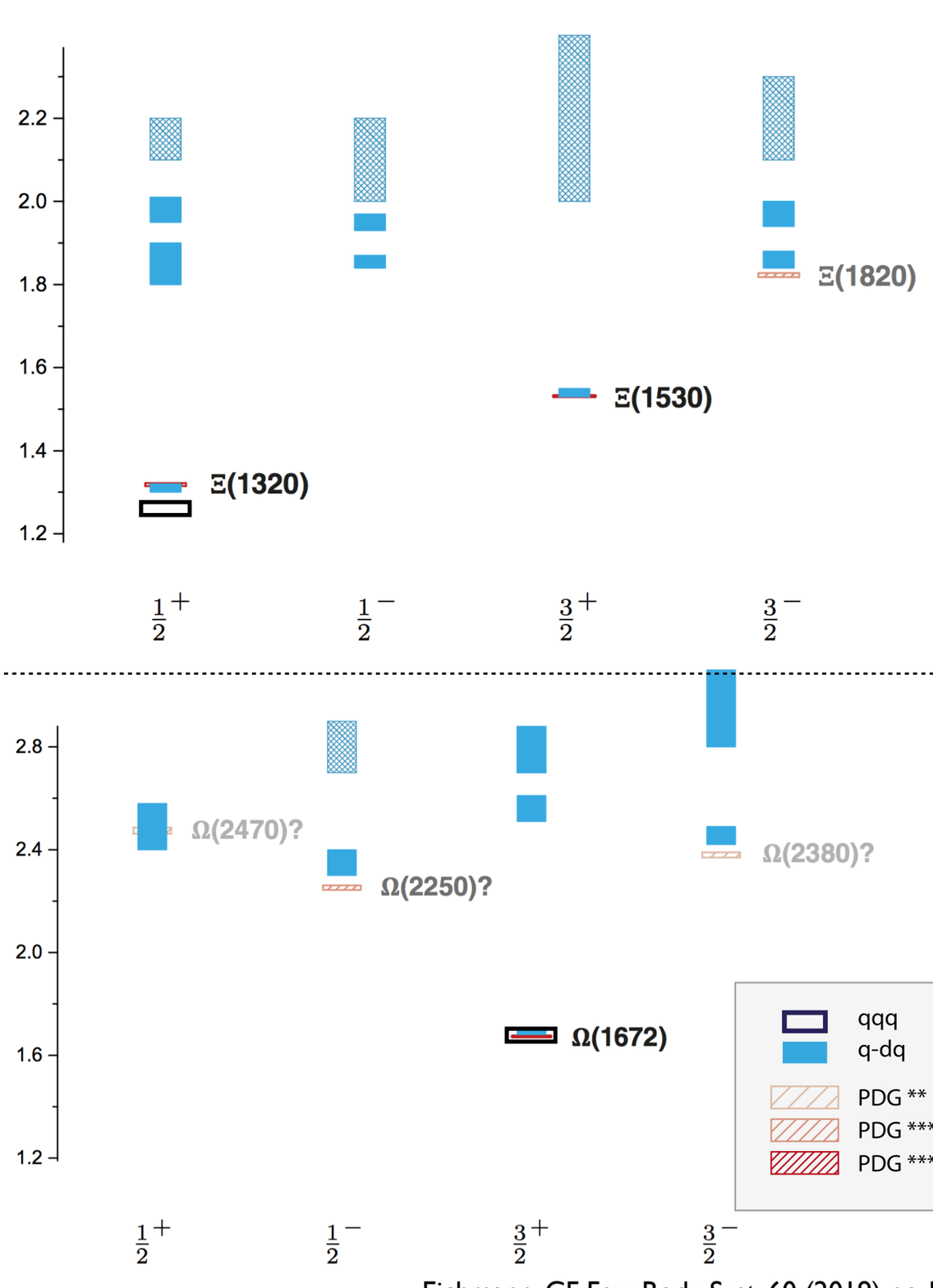

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