Cluster of Excellence PRISMA+ Precision Physics, Fundamental Interactions and Structure of Matter

MPA MAINZ PHYSICS ACADEMY

How to detect:
Ultra-High Frequency Gravitational Waves

MPA Retreat, , 2024

Kristof Schmieden*, Tim Schneemann*, Matthias Schott **

*: University of Mainz, **: University of Bonn

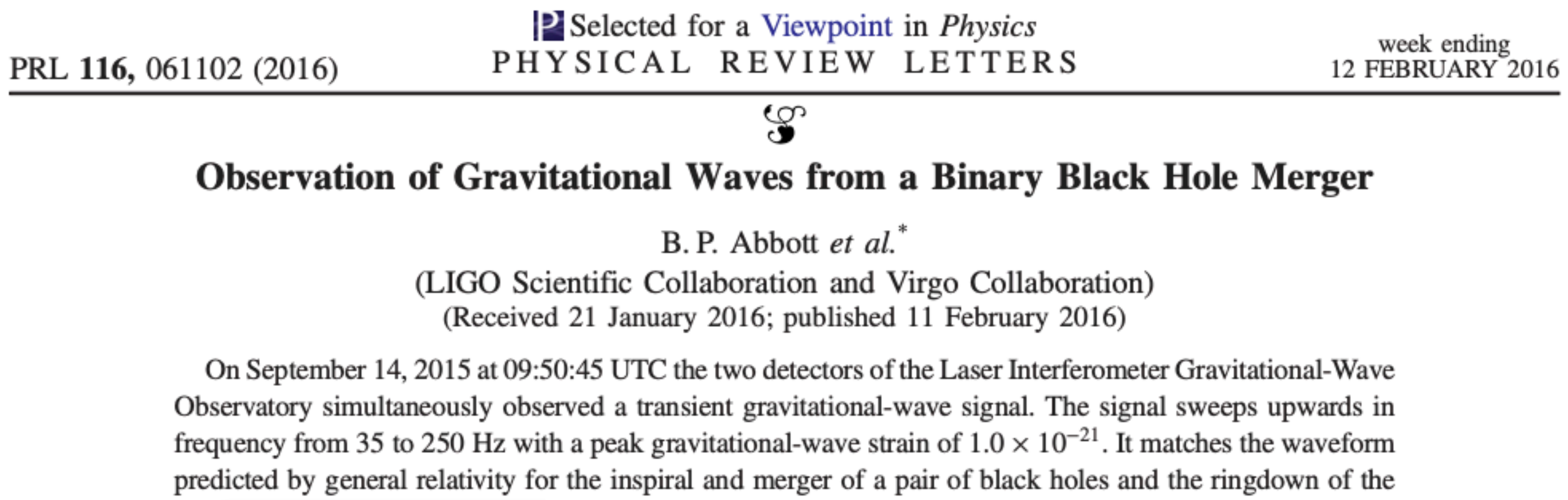
SUPAX

JG|U

Based on [arXiv:2308.11497]

Introduction - Gravitational Waves

- 2016 breakthrough in fundamental physics:
  - Observation of gravitational waves by LIGO / Virgo



Selected for a Viewpoint in *Physics*

PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.*[*]

(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the

The New York Times

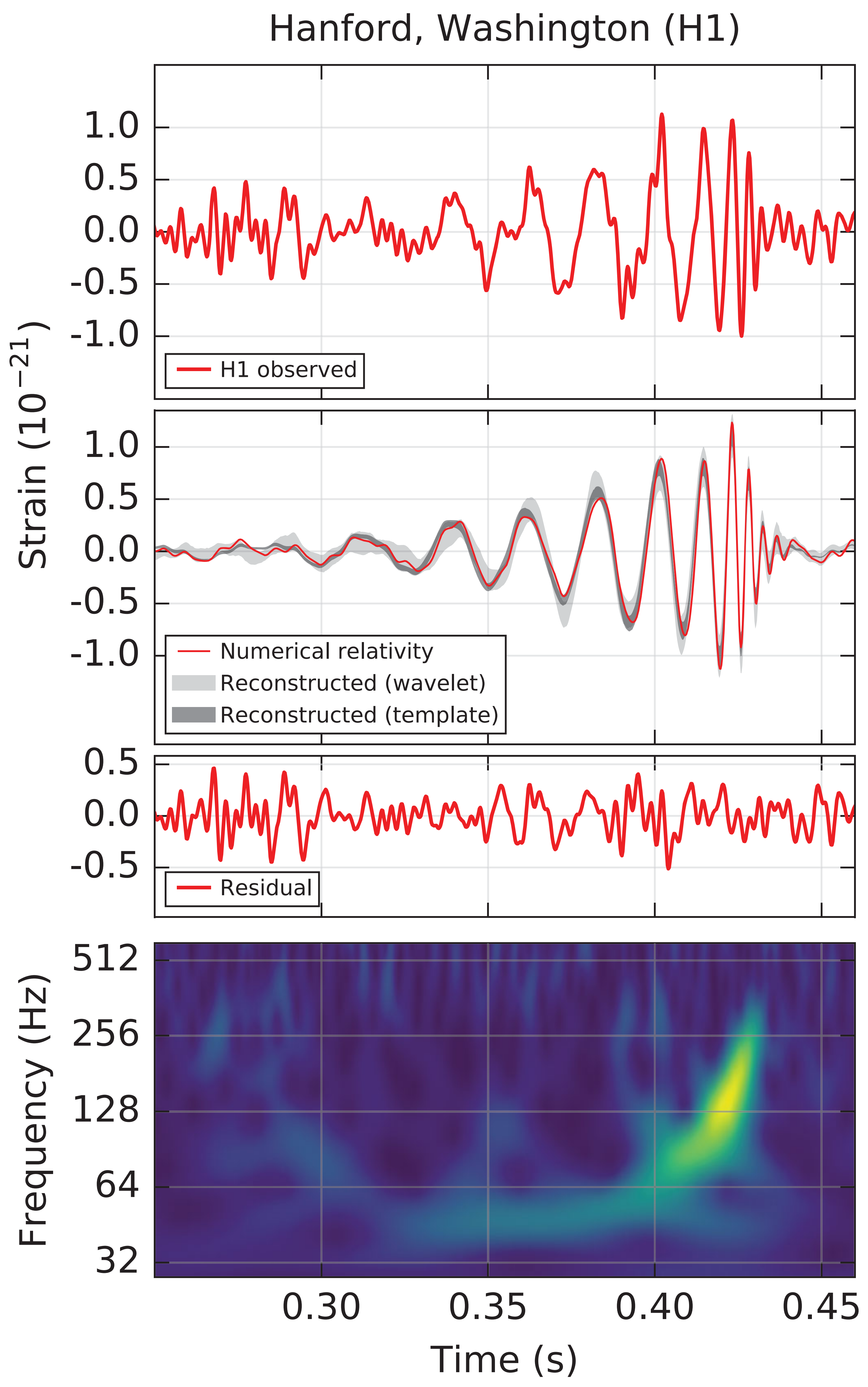
OUT THERE

Gravitational Waves Detected, Confirming Einstein's Theory

Gravitational waves: breakthrough discovery after a century of expectation

Scientists announce discovery of clear gravitational wave signal, ripples in spacetime first predicted by Albert Einstein

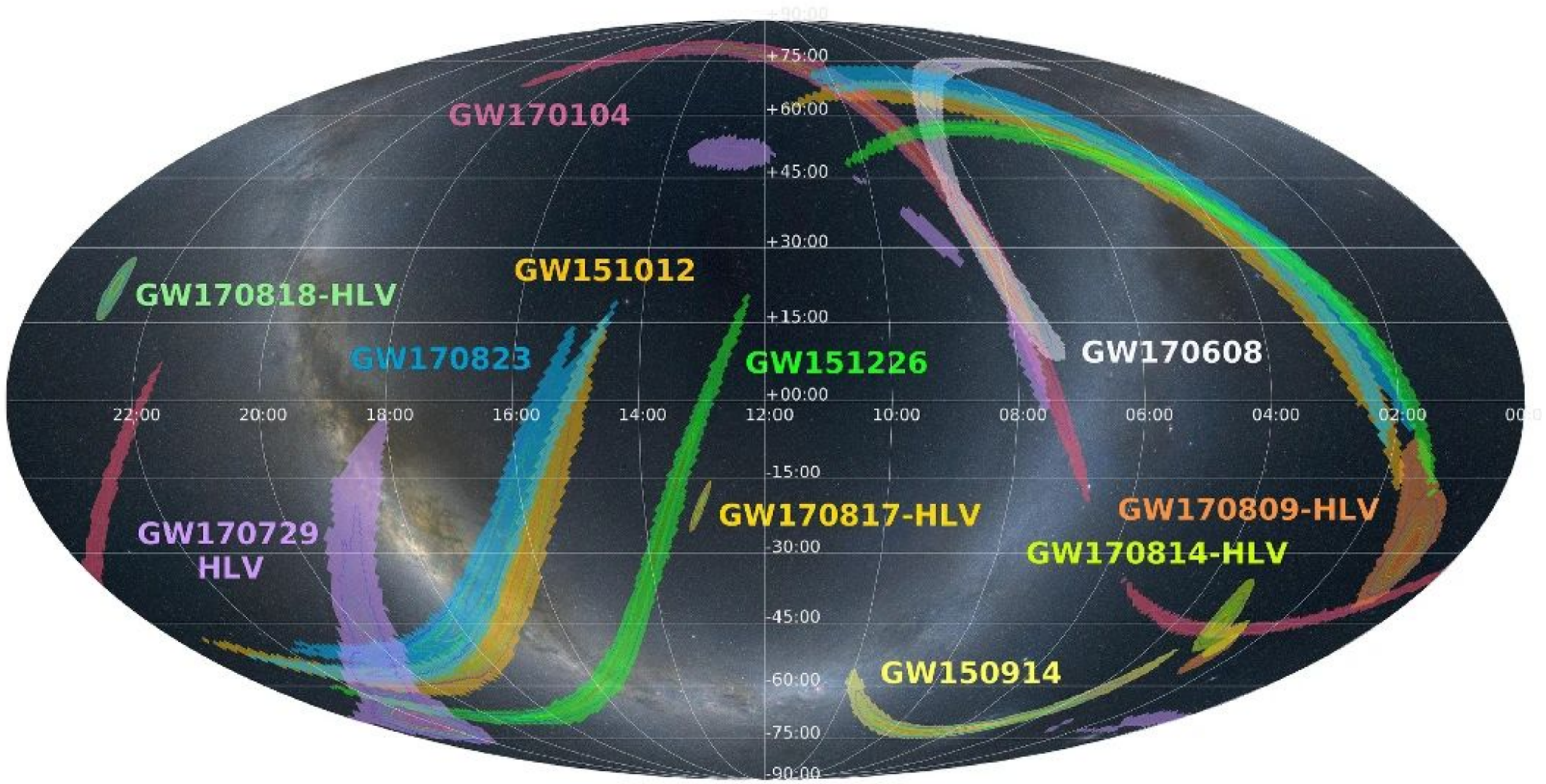
- Frequency range: 10-1000 Hz

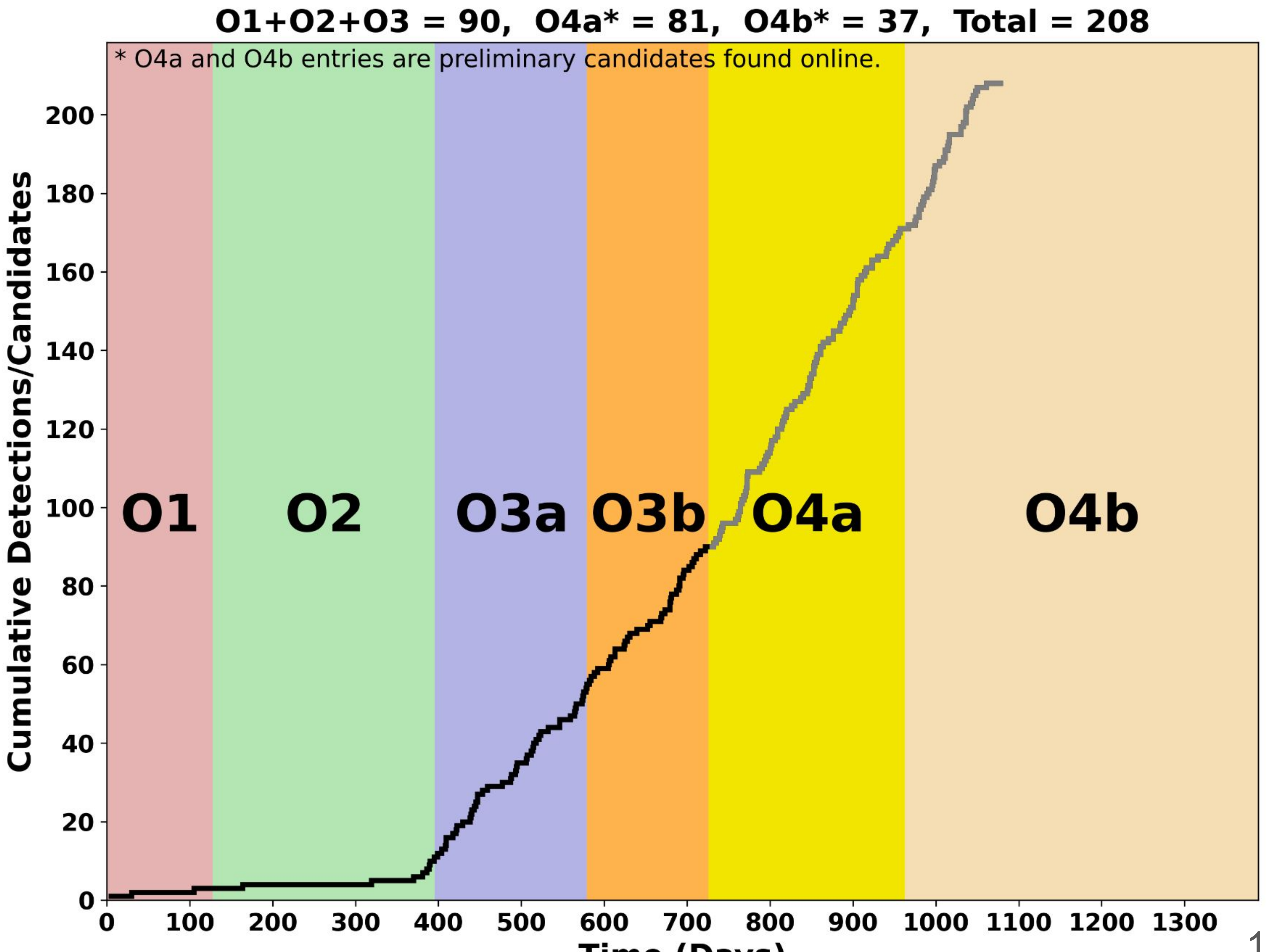


[PRL 116, 061102 (2016)]

Introduction - Gravitational Waves

- 8 years later:
  - 90 observed GW events, > 200 Candidate events
  - Able to start statistical analysis
  - New observational window into the universe established





LIGO-G2302098(527814e7), updated on 5 August, 2024

Credit: LIGO-Virgo-KAGRA Collaboration

1

Introduction - Gravitational Waves

- 2023: First observation of GW in Pulsar timing array data

[Gabriella Agazie et al 2023 ApJL 951 L8]

The Astrophysical Journal Letters, 951:L8 (24pp), 2023 July 1 https://doi.org/10.3847/2041-8213/acdac6

© 2023. The Author(s). Published by the American Astronomical Society.

OPEN ACCESS

CrossMark

The NANOGrav 15 yr Data Set: Evidence for a Gravitational-wave Background

Gabriella Agazie[1], Akash Anumarlapudi[1], Anne M. Archibald[2], Zaven Arzoumanian[3], Paul T. Baker[4], Bence Bécsy[5], Laura Blecha[6], Adam Brazier[7,8], Paul R. Brook[9], Sarah Burke-Spolaor[10,11], Rand Burnette[5], Robin Case[5], Maria Charisi[12], Shami Chatterjee[7], Katerina Chatziioannou[13], Belinda D. Cheeseboro[10,11], Siyuan Chen[14]

Introduction - Gravitational Waves

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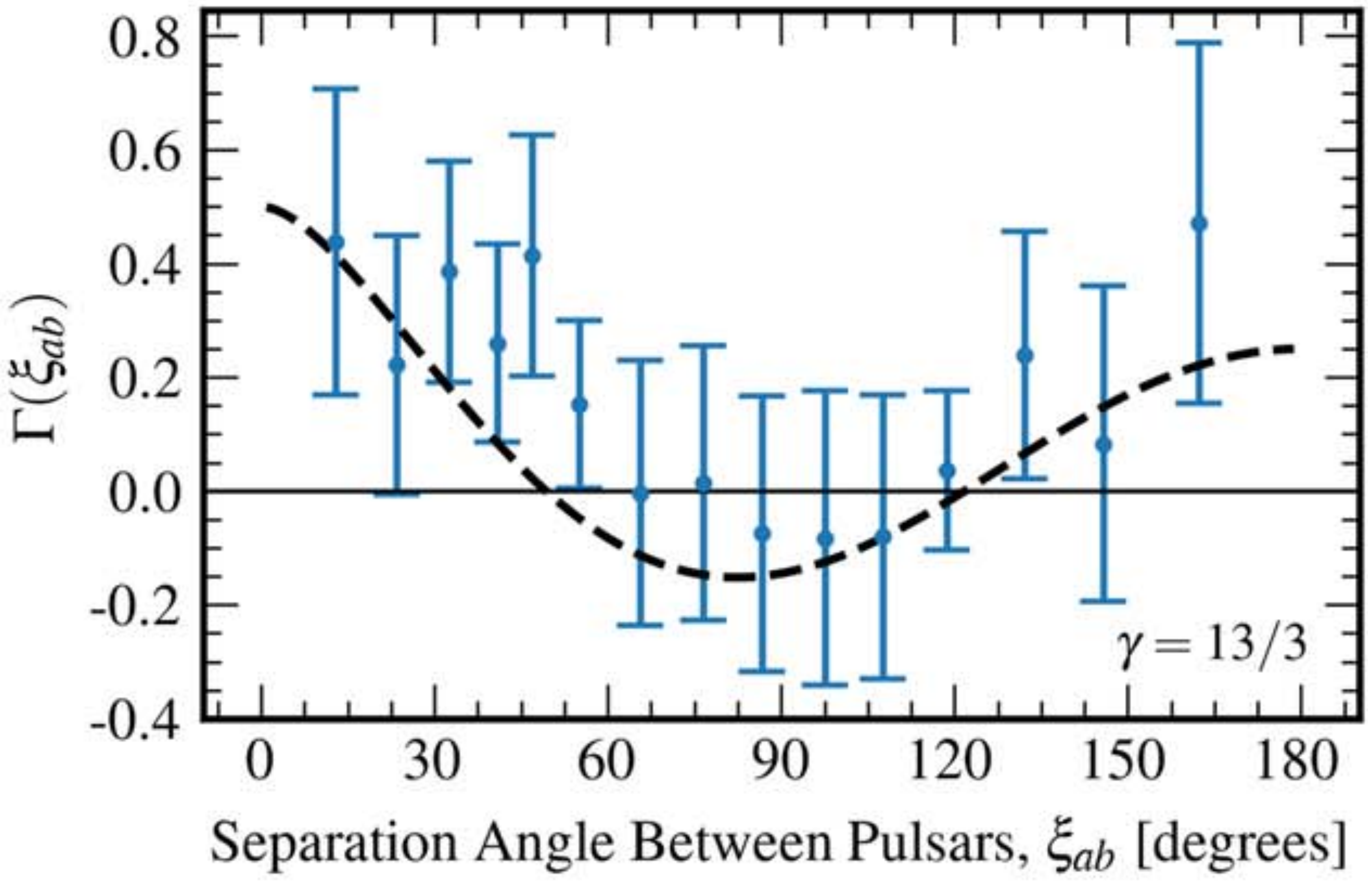
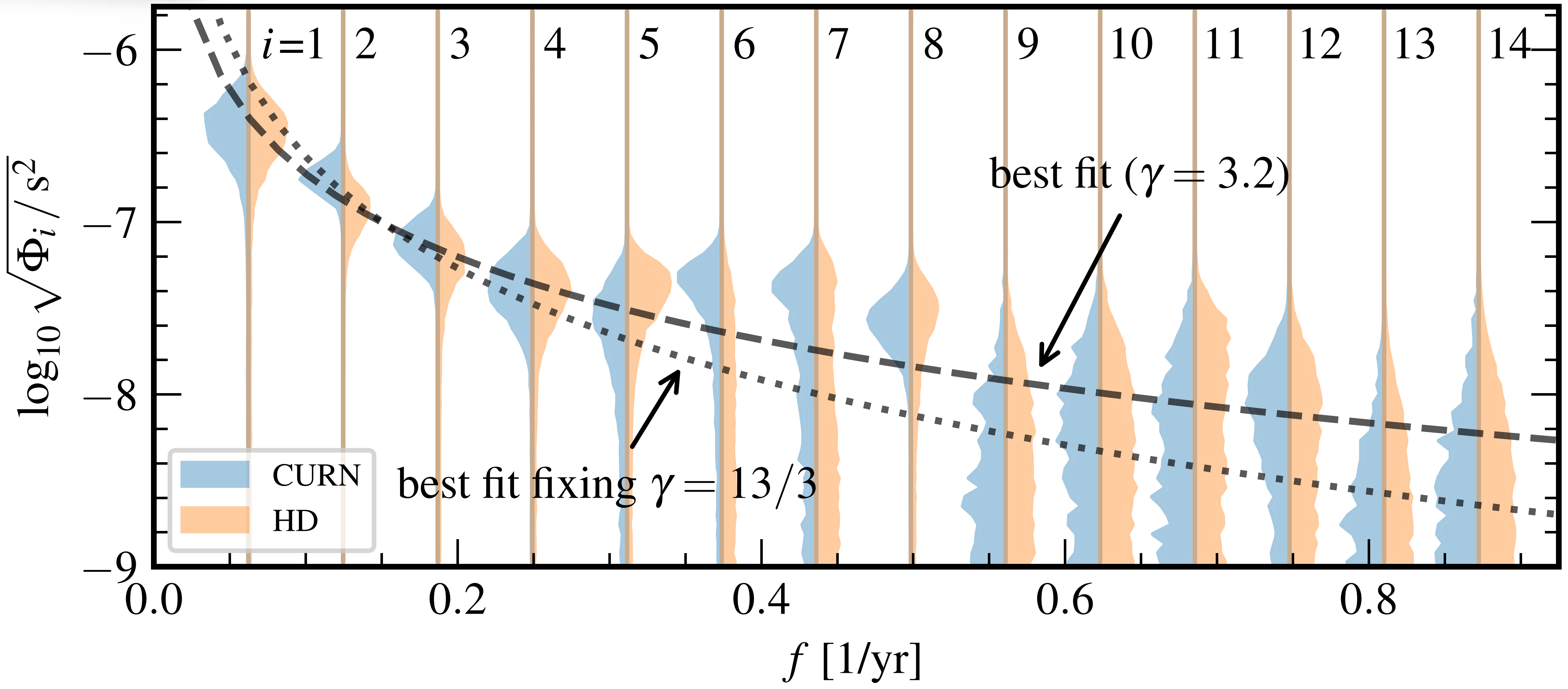
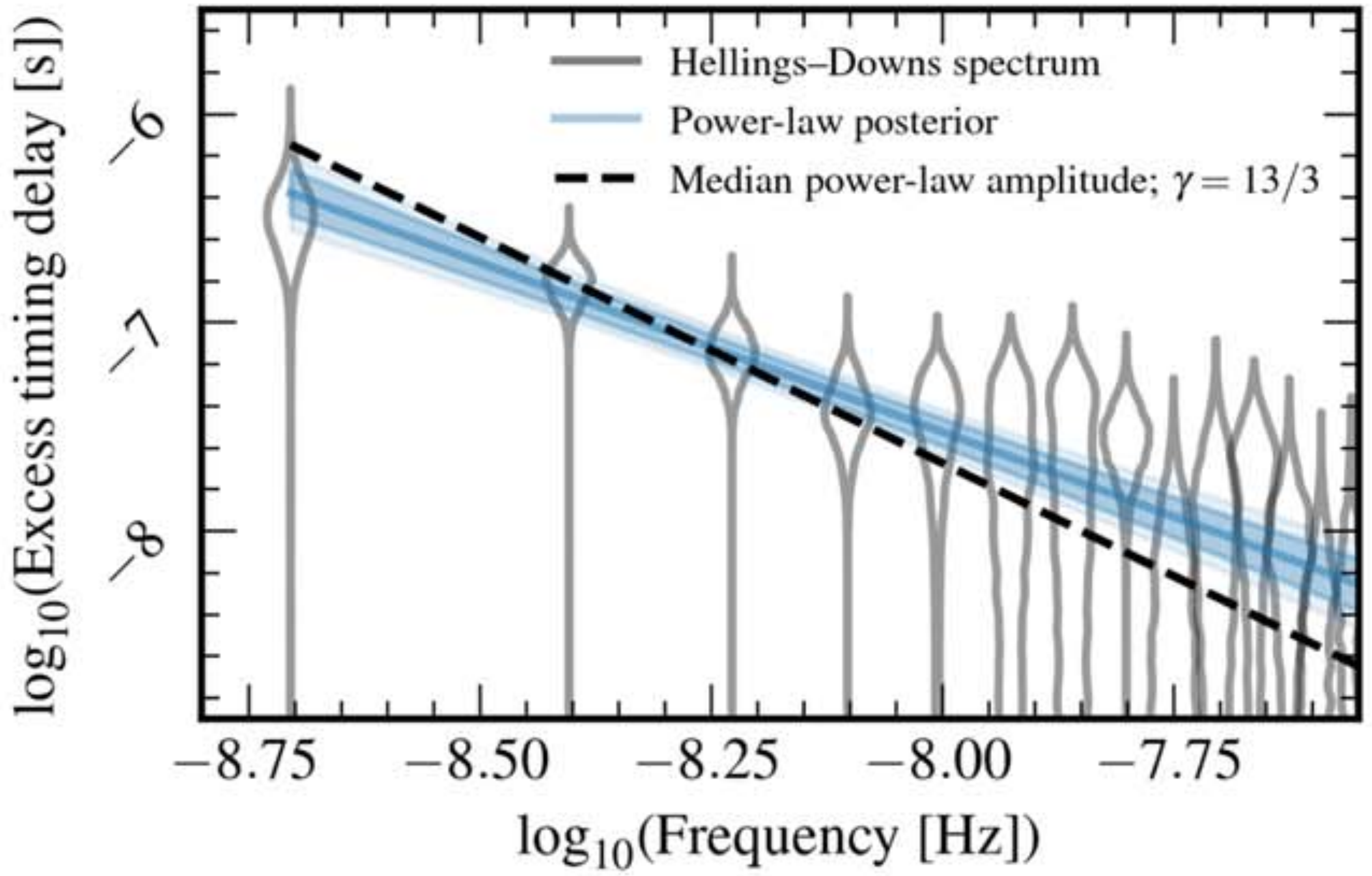
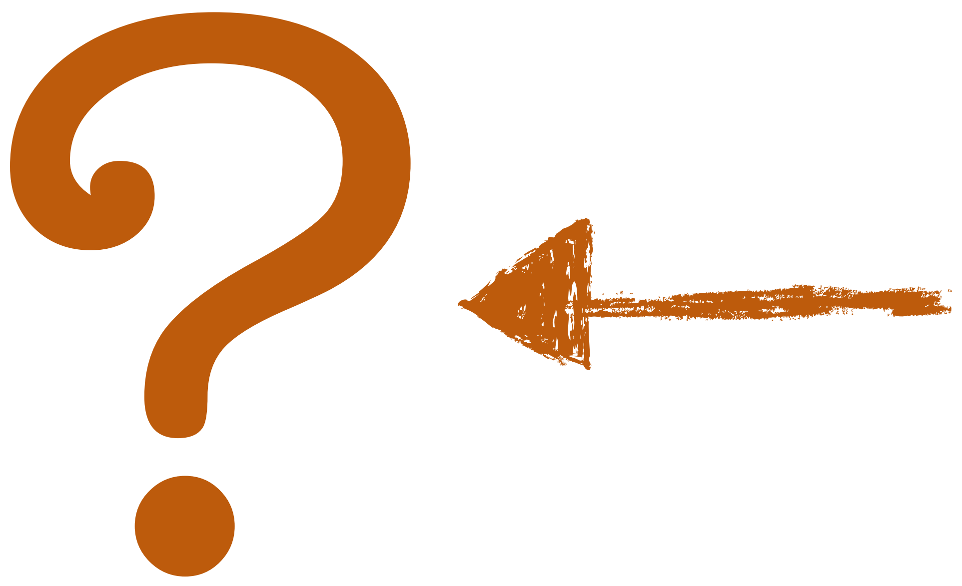
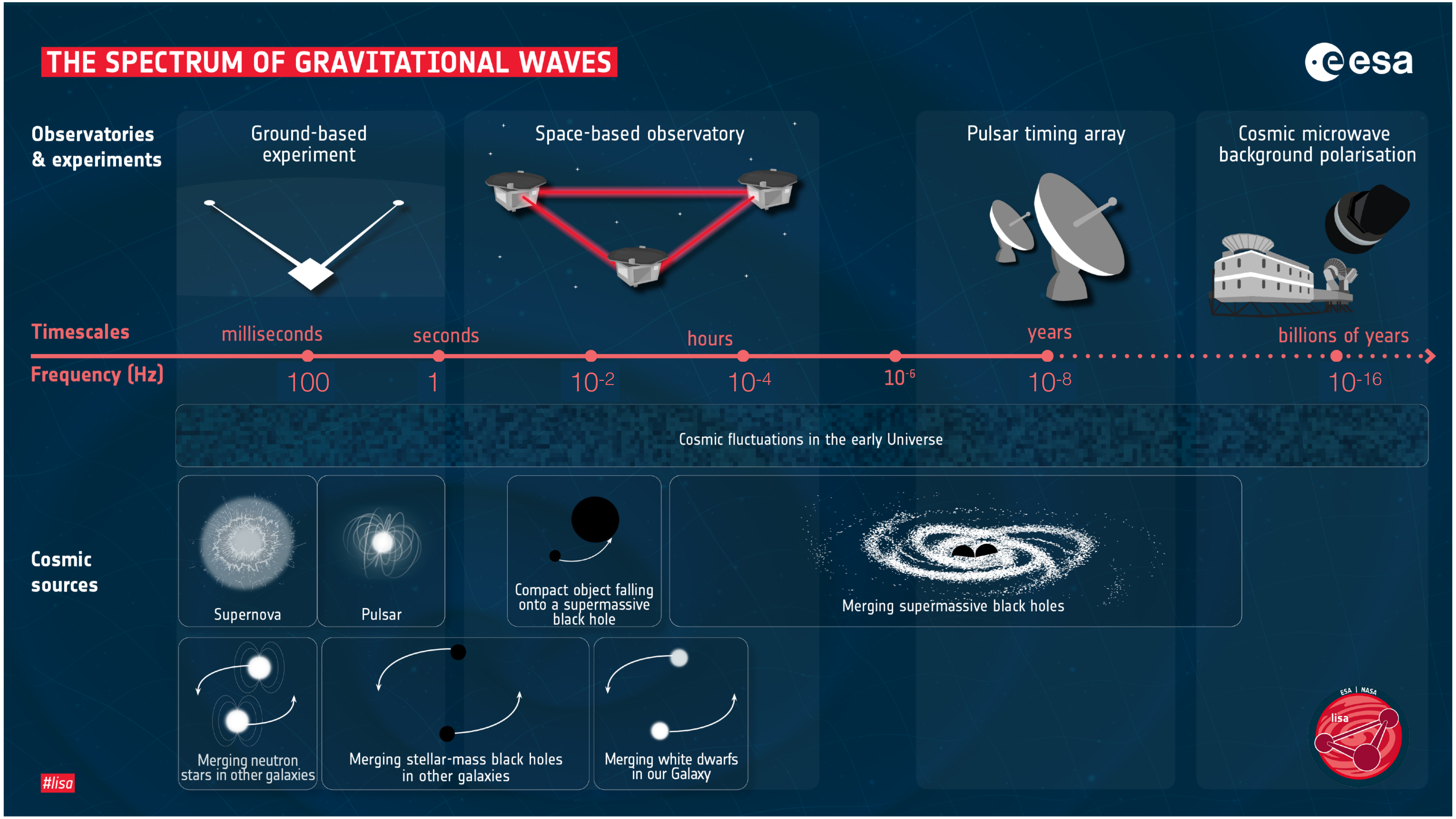






Introduction - Gravitational Waves





[https://www.esa.int/]

Introduction - Gravitational Waves

What are gravitational waves?

Introduction - Gravitational Waves

Cosmological constant * metric tensor

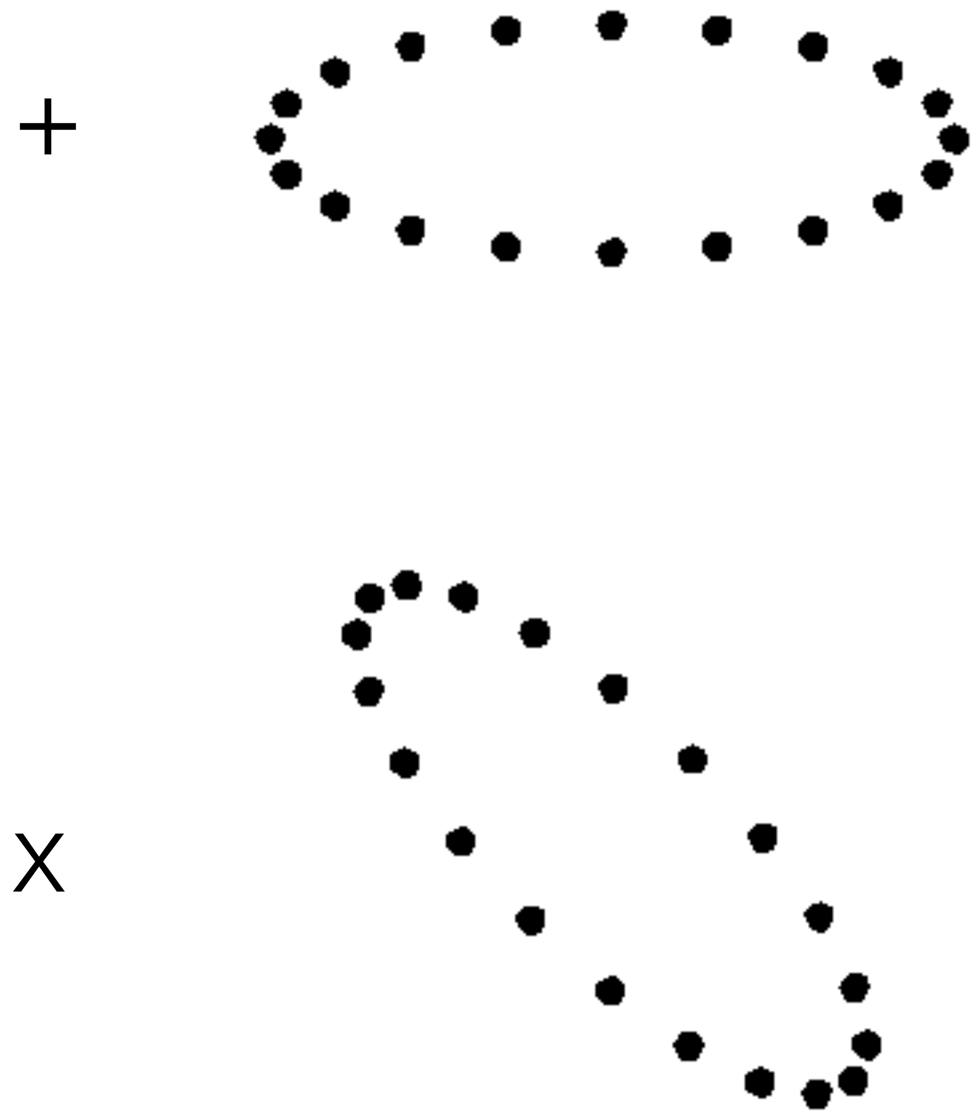
$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

Energy-Momentum tensor

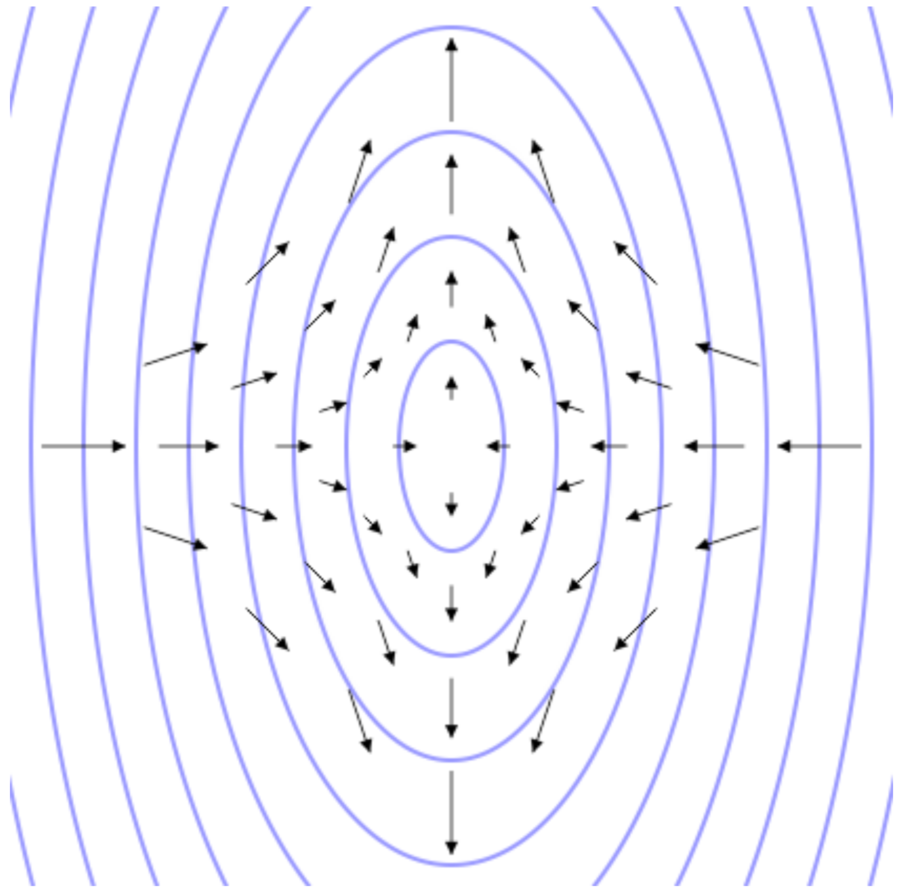
Einstein tensor

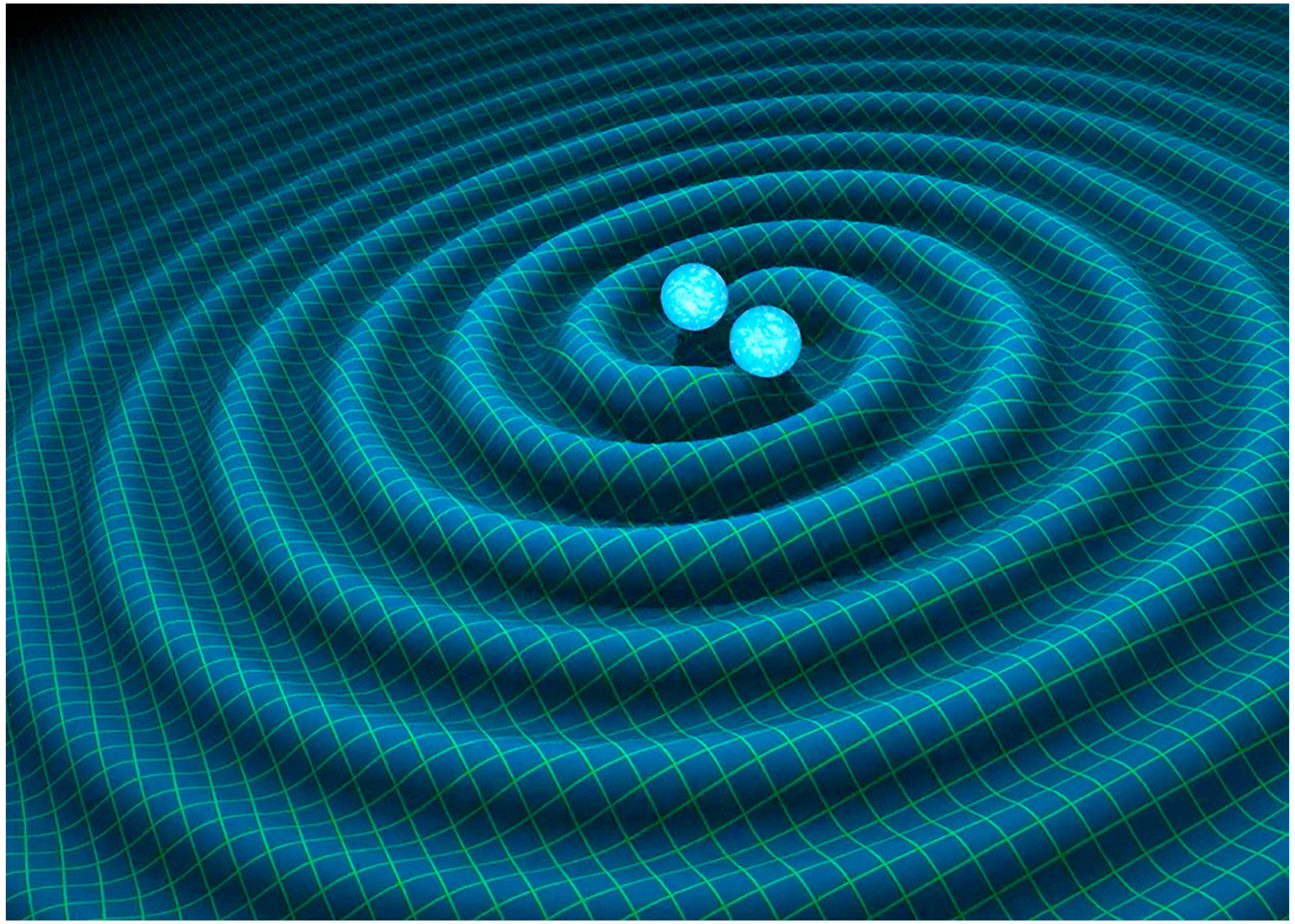
$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu}$$

- Wave solution of Einstein equations:
  - 2 Polarisations



Quadrupole structure





Introduction - Gravitational Waves

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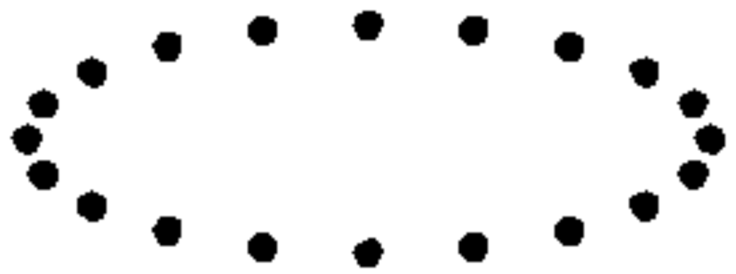
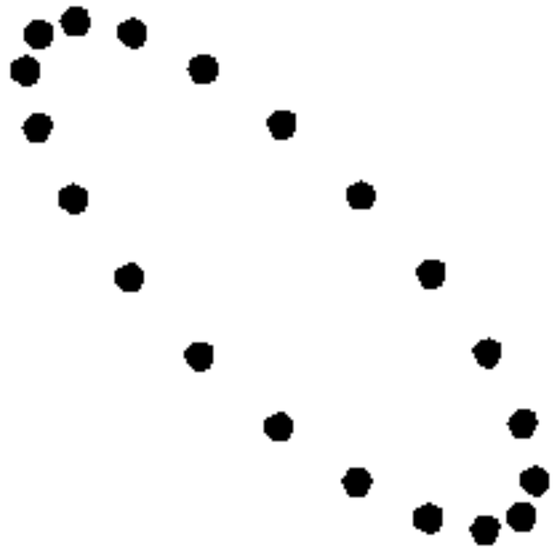
+



X



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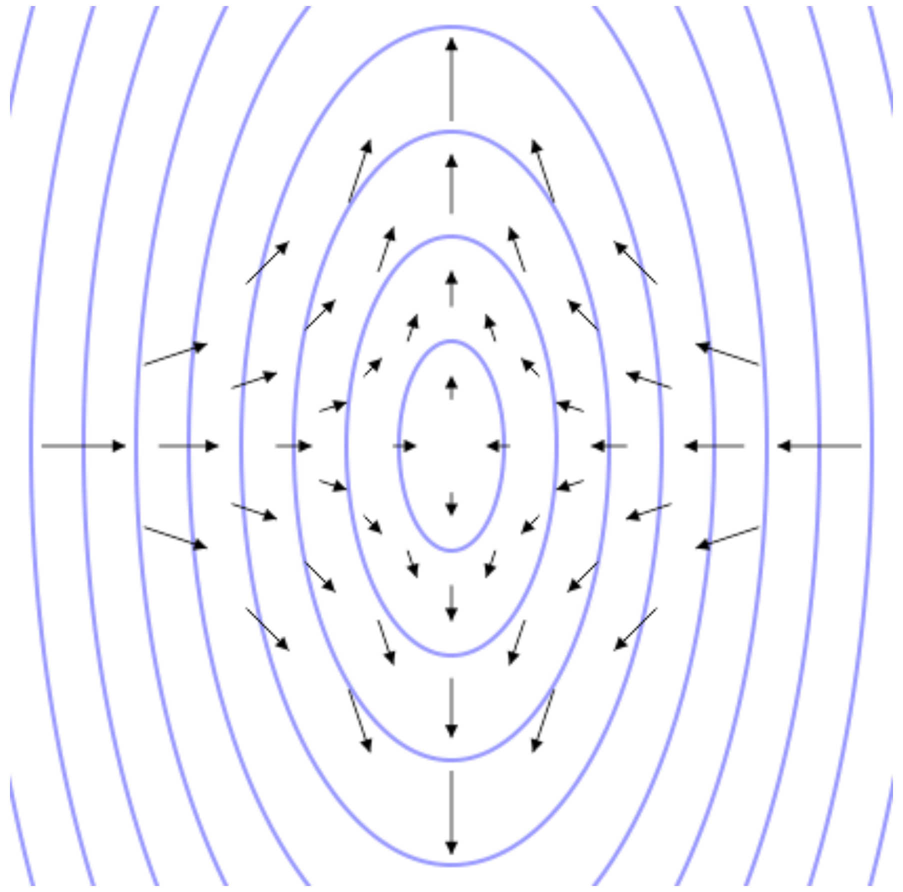
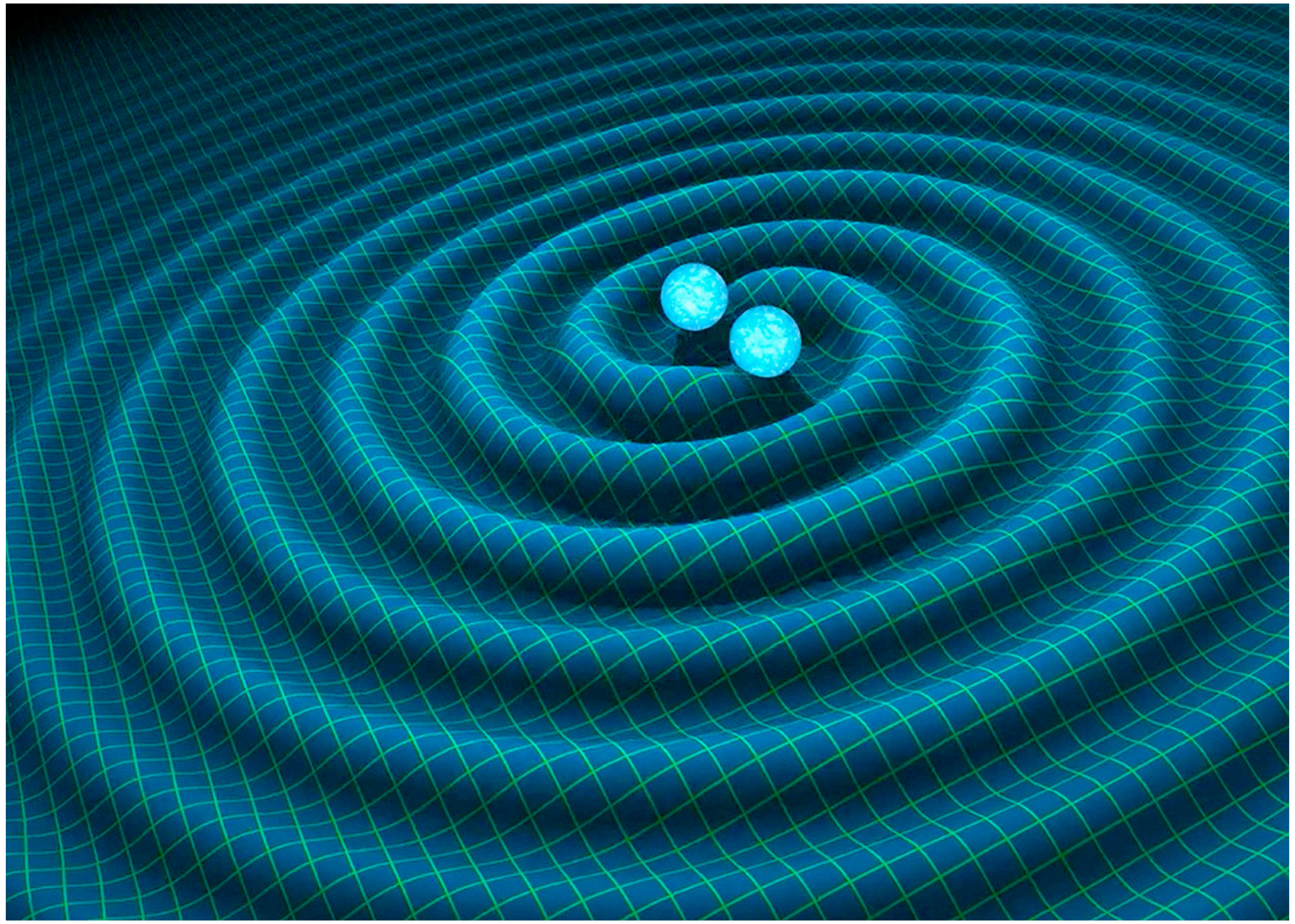
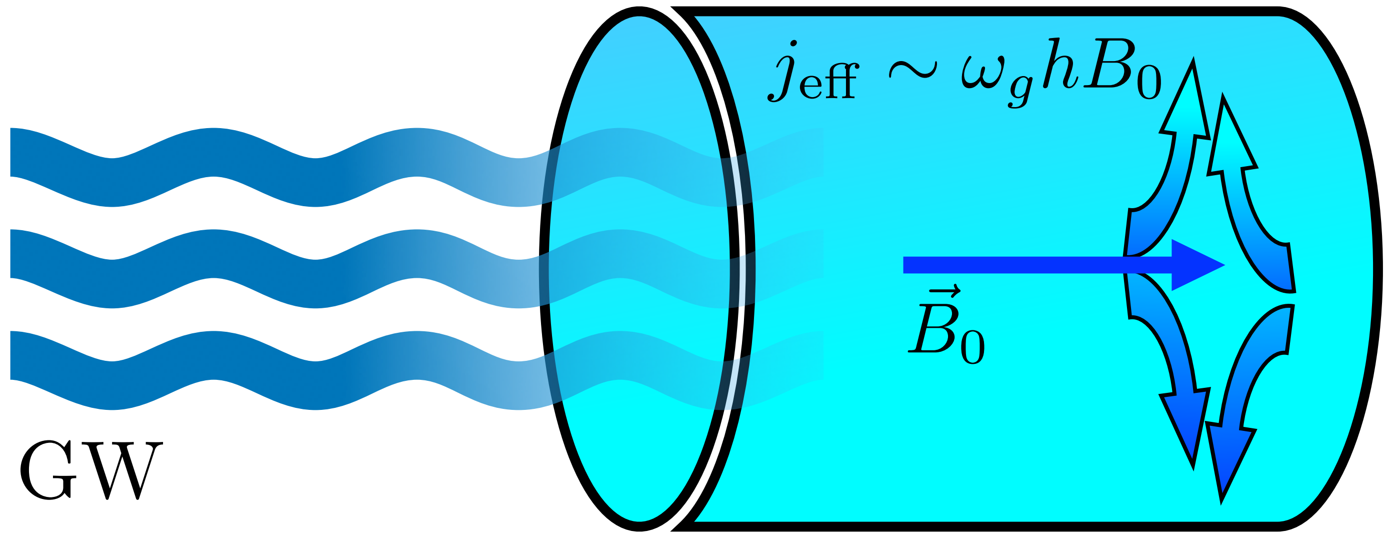




Introduction - Gravitational Waves

- GW leads to source of effective current in Maxwell's equation

$$j_{eff} \propto \omega_g h B_0 e^{i(k_g z - \omega_g t)}$$

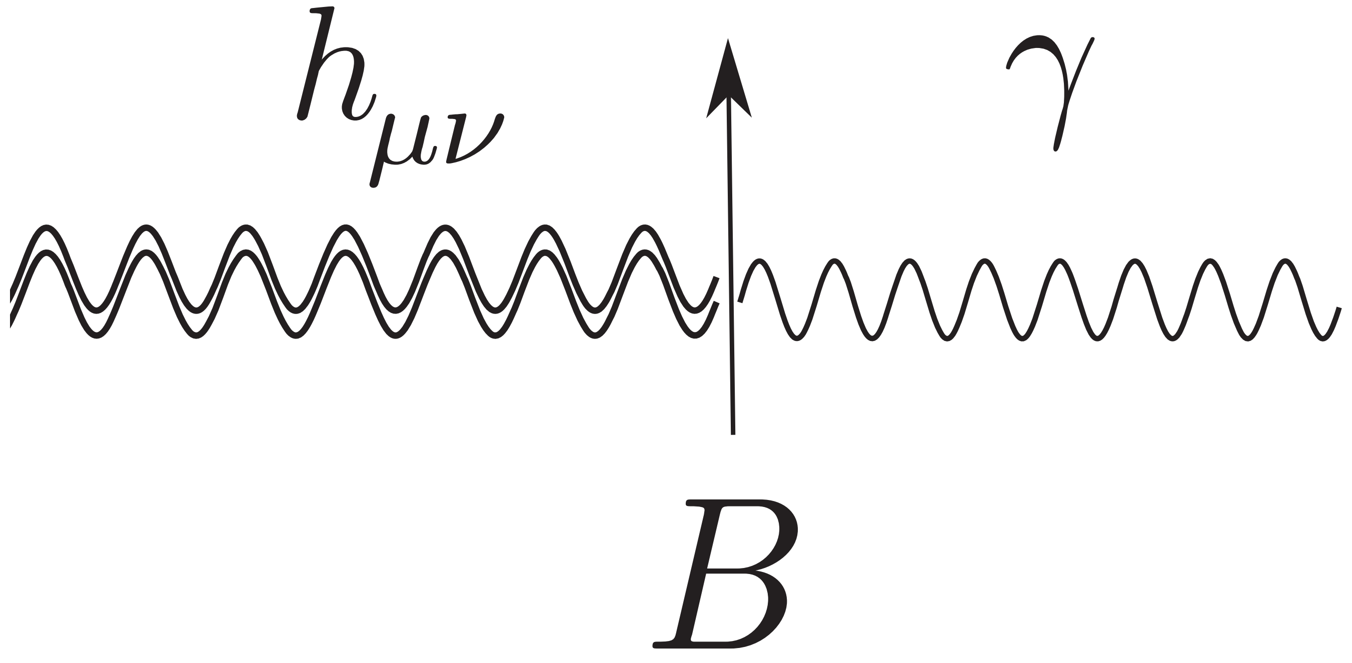


- Conversion of GW energy into Photons and vice-versa!

- => GW can excite EM field within RF resonator!

