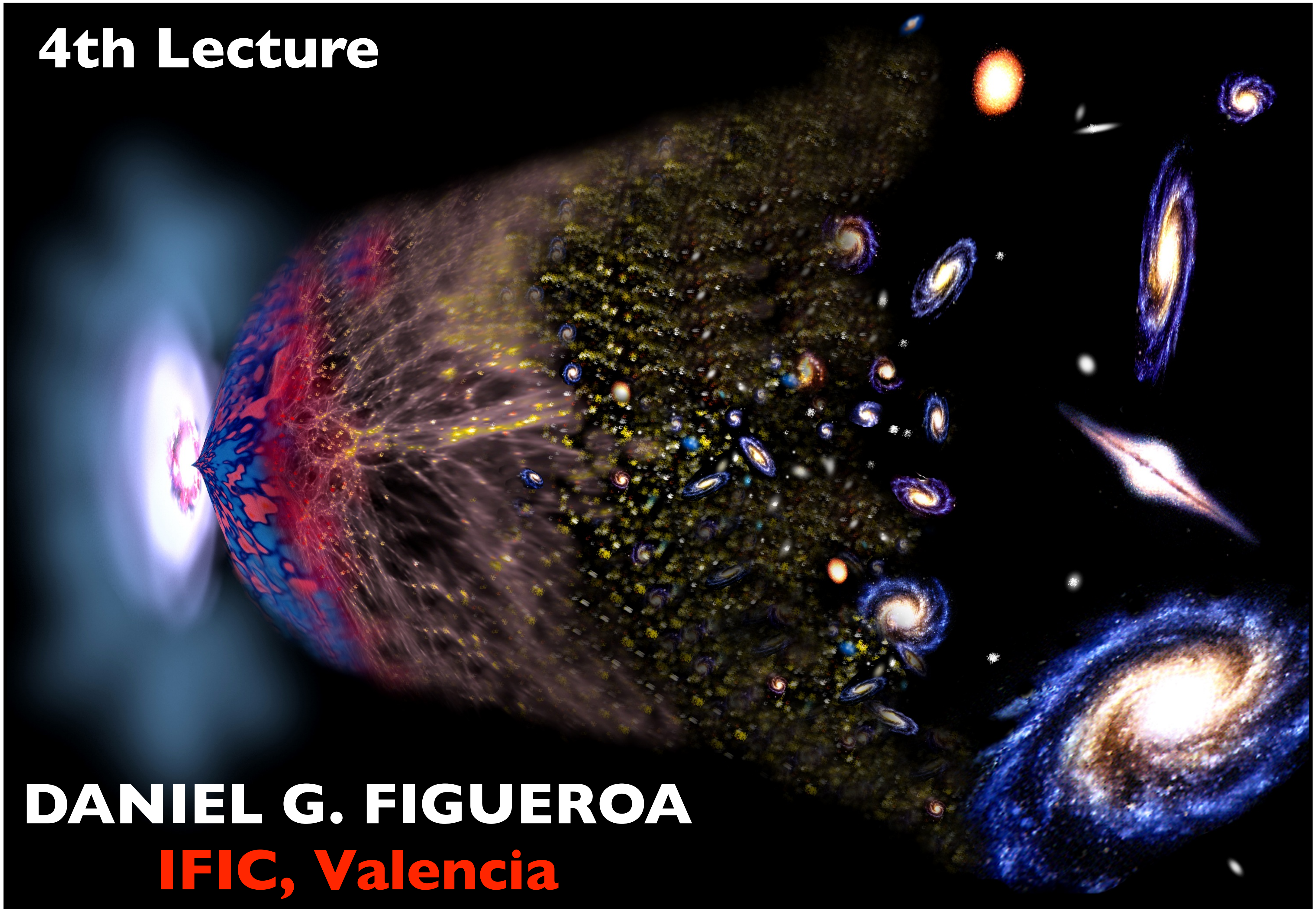
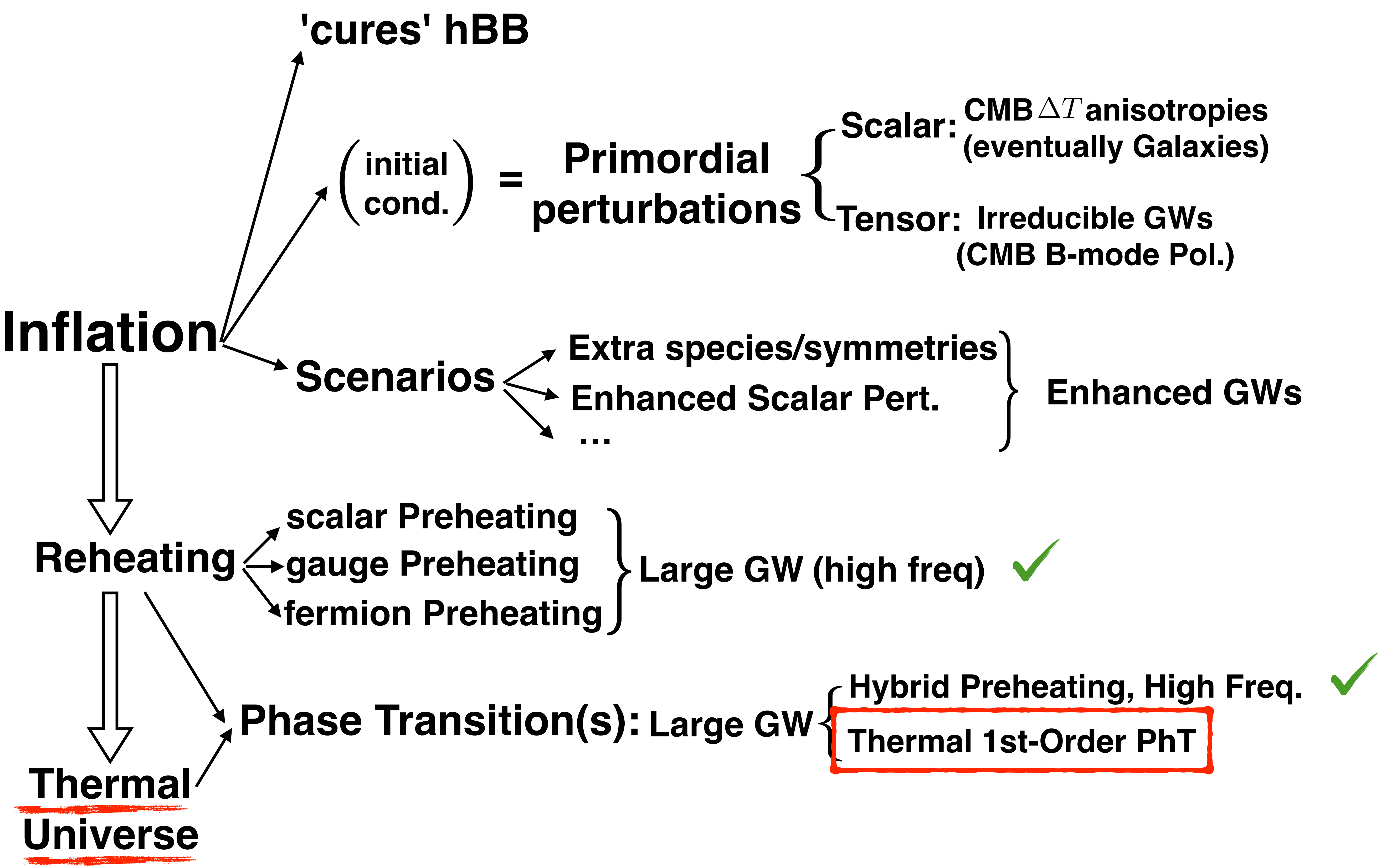
GRAVITATIONAL WAVE
— BACKGROUNDS —



ICTP's Summer School on Cosmology, 17-28 June 2024, Trieste, Italy

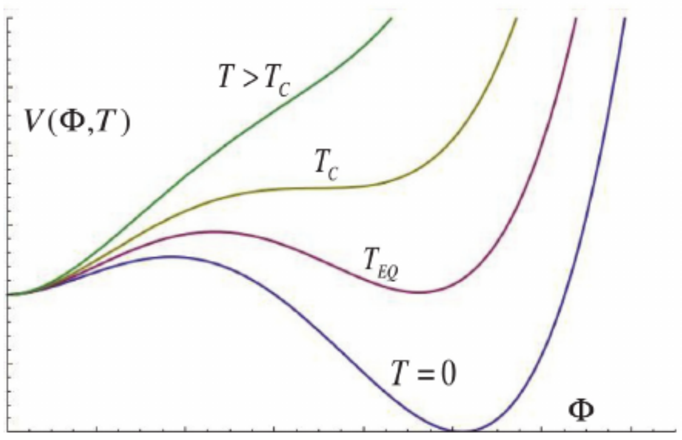
EARLY UNIVERSE



GWs from first order phase transitions

Universe expands, temperature decreases: phase transition triggered !

* Potential barrier separates
true and **false** vacua

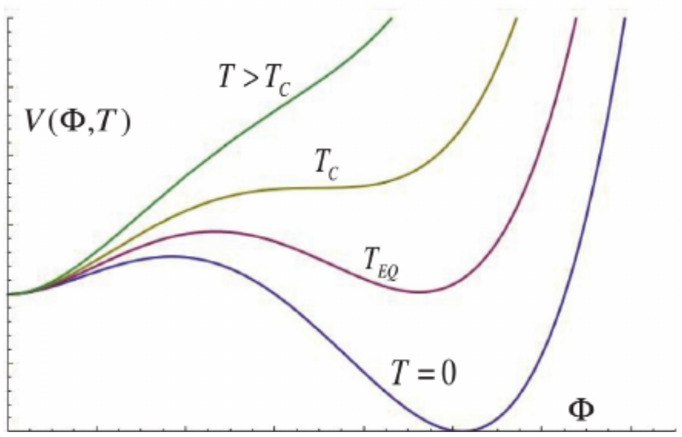


$$V(\Phi, T) = \begin{cases} \textbf{Bare potential} \\ + \\ \textbf{thermal corrections} \end{cases}$$

GWs from first order phase transitions

Universe expands, temperature decreases: phase transition triggered !

* Potential barrier separates **true** and **false** vacua



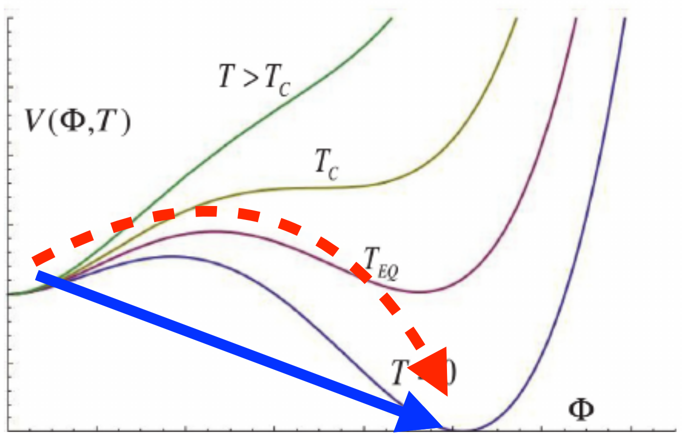
$$V(\Phi, T) = \begin{cases} \textbf{Bare potential} \\ + \\ \textbf{thermal corrections} \end{cases}$$

$V(\Phi, T) \to T(t) \downarrow\downarrow$

GWs from first order phase transitions

Universe expands, temperature decreases: phase transition triggered !

* Potential barrier separates
true and **false** vacua

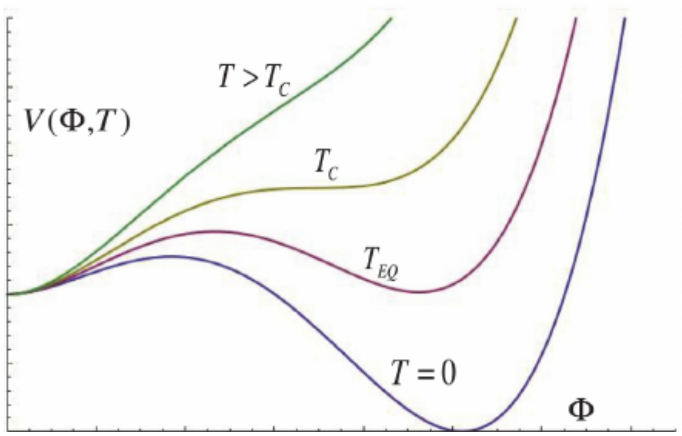


Thermal Fluctuations
or
Quantum Tunnelling

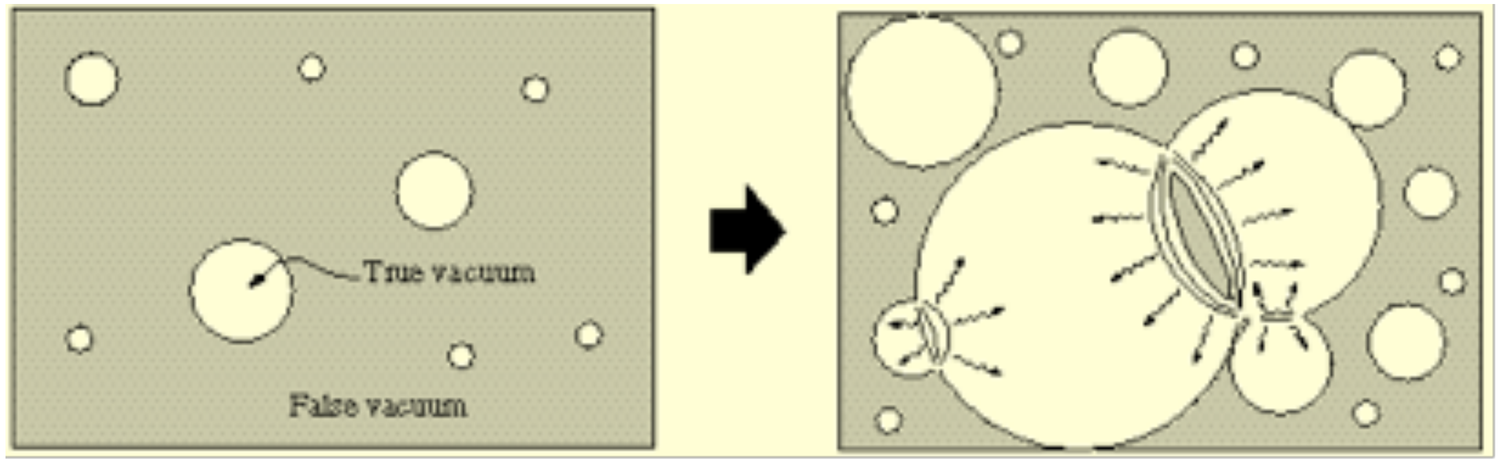
GWs from first order phase transitions

Universe expands, temperature decreases: phase transition triggered !

* Potential barrier separates **true** and **false** vacua



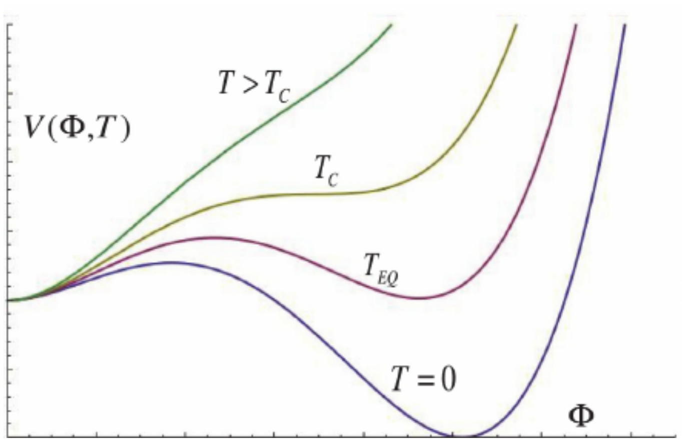
"bubble" formation



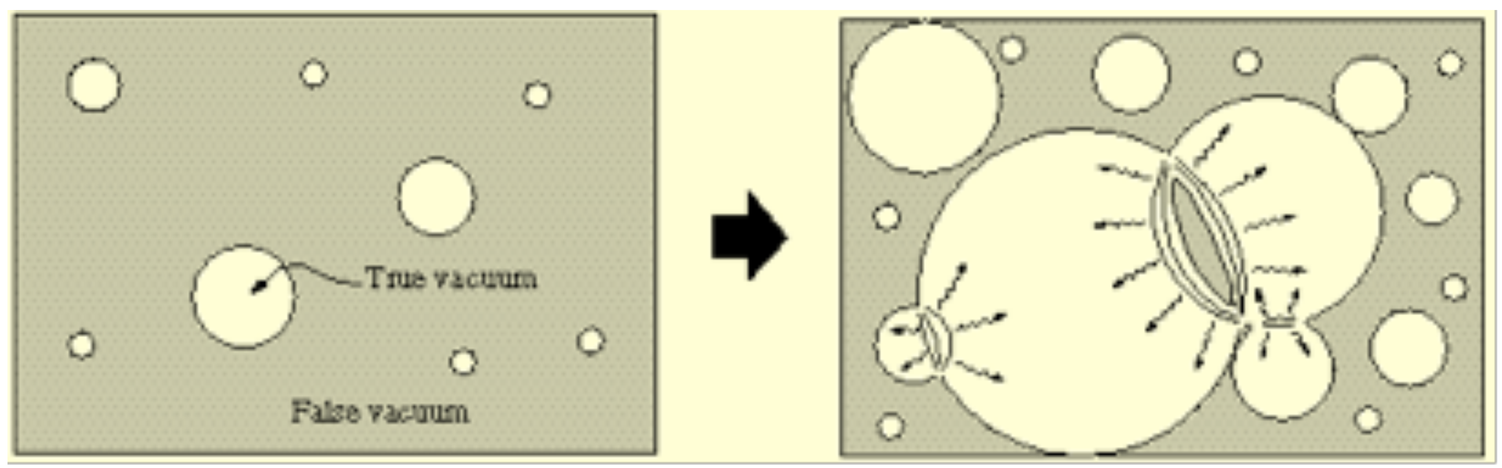
GWs from first order phase transitions

Universe expands, temperature decreases: phase transition triggered !

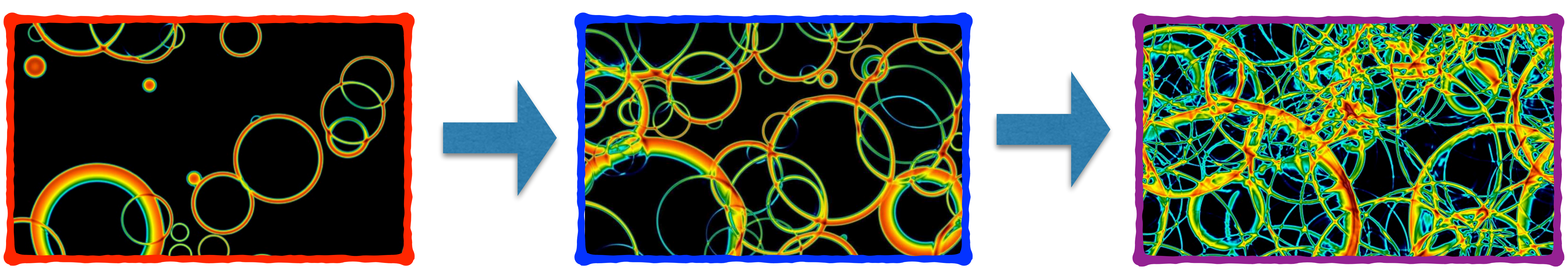
* Potential barrier separates **true** and **false** vacua



"bubble" formation



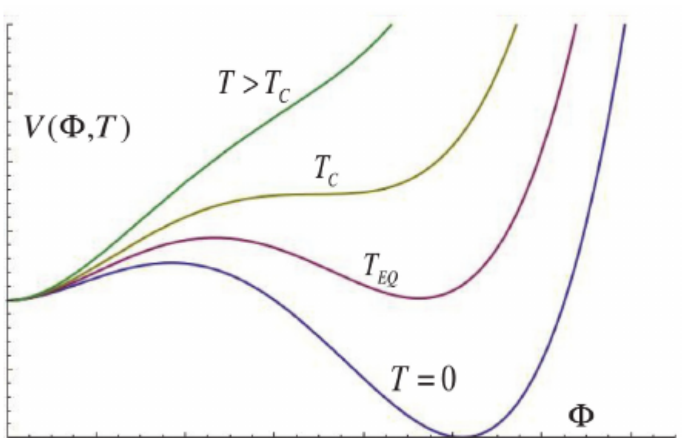
Real Simulation (Helsinki Group: **Hindmarsh, Rummukainen, Weir, ...**)



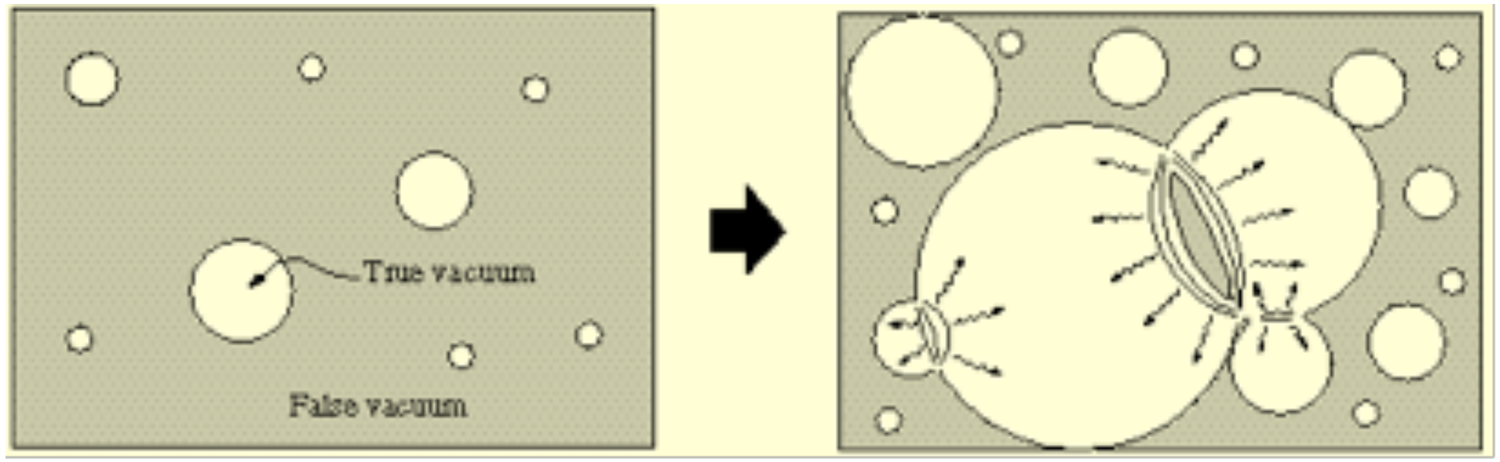
GWs from first order phase transitions

Universe expands, temperature decreases: phase transition triggered !

* Potential barrier separates **true** and **false** vacua



"bubble" formation



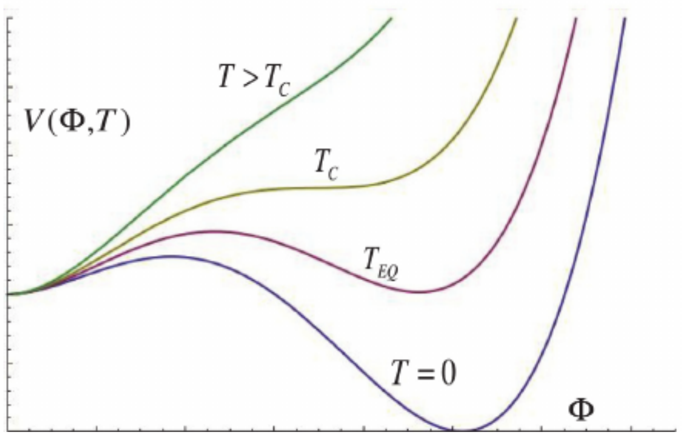
source: Π_{ij} tensor
anisotropic stress

?

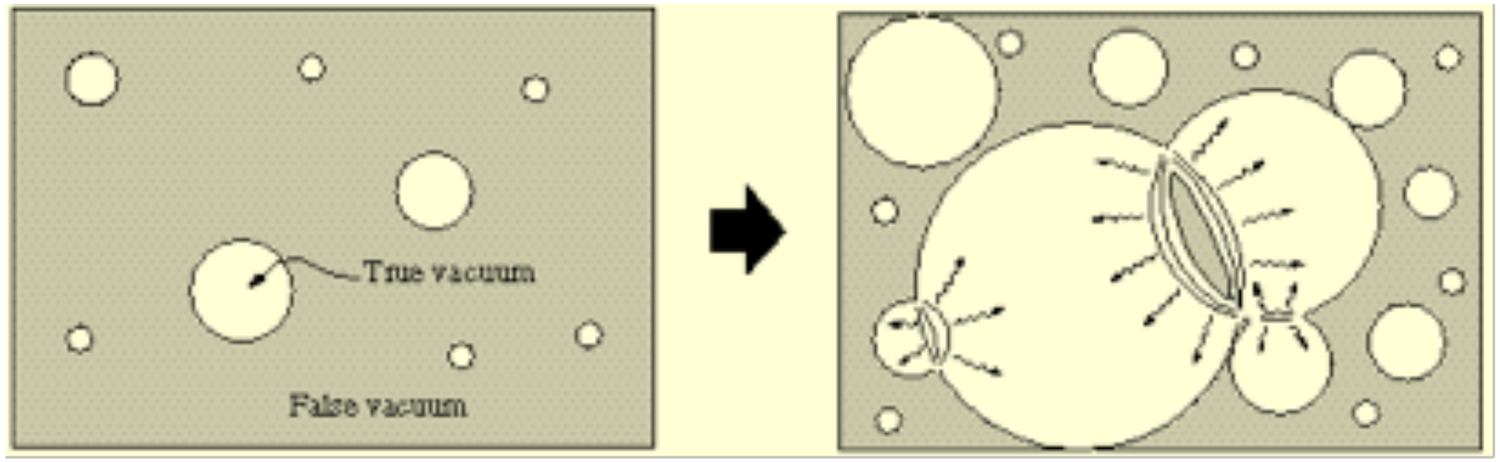
GWs from first order phase transitions

Universe expands, temperature decreases: phase transition triggered !

* Potential barrier separates **true** and **false** vacua



"bubble" formation



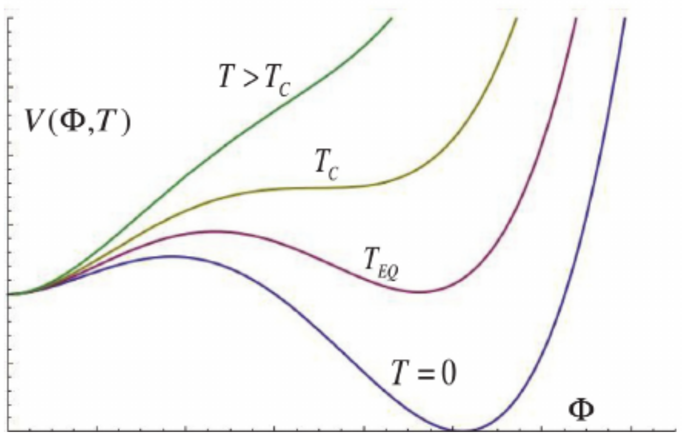
source: Π_{ij} tensor
anisotropic stress

- collisions of bubble walls

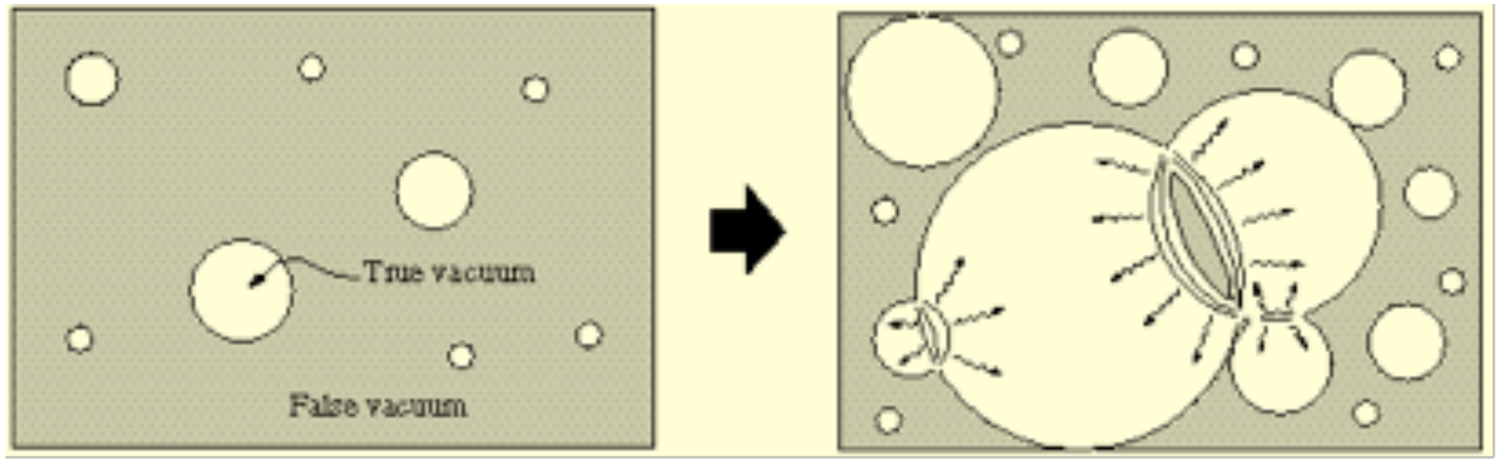
GWs from first order phase transitions

Universe expands, temperature decreases: phase transition triggered !

