Revisiting Isocurvature Evolution

Perturbations emerging from the dark

MPA Retreat Kloster Höchst

Monday, 30.09.24

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Based on work with Wolfram Ratzinger & Pedro Schwaller
in preparation

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1. What is isocurvature?

Cosmic perturbation theory - The basics

- Metric in perturbed spacetime:

$$\mathrm{d}s^2 = a^2(\tau)\left[-(1+2\phi)\mathrm{d}\tau^2 + 2B_i\mathrm{d}x^i\mathrm{d}\tau + \left((1-2\psi)\delta_{ij} + E_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j\right]$$

- Matter perturbations in the stress-energy tensor:

$$T^0{}_0 = -\left(\rho + \delta\rho\right)$$
$$T^i{}_0 = -\left(\rho + P\right)v^i$$
$$T^i{}_j = \left(P + \delta P\right)\delta^i{}_j + \Pi^i{}_j$$

- ▸ Solve perturbed Einstein-Boltzmann equations [Ma, Bertschinger 9506072]
- ▸ Follow perturbations through cosmic history: $h, \eta, \delta_i, \theta_i, \sigma_i, F_{3i}, F_{4i}$

1. What is isocurvature?

Perturbed Einstein-Boltzmann equations

[Ma, Bertschinger 9506072]

$$k^2\eta - \frac{1}{2}\mathscr{H}h' = -4\pi G a^2 \delta\rho,$$
$$k^2\eta' = 4\pi G a^2 (\rho + P)\theta,$$
$$h'' + 2\mathscr{H}h' - 2k^2\eta = -24\pi G a^2 \delta P,$$
$$h'' + 6\eta'' + 2\mathscr{H}(h' + 6\eta') - 2k^2\eta = -24\pi G a^2 (\rho + P)\sigma\,.$$

Synchronous gauge Einstein eqns.

$$\delta_c' = -\theta_c - \frac{1}{2}h'$$

CDM

$$\delta_\gamma' = -\frac{4}{3}\theta_\gamma - \frac{2}{3}h'$$

Photons+baryons (tight coupling)

$$\delta_b' = -\theta_b - \frac{1}{2}h' \qquad \left(\Omega_b + \frac{4}{3}\Omega_\gamma\right)\theta_{\gamma b}' = \Omega_\gamma \frac{k^2}{3}\delta_\gamma - \Omega_b \mathscr{H}\theta_{\gamma b}$$

$$\delta_\nu' = -\frac{4}{3}\theta_\nu - \frac{2}{3}h'$$

Neutrinos & FDR

$$\theta_\nu' = k^2\left(\frac{1}{4}\delta_\nu - \sigma_\nu\right)$$
$$\sigma_\nu' = \frac{4}{15}\theta_\nu - \frac{3}{10}kF_{\nu 3} + \frac{2}{15}h' + \frac{4}{5}\eta'$$
$$F_{\nu 3}' = \frac{k}{7}\left[6\theta_\nu - 4F_{\nu 4}\right]$$
$$F_{\nu 4}' = \frac{k}{9}\left[4F_{\nu 3}\right]$$

But what about initial conditions?

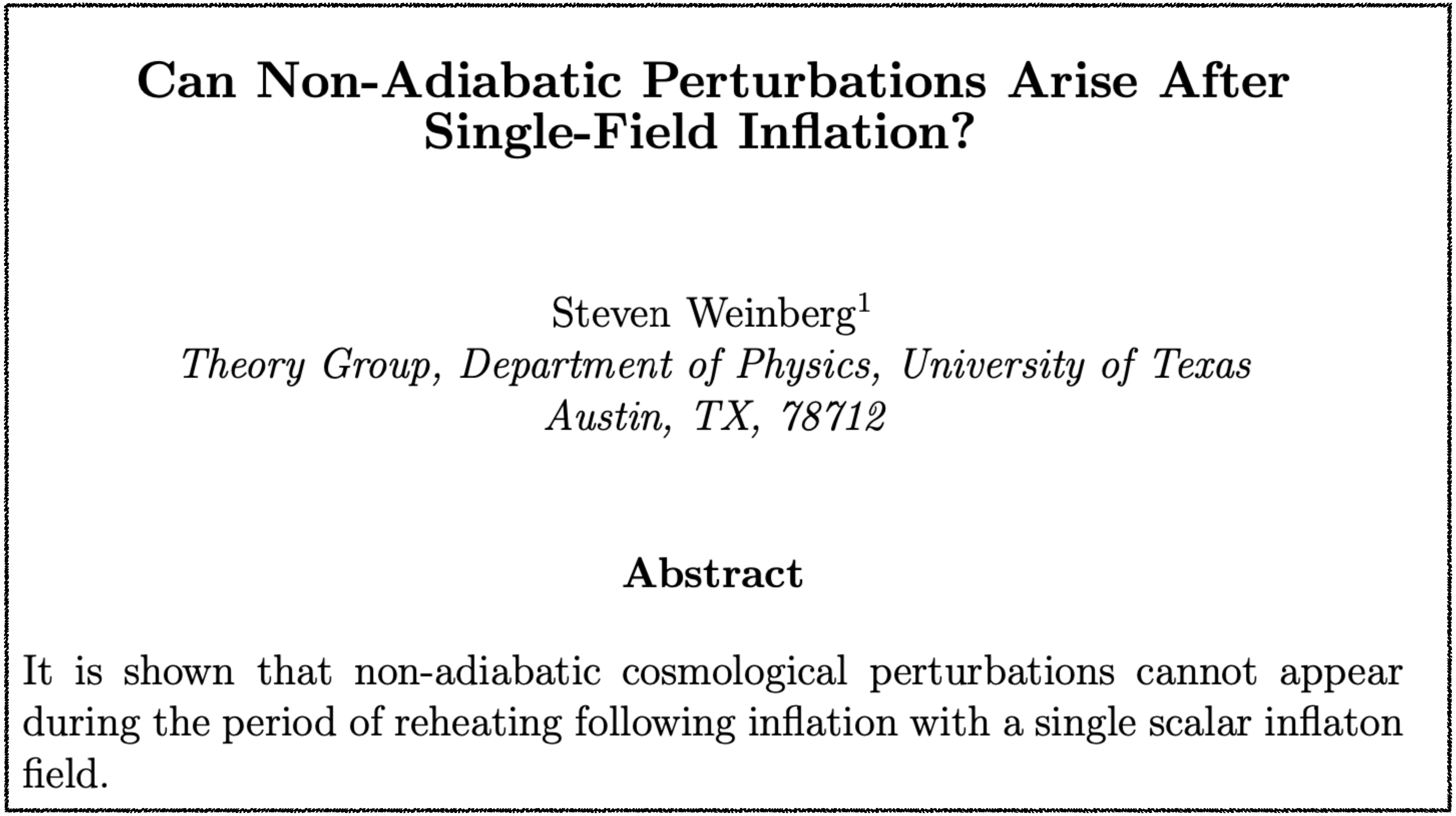
1. What is isocurvature?

Adiabatic perturbations

- Single-field inflation: time shift $\delta\tau(\vec{x})$ common to all fluids **(one clock)**

  = Adiabatic perturbations

[Weinberg 0401313]



Can Non-Adiabatic Perturbations Arise After Single-Field Inflation?