Gravitational Waves & Haloscopes

- Direct conversion of GW to photons: **inverse Gertsenshtein effect**
  - Gertsenshtein effect described 1962
  - Inverse effect calculated in 70ies
    [Ya. B. Zel'dovich]
  - White-paper on HFGW detection: 2020
    [Living Rev. Rel. 24 (2021) no.1, 4]



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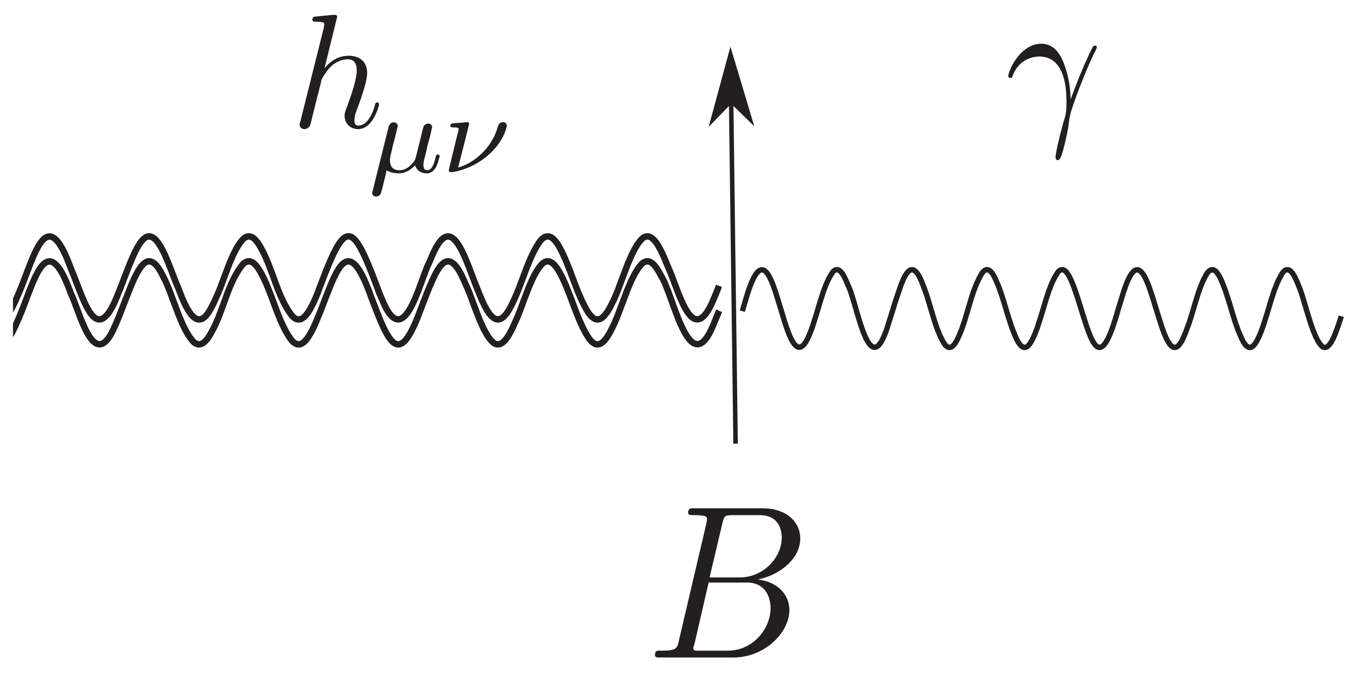
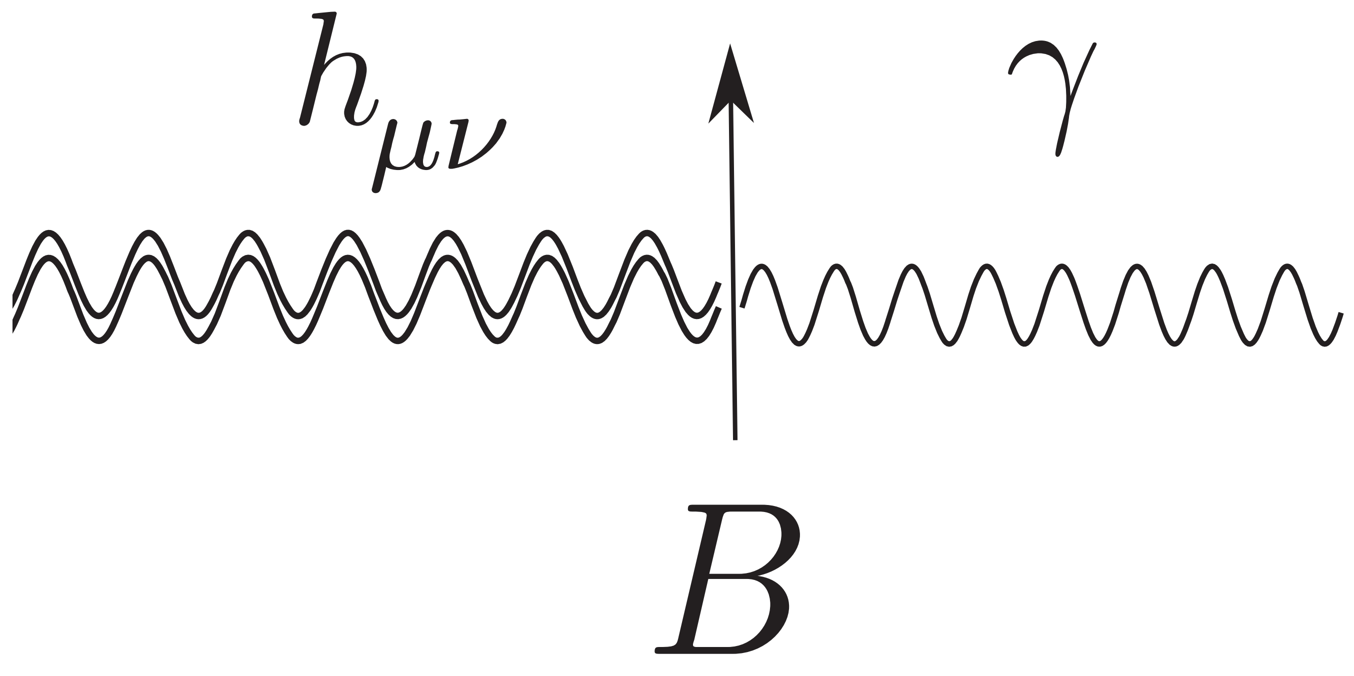
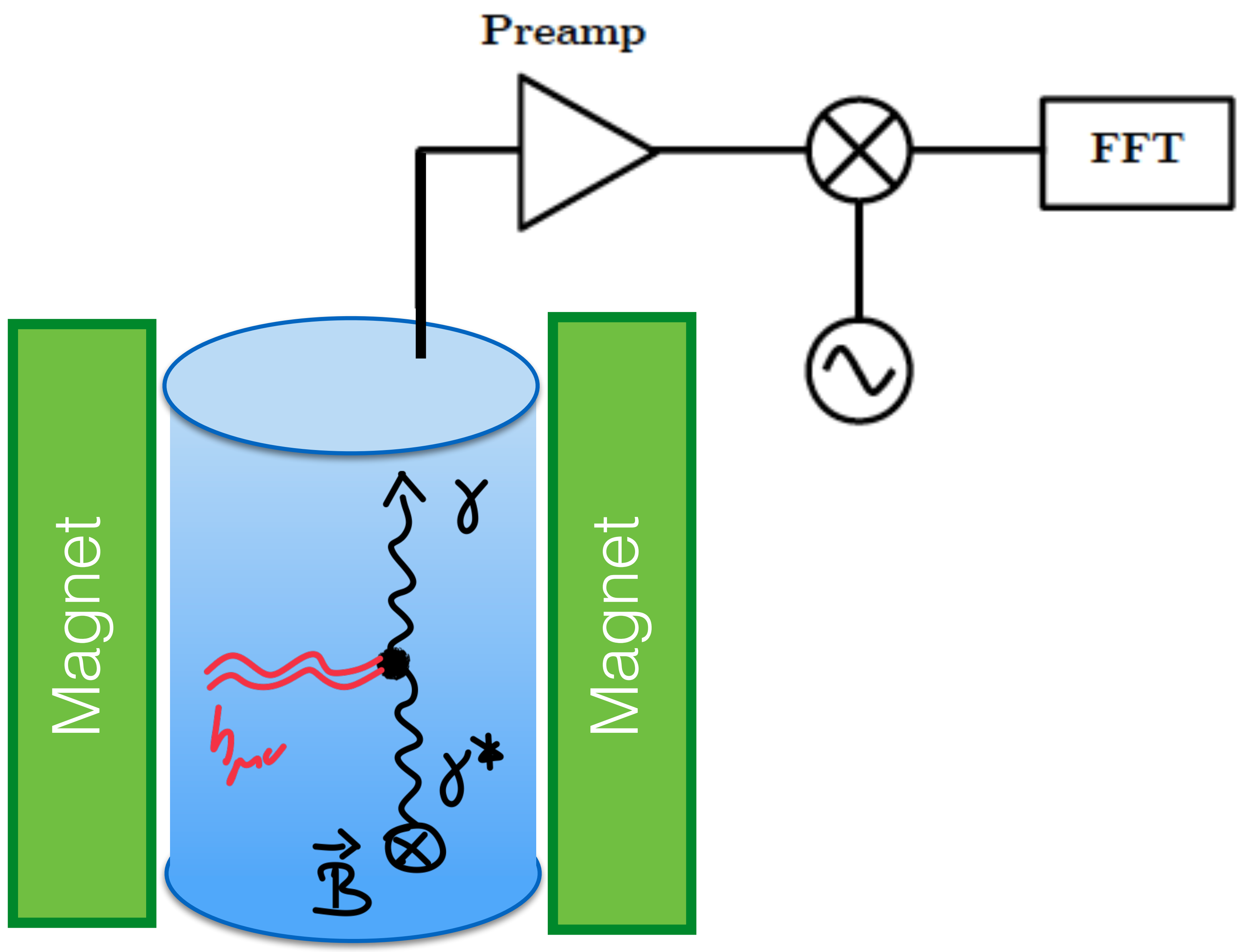


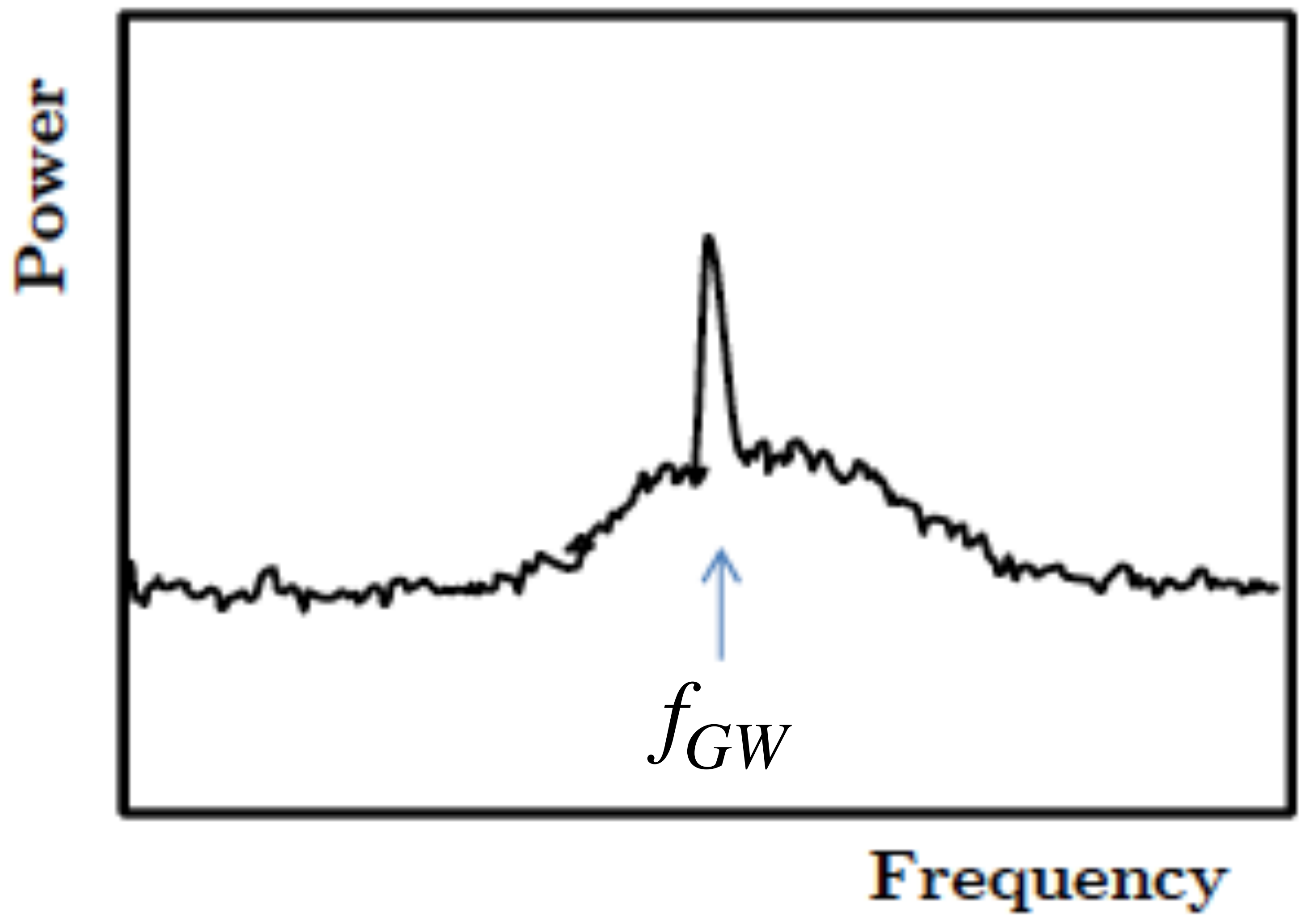
How Can this be used to detect GWs ?

Typical setup

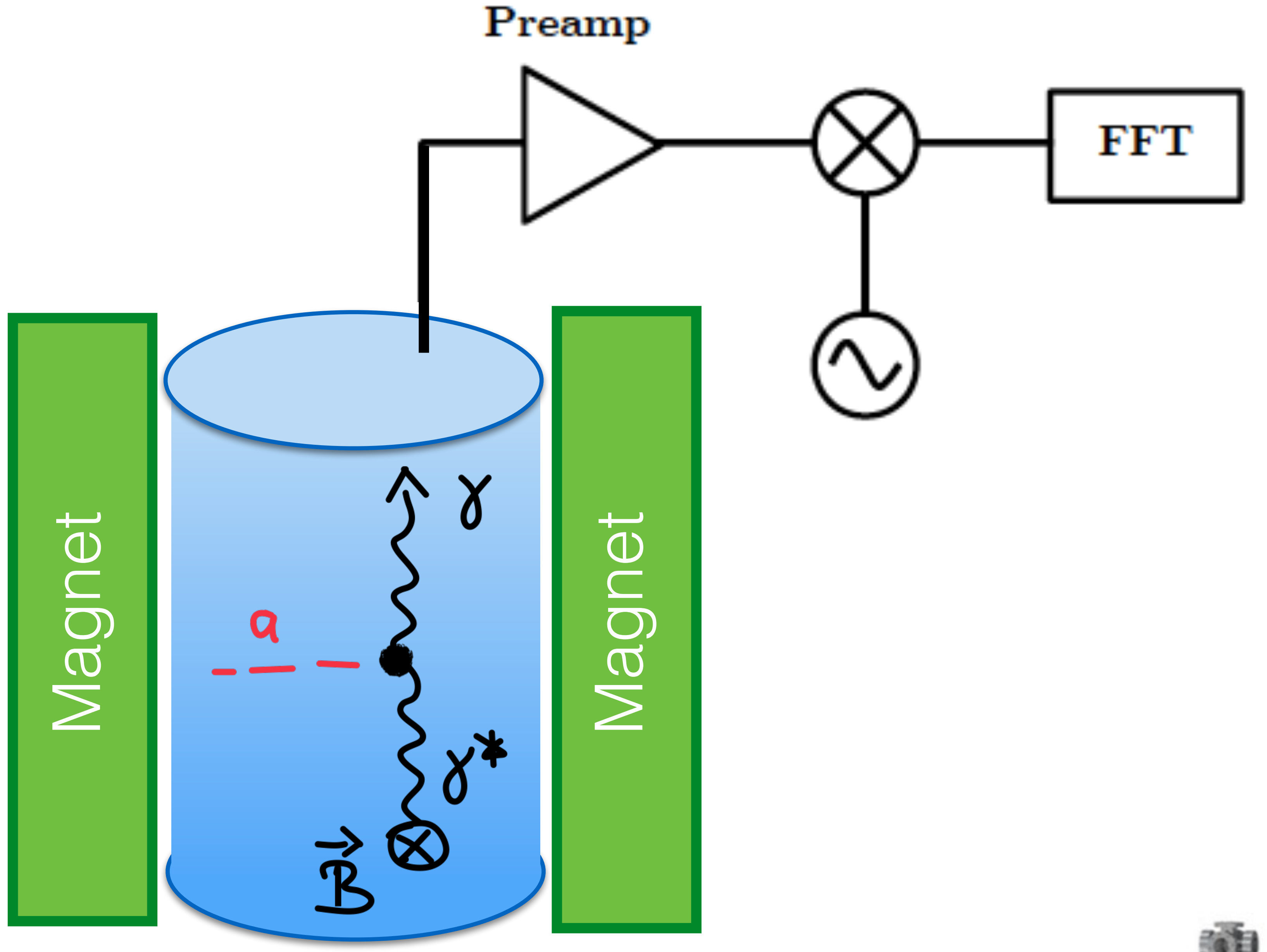




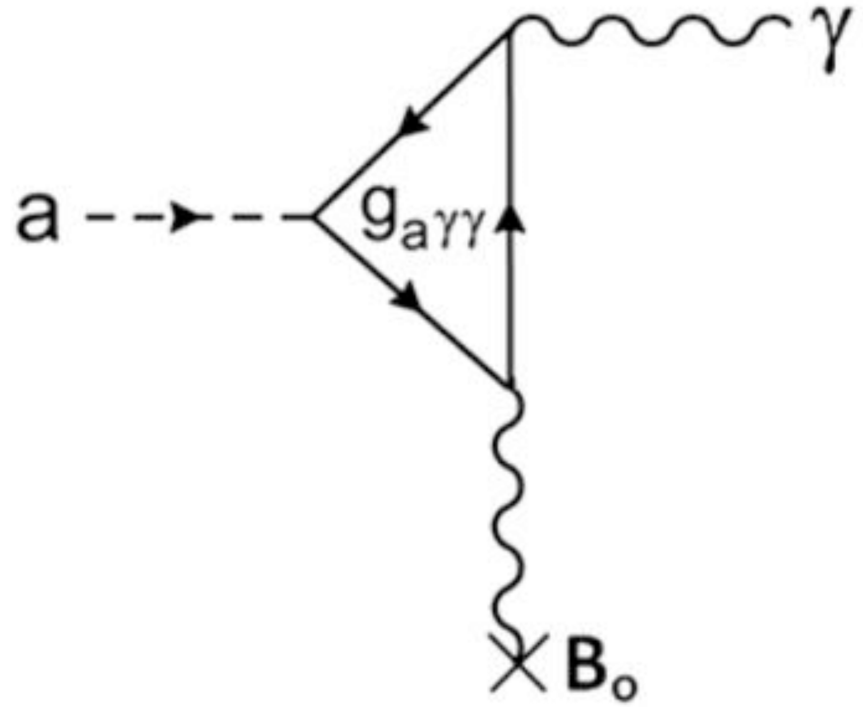
- Electromagnetoc field
  - (Source of virtual photons) => B-Field
- Resonant enhancement of signal => RF-resonator
  - Narrow band experiment
- Noise suppression => Cryogenic setup
- High sensitivity => Low Noise, high gain DAQ



Typical setup



- Suspicious similarity with axion haloscopes
  - Indeed: Identical setup



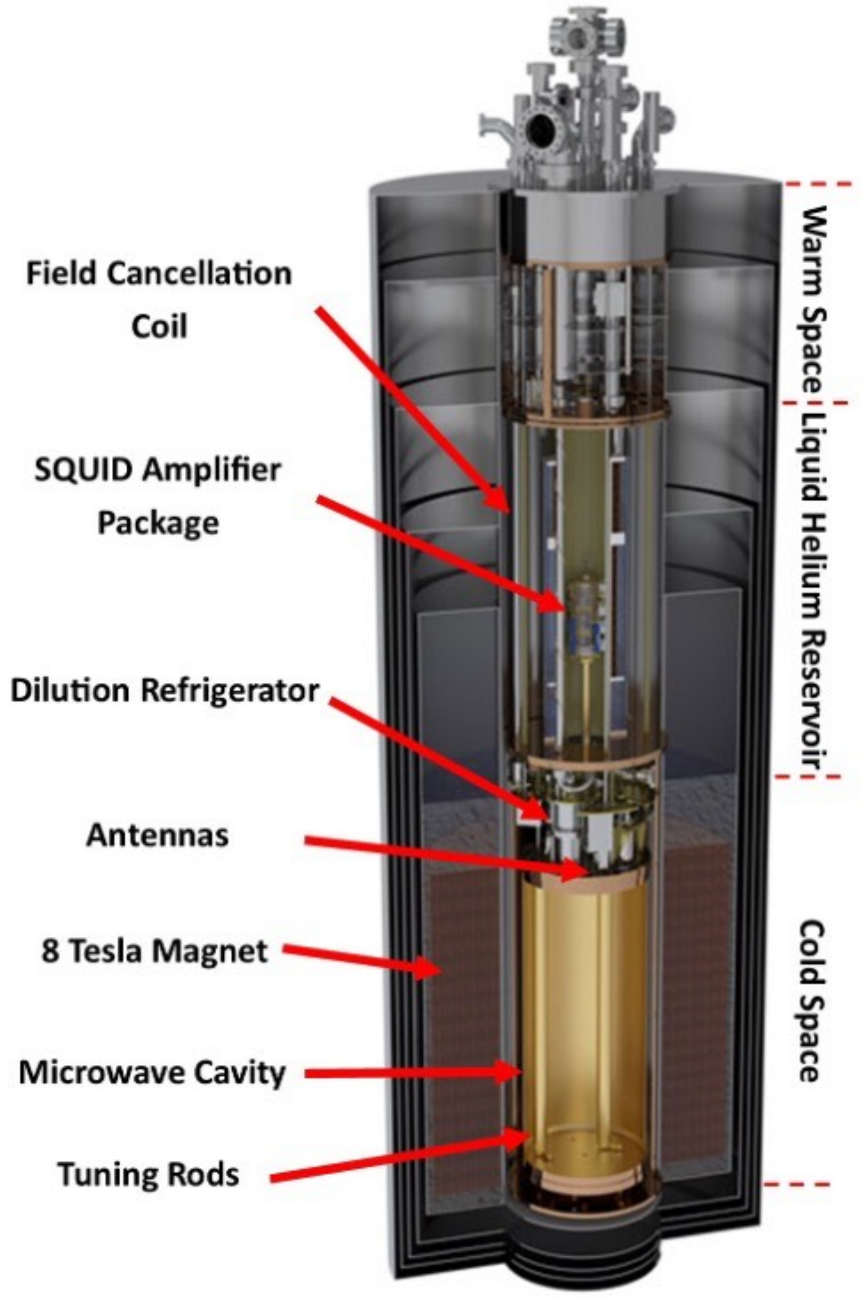
Flash

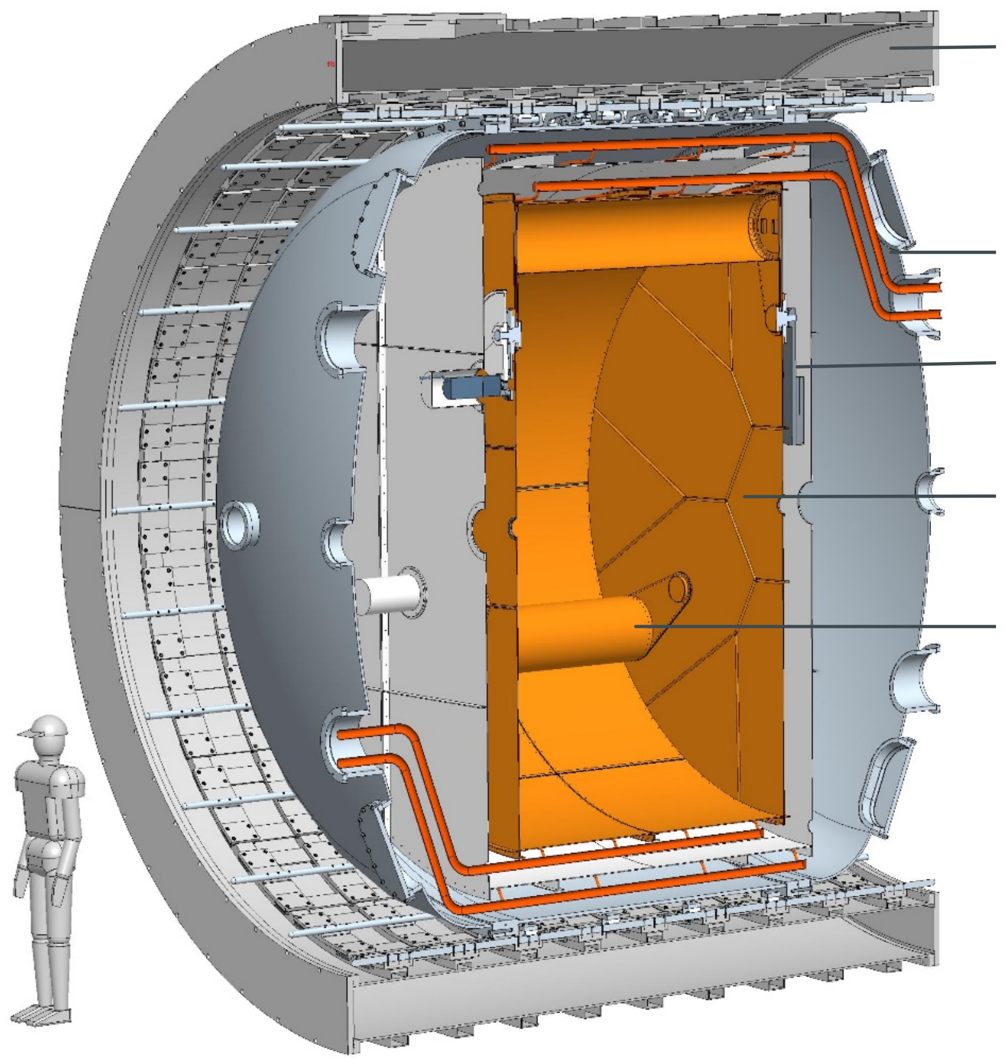
100 MHz

Organ

100 GHz

ADMX 1 GHz





Interlude: Axions

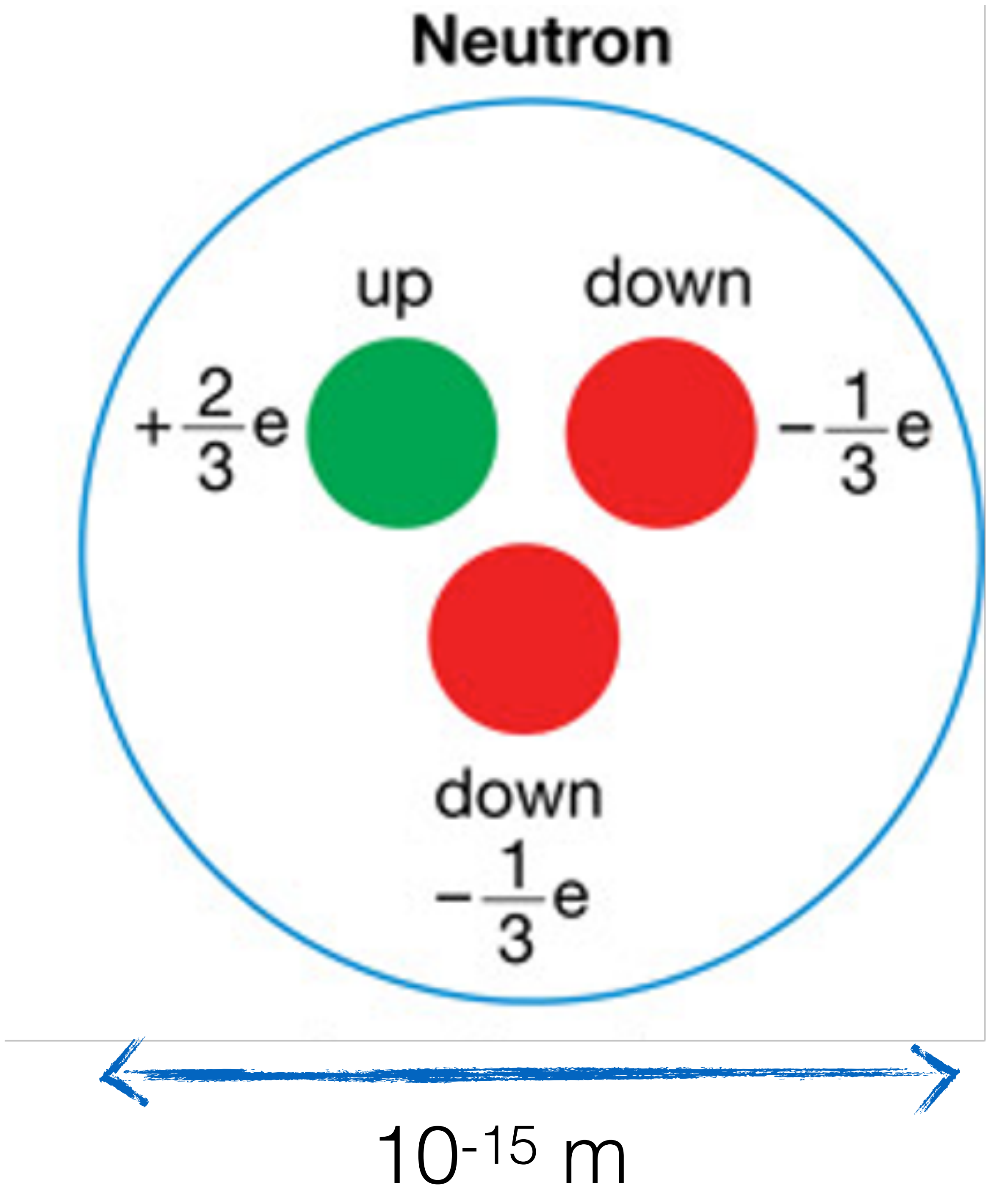
Axions

CPV in QCD

Naive estimate gives:

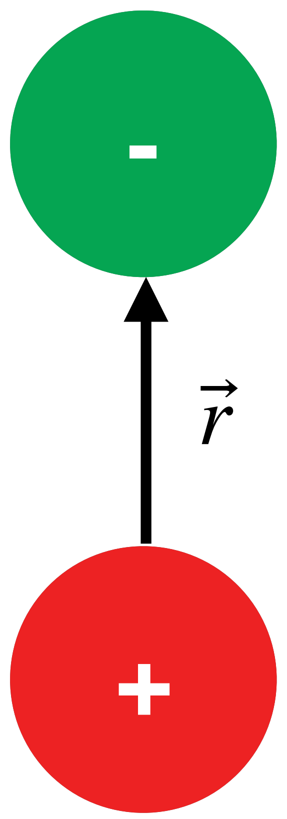
EDM$_n$ $\approx 10^{-16}\ e\,cm$

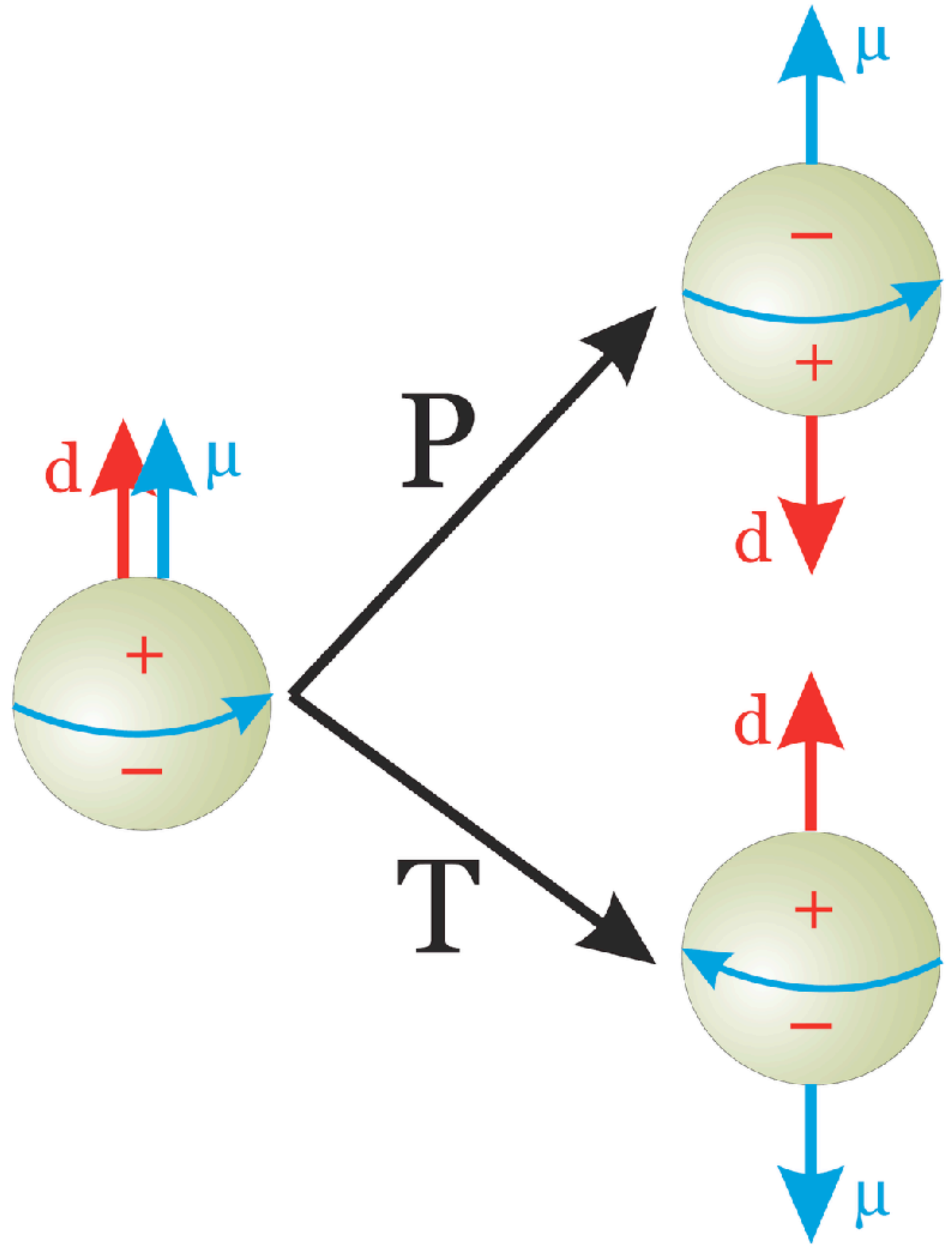
[https://cds.cern.ch/record/198102/]



Dipole moment

$\vec{d} = q \cdot \vec{r}$





$CPT|n> = |n>$

CPV in QCD

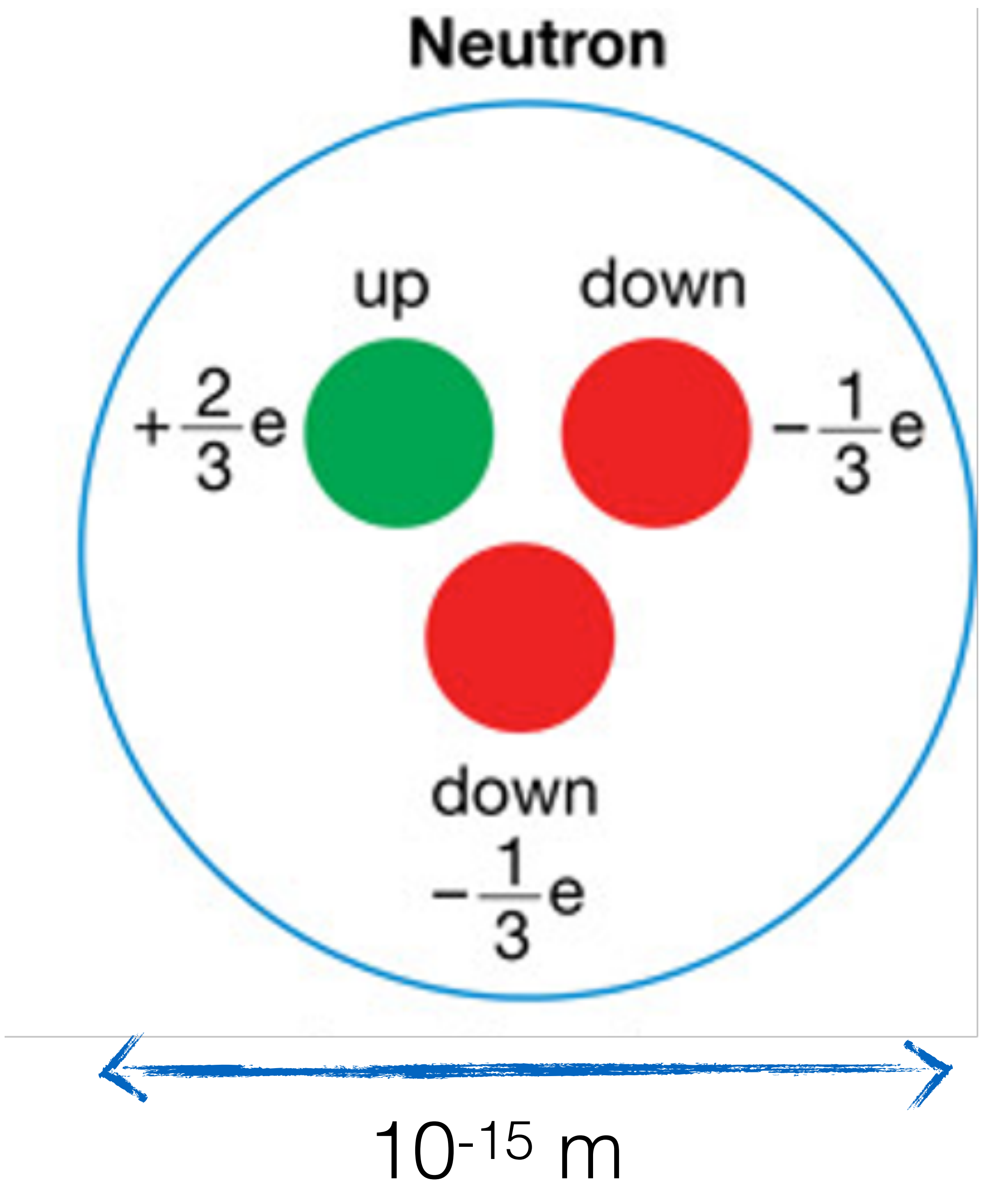
Naive estimate gives:

EDMₙ $\approx 10^{-16}\ e\,cm$

[https://cds.cern.ch/record/198102/]

Phys. Rev. Lett. 124 (8) 081803:

EDMn $= 0.0 \pm 1.1_{stat} \pm 0.2_{sys} \times 10^{-26} e\,cm$



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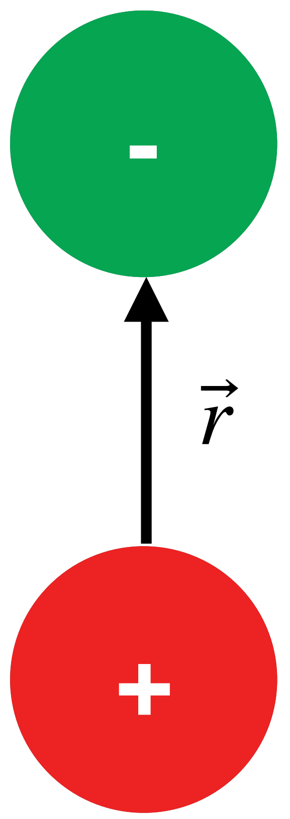
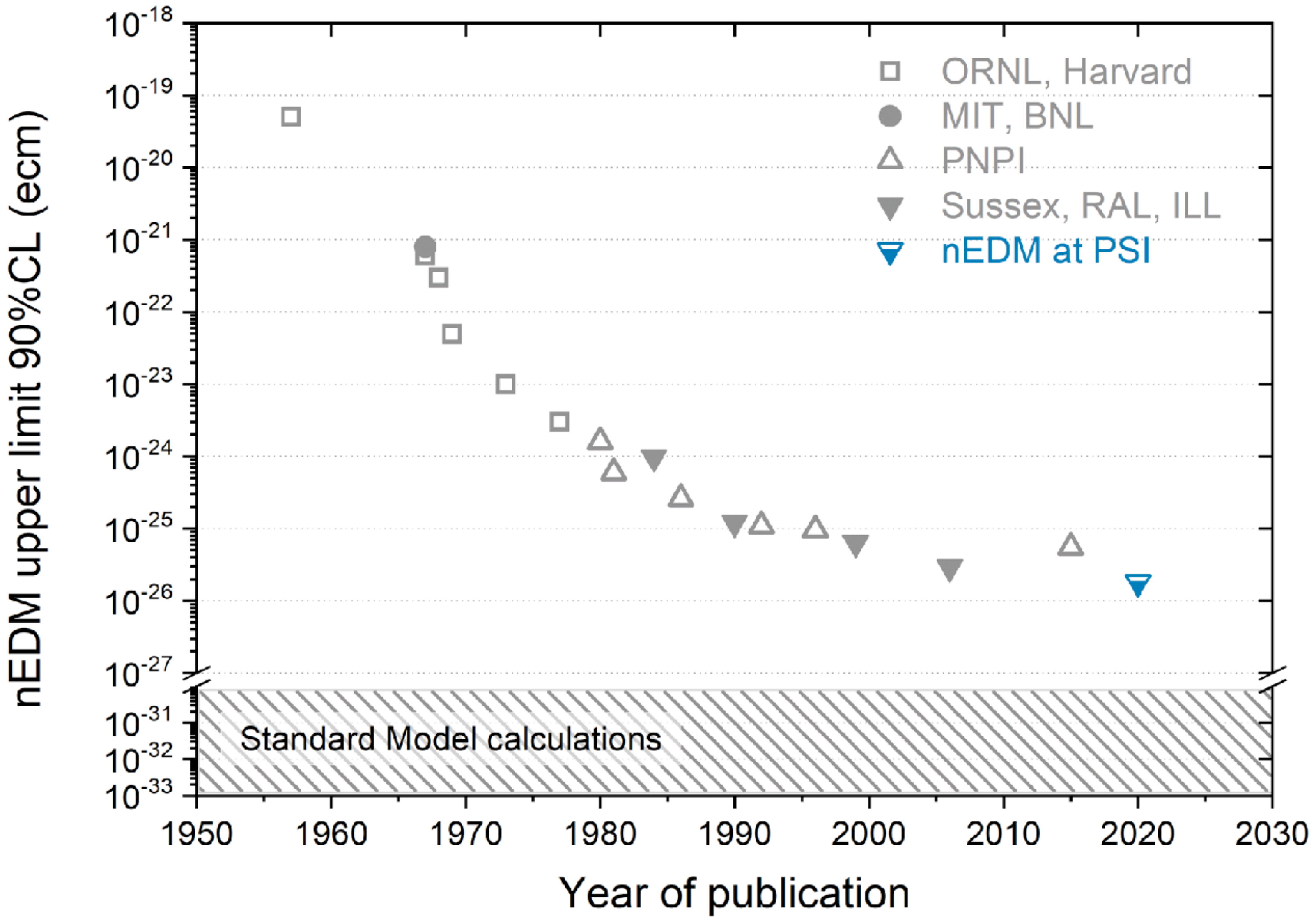
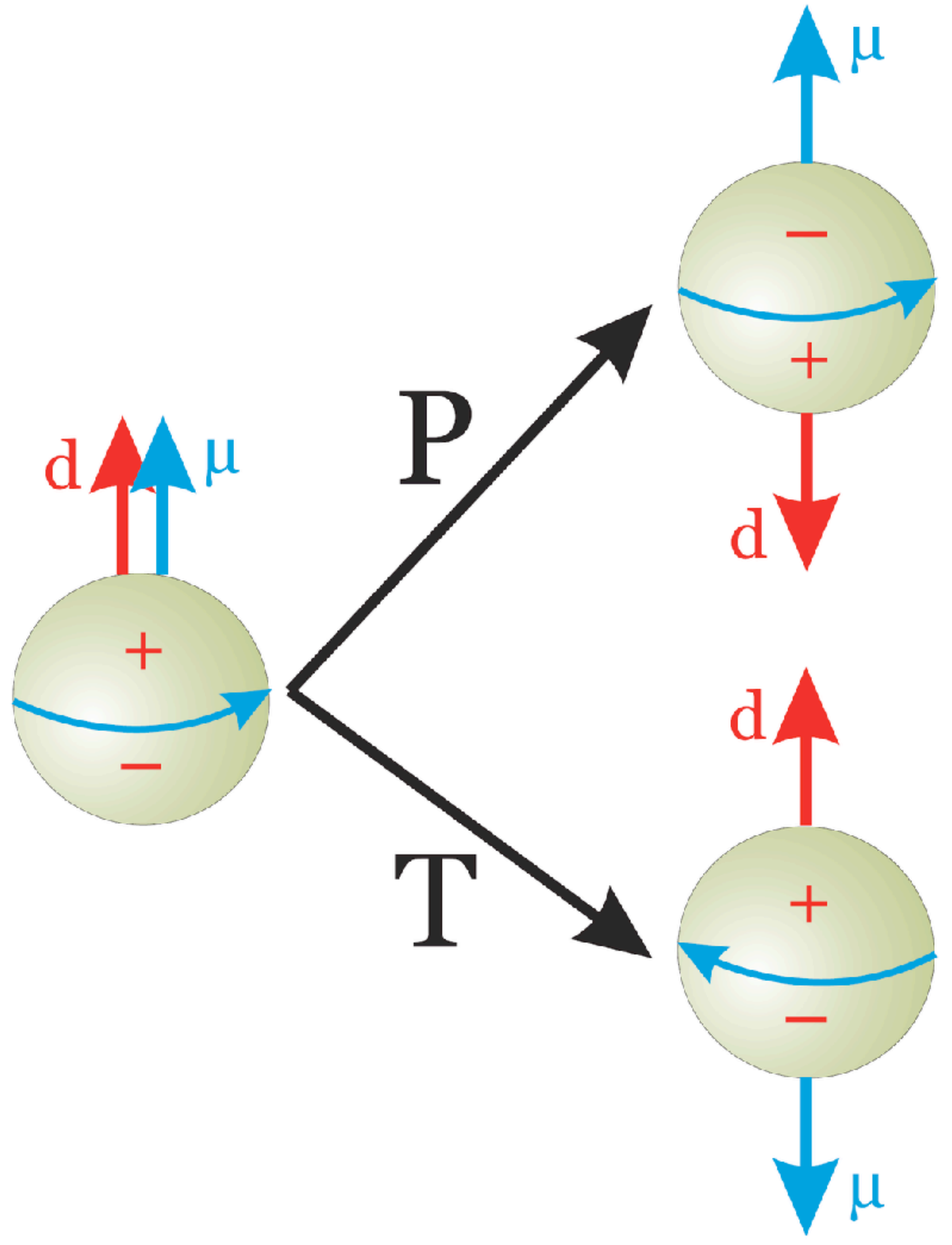




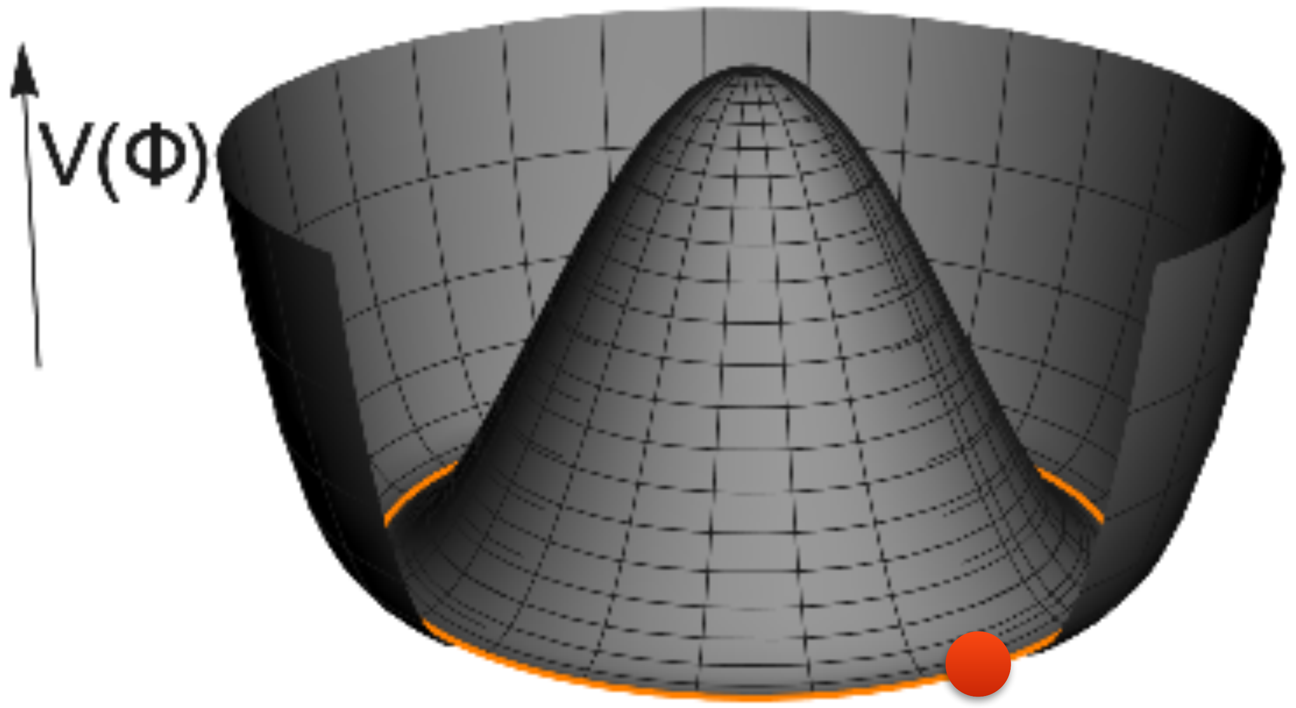


Why is EDM so small ?