* Potential barrier separates **true** and **false** vacua



"bubble" formation



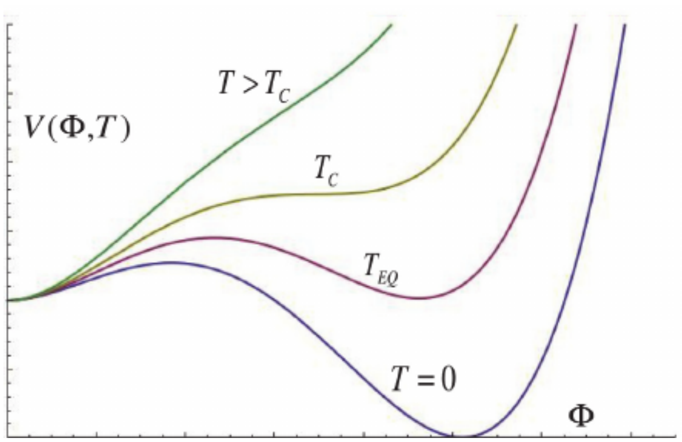
source: Π_{ij} tensor
anisotropic stress

- collisions of bubble walls
- sound waves and turbulence in the fluid

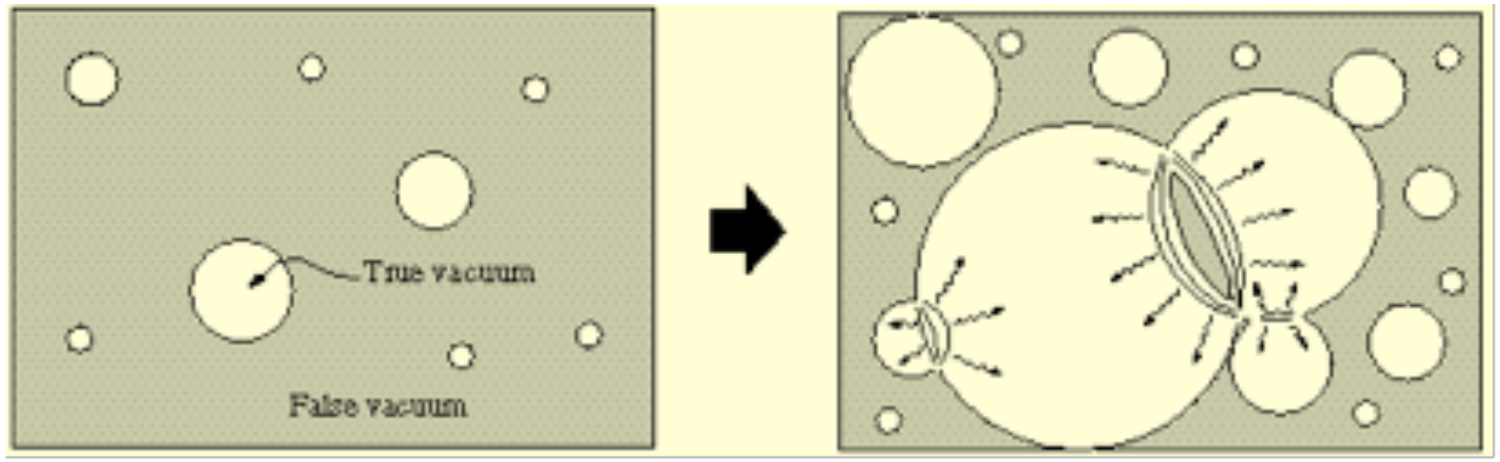
GWs from first order phase transitions

Universe expands, temperature decreases: phase transition triggered !

* Potential barrier separates **true** and **false** vacua



"bubble" formation



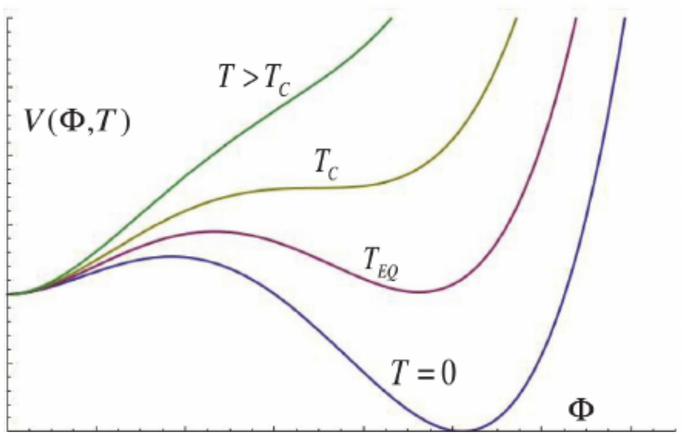
source: Π_{ij} tensor anisotropic stress

- collisions of bubble walls
- sound waves and turbulence in the fluid
- primordial magnetic fields (MHD)

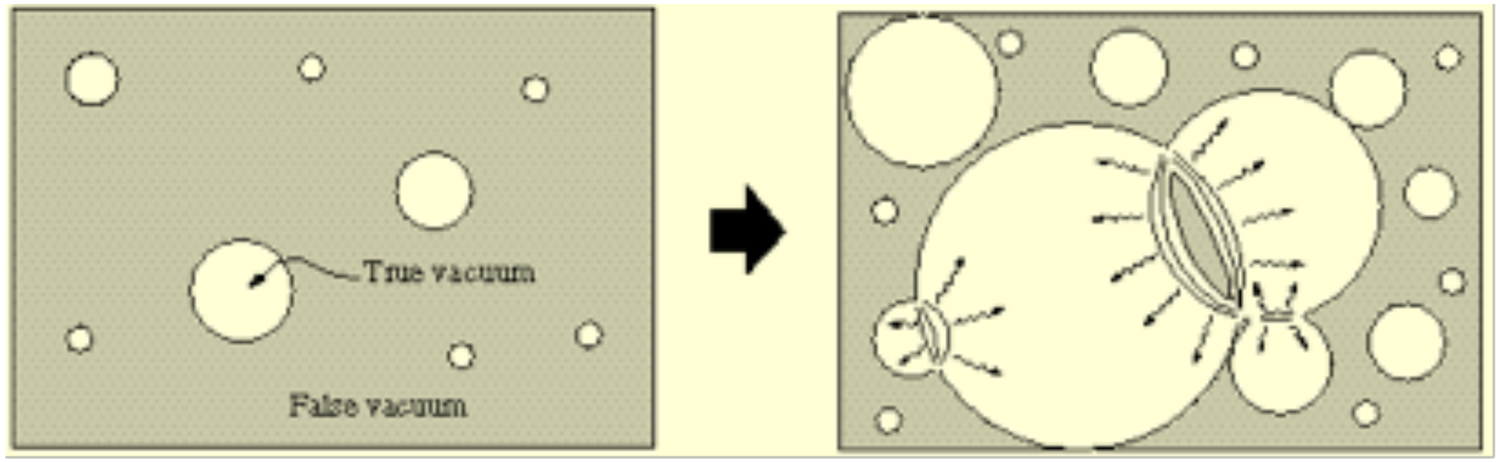
GWs from first order phase transitions

Universe expands, temperature decreases: phase transition triggered !

* Potential barrier separates **true** and **false** vacua



"bubble" formation



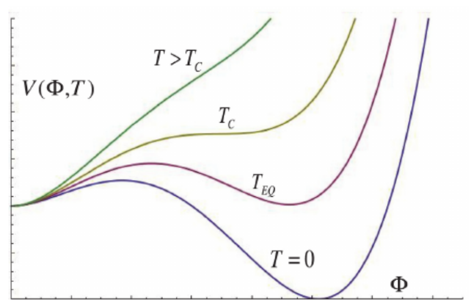
source: Π_{ij} tensor
anisotropic stress

- **collisions** of bubble walls
- **sound waves** and turbulence in the fluid
- primordial magnetic fields (**MHD**)

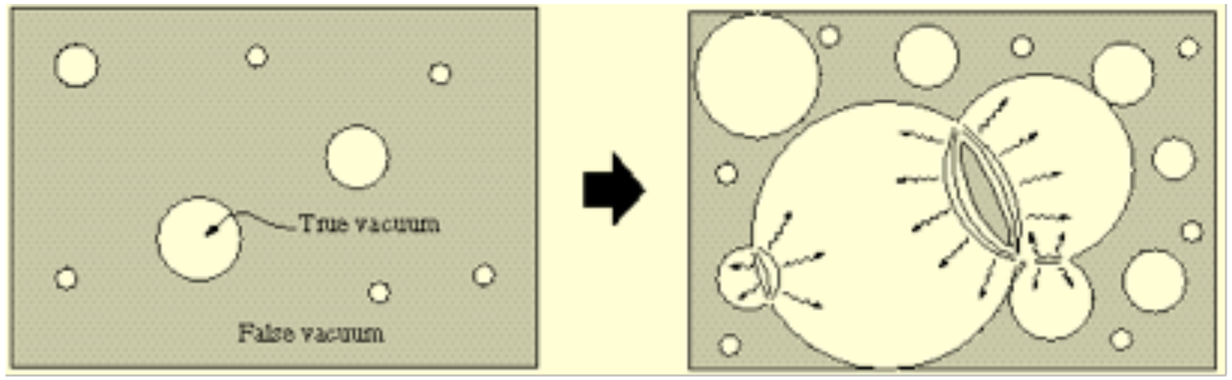
GWs from first order phase transitions

Universe expands, temperature decreases: phase transition triggered !

* Potential barrier separates **true** and **false** vacua



"bubble" formation



Technically

source: Π_{ij} tensor
anisotropic stress

$$\Pi_{ij} \sim \partial_i\phi\,\partial_j\phi$$

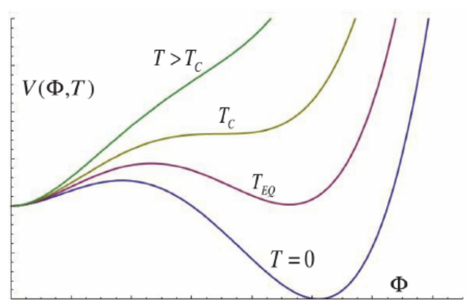
$$\Pi_{ij} \sim \gamma^2(\rho+p)\,v_i v_j$$ **+ Scalar-Fluid coupling**

$$\Pi_{ij} \sim \frac{(E^2+B^2)}{3} - E^iE^j - B^iB^j$$ **+ MHD**

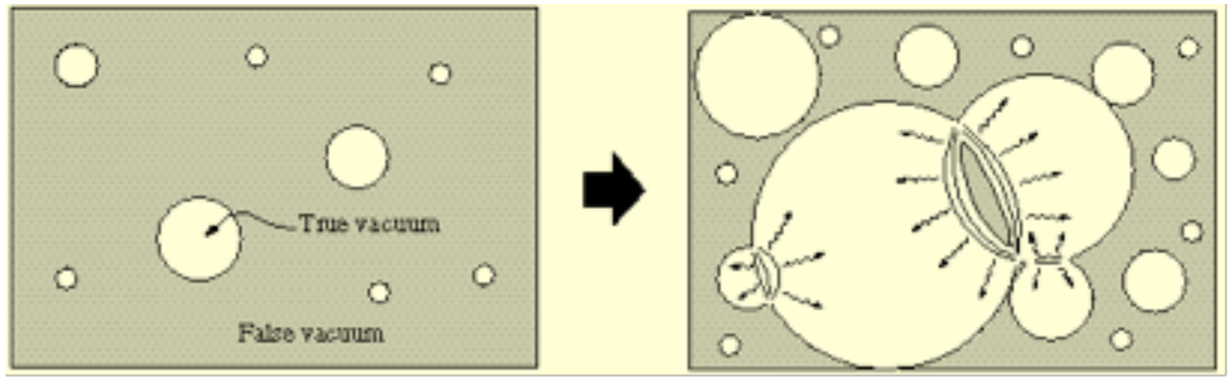
GWs from first order phase transitions

Universe expands, temperature decreases: phase transition triggered !

* Potential barrier separates **true** and **false** vacua



"bubble" formation



Technically

source: Π_{ij} tensor
anisotropic stress

$$\Pi_{ij} \sim \partial_i\phi\,\partial_j\phi$$

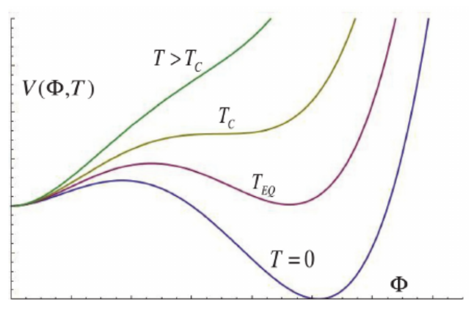
$$\Pi_{ij} \sim \gamma^2(\rho+p)\,v_i v_j$$ **+ Scalar-Fluid coupling**

$$\Pi_{ij} \sim \frac{(E^2+B^2)}{3} - E^iE^j - B^iB^j$$ **+ MHD**

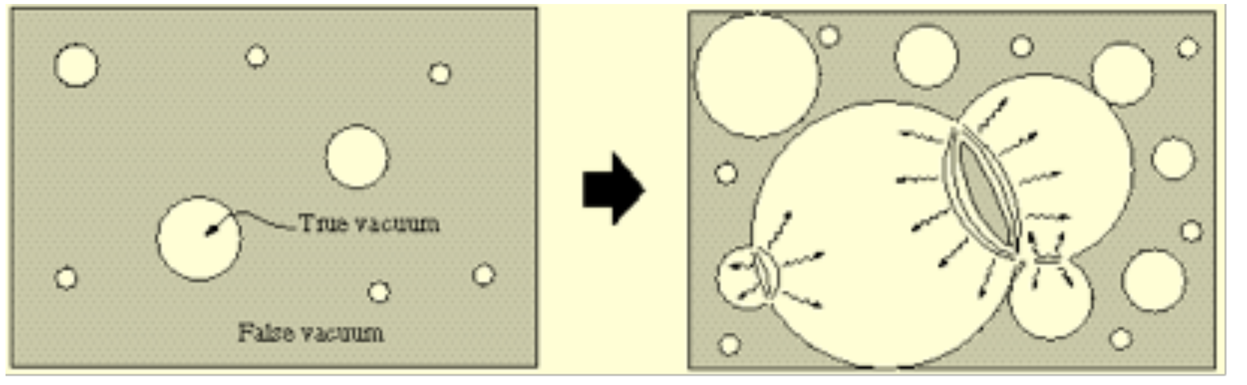
GWs from first order phase transitions

Universe expands, temperature decreases: phase transition triggered !

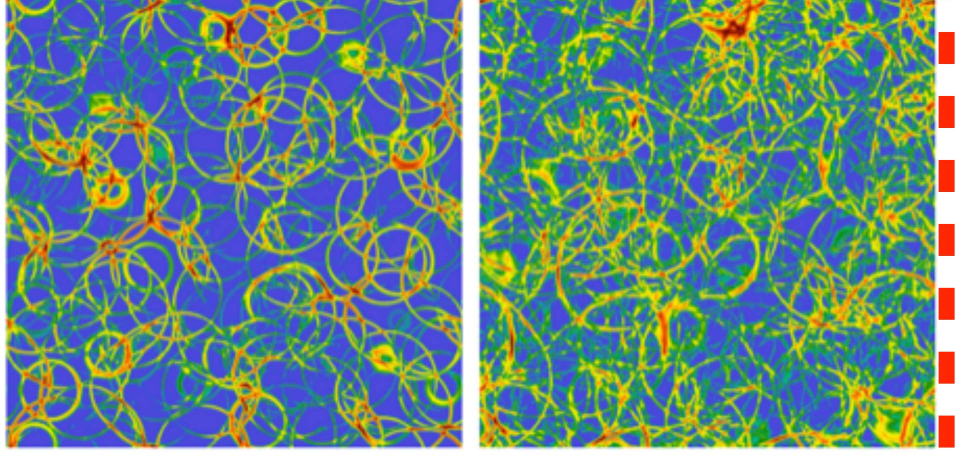
* Potential barrier separates
true and **false** vacua



"bubble" formation



Sounds waves (relativistic fluids) + Magneto-hydrodynamic (MHD) effects + Turbulence

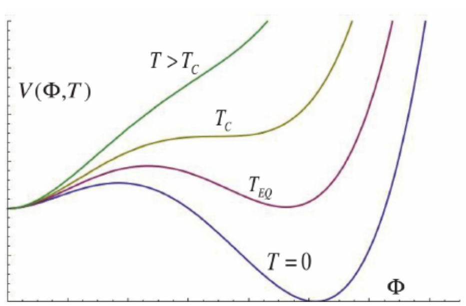


M. Hindmarsh *et al*,
Phys.Rev.D 92 (2015) 12, 123009;
Arxiv: 1504.03291
(Series of papers 2012-2023)

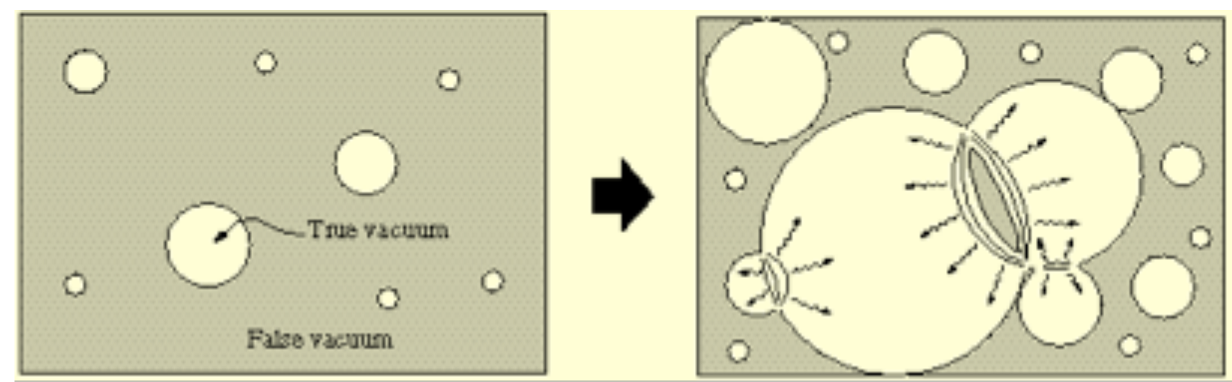
GWs from first order phase transitions

Universe expands, temperature decreases: phase transition triggered !

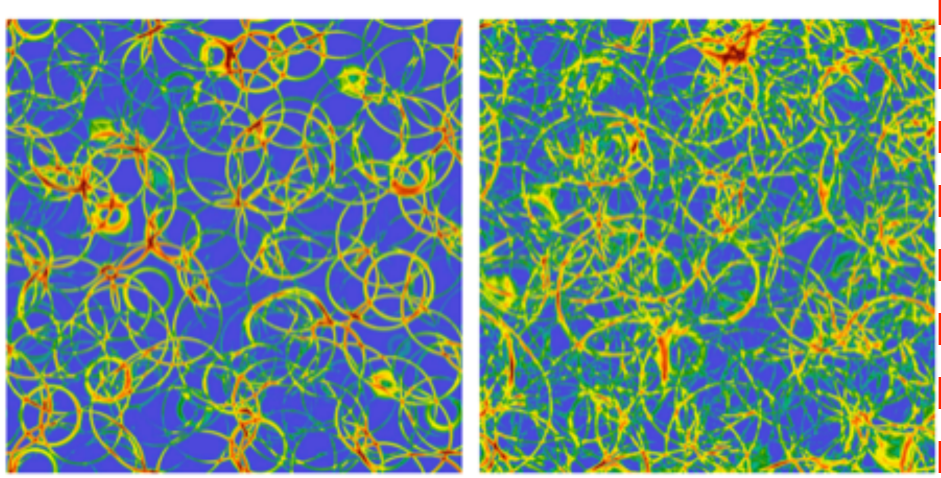
* Potential barrier separates
true and **false** vacua



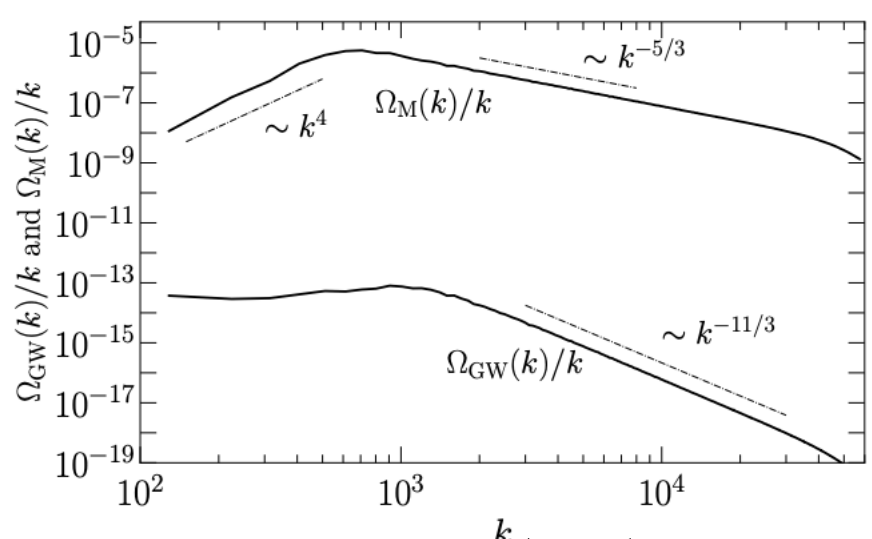
"bubble" formation



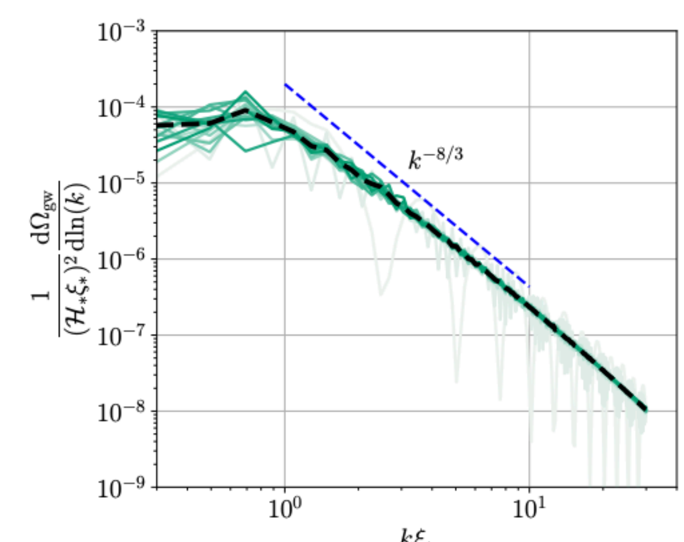
Sounds waves (relativistic fluids) + **Magneto-hydrodynamic (MHD) effects** + **Turbulence**



M. Hindmarsh *et al*,
Phys.Rev.D 92 (2015) 12, 123009;
Arxiv: 1504.03291
(Series of papers 2012-2023)



A. Roper Pol *et al*,
Phys.Rev.D 102 (2020) 8, 083512
Arxiv: 1903.08585



P. Auclair *et al*,
JCAP 09 (2022) 029
Arxiv: 2205.02588

GWs from first order phase transitions

* **GW causal source**: cannot 'operate' beyond the **horizon**

$$f_* = \frac{H(T_*)}{\epsilon_*}$$

$$\epsilon_* \leq 1$$

parameter characterising source

GWs from first order phase transitions

* **GW causal source**: cannot 'operate' beyond the **horizon**

$$f_* = \frac{H(T_*)}{\epsilon_*}$$

$$\epsilon_* \leq 1$$

parameter characterising source

@ Today

@ Emission time

Hubble rate ↕ temperature

$$f_c = f_* \frac{a_*}{a_0} = \frac{2 \cdot 10^{-5}}{\epsilon_*} \frac{T_*}{1\,\mathrm{TeV}}\,\mathrm{Hz}$$

GWs from first order phase transitions

* **GW causal source**: cannot 'operate' beyond the **horizon**

$$f_* = \frac{H(T_*)}{\epsilon_*}$$

$\epsilon_* \leq 1$ parameter characterising source

@ Today

@ Emission time

Hubble rate ↕ temperature

$$f_c = f_* \frac{a_*}{a_0} = \frac{2\cdot 10^{-5}}{\epsilon_*}\frac{T_*}{1\,\mathrm{TeV}}\,\mathrm{Hz} \qquad \simeq \mathrm{mHz}$$

for $\epsilon_* \simeq 10^{-2}$ $T_* \simeq 1\,\mathrm{TeV}$

GWs from first order phase transitions

* **GW causal source**: cannot 'operate' beyond the **horizon**

$$f_* = \frac{H(T_*)}{\epsilon_*}$$

$$\epsilon_* \leq 1$$ parameter characterising source

@ Today

@ Emission time

Hubble rate ↕ temperature

$$f_c = f_* \frac{a_*}{a_0} = \frac{2 \cdot 10^{-5}}{\epsilon_*} \frac{T_*}{1\,\mathrm{TeV}}\,\mathrm{Hz} \qquad \simeq \mathrm{mHz}$$

LISA Freq !

for $\epsilon_* \simeq 10^{-2}$ $\quad T_* \simeq 1\,\mathrm{TeV}$ **~ EW scale !**

What is ϵ in 1st Order PhT's?

$$f_c = f_* \frac{a_*}{a_0} = \frac{2 \cdot 10^{-5}}{\epsilon_*} \frac{T_*}{1\,\mathrm{TeV}}\,\mathrm{Hz}$$

What is ϵ in 1st Order PhT's?

$$f_c = f_* \frac{a_*}{a_0} = \frac{2 \cdot 10^{-5}}{\epsilon_*} \frac{T_*}{1\,\mathrm{TeV}}\,\mathrm{Hz}$$

GW generation <—> bubbles properties

What is ϵ in 1st Order PhT's?

$$f_c = f_* \frac{a_*}{a_0} = \frac{2 \cdot 10^{-5}}{\epsilon_*} \frac{T_*}{1\,\text{TeV}}\,\text{Hz}$$

GW generation <—> bubbles properties

β^{-1} : duration of PhT

$v_b \leq 1$: speed of bubble walls

$\longrightarrow R_* = v_b\,\beta^{-1}$ size of bubbles at collision

What is ϵ in 1st Order PhT's?

$$f_c = f_* \frac{a_*}{a_0} = \frac{2 \cdot 10^{-5}}{\epsilon_*} \frac{T_*}{1\,\mathrm{TeV}}\,\mathrm{Hz}$$

GW generation <—> bubbles properties

β^{-1} : duration of PhT

$v_b \leq 1$: speed of bubble walls

→ $R_* = v_b\, \beta^{-1}$ size of bubbles at collision

$$\epsilon \simeq \frac{H_*}{\beta}\,,\ H_*\, R_*$$

BUBBLE COLLISION

SOUND WAVES AND MDH TURBULENCE

Parameters determining the GW spectrum

Freq. (today)

$$f_c = f_* \frac{a_*}{a_0} = \frac{2 \cdot 10^{-5}}{\epsilon_*} \frac{T_*}{1\,\mathrm{TeV}}\,\mathrm{Hz}$$

$$\epsilon \simeq \frac{H_*}{\beta}\,, \quad H_*\,R_*$$

Parameter List

(not independent)

$$\frac{\beta}{H_*}\,, \quad v_b\,, \quad T_*$$

Parameters determining the GW spectrum

Freq. (today)

$$f_c = f_* \frac{a_*}{a_0} = \frac{2\cdot 10^{-5}}{\epsilon_*}\frac{T_*}{1\,\mathrm{TeV}}\,\mathrm{Hz}$$

$$\epsilon \simeq \frac{H_*}{\beta}\,,\quad H_*\,R_*$$

Parameter List

(not independent)

$$\frac{\beta}{H_*}\,,\quad v_b\,,\quad T_*$$

Amplitude (today)

$$\Omega_{\mathrm{GW}} \sim \Omega_{\mathrm{rad}}\,\epsilon_*^2\left(\frac{\rho_{\mathrm{s}}^*}{\rho_{\mathrm{tot}}^*}\right)^2$$

$$\frac{\rho_{\mathrm{s}}^*}{\rho_{\mathrm{tot}}^*} = \frac{\kappa\,\alpha}{1+\alpha}$$

$$\alpha = \frac{\rho_{\mathrm{vac}}}{\rho_{\mathrm{rad}}^*}$$

$$\kappa = \frac{\rho_{\mathrm{kin}}}{\rho_{\mathrm{vac}}}$$

Parameters determining the GW spectrum

Freq. (today)

$$f_c = f_* \frac{a_*}{a_0} = \frac{2 \cdot 10^{-5}}{\epsilon_*} \frac{T_*}{1\,\mathrm{TeV}}\,\mathrm{Hz}$$

$$\epsilon \simeq \frac{H_*}{\beta}\,, \quad H_*\,R_*$$

Parameter List (not independent)

$$\frac{\beta}{H_*}\,, \quad v_b\,, \quad T_*$$

Amplitude (today)

$$\Omega_{\mathrm{GW}} \sim \underbrace{\Omega_{\mathrm{rad}}\,\epsilon_*^2 \left(\frac{\rho_s^*}{\rho_{\mathrm{tot}}^*}\right)^2}_{\sim 10^{-4} \times (10^{-2})^2 \times 10^{-X}}$$

$$\alpha = \frac{\rho_{\mathrm{vac}}}{\rho_{\mathrm{rad}}^*}$$

$$\kappa = \frac{\rho_{\mathrm{kin}}}{\rho_{\mathrm{vac}}}$$

Parameters determining the GW spectrum

Freq. (today)

$$f_c = f_* \frac{a_*}{a_0} = \frac{2 \cdot 10^{-5}}{\epsilon_*} \frac{T_*}{1\,\mathrm{TeV}}\,\mathrm{Hz}$$

$$\epsilon \simeq \frac{H_*}{\beta}\,, \quad H_*\,R_*$$

Parameter List
(not independent)

$$\frac{\beta}{H_*}\,, \quad v_b\,, \quad T_*$$

Amplitude (today)

$$\Omega_{\rm GW} \sim \underbrace{\Omega_{\rm rad}\,\epsilon_*^2 \left(\frac{\rho_{\rm s}^*}{\rho_{\rm tot}^*}\right)^2}_{\sim 10^{-8} \times 10^{-X}}$$

$$\alpha = \frac{\rho_{\rm vac}}{\rho_{\rm rad}^*}$$

$$\kappa = \frac{\rho_{\rm kin}}{\rho_{\rm vac}}$$

Parameters determining the GW spectrum

Freq. (today)

$$f_c = f_* \frac{a_*}{a_0} = \frac{2 \cdot 10^{-5}}{\epsilon_*} \frac{T_*}{1\,\mathrm{TeV}}\,\mathrm{Hz}$$

$$\epsilon \simeq \frac{H_*}{\beta}\,, \quad H_*\,R_*$$

Parameter List
(not independent)

$$\frac{\beta}{H_*}\,, \quad v_b\,, \quad T_*$$

Amplitude (today)

$$\Omega_{\mathrm{GW}} \sim \underbrace{\Omega_{\mathrm{rad}}\,\epsilon_*^2 \left(\frac{\rho_{\mathrm{s}}^*}{\rho_{\mathrm{tot}}^*}\right)^2}_{\sim 10^{-10} - 10^{-13}}$$

(Plausible values)

$$\alpha = \frac{\rho_{\mathrm{vac}}}{\rho_{\mathrm{rad}}^*}$$

$$\kappa = \frac{\rho_{\mathrm{kin}}}{\rho_{\mathrm{vac}}}$$

Parameters determining the GW spectrum

Freq. (today)

$$f_c = f_* \frac{a_*}{a_0} = \frac{2 \cdot 10^{-5}}{\epsilon_*} \frac{T_*}{1\,\mathrm{TeV}}\,\mathrm{Hz}$$

$$\epsilon \simeq \frac{H_*}{\beta}\,, \quad H_*\,R_*$$

Parameter List
(not independent)

$$\frac{\beta}{H_*}\,, \quad v_b\,, \quad T_*$$

Amplitude (today)

Ω_{GW} ...

not most general !