Steven Weinberg[1]
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Austin, TX, 78712

Abstract

It is shown that non-adiabatic cosmological perturbations cannot appear during the period of reheating following inflation with a single scalar inflaton field.

$$\delta = \frac{\delta\rho}{\rho}$$

1. What is isocurvature?

Adiabatic perturbations

- Single-field inflation: time shift $\delta\tau(\vec{x})$ common to all fluids **(one clock)**

  = Adiabatic perturbations

[Weinberg 0401313]

$$\delta\tau = \frac{\delta\rho_\alpha}{\rho'_\alpha} = \frac{\delta_\alpha}{1+w_\alpha} \qquad \Rightarrow \qquad \delta_\gamma = \delta_\nu = \frac{4}{3}\delta_b = \frac{4}{3}\delta_c$$

- Outside horizon: gauge-invariant quantities

$$\delta = \frac{\delta\rho}{\rho}$$

$$\psi \rightarrow \psi + H\,\delta\tau$$

$$\delta\rho \rightarrow \delta\rho - \dot{\rho}\delta\tau$$

$$\boxed{\text{Curvature perturbation* } \zeta = -\psi - H\frac{\delta\rho}{\dot{\rho}}}$$

*on uniform-density hypersurfaces

- Adiabatic: Constant outside horizon, same ζ for all fluids

- Only metric perturbations at horizon entry (as „initial condition“)

1. What is isocurvature?

Isocurvature perturbations

- Multi-field inflation: Different density perturbations for different fluids
  = Isocurvature perturbations

$$\zeta_\alpha = -\psi - H\frac{\delta\rho_\alpha}{\dot{\rho}_\alpha}$$

[Wands, Malik, Lyth, Liddle 0003278]

- Define: $\boxed{\text{Isocurvature perturbation } \mathcal{S}_{\alpha\beta} = 3(\zeta_\alpha - \zeta_\beta)}$

- Simple approach: „Neutrino density isocurvature": $\mathcal{S}_{\gamma\nu} = \frac{3}{4}(\delta_\gamma - \delta_\nu) \neq 0$

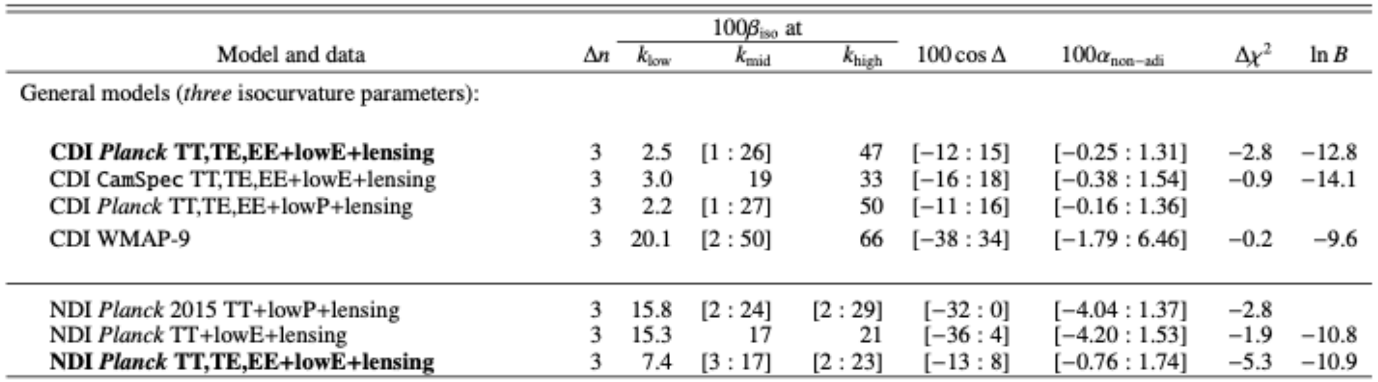
- Common initial conditions: δ_ν=1 and $\delta_\gamma = -\rho_\nu/\rho_\gamma$,
  other perturbations zero at horizon entry

2. Why care about isocurvature?

$$k_{low,mid,high} = 0.002, 0.050, 0.100\,\mathrm{Mpc}^{-1}$$

Motivation

- Planck constrains isocurvature perturbations on CMB scales:



Model and data	Δn	$100\beta_{\rm iso}$ at $k_{\rm low}$	$k_{\rm mid}$	$k_{\rm high}$	$100\cos\Delta$	$100\alpha_{\rm non-adi}$	$\Delta\chi^2$	$\ln B$
General models (*three* isocurvature parameters):								
CDI *Planck* TT,TE,EE+lowE+lensing	3	2.5	[1 : 26]	47	[−12 : 15]	[−0.25 : 1.31]	−2.8	−12.8
CDI CamSpec TT,TE,EE+lowE+lensing	3	3.0	19	33	[−16 : 18]	[−0.38 : 1.54]	−0.9	−14.1
CDI *Planck* TT,TE,EE+lowP+lensing	3	2.2	[1 : 27]	50	[−11 : 16]	[−0.16 : 1.36]		
CDI WMAP-9	3	20.1	[2 : 50]	66	[−38 : 34]	[−1.79 : 6.46]	−0.2	−9.6
NDI *Planck* 2015 TT+lowP+lensing	3	15.8	[2 : 24]	[2 : 29]	[−32 : 0]	[−4.04 : 1.37]	−2.8	
NDI *Planck* TT+lowE+lensing	3	15.3	17	21	[−36 : 4]	[−4.20 : 1.53]	−1.9	−10.8
NDI *Planck* TT,TE,EE+lowE+lensing	3	7.4	[3 : 17]	[2 : 23]	[−13 : 8]	[−0.76 : 1.74]	−5.3	−10.9

[Planck Collaboration 1807.06211]

$$\beta_{iso}(k) = \frac{\mathscr{P}_{II}(k)}{\mathscr{P}_{II}(k) + \mathscr{P}_{RR}(k)}$$

- Recent discussions on sourcing isocurvature during freeze-in:

  [Bellomo, Berghaus, Boddy 2210.15691v2]
  [Strumia 2211.08359]
  [Racco, Riotto 2211.08719]
  [Holst, Hu, Jenks 2311.17164]
  [Stebbins 2311.17379]

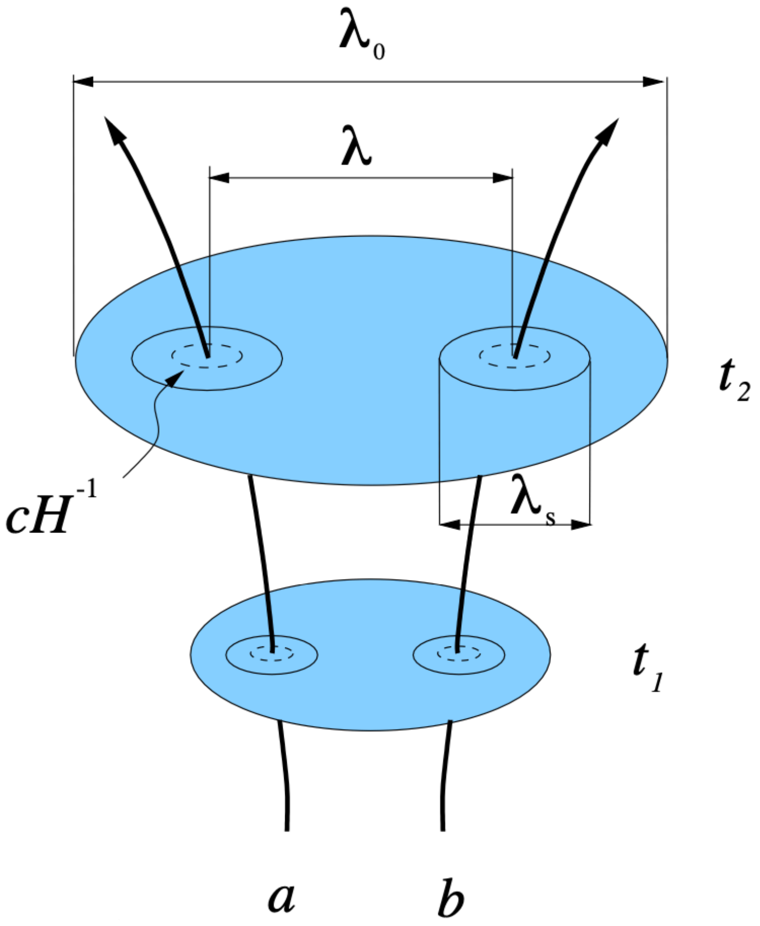
- Evolution of perturbations useful for: LSS, PBH, GWs

3. Isocurvature evolution

[Wands, Malik, Lyth, Liddle 0003278]
[Lyth, Wands 0306498]
[Lyth, Rodriguez 0504045]

The separate-universe picture

- Region of sufficient size evolves like independent & unperturbed universe in super-horizon era
  - Individual but locally homogeneous density + pressure



[Credit: Wands, Malik, Lyth, Liddle 0003278]

3. Isocurvature evolution

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The separate-universe picture