$CPT|n> = |n>$

Possible Solution to strong CP problem

- 1977 by Peccei and Quinn:
  - Postulated **new** U(1) **symmetry**
  - Generic coupling to quarks



Robert Peccei

Helen Quinn

- Symmetry **spontaneously broken** at scale f_a
  - New massive Goldstone boson: **Axion**

Steven Weinberg

Frank Wilczek

- Exact way of symmetry breaking (structure of QCD vacuum)
  - **CP violating term nulled dynamically**

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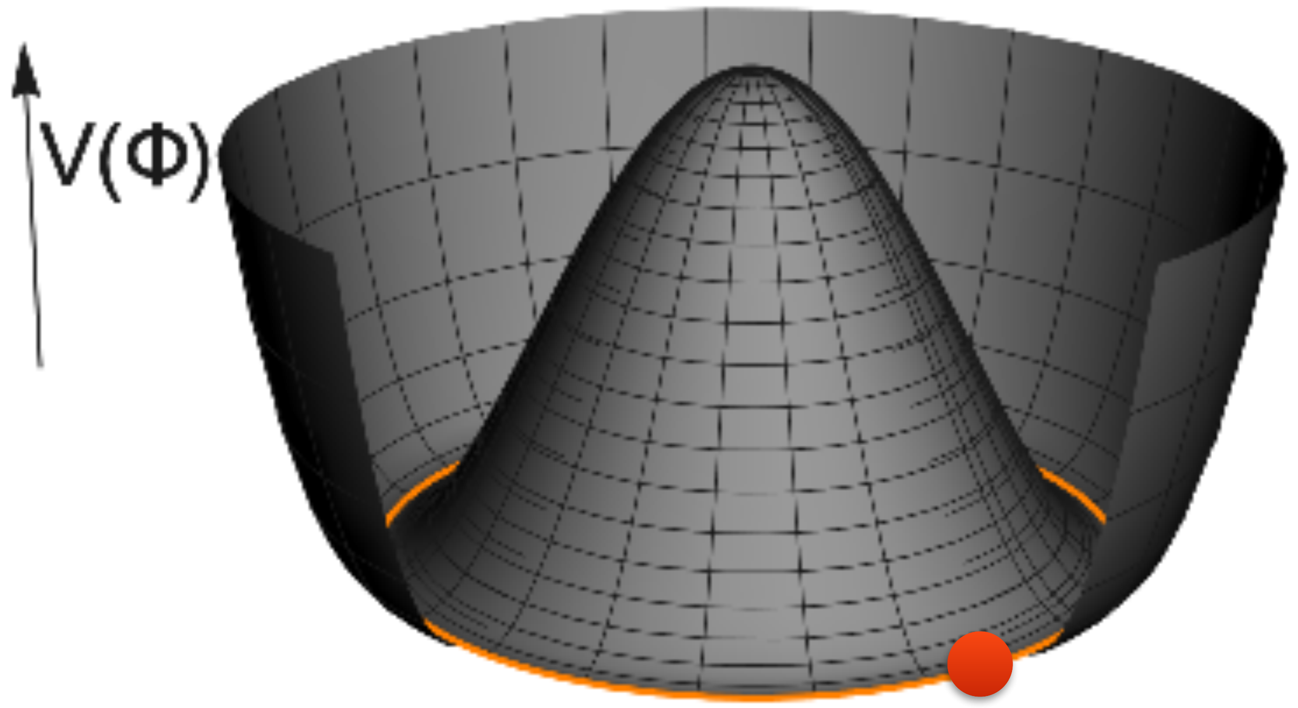

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Axion:
'Washes away all problems'

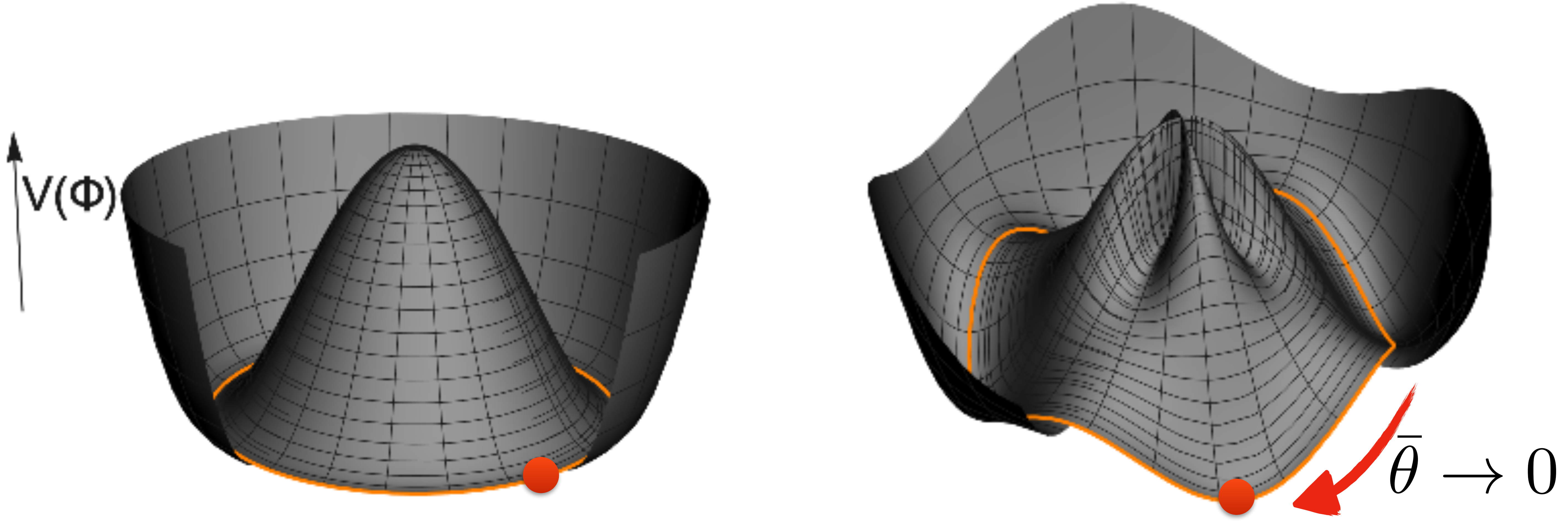
Here:
Massive, pseudo-scalar particle

Possible Solution to strong CP problem

- a: Axion field
- f_a: “Peccei-Quinn scale”

$$\mathcal{L}_{\text{tot}} = \mathcal{L}_{\text{SM,axion}} + \underbrace{\bar{\theta}\frac{g_s^2}{32\pi^2}G_a^{\mu\nu}\tilde{G}^a_{\alpha\beta}}_{\text{QCD term}} + \underbrace{\xi\frac{a}{f_a}\frac{g_s^2}{32\pi^2}G_b^{\mu\nu}\tilde{G}^b_{\alpha\beta}}_{\text{Axion term}}$$

- E ~ f_a (large)
  - Spontaneously broken symmetry
  - Axion = Nambu-Goldstone boson (massless)



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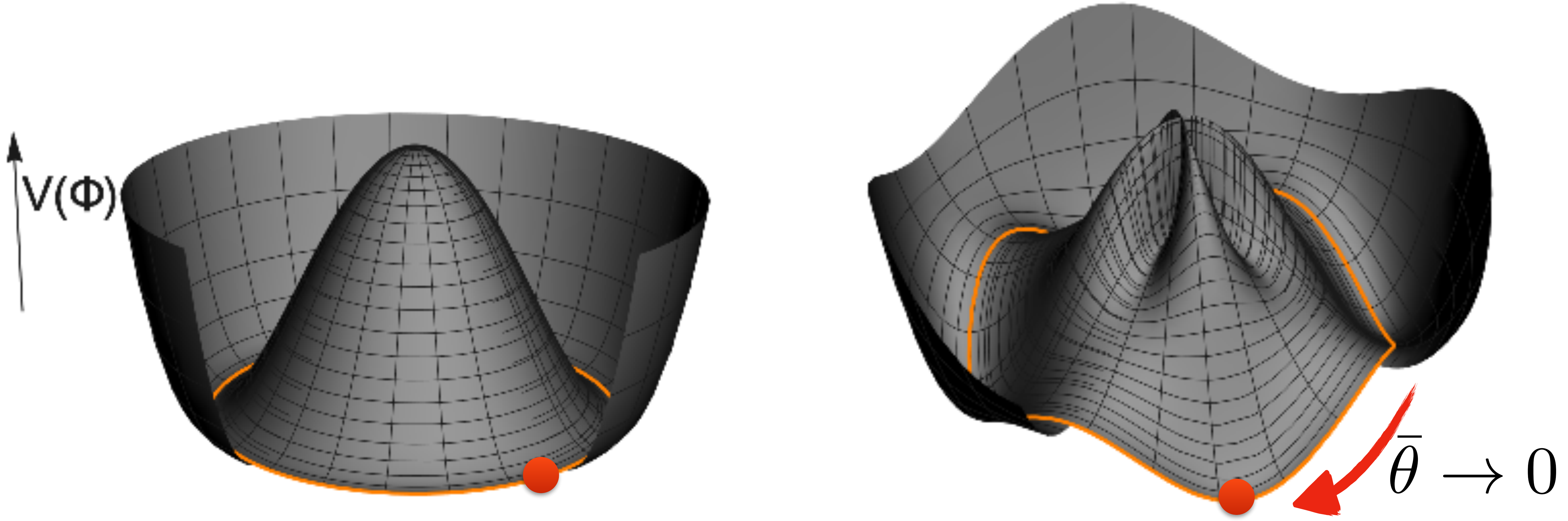
- E ~ Λ_{QCD}
  - **QCD instanton** effects break U(1) explicitely
    - "tilted mexican hat"
  - Axion becomes massive
  - Drives potential to $\theta = 0$
    - CP symmetry restored



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QCD term

Axion term

- E ~ Λ_{QCD}
  - **QCD instanton** effects break U(1) explicitely
    - “tilted mexican hat”
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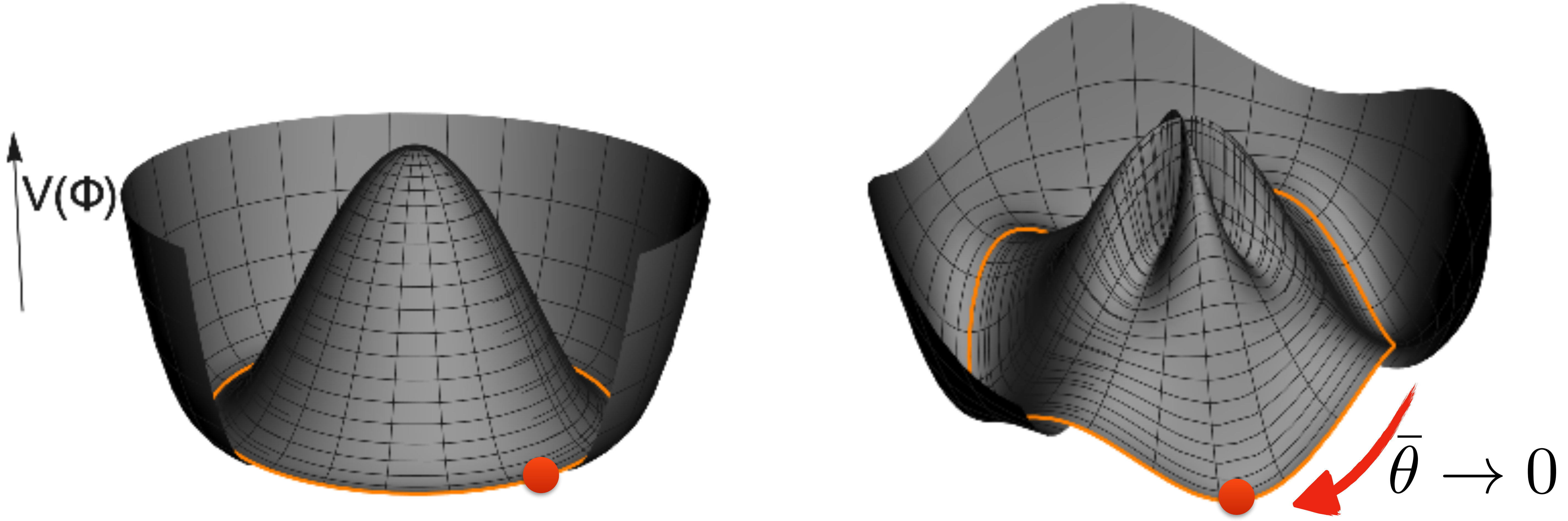
- Only free parameter:
  - Scale of symmetry breaking

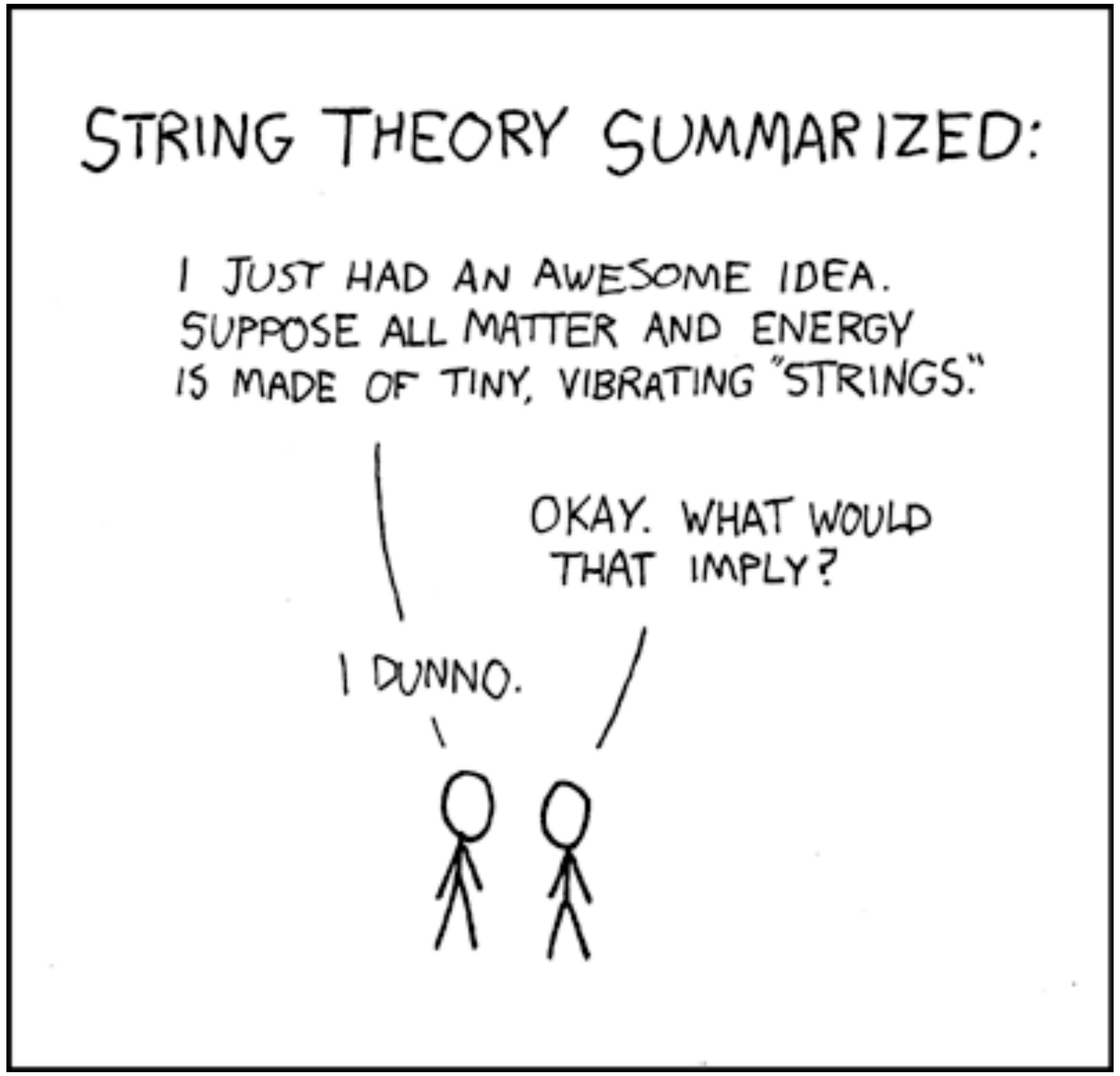
Coupling: $g_i \propto \frac{1}{f_a}$ $g_i \propto m_a$



VI Reasons to like Axions

1. … may solve the strong CP problem
2. … may be Dark Matter
3. … may explain anomalous star cooling
4. … may explain TeV transparency of intergalactic space
5. … may contribute to $(g\text{-}2)_\mu$
6. … are **well motivated** by string theory

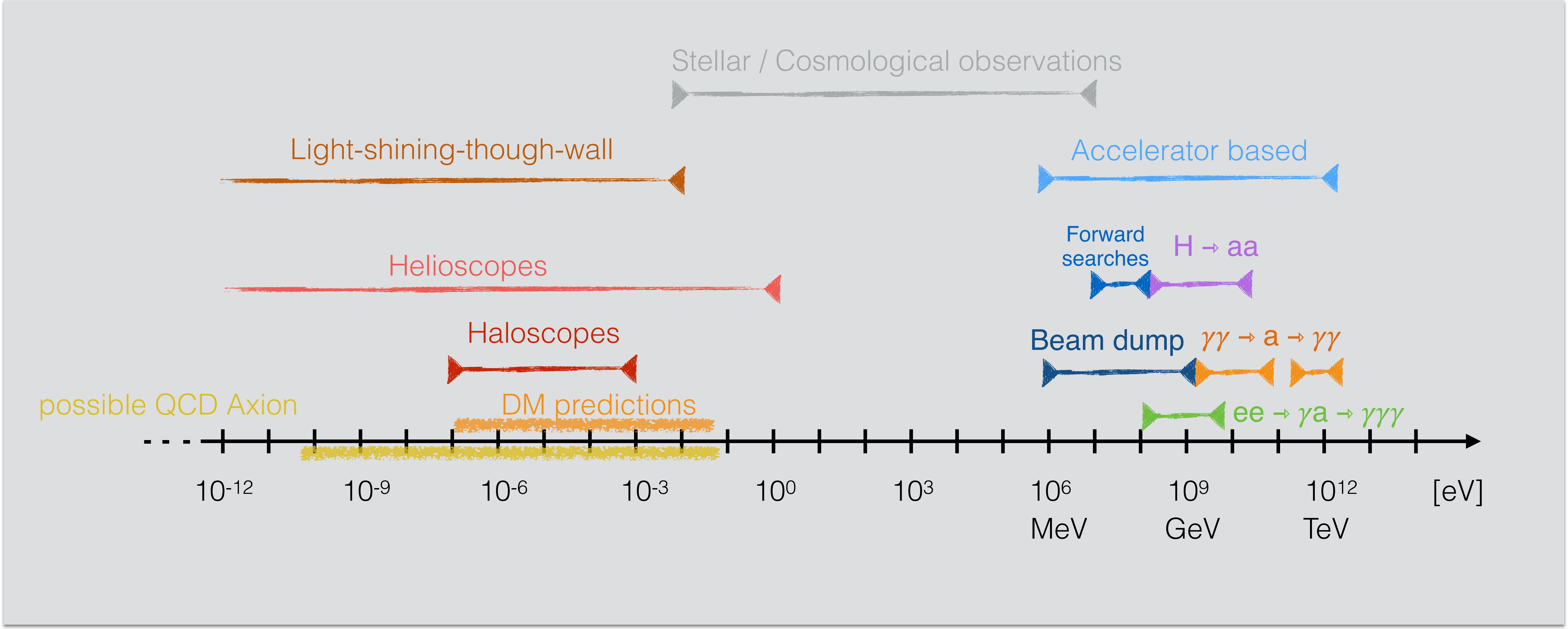
[arXiv:0605206]



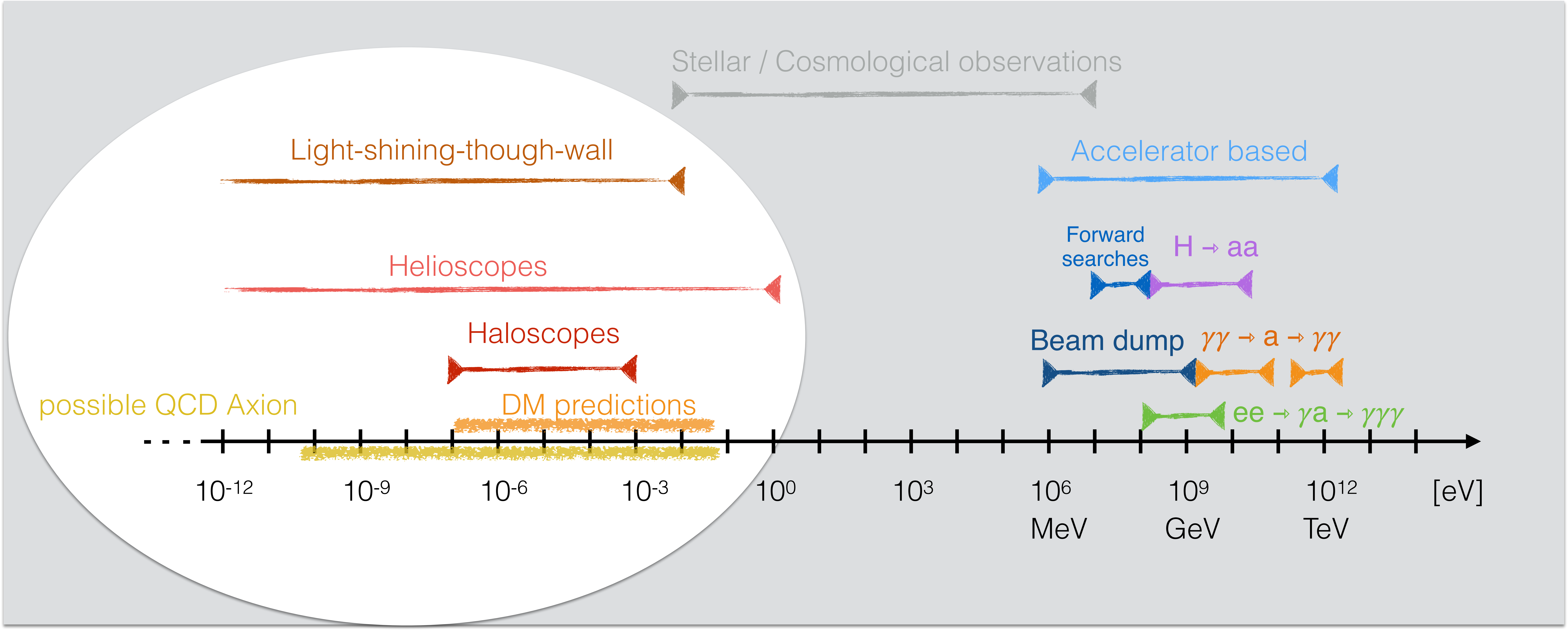
- Axion-like fields emerge in string theory in 10D -> 4D compactifications as Kaluza-Klein zero modes of ten-dimensional form fields

[A. Ringwald 2014 J. Phys.: Conf. Ser. 485 012013]

Light Axions and ALPs



Light Axions and ALPs



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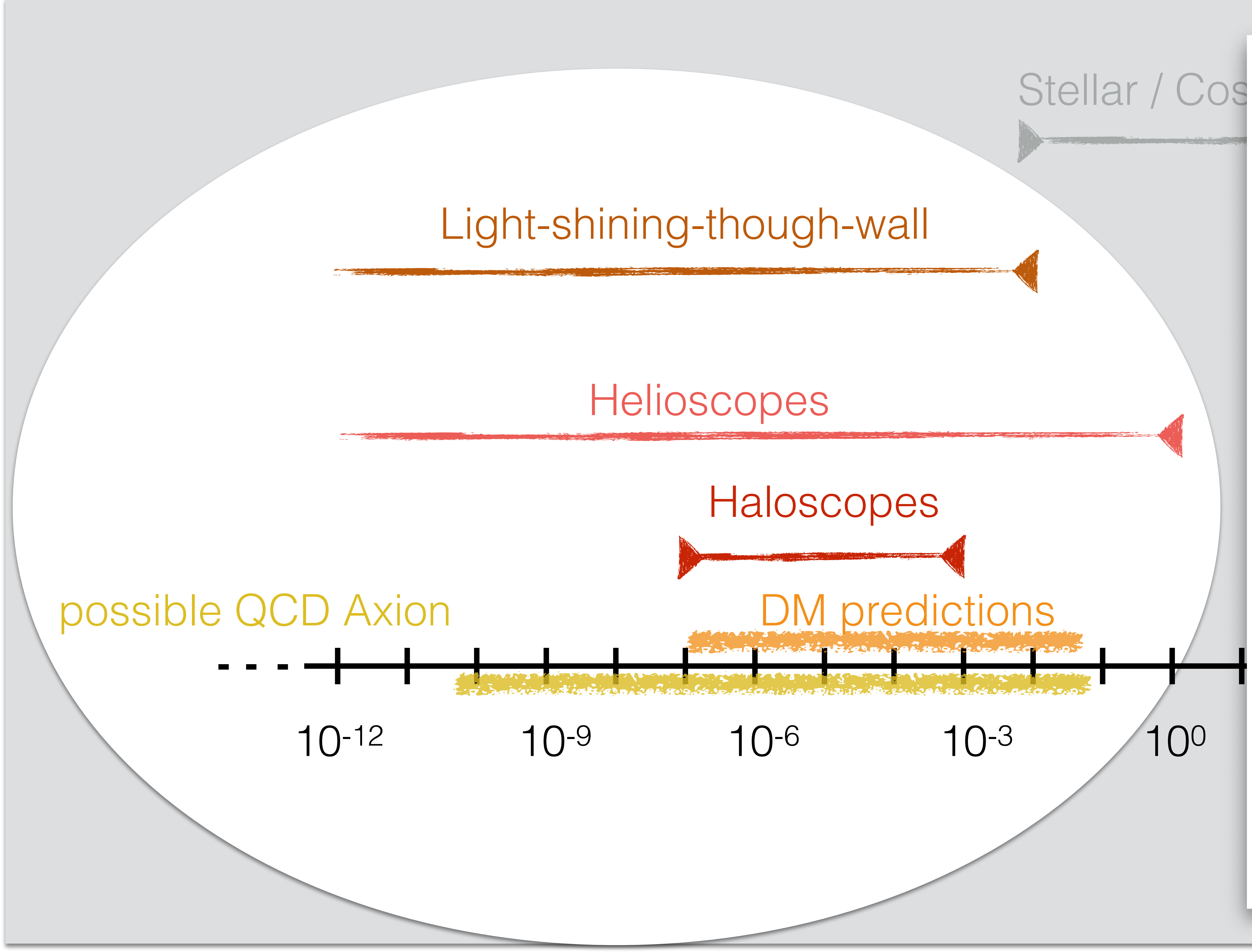


Stellar / Cos

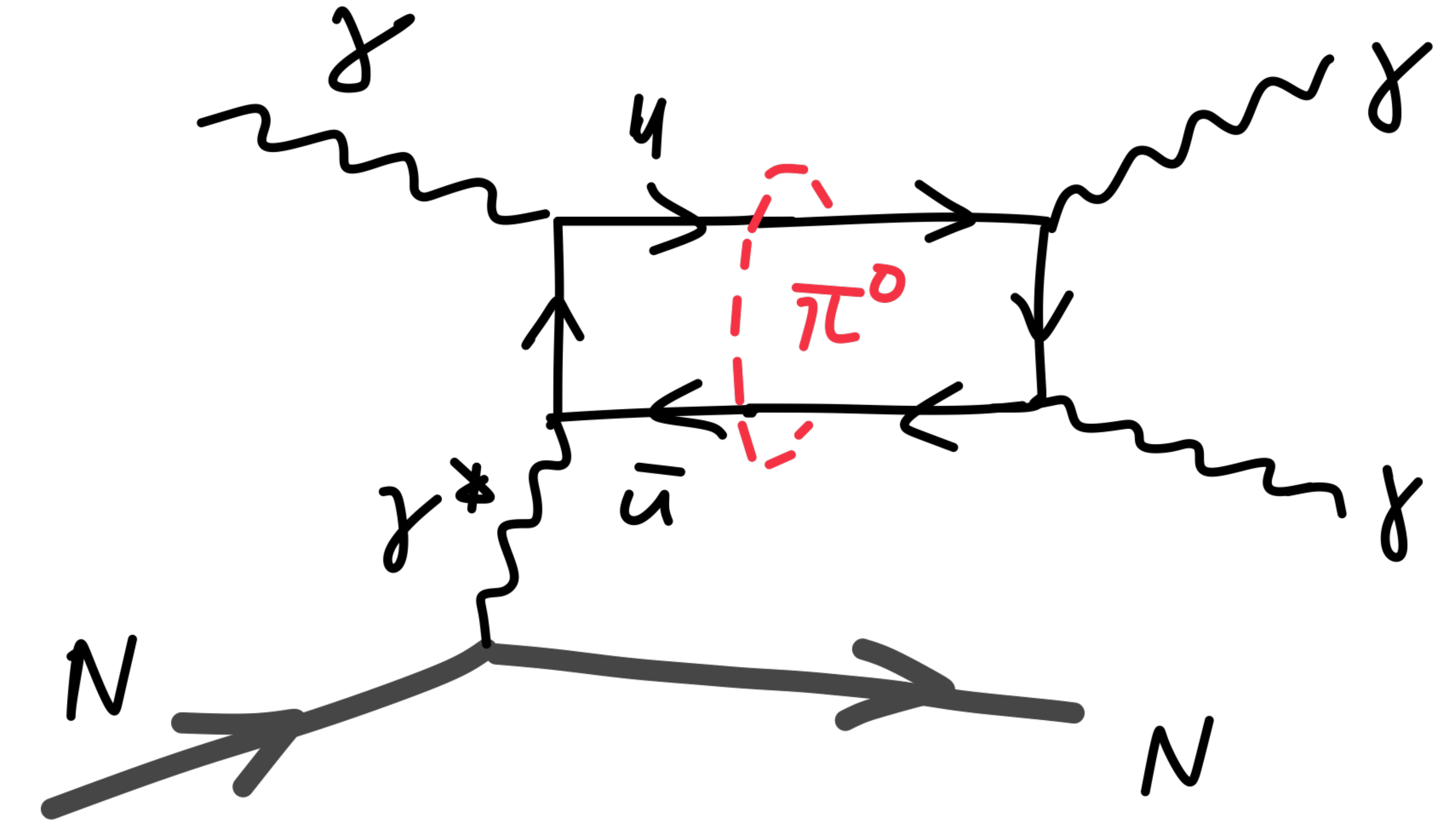
Light-shining-though-wall

Helioscopes

Haloscopes

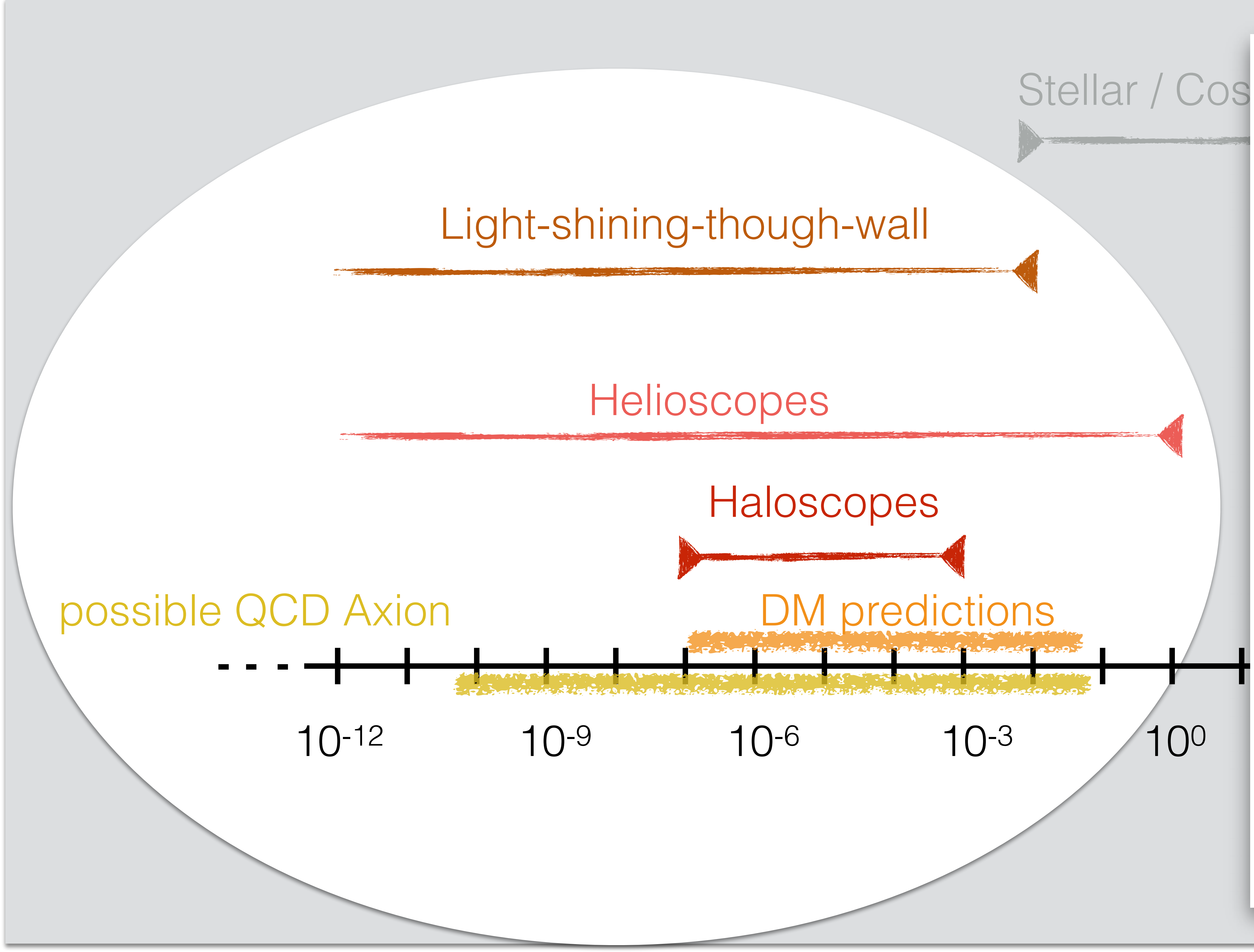
possible QCD Axion

DM predictions



- Primakoff effect:
  Resonant photo-production of **pseudoscalar mesons**

Light Axions and ALPs



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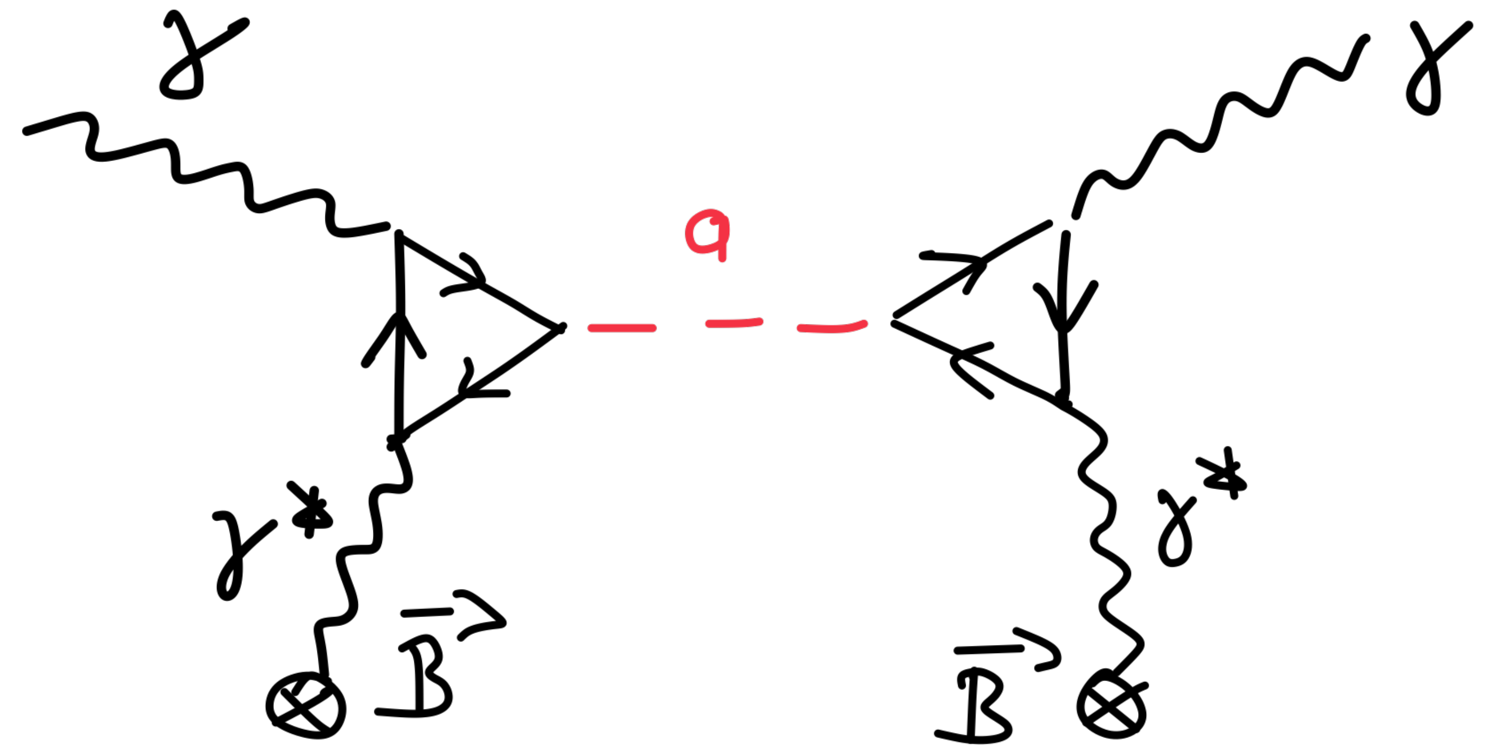
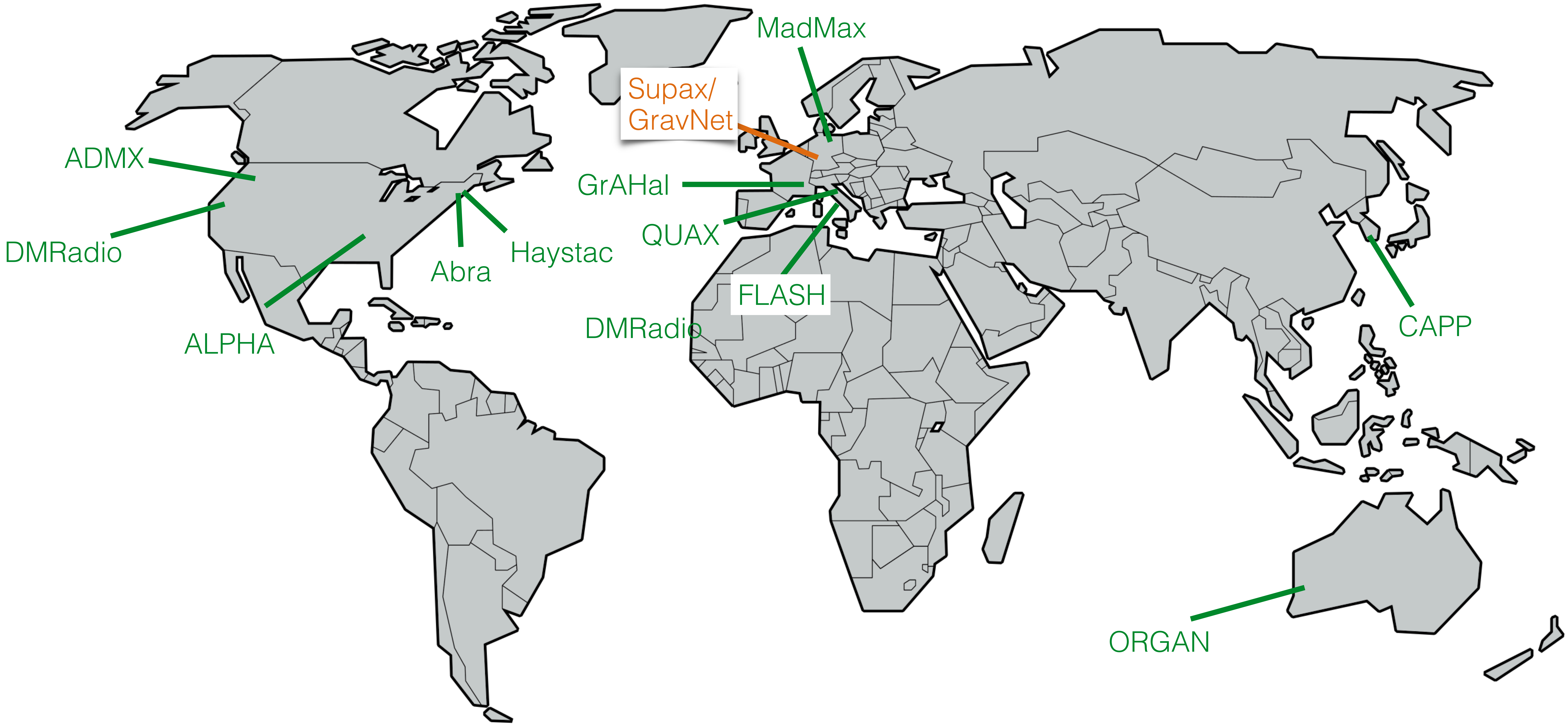


- Axion to photon conversion in magnetic field
  (inverse Primakoff effect)

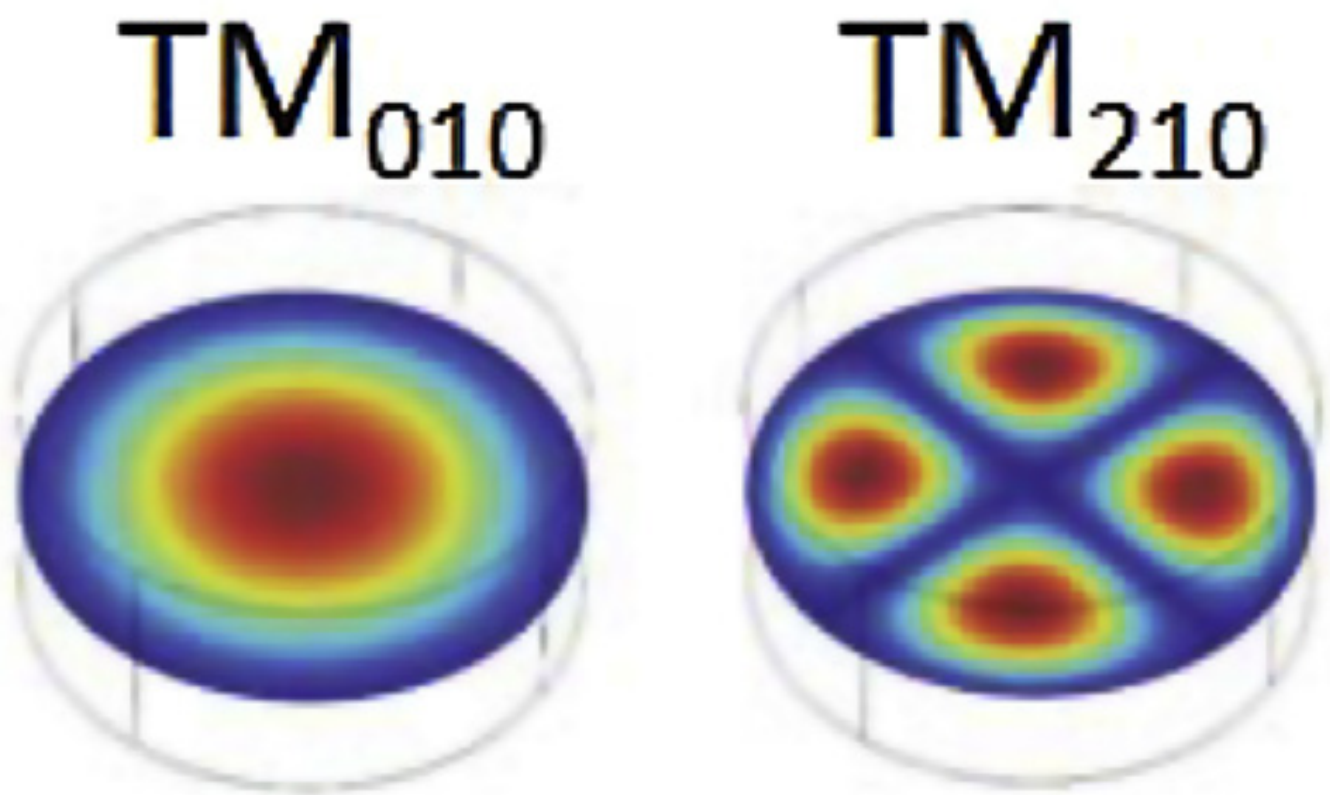
Axion Haloscope Experiments



GW waves exited higher order mode(s) in cavity

- Fundamental differences to Axions:
  - **Quadrupol** vs. **Dipole** structure
  - GWs are **transient signals**!
    - Long integration times not useful

E - field distribution



- Dedicated cavities needed for GW detection
  - Optimal geometry?

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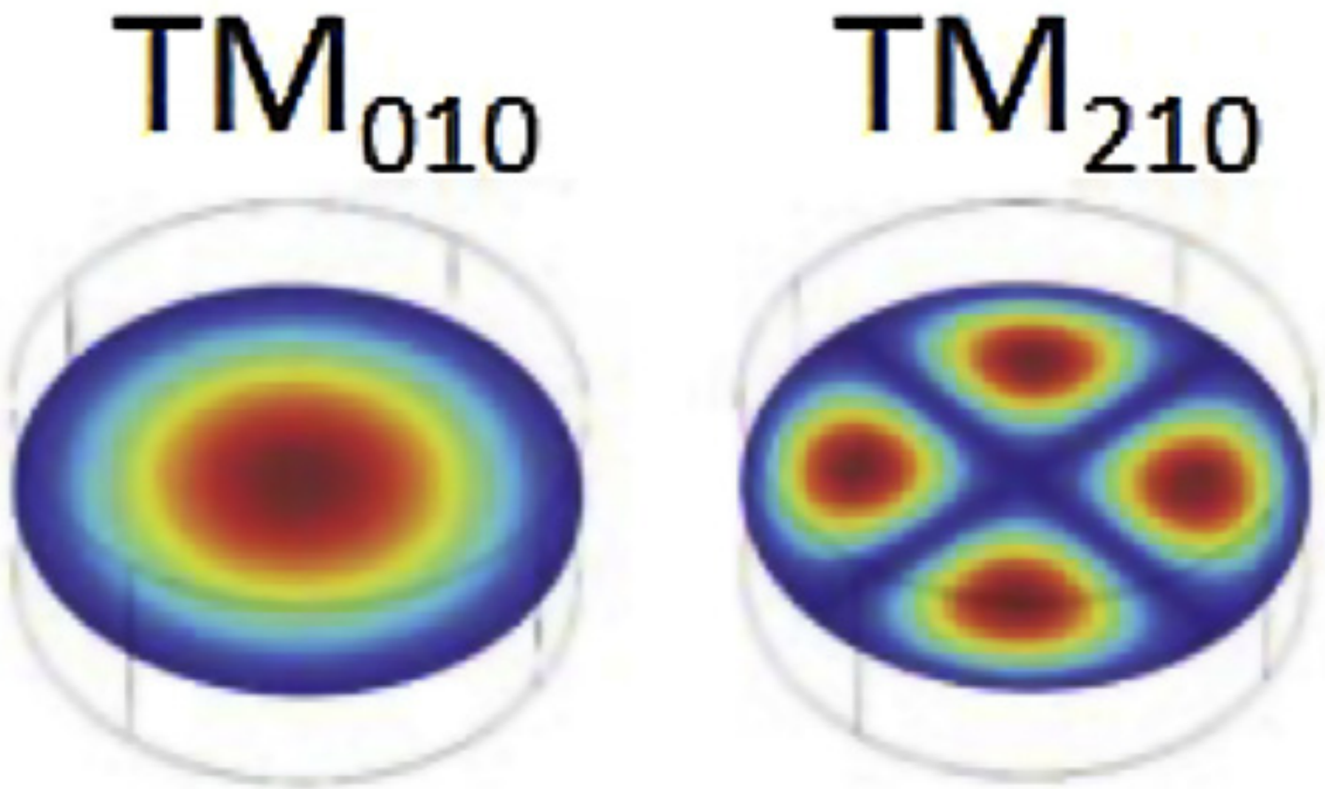
    - Long integration times not useful

E - field distribution

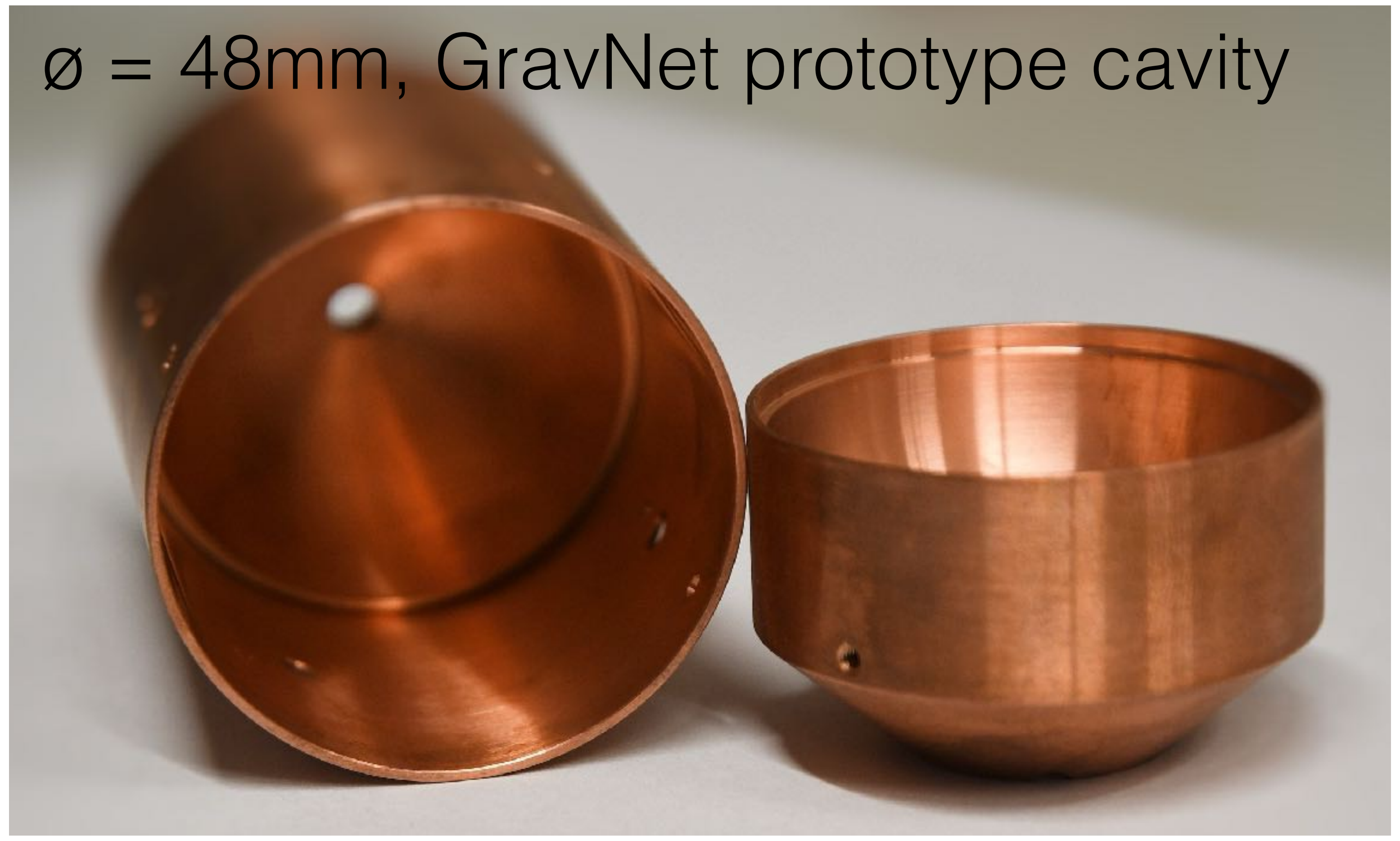


- Dedicated cavities needed for GW detection
  - Optimal geometry?

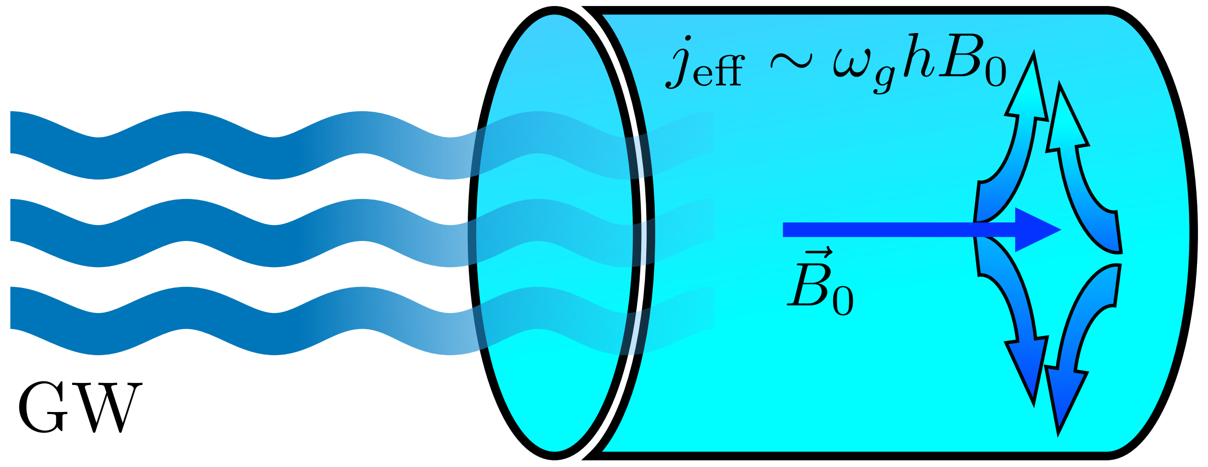
- Collaboration with our theory colleagues at Mainz:

P. Schwaller et. al. arXiv:2404.08572

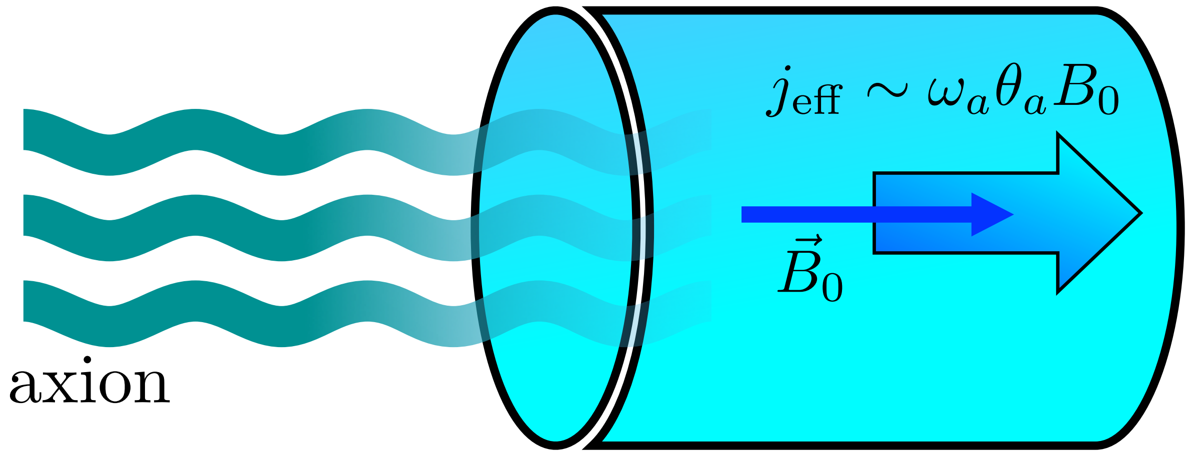
ø = 48mm, GravNet prototype cavity



Difference w.r.t. Axion Searches



- GW:
  - Typical quadruple structure
  - Preferred mode: **TM 020**
  - Current direction dependent on GW



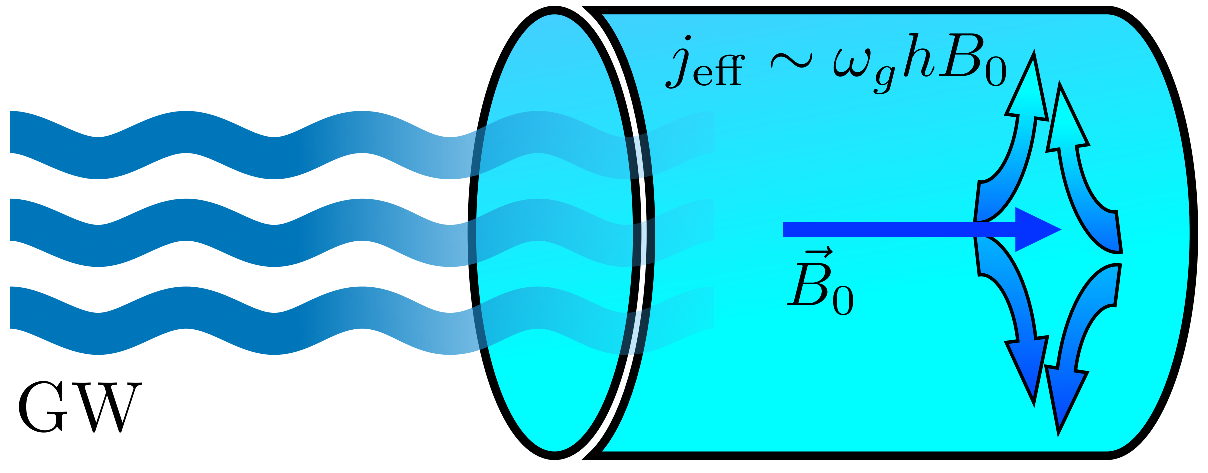
- Axions:
  - Preferred mode: **TM 010**
  - Current dependent on B-field direction
  - Litle overlap with GW mode

[arXiv:2112.11465]

- Ideal setup:
  - Axion setup has NO overlap with GW mode!