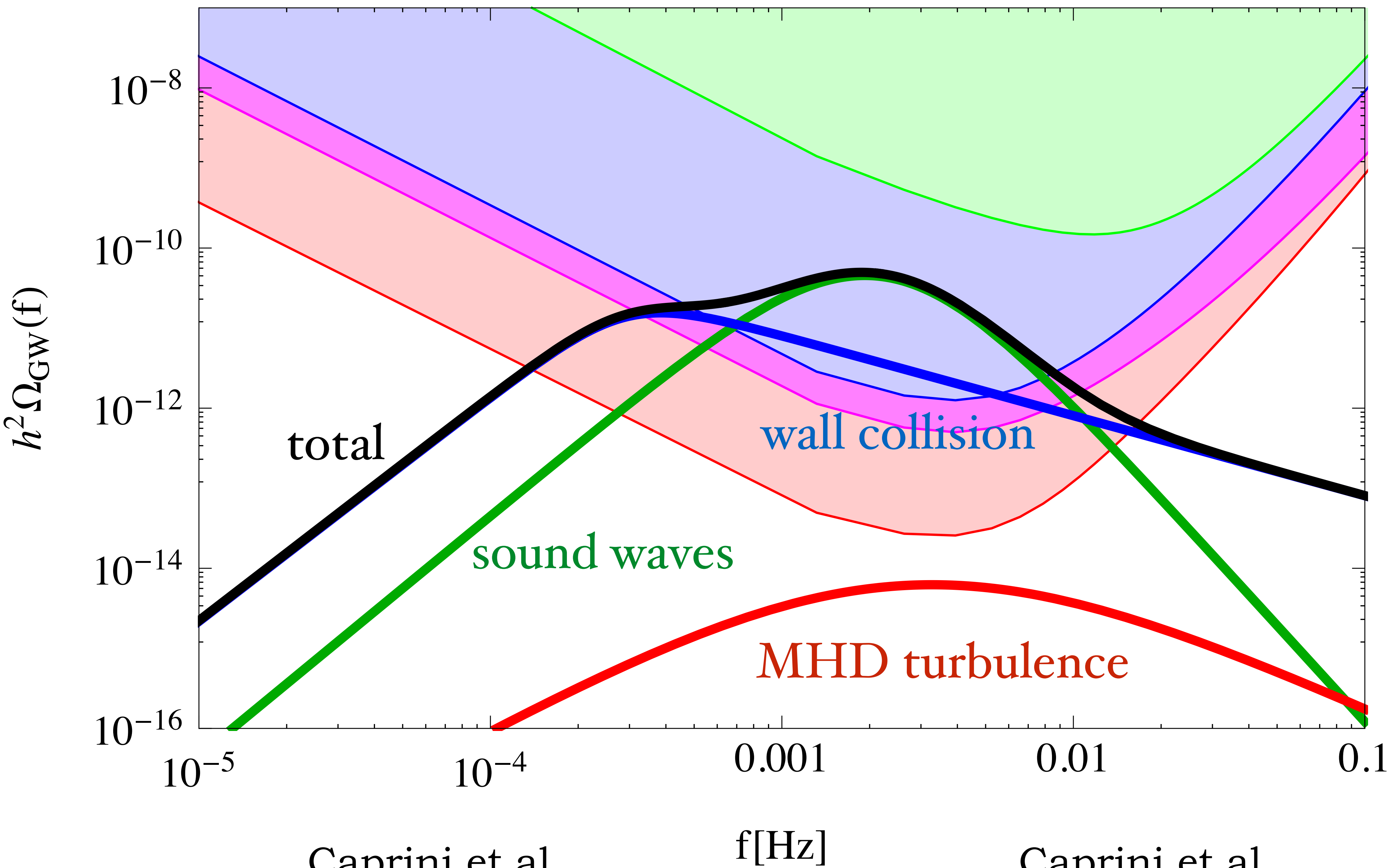
Hydro-dynamical effects & turbulence separate treatment

$$\alpha = \frac{\rho_{\mathrm{vac}}}{\rho^*_{\mathrm{rad}}}$$

$$\kappa = \frac{\rho_{\mathrm{kin}}}{\rho_{\mathrm{vac}}}$$

Example of spectrum

(From Collisions + Sound Waves + Turbulence)

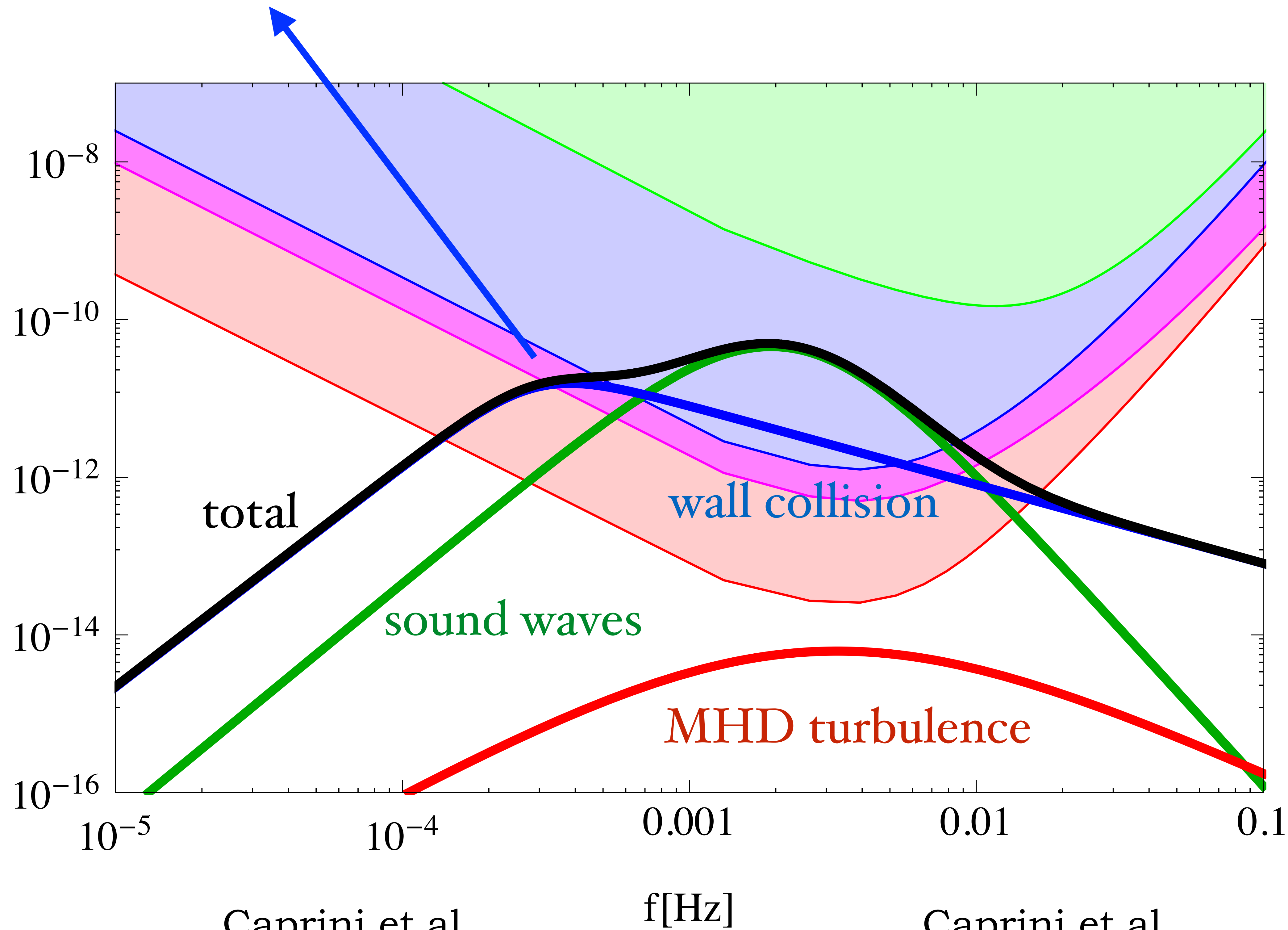


Caprini et al, arXiv:1512.06239

Caprini et al, arXiv:1910.13125

Example of spectrum

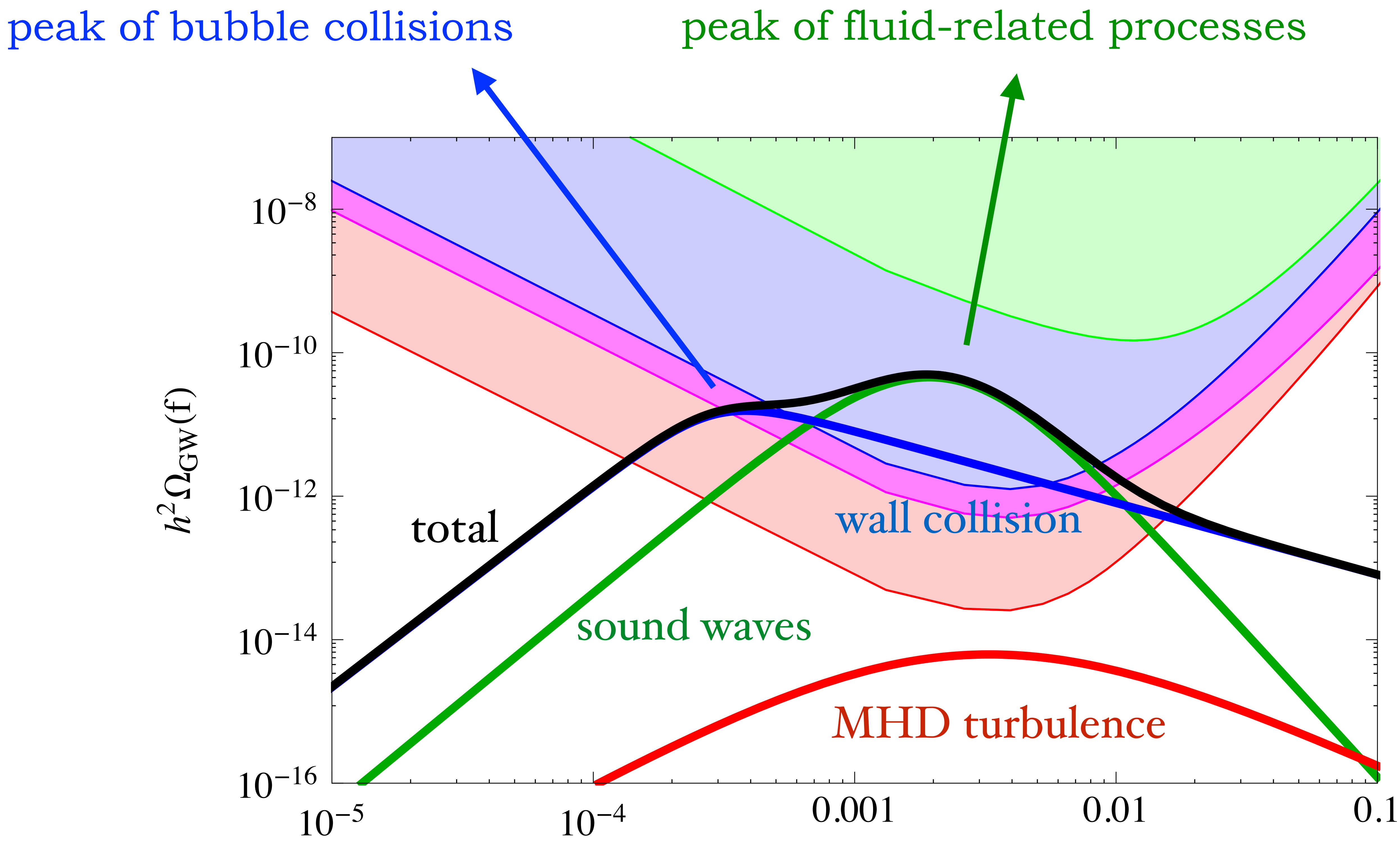
peak of bubble collisions



Caprini et al, arXiv:1512.06239

Caprini et al, arXiv:1910.13125

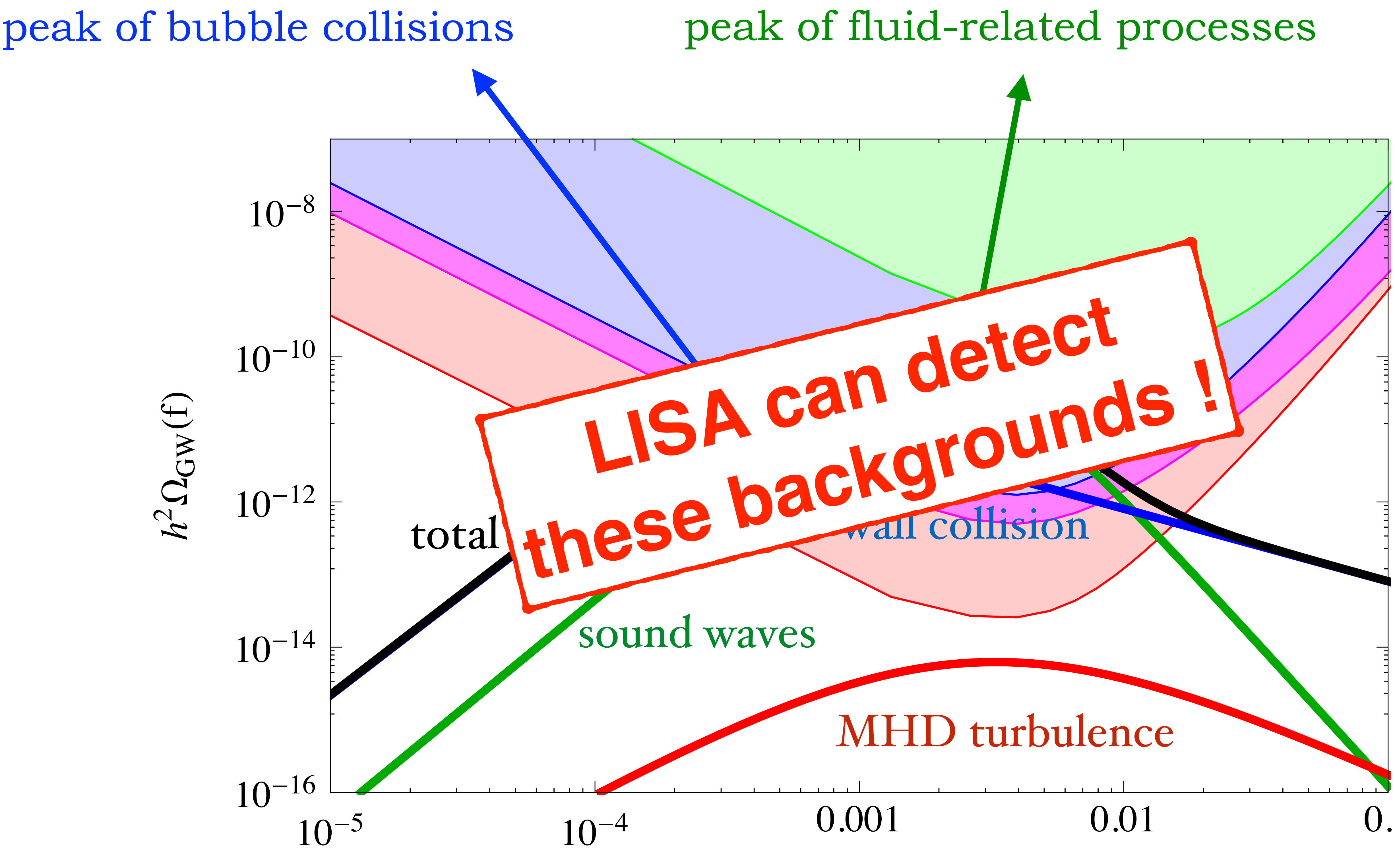
Example of spectrum



Caprini et al, arXiv:1512.06239

Caprini et al, arXiv:1910.13125

Example of spectrum



Caprini et al, arXiv:1512.06239

Caprini et al, arXiv:1910.13125

Evaluation of the signal

- **bubble collisions**: **analytical** and **numerical** simulations

  Huber, Konstandin '08   Cutting, Hindmarsh et al 2018, …

Evaluation of the signal

- **bubble collisions**: **analytical** and **numerical** simulations

  Huber, Konstandin '08 Cutting, Hindmarsh et al 2018, ...

- **sound waves**: **numerical** simulations of scalar field and fluid

  Hindmarsh, Weir et al 2012 - 2019,

  analytical Hindmarsh 2016, 2019,

Evaluation of the signal

- **bubble collisions**: **analytical** and **numerical** simulations
  Huber, Konstandin '08   Cutting, Hindmarsh et al 2018, ...

- **sound waves**: **numerical** simulations of scalar field and fluid
  Hindmarsh, Weir et al 2012 - 2019,
  analytical Hindmarsh 2016, 2019,

- **MDH turbulence**: **analytical** evaluation
  Kosowsky et al '07, Caprini et al '09, Niksa et al '18
  numerical Pol et al 2019

Models for EWPT and beyond

- **LISA** sensitive to energy scale **10 GeV - 100 TeV** !
  (mHZ)

Models for EWPT and beyond

- **LISA** sensitive to energy scale **10 GeV - 100 TeV** !
  (mHZ)

- LISA can probe the EWPT in BSM models …
  - singlet extensions of (MS)SM (e.g. Huber et al 2015)
  - direct coupling of Higgs to scalars (e.g. Kozackuz et al 2013)
  - SM + dimension six operator (e.g. Grojean et al 2004)

Models for EWPT and beyond

- **LISA** sensitive to energy scale **10 GeV - 100 TeV** !
  (mHZ)

- LISA can probe the EWPT in BSM models …
  - singlet extensions of (MS)SM (e.g. Huber et al 2015)
  - direct coupling of Higgs to scalars (e.g. Kozackuz et al 2013)
  - SM + dimension six operator (e.g. Grojean et al 2004)

- … and beyond the EWPT
  - Dark sector: provides DM candidate and confining PhT (Schwaller 2015)
  - Warped extra dimensions : PT from the dilaton/radion stabilisation in RS-like models (Randall and Servant 2015)

Models for EWPT and beyond

- **LISA** sensitive to energy scale **10 GeV - 100 TeV** !
  (mHZ)

- LISA can probe the EWPT
  - ... et al 2015)
  - ... scalars (e.g. Kozackuz et al 2013)
  - ... six operator (e.g. Grojean et al 2004)

Cosmology and Particle Physics interplay!
Connections with baryon asymmetry & dark matter

- … and beyond the EWPT
  - Dark sector: provides DM candidate and confining PhT (Schwaller 2015)
  - Warped extra dimensions : PT from the dilaton/radion stabilisation in RS-like models (Randall and Servant 2015)

Models for EWPT and beyond

- **LISA** sensitive to energy scale **10 GeV - 100 TeV** !
  (mHZ)

- LISA can p
  - sin
    al 2015)
    ackuz et al 2013)
    t al 2004)

Conn

nterplay!
& dark matter

Problem: LHC is putting great pressure over BSM scenarios

- … and beyond the EWPT
  - Dark sector: provides DM candidate and confining PhT (Schwaller 2015)
  - Warped extra dimensions : PT from the dilaton/radion stabilisation in RS-like models (Randall and Servant 2015)

Models for EWPT and beyond

- **LISA** sensitive to energy scale **10 GeV - 100 TeV !**
  (mHZ)

- LISA can p
  - sin
    al 2015)
    ackuz et al 2013)
    t al 2004)

Conn

Problem: LHC is putting great pressure over BSM scenarios

nterplay!
& dark matter

- … and beyond the EWPT
  - Dark sector: provides DM … PhT (Schwa…
  - … the dilaton/radion
    … like models (Randall and Servant 2015)

GW —> new probe of BSM physics!
(complementary to particle colliders)

Can we really detect a 1st-O Ph-T ?

* LISA/ET can, but LHC pressures typical BSM extensions to promote EW-PhT into First Order

* Assuming LHC does not rule out models, LISA/ET can detect/constrain significant fraction of Param Space

Can we really detect a 1st-O Ph-T ?

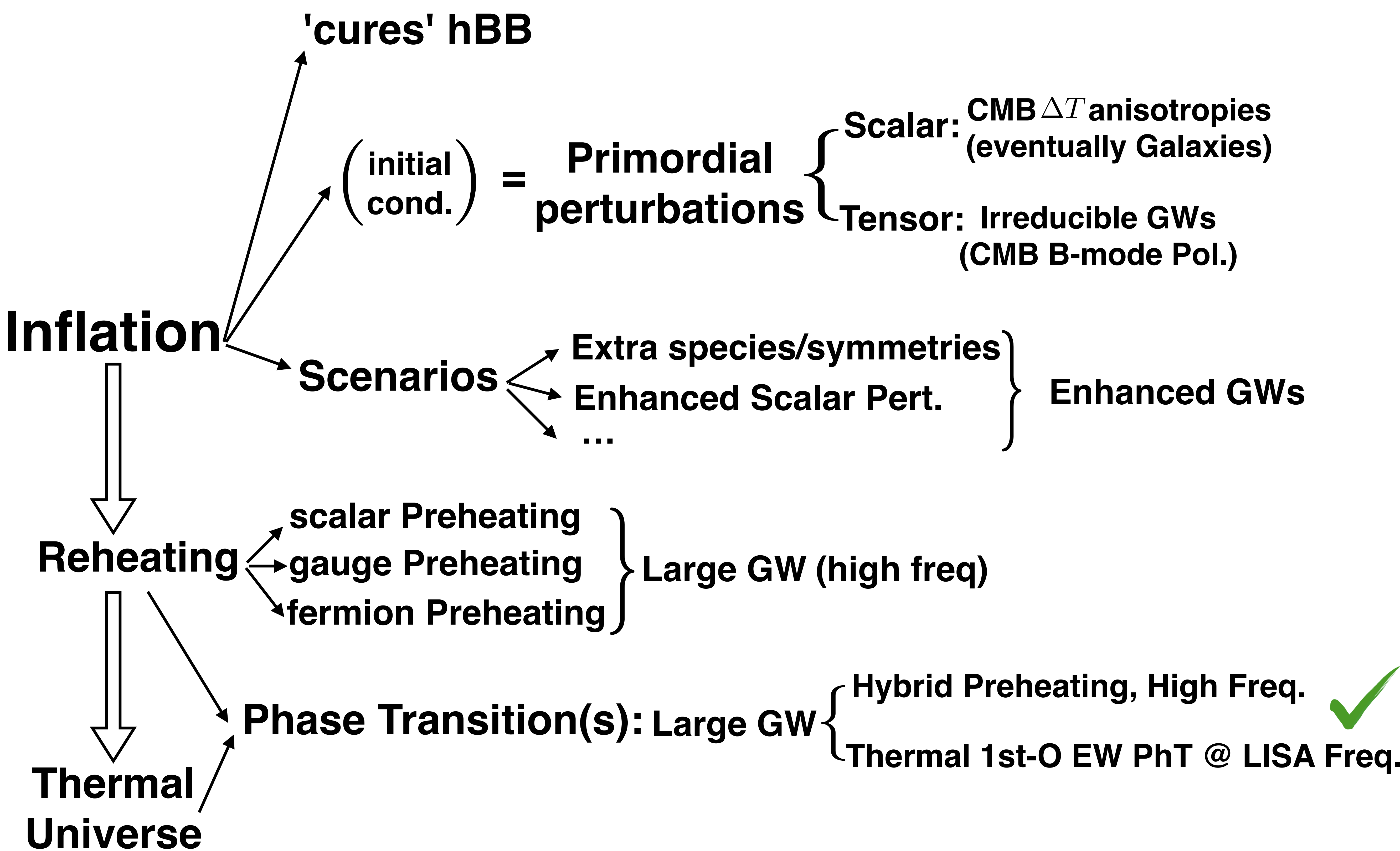
* LISA/ET can, but LHC pressures typical BSM extensions to promote EW-PhT into First Order

* Assuming LHC does not rule out models, LISA/ET can detect/constrain significant fraction of Param Space

* Predictions depend on many assumptions (particularly in sound waves), so is our modelling correct?

* Even if we detect it, then we infer α and β , but what BSM model is behind? not univocal !

EARLY UNIVERSE



Gravitational Wave Backgrounds

OUTLINE

1) Grav. Waves (GWs) ✓ — 1st Topic

Early Universe Sources — Core Topics

2) GWs from Inflation ✓

3) GWs from Preheating ✓

4) GWs from Phase Transitions ✓

5) GWs from Cosmic Defects

6) Astrophysical Background(s)

7) Observational Constraints/Prospects

← (Briefly)

Gravitational Wave Backgrounds

OUTLINE

1) Grav. Waves (GWs) — 1st Topic

Early Universe Sources:

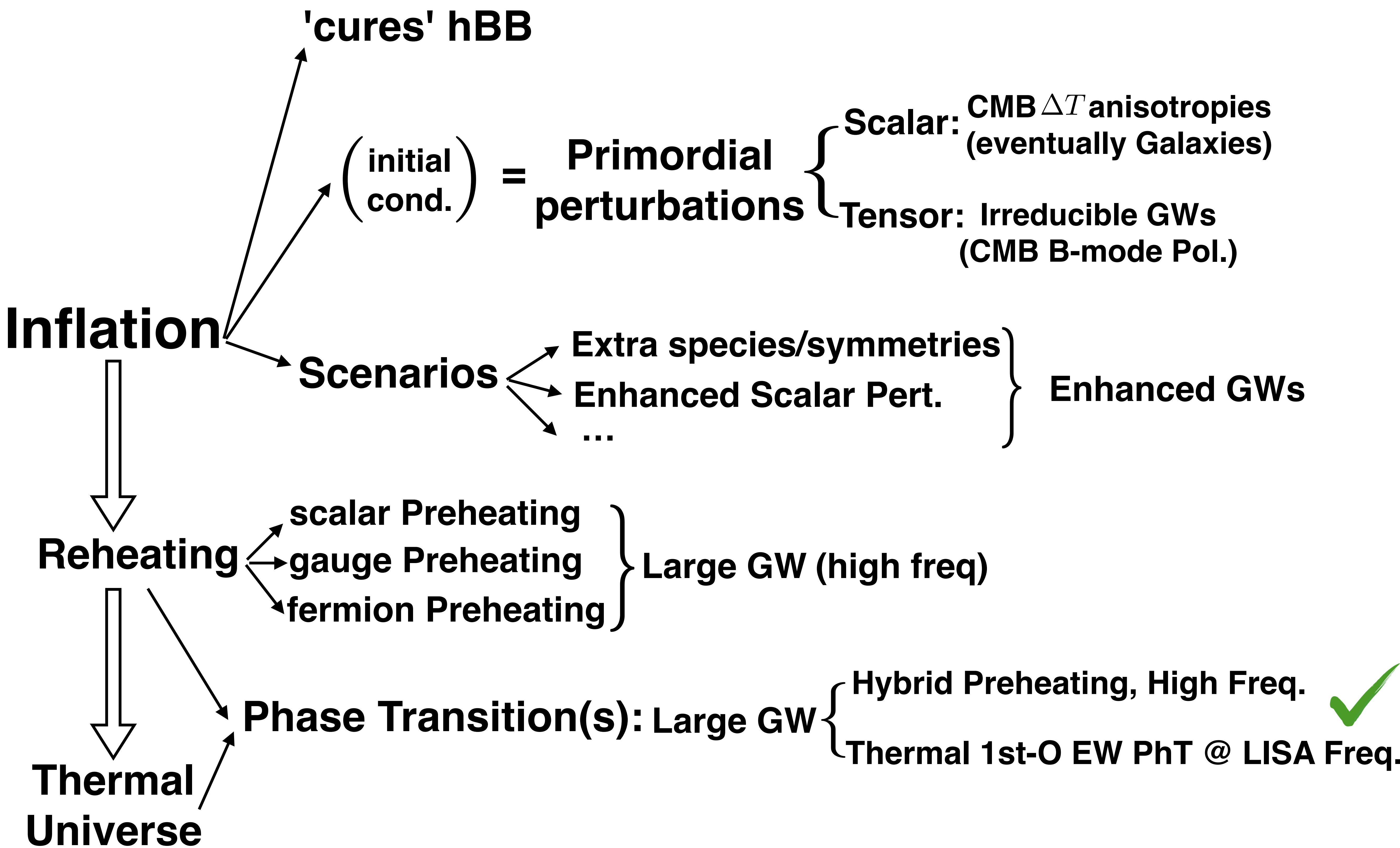
2) GWs from Inflation
3) GWs from Preheating
4) GWs from Phase Transitions
5) GWs from Cosmic Defects