- Region of sufficient size evolves like independent & unperturbed universe in super-horizon era
  - Individual but locally homogeneous density + pressure
- Equivalent to long-wavelength limit: spatial gradients negligible
  - Perturbed continuity equation for individual fluid α
  - With energy transfer $\nabla_\mu T^{\mu\nu}_{(\alpha)} = Q^\nu_{(\alpha)}$ $\sum Q^\nu_{(\alpha)} = 0$

$$\delta\dot{\rho}_\alpha + 3H\left(\delta\rho_\alpha + \delta P_\alpha\right) - \left(\rho_\alpha + P_\alpha\right)3\dot{\psi} + \cancel{\frac{\nabla^2}{a^2}\left[\delta q_\alpha + \left(\rho_\alpha + P_\alpha\right)\chi\right]} = Q_\alpha\phi + \delta Q_\alpha$$

- Valid at least for slow-roll. Expansion breaks in case of USR phase during slow-roll [Jackson et al. 2311.03281]

3. Isocurvature evolution

Long-wavelength evolution

- Total curvature perturbation: $\dot{\zeta} = -\frac{H}{\rho + P}\left(\delta P - \frac{\dot{P}}{\dot{\rho}}\delta\rho\right)$
- What about $\dot{\zeta}_\alpha$? → Compute derivative from definition + pert. continuity with heat transfer:

$$\dot{\zeta}_\alpha = \frac{3H^2}{\dot{\rho}_\alpha}\left(\delta P_\alpha - c_\alpha^2 \delta\rho_\alpha\right) - \frac{HQ_\alpha}{\dot{\rho}_\alpha}\left[\frac{\delta Q_\alpha}{Q_\alpha} + \phi + H^{-1}\dot{\psi} + \frac{\delta\rho_\alpha}{\dot{\rho}_\alpha}\left(\frac{\dot{H}}{H} - \frac{\dot{Q}_\alpha}{Q_\alpha}\right)\right]$$

- Find evolution of isocurvature:

$$\boxed{\dot{\mathcal{S}}_{\alpha\beta} = 3H\left(\frac{3H\delta P_{int,\alpha} - \delta Q_{int,\alpha}}{\dot{\rho}_\alpha} - \frac{3H\delta P_{int,\beta} - \delta Q_{int,\beta}}{\dot{\rho}_\beta}\right) + \frac{1}{2\rho}\sum_\gamma \dot{\rho}_\gamma\left(\frac{Q_\alpha}{\dot{\rho}_\alpha}\mathcal{S}_{\alpha\gamma} - \frac{Q_\beta}{\dot{\rho}_\beta}\mathcal{S}_{\beta\gamma}\right)}$$

[Malik, Wands, Ungarelli 0211602]

3. Isocurvature evolution

A simpler formalism - Separate-universe picture

[Wands, Malik, Lyth, Liddle 0003278]
[Lyth, Wands 0306498]
[Lyth, Rodriguez 0504045]

- Background evolution: $\rho' = F(\rho)$
  - Autonomous! Solutions: $\rho(N) = \rho(\rho_0; N - N_0)$, $\rho(\rho_0; 0) = \rho_0$
- Consider second homogeneous path with $\rho_\alpha + \delta\rho_\alpha$
  - $\rho(N) + \delta\rho(N) \simeq \rho(\rho_0 + \delta\rho_0; N - N_0)$
  - Perturbation $\delta\rho(N)$ as solution to initial value problem

$$\frac{d}{dN}\delta\rho_\alpha = \sum_\beta \delta\rho_\beta \frac{\partial F_\alpha}{\partial \rho_\beta}(\rho(N)) \ , \qquad \delta\rho_\alpha(N_0) = \delta\rho_{0,\alpha}$$

- By expanding one finds:

$$\delta\rho_\alpha(N) = \sum_\beta \delta\rho_{0,\beta} \frac{\partial \rho_\alpha(N)}{\partial \rho_{0,\beta}}$$

3. Isocurvature evolution

A simpler formalism - Results

- Results can be connected to curvature perturbation!
- Normalize with ρ':

$$\boxed{\frac{d}{dN}\left(\frac{\delta\rho_\alpha}{\rho'_\alpha}\right) = \frac{1}{\rho'_\alpha}\sum_\beta \partial_\beta F_\alpha \left(\delta\rho_\beta - \frac{\delta\rho_\alpha}{\rho'_\alpha}\rho'_\beta\right)}$$

  - Equivalent to ★ !

- Curvature perturbation result:

$$\boxed{\zeta_\alpha(N) = \sum_\beta \frac{1}{\rho'_\alpha(N)}\frac{\partial\rho_\alpha(N)}{\partial\rho_{0,\beta}}\rho'_\beta(N_0)\ \zeta_\beta(N_0)}$$

  - Integrated solution - simply by varying initial condition of the background!

4. Back to the decaying curvaton

[Malik, Wands, Ungarelli 0211602]

Decaying curvaton example - Input

- Curvaton: starts with curvature perturbation
- Radiation dominates initial energy density

$$\boxed{\zeta_\gamma = 0 \quad \zeta_\sigma = \zeta_{\sigma,0} \quad \mathcal{S}_{\sigma\gamma} = 3\zeta_{\sigma,0}}$$

Initial conditions

$$\boxed{\begin{aligned} \rho'_\sigma &= -(3+\tilde{\Gamma})\rho_\sigma \\ \rho'_\gamma &= -4\rho_\gamma + \tilde{\Gamma}\rho_\sigma \end{aligned}}$$

Background

$$\tilde{\Gamma} = \Gamma/H, \quad \Gamma = \text{const}\,.$$

- Evolution equations:

$$\zeta_\sigma(N) = \frac{1}{\rho'_\sigma(N)}\left(\frac{\partial\rho_\sigma}{\partial\rho_{\sigma,0}}\rho'_\sigma(N_0)\zeta_\sigma(N_0) + \frac{\partial\rho_\sigma}{\partial\rho_{\gamma,0}}\rho'_\gamma(N_0)\zeta_\gamma(N_0)\right)$$

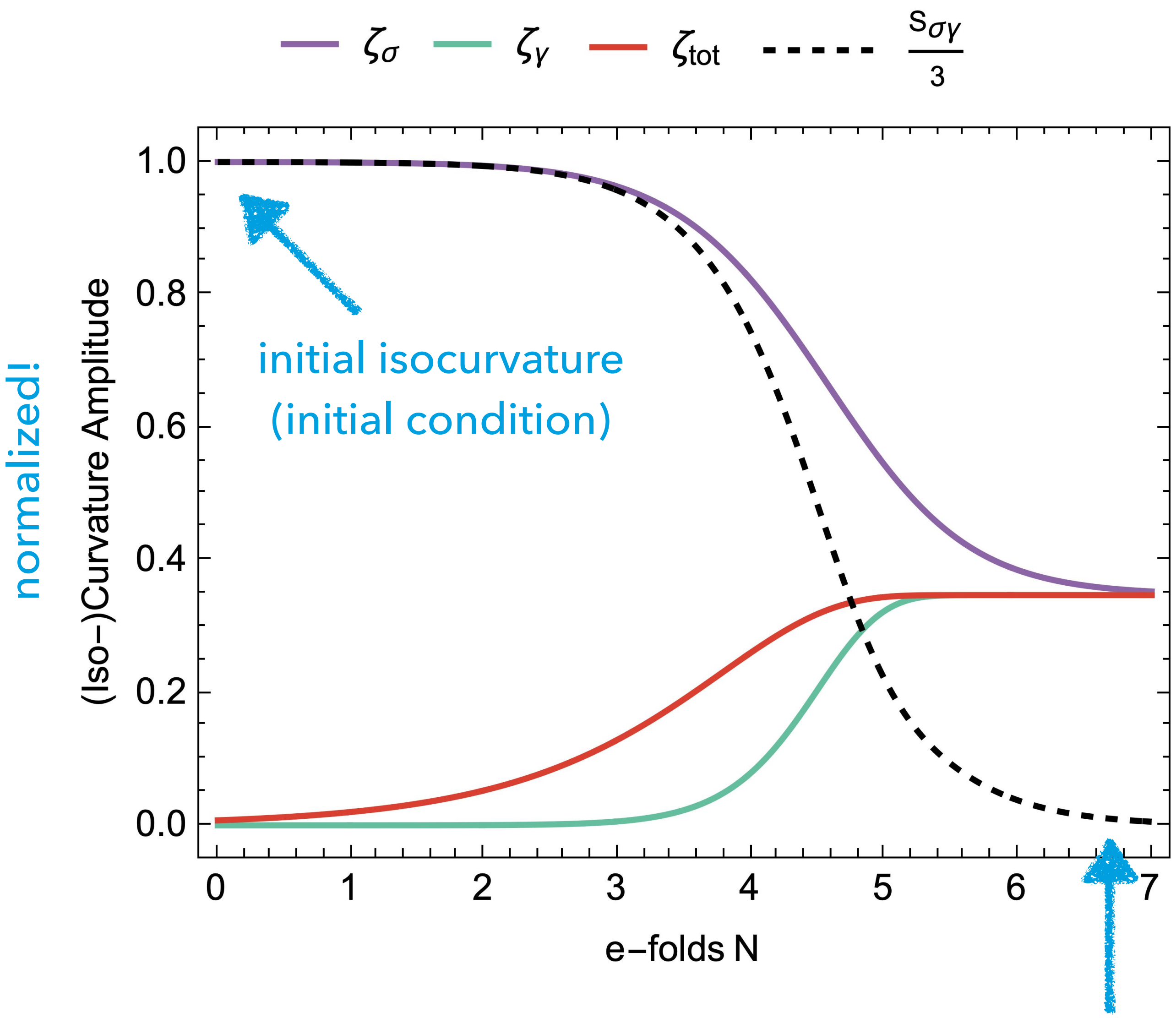
$$\zeta_\gamma(N) = \frac{1}{\rho'_\gamma(N)}\left(\frac{\partial\rho_\gamma}{\partial\rho_{\sigma,0}}\rho'_\sigma(N_0)\zeta_\sigma(N_0) + \frac{\partial\rho_\gamma}{\partial\rho_{\gamma,0}}\rho'_\gamma(N_0)\zeta_\gamma(N_0)\right)$$