- Signal **lifetime**:
  - Axions: infinite => Integration time O(100s)
  - GW merging events: µs - ms => Need new analysis techniques

Difference w.r.t. Axion Searches



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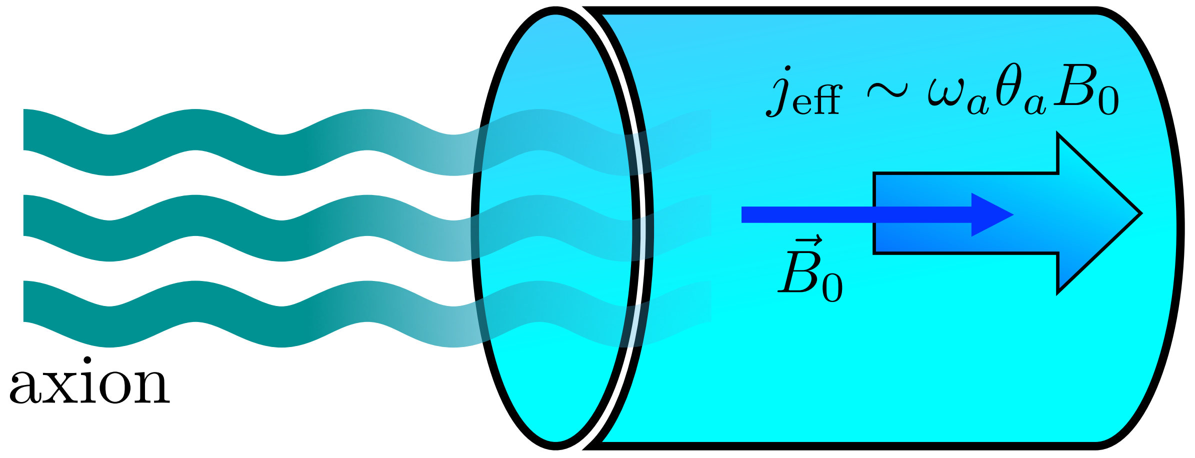

- Axions:
  - Preferred mode: **TM 010**
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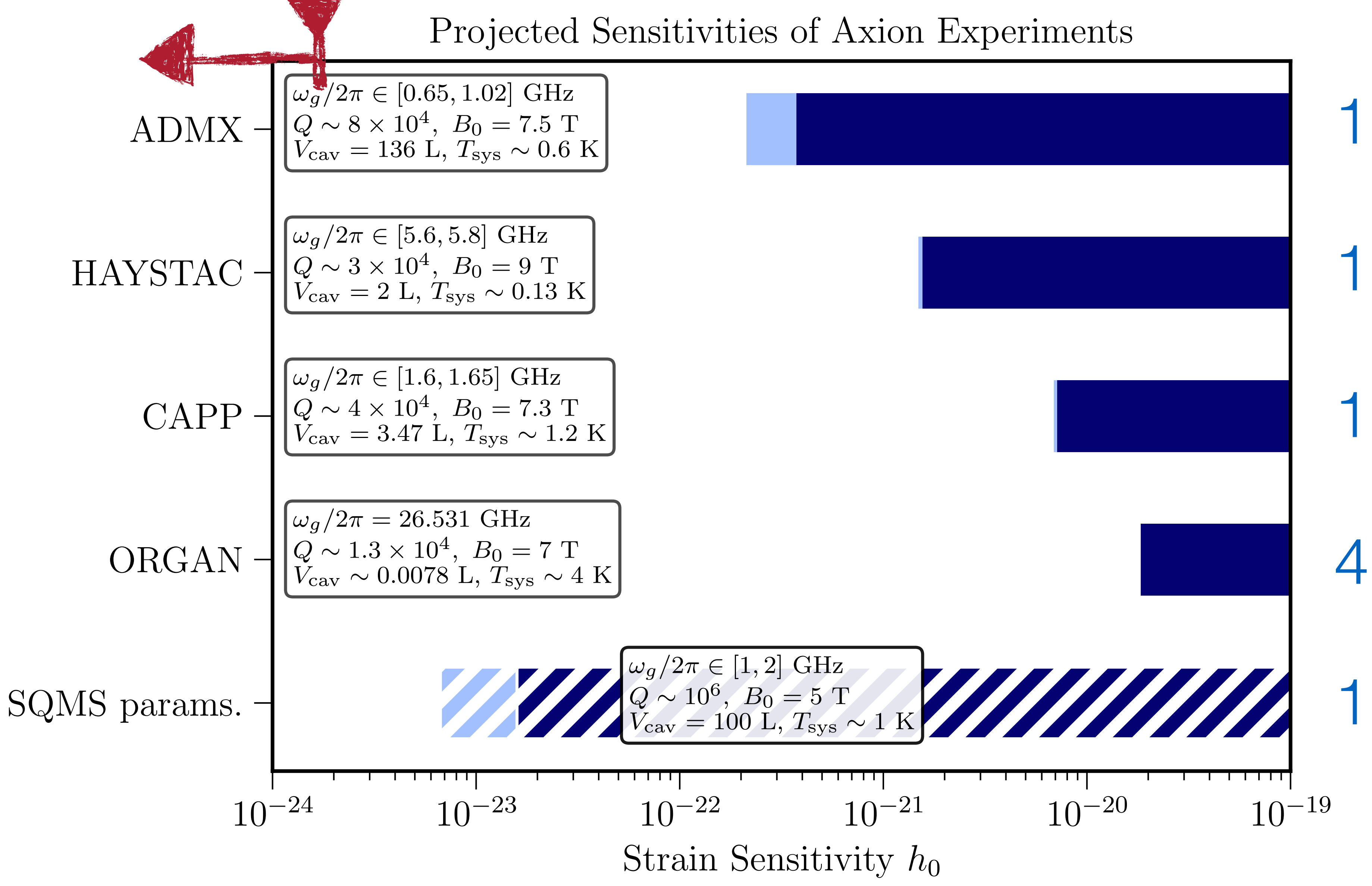
[arXiv:2112.11465]

Interesting sensitivity range for PBH



t_int

120s

120s

120s

4d

1d

[Detecting high-frequency gravitational waves with microwave cavities
Asher Berlin, Diego Blas, Raffaele Tito D'Agnolo , Sebastian A.R. Ellis
arXiv:2112.11465]

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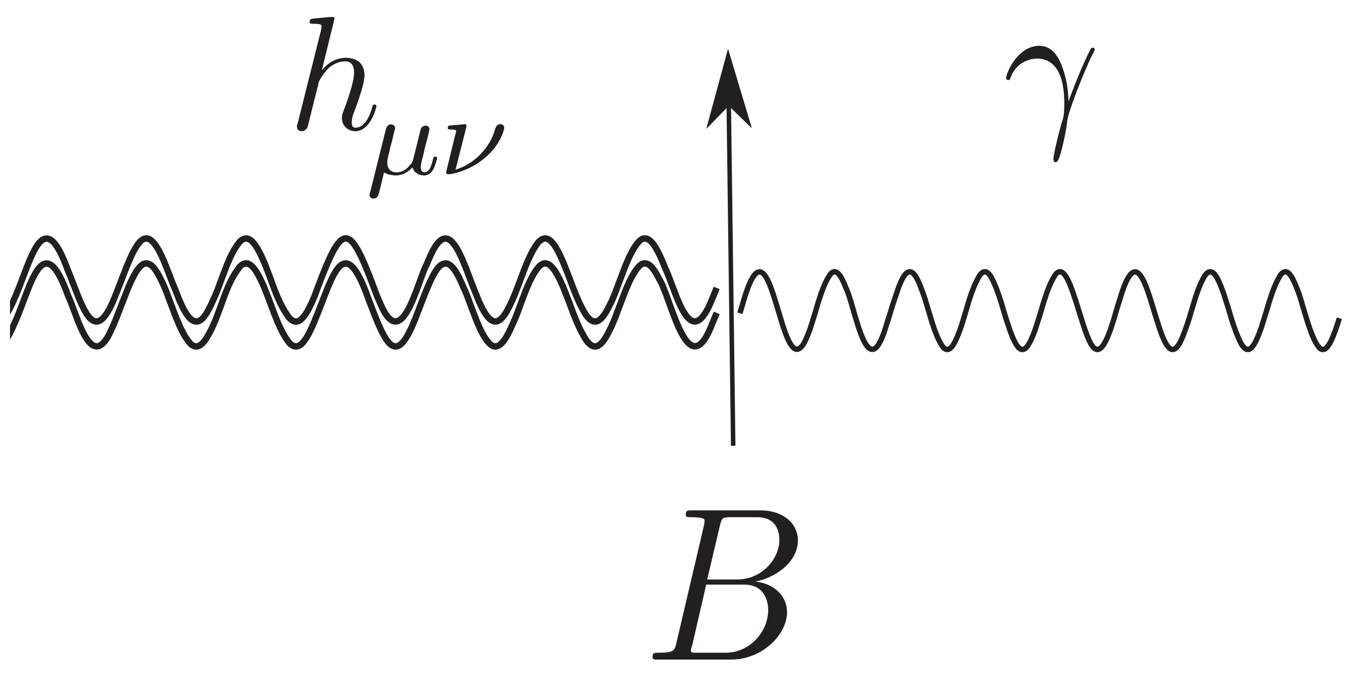
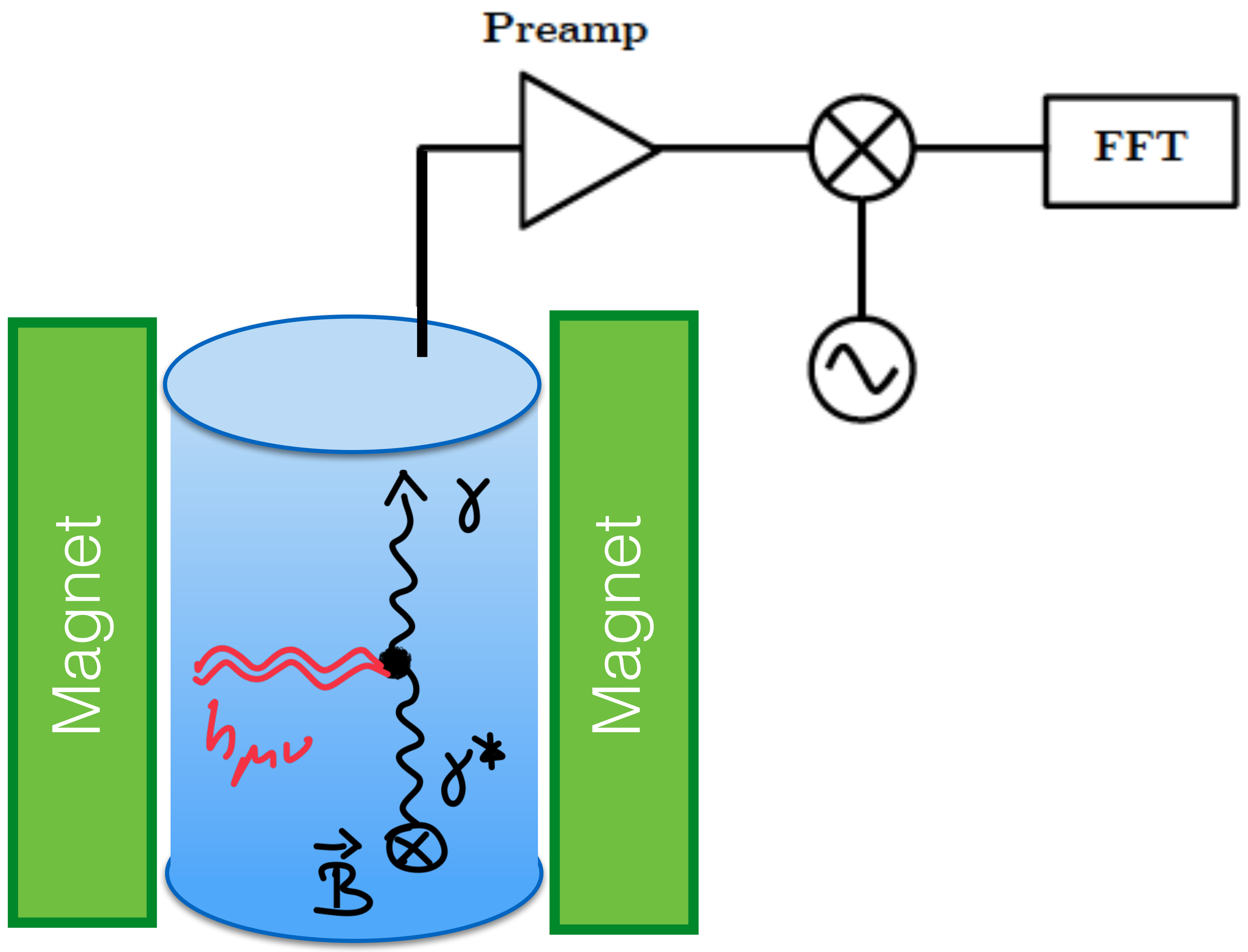
What we do at Mainz





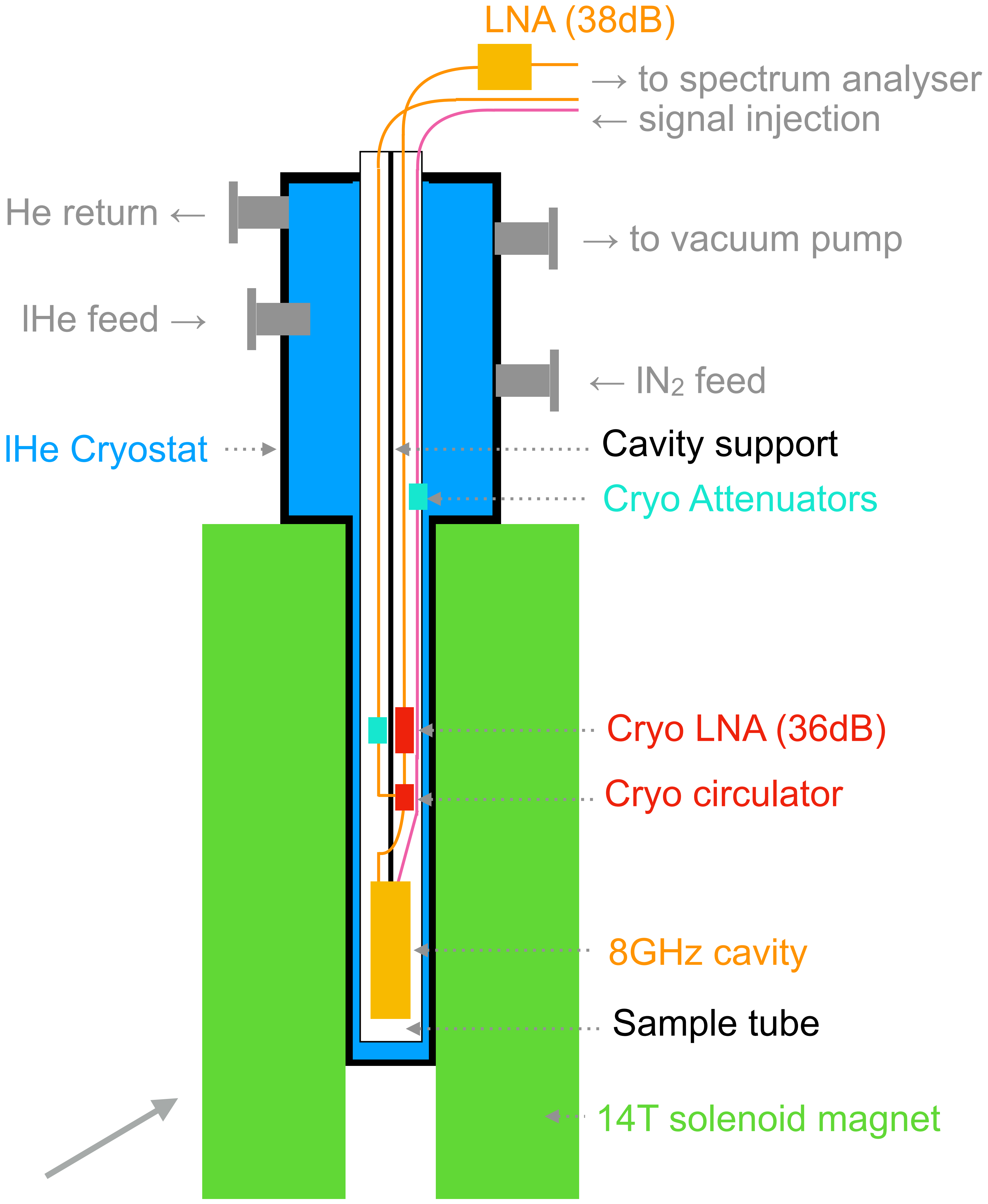
Niche for experiments optimised for EM detection of GW signals

Exploited @ Mainz

What we do at Mainz

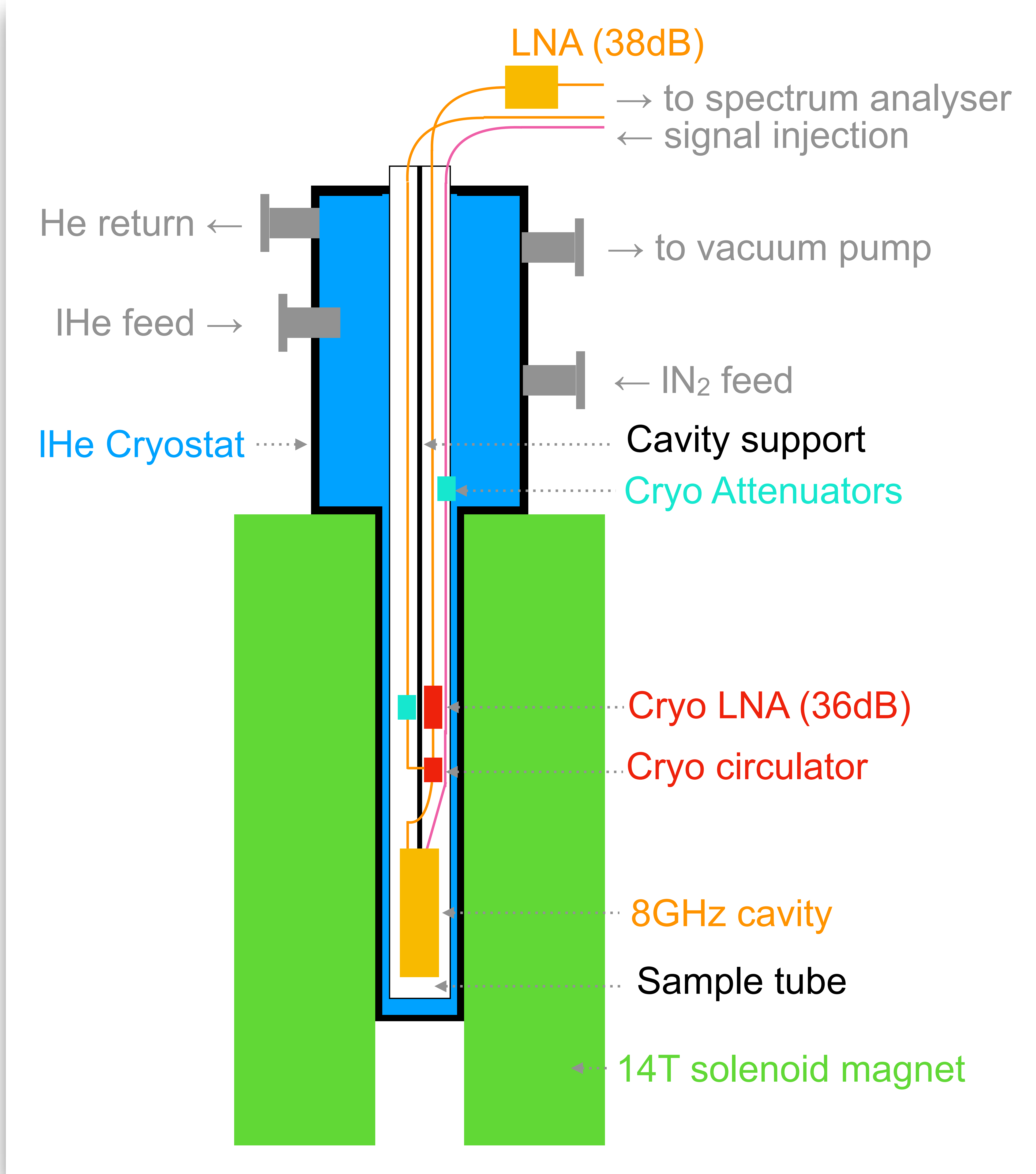
- **Supax:** superconducting axion search @ Mainz
  - First results on dark photons (~commissioning) [arXiv:2308.08337]
  - **Goals**:
    - Study of new **SC materials** for resonant cavity experiments
    - Study of **cavity geometries** optimised for **GW** searches
      - Together with Mainz theory section (P. Schwaller)

[arXiv:2308.08337]



Magnet @ HIM in D. Budkers group

Current Efforts at Mainz - SupAx / GravNet



- New haloscope setup for R&D and physics

- Magnet bore: 89mm
  - Inner cryostat diameter: 50 mm

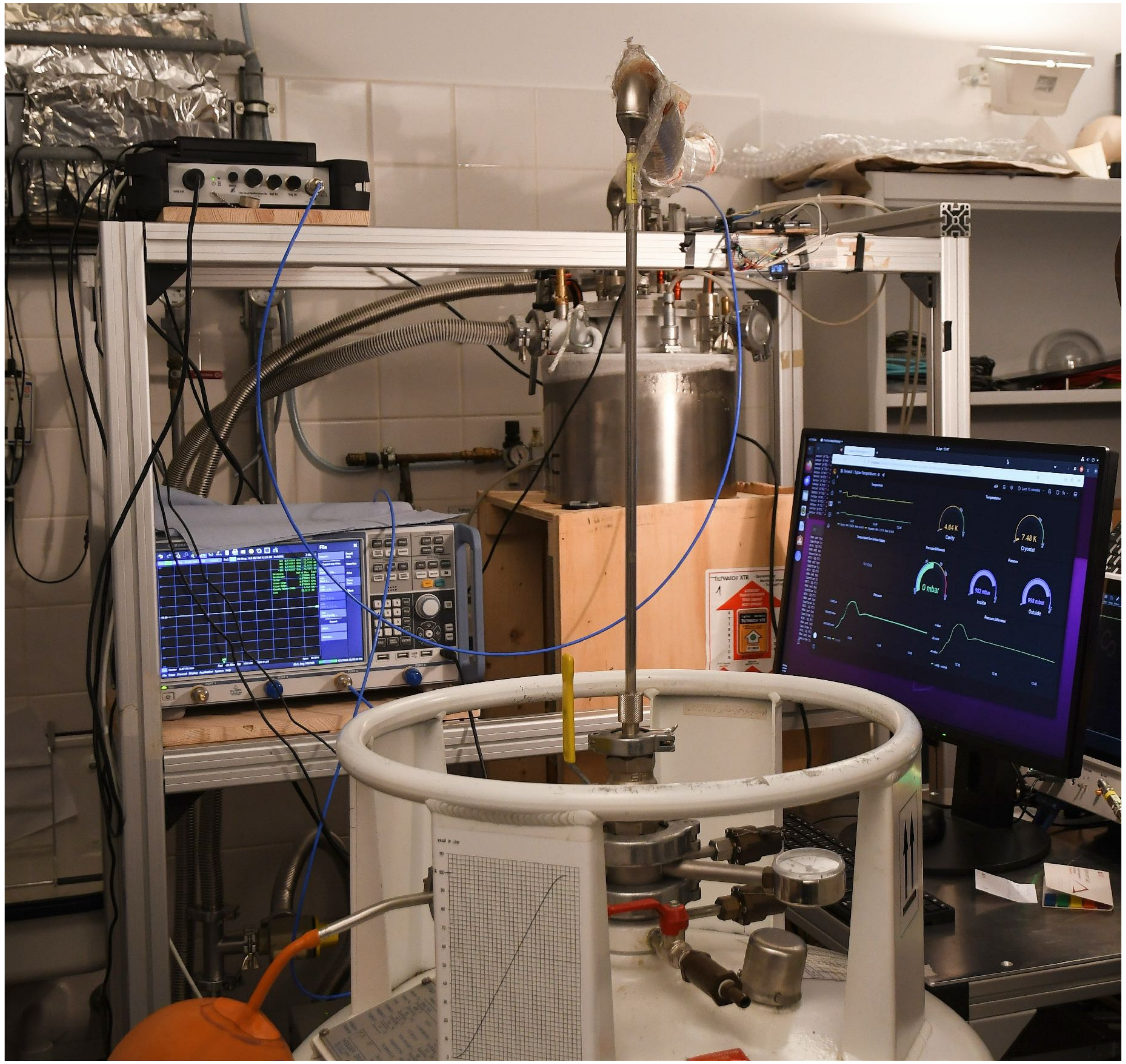
- Suppression of 300K noise from outside:
  - Attenuators on input lines @ 4K

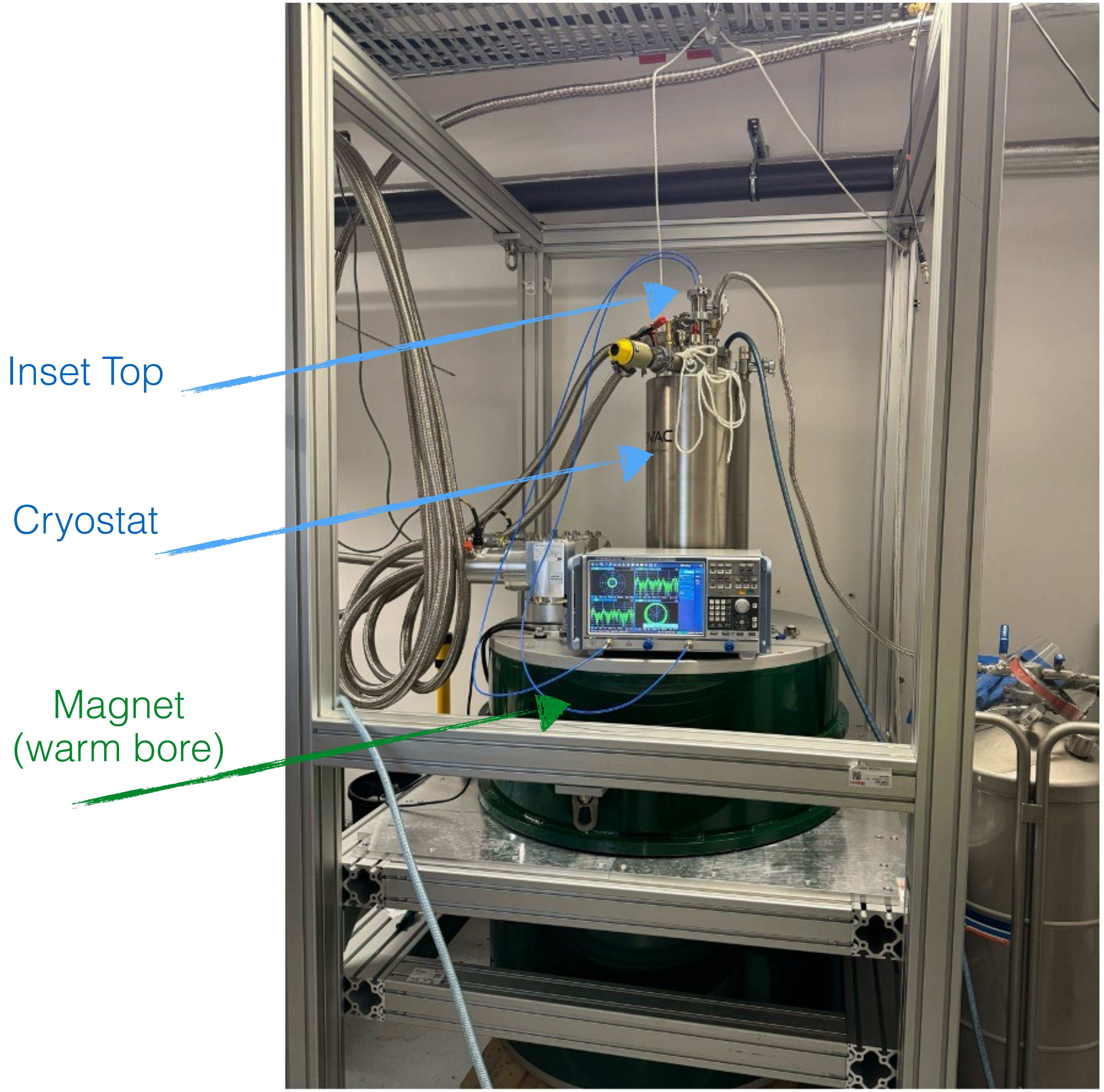
- Isolator (Circulator) before Preamp
  - Reduction of residual RF reflection

- Cryo Preamp @ 4K, 10GHz:
  - Gain: 36 dB
  - Noise: 3.8K (0.06dB)

- Cavity resonance frequency:
  - 8.4 GHz

Supax / GravNet - Measurements





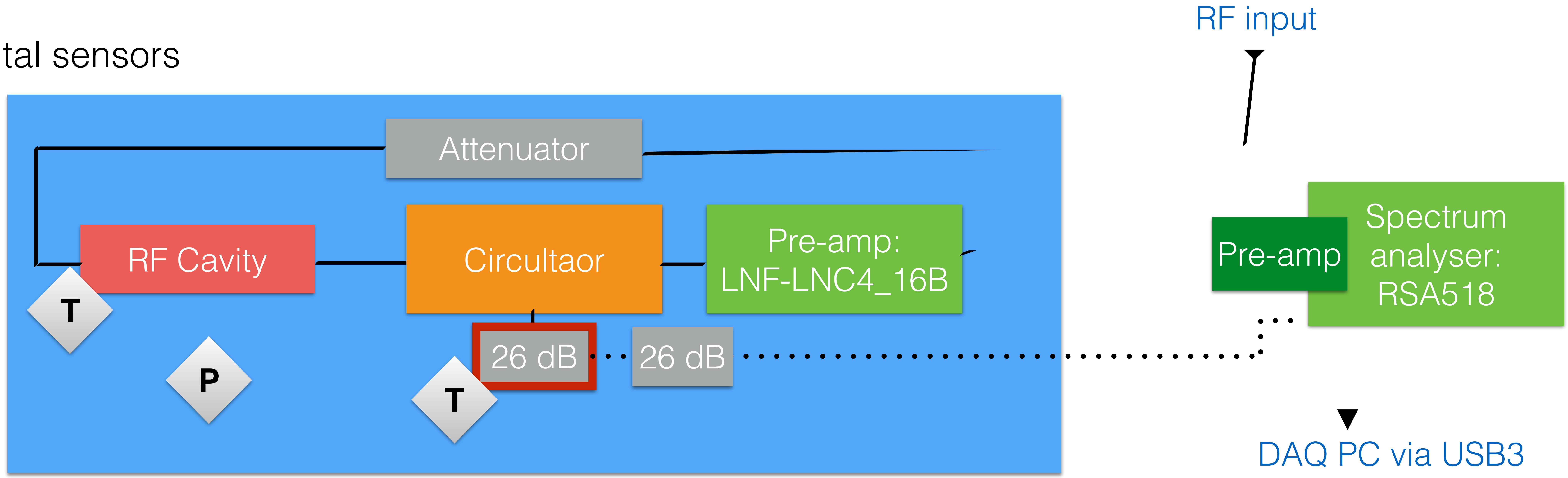
Inset Top

Cryostat

Magnet
(warm bore)

Supax / GravNet - DAQ system

T : Environmental sensors



- **Slowcontrol**
  - Temperature and pressure sensors
  - Monitoring with Influx + Grafana

  - T: PID control
  - P: Actuator in development

- **Readout**
  - 40 MHz realtime IQ data: 200MB/s
  - Realtime FFT, averaging and DQ

Supax / GravNet - DAQ system

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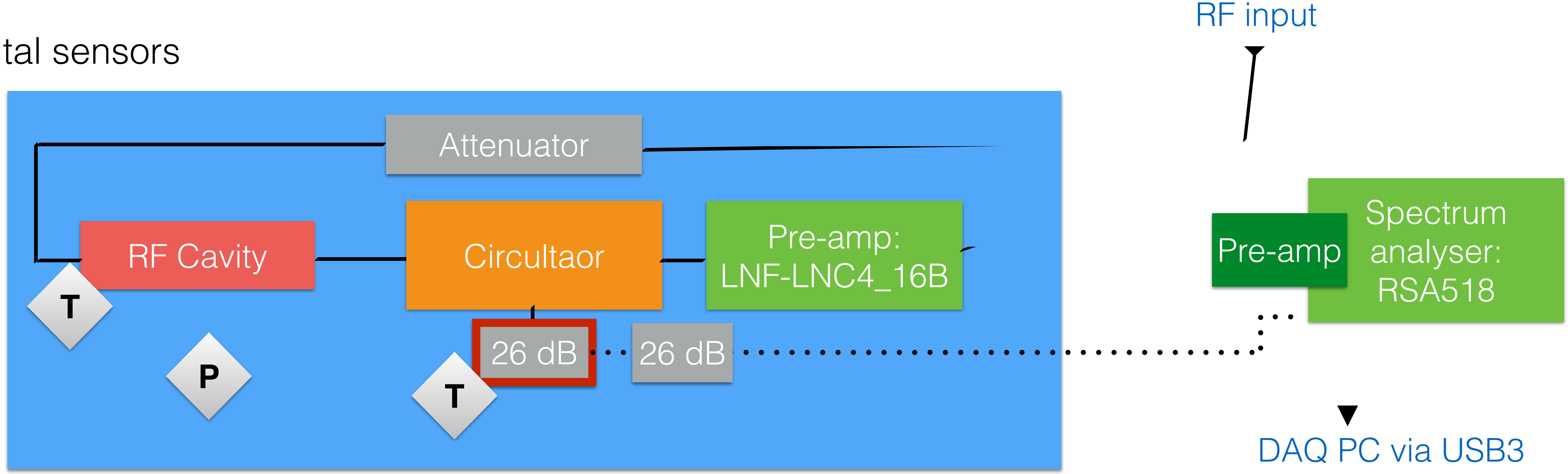

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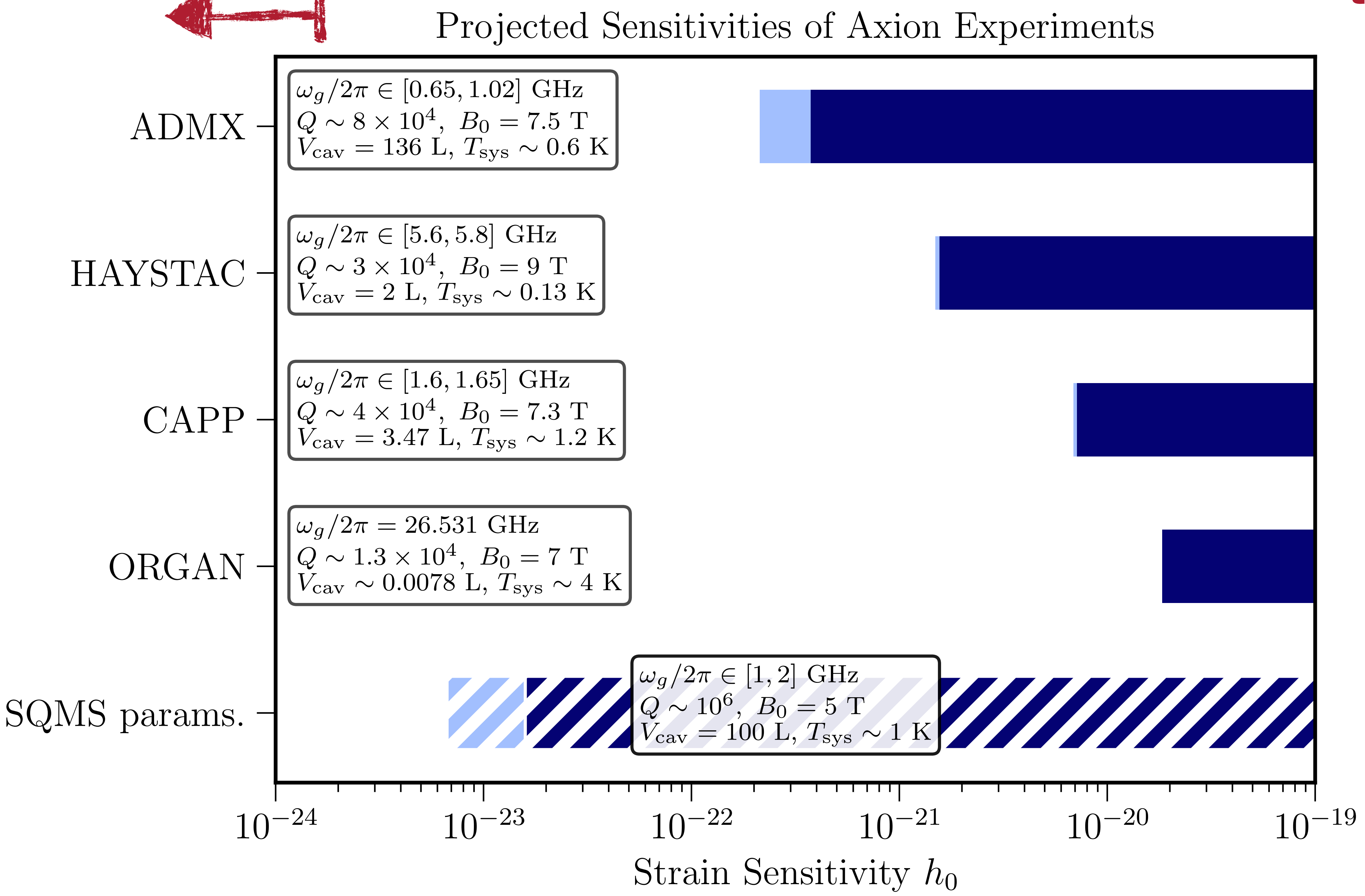
- **Readout - Future**
  - JPA based readout
  - Eventually: **Quantum detectors** for single photons

Axions vs. Gravitational waves in haloscopes

Interesting sensitivity range



Integration time:

120s

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4d

1d

[Detecting high-frequency gravitational waves with microwave cavities
Asher Berlin, Diego Blas, Raffaele Tito D'Agnolo , Sebastian A.R. Ellis
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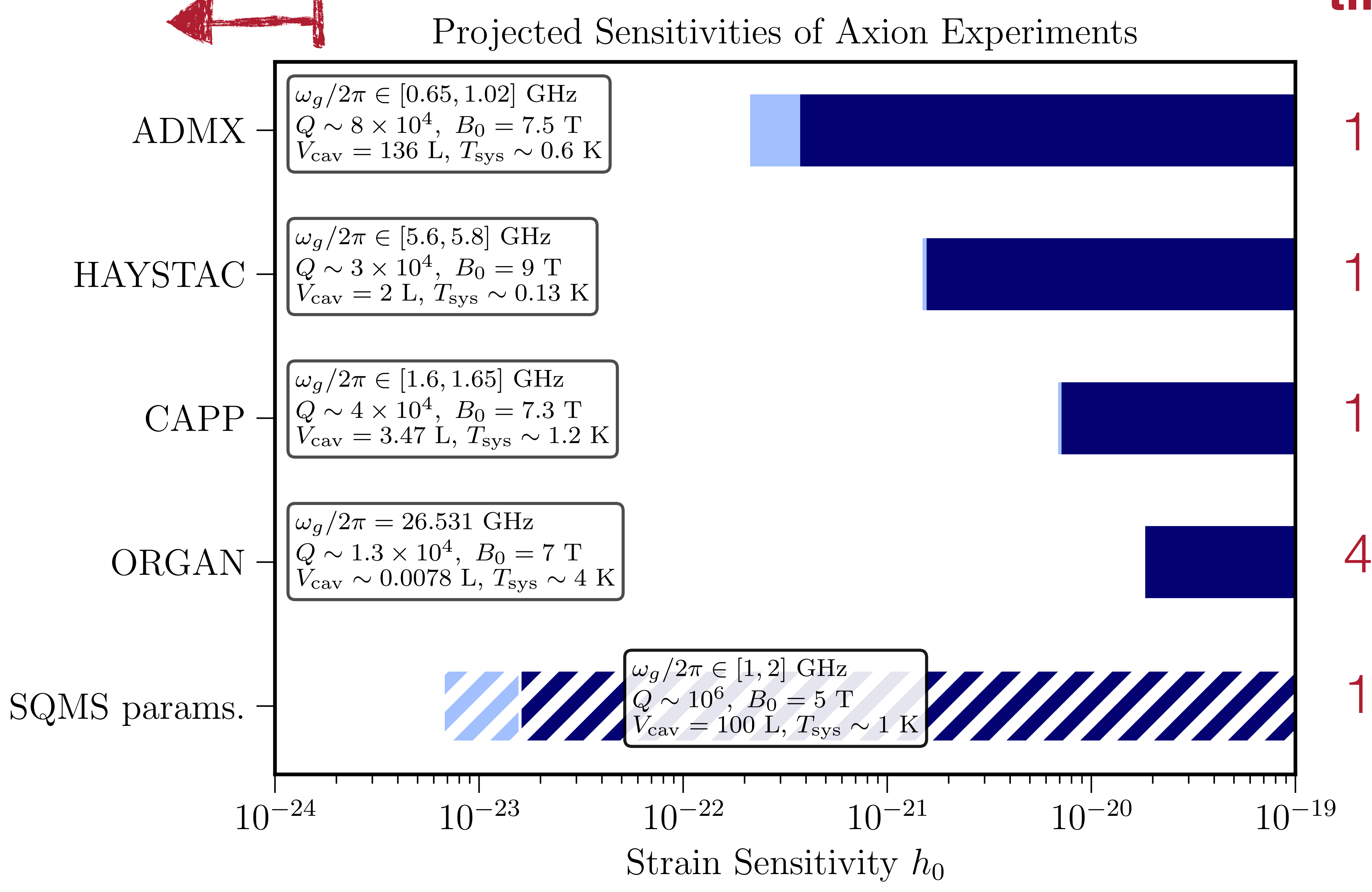
Axions vs. Gravitational waves in haloscopes

Interesting sensitivity range

- Resonant excitation of EM field in Cavity
  - Produced EM power given by:

$$P_{sig}^{GW} \propto \omega_g^3 Q V^{5/3} (\eta_q h_0 B_0)^2$$

$$P_{sig}^{a} \propto \quad Q V \quad (\eta_d g_\gamma B_0)^2$$



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Supax / GravNet - Signal Power

$$P_{sig} = \frac{1}{2} Q \omega_g^3 V^{5/3} (\eta_n h_0 B_0)^2 \frac{1}{\mu_0 c^2}$$

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- Up to 14T magnets in use
  - Up to 20T envisioned
- Larger fields - smaller volume

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  - Resonance frequency
  - Tuning elements

Supax / GravNet - Signal Power

- High purity copper: ~$5 \cdot 10^4$

- Superconducting: difficult in high magnetic field!
  - Target: 10^6
  - Achieved: $3 \cdot 10^5$ (CAPP, non tunable)
    - Materials under study: Nb_3Sn, **NbN**, HTS materials (YBCO)

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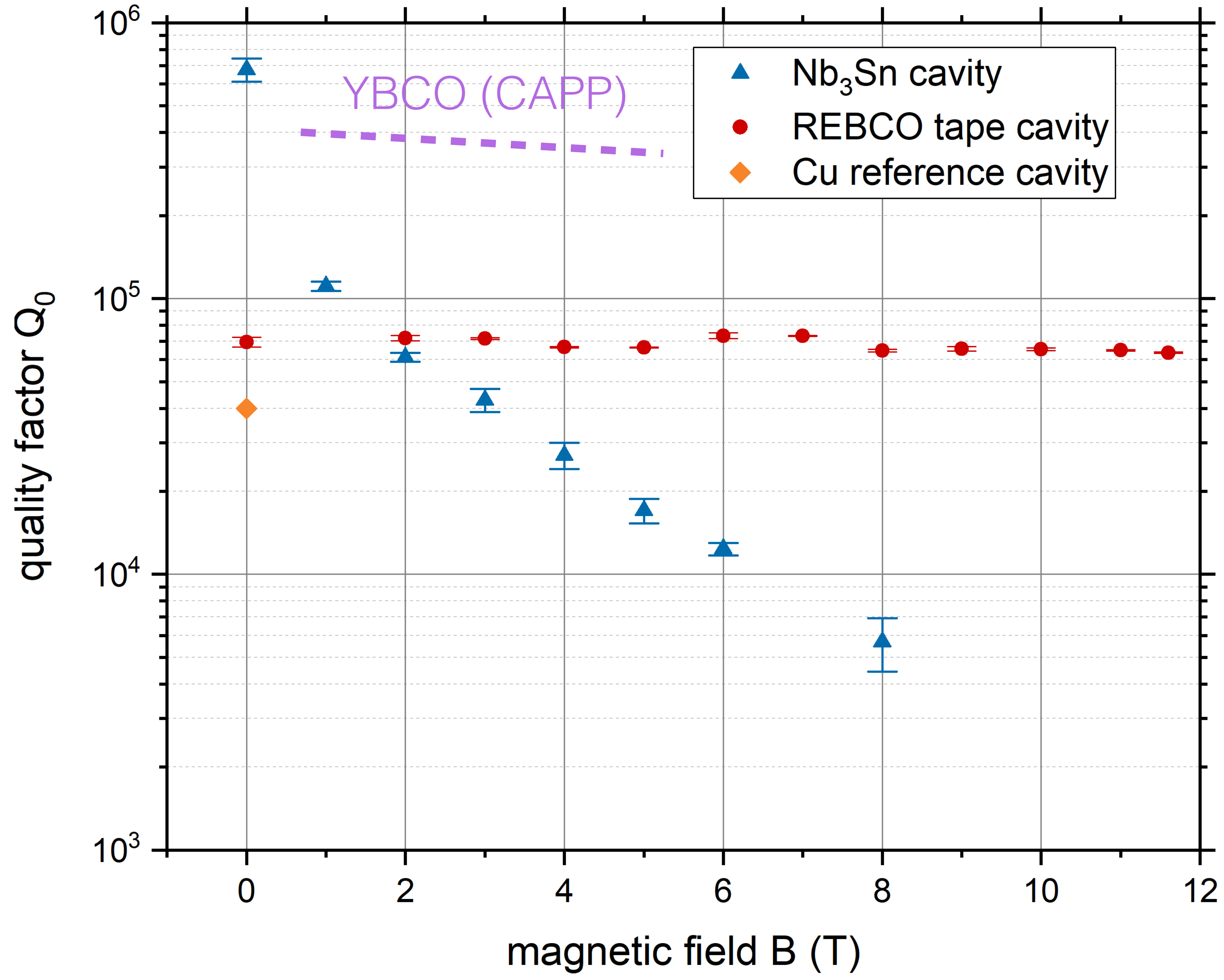

- D. Ahn et. al (CAPP), ~7 GHz
  https://arxiv.org/abs/2002.08769
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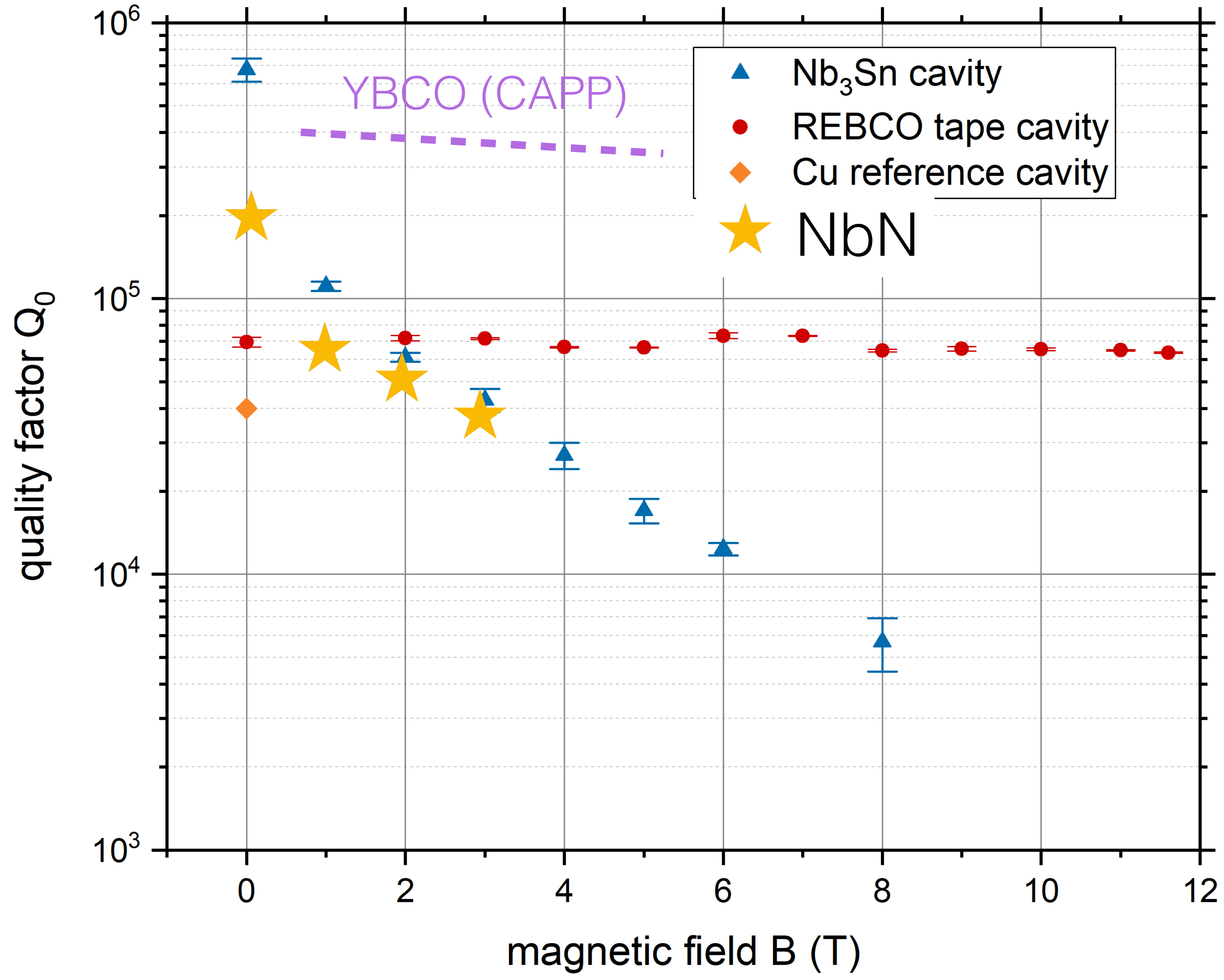

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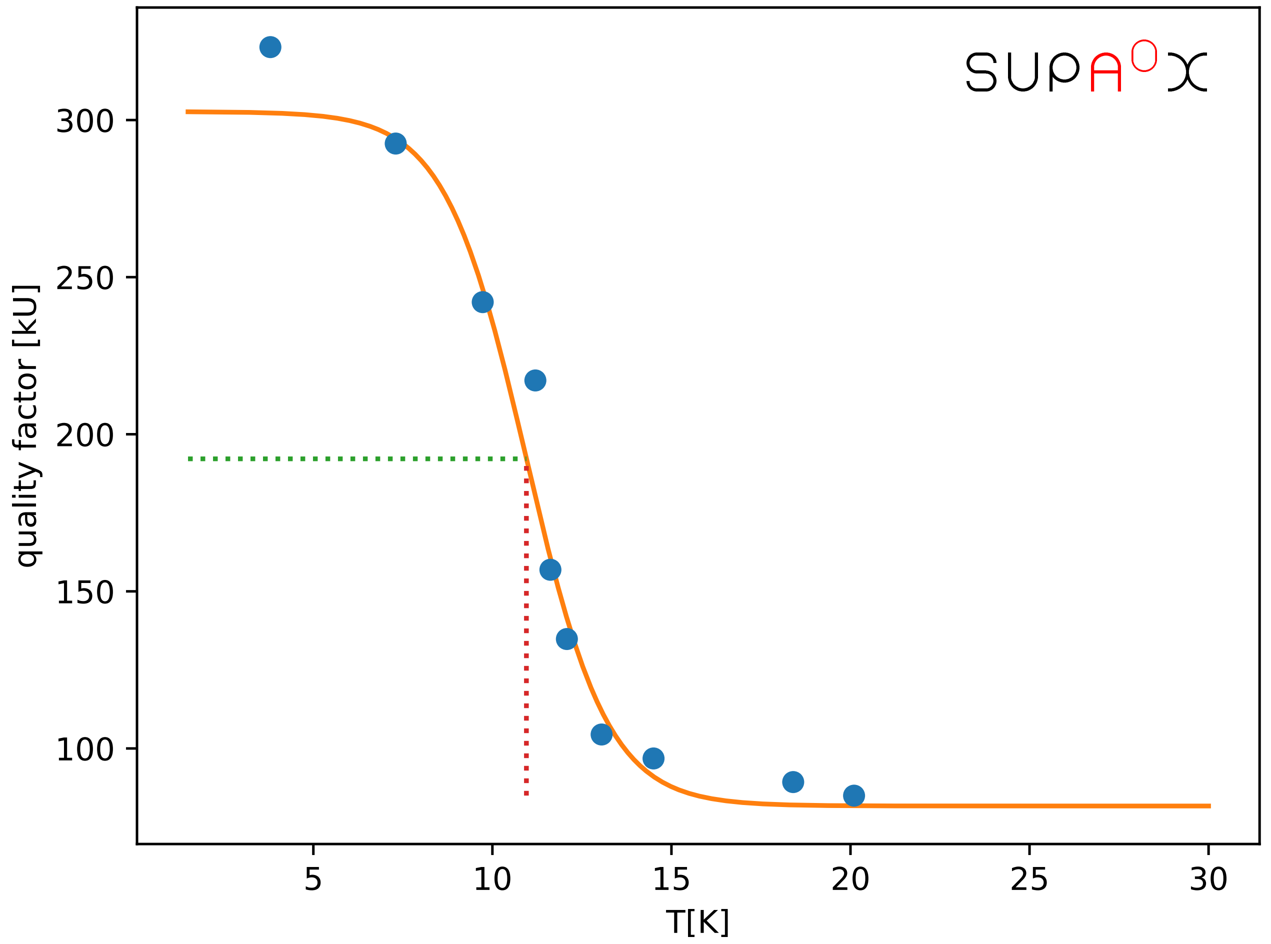
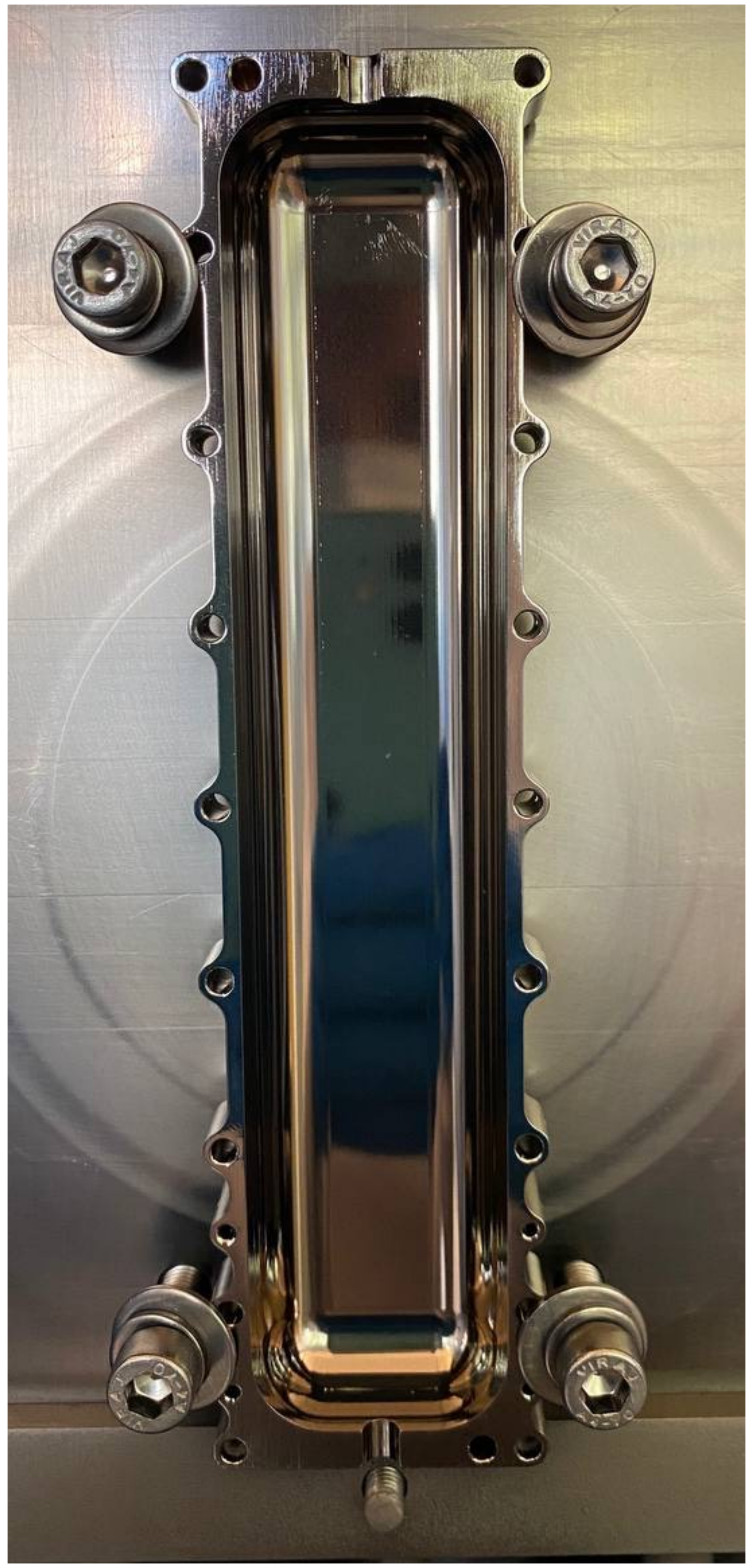
Supax / GravNet - Cavities

- Test of various cavity geometries and coatings



Cu coated with NbN
Coating by Zubtsovskii @ Uni Siegen



Sources of HF GW

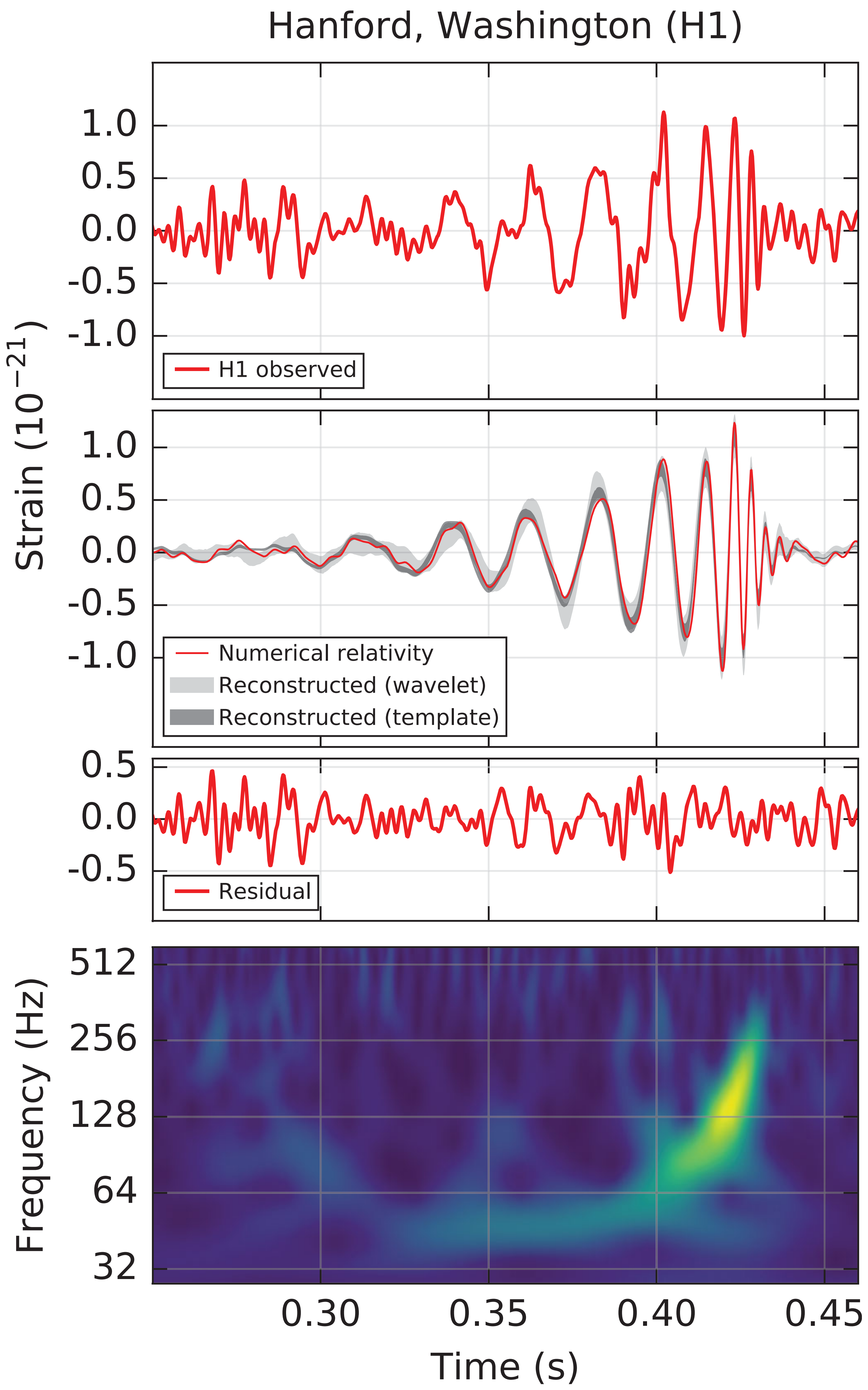
- **Working setup**
- Sensitive to HFGW (~ GHz)
- Which **sources** can be seen?
- Is there anything emitting GHz gravitational waves?