Core Topics

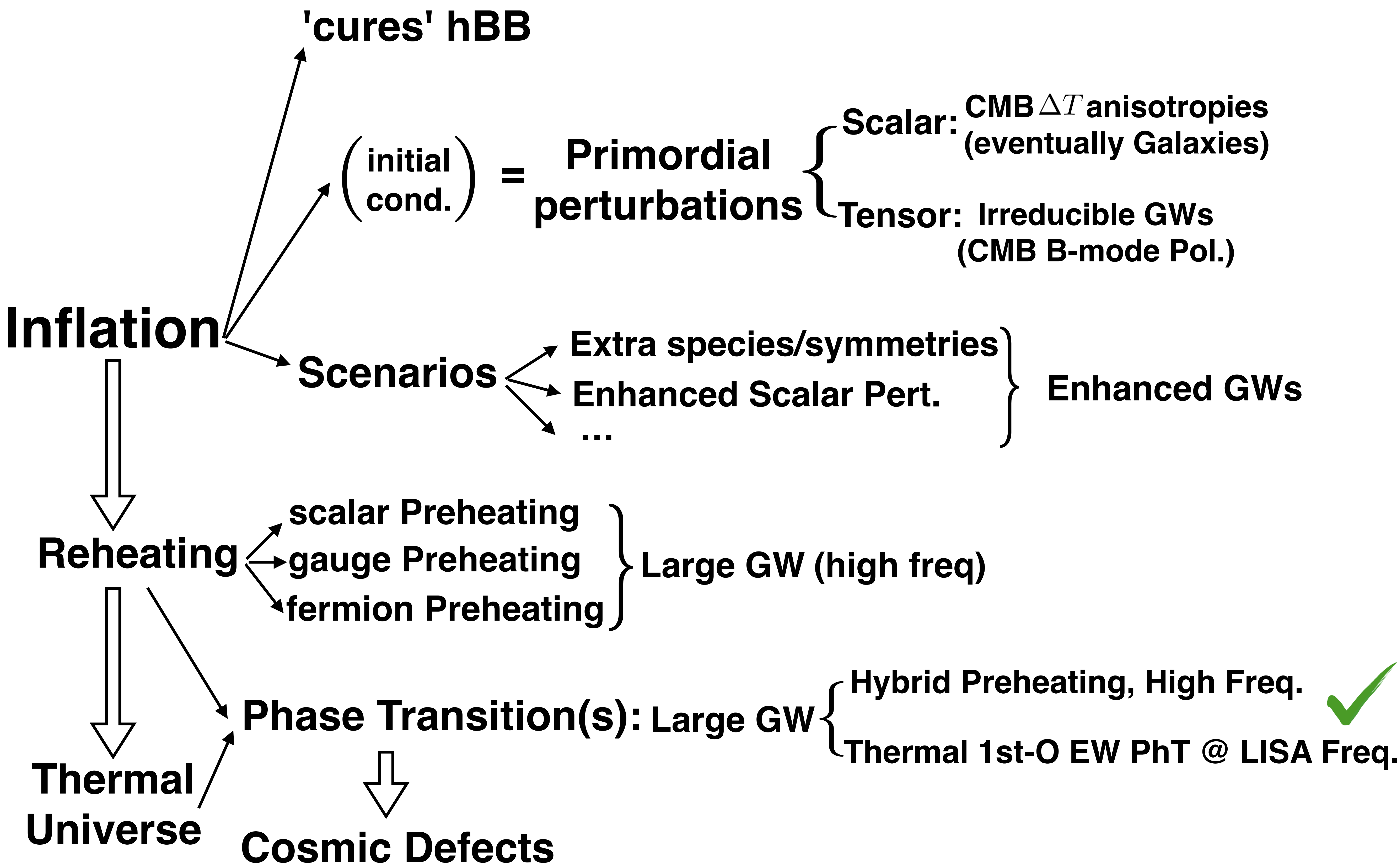
6) Astrophysical Background(s)
7) Observational Constraints/Prospects

(Briefly)

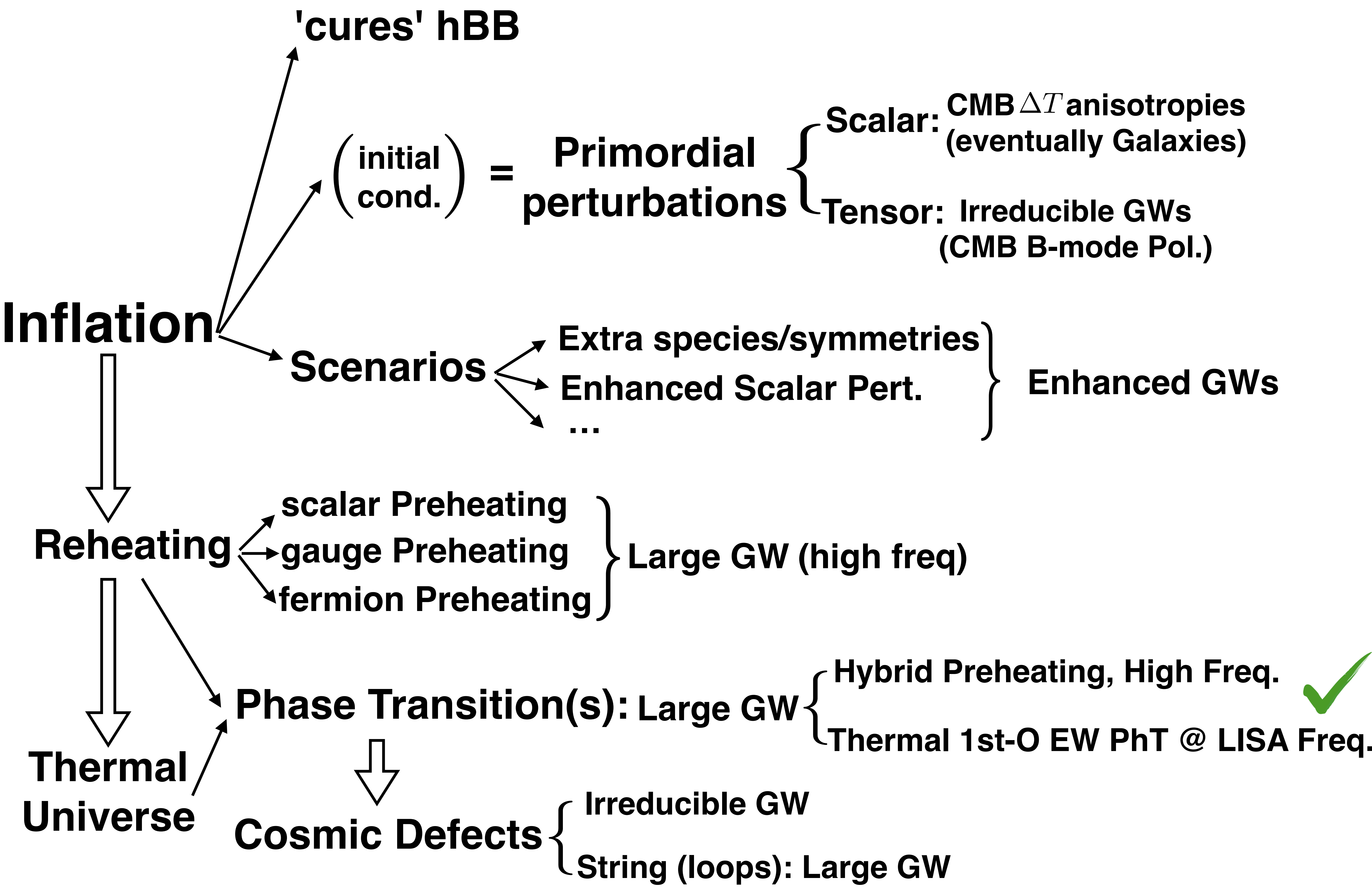
EARLY UNIVERSE



EARLY UNIVERSE



EARLY UNIVERSE



Gravitational Wave Backgrounds

OUTLINE

1) Grav. Waves (GWs) ✓ — 1st Topic

Early Universe Sources:

2) GWs from Inflation ✓

3) GWs from Preheating ✓

4) GWs from Phase Transitions

5) GWs from Cosmic Defects

Core Topics

6) Astrophysical Background(s)

7) Observational Constraints/Prospects

(Briefly)

EARLY UNIVERSE

Inflation

- 'cures' hBB
- (initial cond.) = Primordial perturbations
  - Scalar: CMB ΔT anisotropies (eventually Galaxies)
  - Tensor: Irreducible GWs (CMB B-mode Pol.)
- Scenarios
  - Extra species/symmetries
  - Enhanced Scalar Pert.
  - ...
  - } Enhanced GWs

Reheating

- scalar Preheating
- gauge Preheating
- fermion Preheating
- } Large GW (high freq)

Thermal Universe

Phase Transition(s): Large GW

- Hybrid Preheating, High Freq. ✓
- Thermal 1st-O EW PhT @ LISA Freq.

Cosmic Defects

- Irreducible GW
- String (loops): Large GW

Cosmic Defects

Aftermath product of a Ph.T.

Introduction to Cosmic Defects

Topology of cosmic domains and strings

T W B Kibble

Blackett Laboratory, Imperial College, Prince Consort Road, London SW7 2BZ, UK

Received 11 March 1976

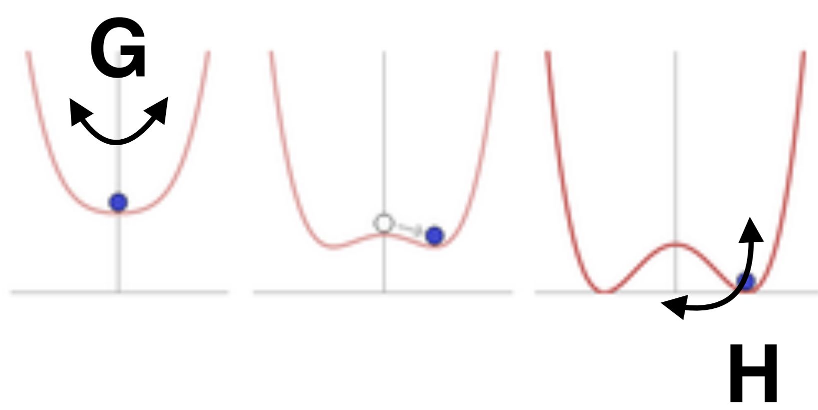
Abstract. The possible domain structures which can arise in the universe in a spontaneously broken gauge theory are studied. It is shown that the formation of domain walls, strings or monopoles depends on the homotopy groups of the manifold of degenerate vacua. The subsequent evolution of these structures is investigated. It is argued that while theories generating domain walls can probably be eliminated (because of their unacceptable gravitational effects), a cosmic network of strings may well have been formed and may have had important cosmological effects.

Kibble pioneered the study of topological defect generation in the early universe.

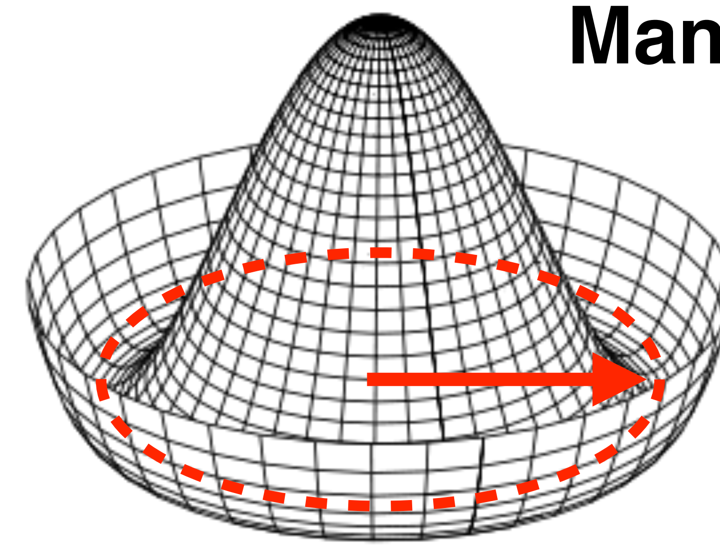
Introduction to Cosmic Defects

Kibble'76

Spontaneous Symm. Break.



Vacuum Manifold

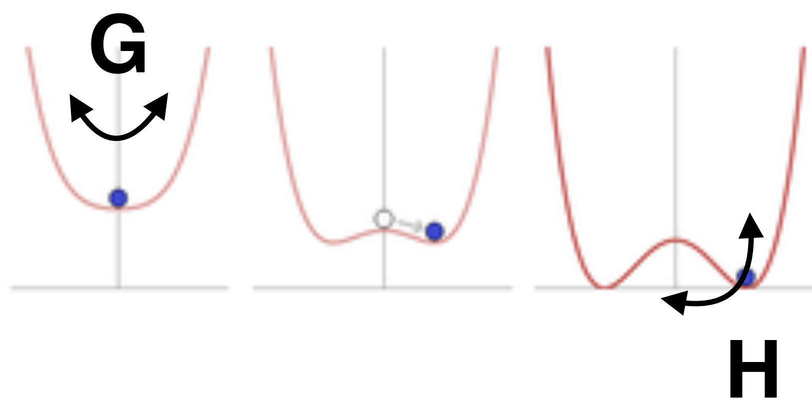


M = G/H

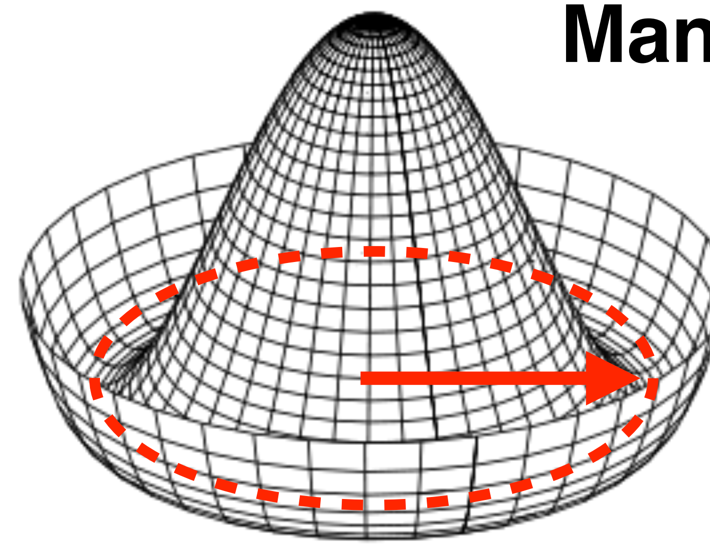
Introduction to Cosmic Defects

Kibble'76

Spontaneous Symm. Break.

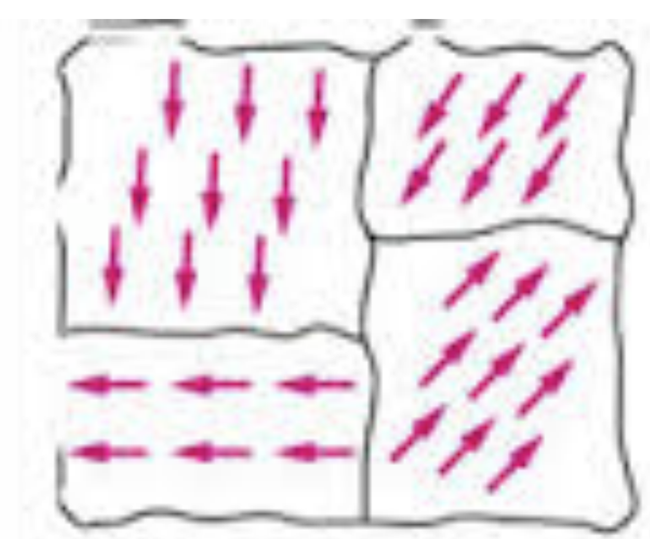


Vacuum Manifold



M = G/H

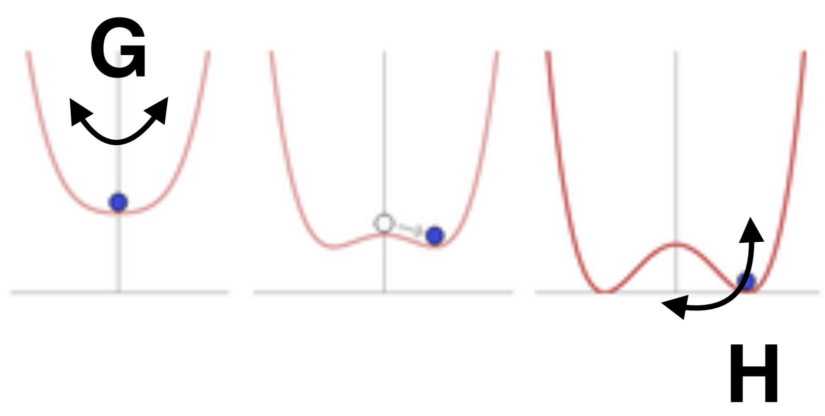
Spatial Dist.



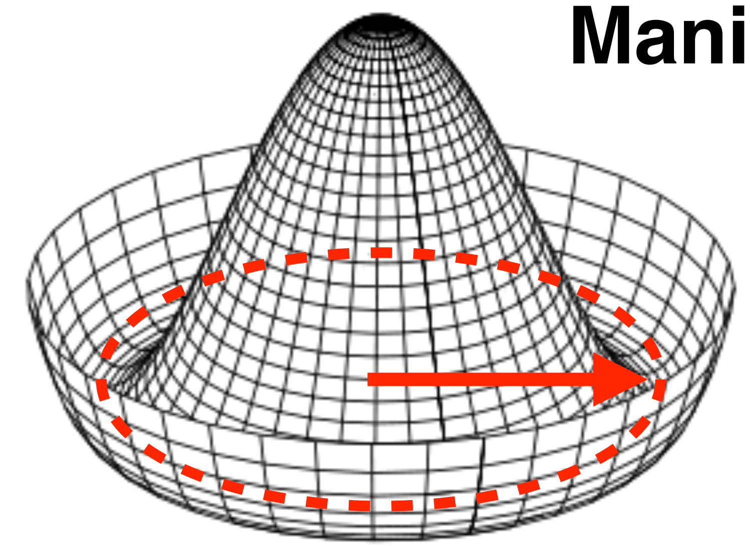
Introduction to Cosmic Defects

Kibble'76

Spontaneous Symm. Break.

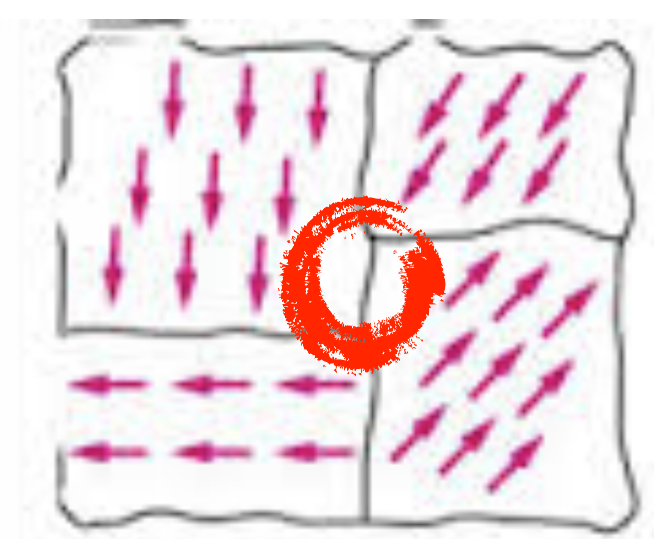


Vacuum Manifold



M = G/H

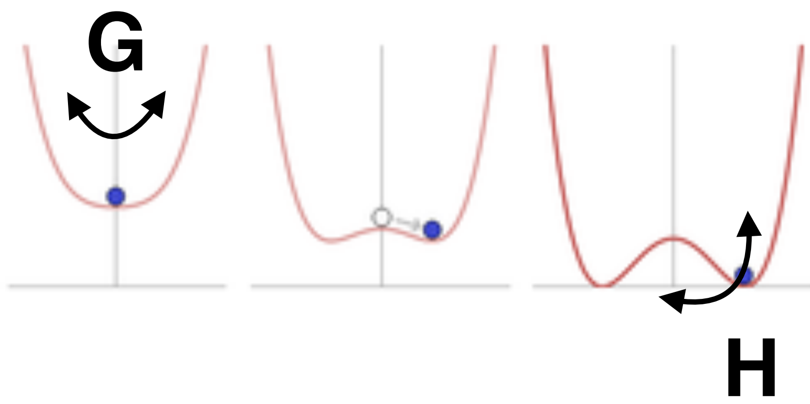
Spatial Dist.



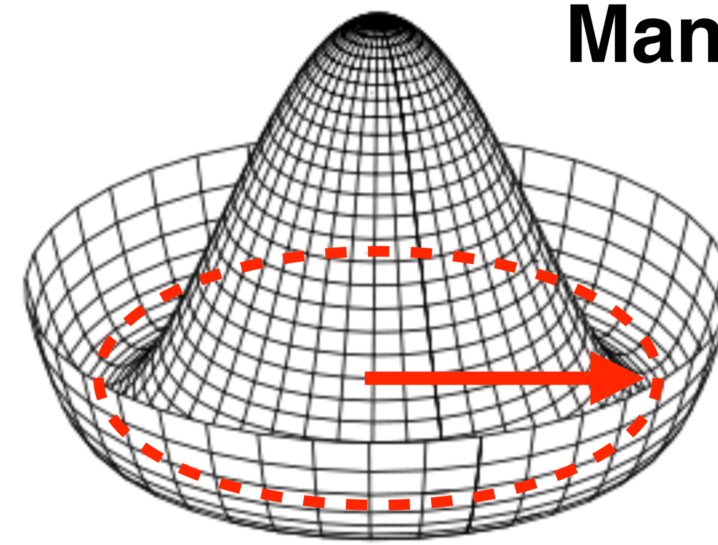
Introduction to Cosmic Defects

Kibble'76

Spontaneous Symm. Break.

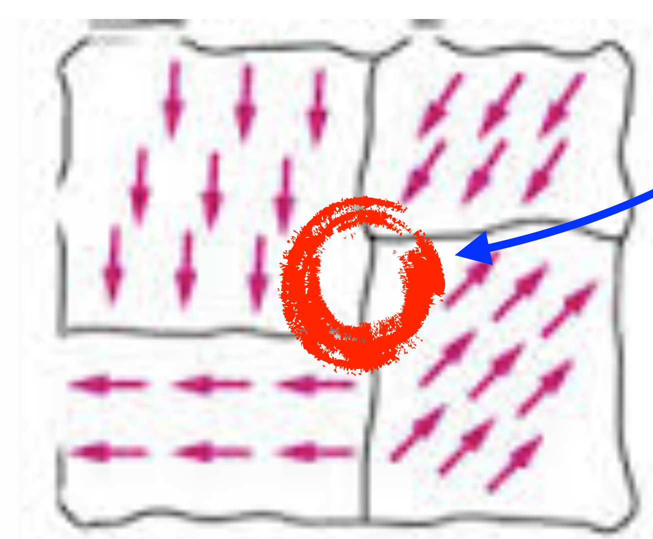


Vacuum Manifold



M = G/H

Spatial Dist.



Topological Properties

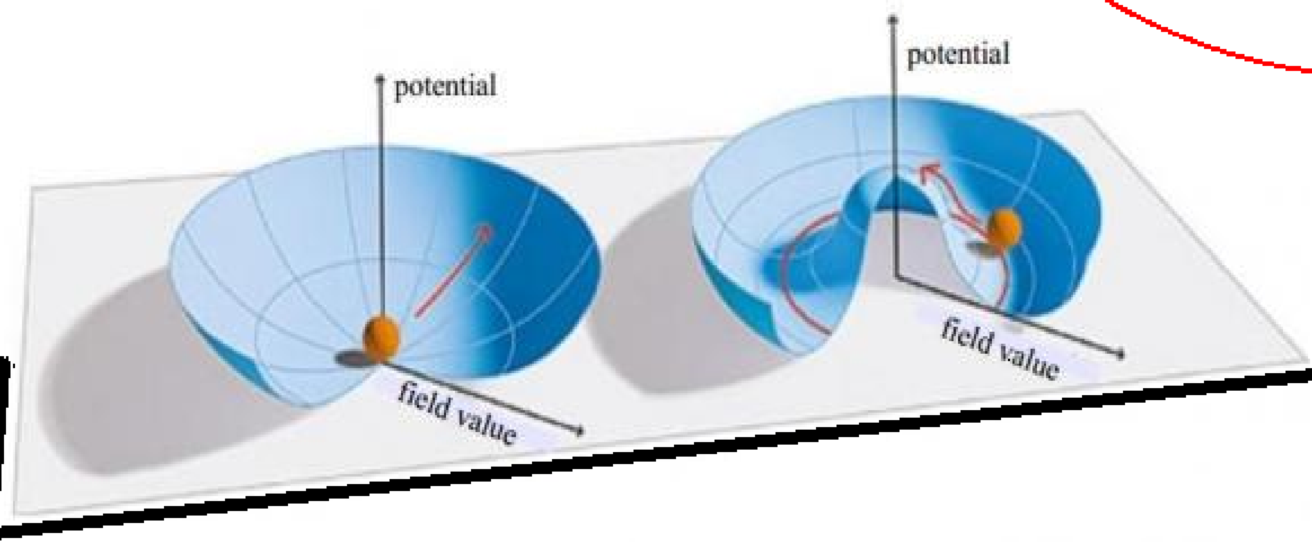
(Homotopy Groups $\Pi_n(M) \neq I$)

Introduction to Cosmic Defects

$$V = \frac{\lambda}{4}\left(|\Phi|^2 - v^2\right)^2 + V_{\rm int}(\Phi, \chi, T)$$

(1[st] Order, 2[nd] Order, Cross-Over)

$$V_{\rm int} \sim \begin{cases} g_{\rm T}^2|\Phi|^2T^2 & (\text{ THERMAL }) \\ g^2|\Phi|^2\chi^2 & (\text{ FIELD INT.}) \end{cases}$$

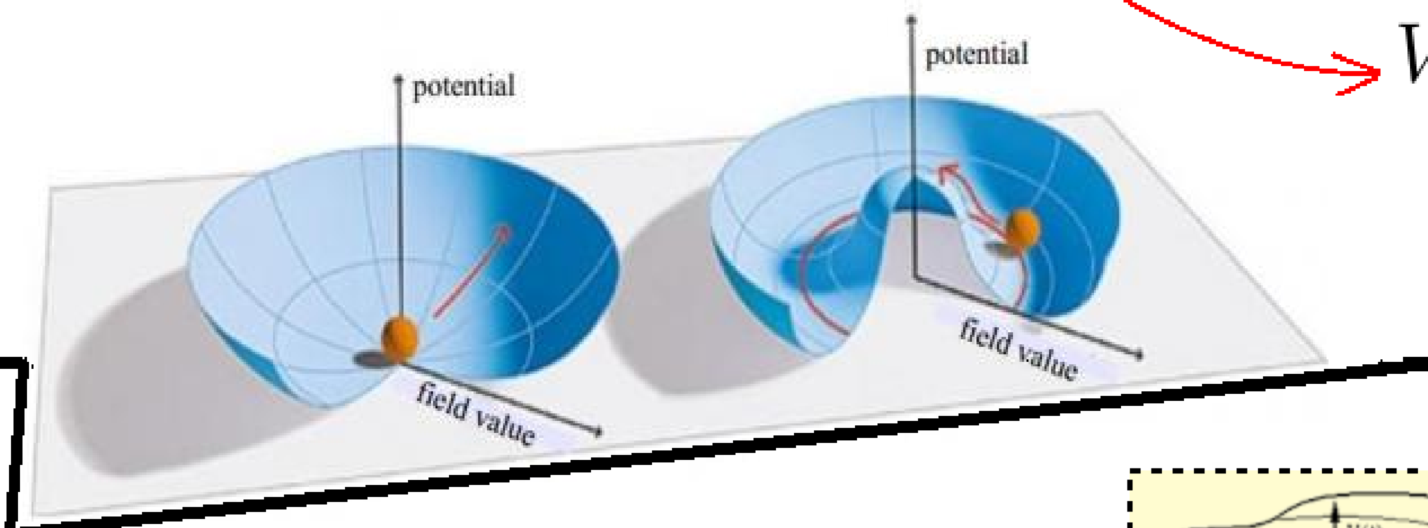


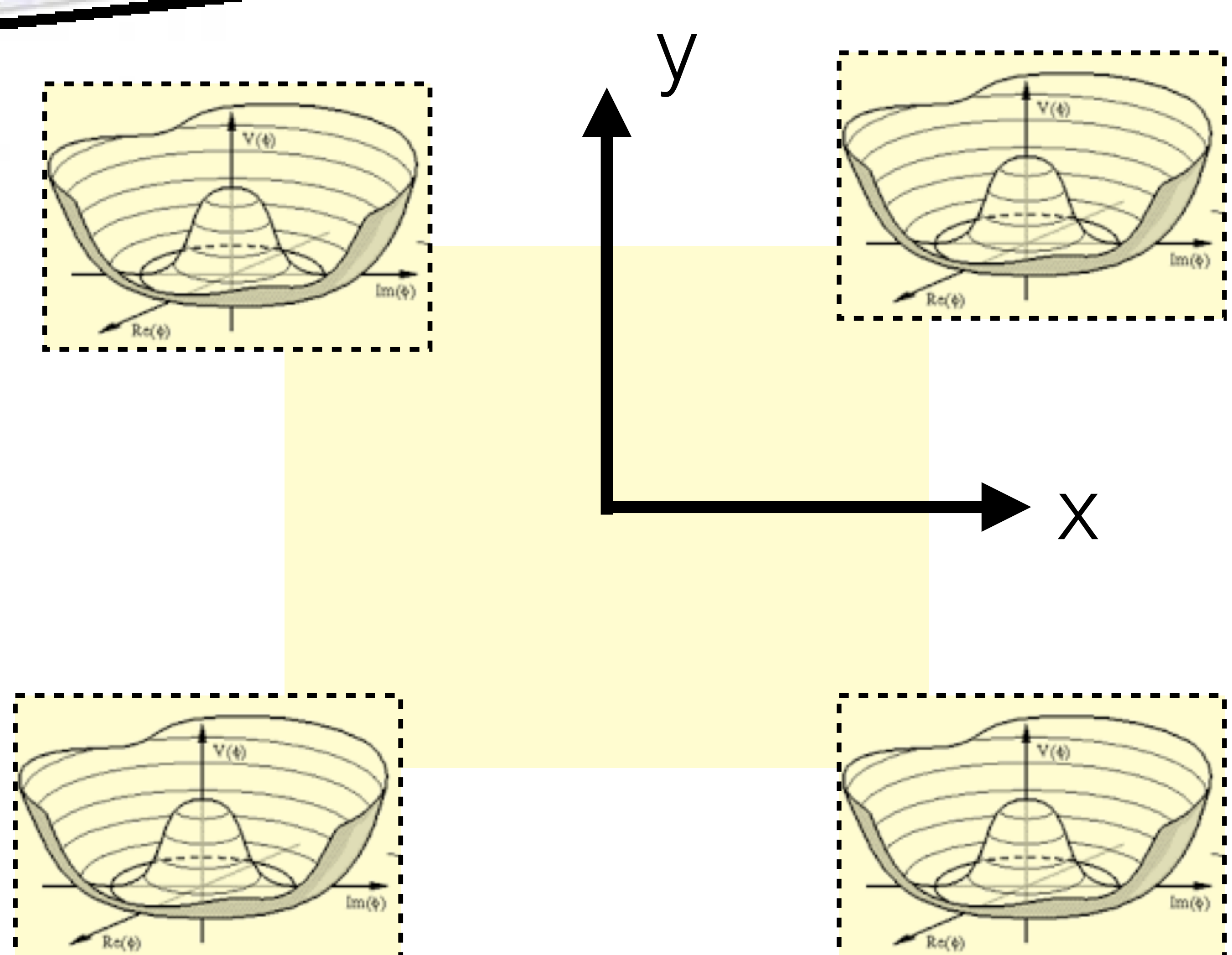
Introduction to Cosmic Defects

$$V = \frac{\lambda}{4}\left(|\Phi|^2 - v^2\right)^2 + V_{\rm int}(\Phi, \chi, T)$$