4. Back to the decaying curvaton

[Malik, Wands, Ungarelli 0211602]

Decaying curvaton example - Results

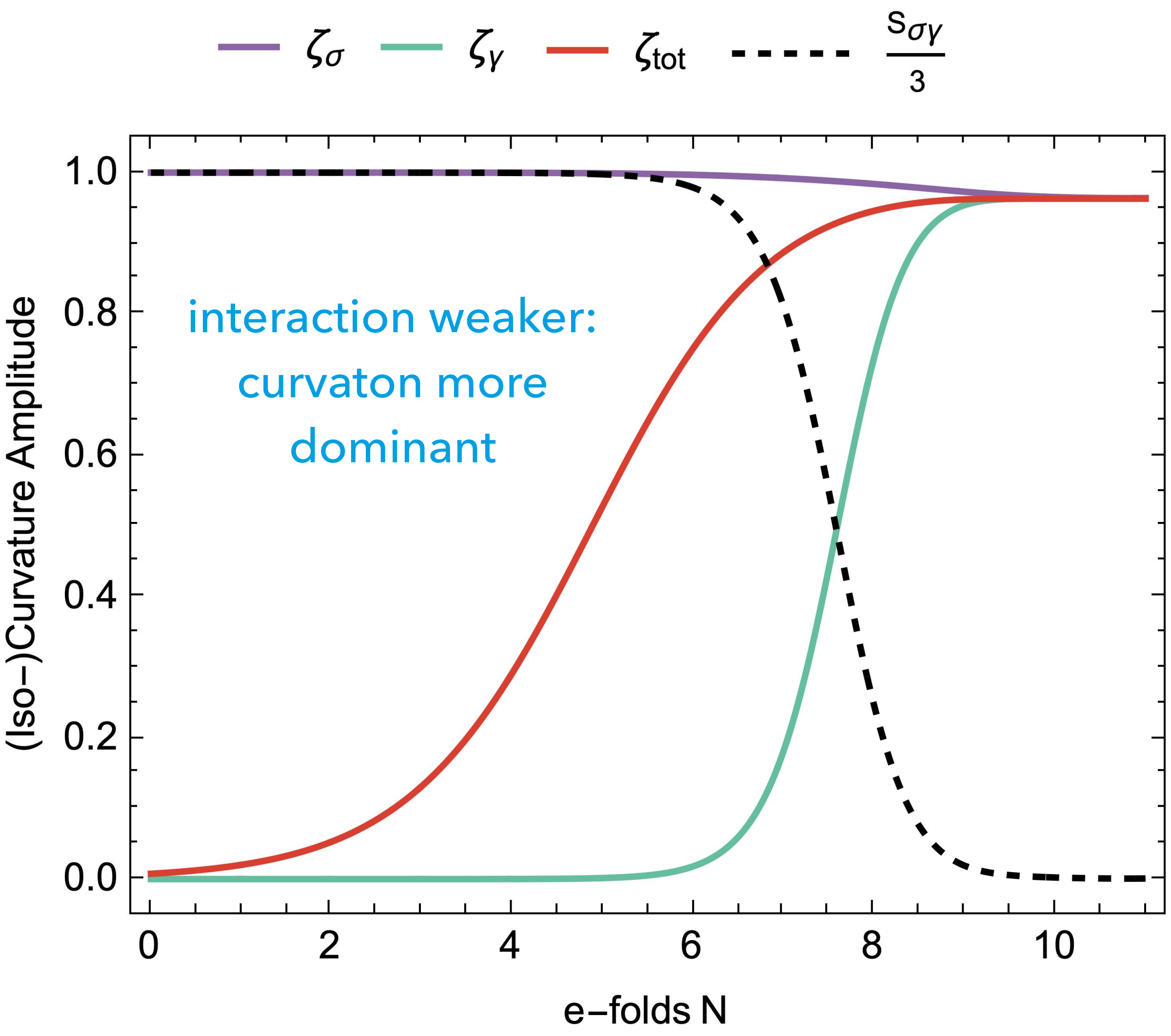
Numerically solving the background + computing curvature:



normalized!

initial isocurvature (initial condition)

zero final iso.



interaction weaker: curvaton more dominant

5. Isocurvature out of the dark

Advanced toy models - Input

- Two sectors Φ and Ψ - allow iso. in between
- Three radiation fluids:

$$\left.\begin{aligned}\rho_1'(N) &= -3(1+w_1)\rho_1 - Q(N)\\ \rho_2'(N) &= -3(1+w_2)\rho_2 + Q(N)\end{aligned}\right\}$$