Sources of GW

- First observed sources: **Black hole merging events**
  - Chirp signals
    - m_{BH} ~ O(10$M_\odot$): frequency in acoustic range

$$f \approx 100\,Hz \rightarrow m_{BH} \approx 30\,M_\odot \text{ , Duration: } 0.1s$$

- Frequency range: 10-1000 Hz



PRL 116, 061102 (2016)

Sources of GW

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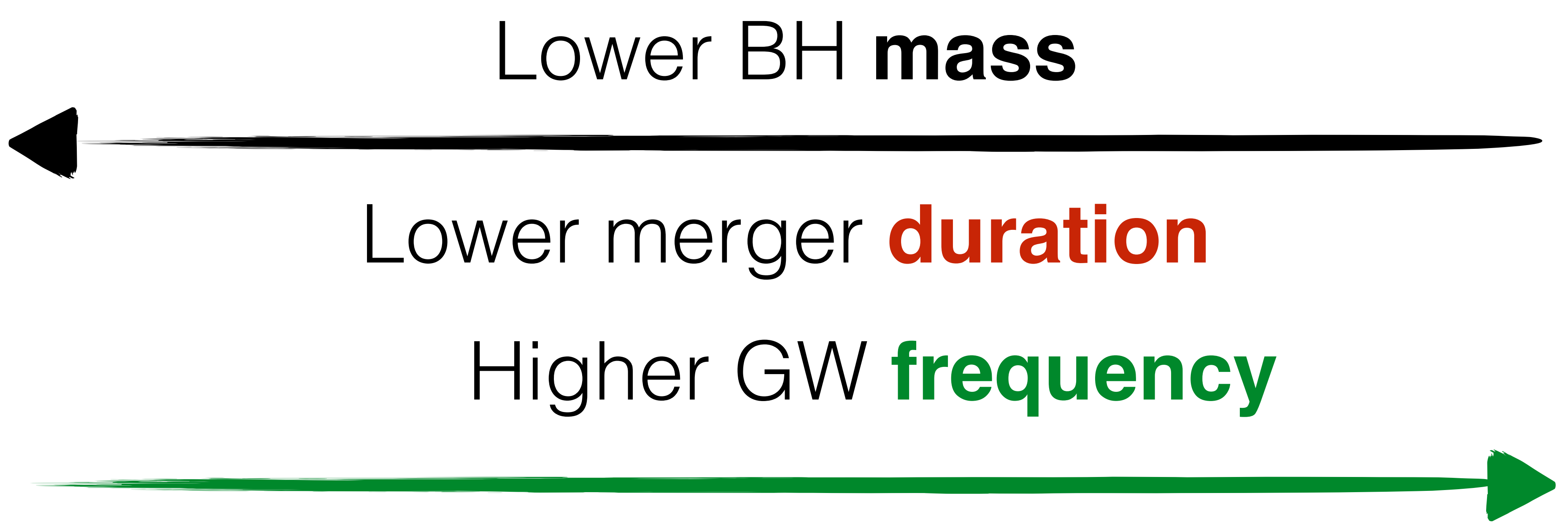
- Lighter BHs => higher frequencies



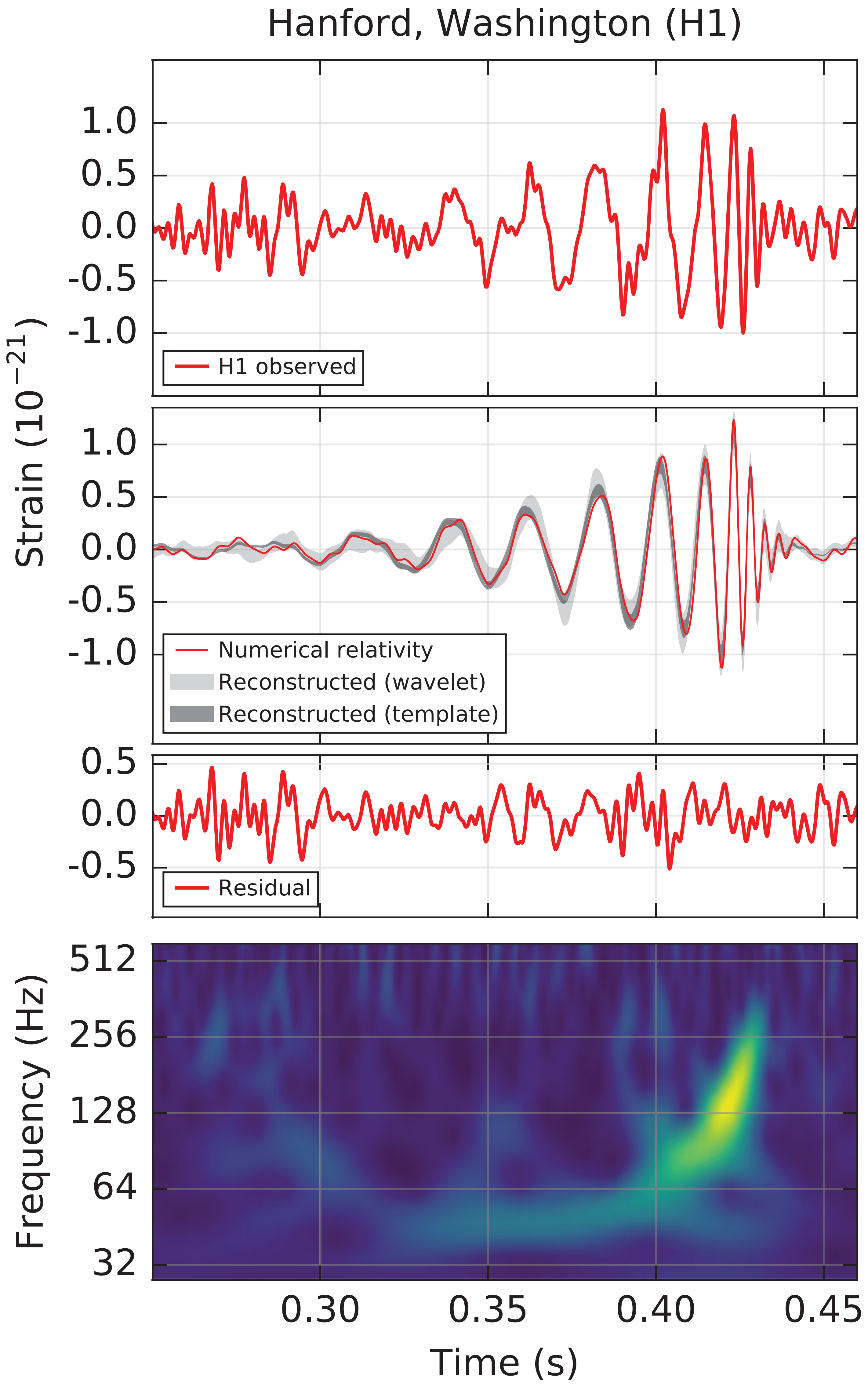
Lower BH **mass**

Lower merger **duration**

Higher GW **frequency**

$$f \approx GHz \rightarrow m_{BH} < 10^{-6} M_\odot \text{ , Duration: } \mu s$$

- Frequency range: 10-1000 Hz


Hanford, Washington (H1)

PRL 116, 061102 (2016)

Sources of GW

Any issues with black hole masses of $10^{-6} M_\odot$?

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- Chandrasekhar limit: Up to 1.4 $M_\odot$ white dwarfs are stable
- Tolman–Oppenheimer–Volkoff limit: Neutron stars stable up to 2 - 3 $M_\odot$
  - Corresponding to stellar progenitor masses O(10$M_\odot$)

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  - Corresponding to stellar progenitor masses O(10$M_\odot$)

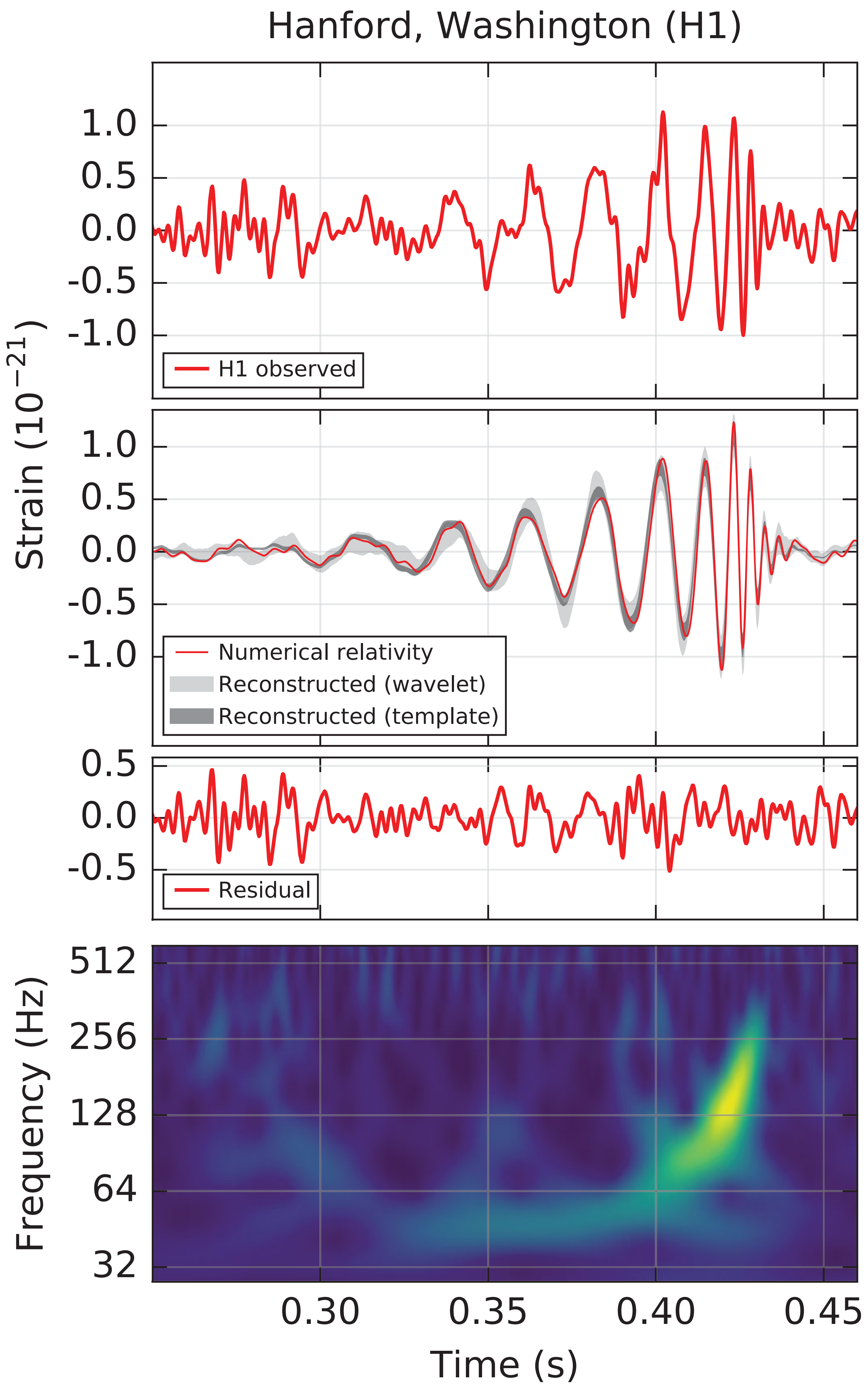
Lightest BH should be around $2-3\,M_\odot$

(Lightest currently observed: $3\,M_\odot$)

Sources of HF GW

- **Primordial black hole mergers**
  - Chirp signals

- Frequency range: 10-1000 Hz



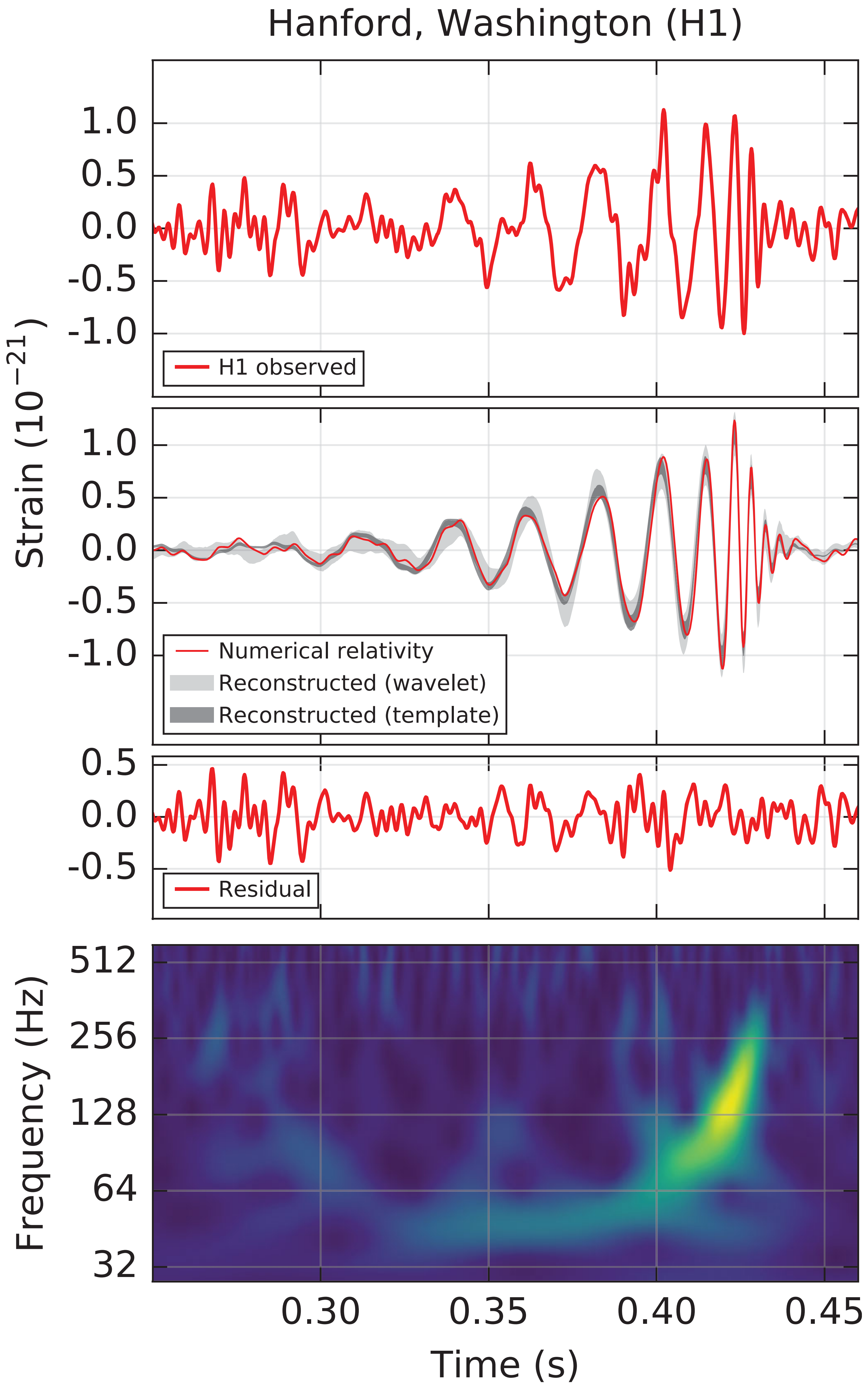
PRL 116, 061102 (2016)

Sources of HF GW

- **Primordial black hole mergers**
  - Chirp signals
    - $f \approx GHz \rightarrow m_{BH} < 10^{-6} M_{\odot}$, Duration: μs

- **Primordial:**
  - Hypothetical BHs created shortly after the big bang, before the first stars were formed
  - Not limited to the narrow mass range of stellar BHs

- Frequency range: 10-1000 Hz



PRL 116, 061102 (2016)

Sources of HF GW - Primordial Black Holes

- Sources for HFGWs:

  - **Primordial black hole merges**
  - Boson clouds (BH superradiance)
  - …

Primordial black holes:

- Black holes created in the early universe
  - Unlike stellar BH: No minimum mass requirement
  - Expected Mass range: 10^{-10} - $10^{-16}\,M_{\odot}$
  - Density unknown

- Merging events expected
  - Low mass -> High frequency
  - Fast transients (μs - ms)

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- Small scale perturbation in early universe

- Amplitude of space-time curvature perturbations enhanced by some mechanism

- Perturbation freeze in during inflation

- Post-inflation collapse if larger than some threshold

  - Population of PBHs
  - Masses controlled by energy in one Hubble volume

Sources of HF GW - Primordial Black Holes

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Why are PBH interesting objects?

Sources of HF GW - Primordial Black Holes

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Why are PBH interesting objects?

Could be dark matter

Sources of HF GW - Primordial Black Holes

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  - Boson clouds (BH superradiance)
  - …

Why are PBH interesting objects?

Could be dark matter

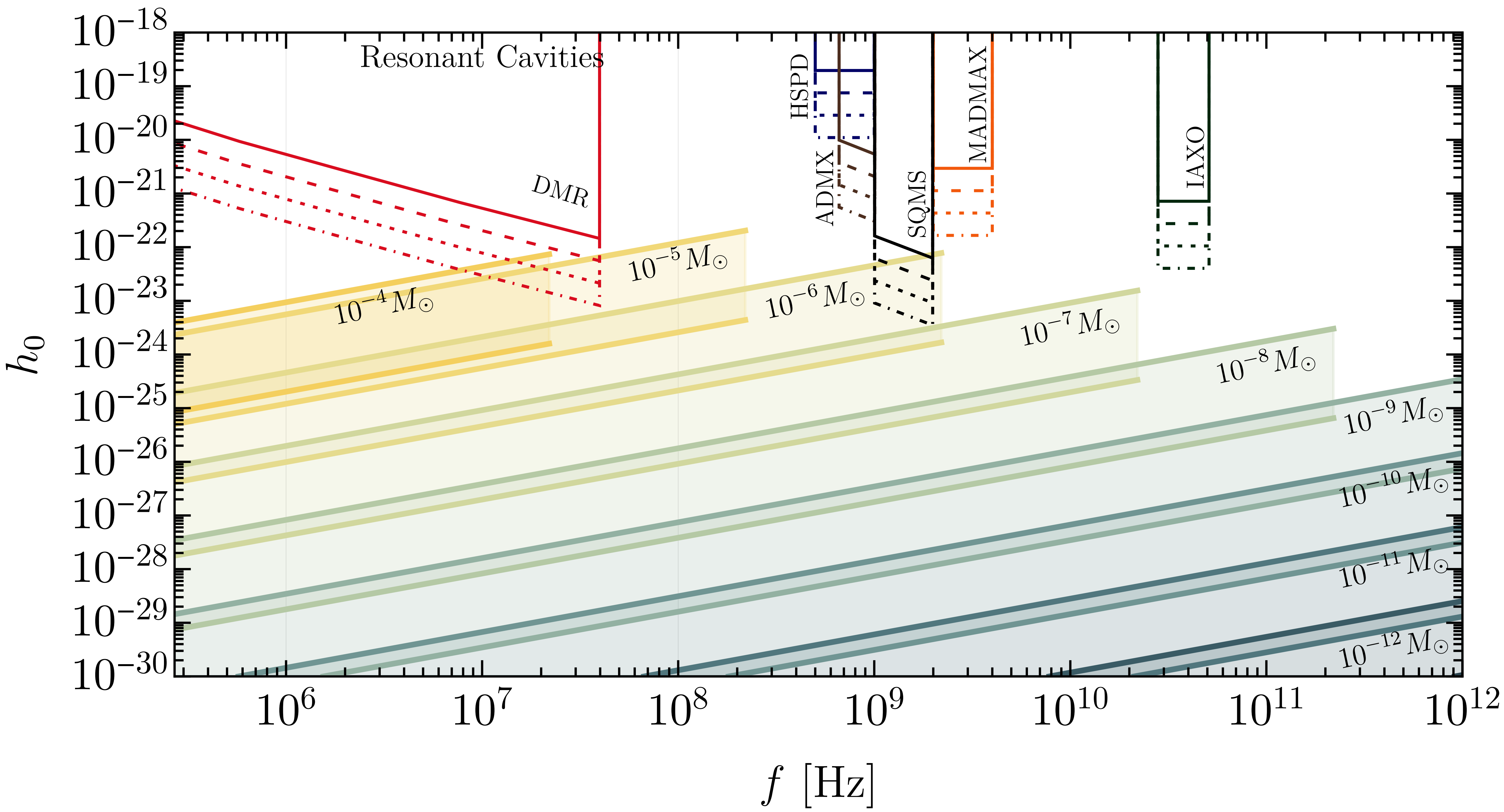
- How many could we possibly expect?

Sources of HF GW - Primordial Black Holes

$$h_0 \simeq 9.77 \times 10^{-34} \left(\frac{f}{1\,\mathrm{GHz}}\right)^{2/3} \left(\frac{m_{\mathrm{PBH}}}{10^{-12}\,M_\odot}\right)^{5/3} \left(\frac{d_L}{1\,\mathrm{kpc}}\right)^{-1}$$

- Assuming $f_{PBH} = \Omega_{PBH}/\Omega_{DM} = 1$
  - Allowed for: $m_{\mathrm{PBH}} \subset (10^{-16} \div 10^{-10})\, M_\odot$

[arXiv:1906.05950]



[Gabriele Franciolini, Anshuman Maharana, Francesco Muia; arXiv:2205.02153v1]

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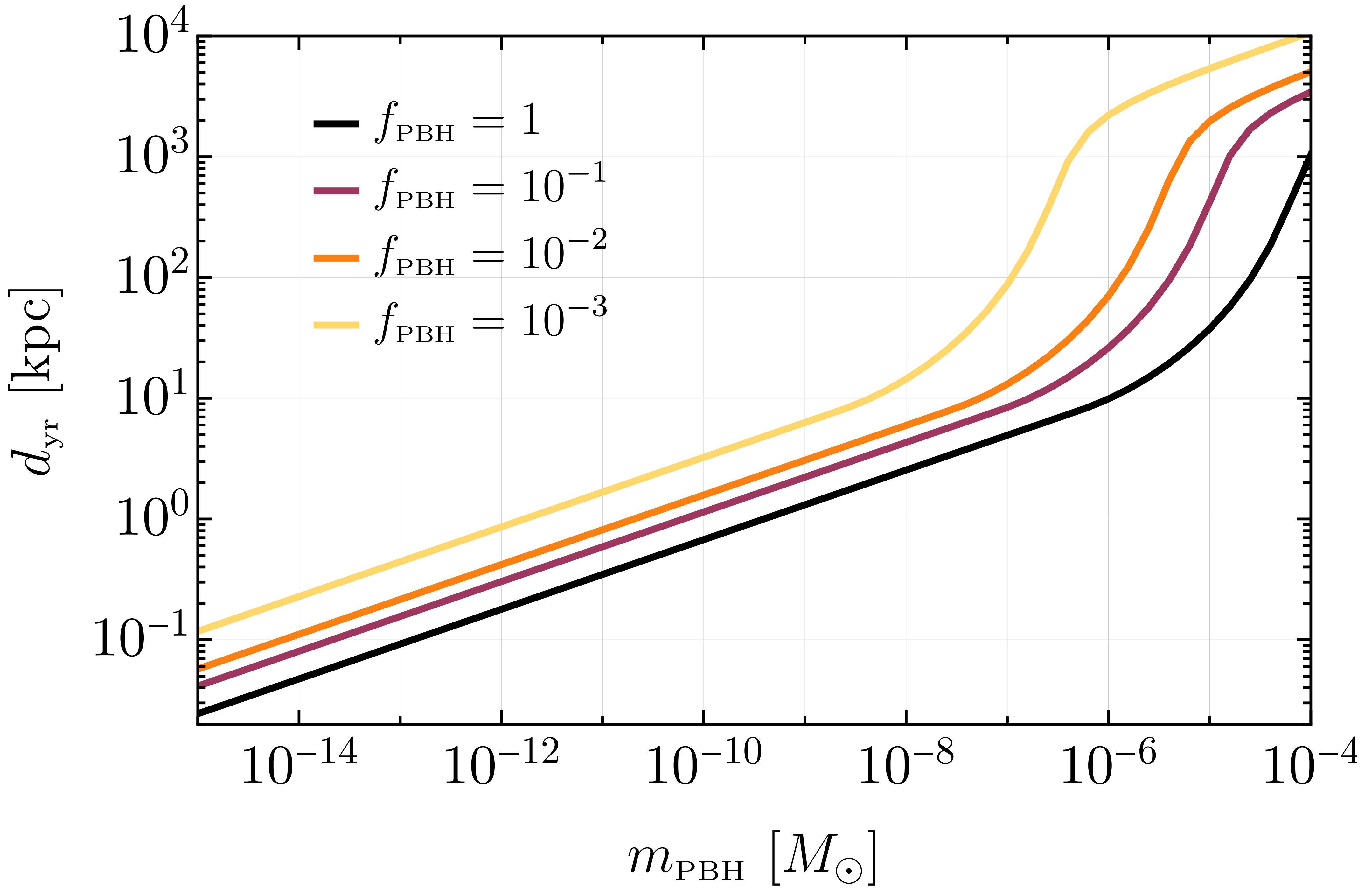
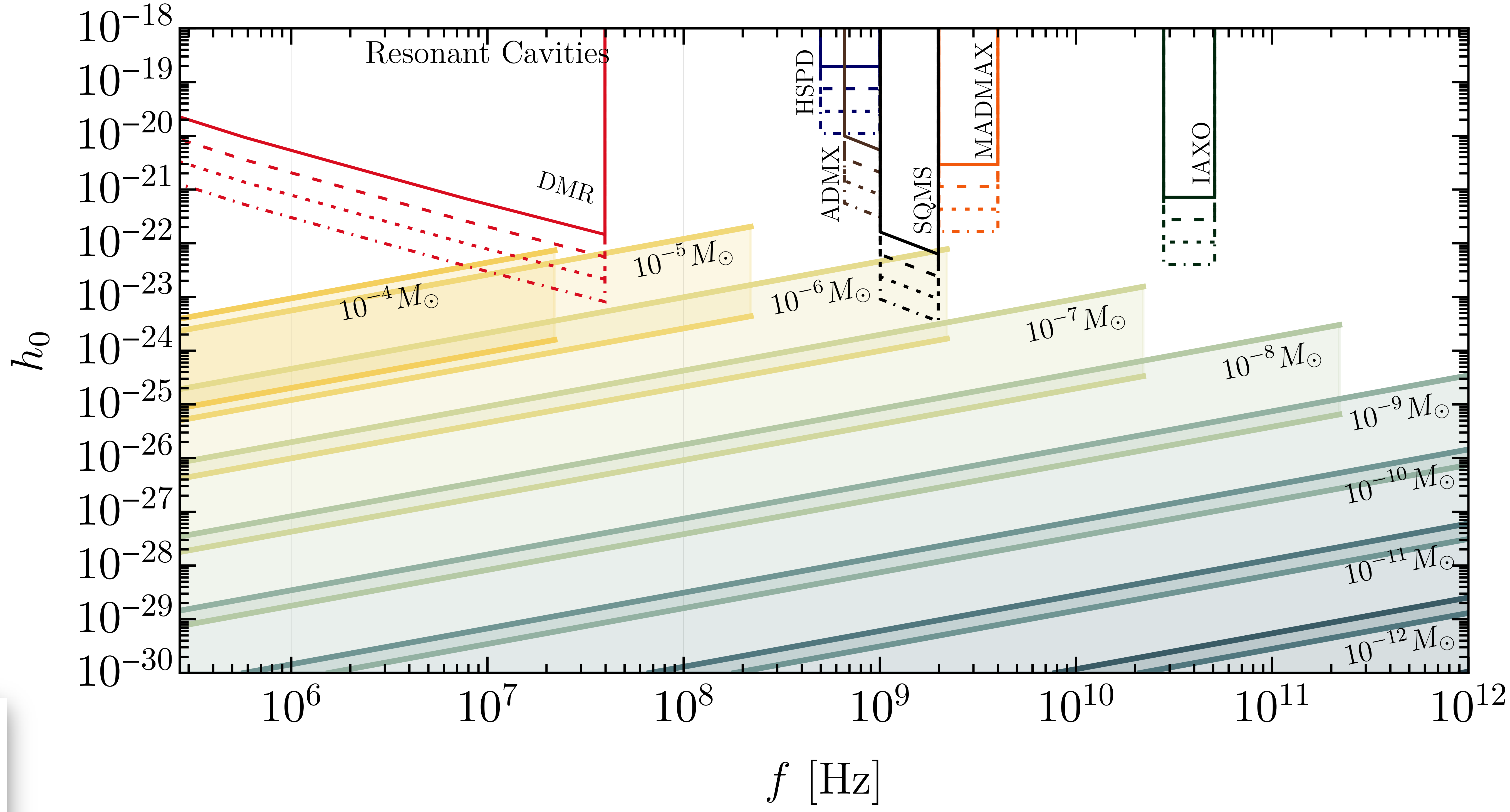




- Distance = d_{yr} = radius of sphere with ≥ 1 PBH merger / year

- Slope change: impact of local DM over density

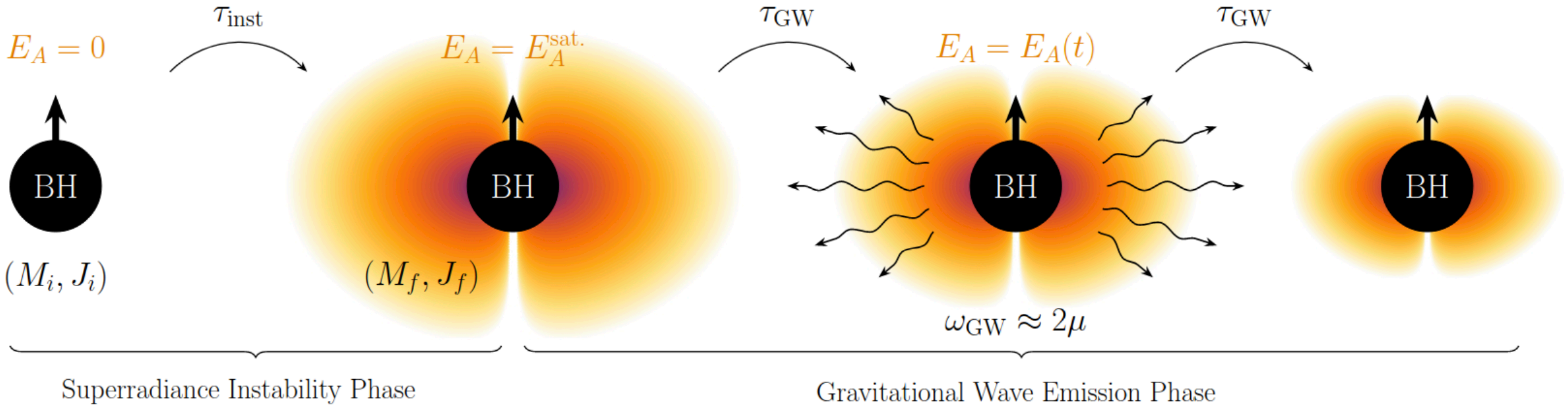
[Gabriele Franciolini, Anshuman Maharana, Francesco Muia; arXiv:2205.02153v1]

Sources of HF GW - Axion Superradiance

- Sources for HFGWs:
  - Primordial black hole merges
  - **Boson clouds (BH superradiance)**
  - …

Axion superradiance:

- Compton wavelength of boson = size of BH
  - Boson accumulates outside BH event horizon
  - Annihilation into gravitons if mass > threshold
    - $\omega_a < m\Omega_H$



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- Requires **light, spinning BHs**
- Requires **axion (-like) bosons**

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  - Annihilation into gravitons if mass > threshold
- Requires **light, spinning BHs**
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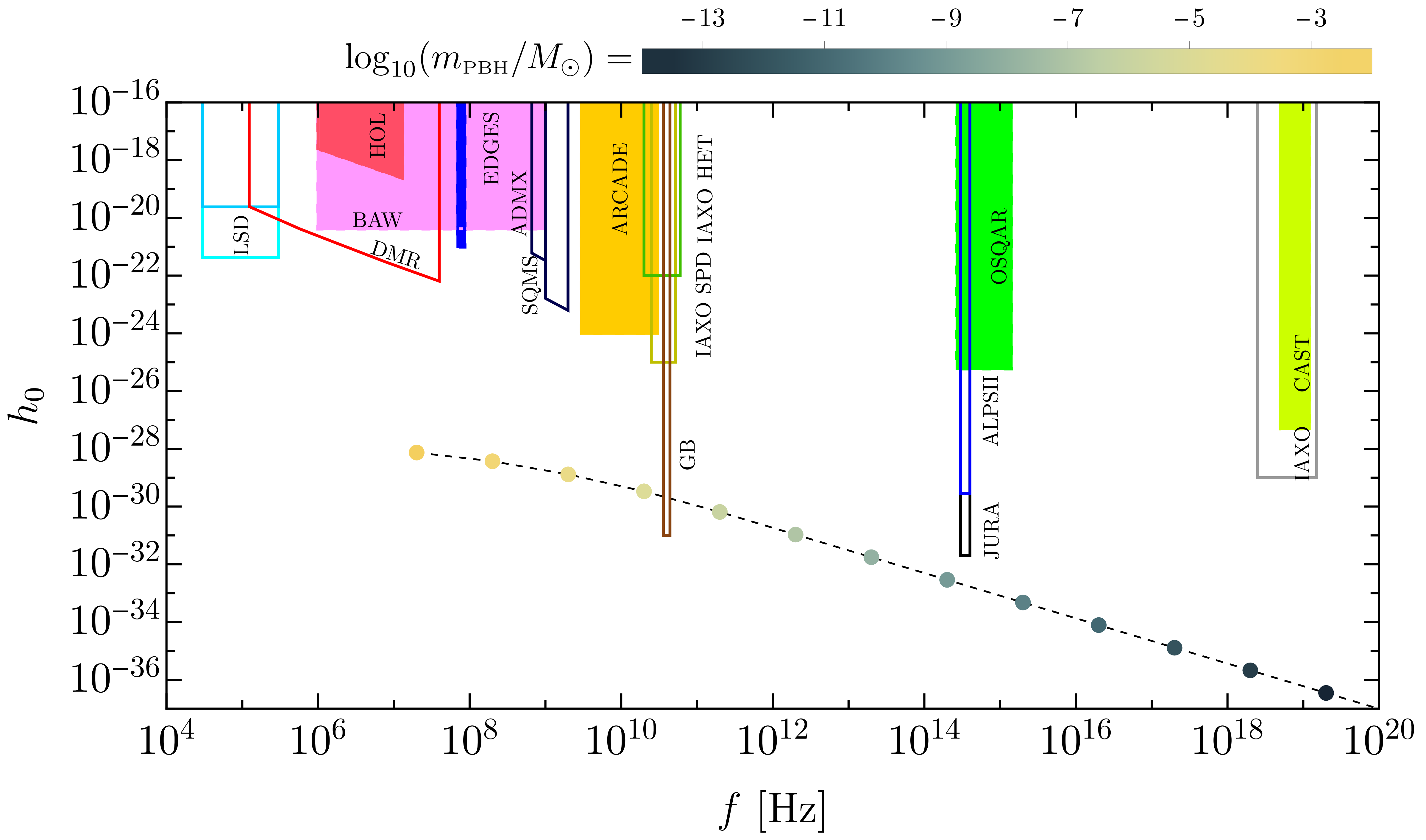
- **Monochromatic**, coherent signal!
- **Decay times of min. to years** (depending on BH mass)
- Strain assuming distance = radius of sphere with one event per year



Sources of HF GW - Stochastic Background

- Sources for HFGWs:

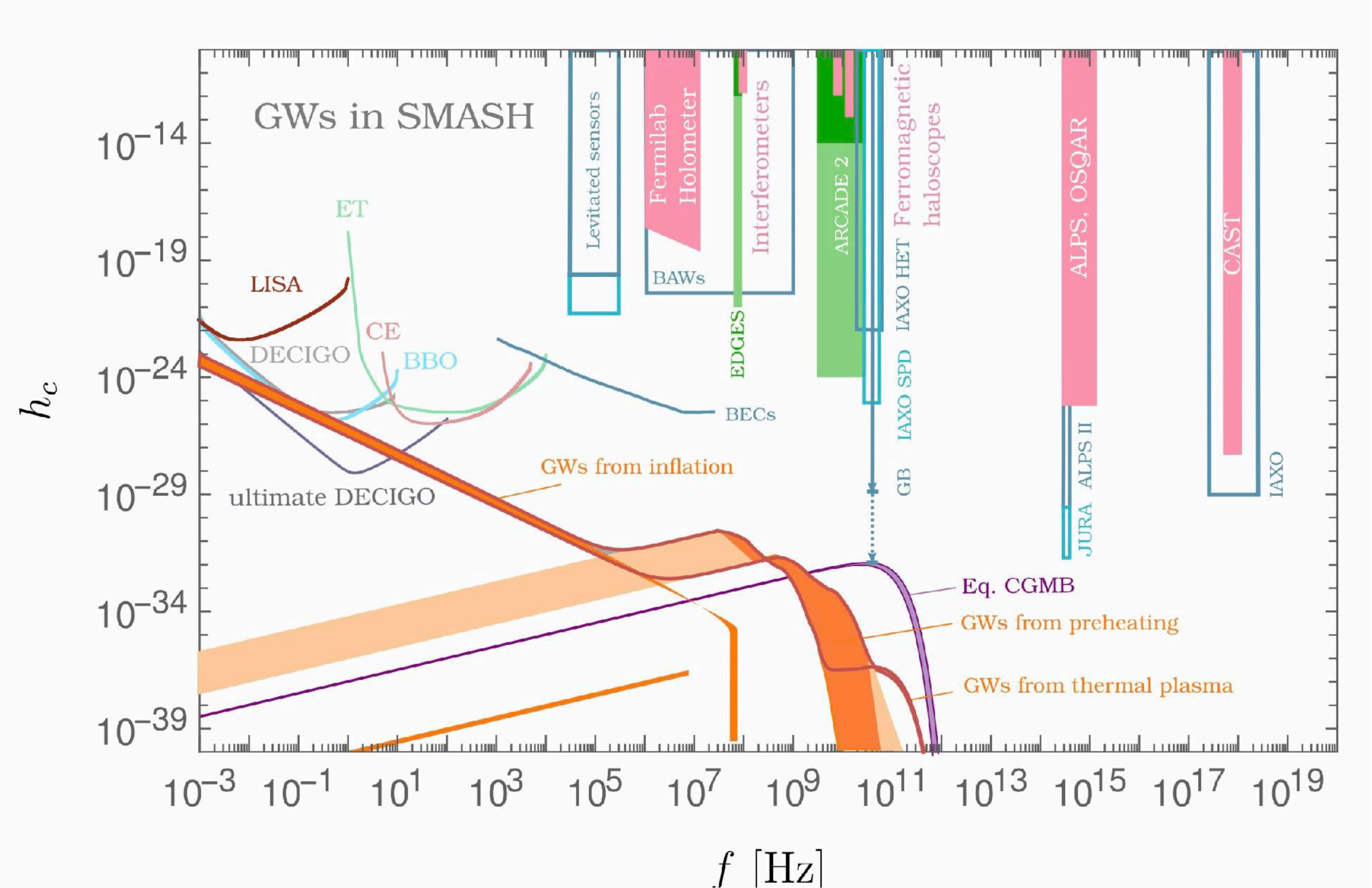
  - Primordial black hole merges
  - Boson clouds (BH superradiance)
  - **Stochastic GW sources**

Several sources possible:

- Phase transitions in the early universe
- Dynamics of inflation and subsequent (p-)reheating
- Fluctuations in the thermal plasma
- Cosmic strings

Very low strain expected:

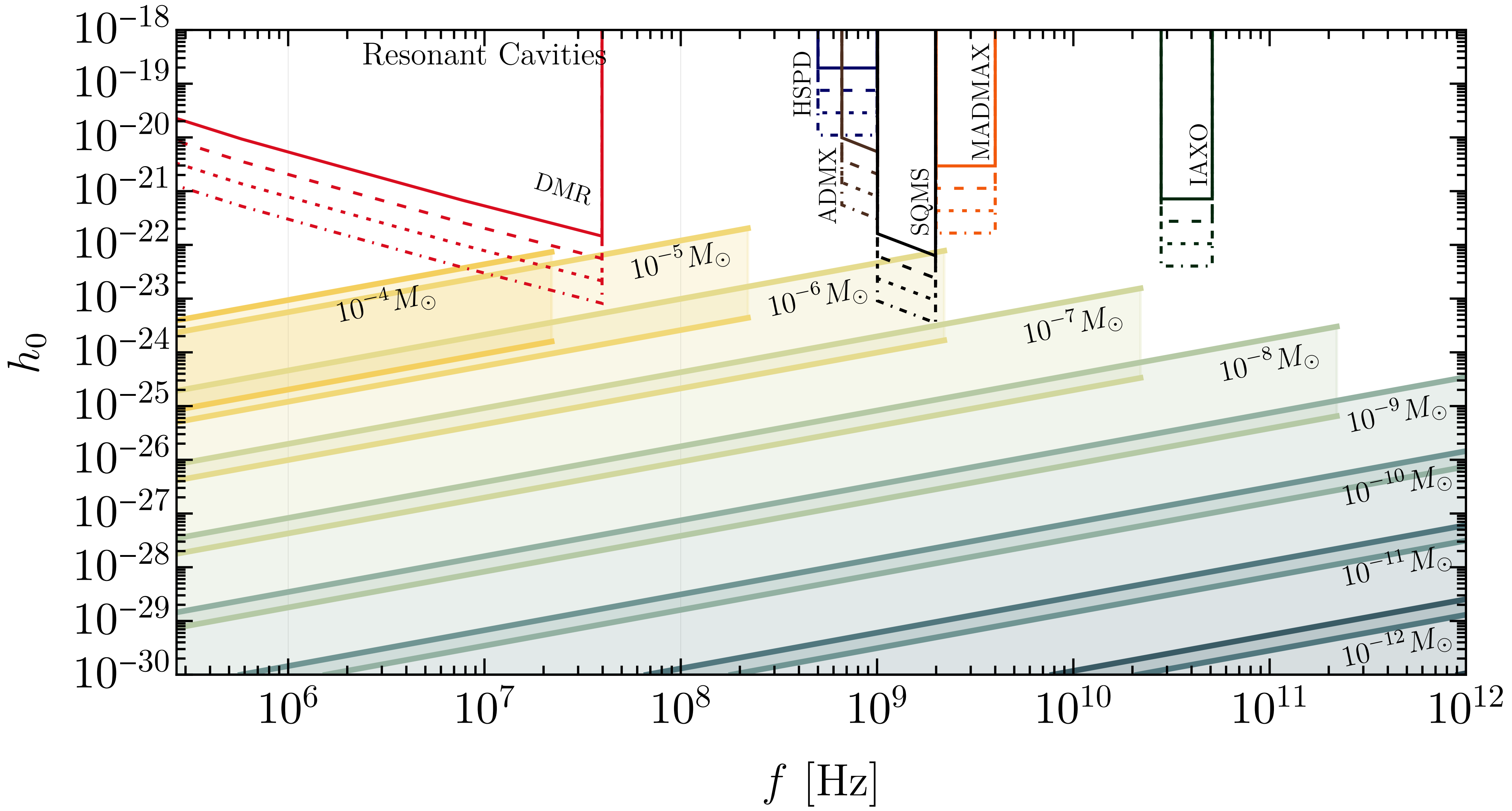
Sources of HF GW - Stochastic Background

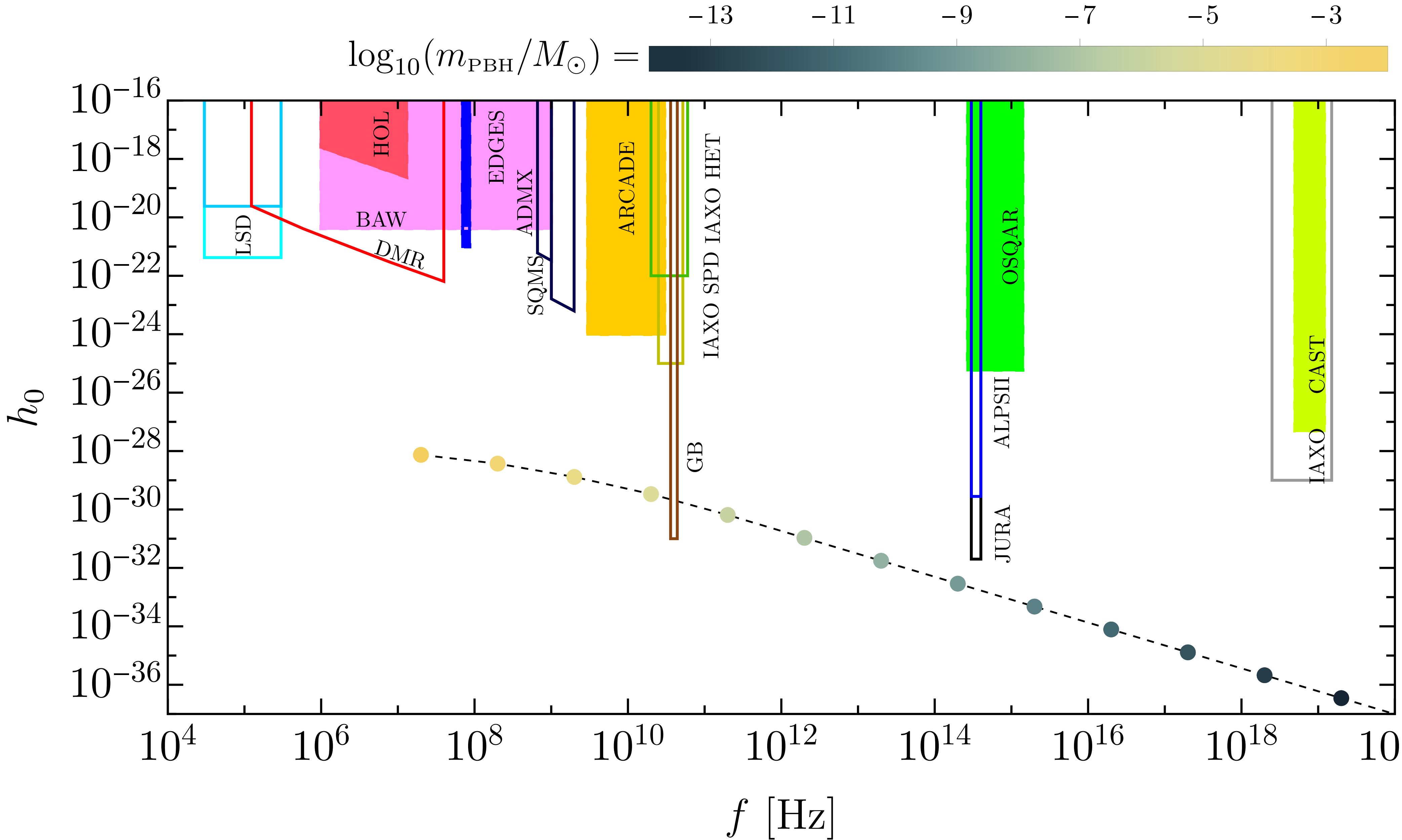


[arXiv:2312.14679]

High Frequency Gravitational Waves - Sources

- Several well motivated beyond the standard model sources:
  - Primordial black hole mergers
    - Chirp signals
  - GW from boson clouds around BHs
    - (BH super radiance)
    - Monochromatic over long timescales
  - Stochastic GW background
    - Even lower strains …





[Gabriele Franciolini, Anshuman Maharana, Francesco Muia; arXiv:2205.02153v1]

High Frequency Gravitational Waves - Signals

- Most interesting UHFGW source: **Primordial black hole merges**
  - **Fast transient** signal!
  - Typically **~10ms - 100ms** in GHz range
  - Long integration times are not applicable!
  - **Analysis strategies:**
    - Frequency domain analysis with short integration intervals
    - Time-domain analysis

$$\dot{f} = 4.62 \cdot 10^{11} Hz^2 \left(\frac{m_{PBH}}{10^{-9} M_\odot} \right)^{5/3} \left(\frac{f}{GHz} \right)^{11/3}$$

- To resonantly excite a cavity:
  - GW frequency must stay within resonator bandwidth
    - $\omega / Q \approx 10^9 Hz / 10^5 = 10 kHz$
  - Very short integration times O(ms) or below for larger PBH masses

High Frequency Gravitational Wave - Strains

Expected Strain

- Primoridal black hole mergers
  - Chirp signals
- $h_0 < 10^{-24}$
- GW from boson superradiance
  - Monochromatic over long timescales
- $h_0 < 10^{-29}$
- Stochastic GW background
  - Even lower strains …
- $h_0 < 10^{-32}$

Expected Sensitivity:

- 1 cavity
- T = 100 mK
- B = 14 T
- f_0 = 8 GHz

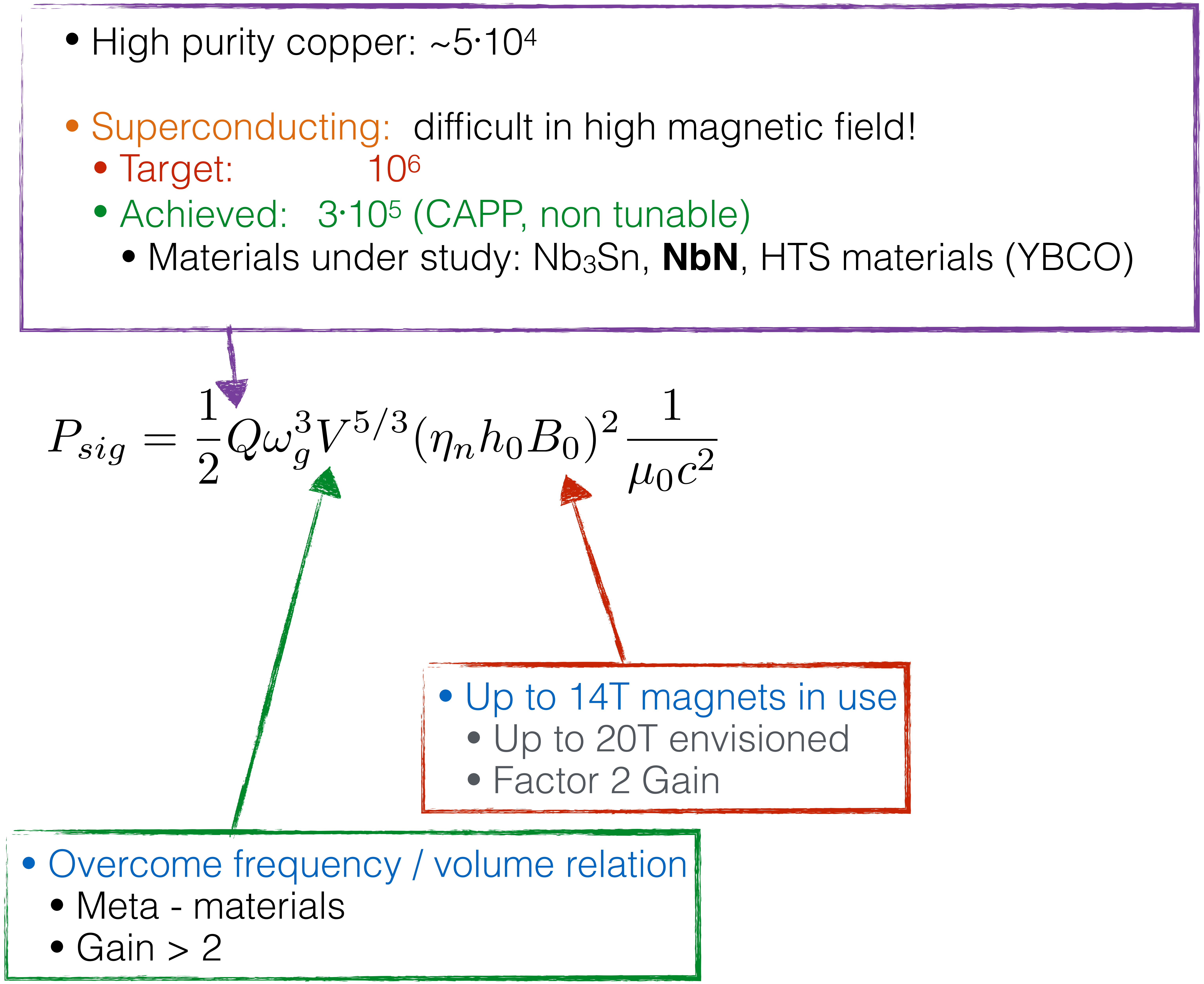
$$h_0 > 10^{-22}$$

Observed Strain

- Ligo / Virgo Signals
  - BH mergers
- $h_0 < 10^{-21}$

How to improve the sensitivity?

Getting more sensitive



- High purity copper: ~5·10⁴

- Superconducting: difficult in high magnetic field!
  - Target: 10^6
  - Achieved: $3 \cdot 10^5$ (CAPP, non tunable)
    - Materials under study: Nb_3Sn, **NbN**, HTS materials (YBCO)

$$P_{sig} = \frac{1}{2} Q \omega_g^3 V^{5/3} (\eta_n h_0 B_0)^2 \frac{1}{\mu_0 c^2}$$

- Up to 14T magnets in use
  - Up to 20T envisioned
  - Factor 2 Gain

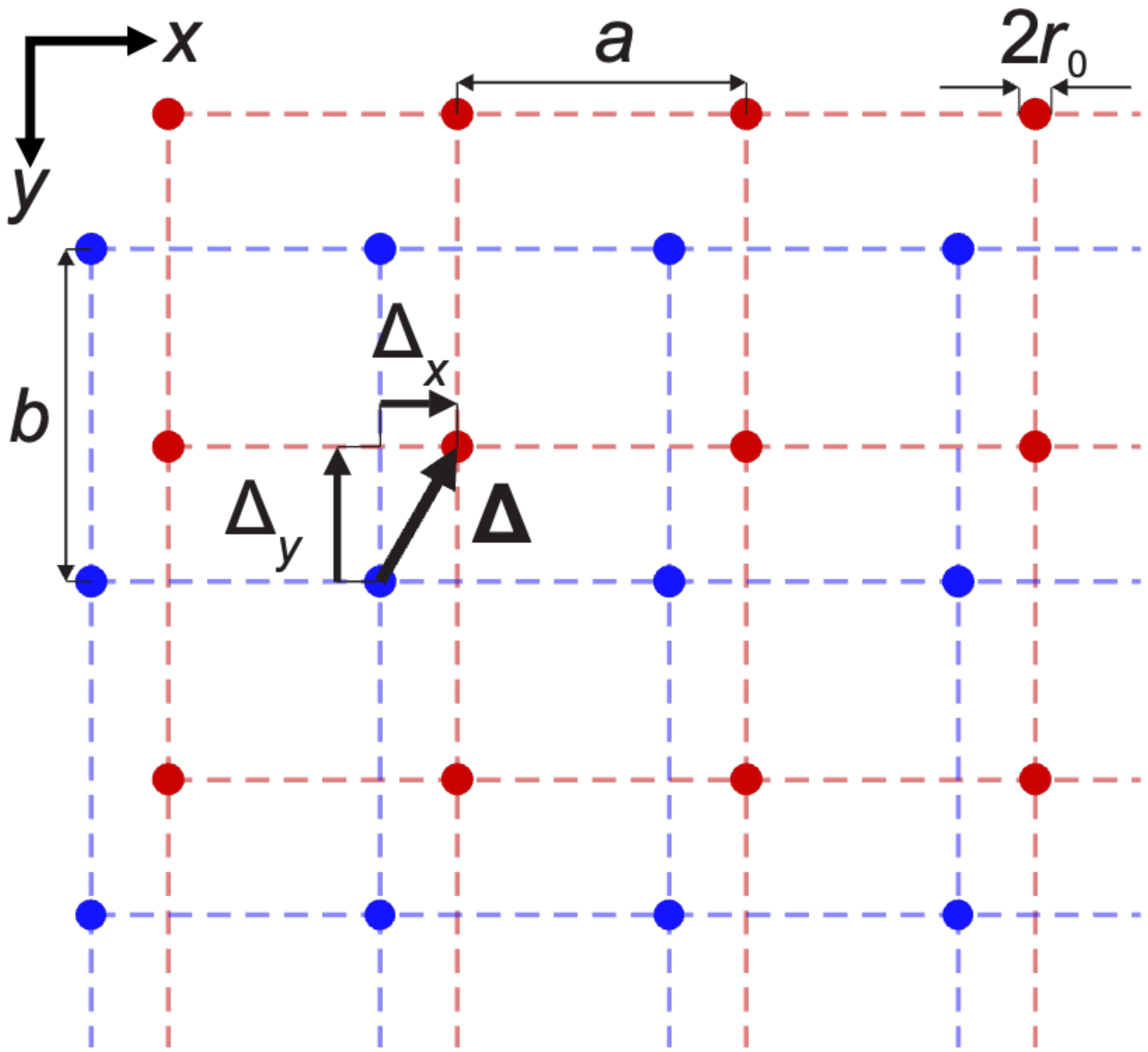
- Overcome frequency / volume relation
  - Meta - materials
  - Gain > 2

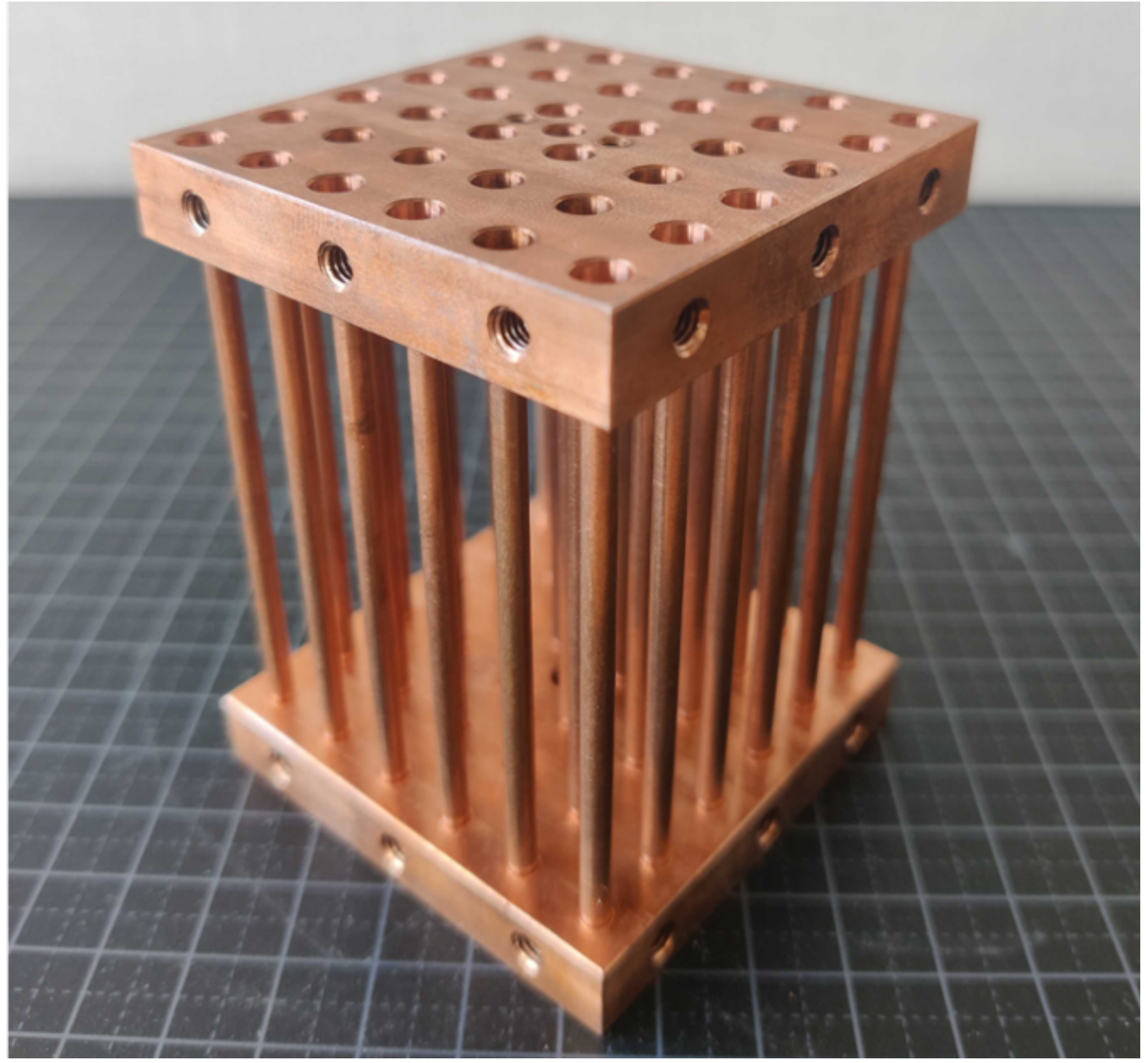
- Expect > 1 order of magnitude gain in strain sensitivity:

$$h_0 > 10^{-23}$$

Meta-Materials for cavities

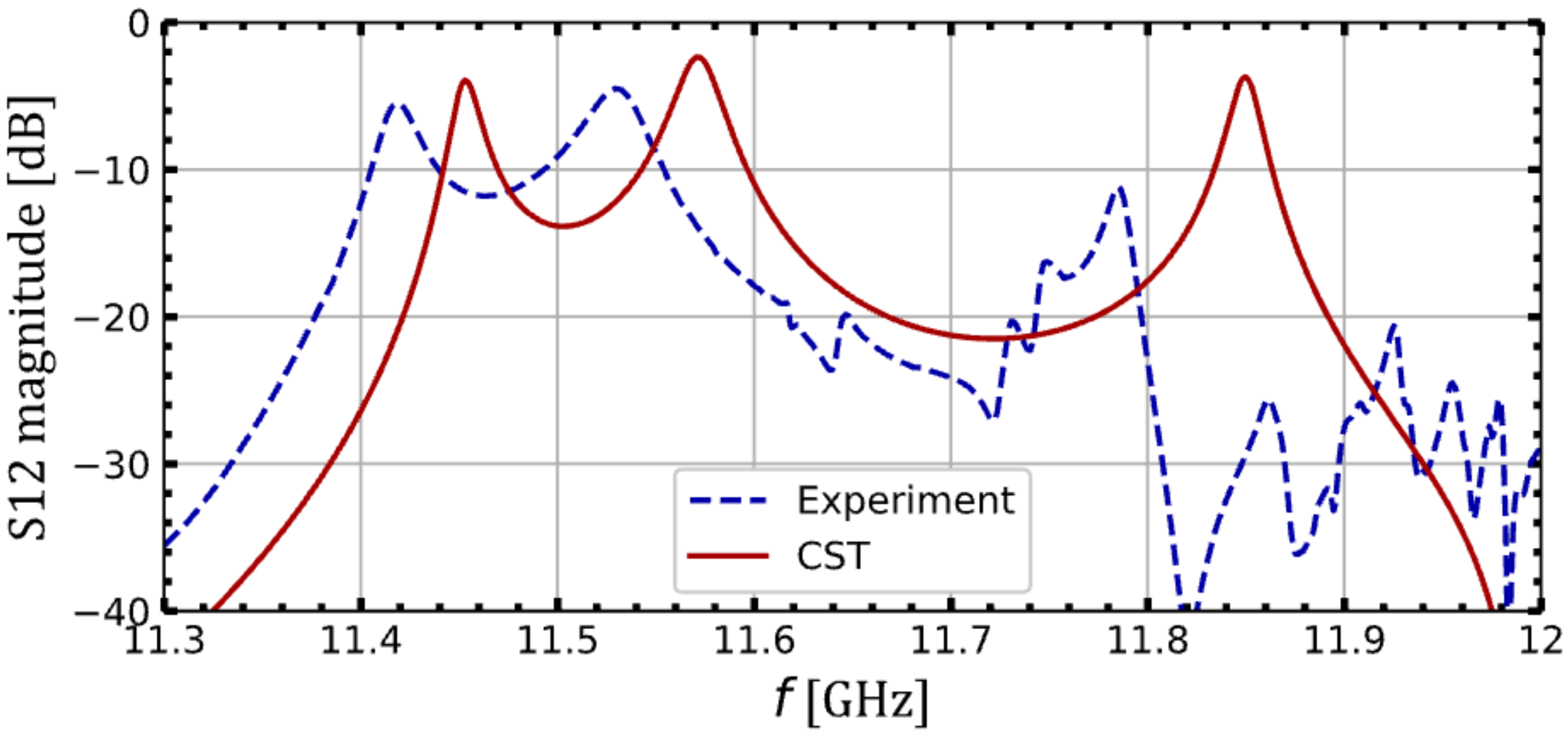
- Wire medium can be mechanically tuned by changing the lattice period



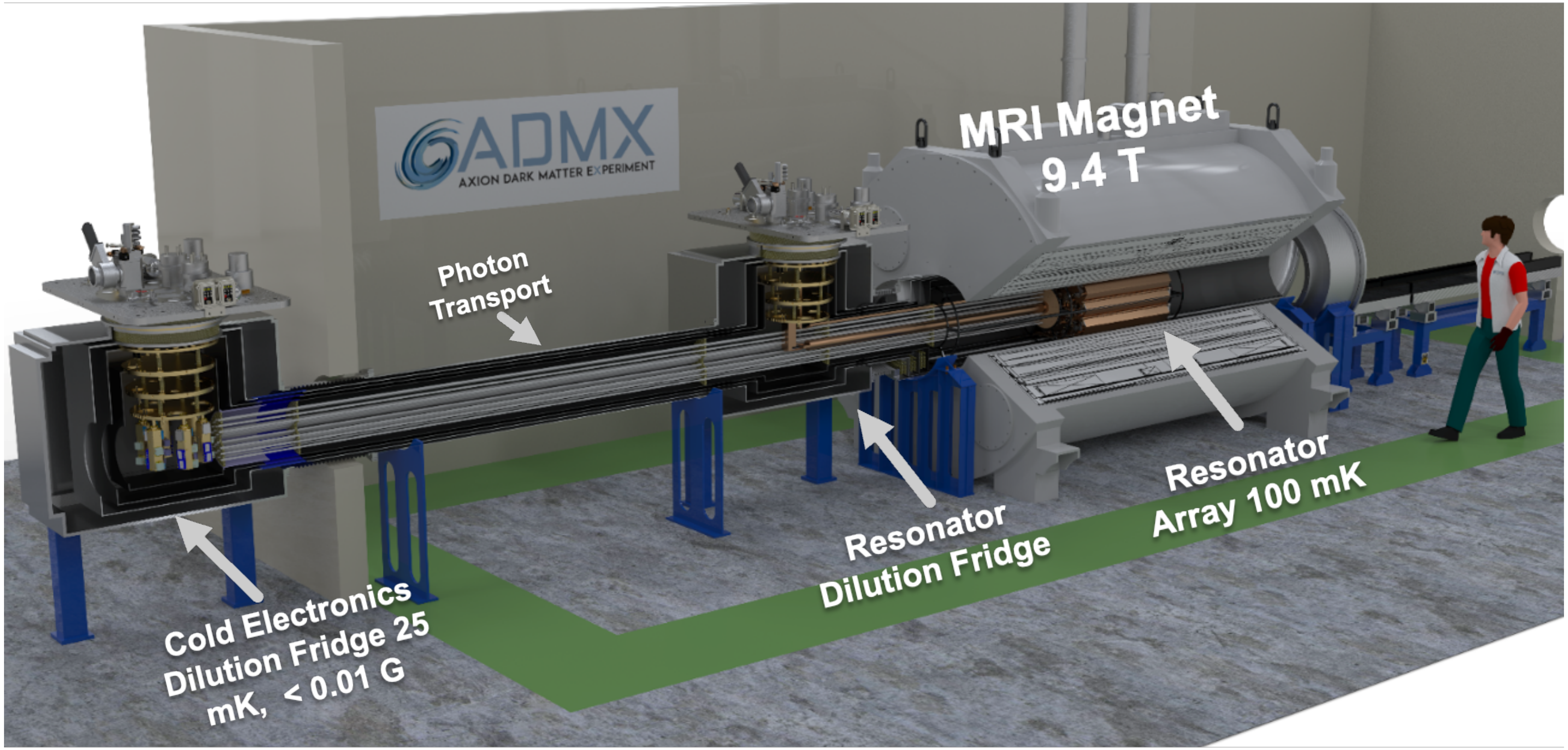


Meta-Materials for cavities

- Tuneable over large frequency range!