(1st Order, 2nd Order, Cross-Over)

$$V_{\rm int} \sim \begin{cases} g_{\rm T}^2 |\Phi|^2 T^2 & (\text{ THERMAL }) \\ g^2 |\Phi|^2 \chi^2 & (\text{ FIELD INT.}) \end{cases}$$



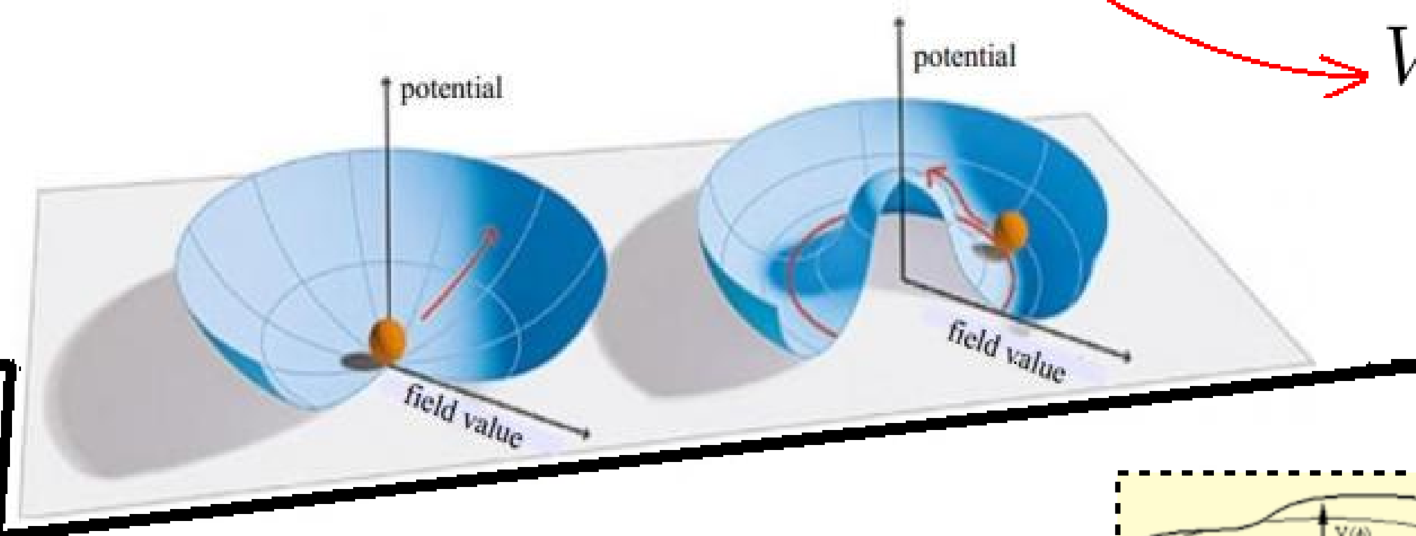


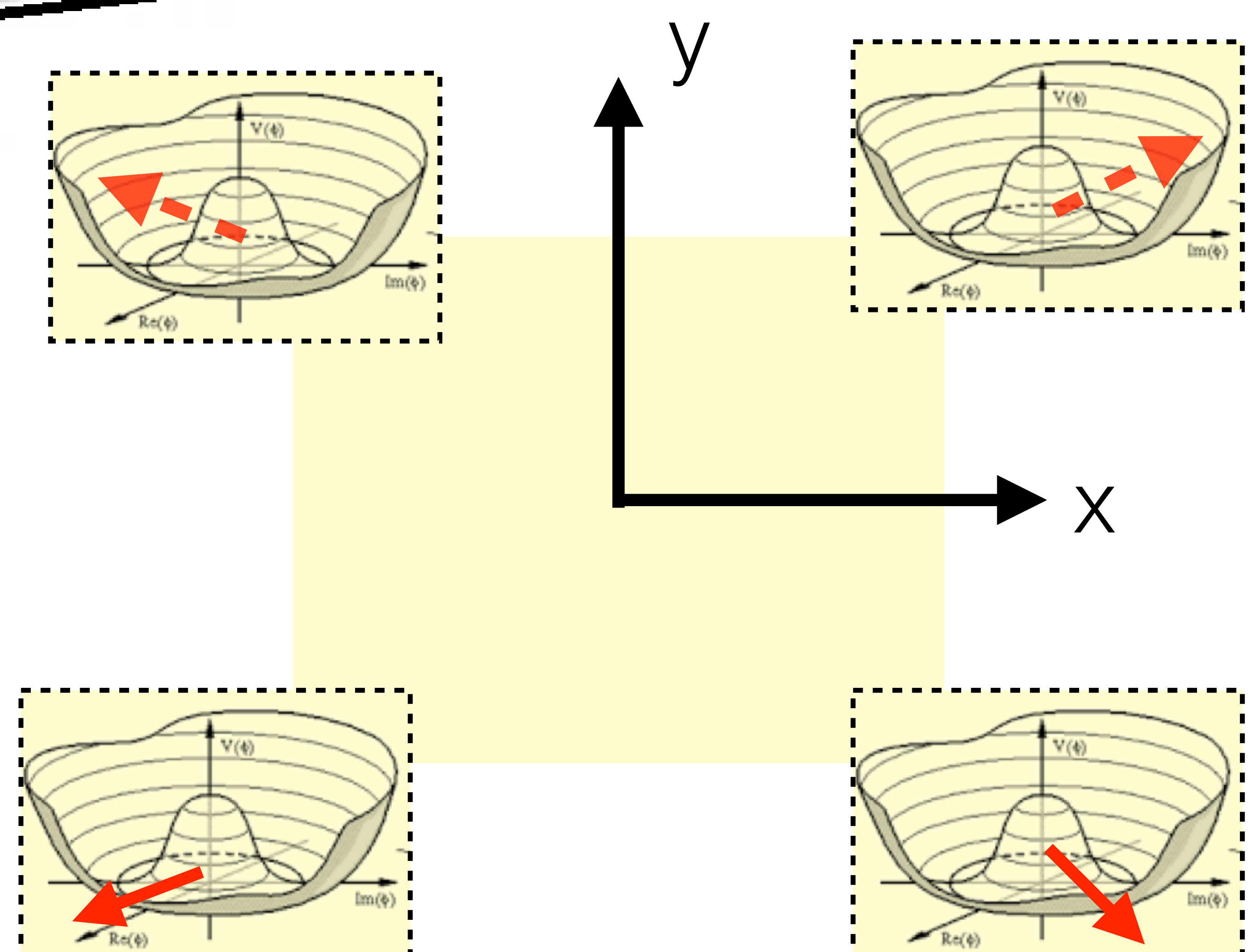
Introduction to Cosmic Defects

(1st Order, 2nd Order, Cross-Over)

$$V = \frac{\lambda}{4}\left(|\Phi|^2 - v^2\right)^2 + V_{\rm int}(\Phi,\chi,T)$$

$$V_{\rm int} \sim \begin{cases} g_{\rm T}^2|\Phi|^2T^2 & (\text{ THERMAL }) \\ g^2|\Phi|^2\chi^2 & (\text{ FIELD INT.}) \end{cases}$$



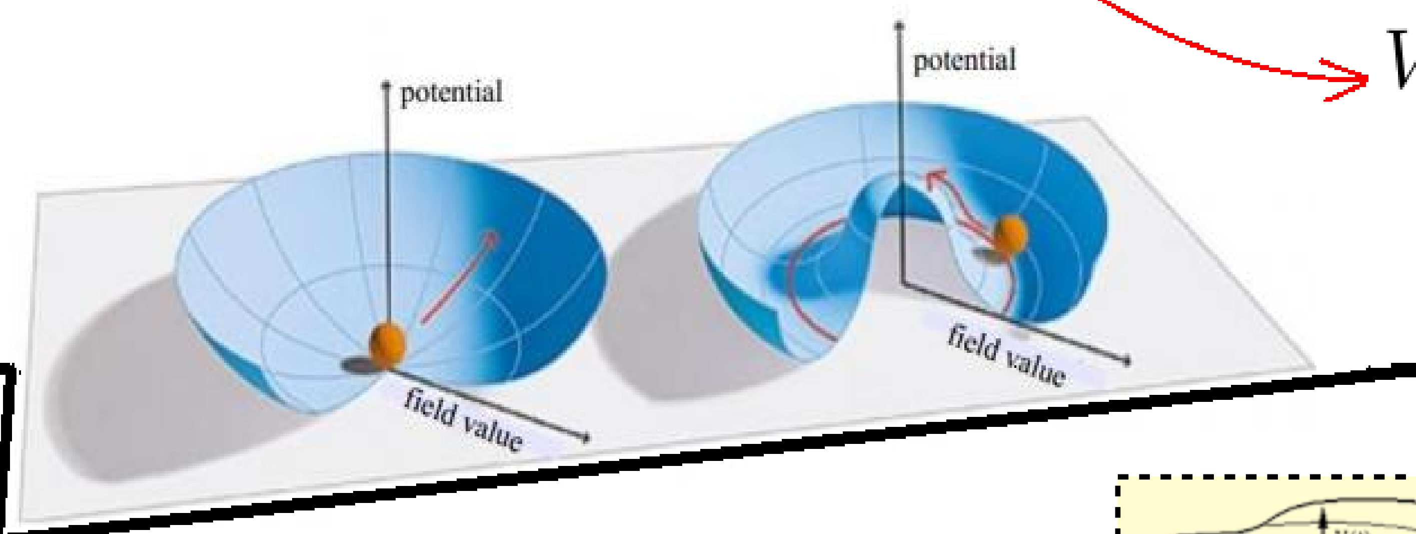


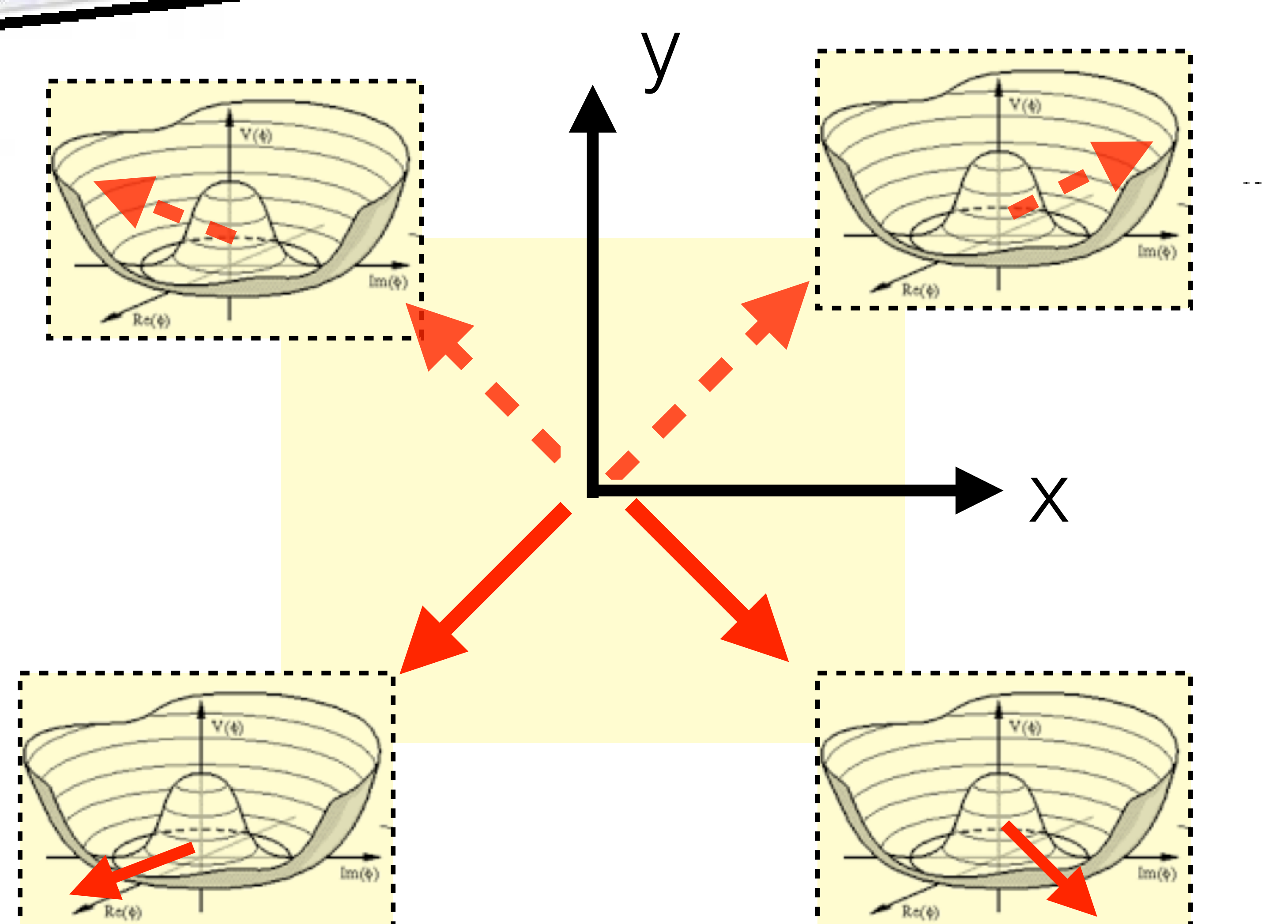
Introduction to Cosmic Defects

$$V = \frac{\lambda}{4}\left(|\Phi|^2 - v^2\right)^2 + V_{\rm int}(\Phi, \chi, T)$$

(1st Order, 2nd Order, Cross-Over)

$$V_{\rm int} \sim \begin{cases} g_{\rm T}^2 |\Phi|^2 T^2 & \text{(THERMAL)} \\ g^2 |\Phi|^2 \chi^2 & \text{(FIELD INT.)} \end{cases}$$



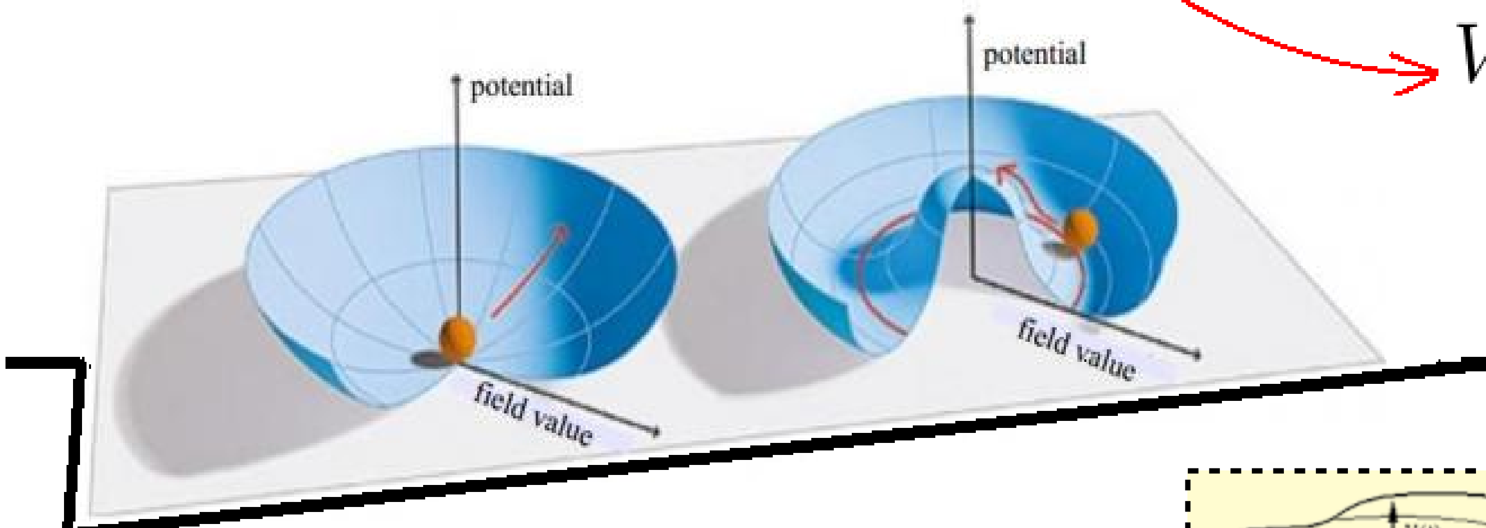


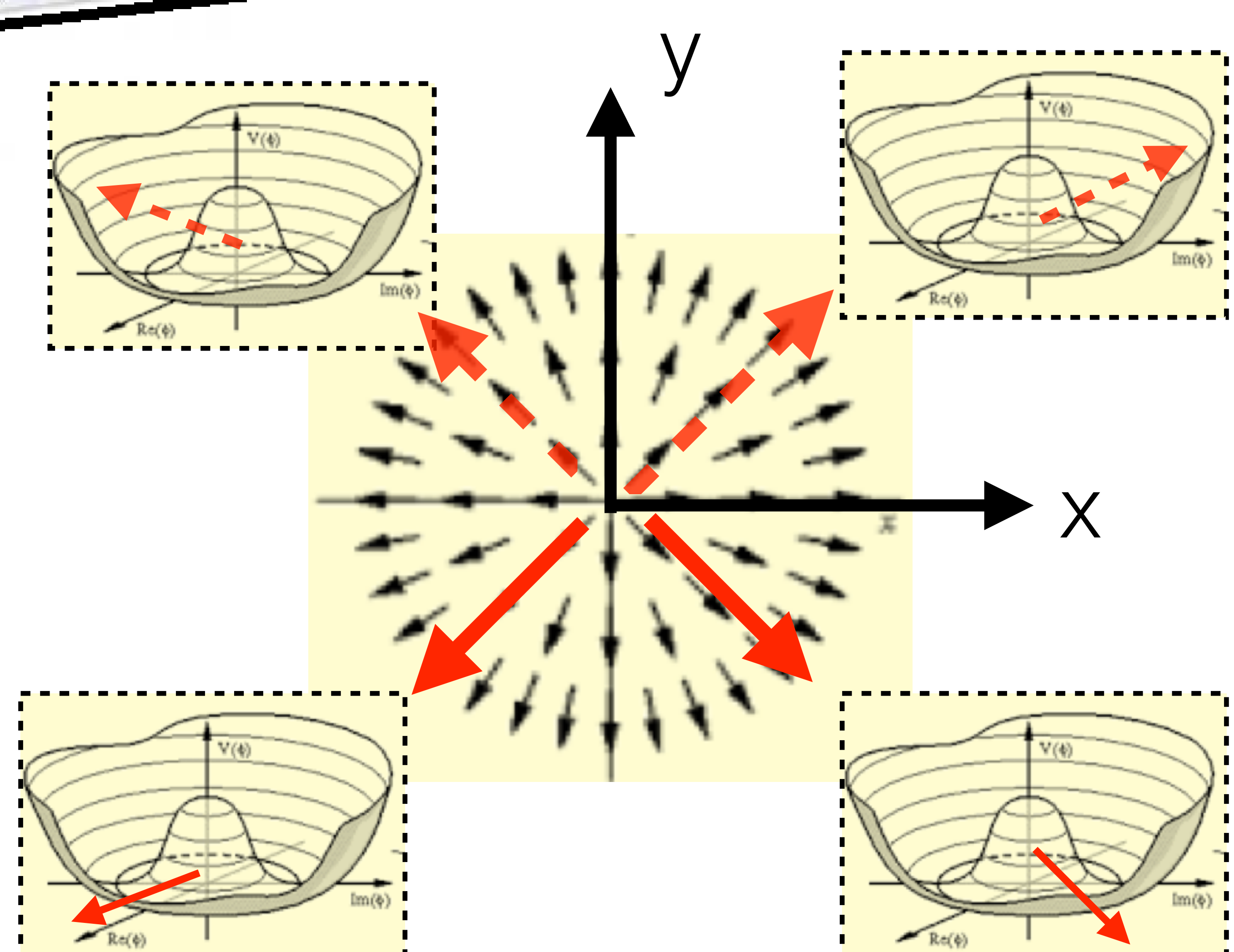
Introduction to Cosmic Defects

$$V = \frac{\lambda}{4}\left(|\Phi|^2 - v^2\right)^2 + V_{\rm int}(\Phi, \chi, T)$$

(1st Order, 2nd Order, Cross-Over)

$$V_{\rm int} \sim \begin{cases} g_T^2 |\Phi|^2 T^2 & \text{(THERMAL)} \\ g^2 |\Phi|^2 \chi^2 & \text{(FIELD INT.)} \end{cases}$$



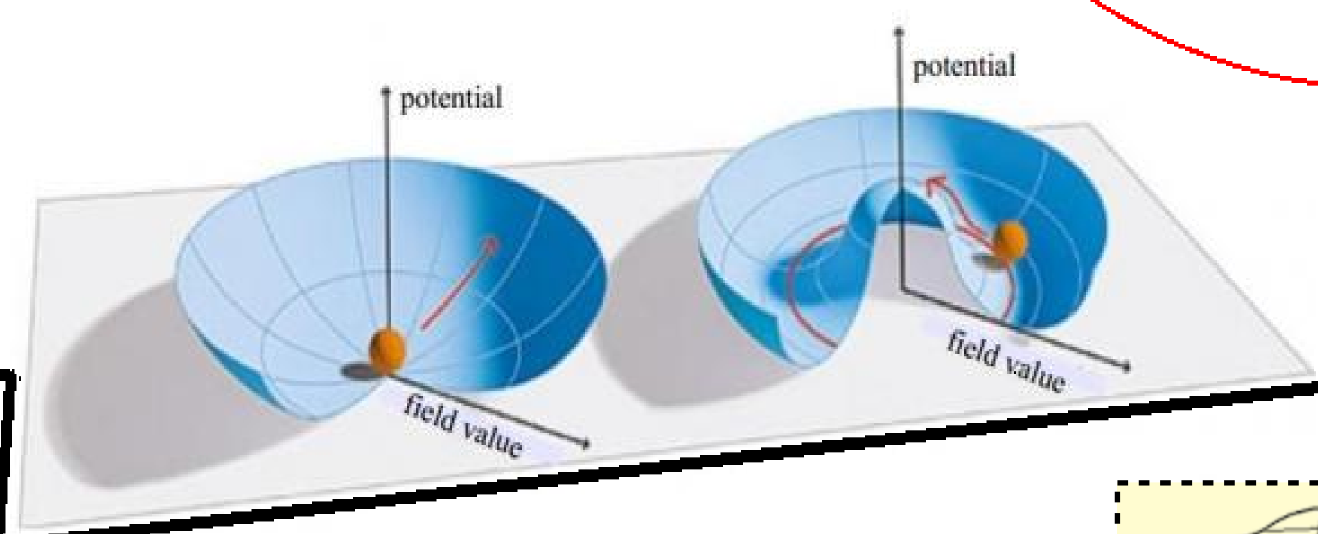


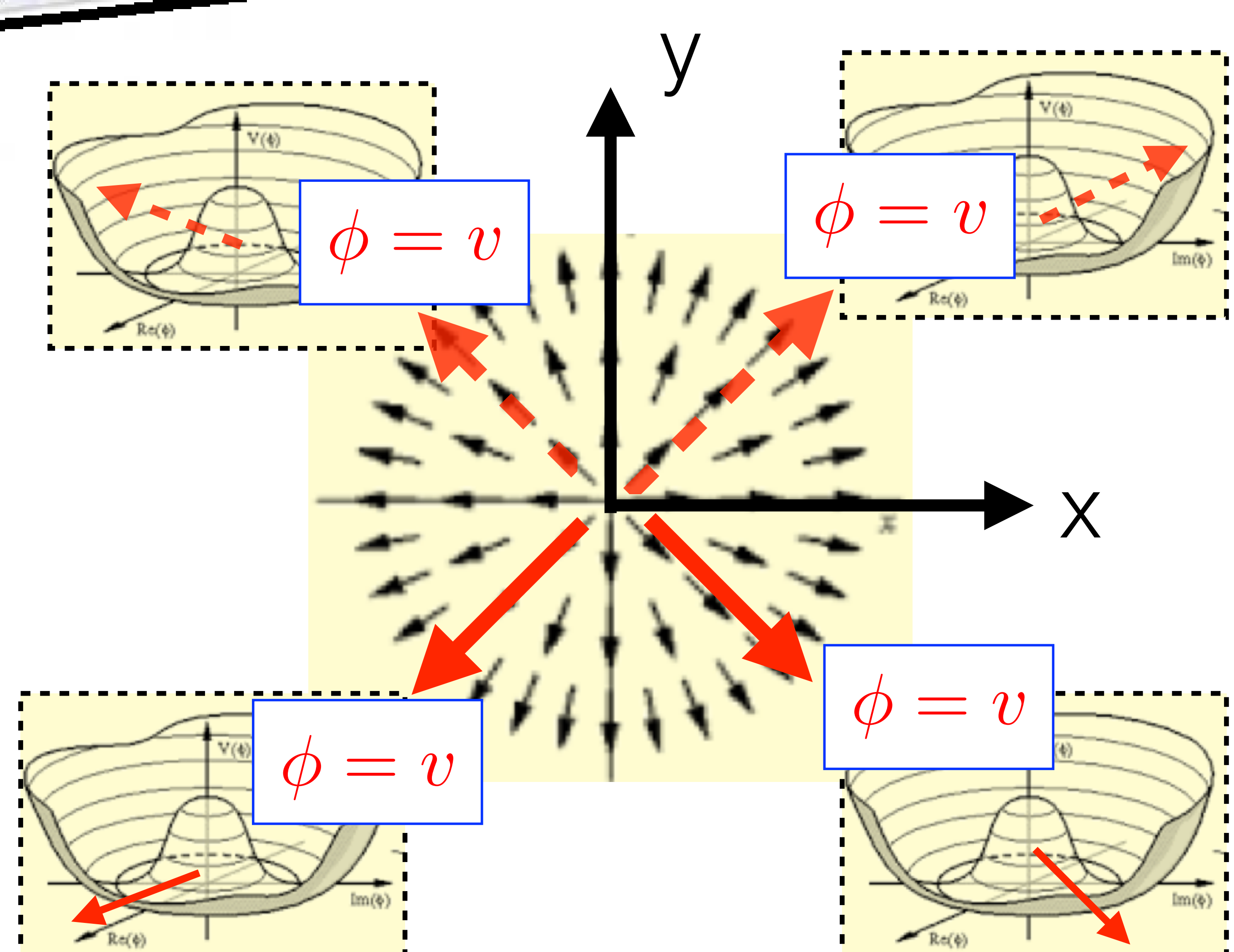
Introduction to Cosmic Defects

$$V = \frac{\lambda}{4}\left(|\Phi|^2 - v^2\right)^2 + V_{\rm int}(\Phi, \chi, T)$$

(1st Order, 2nd Order, Cross-Over)

$$V_{\rm int} \sim \begin{cases} g_{\rm T}^2 |\Phi|^2 T^2 & \text{(THERMAL)} \\ g^2|\Phi|^2\chi^2 & \text{(FIELD INT.)} \end{cases}$$