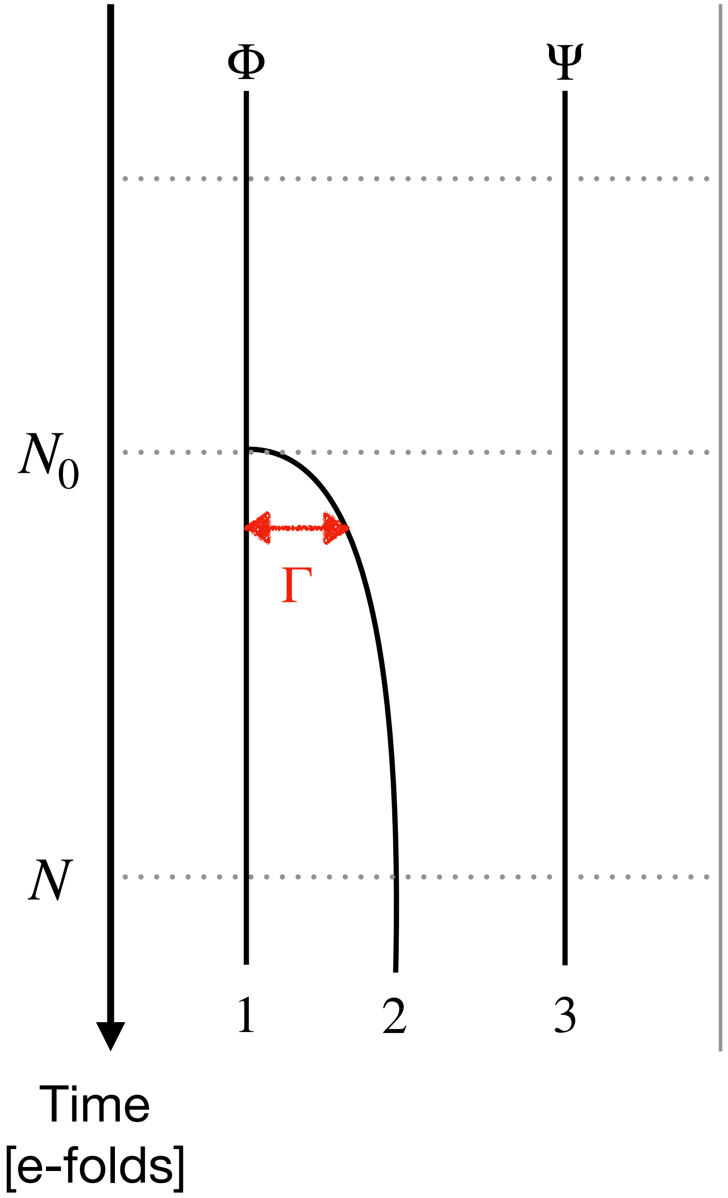
start with over-abundance in 1

$$\rho_3'(N) = -3(1+w_3)\rho_3$$

→ initially dominating background field

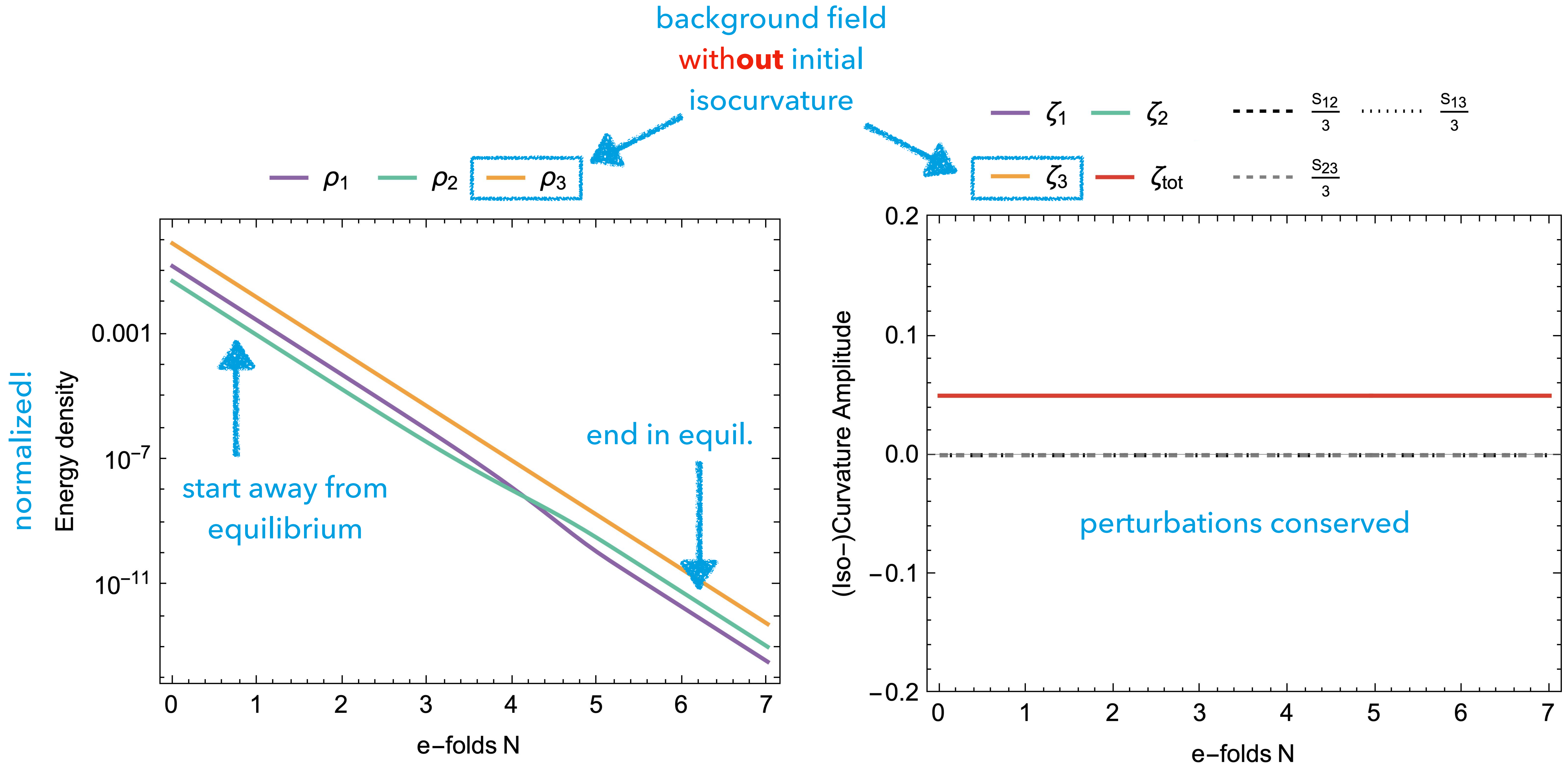
- Interaction:

$$Q(N) = \left(\underbrace{f_{eq,2}\rho_1(N) - f_{eq,1}}_{\text{artificial equilibrium}}\rho_2(N)\right)\cdot\tilde{\Gamma}(N)$$



5. Isocurvature out of the dark

Advanced toy models - Result for no initial isocurvature



background field **without** initial isocurvature

normalized!

start away from equilibrium

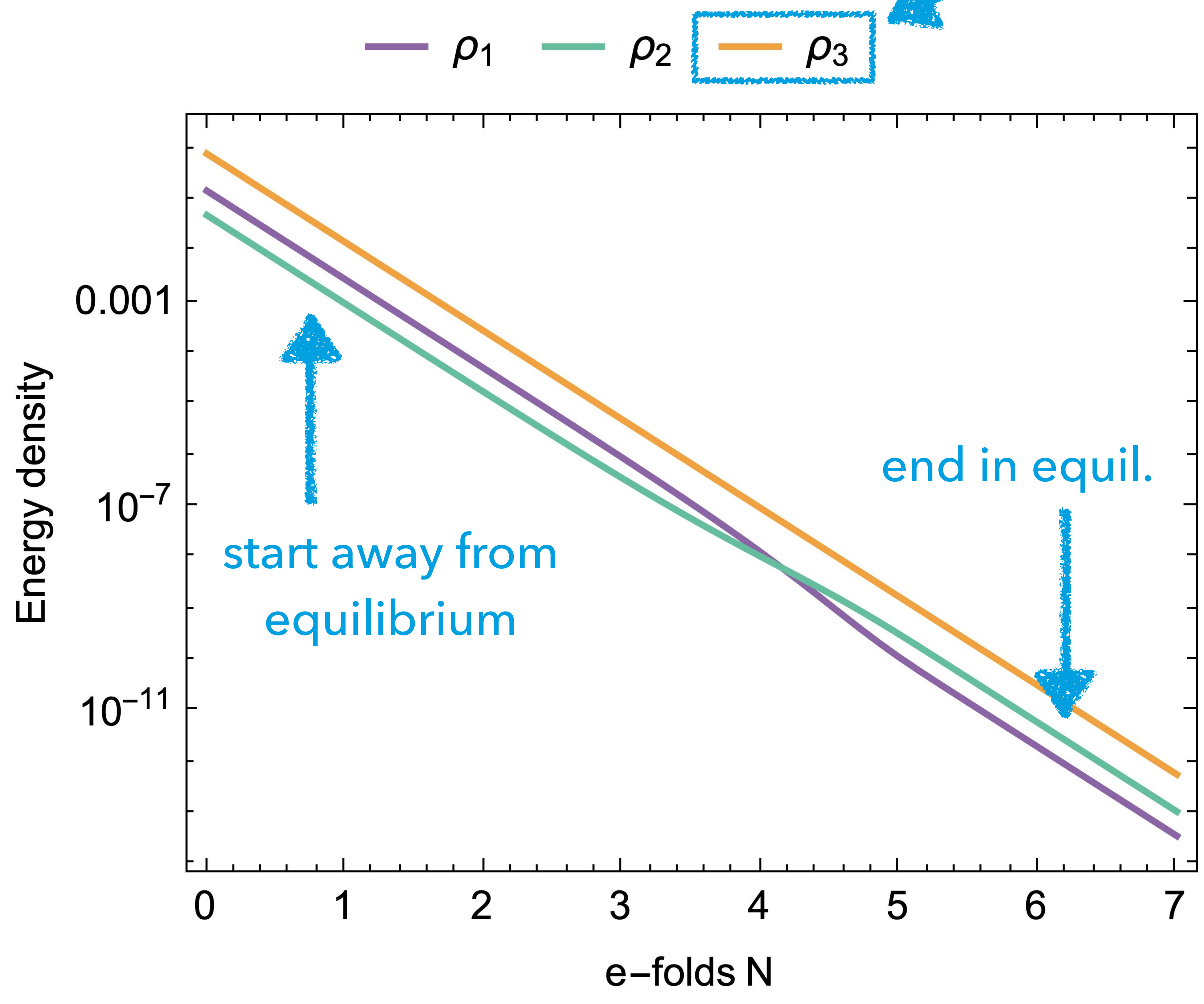
end in equil.