Large scale setups: ADMX as example



[Gianpaolo Carossi]

How to become more sensitive?

- Current efforts focus on **improving single cavity** sensitivity

[Tim Schneemann]

- But what about **combining various setups**?

How to become more sensitive?

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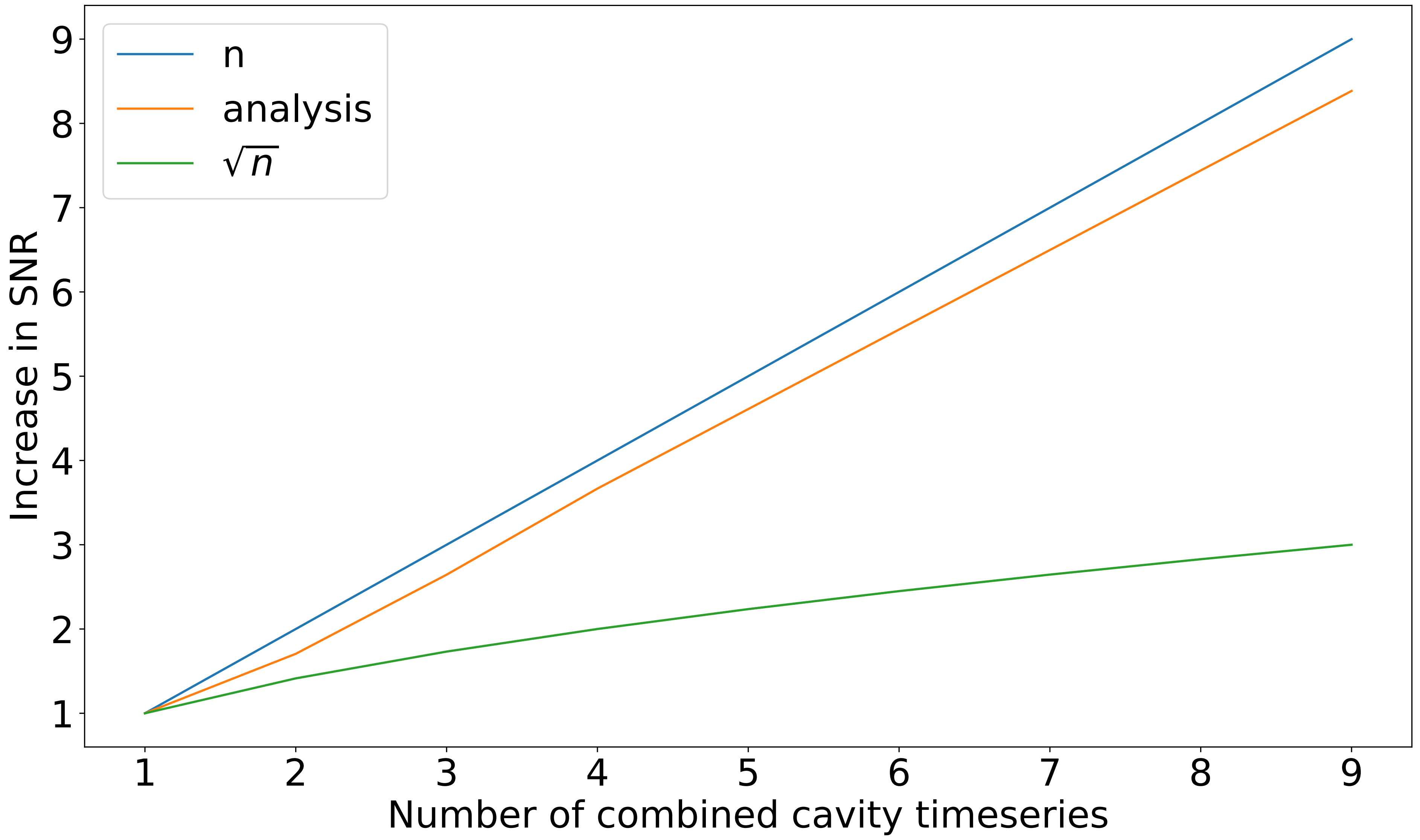
- But what about **combining various setups**?



[Tim Schneemann]

- Phase aligned combination voltages from of N cavities
  - RF amplitude (voltage):

$$V_{comb} = \frac{it\omega}{\sqrt{N}} \sum_i V_i e^{i\phi_i} \propto \sqrt{N} V_0$$

$$\uparrow$$

$$V_i = V, \phi_i = \phi$$

- Hence the **signal power scales linearly in N**!

$$P_{sig} = \frac{1}{2} Q \omega_g^3 V^{5/3} (\eta_n h_0 B_0)^2 \frac{1}{\mu_0 c^2}$$

- Sensitivity on h_0 scales with $\sqrt{N}$
  - Allows for new analysis techniques exploitng phase / timing relations

How to become more sensitive? — GravNet

- Target sensitivity: $\mathbf{h_0 < 10^{-24}}$ with **ms - *μ*s** time resolution

Network of distributed GW detectors

- Various possibilities
  - **Time evolution of ms spectra**
    - Easy, very noisy
  - **Simultaneous fit of time series data**
    - Computationally challenging
  - **Coincident experiment**
    - Requires **single RF photon detection**
    - Technique developed here at KIT

Combining information of distributed detectors at various frequencies!

GravNet Idea

[arXiv:2308.11497]

GravNet: A global network for HFWG detection

- How sensitive can we get with **10 setups**, scattered around the globe

- Assumptions:
  - Sampling of Waveform -> offline combination of phase aligned IQ data
  - Setups as shown before
    - Effective signal power increased by factor 10
    - Strain sensitivity increased by factor $\sqrt{10} \approx 3$

$h_0 < 10^{-24}$, 1 second integration time

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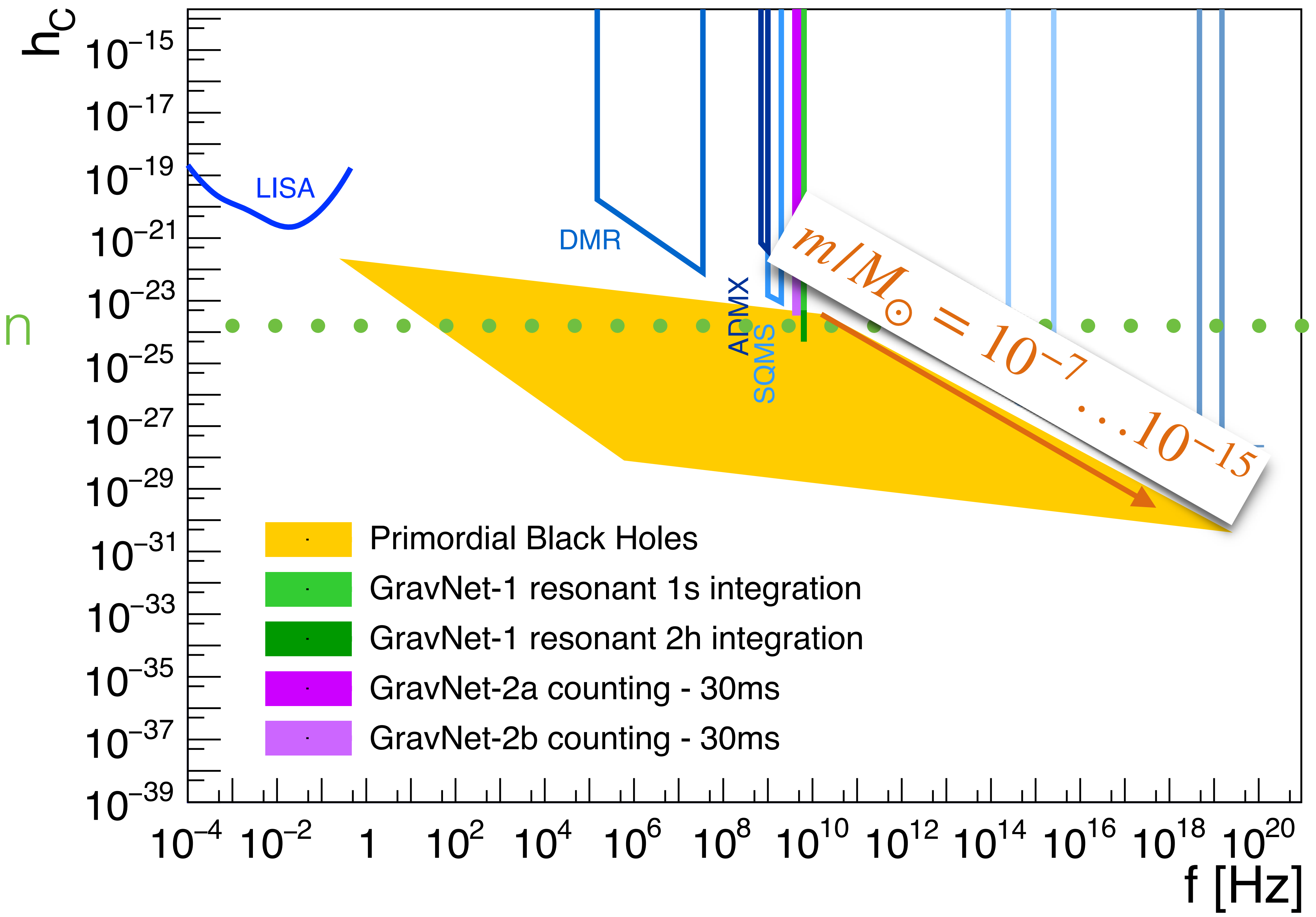
$$h_0 < 10^{-24}\text{, 1 second integration time}$$

- Phase alignment for distributed setups:
  - If signal seen in 3 cavities:
    - Direction of GW can be reconstructed
  - Otherwise:
    - Scan through all possible directions and repeat combinations

1s integration



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$$h_0 < 10^{-24}, \text{ 1 second integration time}$$

- No frequency tuning needed:
  - PBH signals are fast transients
    - Single frequency sufficiency

- Phase alignment for distributed setups:
  - If signal seen in 3 cavities:
    - Direction of GW can be reconstructed
  - Otherwise:
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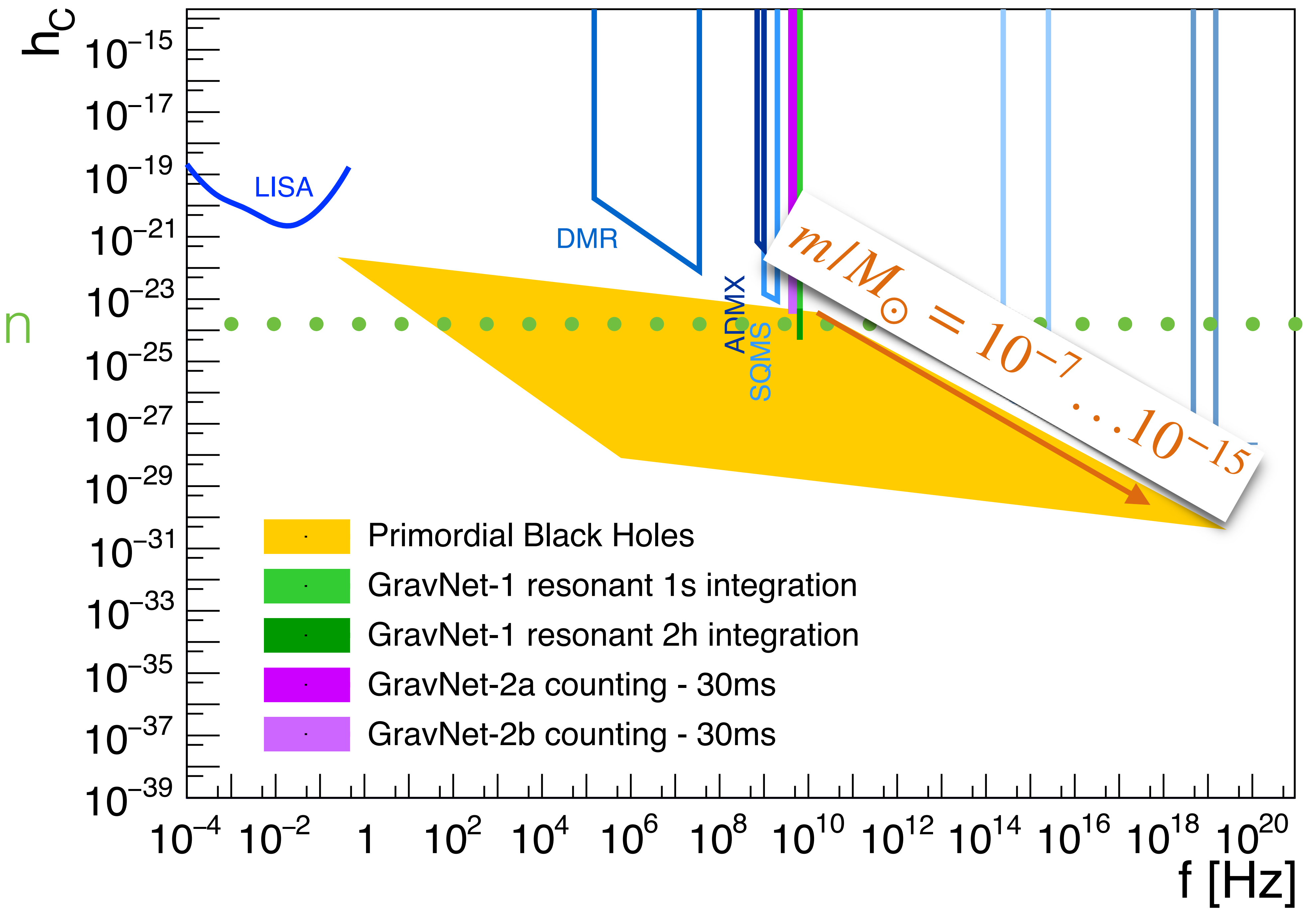
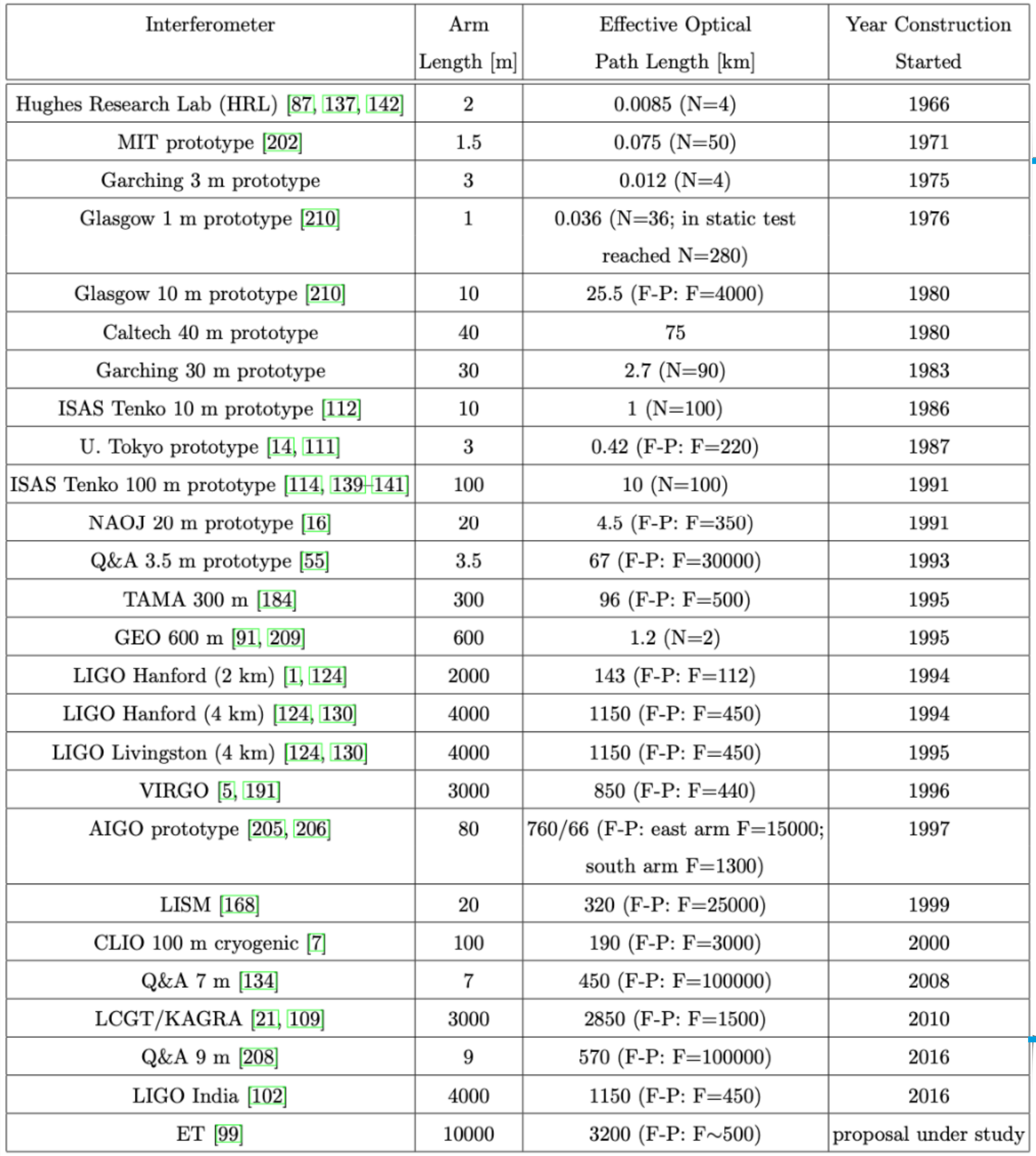


1s integration

Ambitious, but rewarding goal

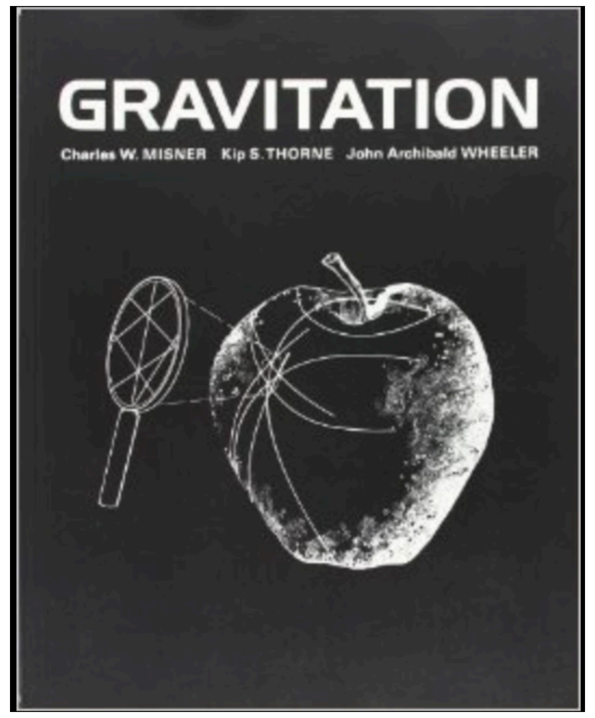


Interferometer	Arm Length [m]	Effective Optical Path Length [km]	Year Construction Started
Hughes Research Lab (HRL) [87, 137, 142]	2	0.0085 (N=4)	1966
MIT prototype [202]	1.5	0.075 (N=50)	1971
Garching 3 m prototype	3	0.012 (N=4)	1975
Glasgow 1 m prototype [210]	1	0.036 (N=36; in static test reached N=280)	1976
Glasgow 10 m prototype [210]	10	25.5 (F-P: F=4000)	1980
Caltech 40 m prototype	40	75	1980
Garching 30 m prototype	30	2.7 (N=90)	1983
ISAS Tenko 10 m prototype [112]	10	1 (N=100)	1986
U. Tokyo prototype [14, 111]	3	0.42 (F-P: F=220)	1987
ISAS Tenko 100 m prototype [114, 139–141]	100	10 (N=100)	1991
NAOJ 20 m prototype [16]	20	4.5 (F-P: F=350)	1991
Q&A 3.5 m prototype [55]	3.5	67 (F-P: F=30000)	1993
TAMA 300 m [184]	300	96 (F-P: F=500)	1995
GEO 600 m [91, 209]	600	1.2 (N=2)	1995
LIGO Hanford (2 km) [1, 124]	2000	143 (F-P: F=112)	1994
LIGO Hanford (4 km) [124, 130]	4000	1150 (F-P: F=450)	1994
LIGO Livingston (4 km) [124, 130]	4000	1150 (F-P: F=450)	1995
VIRGO [5, 191]	3000	850 (F-P: F=440)	1996
AIGO prototype [205, 206]	80	760/66 (F-P: east arm F=15000; south arm F=1300)	1997
LISM [168]	20	320 (F-P: F=25000)	1999
CLIO 100 m cryogenic [7]	100	190 (F-P: F=3000)	2000
Q&A 7 m [134]	7	450 (F-P: F=100000)	2008
LCGT/KAGRA [21, 109]	3000	2850 (F-P: F=1500)	2010
Q&A 9 m [208]	9	570 (F-P: F=100000)	2016
LIGO India [102]	4000	1150 (F-P: F=450)	2016
ET [99]	10000	3200 (F-P: F∼500)	proposal under study

MTW book

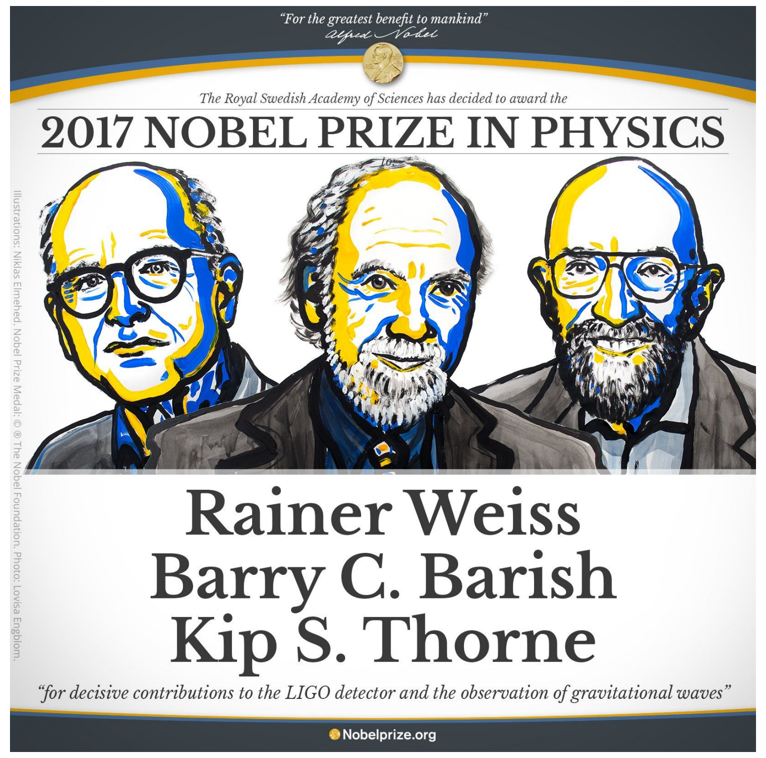
“[interferometers] have so low sensitivity that they are of little experimental interest”

page 1014



50 years
23 attempts

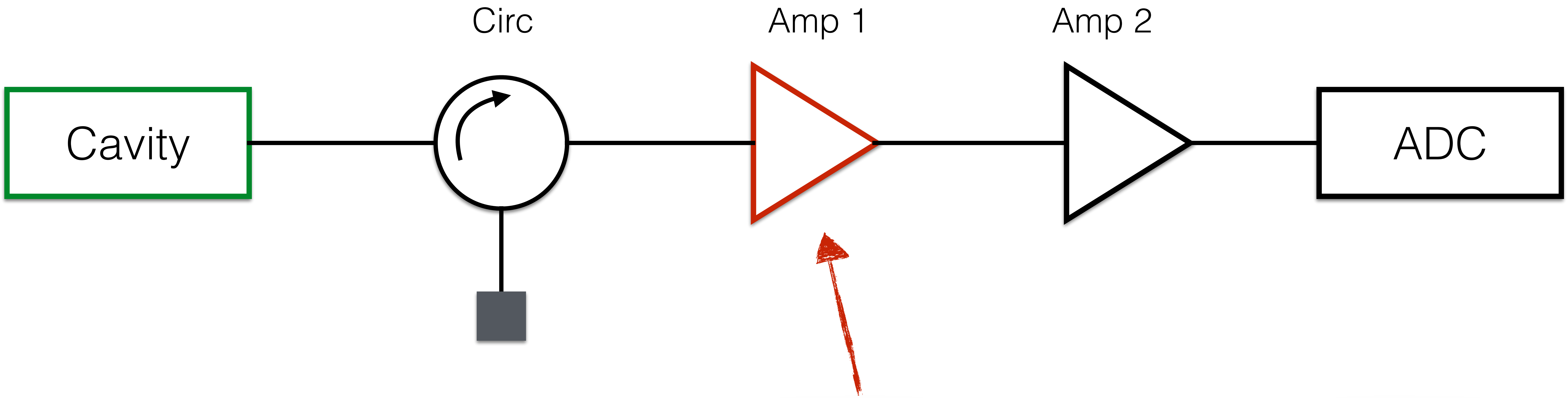
First direct detection



What will be the next step in sensitivity?

Possibly on photon detection side!

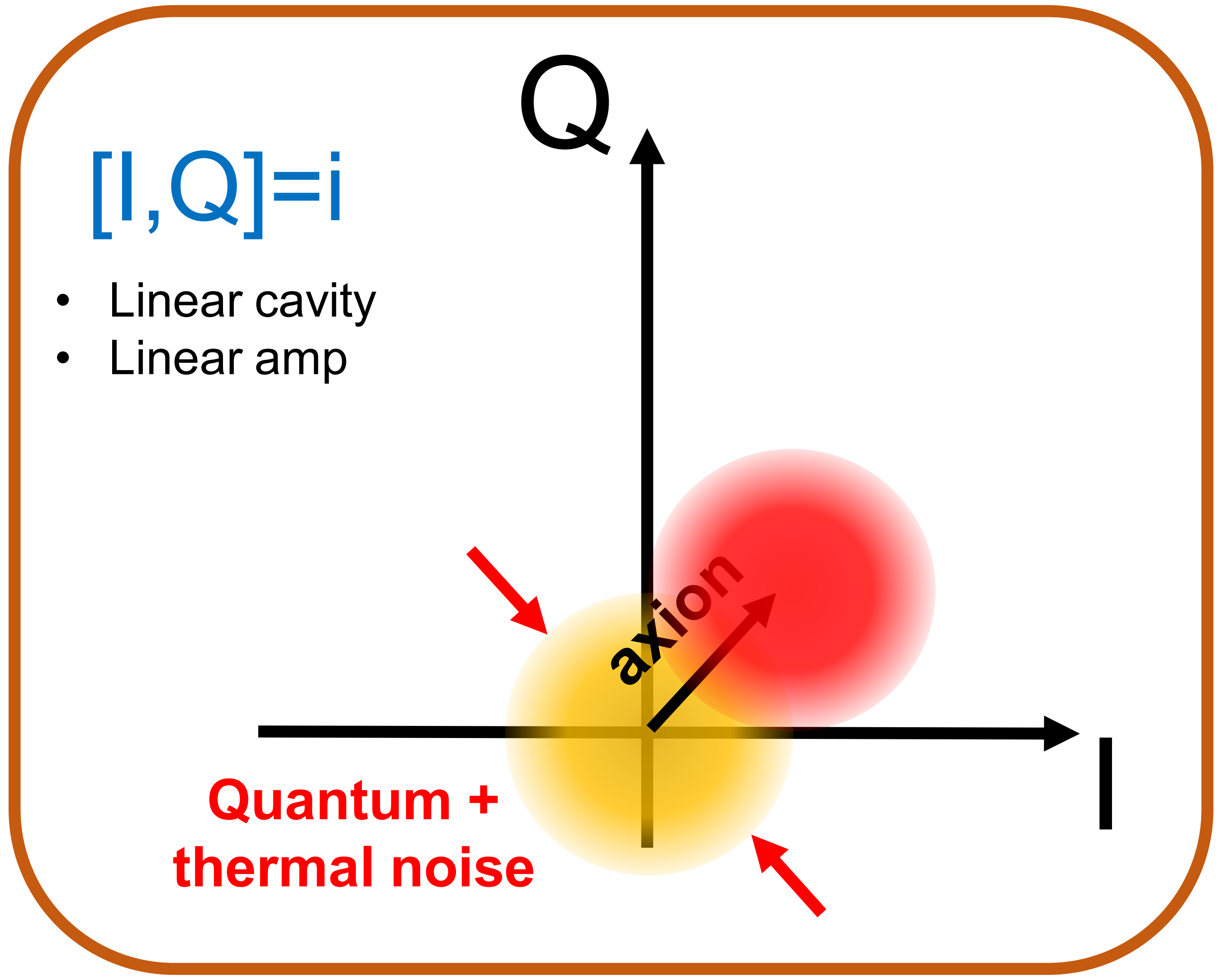
Conventional Read-Out System

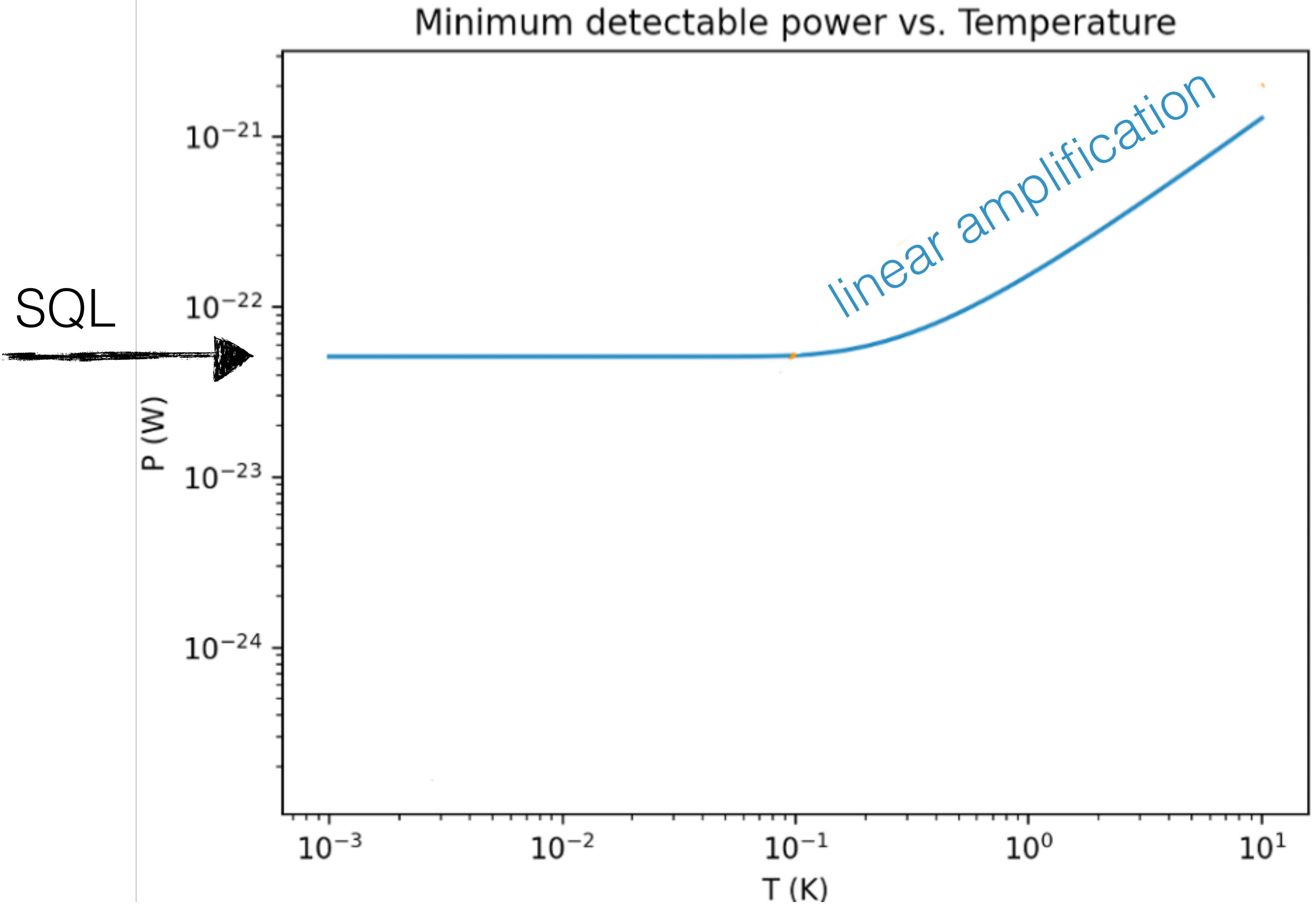


Noise from 1st amplifier is **limiting** the measurement!

Standard Quantum Limit

- Conventional haloscope:
  - In-phase(I) and Out of phase (Q) conjugates limited by SQL
- Measurement of amplitude and phase of EM wave:
  - Minimum noise corresponding to one quantum (c.f. zero point energy)





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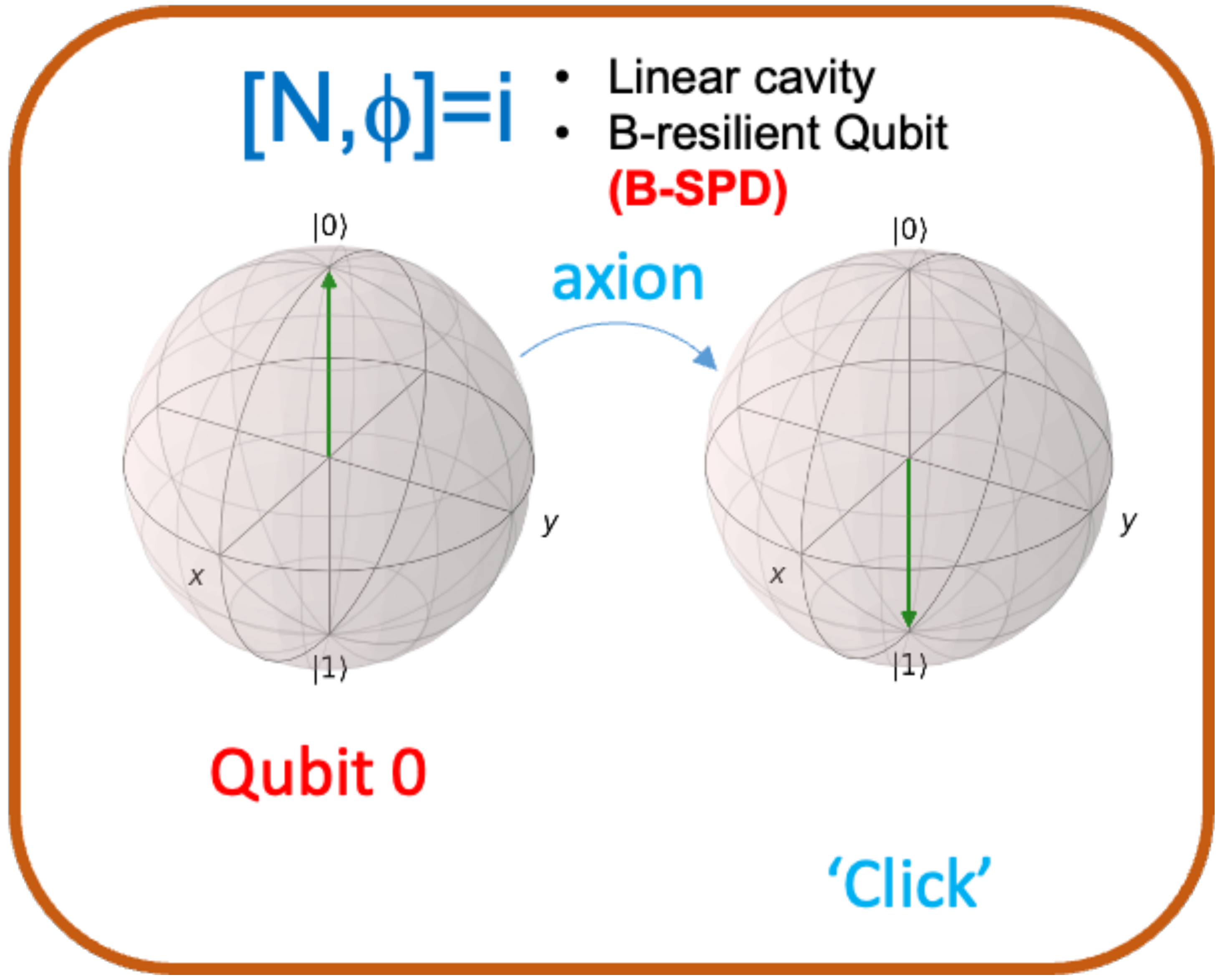
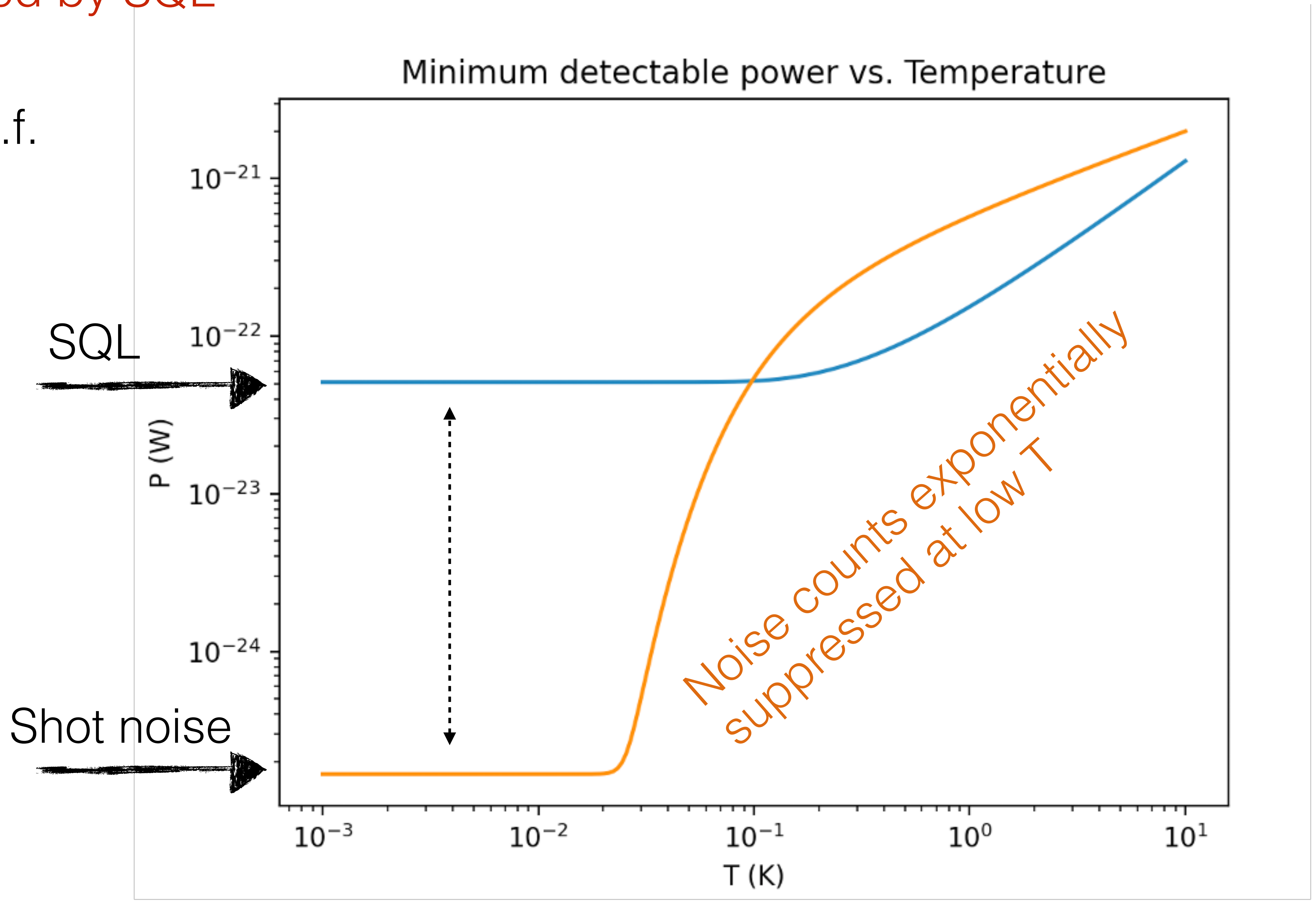




Noise counts exponentially suppressed at low T

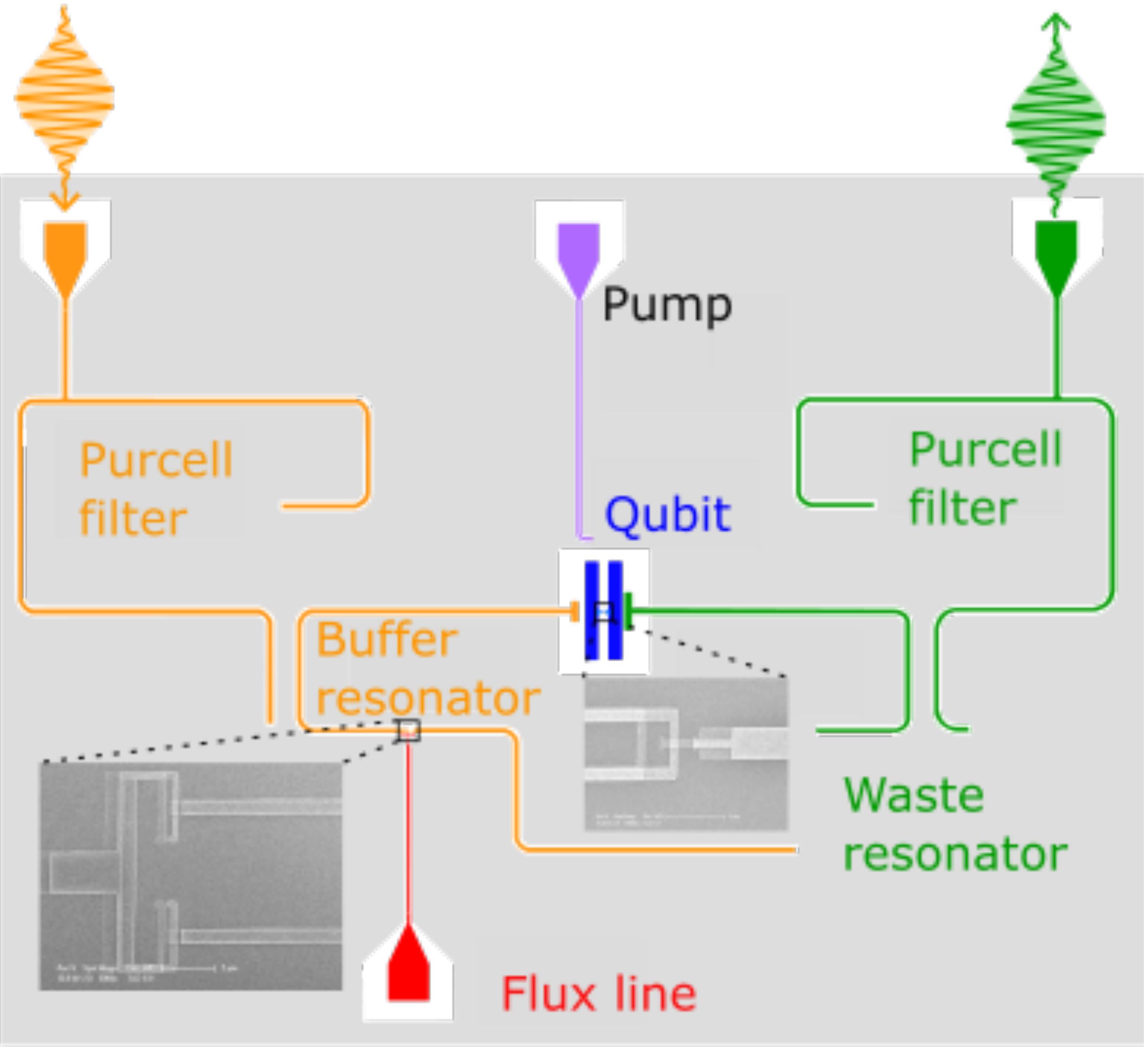
- Change of paradigm
    - Number-Phase conjugates **evade the SQL**

Single RF Photon Detection

- Recent progress in R&D for single RF photon counters

- Several technologies under study

  - Current Biased Josephson Junctions [arXiv:2302.07556]
  - Kerr Josephon Parametric amplifiers [arXiv:2308.07084]
  - Transmon **Q-Bit readout** [arXiv:2307.03614]



[arXiv:2307.03614]

- Shown single photon efficiency: 43% @ 90 Hz dark count rate
  - Big R&D effort ongoing [ERC syn.: “Dark Quantum”]