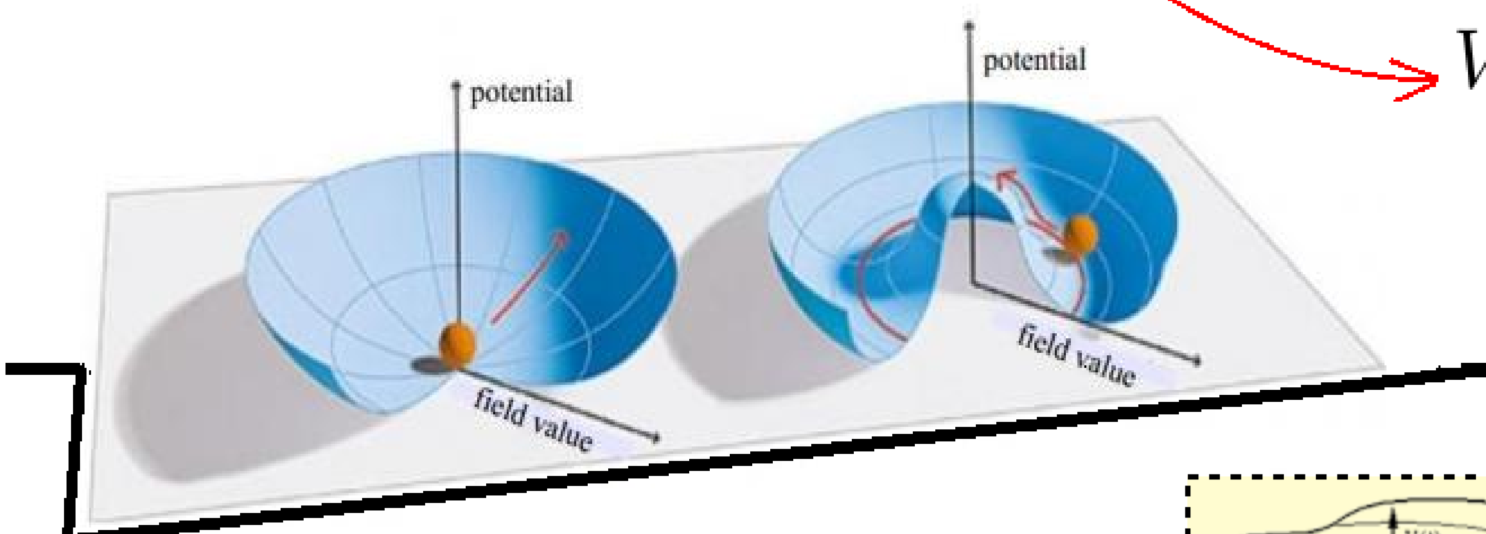


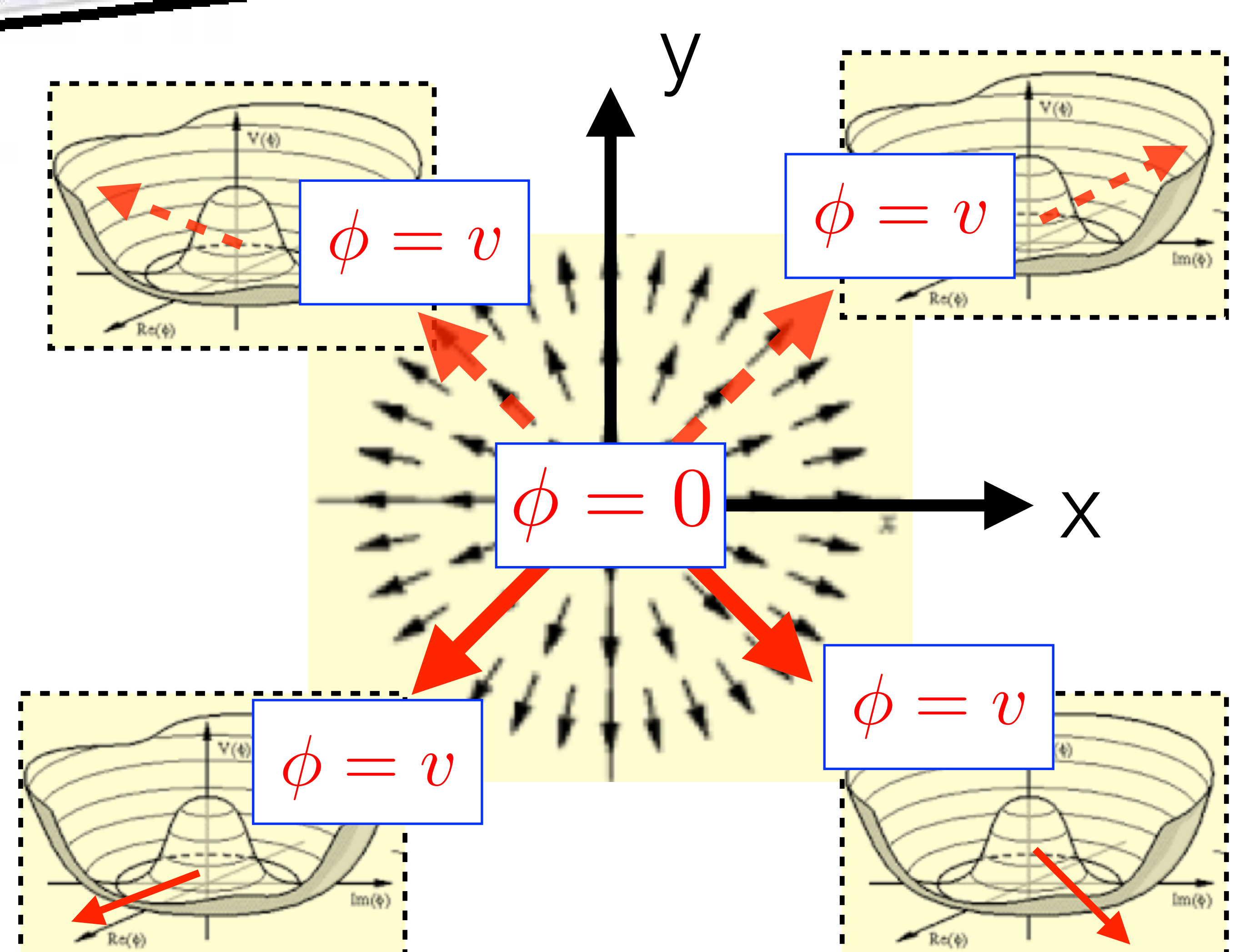
Introduction to Cosmic Defects

$$V = \frac{\lambda}{4}\left(|\Phi|^2 - v^2\right)^2 + V_{\rm int}(\Phi, \chi, T)$$

(1st Order, 2nd Order, Cross-Over)

$$V_{\rm int} \sim \begin{cases} g_T^2|\Phi|^2T^2 & \text{(THERMAL)} \\ g^2|\Phi|^2\chi^2 & \text{(FIELD INT.)} \end{cases}$$



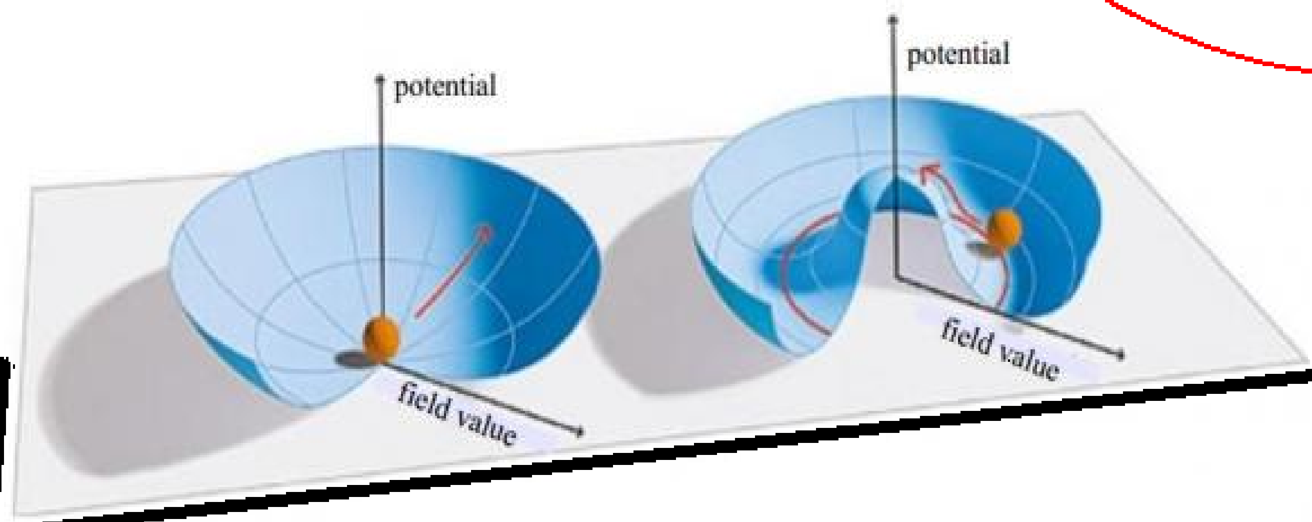


Introduction to Cosmic Defects

$$V = \frac{\lambda}{4}\left(|\Phi|^2 - v^2\right)^2 + V_{\rm int}(\Phi, \chi, T)$$

(1st Order, 2nd Order, Cross-Over)

$$V_{\rm int} \sim \begin{cases} g_{\rm T}^2|\Phi|^2T^2 & (\text{ THERMAL }) \\ g^2|\Phi|^2\chi^2 & (\text{ FIELD INT.}) \end{cases}$$

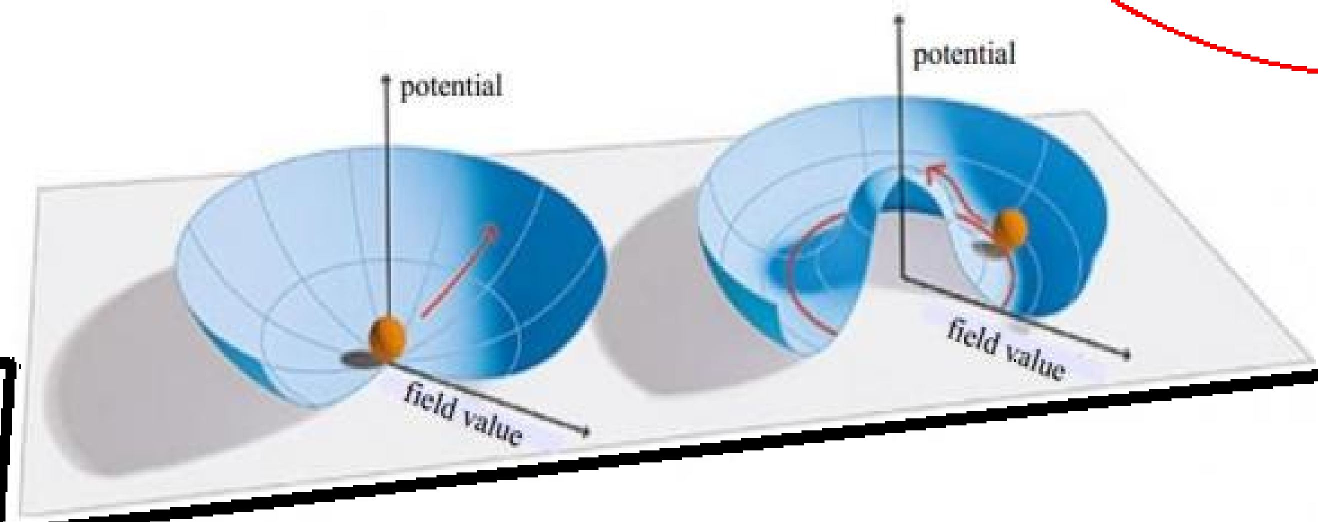


Introduction to Cosmic Defects

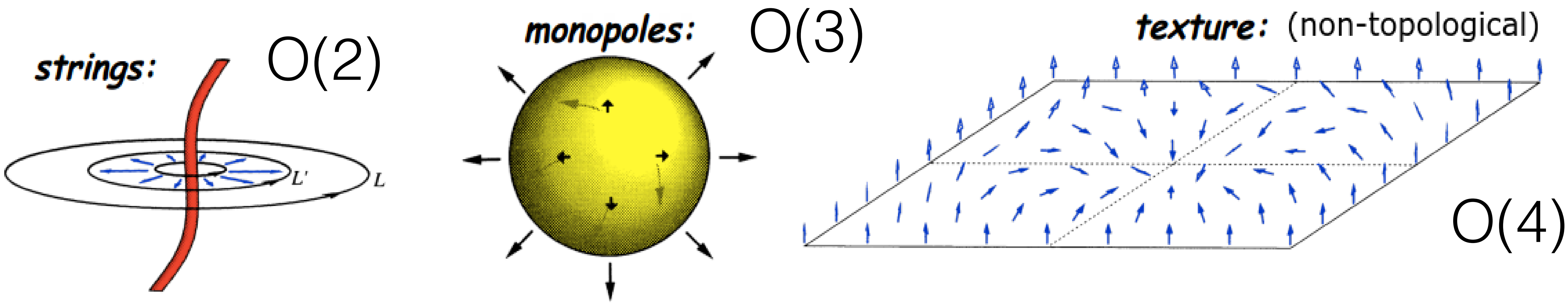
$$V = \frac{\lambda}{4}\left(|\Phi|^2 - v^2\right)^2 + V_{\rm int}(\Phi, \chi, T)$$

(1st Order, 2nd Order, Cross-Over)

$$V_{\rm int} \sim \begin{cases} g_{\rm T}^2|\Phi|^2T^2 & \text{(THERMAL)} \\ g^2|\Phi|^2\chi^2 & \text{(FIELD INT.)} \end{cases}$$



ZOOLOGY:

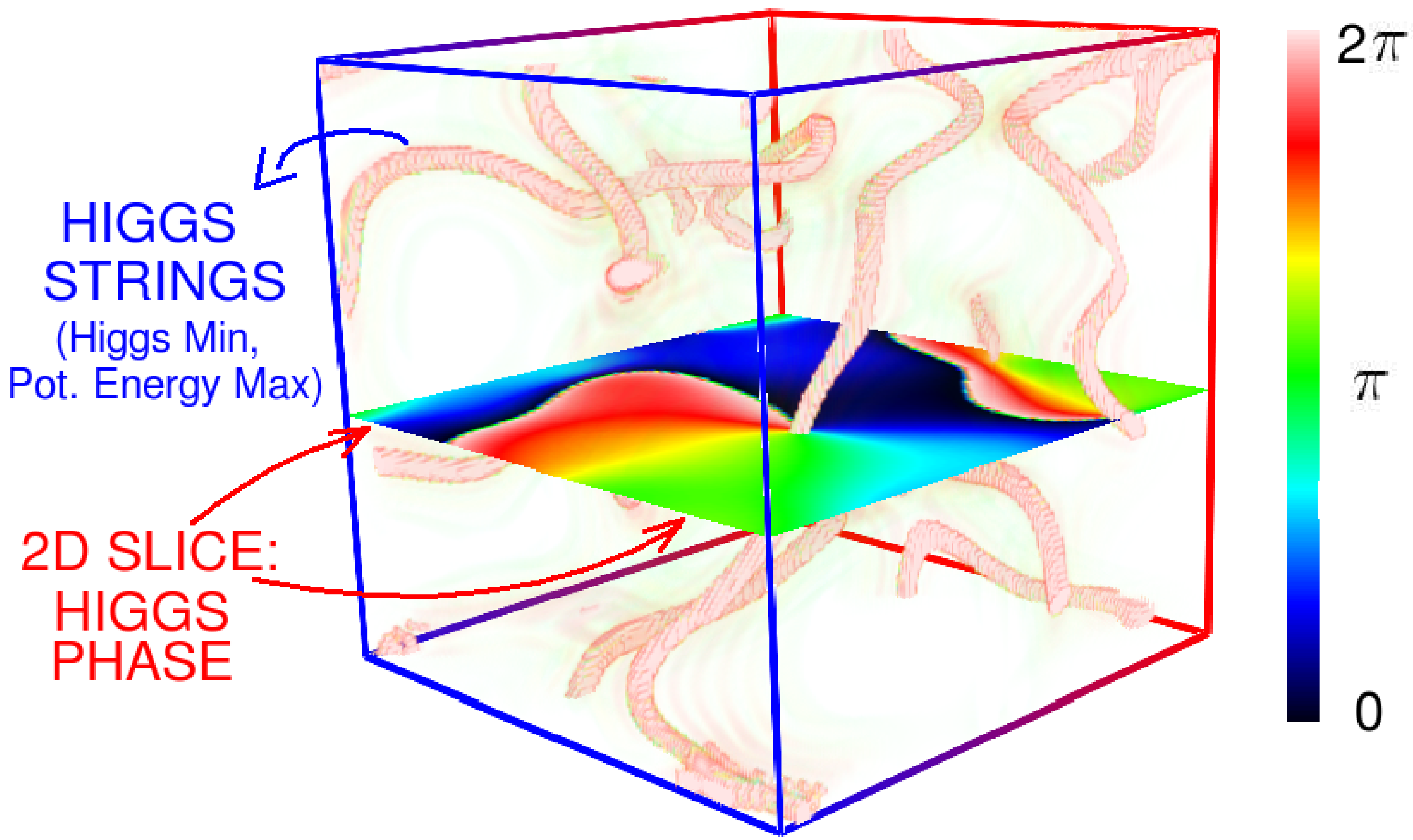


MICRO-PHYSICS → COSMIC DEFECTS

(M = G/H)

Example: Cosmic Strings

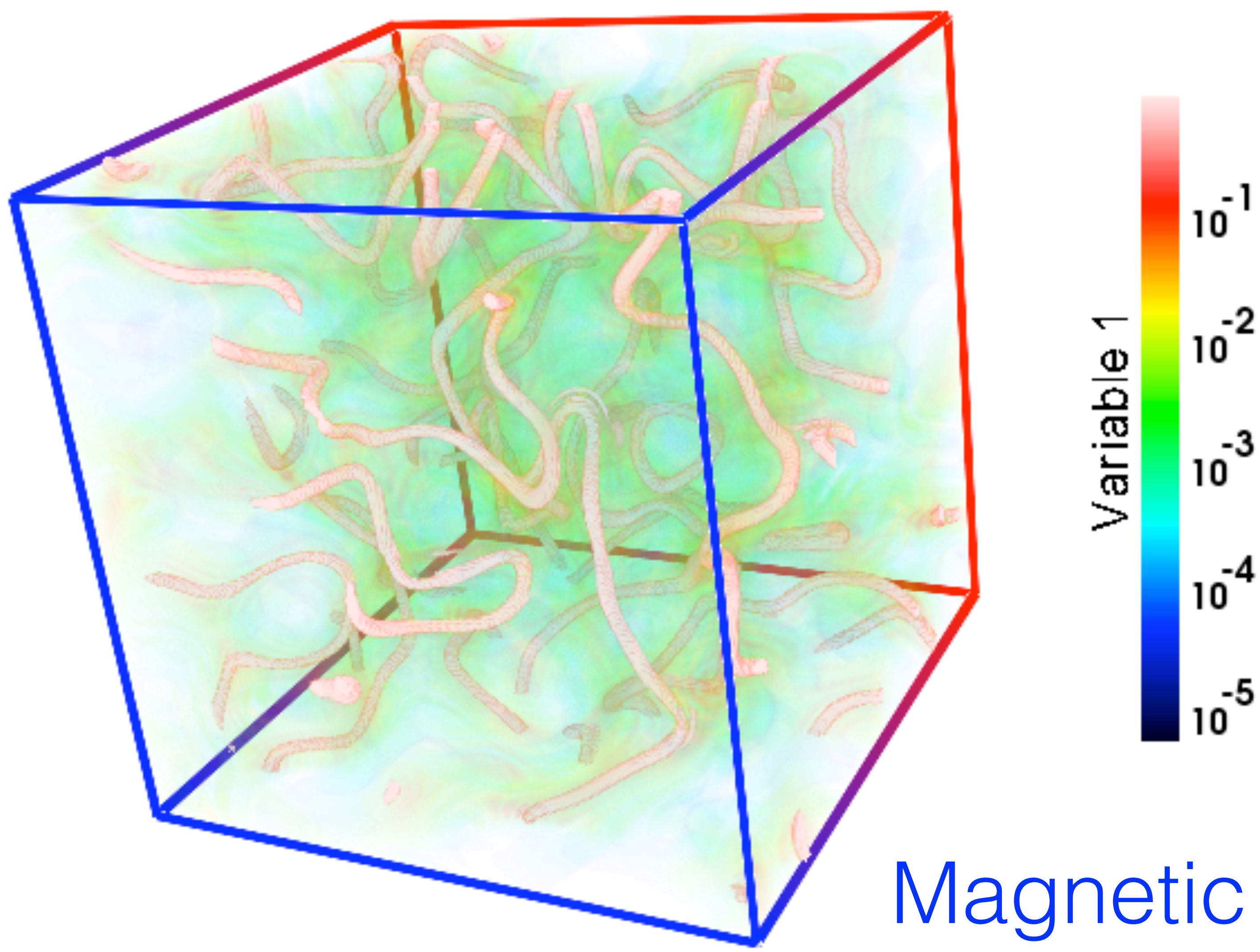
(e.g. From PhT after Hybrid Inflation)



Dufaux et al PRD 2010

Example: Cosmic Strings

(e.g. From PhT after Hybrid Inflation)



Magnetic Field
energy density

Dufaux et al PRD 2010

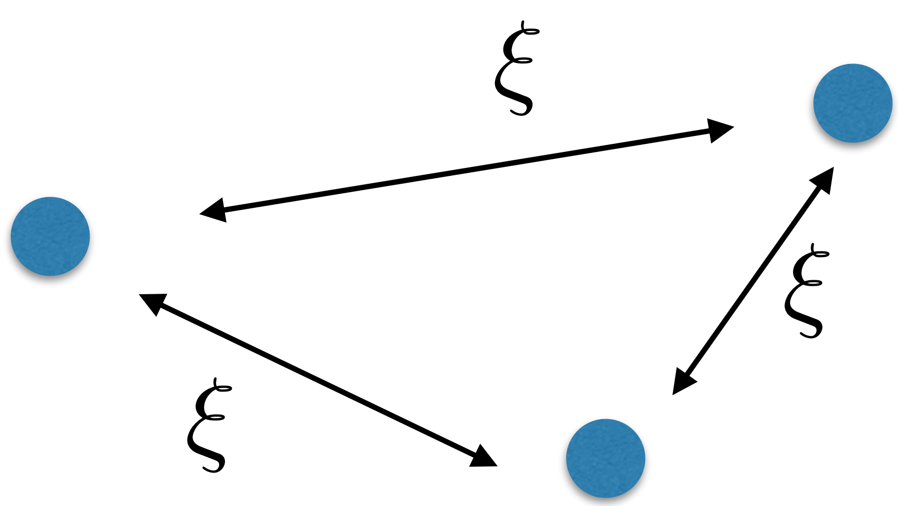
Introduction to Cosmic Defects

$$\text{DEFECTS: Aftermath of PhT} \rightarrow \begin{cases} \begin{cases} \text{Domain Walls} \\ \text{Cosmic Strings} \\ \text{Cosmic Monopoles} \end{cases} \\ \text{Non} - \text{Topological} \end{cases}$$

Introduction to Cosmic Defects

DEFECTS: Aftermath of PhT $\rightarrow \begin{cases} \begin{cases} \text{Domain Walls} \\ \text{Cosmic Strings} \\ \text{Cosmic Monopoles} \end{cases} \\ \text{Non} - \text{Topological} \end{cases}$

CAUSALITY & MICROPHYSICS $\Rightarrow$ Corr. Length: $\xi(t) = \lambda(t)\, H^{-1}(t)$



Introduction to Cosmic Defects

DEFECTS: Aftermath of PhT $\rightarrow \begin{cases} \begin{cases} \text{Domain Walls} \\ \text{Cosmic Strings} \\ \text{Cosmic Monopoles} \end{cases} \\ \text{Non-Topological} \end{cases}$

CAUSALITY & MICROPHYSICS $\Rightarrow$ Corr. Length: $\xi(t) = \lambda(t)\, H^{-1}(t)$

(Kibble' 76)

SCALING: $\lambda(t) = \text{const.} \rightarrow \lambda \sim 1 \quad \Longrightarrow \quad k/\mathcal{H} = kt$

comoving momentum

conformal time

Introduction to Cosmic Defects

DEFECTS: Aftermath of PhT $\rightarrow \begin{cases} \begin{cases} \text{Domain Walls} \\ \text{Cosmic Strings} \\ \text{Cosmic Monopoles} \end{cases} \\ \text{Non} - \text{Topological} \end{cases}$

CAUSALITY & MICROPHYSICS $\Rightarrow$ Corr. Length: $\xi(t) = \lambda(t)\, H^{-1}(t)$

(Kibble' 76)

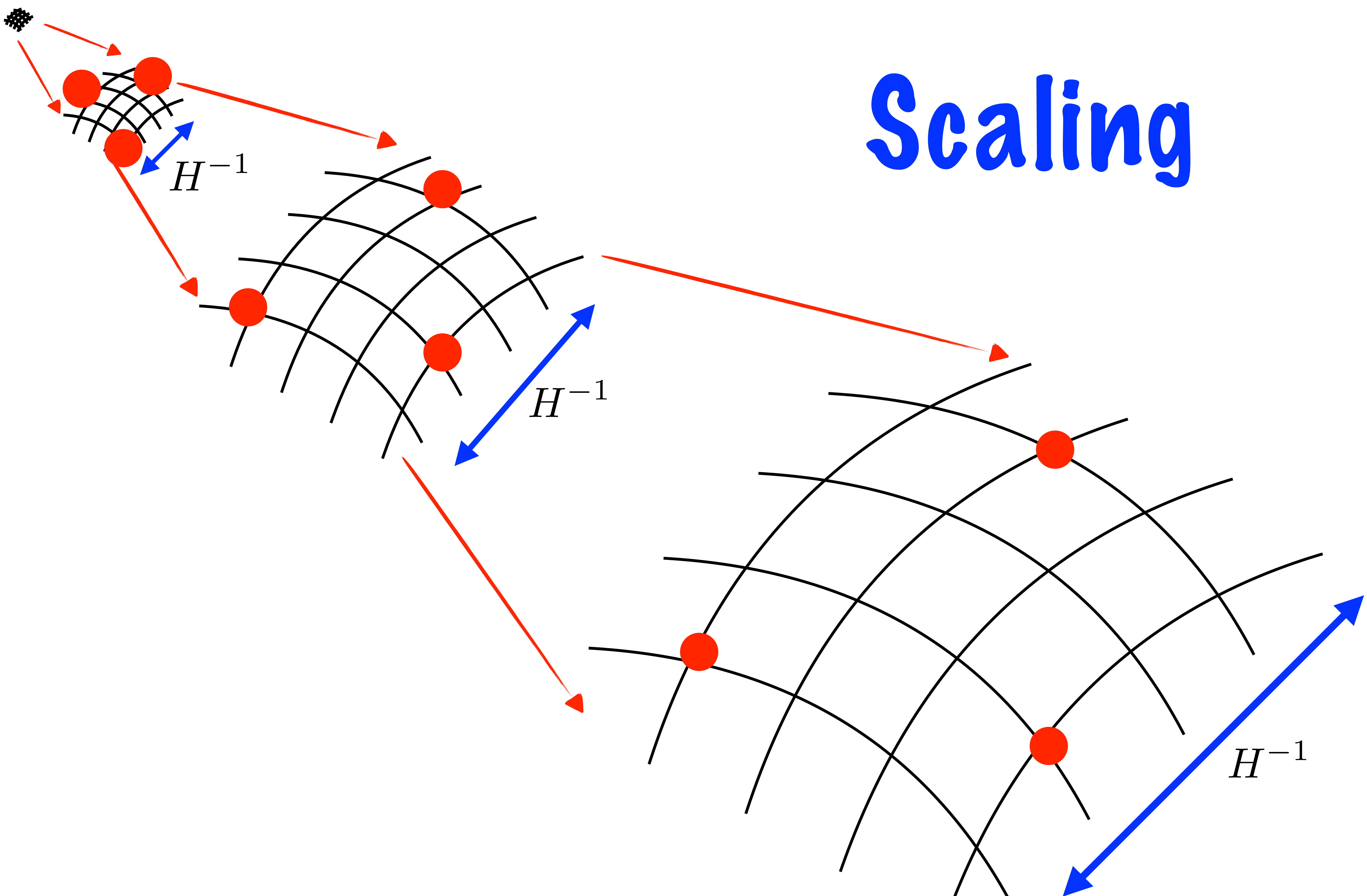
SCALING:* $\lambda(t) = \text{const.} \quad \rightarrow \quad \lambda \sim 1 \quad \Rightarrow \quad k/\mathcal{H} = kt$

(k: comoving momentum; t: conformal time)