perturbations conserved

5. Isocurvature out of the dark

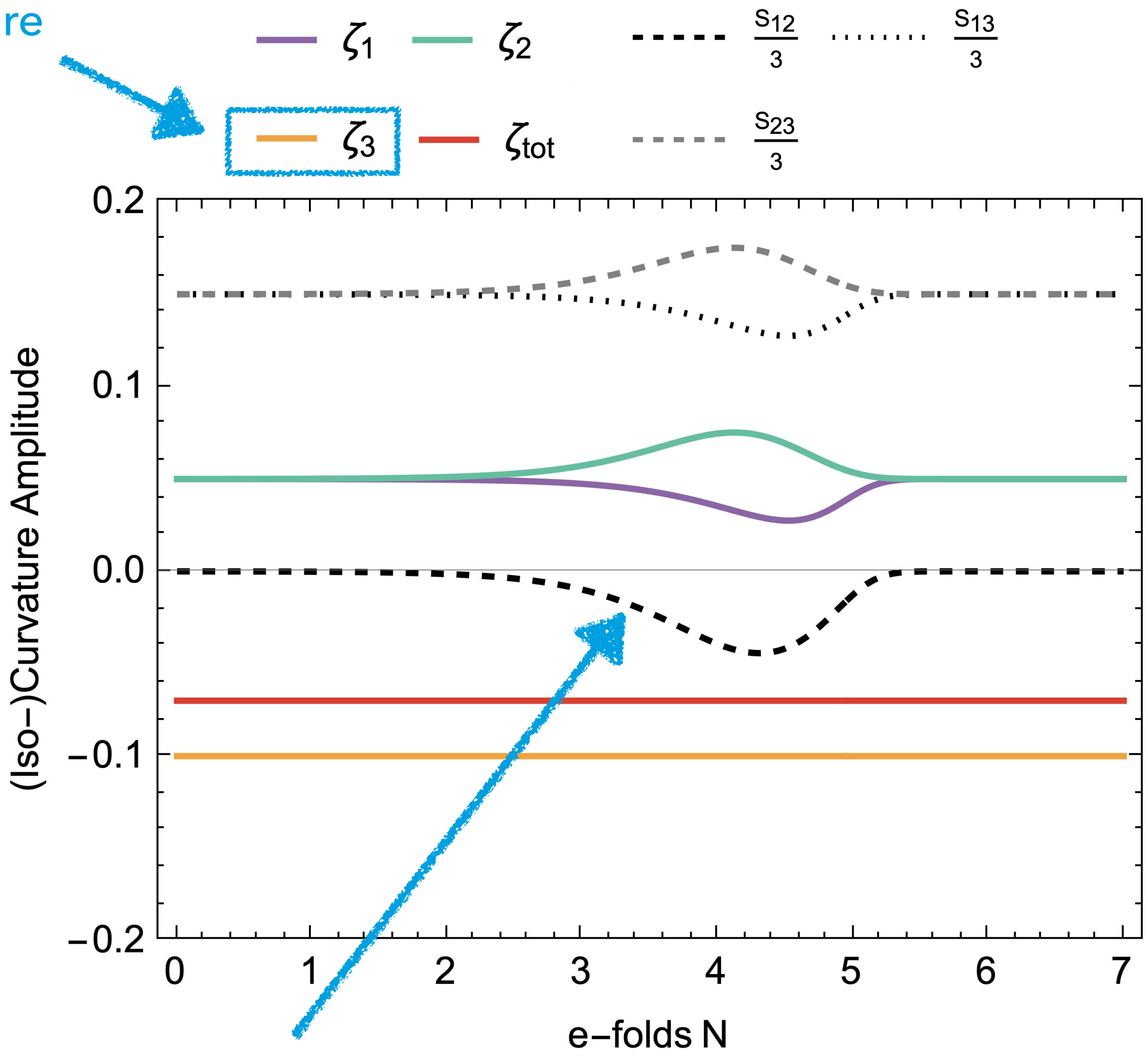
Advanced toy model - Result with initial isocurvature

background field **with** initial isocurvature



start away from equilibrium

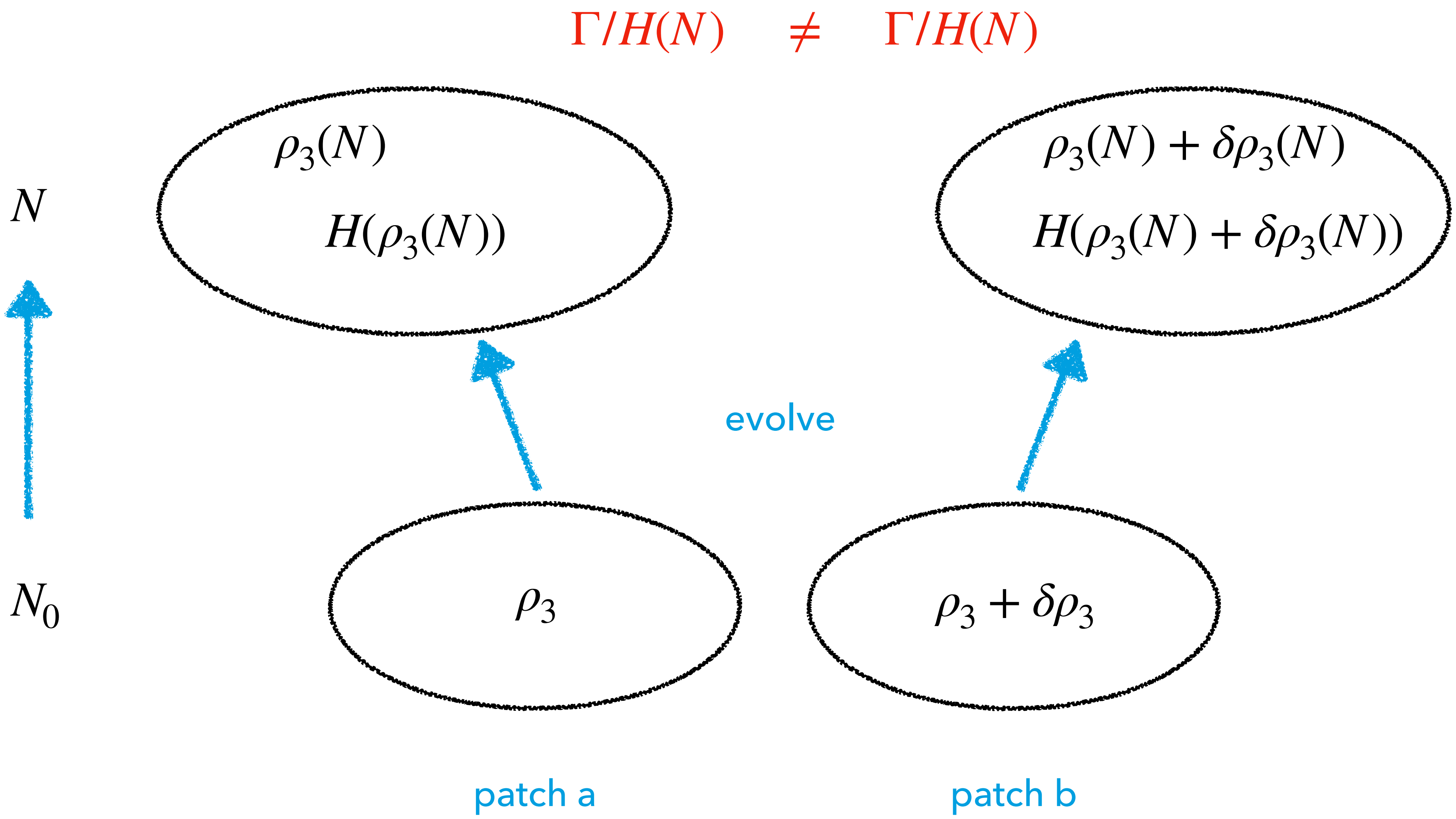
end in equil.