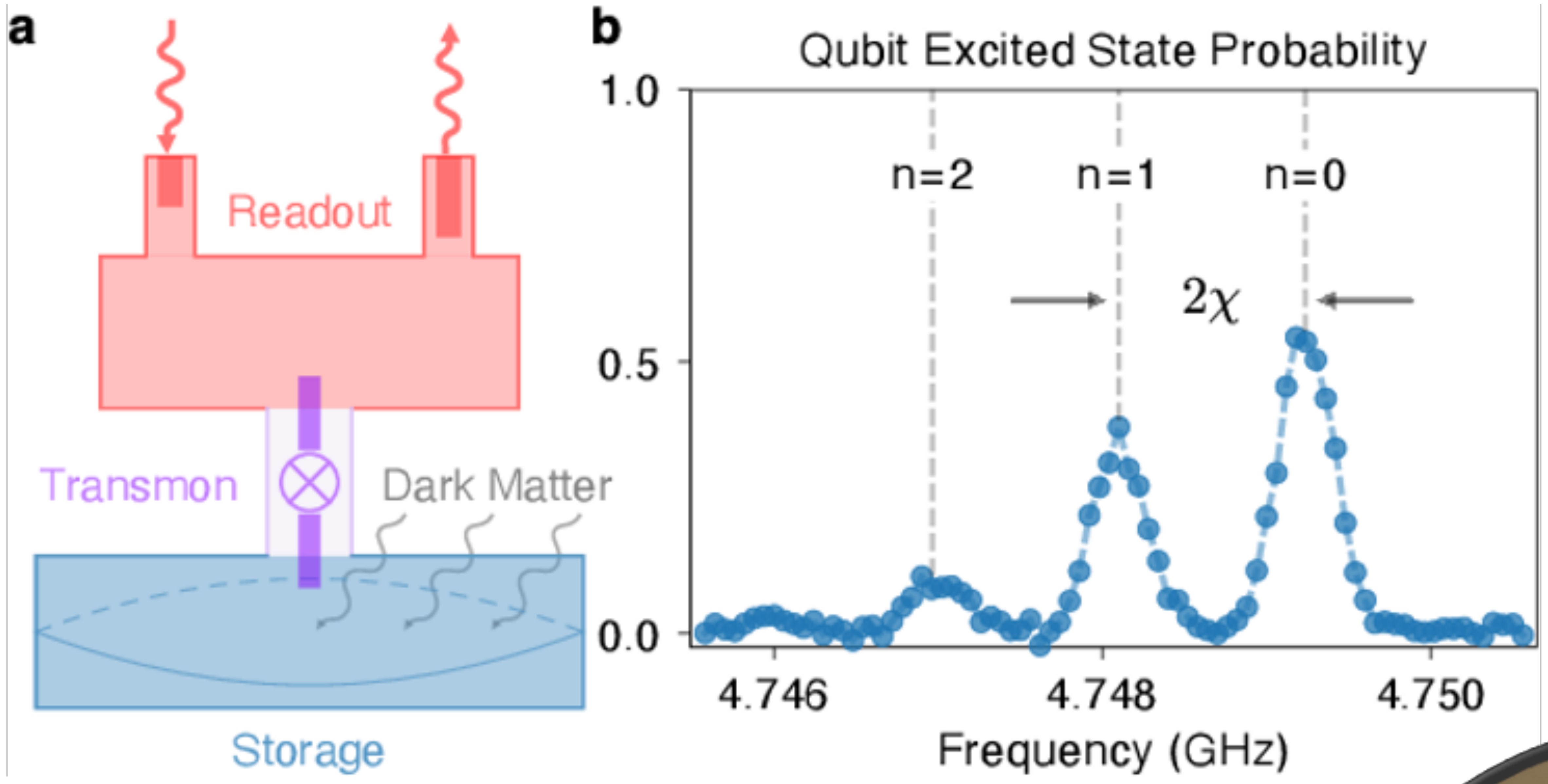
- Using **Q-bits** for single **RF photon sensing**

https://doi.org/10.1103/PhysRevLett.126.141302

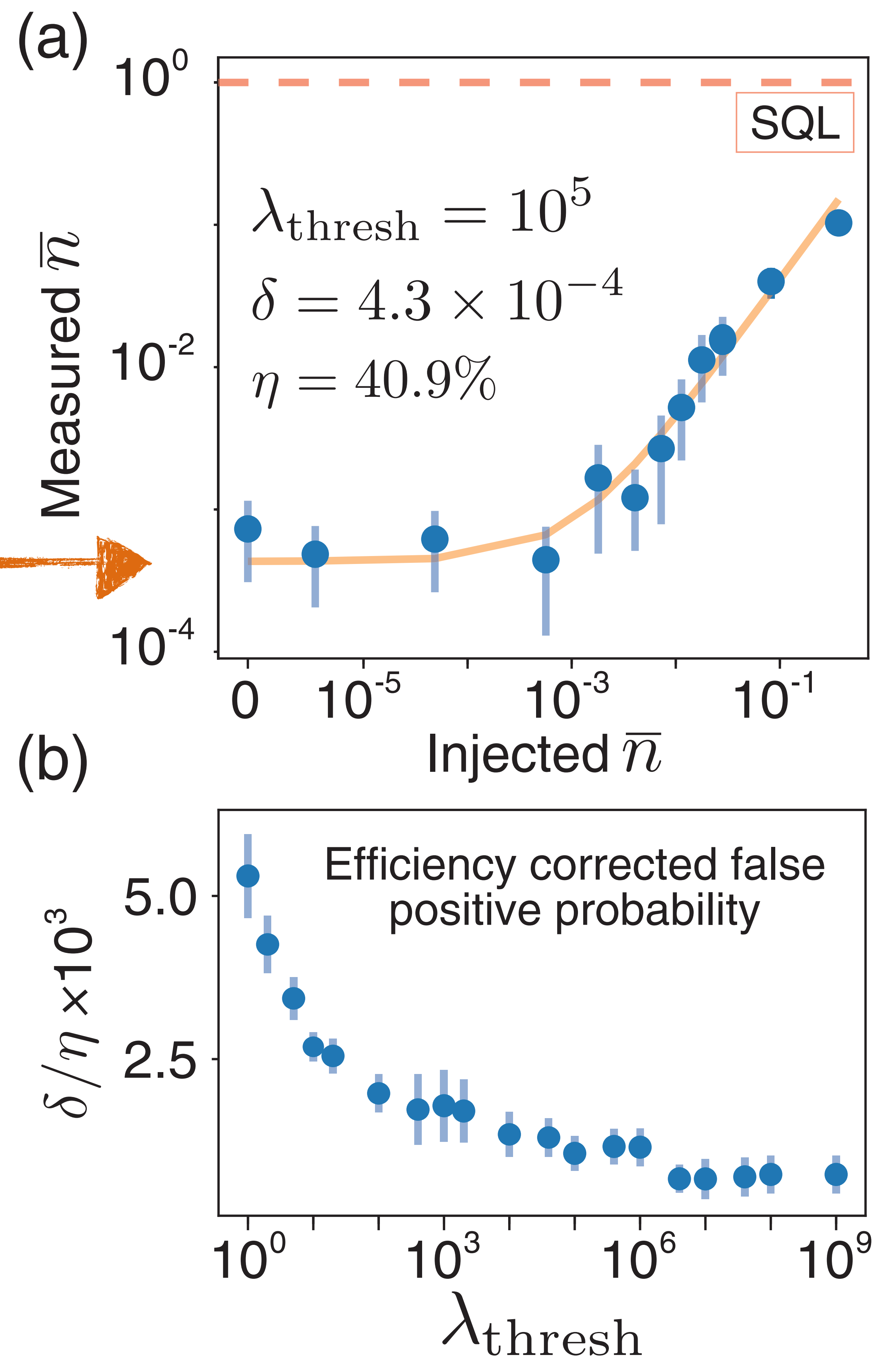
Transmon Q-Bit

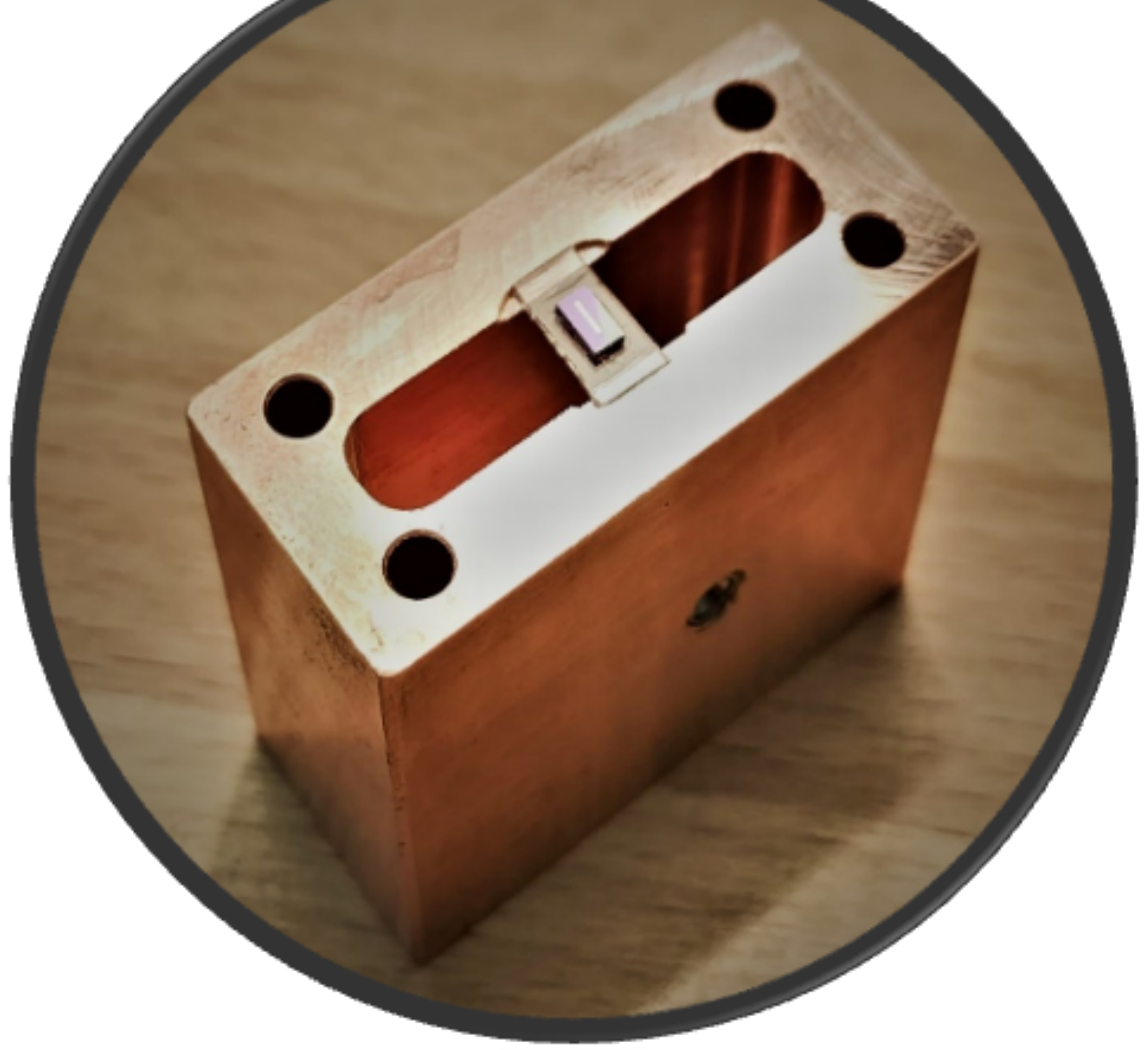
https://doi.org/10.1103/PhysRevLett.126.141302

SQL: average occupation number: $\bar{n} = 1$



Effective noise
$T_{noise} = 40\,mK$





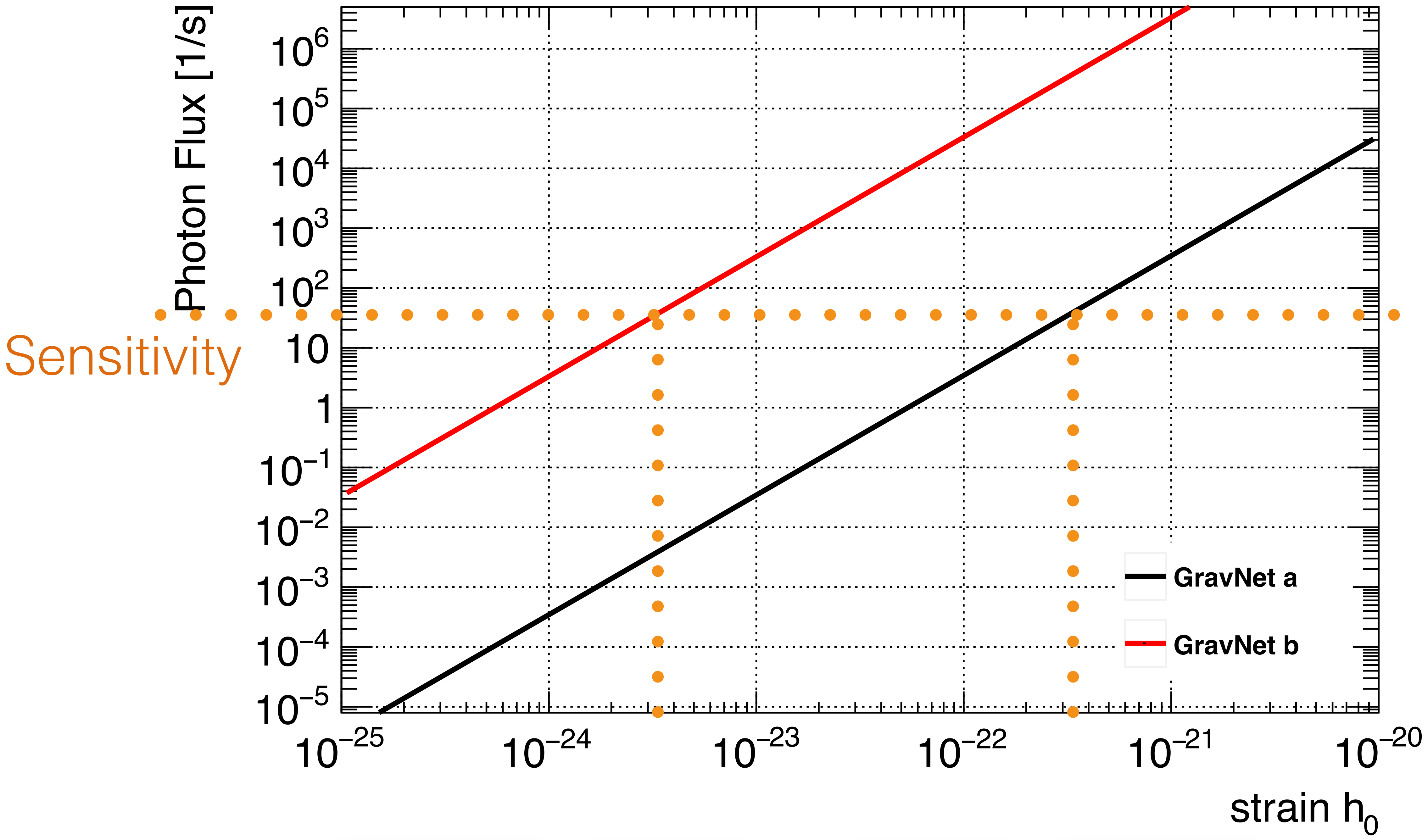
Detection efficiency: $\epsilon = 0.41$
False positive probability: $\delta = 4.3 \cdot 10^{-4}$

GravNet: Photon Counting - Sensitivity on GW

- With 20 detectors a photon flux of 40 Hz can be detected with an efficiency of 1 within a coincidence interval of 32ms

- Signal photon flux depends on conversion region:
  - a) Magnet dimensions as before (9cm diameter), B = 14T
  - b) Assuming large NMR magnet (80cm diameter), B = 9T

Setup	GravNet-a	GravNet-b
radius	40 mm	40 cm
length	12cm	50 cm
Volume $[m^3]$	6×10^{-4}	0.25
Q_0	10^6	10^5
T_{sys} [K]	0.1	0.1
B [T]	14	9



- Achievable sensitivity:
  - **$h_0 < 3x10^{-22}$ …. $3x10^{-24}$**
  - With coincidence time of 32ms!

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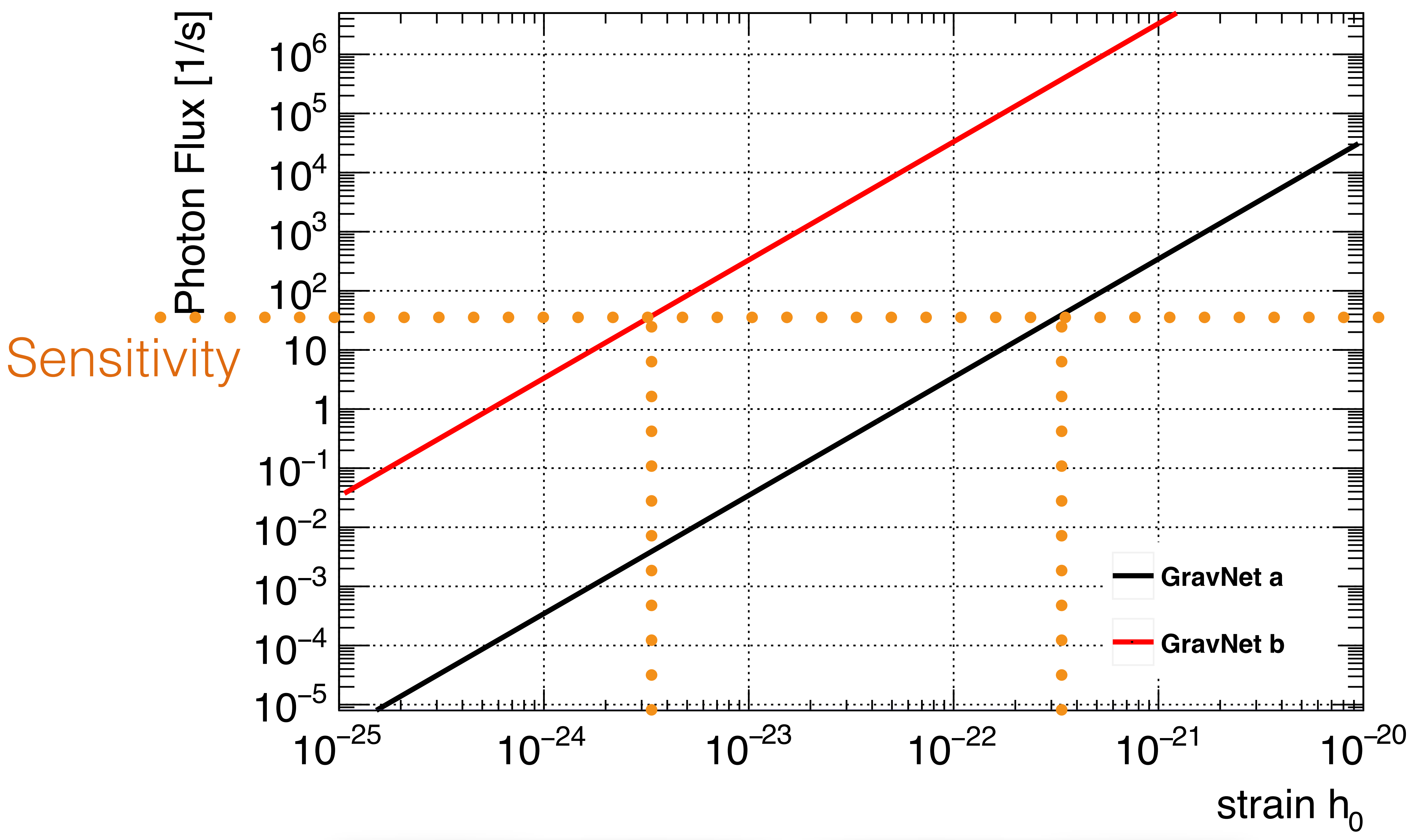

Global network of HFGW detectors will be able to reach into the interesting region for PBH with existing technologies!

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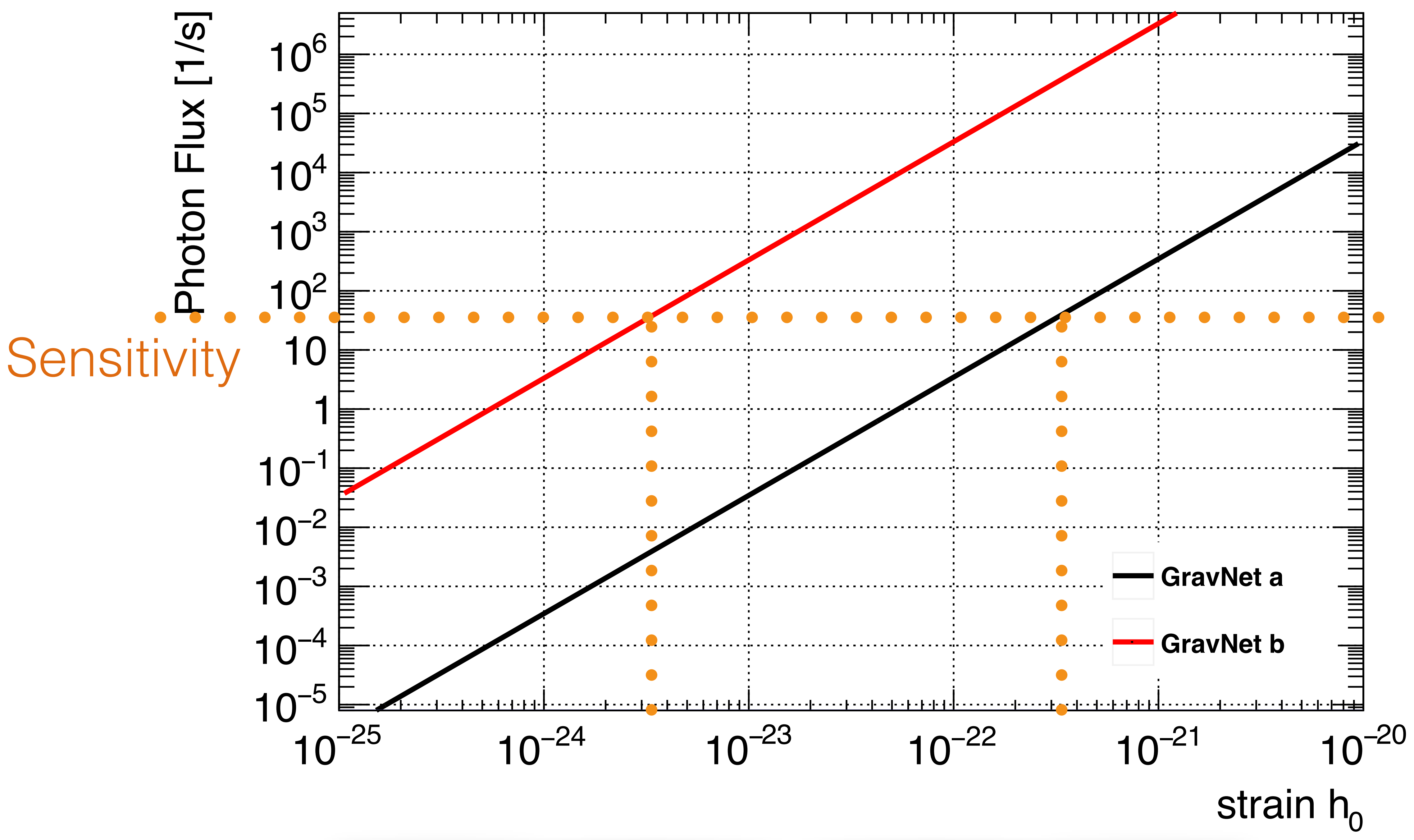

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  - **$h_0 < 3 \times 10^{-22}$ .... 3×10^{-24}**
  - With coincidence time of 32ms!

Global network of HFGW detectors will be able to reach into the interesting region for PBH with existing technologies!

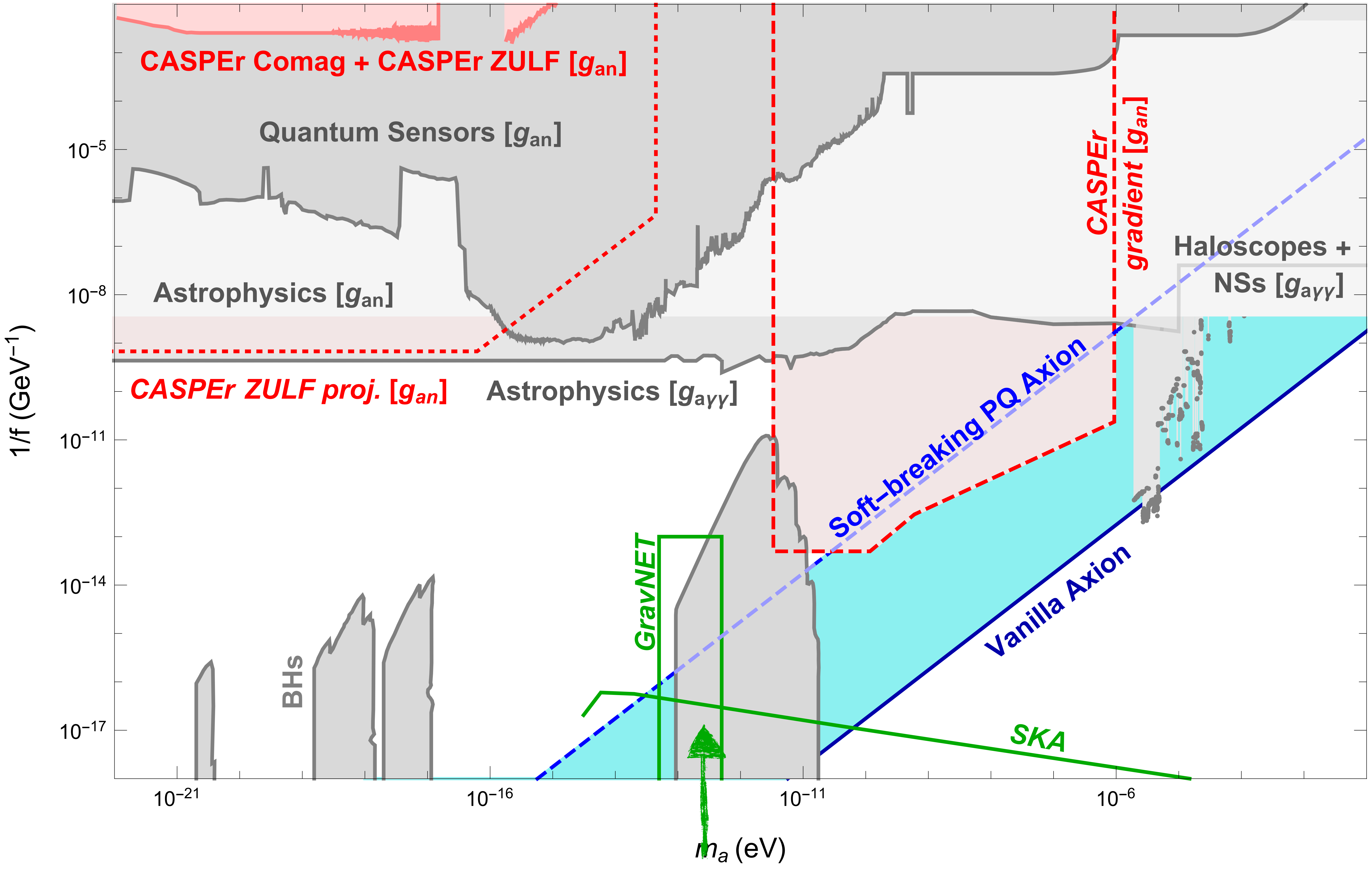
Significant room for improvements:
more detectors, larger volumes, higher detector efficiency, lower dark count rate

Connection of UHFGW to Axions

- GW from **axion super-radiance**
  - Requires light black holes
  - Requires axions (or similar)

- If monochromatic GW signal is found:
  - Strong indication for super-radiance
  - New boson!

[P. Schwaller]



$10^{-13} - 10^{-12}$ eV
(Expected sensitivity of GravNet)

Alternative Detection Approaches

The HFGW Community

https://indico.cern.ch/event/1257532/

Ultra-high frequency gravitational waves: where to next ?

Dec 4, 2023, 9:00 AM → Dec 8, 2023, 7:00 PM Europe/Zurich

4/3-006 - TH Conference Room (CERN)

Gravitational Wave Probes of Fundamental Physics

A cross-cutting initiative for a common platform to:

- Foster synergies among astroparticle, atomic, nuclear, high-energy, and gravitational physics, cosmology, and GW and multi-messenger astronomy
- Strengthen the connection between the theoretical and experimental/observational communities
- Share expertise, tools, cutting edge technologies to attack multidisciplinary problems
- Train a new generation of researchers with diverse expertise and background
- Share and disseminate knowledge in fundamental physics

EPS-HEP2023 conference

Aug 20 – 25, 2023
Universität Hamburg
Europe/Berlin timezone

A Global Network of Cavities to Search for Gravitational Waves (GravNet): A novel scheme to hunt gravitational waves signatures from the early universe

The Stephen Hawking Centre for Theoretical Cosmology

Home › Activities › Ultra-High-Frequency Gravitational Waves Initiative

Ultra-High-Frequency Gravitational Waves

19th Patras Workshop on Axions, WIMPs and WISPs

Sep 16 – 20, 2024
Europe/Rome timezone

How And Where Can We Detect High Frequency Gravitational Waves?

Variety of Detection Mechanisms

- Very few experiments with any interesting sensitivity

- Small community, very active
- Growing field of research!

- Currently driven by theory efforts

Variety of Detection Mechanisms

- But some experimental efforts ongoing ;)

- Very few experiments with any interesting sensitivity

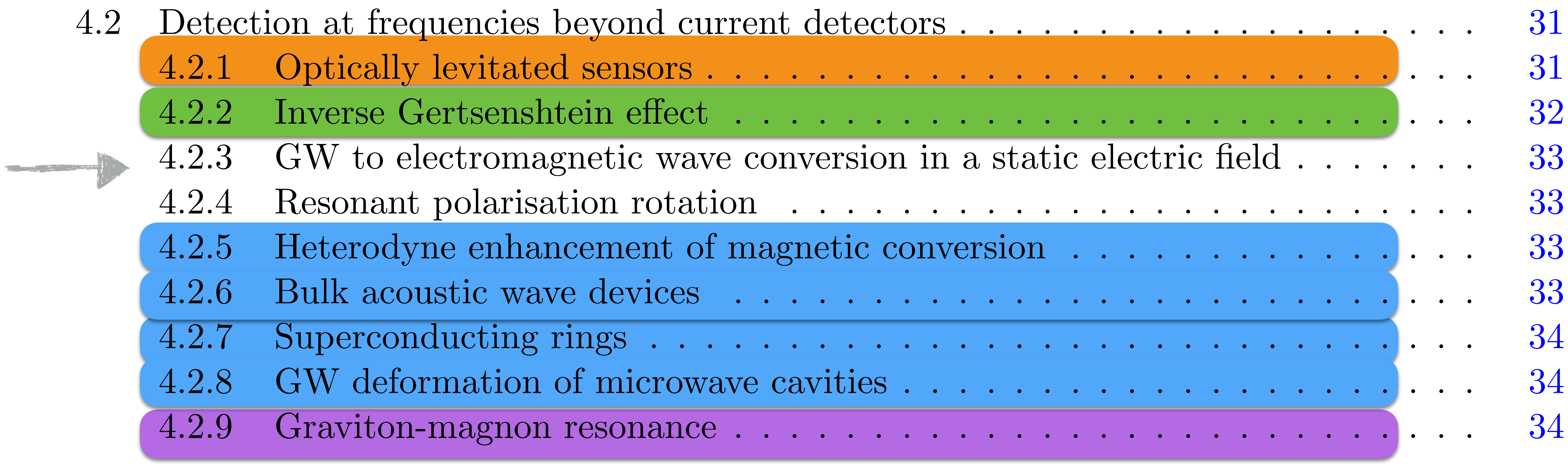
- Small community, very active
- Growing field of research!

- Currently driven by theory efforts

- Classes of **principle of detection**:
  - Movement of a test-mass
  - Deformation of detector
  - Direct conversion into photons
  - Graviton — Magnon resonance

Conversion of GWs into Photons

- Most recent overview from 2020, currently being updated: [arXiv:2011.12414]
  - "*Challenges and Opportunities of Gravitational Wave Searches at MHz to GHz Frequencies*"



Conversion of GWs into Photons

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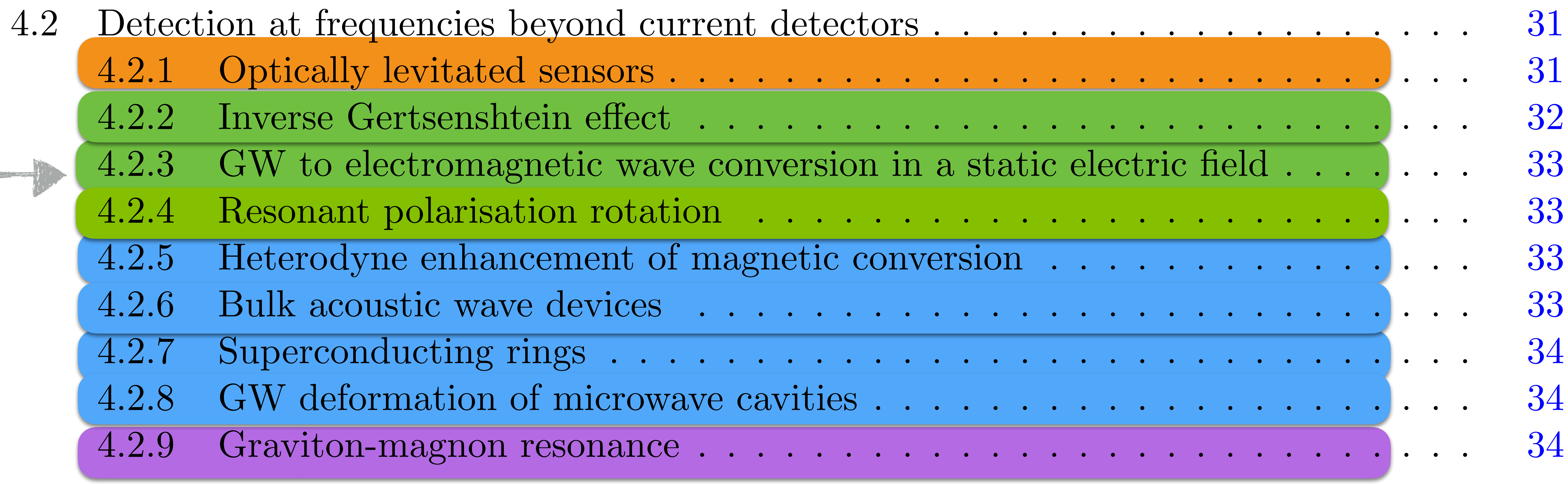
  - "*Challenges and Opportunities of Gravitational Wave Searches at MHz to GHz Frequencies*"

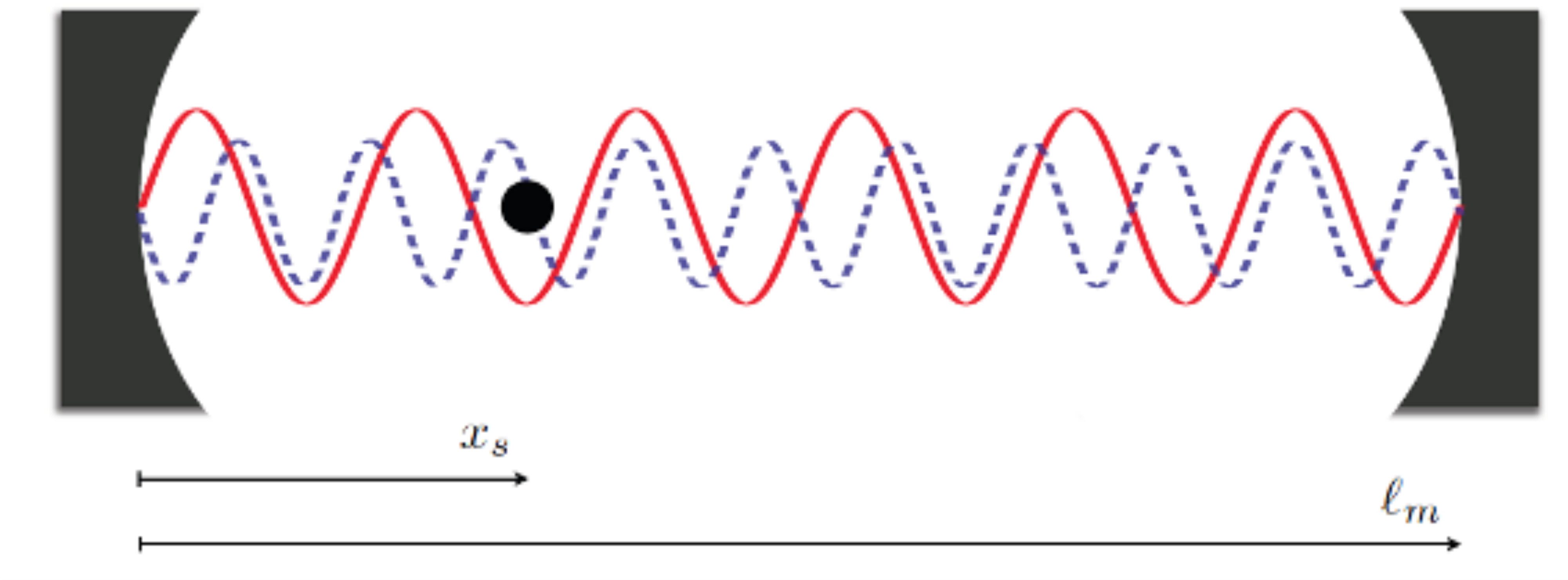
- Energy density in E-field about 10^{-6} compared to B-fields due to electron release



Conversion of GWs into Photons

- Levitated Sensors

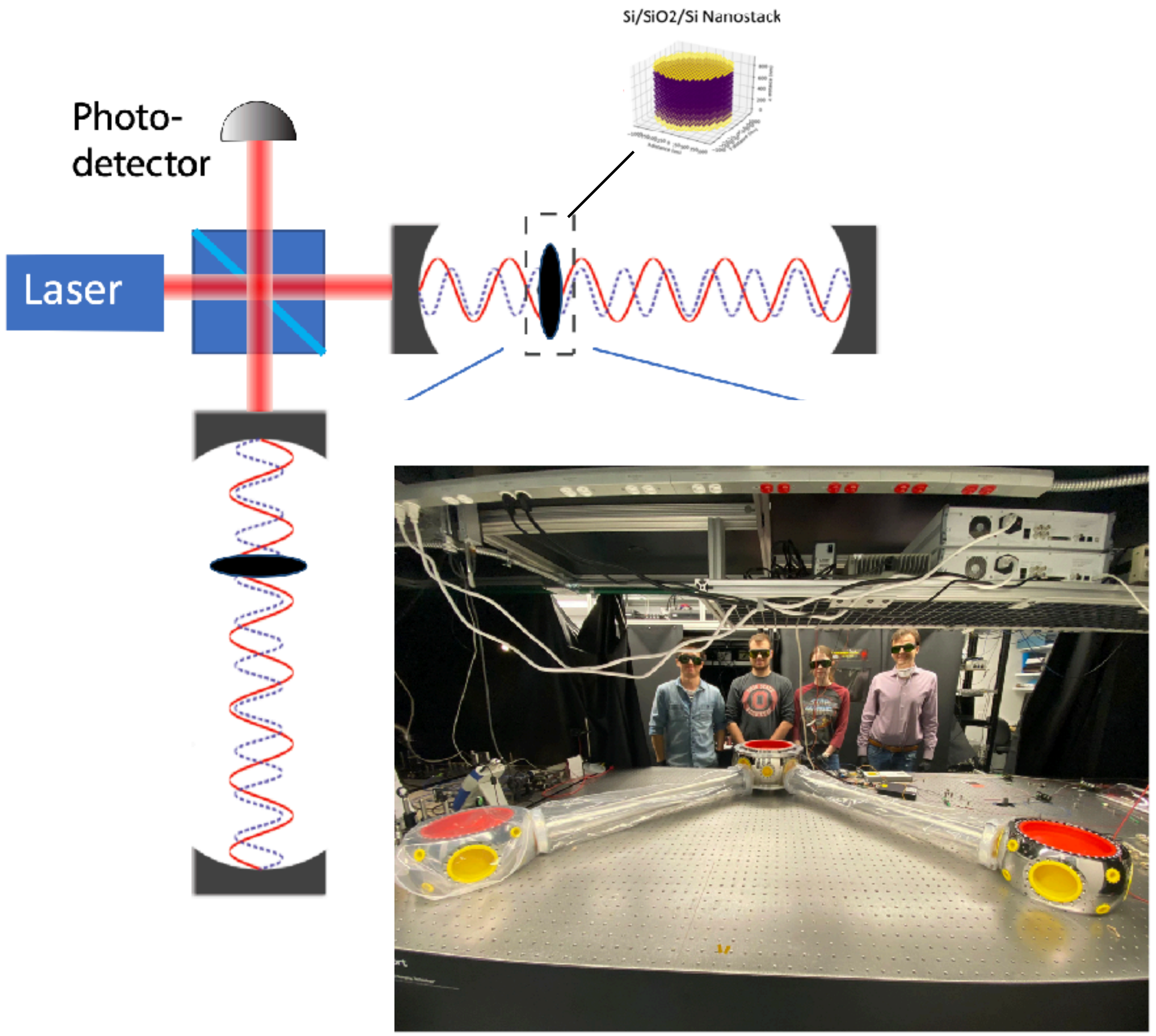
Arvanitaki, Geraci, PRL 110 (2013) 7



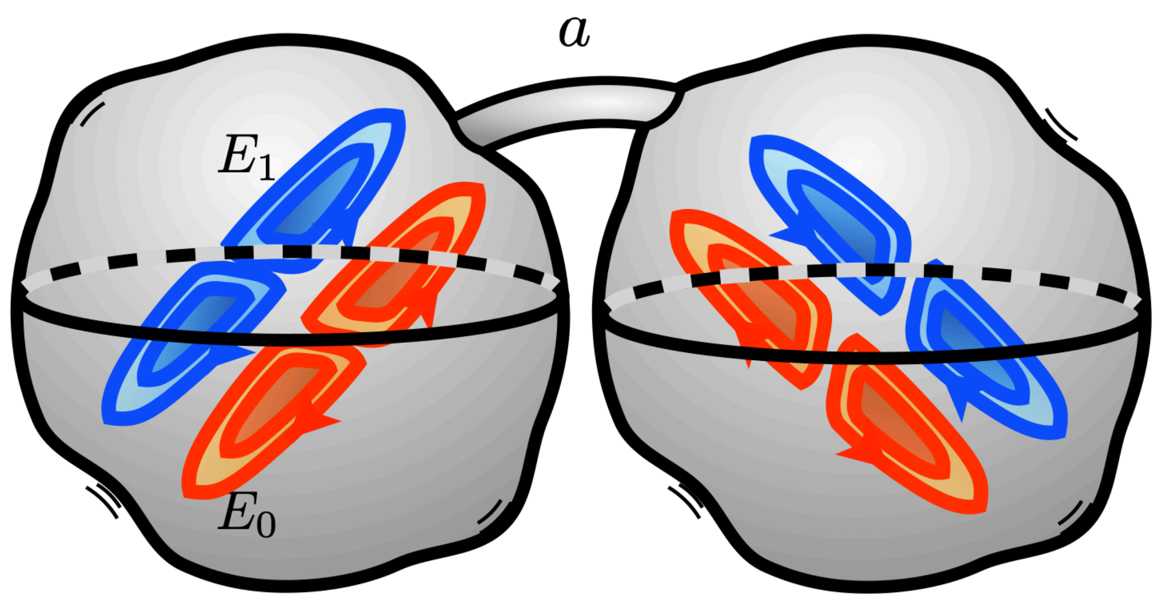
- Limited by thermal noise & Laser heating of levitated particle

- Sensitivity from 10 kHz - 100kHz
- $h_0 > 10^{-21}$

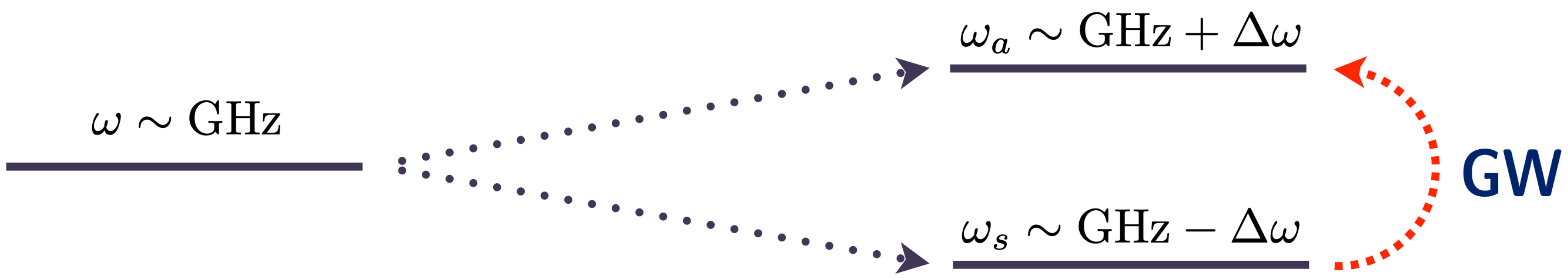
- Trapping dielectric nano-particles in Laser-field
- Second beam for cooling and readout
- GW displaces nanoparticle w.r.t. trap minimum



Conversion of GWs into Photons - Heterodyne detection



- **MAGO experiment @ Desy**



$\omega_a \sim \mathrm{GHz} + \Delta\omega$

$\omega \sim \mathrm{GHz}$

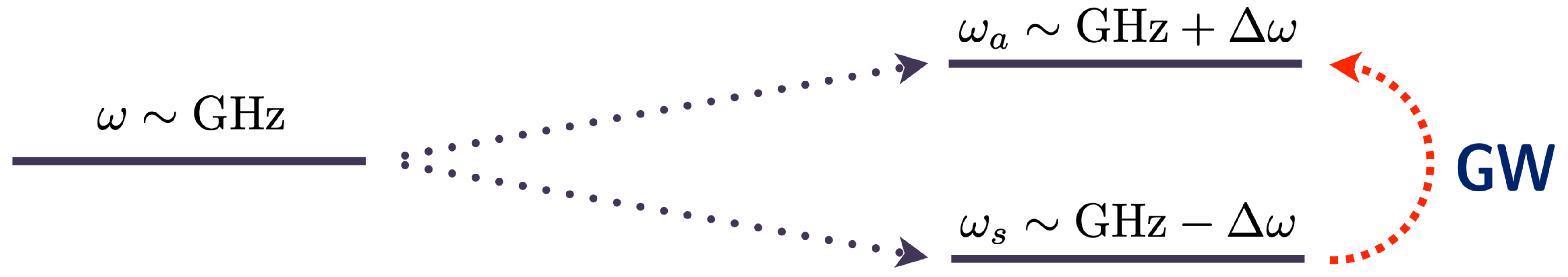
GW

$\omega_s \sim \mathrm{GHz} - \Delta\omega$

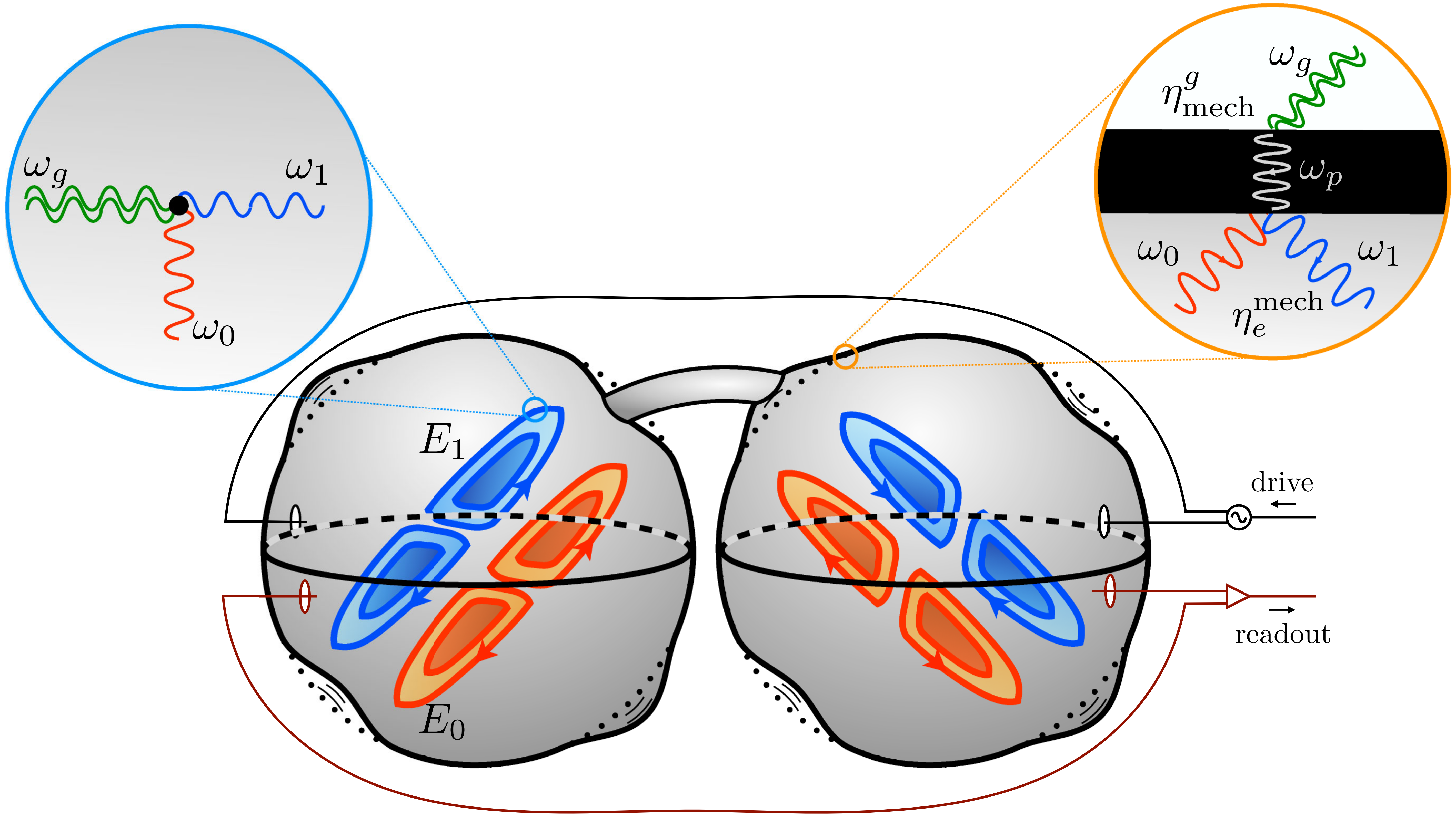
$$\Delta\omega \sim (a/R)^3 \sim kHz - MHz$$

- Two EM levels achieved by coupling identical cavities
- Different spacial field distribution (ω_0 and ω_π, symmetric and anti-symmetric modes)

Conversion of GWs into Photons - Heterodyne detection



$$\Delta\omega \sim (a/R)^3 \sim kHz - MHz$$

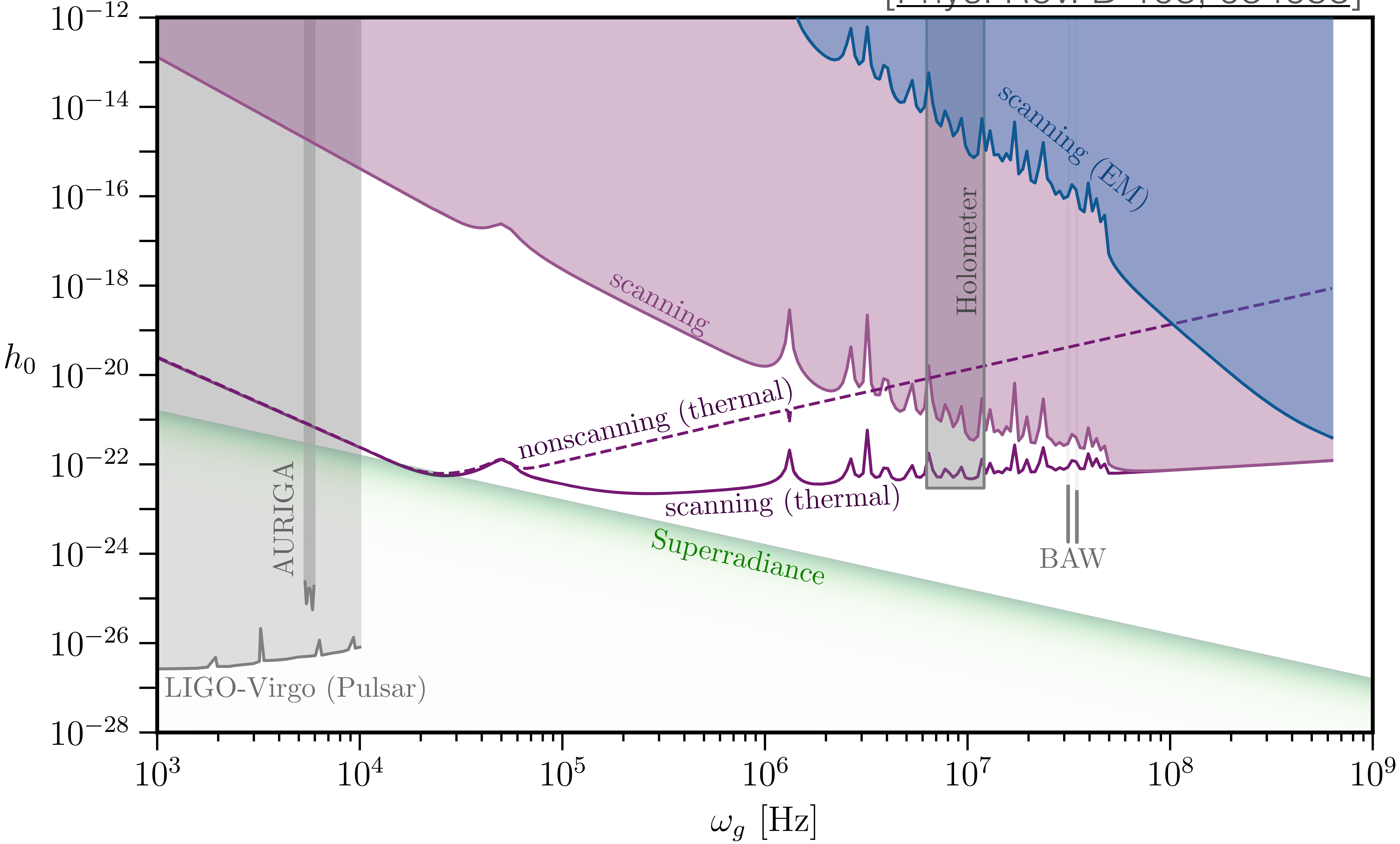


- **Pro**:
  - Amplification linear in Pump Power
- **Con**:
  - Frequency stability of modes
  - RF leakage into signal mode

Kristof Schmieden

Conversion of GWs into Photons - Heterodyne detection

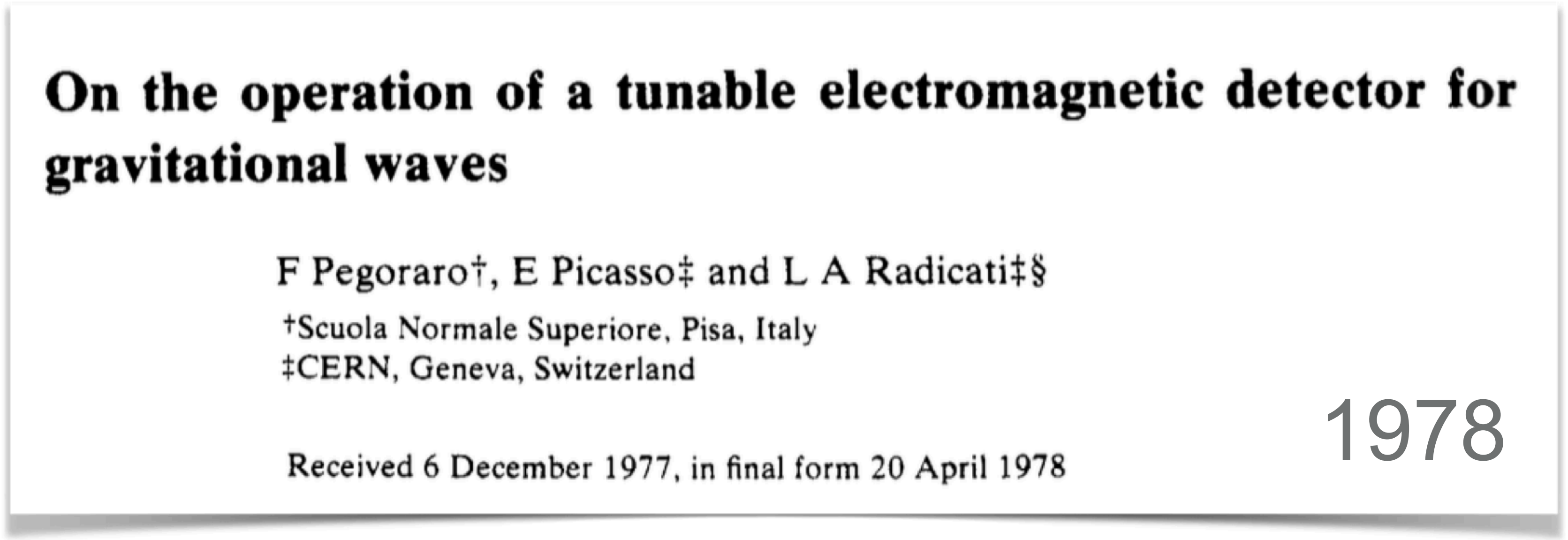
[Phys. Rev. D 108, 084058]



- Sensitivity from 10 kHz - 100MHz (with various cavities

$$h_0 > 10^{-22} \qquad h_0 > 10^{-21}$$

The MAGO proposal



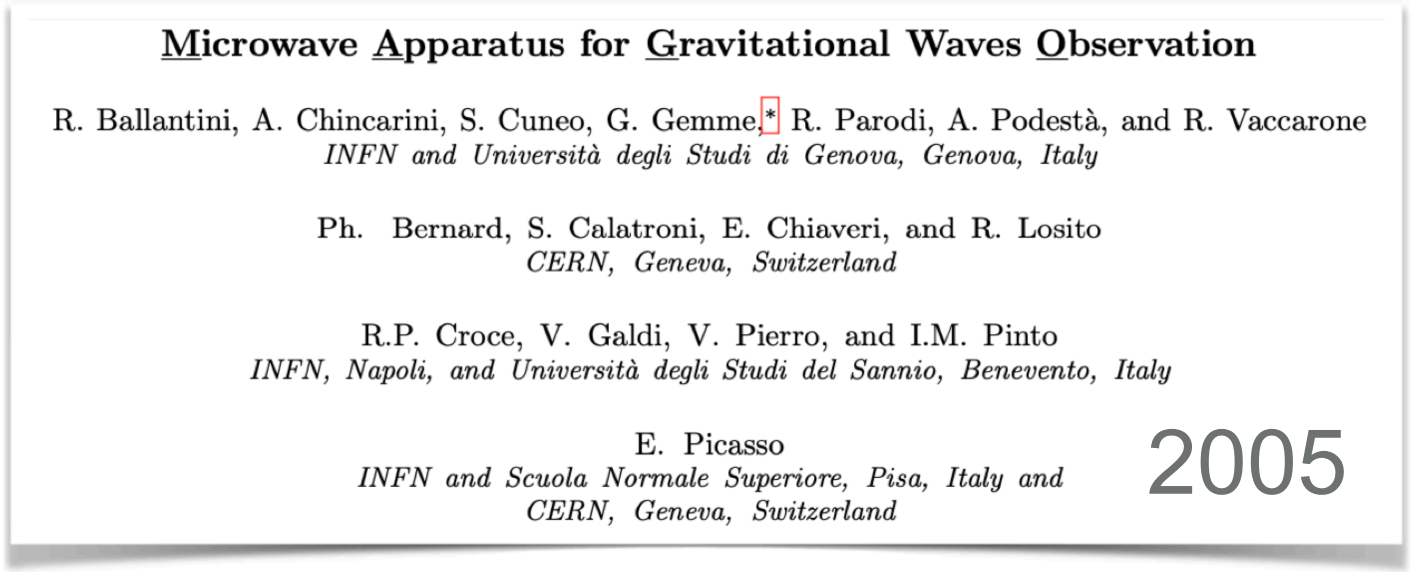
On the operation of a tunable electromagnetic detector for gravitational waves

F Pegoraro†, E Picasso‡ and L A Radicati‡§

†Scuola Normale Superiore, Pisa, Italy
‡CERN, Geneva, Switzerland

Received 6 December 1977, in final form 20 April 1978

1978



Microwave Apparatus for Gravitational Waves Observation

R. Ballantini, A. Chincarini, S. Cuneo, G. Gemme,* R. Parodi, A. Podestà, and R. Vaccarone
INFN and Università degli Studi di Genova, Genova, Italy