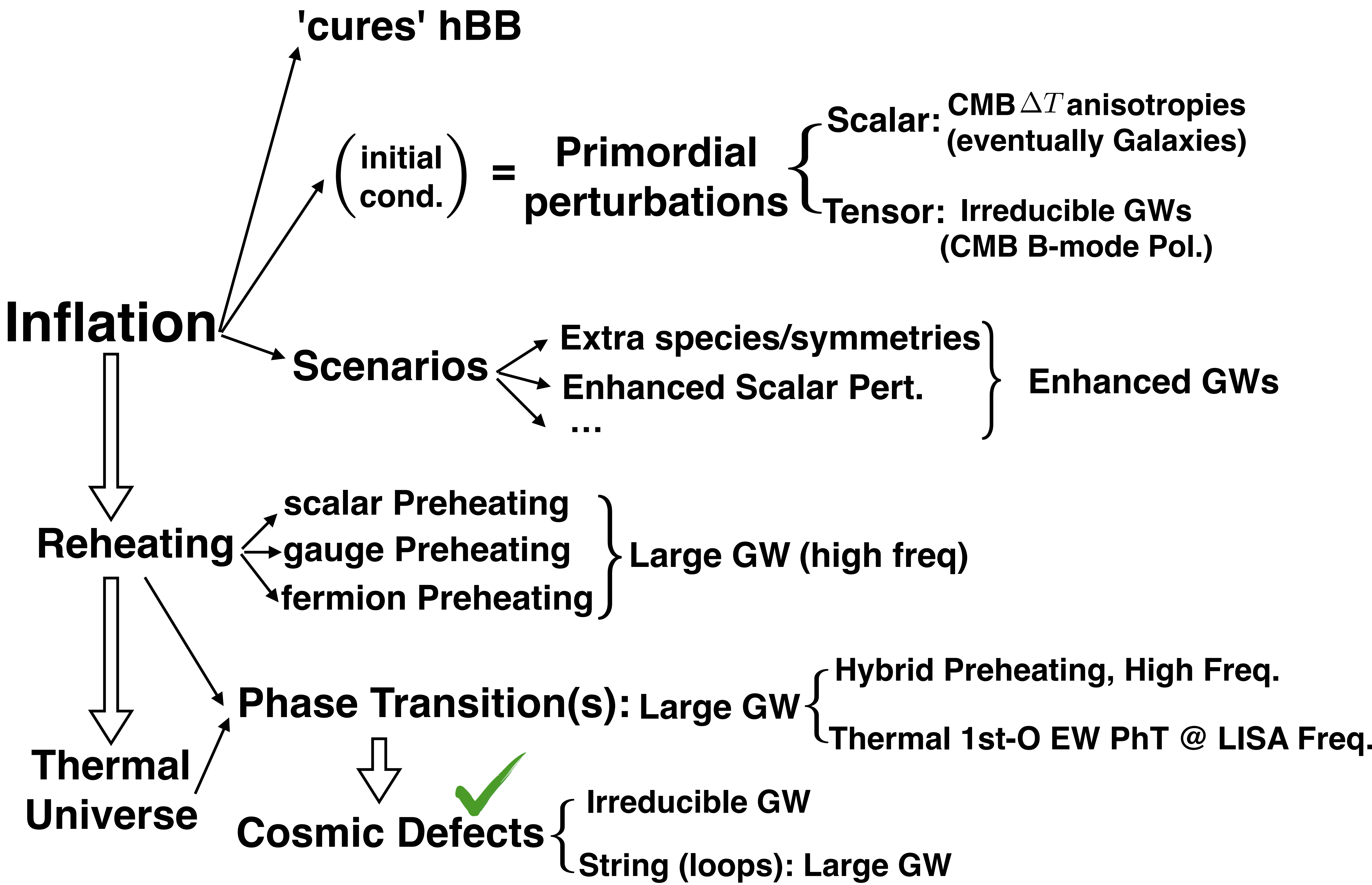
* Could be not exact in some cases

Introduction to Cosmic Defects

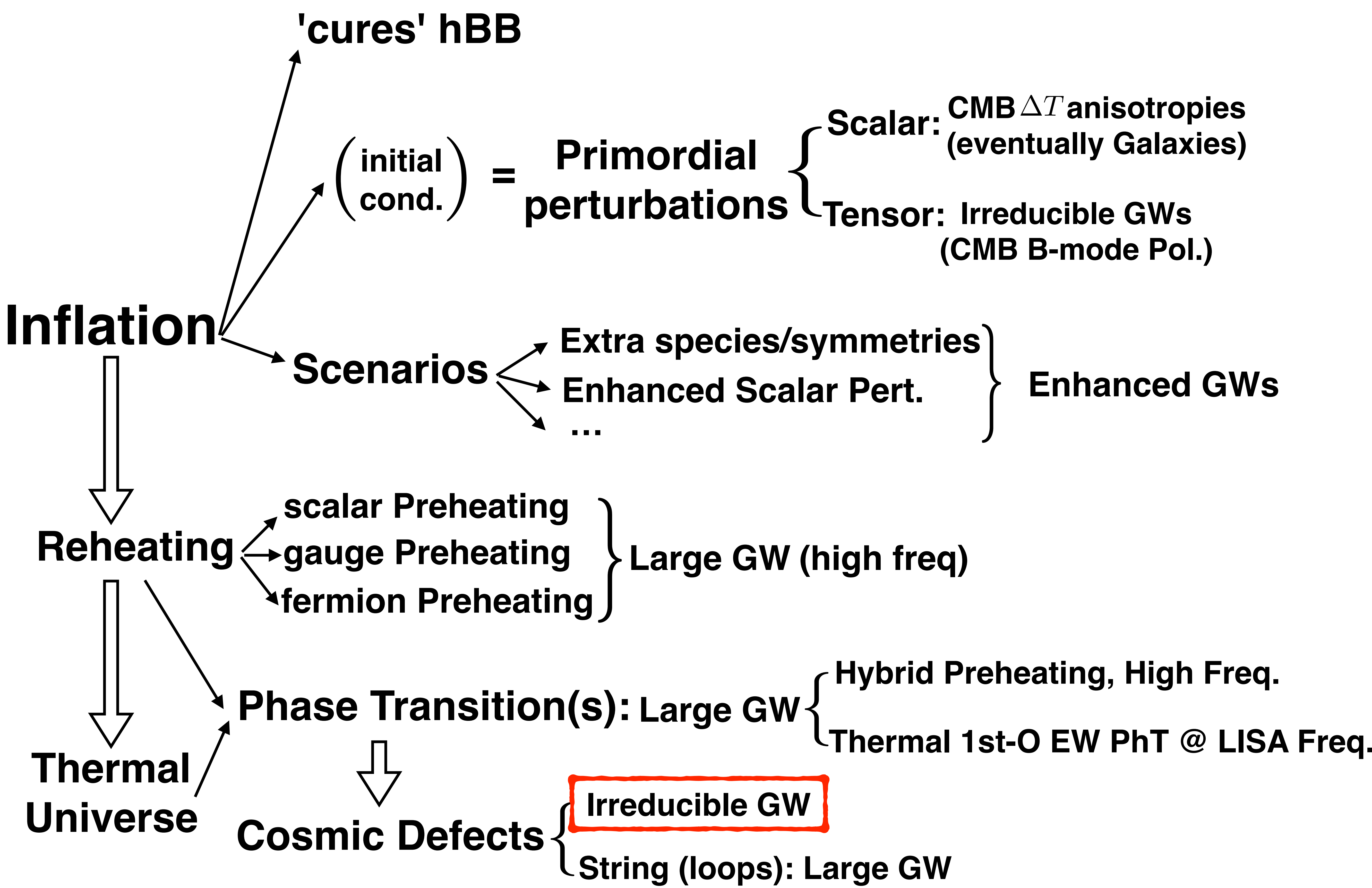
Scaling



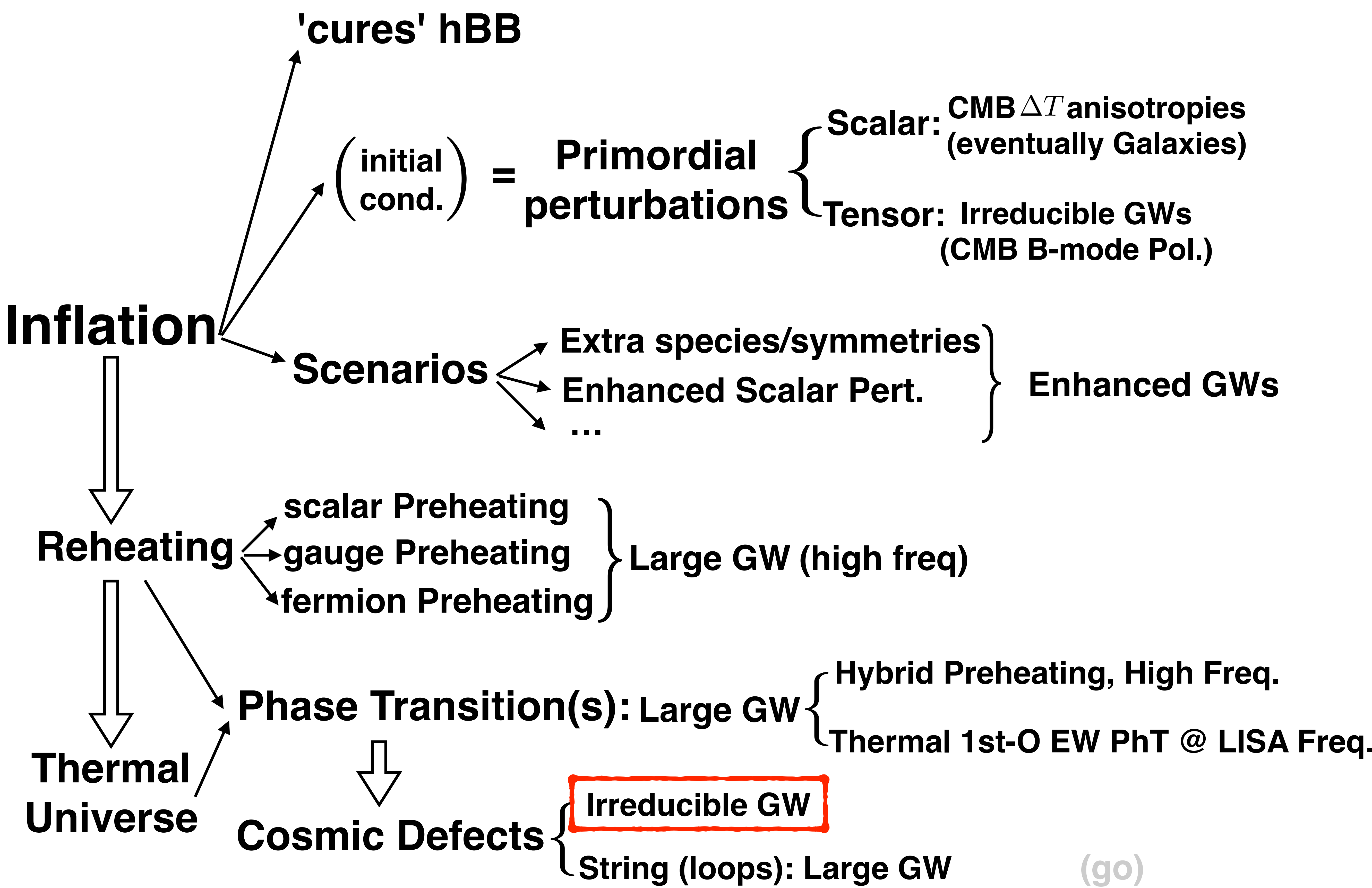
EARLY UNIVERSE



EARLY UNIVERSE



EARLY UNIVERSE



GWs from a Scaling network of cosmic defects

DEFECTS: GW Source $\rightarrow \{T_{ij}\}^{\rm TT} \propto \{\partial_i\phi\partial_j\phi, E_iE_j, B_iB_j\}^{\rm TT}$

GWs from a Scaling network of cosmic defects

DEFECTS: GW Source $\rightarrow \{T_{ij}\}^{\rm TT} \propto \{\partial_i\phi\partial_j\phi, E_iE_j, B_iB_j\}^{\rm TT}$

UTC: $\langle T_{ij}^{\rm TT}(\mathbf{k},t)T_{ij}^{\rm TT}(\mathbf{k}',t')\rangle = (2\pi)^3\,\Pi^2(k,t_1,t_2)\;\delta^3(\mathbf{k}-\mathbf{k}')$

(Unequal Time Correlator)

GWs from a Scaling network of cosmic defects

DEFECTS: GW Source $\rightarrow \{T_{ij}\}^{\rm TT} \propto \{\partial_i\phi\partial_j\phi, E_iE_j, B_iB_j\}^{\rm TT}$

UTC: $\langle T_{ij}^{\rm TT}(\mathbf{k},t)T_{ij}^{\rm TT}(\mathbf{k}',t')\rangle = (2\pi)^3\,\Pi^2(k,t_1,t_2)\;\delta^3(\mathbf{k}-\mathbf{k}')$

(Unequal Time Correlator)

GW spectrum: Expansion UTC

$$\frac{d\rho_{\rm GW}}{d\log k}(k,t) \propto \frac{k^3}{M_p^2a^4(t)}\int dt_1dt_2\;a(t_1)a(t_2)\;\cos(k(t_1-t_2))\;\Pi^2(k,t_1,t_2)$$

GWs from a Scaling network of cosmic defects

DEFECTS: GW Source $\rightarrow \{T_{ij}\}^{\rm TT} \propto \{\partial_i\phi\partial_j\phi, E_iE_j, B_iB_j\}^{\rm TT}$

UTC: $\langle T_{ij}^{\rm TT}(\mathbf{k},t)T_{ij}^{\rm TT}(\mathbf{k}',t')\rangle = (2\pi)^3\, \Pi^2(k,t_1,t_2)\; \delta^3(\mathbf{k}-\mathbf{k}')$

(Unequal Time Correlator)

GW spectrum: Expansion UTC

$$\frac{d\rho_{\rm GW}}{d\log k}(k,t) \propto \frac{k^3}{M_p^2 a^4(t)} \int dt_1 dt_2\; a(t_1)a(t_2)\; \cos(k(t_1-t_2))\; \Pi^2(k,t_1,t_2)$$

Comoving Scale (→ k) Conformal Time (→ t)

GWs from a Scaling network of cosmic defects

DEFECTS: GW Source $\rightarrow \{T_{ij}\}^{\rm TT} \propto \{\partial_i\phi\partial_j\phi, E_iE_j, B_iB_j\}^{\rm TT}$

SCALING

UTC: $$\langle T_{ij}^{\rm TT}(\mathbf{k},t)T_{ij}^{\rm TT}(\mathbf{k}',t')\rangle = (2\pi)^3\,\frac{{\rm V}^4}{\sqrt{tt'}}\,U(kt,kt')\delta^3(\mathbf{k}-\mathbf{k}')$$

GW spectrum: Expansion UTC

$$\frac{d\rho_{\rm GW}}{d\log k}(k,t) \propto \frac{k^3}{M_p^2a^4(t)}\int dt_1dt_2\; a(t_1)a(t_2)\;\cos(k(t_1-t_2))\,\frac{{\rm V}^4}{\sqrt{t_1t_2}}\,U(kt_1,kt_2)$$

SCALING

Comoving Scale

Conformal Time

GWs from a Scaling network of cosmic defects

DEFECTS: GW Source $\rightarrow \{T_{ij}\}^{\rm TT} \propto \{\partial_i\phi\partial_j\phi, E_iE_j, B_iB_j\}^{\rm TT}$

SCALING

UTC: $\langle T^{\rm TT}_{ij}(\mathbf{k},t)T^{\rm TT}_{ij}(\mathbf{k}',t')\rangle = (2\pi)^3\,\frac{{\rm V}^4}{\sqrt{tt'}}\,U(kt,kt')\delta^3(\mathbf{k}-\mathbf{k}')$

GW spectrum: Expansion UTC

$$\frac{d\rho_{\rm GW}}{d\log k}(k,t) \propto \frac{k^3}{M_p^2a^4(t)}\int dt_1dt_2 \quad t_1t_2 \quad \cos(k(t_1-t_2))\,\frac{{\rm V}^4}{\sqrt{t_1t_2}}\,U(kt_1,kt_2)$$

Rad. Dom **SCALING**

Comoving Scale (k) — Conformal Time (t)

GWs from a Scaling network of cosmic defects

DEFECTS: GW Source $\rightarrow \{T_{ij}\}^{\rm TT} \propto \{\partial_i\phi\partial_j\phi, E_iE_j, B_iB_j\}^{\rm TT}$

SCALING

UTC: $\langle T_{ij}^{\rm TT}(\mathbf{k},t)T_{ij}^{\rm TT}(\mathbf{k}',t')\rangle = (2\pi)^3\,\frac{\rm V^4}{\sqrt{tt'}}\,U(kt,kt')\,\delta^3(\mathbf{k}-\mathbf{k}')$

GW spectrum: $(x_i \equiv kt_i)$ Expansion UTC

$$\frac{d\rho_{\rm GW}}{d\log k}(k,t) \propto \left(\frac{\rm V}{M_p}\right)^4 \frac{M_p^2}{a^4(t)}\left[\int dx_1dx_2\,\sqrt{x_1x_2}\,\cos(x_1-x_2)\,\,U(x_1,x_2)\right]$$

Rad. Dom **SCALING**

GWs from a Scaling network of cosmic defects

DEFECTS: GW Source $\rightarrow \{T_{ij}\}^{\rm TT} \propto \{\partial_i\phi\partial_j\phi, E_iE_j, B_iB_j\}^{\rm TT}$

SCALING

UTC: $\langle T_{ij}^{\rm TT}(\mathbf{k},t)T_{ij}^{\rm TT}(\mathbf{k}',t')\rangle = (2\pi)^3\ \frac{\mathrm{V}^4}{\sqrt{tt'}}\,U(kt,kt')\delta^3(\mathbf{k}-\mathbf{k}')$

$(x_i \equiv kt_i)$

GW spectrum: Expansion UTC

$$\frac{d\rho_{\rm GW}}{d\log k}(k,t) \propto \left(\frac{\mathrm{V}}{M_p}\right)^4 \frac{M_p^2}{a^4(t)}\left[\int dx_1dx_2\ \sqrt{x_1x_2}\ \cos(x_1-x_2)\ U(x_1,x_2)\right]$$

Rad. Dom **SCALING**

$F_U \sim$ Const. (Dimensionless)

GWs from a Scaling network of cosmic defects

GW today:

VEV

Scaling @ RD

$$\Omega_{GW}^{(o)} \equiv \frac{1}{\rho_c^{(o)}}\left(\frac{d\rho_{\rm GW}}{d\log k}\right)_o = \frac{32}{3}\left(\frac{\rm V}{M_p}\right)^4 \Omega_{\rm rad}^{(o)}\; F_U\,, \quad \text{(SCALE INV.!)}$$

Defect type

$$F_U \equiv \int_0^x dx_1 dx_2 \sqrt{x_1 x_2} cos(x_1 - x_2) U(x_1, x_2)$$

DGF, Hindmarsh, Urrestilla, PRL 2013

GWs from a Scaling network of cosmic defects

VEV

Scaling @ RD

GW today:

$$\Omega_{GW}^{(o)} \equiv \frac{1}{\rho_c^{(o)}} \left(\frac{d\rho_{\rm GW}}{d \log k} \right)_o = \frac{32}{3} \left(\frac{\rm V}{M_p} \right)^4 \Omega_{\rm rad}^{(o)} \, F_U \, , \quad \text{(SCALE INV.!)}$$

Defect type

$$F_U \equiv \int_0^x dx_1 dx_2 \sqrt{x_1 x_2} cos(x_1 - x_2) U(x_1, x_2)$$

∀ PhT (1st, 2nd, ...), ∀ Defects (top. or non-top.)

DGF, Hindmarsh, Urrestilla, PRL 2013

GWs from a Scaling network of cosmic defects

Total GW Spectrum

energy scale

constants

$$h^2\Omega_{\rm GW}^{(\rm o)} = h^2\Omega_{\rm rad}^{(\rm o)}\left(\frac{V}{M_p}\right)^4\left[F_U^{(\rm R)} + F_U^{(\rm M)}\left(\frac{k_{\rm eq}}{k}\right)^2\right]$$

RD $$F_U^{(\rm R)} \equiv \frac{32}{3}\int_0^x dx_1 dx_2\,(x_1x_2)^{1/2}\cos(x_1-x_2)\,U_{\rm RD}(x_1,x_2)$$

MD $$F_U^{(\rm M)} \equiv \frac{32}{3}\frac{(\sqrt{2}-1)^2}{2}\int_{x_{\rm eq}}^x dx_1 dx_2\,(x_1x_2)^{3/2}\cos(x_1-x_2)\,U_{\rm MD}(x_1,x_2)$$

DGF, Hindmarsh, Lizarraga, Urrestilla, PRD 2020

GWs from a Scaling network of cosmic defects

energy scale

constants

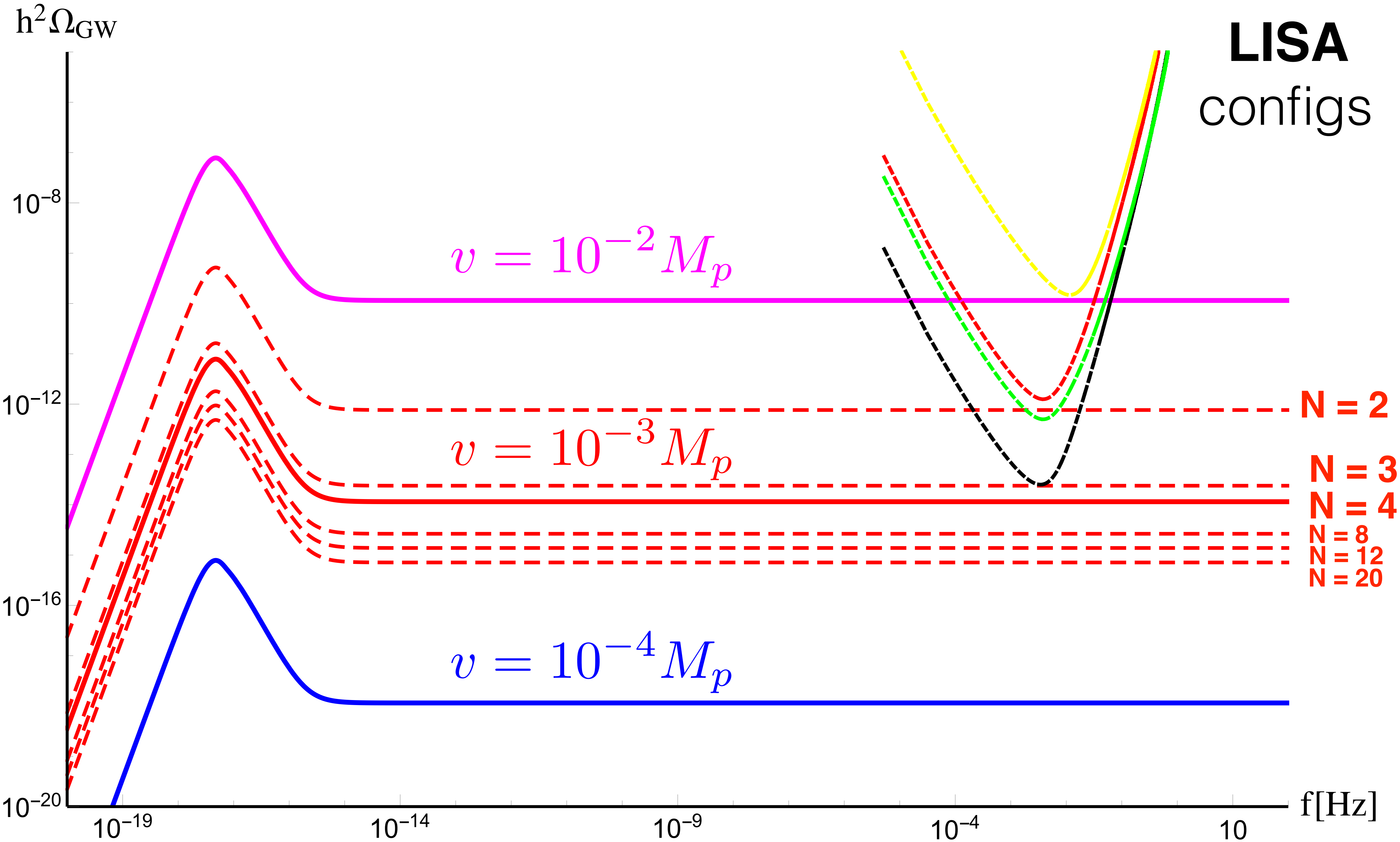
Total GW Spectrum

$$h^2\Omega_{\rm GW}^{(\rm o)} = h^2\Omega_{\rm rad}^{(\rm o)} \left(\frac{V}{M_p}\right)^4 \left[F_U^{(\rm R)} + F_U^{(\rm M)}\left(\frac{k_{\rm eq}}{k}\right)^2\right]$$

However this assumes exact scaling !

DGF, Hindmarsh, Lizarraga, Urrestilla, PRD 2020

GWs from a Scaling network of cosmic defects



DGF, Hindmarsh, Lizarraga, Urrestilla, PRD 2020

GWs from a Scaling network of cosmic defects

Global strings (e.g. axion DM) → **Log corrections argued & found**

Gorghetto, Hardy, Villadoro, JHEP 2018

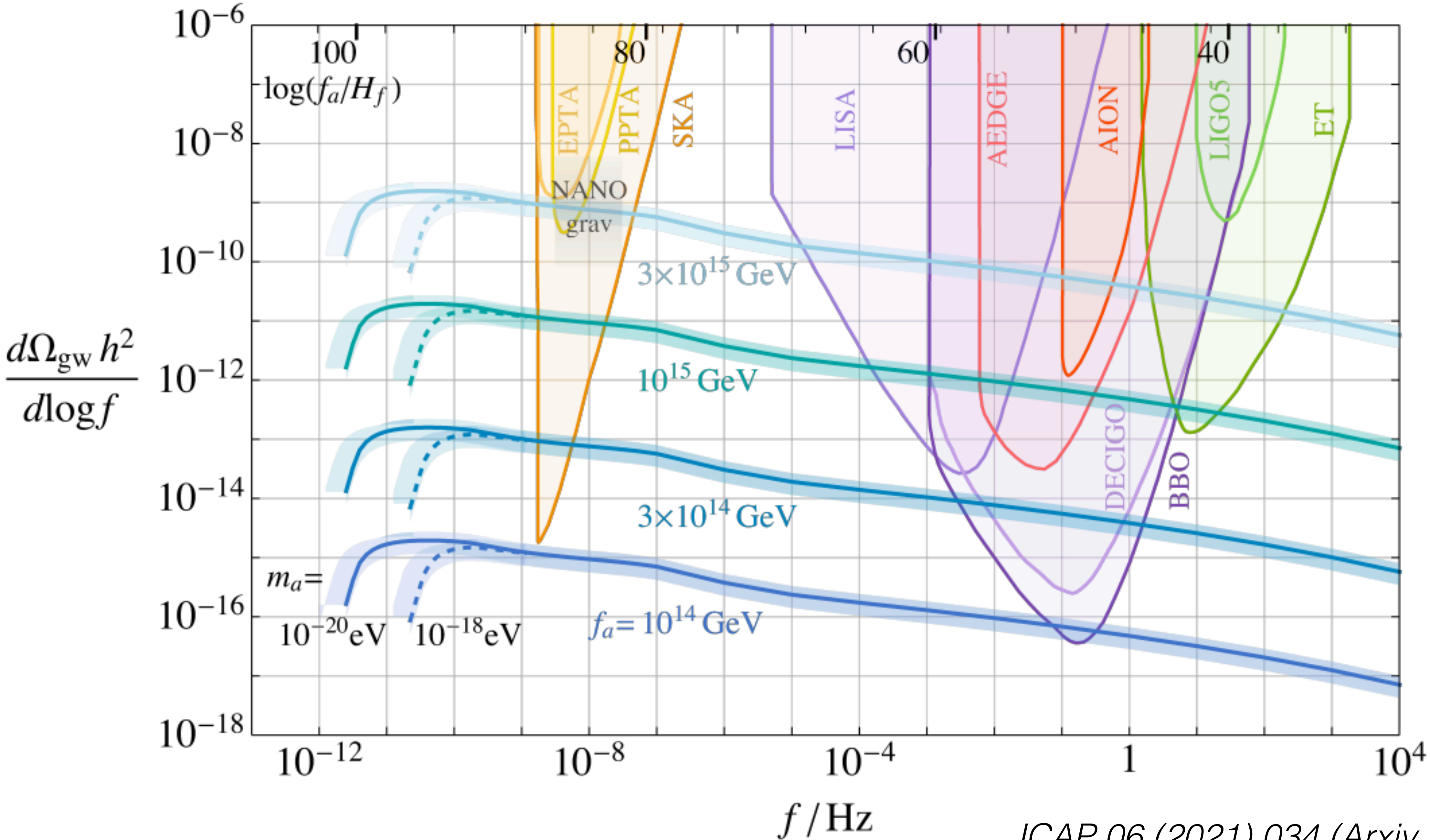
GWs from a Scaling network of Global Strings

Global strings (e.g. axion DM) → **Log corrections argued & found**

Gorghetto, Hardy, Villadoro, JHEP 2018

GWs from a Scaling network of Global Strings

Not scale invariant due to Log enhancements !

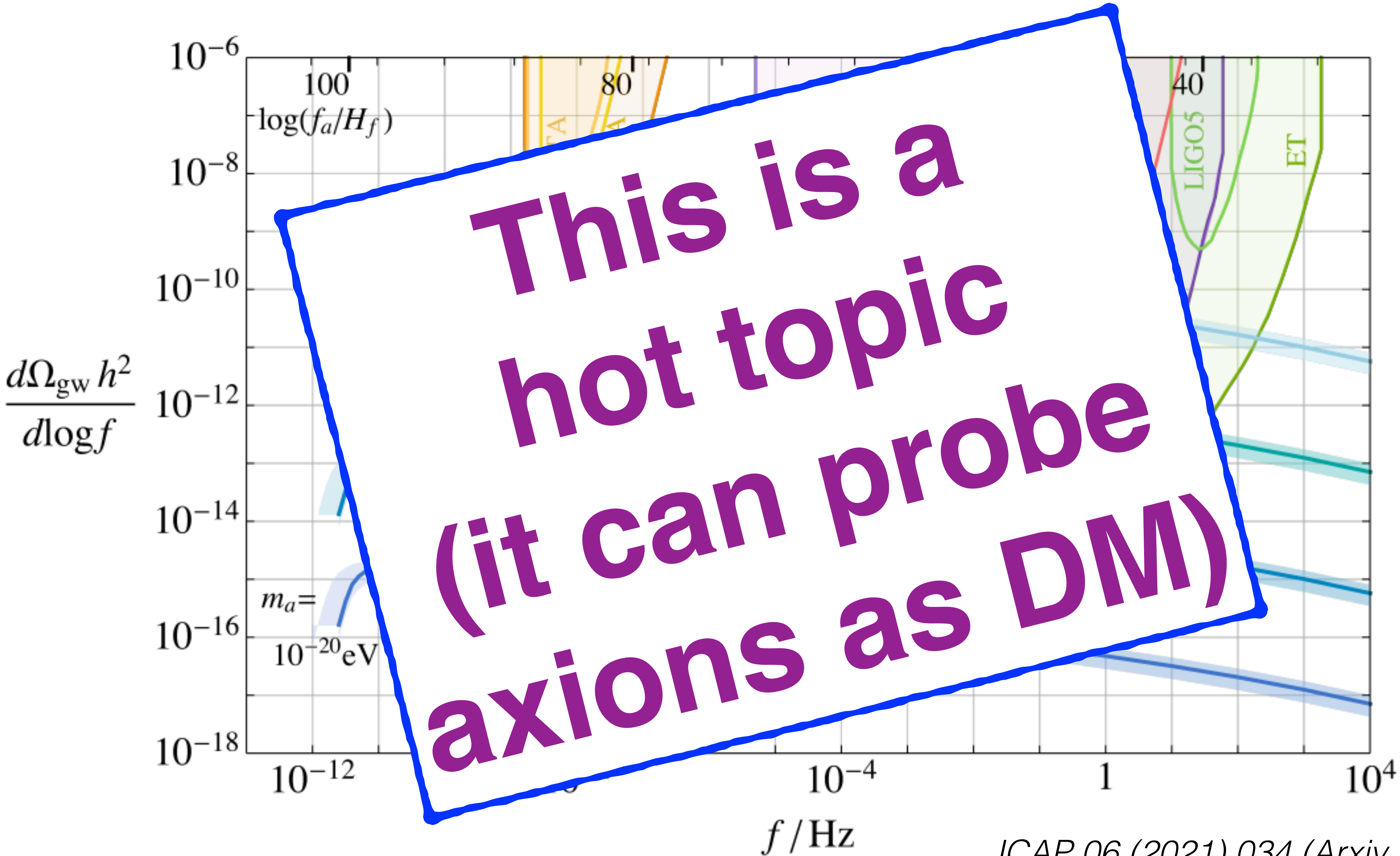


JCAP 06 (2021) 034 (Arxiv 2101.11007)

e.g. Gorghetto et al

GWs from a **Scaling network** of **Global Strings**

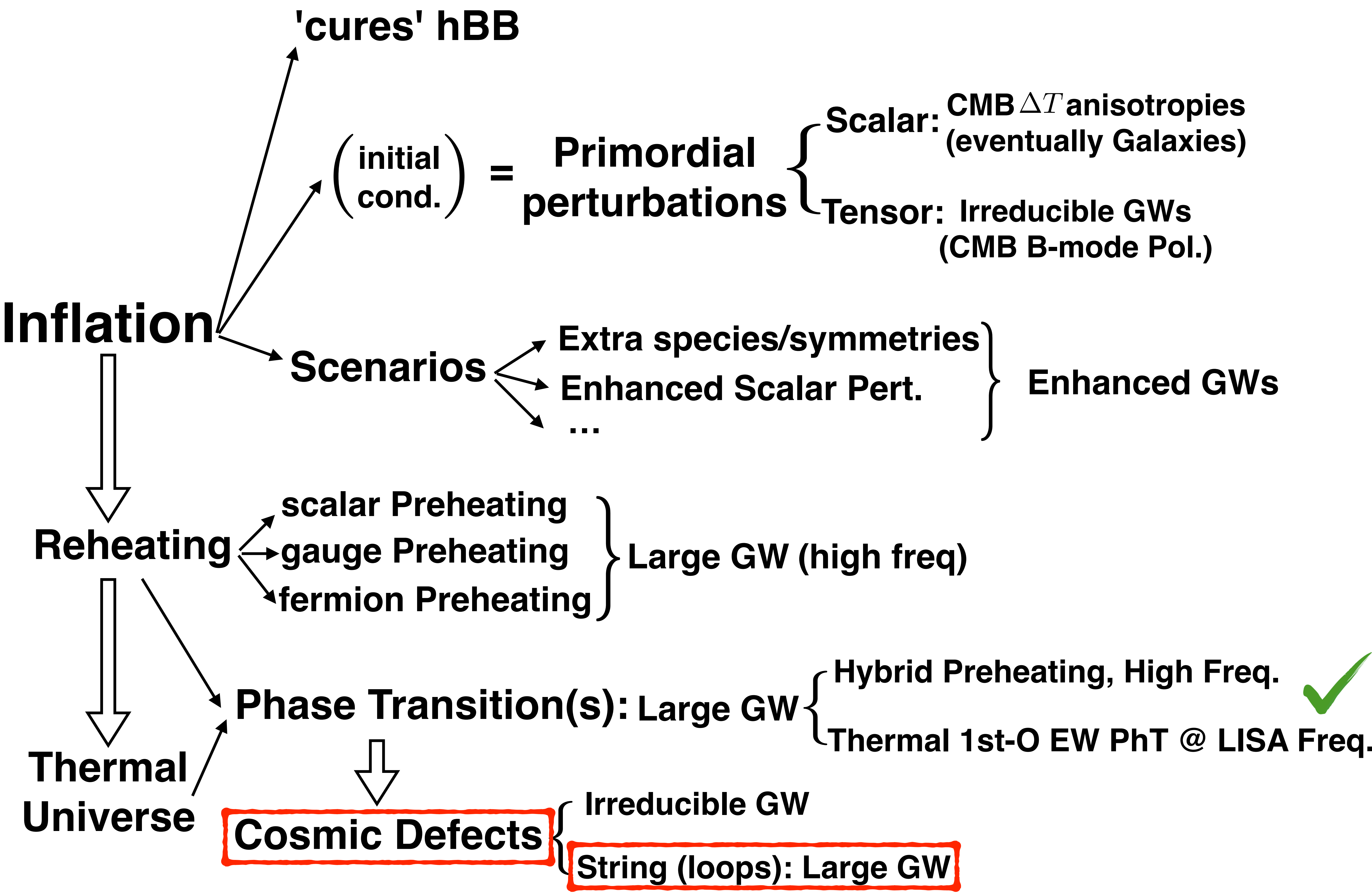
Not scale invariant due to **Log enhancements !**