non-vanishing isocurvature during energy transfer

5. Isocurvature out of the dark

Advanced toy model - Schematic explanation



$\Gamma/H(N) \quad \neq \quad \Gamma/H(N)$

N

N_0

$\rho_3(N)$

$H(\rho_3(N))$

$\rho_3(N) + \delta\rho_3(N)$

$H(\rho_3(N) + \delta\rho_3(N))$

evolve

ρ_3

$\rho_3 + \delta\rho_3$

patch a

patch b

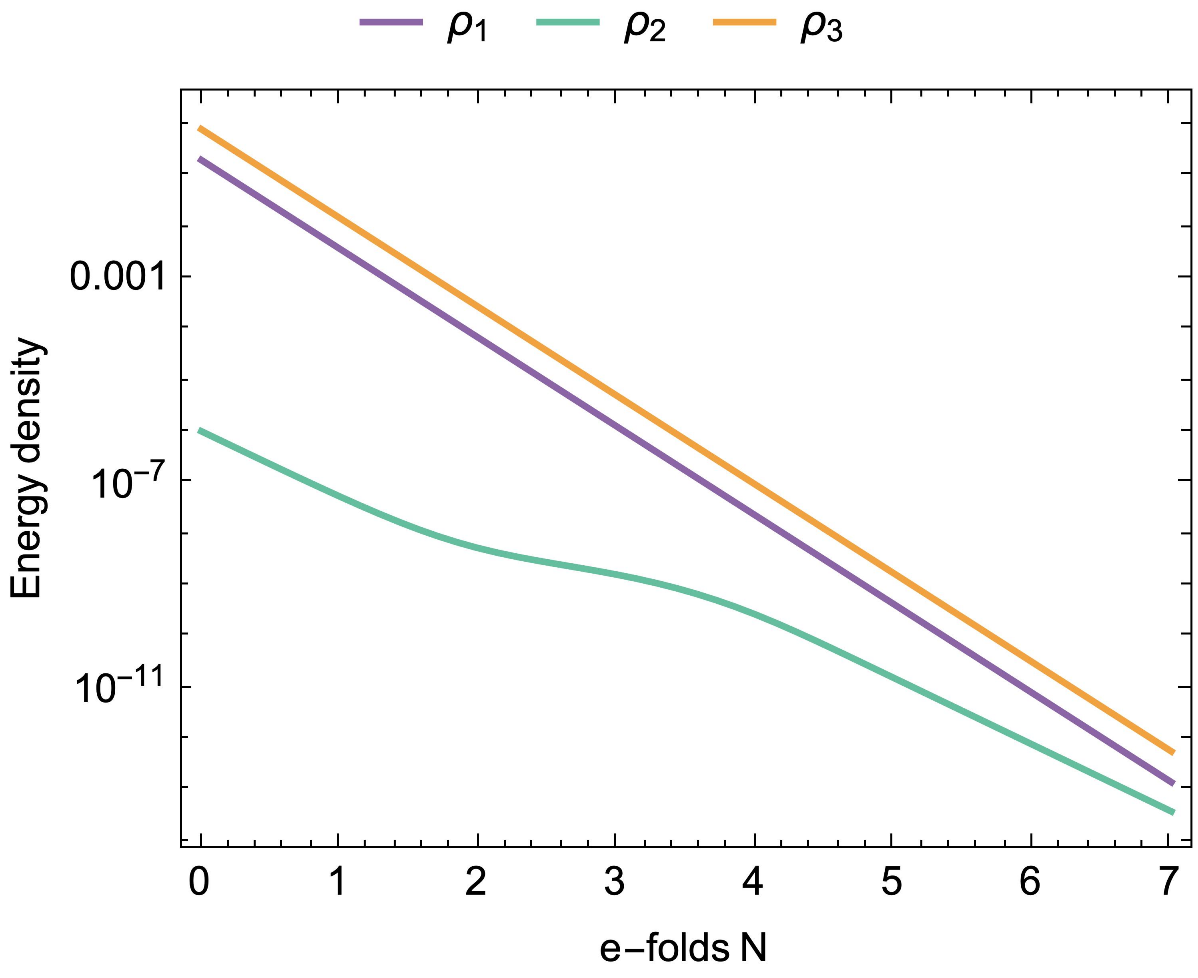
6. Isocurvature in freeze-in

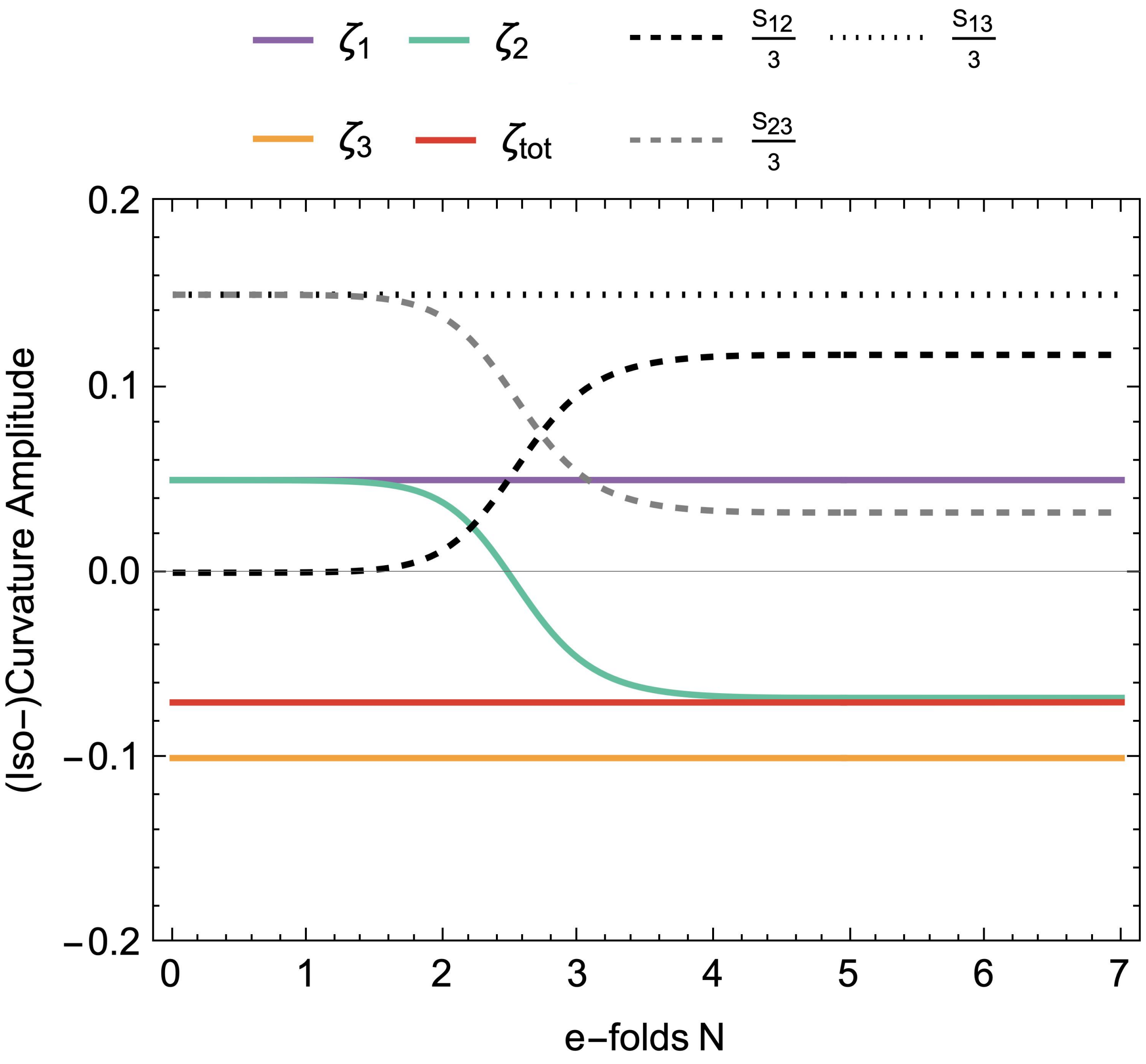
How could a freeze-in scenario look like? [PREVIEW]