Ph. Bernard, S. Calatroni, E. Chiaveri, and R. Losito
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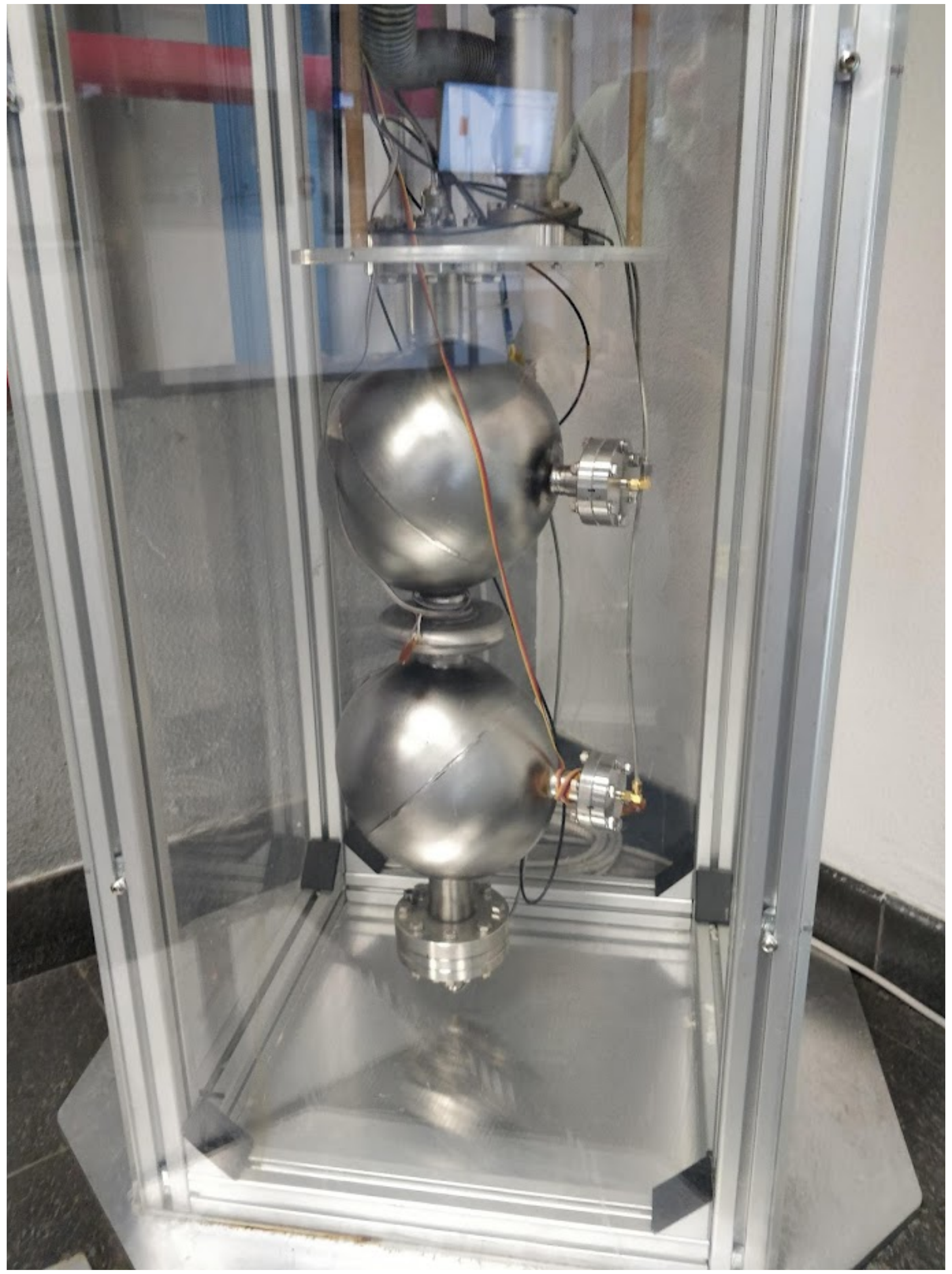
R.P. Croce, V. Galdi, V. Pierro, and I.M. Pinto
INFN, Napoli, and Università degli Studi del Sannio, Benevento, Italy

E. Picasso
INFN and Scuola Normale Superiore, Pisa, Italy and CERN, Geneva, Switzerland

2005

- Initial idea from the 70ies => MAGO proposal
  - Scaled-up experiment with 500 MHz cavities (not funded)
- During the R&D activities **3 SRF cavities were built**, the first one used for a proof-of-principle experiment
- **The third cavity**
  - 2-cell cavity with optimised geometry and variable coupling cell
  - Never treated nor tested – **on shelf for >15y** @ INFN Genova

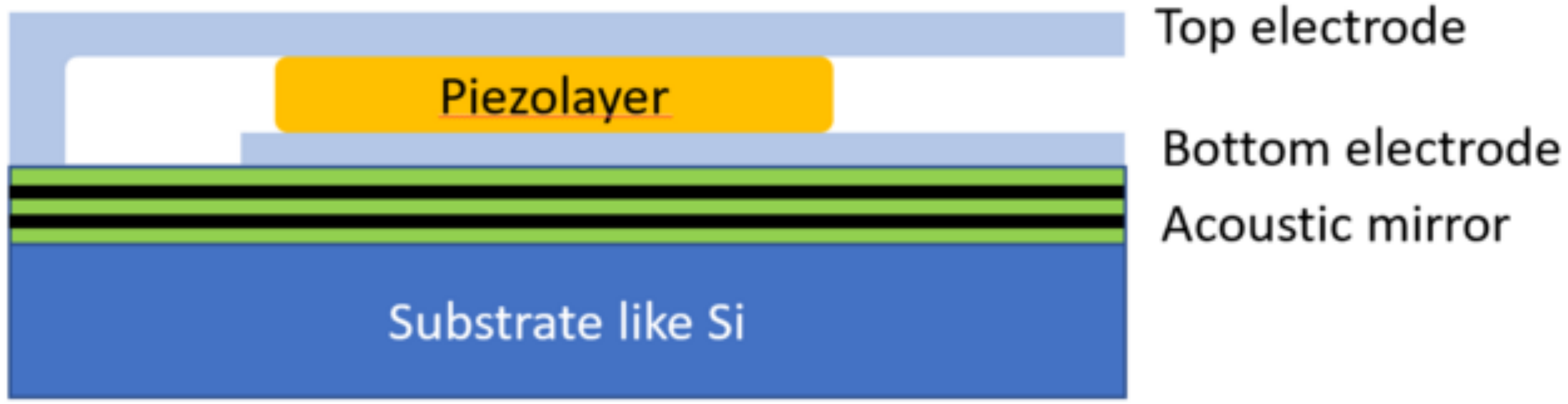
- In a collaborative effort, **DESY**/UHH - FNAL - INFN, continue the R&D studies with a goal to have synchronised observatories



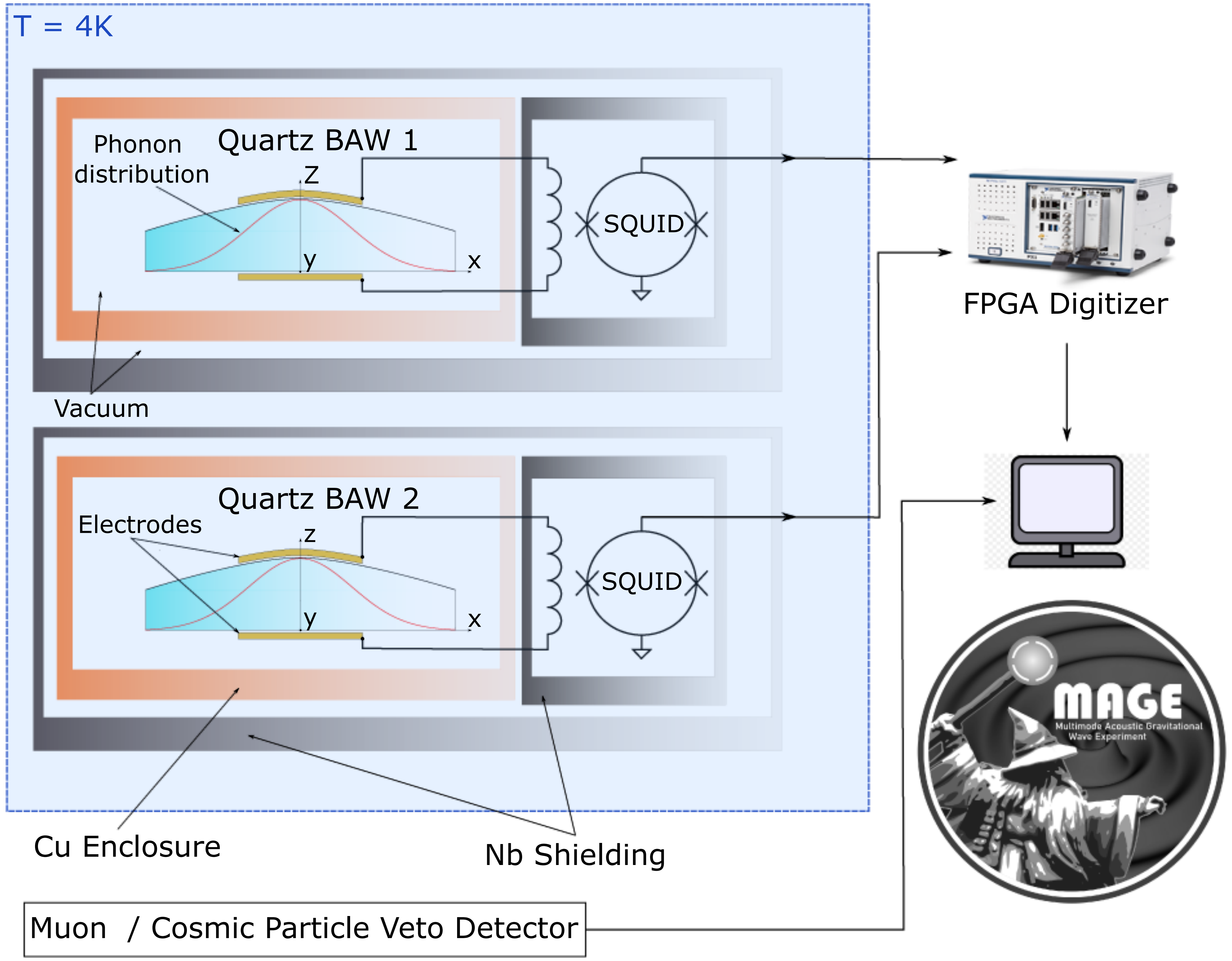
University Genova

Conversion of GWs into Photons

- Bulk acoustic devices



- Piezoelectric resonator
  - Freq: MHz - GHz
  - Consumer product
- GW deforms resonator
  - Periodically changing resonance frequency excites
  - Excitation of resonance



[Sci Rep 13, 10638 (2023). https://doi.org/10.1038/]

- Sensitivity from 5 - 10 MHz
- $h_0 > 10^{-21}$

Conversion of GWs into Photons

- Deformation of cavities

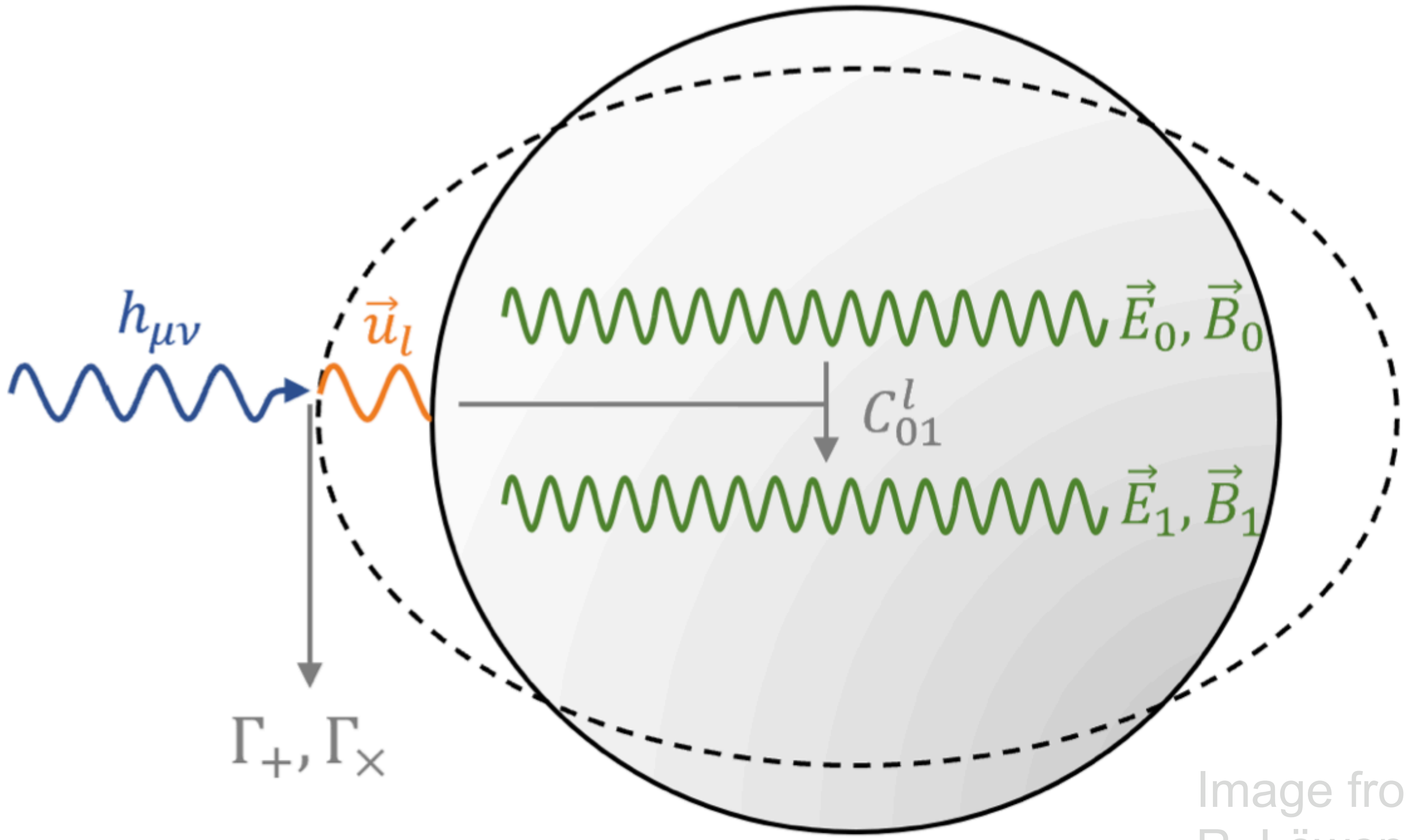
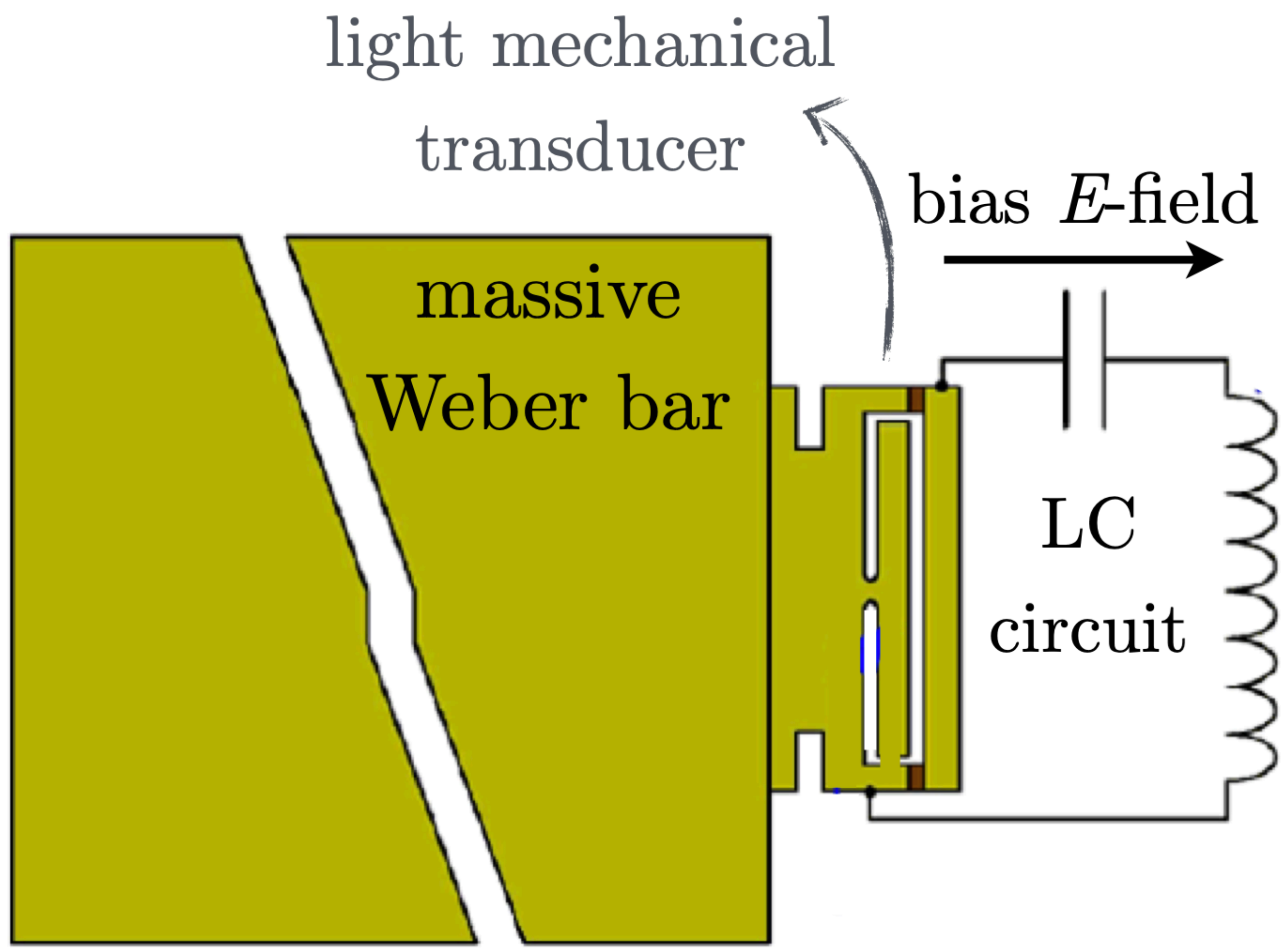


Image from R. Löwenberg thesis

- Transfer of mechanical to EM energy
- Competing process for any cavity based detector

- Exploit mechanical resonances for enhancement
- Noise from environmental vibrations

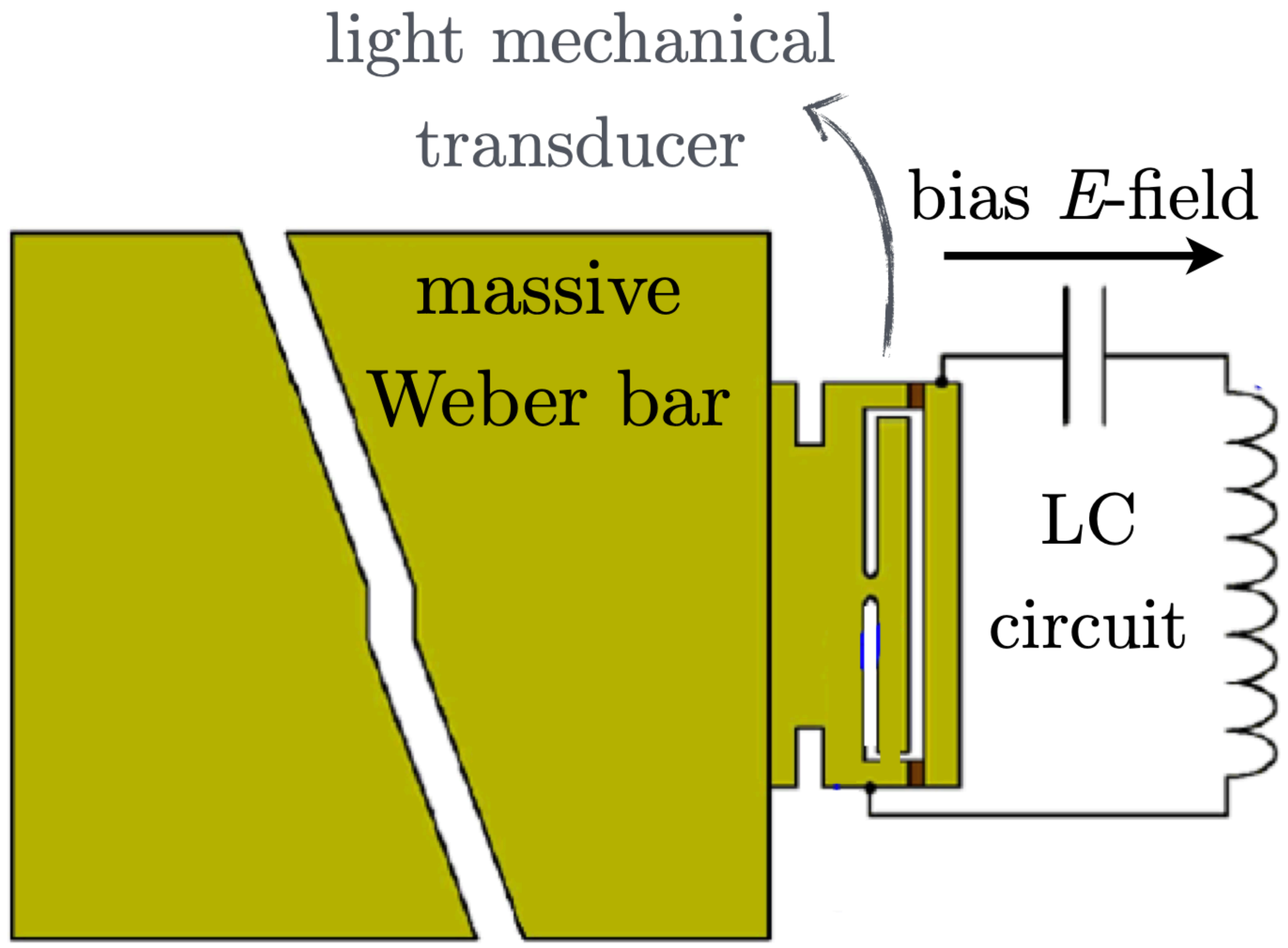
- Original Idea:
  - Weber bar



$$Q_{LC} \sim 10^6 \ll Q_{cav} \sim 10^{11}$$

Conversion of GWs into Photons

- Original Idea:
  - Weber bar: 2m x 1m aluminum rod
  - Sensitivity at ~kHz: $h_0 < 10^{-16}$



$$Q_{LC} \sim 10^6 \ll Q_{cav} \sim 10^{11}$$

Detection of HFGW

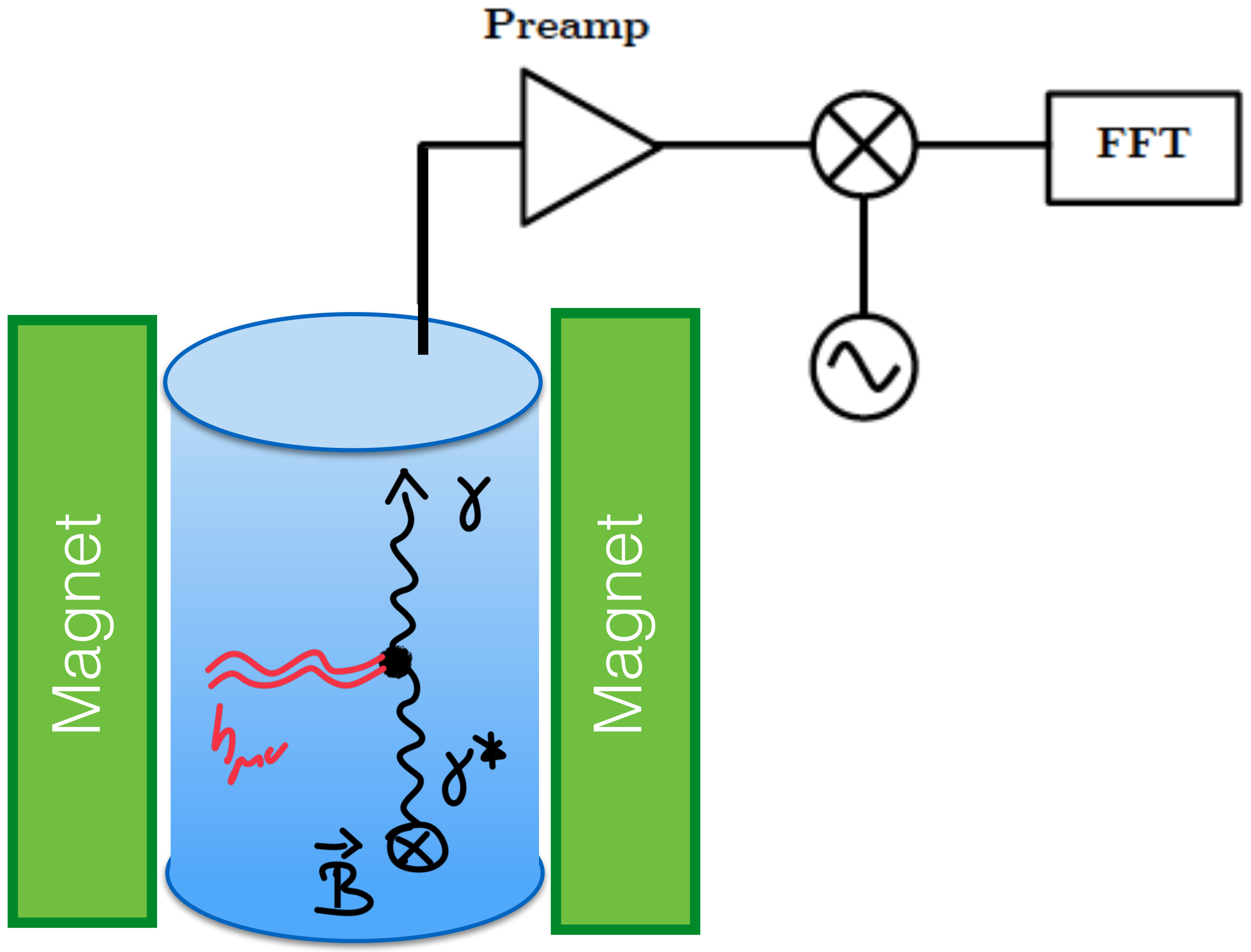
- All experiments face the challenge of
  - **Tiny signals**
  - Ever present **background** (thermal, amplifiers, quantum noise)
- Technological challenges vary with experimental approach
- All technologies will move to **quantum** technology **readout**

Conclusion

MPA
MAINZ PHYSICS
ACADEMY

- New Era of GW astronomy
- Frequencies from nHz to GHz of interest
- Only two frequency windows accessible so far
- Vast variety of experimental approaches
  - **Mainz**: Haloscope style cavity based detector

- Many advantages in **combining efforts searching for HFGWs** in coordinated way
- GravNet will significantly improve the sensitivity on high frequency gravitational waves



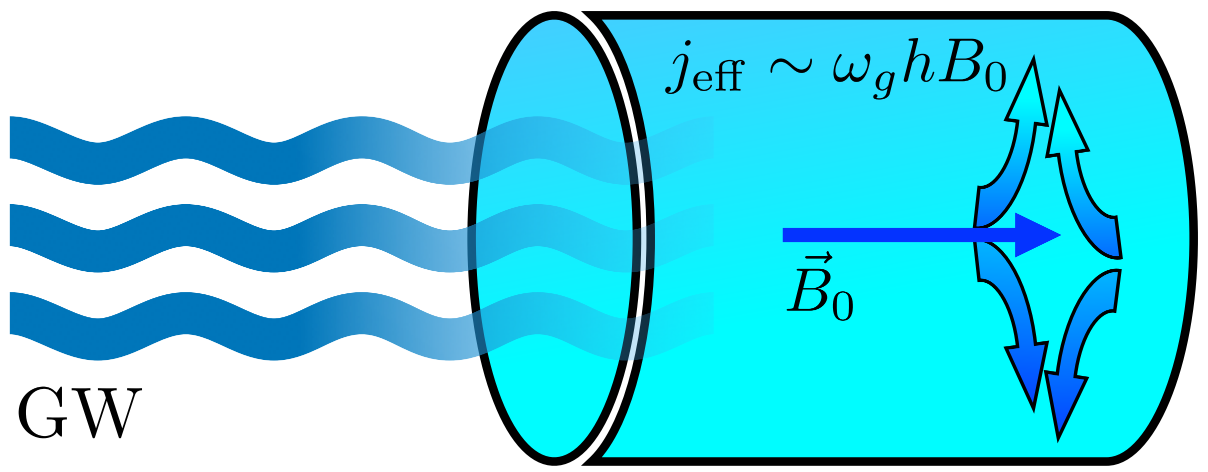
Cluster of Excellence
PRISMA+
Precision Physics, Fundamental Interactions and Structure of Matter

BACKUP

Conversion of GWs into Photons

[arXiv:2112.11465]

- Two contributing effects
  - Assuming conversion cavity with volume V within static B-Field



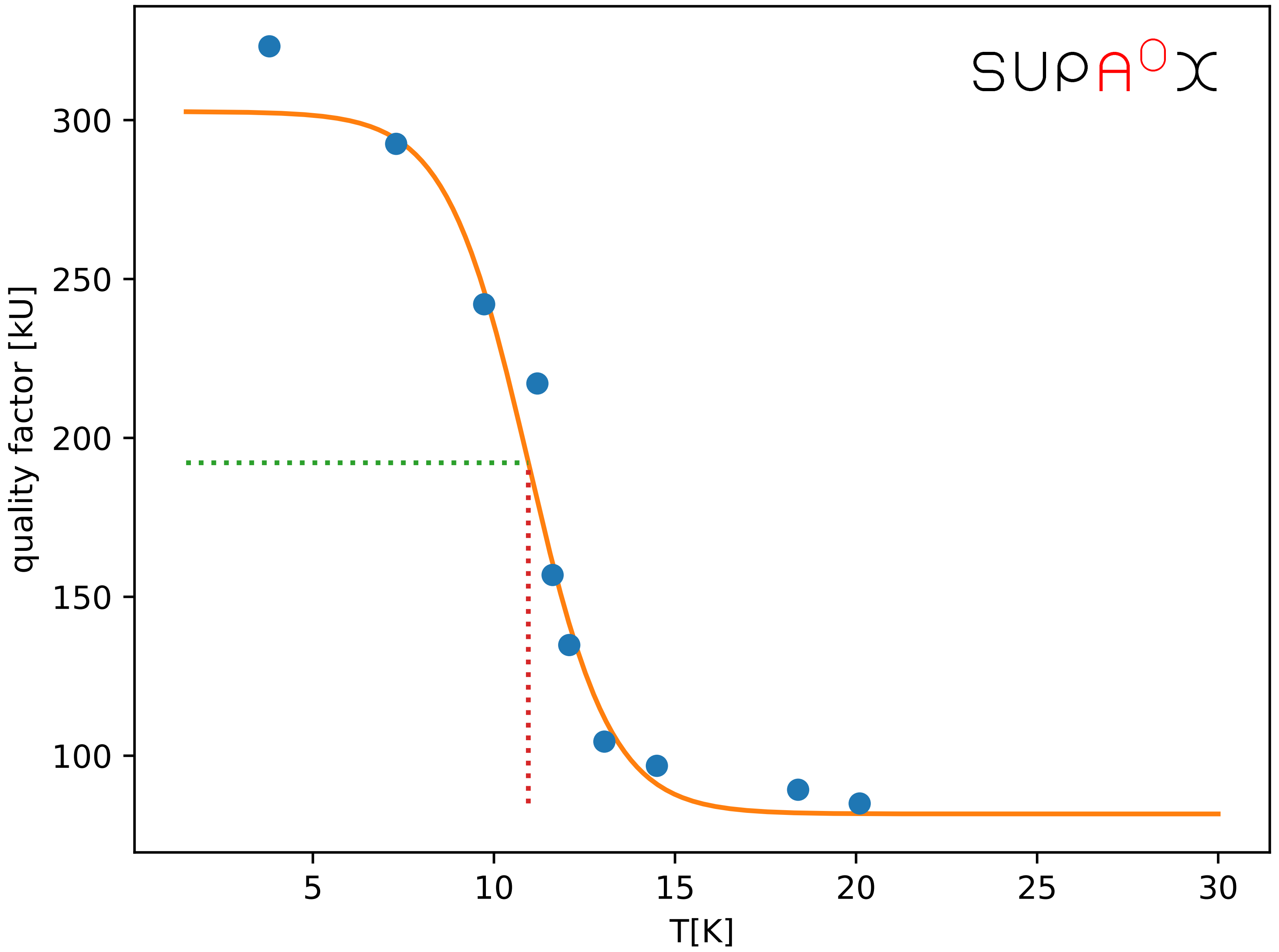
- GW deforms cavity
  - Oscillating change of magnetic flux
  - Excitation of EM field
- Resonant excitation of EM field in Cavity
  - Produced EM power given by:
- Direct conversion of gravitons to photons via the inverse Gertsenshtein effect

$$P_{sig} = \frac{1}{2} Q \omega_g^3 V^{5/3} (\eta_n h_0 B_0)^2 \frac{1}{\mu_0 c^2}$$

Supax SRF cavity

- **Supax:** new superconducting material for RF cavities:
  - **NbN**
  - $Q_0 = 3 \cdot 10^5$ @ 8.4 GHz, 4 K
  - Measurements within B-field currently ongoing

Cu cavity, coated with NbN at university of Siegen



GravNet - a global network for HFWG detection

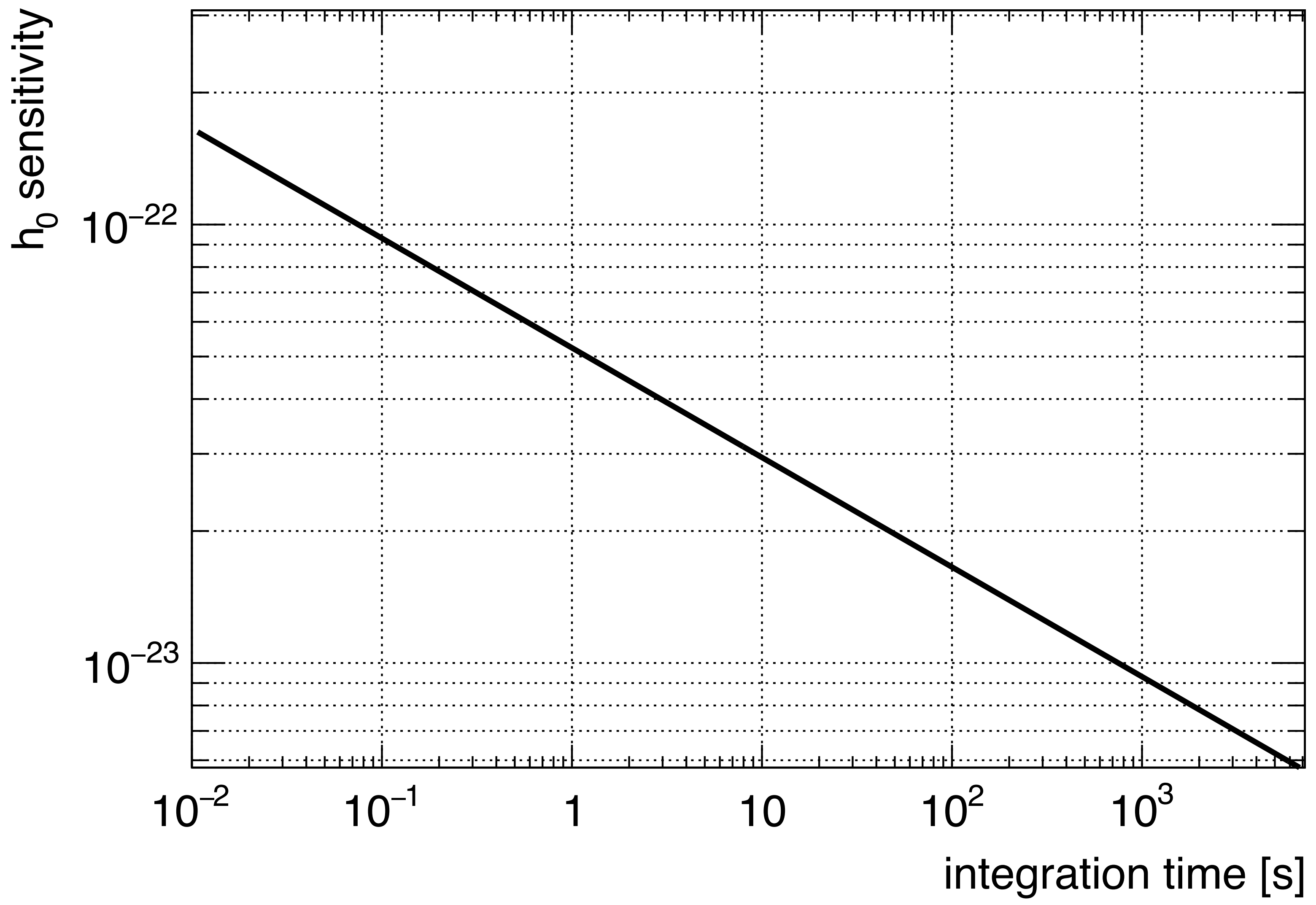
- How sensitive can we get with **10 setups**, scattered around the globe

- Assumptions:
  - Sampling of Waveform -> offline combination of phase aligned IQ data
  - Setups as shown before
    - Effective signal power increased by factor 10
    - Strain sensitivity increased by factor $\sqrt{10} \approx 3$

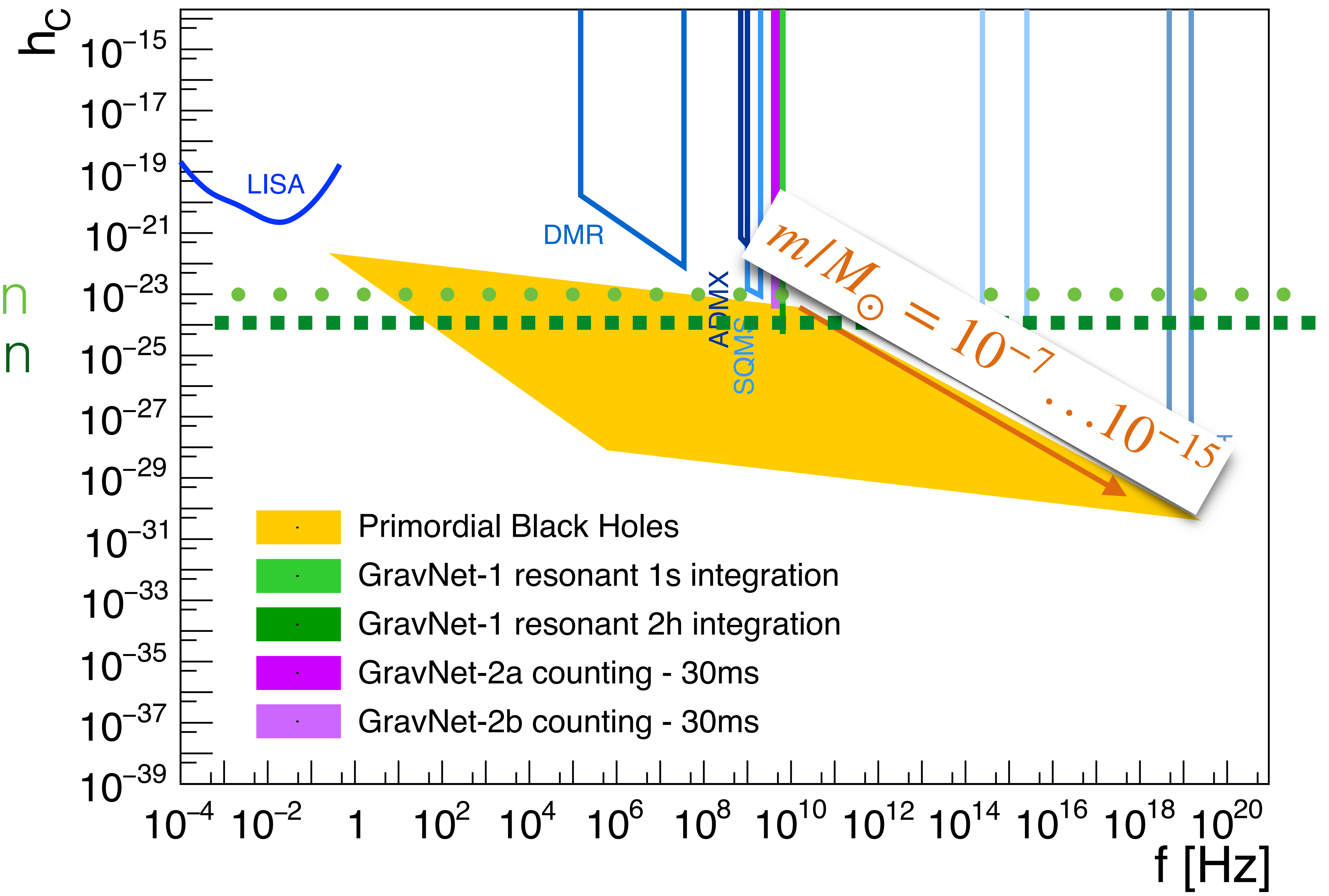
$$h_0 < 10^{-23}\text{, 1 second integration time}$$

- Longer integration times
  - Sensitivity gain with integration time t$^{1/4}$

$$h_0 < 10^{-24}\text{, 2h integration time}$$



1s integration
2h integration

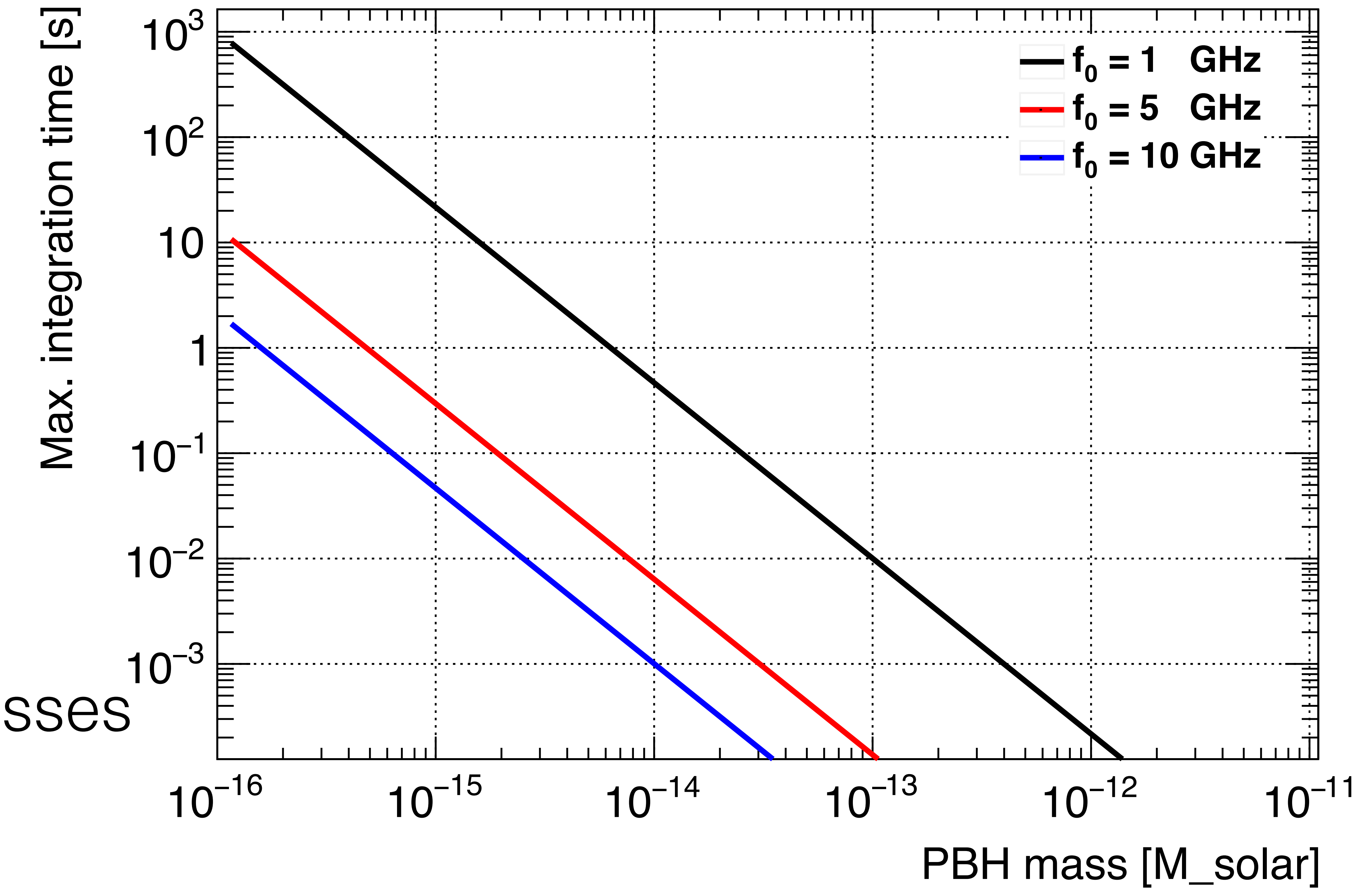


PBH merger & high Q cavities

- GW strain: largest if merging is imminent (closest to innermost stable circular orbit)
  - Frequency drift large

$$\dot{f} = \frac{96}{5}\pi^{8/3} m_c^{5/3} f^{11/3} \simeq 4.62 \times 10^{11}\,\mathrm{Hz}^2 \left(\frac{m_{\mathrm{PBH}}}{10^{-9} M_\odot}\right)^{5/3} \left(\frac{f}{\mathrm{GHz}}\right)^{11/3}$$

- To resonantly excite a cavity:
  - GW frequency must stay within resonator bandwidth
    - $\omega / Q \approx 10^{10} Hz / 10^6 = 10 kHz$
  - Very short integration times O(ms) or below for larger PBH masses



- No improvement with longer integration times!
  - Alternative?

Timeseries analysis

- Similar approach as for low frequency BH mergers:

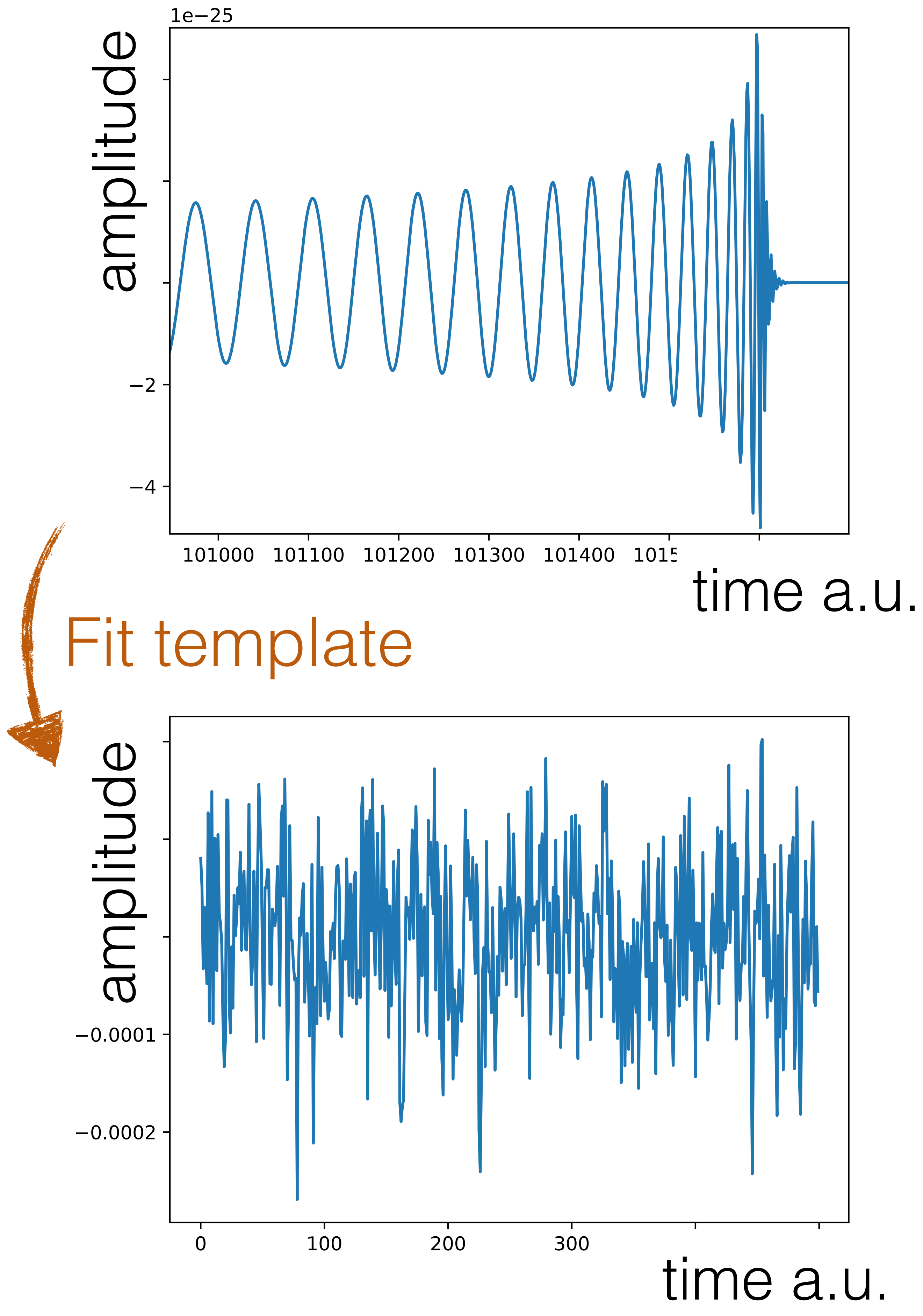
  - Analysis in **time domain**
  - Data rates: ~100MB/s per channel for 10MHz bandwidth

- Simultaneous fit of expected signal shape in all data streams

  - Exploiting all available information

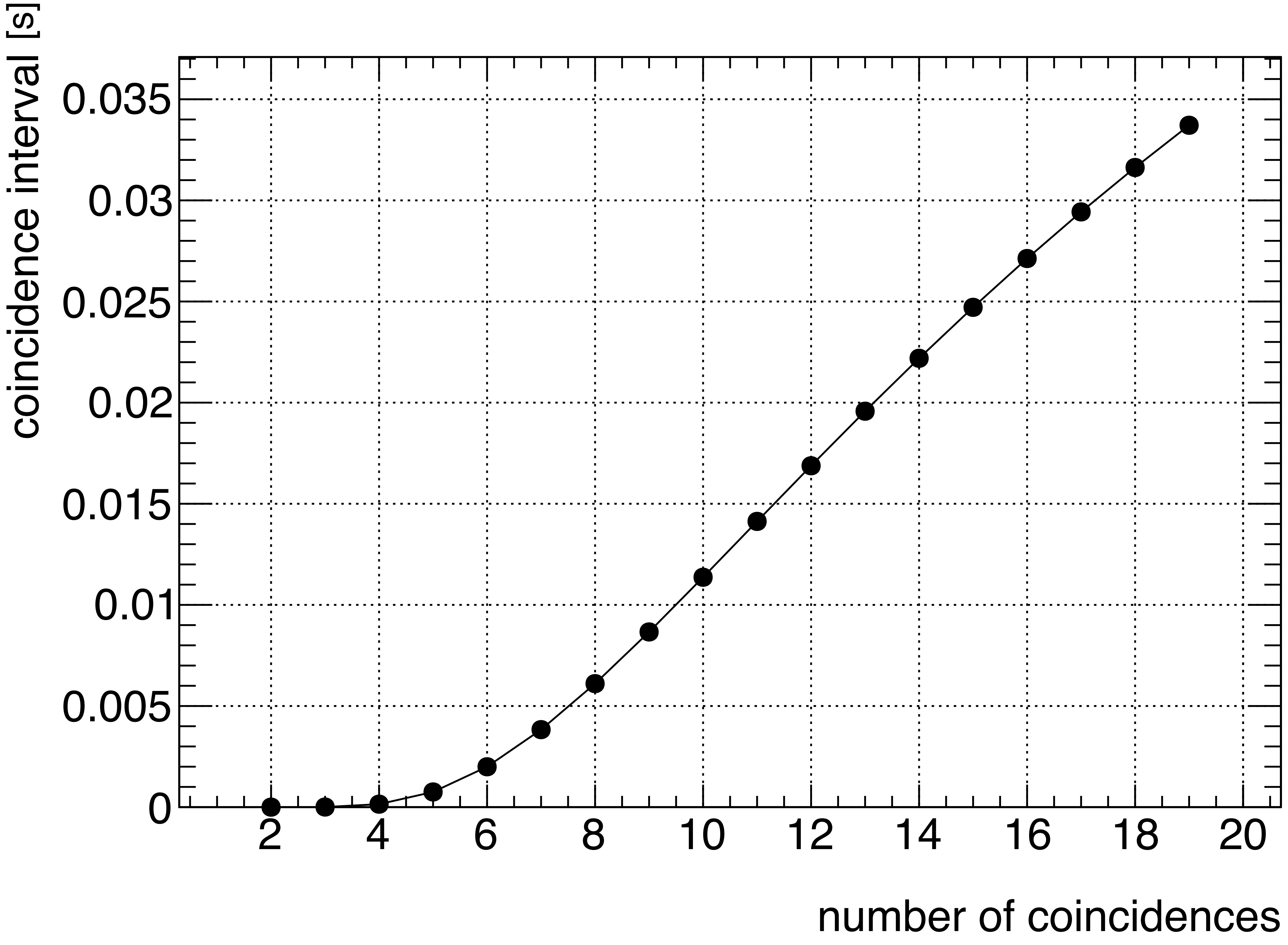
    - \+ Increased sensitivity compared to time domain analysis
    - \- Significant increase in storage & CPU requirement

  - **Sensitive to short transient** signals



Photon Counting - Coincidence Interval

- Background rate:
  - Average thermal power in cavity @ 0.1K ~ 4×10^{-23} W, corresponding to 10 photons / s @ 5 GHz
  - Could be lowered going to lower temperatures
  - Assuming advances in the near future on the single photon sensors:
    - Detector dark count rate will drop significantly -> negligible



- Parameter used for Calculation:
  - Allowed accidental coincidence rate: <= 1/year
  - Background rate: 10 Hz
  - N detectors: 20

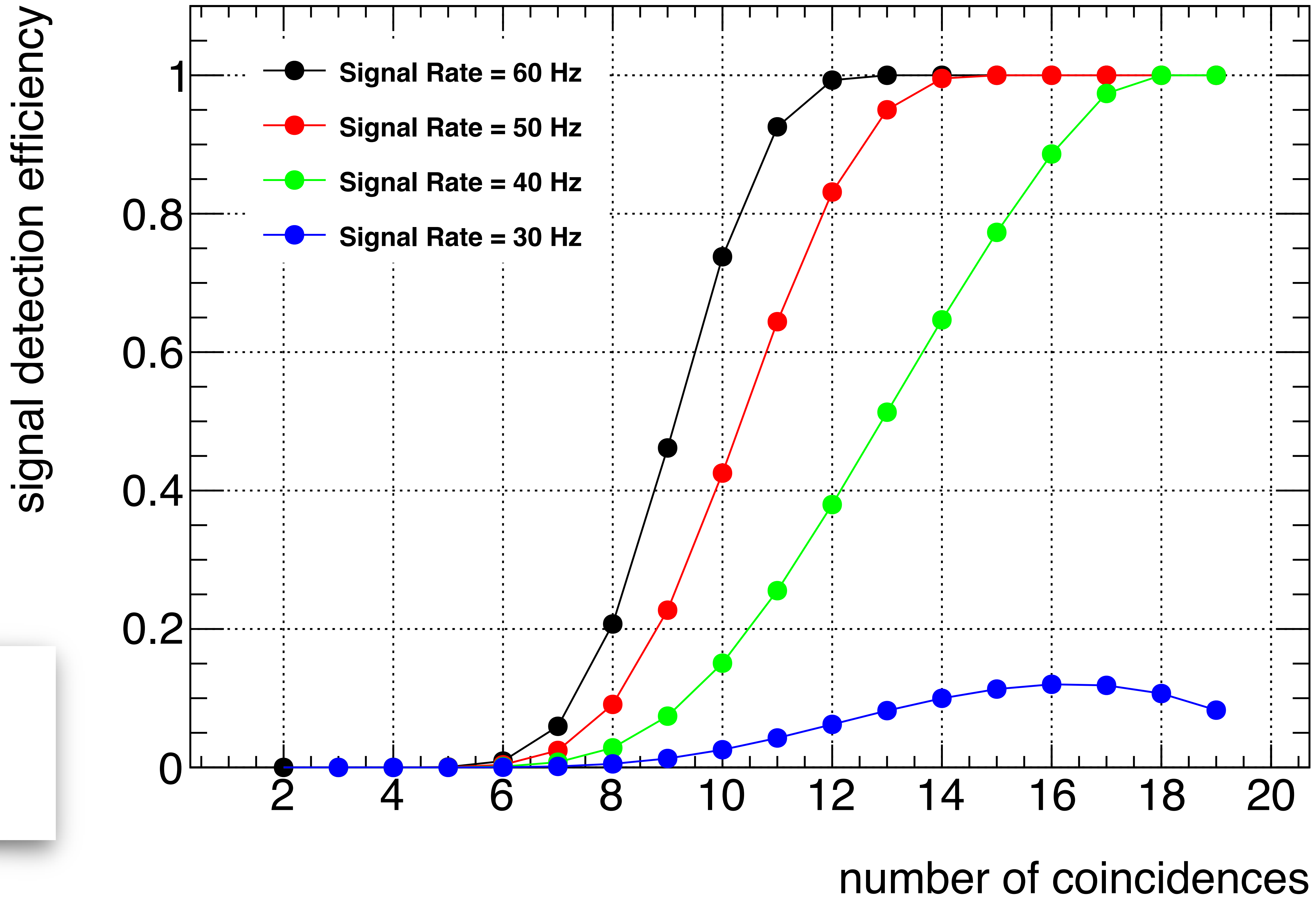
Photon Counting - Signal efficiency

- Overall signal efficiency dependent on detector efficiency, coincidence window and signal photon flux:
  - $\epsilon_{single} = \epsilon_{det} \Delta t_{coincidence} \Phi_{sig}$ Φ_{sig} = signal photon flux
  - $\epsilon_{tot} = \sum_{i>k} \binom{N}{k}, p = \epsilon_{single}$, k = number of required coincidences, N = number of detectors

- Parameter used for Calculation:
  - Allowed accidental coincidence rate: <= 1/year

  - Background rate: 10 Hz
  - N detectors: 20
  - ϵ_{det}: 0.5

- With **20 detectors** a photon flux of **40 Hz** can be detected with an efficiency of 1 within a coincidence interval of **32ms**



Combining Multiple Cavities