- $\rho' + 3\rho \propto m^2 T K_1(m/T) \cdot \Gamma/H$

[Hall, Jedamzik, March-Russell, West 0911.1120v2]

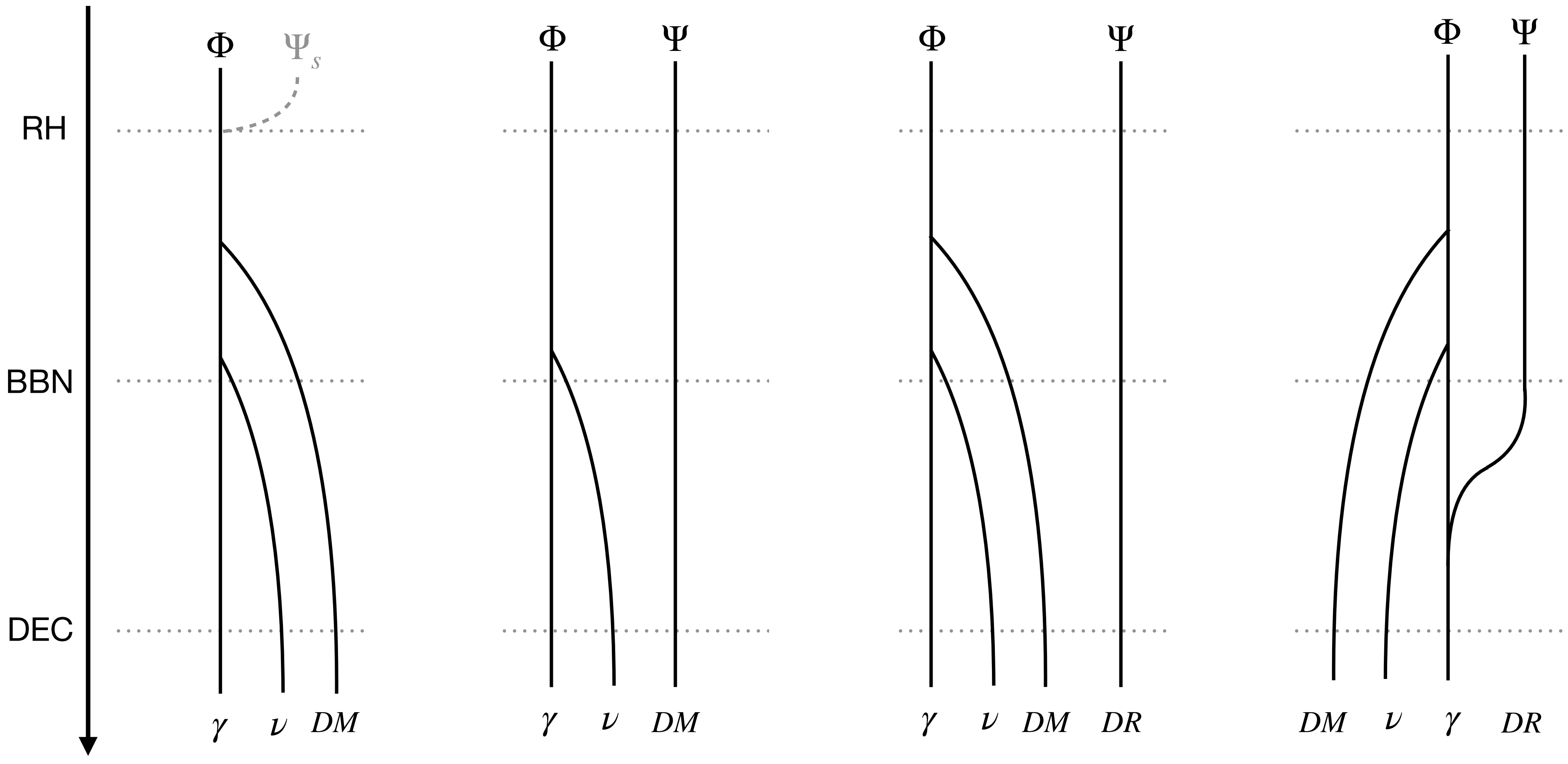
Work in progress!





7. What is neutrino isocurvature?

Three ways to „source" isocurvature



Adiabatic

DM-iso only

Effective ν-iso + iso from background

+ changing iso from DR decay to radiation

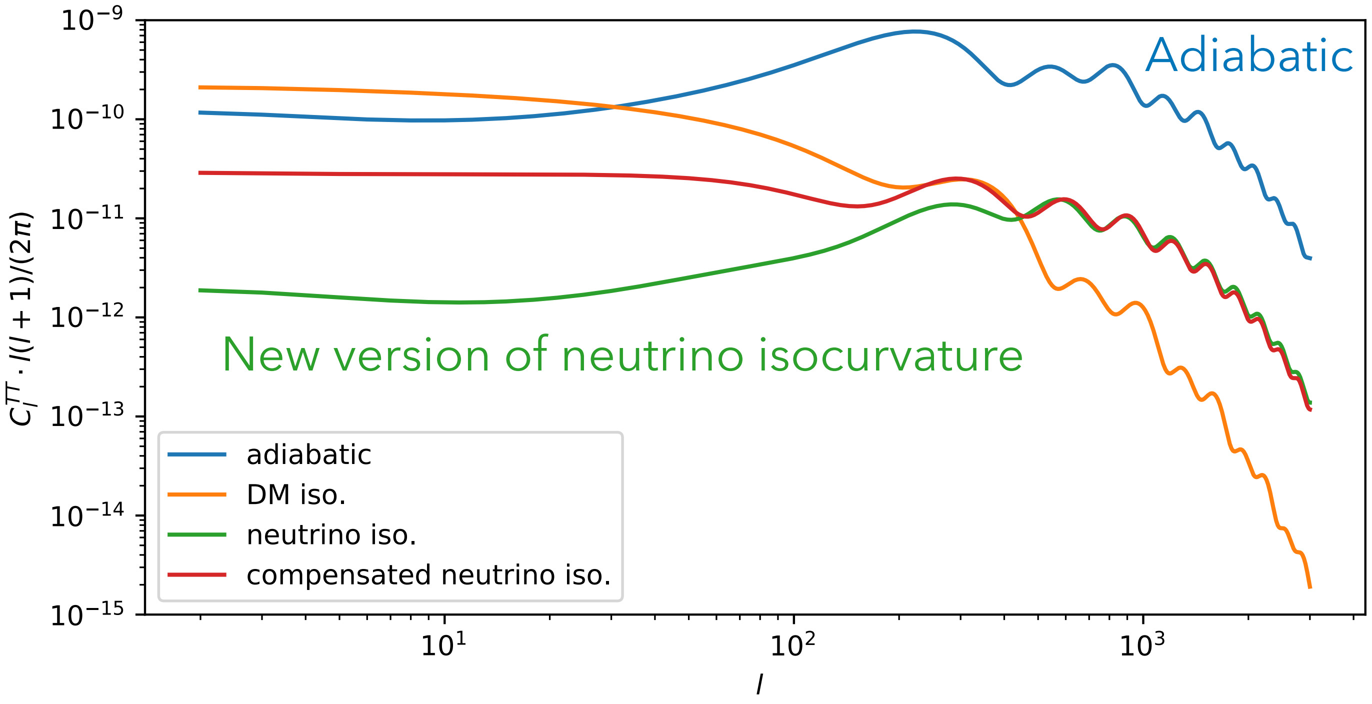
8. Outlook

A reworked version for CLASS

Solved Einstein-Boltzmann eqs. in orders of conformal time with adapted initial conditions (for species DM, neutrinos/FDR, IDR)

- Combine several isocurvature modes by mixing angle
  - $\mathscr{I} = \cos\phi \times \mathrm{DMI} + \sin\phi \times \mathrm{NID}$
  - $\mathscr{I} = \cos\theta \times \mathrm{IDRI} + \sin\theta\cos\varphi \times \mathrm{DMI} + \sin\theta\sin\varphi \times \mathrm{NID}$

TT-spectra from custom CLASS version



8. Outlook

The future

Work in progress:

To be released soon!

- Improving freeze-in model
- What bounds can be found from Planck constraints on isocurvature?
- Provide input for Boltzmann solver

Speculative Remarks:

- Isocurvature could become more interesting with next stage CMB experiments
- Probe multi-field scenarios, if isocurvature is observed (in small amounts)

Thank you for your attention!

Time for questions!