JCAP 06 (2021) 034 (Arxiv 2101.11007)

e.g. Gorghetto et al

EARLY UNIVERSE



Local String Networks

* Scaling dynamics **(exact)**

* Infinitely thin

* Inter-commutation

Local String Networks

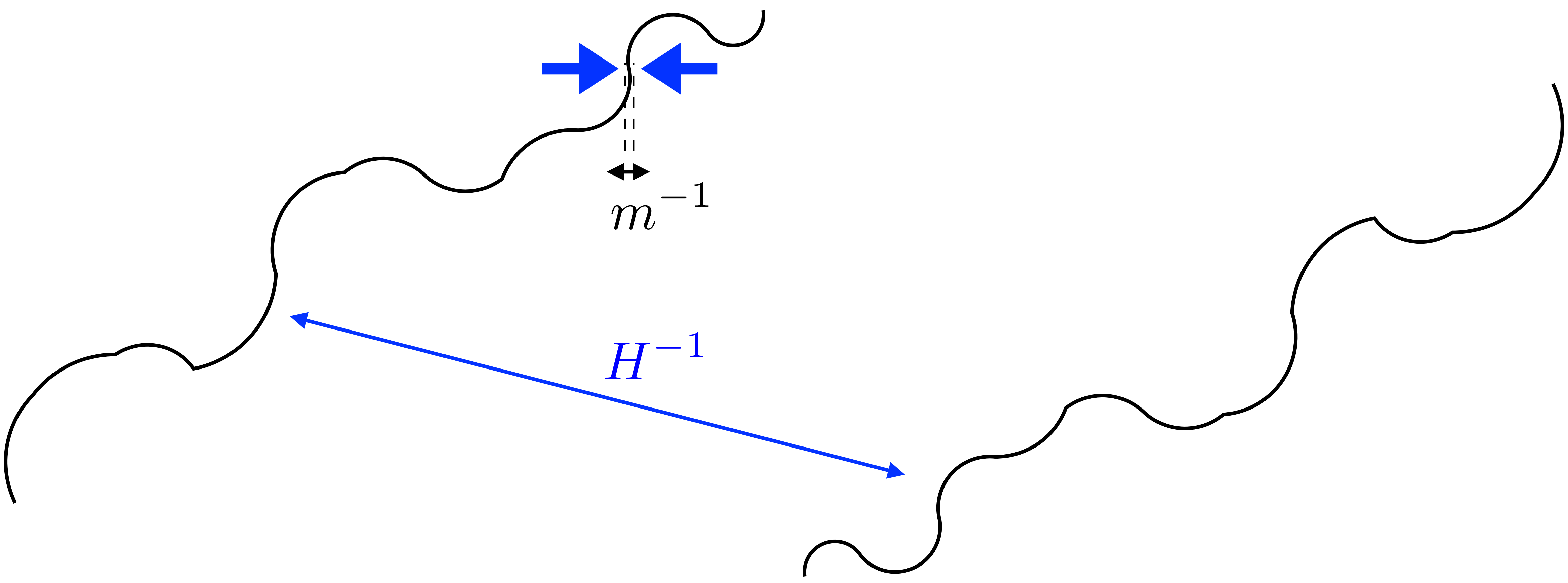
* Scaling dynamics

* Infinitely thin

* Inter-commutation

Local String Networks

Infinitely thin: $H^{-1} \gg m^{-1}$



Nambu-Goto

Local String Networks

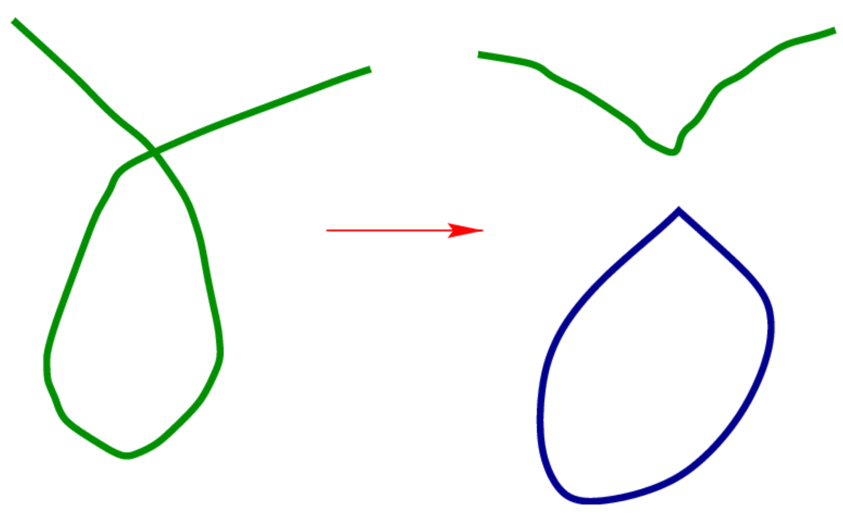
* Scaling dynamics

* Infinitely thin

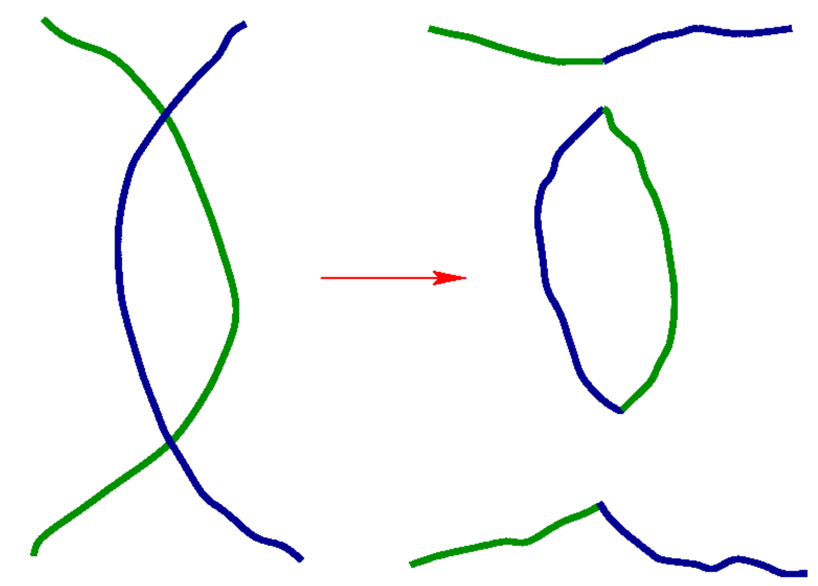
* Inter-commutation

Local String Networks

Intercommutation



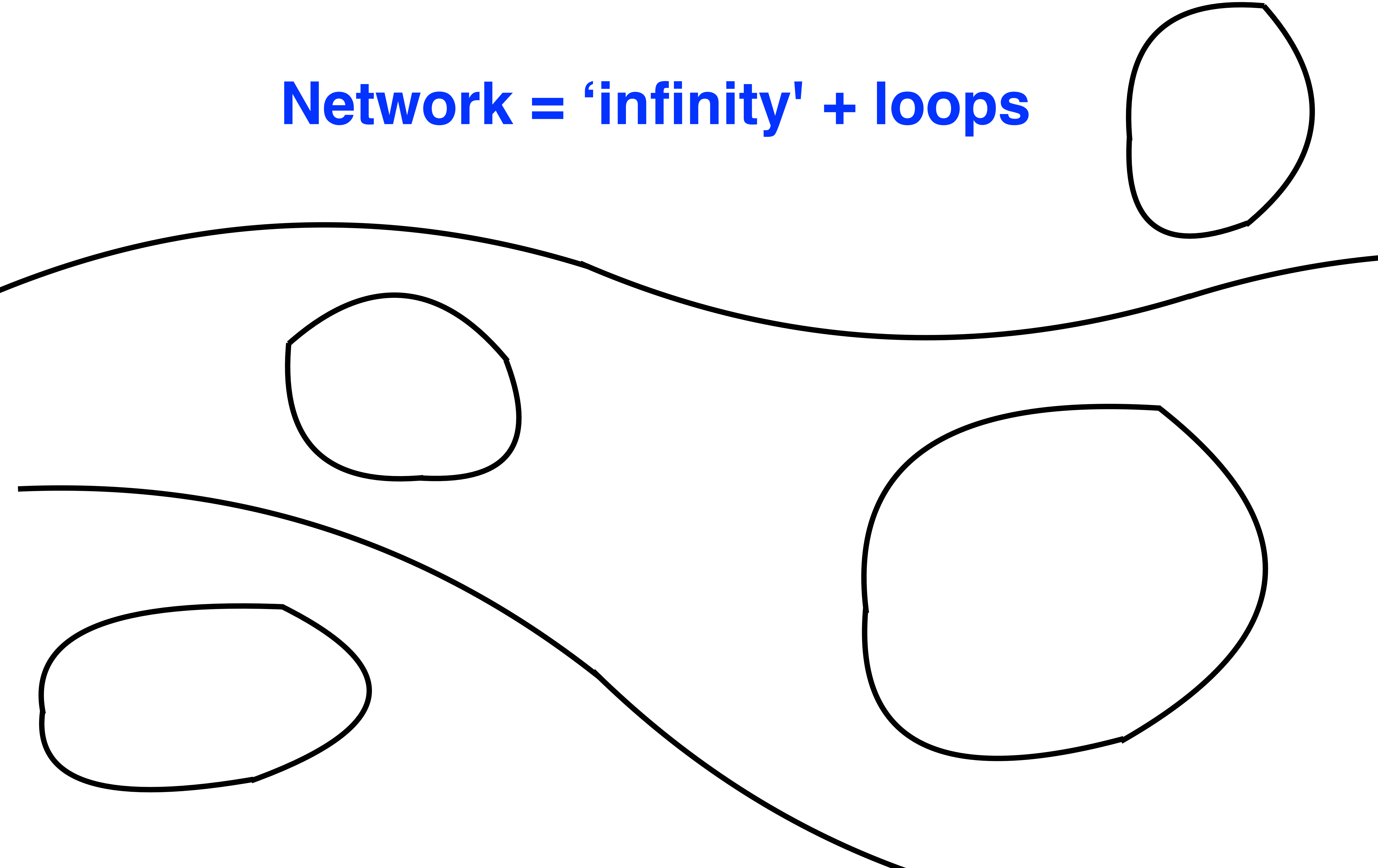
Loops !



Loops !

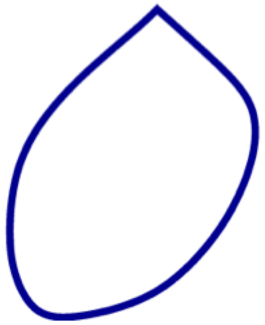
Local String Networks

Network = ‘infinity' + loops



Local String Networks

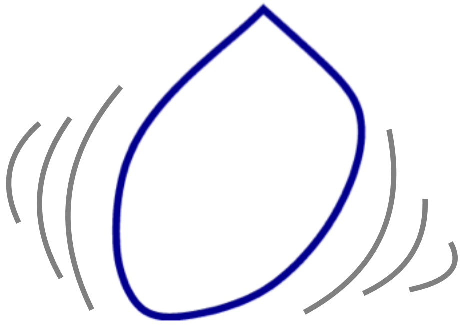
Loops are formed !



Local String Networks

Loops are formed !

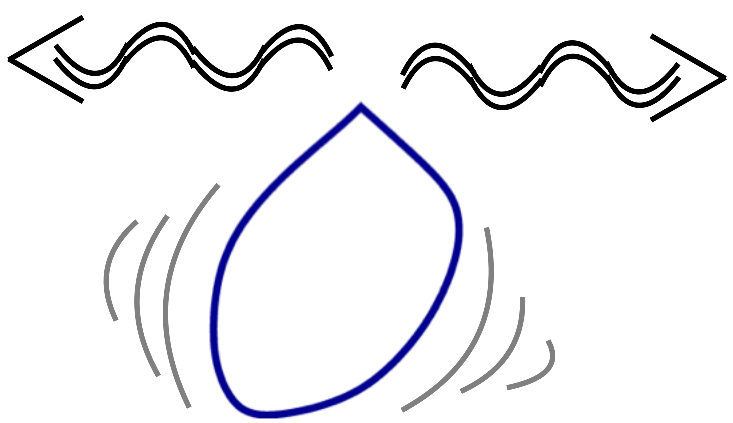
Vibrate under their tension !



Periodic Oscillations

Local String Networks

Loops are formed !



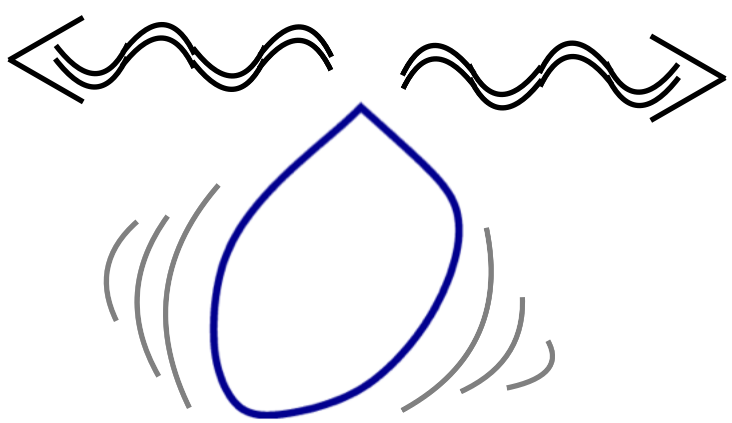
Vibrate under their tension !

Gravitational Waves (GW) are emitted !

Local String Networks

Loops are formed !

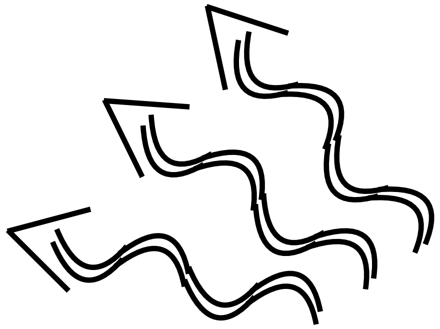
Vibrate under their tension !



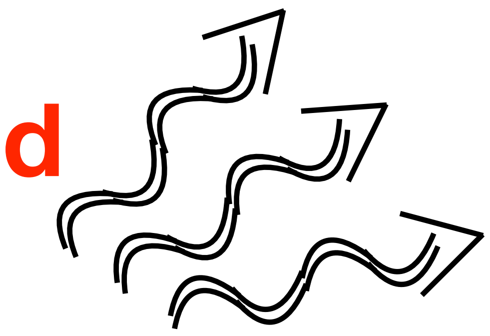
Gravitational Waves (GW) are emitted !

Superposition from many loop signals

=



Gravitational Wave Background



Local String Networks

Traditional picture $\longrightarrow$ **Nambu-Goto approximation** (zero width)

- String networks = Infinite strings + Loops

Local String Networks

Traditional picture ⟶ **Nambu-Goto approximation** (zero width)

- String networks = Infinite strings + Loops

  ↳ 'Decay' to loops

Local String Networks

Traditional picture ⟶ **Nambu-Goto approximation** (zero width)

- String networks = Infinite strings + Loops
  - Loops → Decay to GWs **(Vilenkin '81)**
  - Infinite strings → 'Decay' to loops

Local String Networks

Traditional picture ⟶ **Nambu-Goto approximation** (zero width)

- String networks = Infinite strings + Loops
  - Loops → Decay to GWs
  - Infinite strings → 'Decay' to loops
- Loops decay via GWs radiated in all harmonic frequencies ν_j

$$P_j = \Gamma G\mu^2 \frac{j^{-q}}{\zeta(q)} \longrightarrow P_{\rm GW} = \dot{E}_{\rm GW} = \sum_{j=1}^{\infty} P_j = \Gamma G\mu^2$$

Local String Networks

Traditional picture ⟶ **Nambu-Goto approximation** (zero width)

- String networks = Infinite strings + Loops
  - Loops → Decay to GWs
  - Infinite strings → 'Decay' to loops
- Loops decay via GWs radiated in all harmonic frequencies ν_j

$$P_j = \Gamma G\mu^2 \frac{j^{-q}}{\zeta(q)} \longrightarrow P_{\rm GW} = \dot{E}_{\rm GW} = \sum_{j=1}^{\infty} P_j = \Gamma G\mu^2$$

But …

Field-theory strings can also decay via **particle emission**

Local String Networks

Traditional picture ⟶ **Nambu-Goto approximation** (zero width)

- String networks = Infinite strings + Loops
  - Loops → Decay to GWs
  - Infinite strings → 'Decay' to loops
- Loops decay via GWs radiated in all harmonic frequencies ν_j

$$P_j = \Gamma G\mu^2 \frac{j^{-q}}{\zeta(q)} \longrightarrow P_{\rm GW} = \dot{E}_{\rm GW} = \sum_{j=1}^{\infty} P_j = \Gamma G\mu^2$$

But …

Field-theory strings can also decay via **particle emission**

Following … we assume GW emission dominates !

Local String Networks

Traditional picture ⟶ **Nambu-Goto approximation** (zero width)

- String networks = Infinite strings + Loops
  - Loops ⟶ Decay to GWs
  - Infinite strings ⟶ 'Decay' to loops
- Loops decay via GWs radiated in all harmonic frequencies ν_j

$$P_j = \Gamma G\mu^2 \frac{j^{-q}}{\zeta(q)} \longrightarrow P_{\rm GW} = \dot{E}_{\rm GW} = \sum_{j=1}^{\infty} P_j = \Gamma G\mu^2$$

But …

Field-theory strings can also decay via **particle emission**

Following … we assume GW emission dominates* !

(*Lattice calculations show opposite… not published yet)

Local String Network: Loop configurations

Cosmic string loop (length *l*) oscillates under tension μ

➡ **emits GWs in a series of harmonic modes**

Local String Network: Loop configurations

Cosmic string loop (length l) oscillates under tension μ

➡ **emits GWs in a series of harmonic modes**

Original emission of GWs ! **(Vilenkin ’81)**

and many others !

Local String Network: Loop configurations

Cosmic string loop (length l) oscillates under tension μ

➡ **emits GWs in a series of harmonic modes**

Original emission of GWs ! **(Vilenkin '81)**
and many others !

↓

"extra" emission on top of Irreducible background (only for strings)

Local String Network: Loop configurations

Cosmic string loop (length *l*) oscillates under tension μ

➡ **emits GWs in a series of harmonic modes**

Original emission of GWs ! **(Vilenkin '81)**

and many others !

$$\frac{d\rho^{(\mathrm{o})}}{df} \equiv \Gamma G\mu^2 \int_{t_*}^{t_o} dt \left(\frac{a(t)}{a_o}\right)^3 \int_0^{\alpha/H(t)} dl\, ln(l,t)\, \mathcal{P}((a_o/a(t))fl)$$

Local String Network: Loop configurations

Cosmic string loop (length *l*) oscillates under tension μ

→ emits GWs in a series of harmonic modes

Original emission of GWs ! **(Vilenkin '81)**

and many others !

$$\frac{d\rho^{(\mathrm{o})}}{df} \equiv \Gamma G\mu^2 \int_{t_*}^{t_o} dt \left(\frac{a(t)}{a_o}\right)^3 \int_0^{\alpha/H(t)} dl\, l\, n(l,t)\, \mathcal{P}((a_o/a(t)) f l)$$

expansion history

Local String Network: Loop configurations

Cosmic string loop (length *l*) oscillates under tension μ

→ emits GWs in a series of harmonic modes

Original emission of GWs ! **(Vilenkin '81)**

and many others !

$$\frac{d\rho^{(o)}}{df} \equiv \Gamma G\mu^2 \int_{t_*}^{t_o} dt \left(\frac{a(t)}{a_o}\right)^3 \int_0^{\alpha/H(t)} dl\, l\, n(l,t)\, \mathcal{P}((a_o/a(t))fl)$$

expansion history (← $\left(\frac{a(t)}{a_o}\right)^3$)

length (← l)

number density (← $n(l,t)$) (**Nambu-Goto simulations**)

Local String Network: Loop configurations

Cosmic string loop (length l) oscillates under tension μ

➡ **emits GWs in a series of harmonic modes**

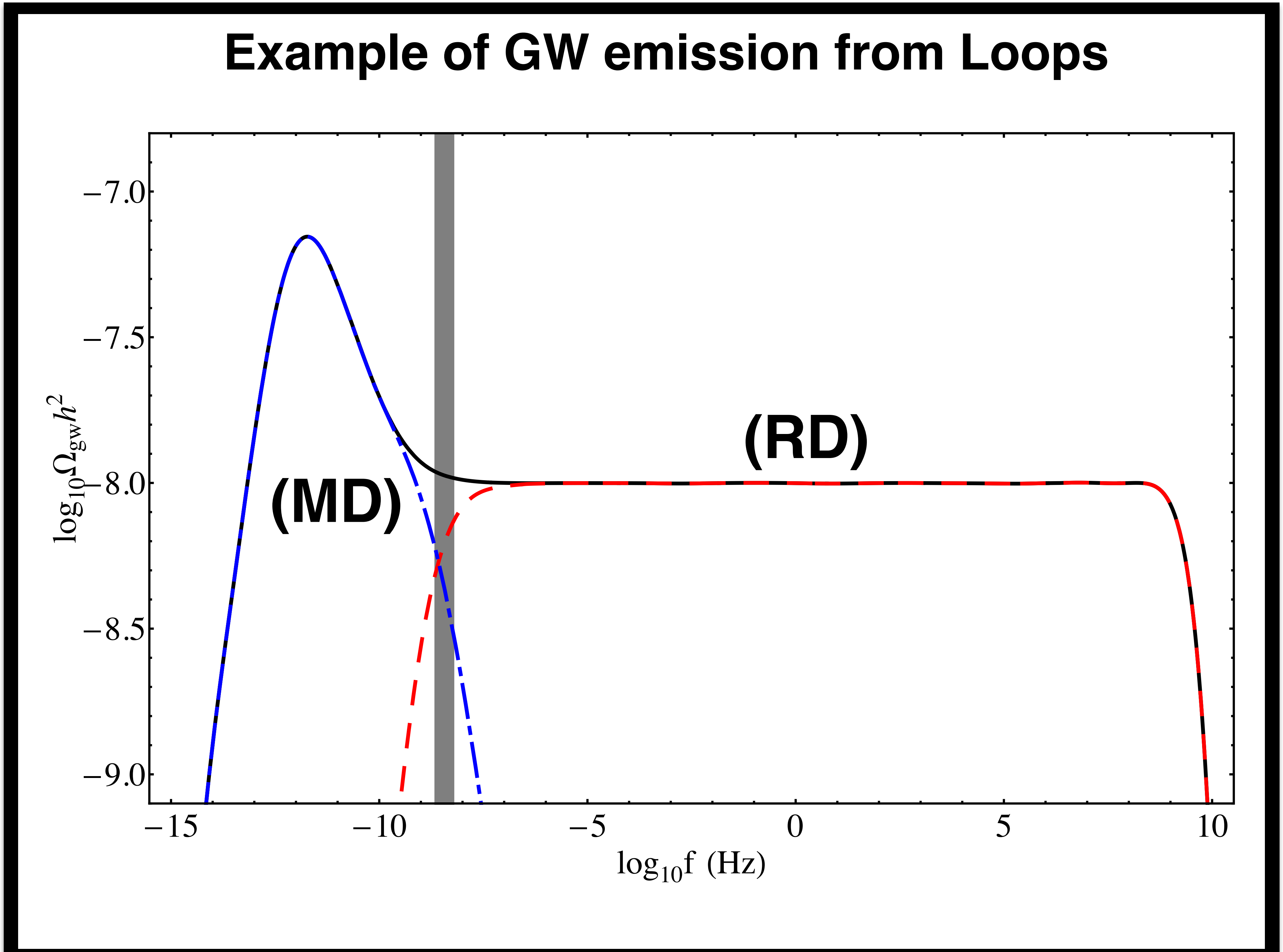
Original emission of GWs ! **(Vilenkin '81)**

and many others !

$$\frac{d\rho^{(o)}}{df} \equiv \Gamma G\mu^2 \int_{t_*}^{t_o} dt \left(\frac{a(t)}{a_o}\right)^3 \int_0^{\alpha/H(t)} dl\, ln(l,t)\, \mathcal{P}((a_o/a(t))fl)$$

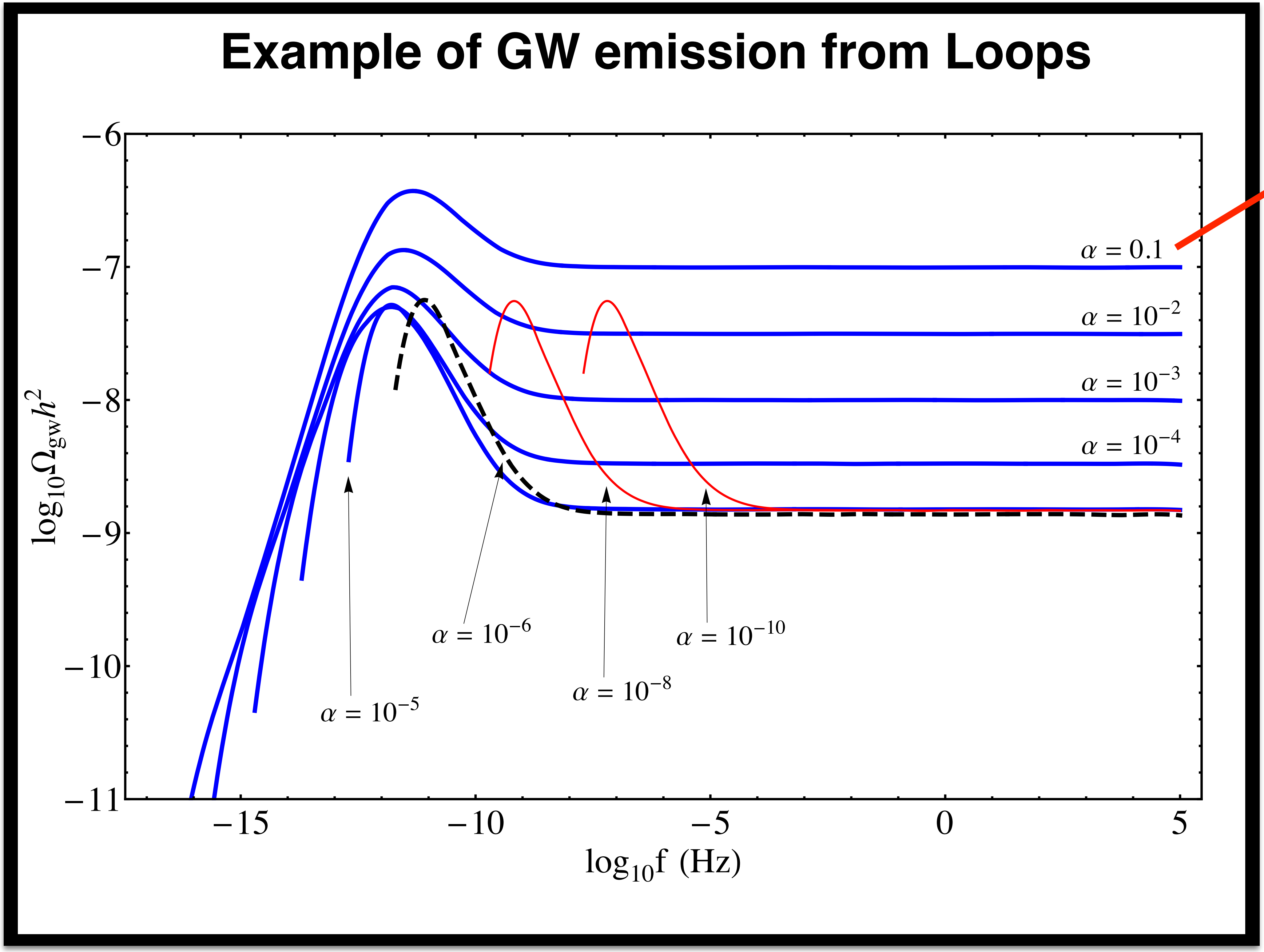
- dt → **expansion history**
- l → **length**
- $n(l,t)$ → **number density** (Nambu-Goto simulations)
- $\mathcal{P}$ → **GW power emission** $\propto 1/(fl)^{q+1}$; $q+1$ → **features (kinks,cusps,...)**

Cosmic strings loops: GW background



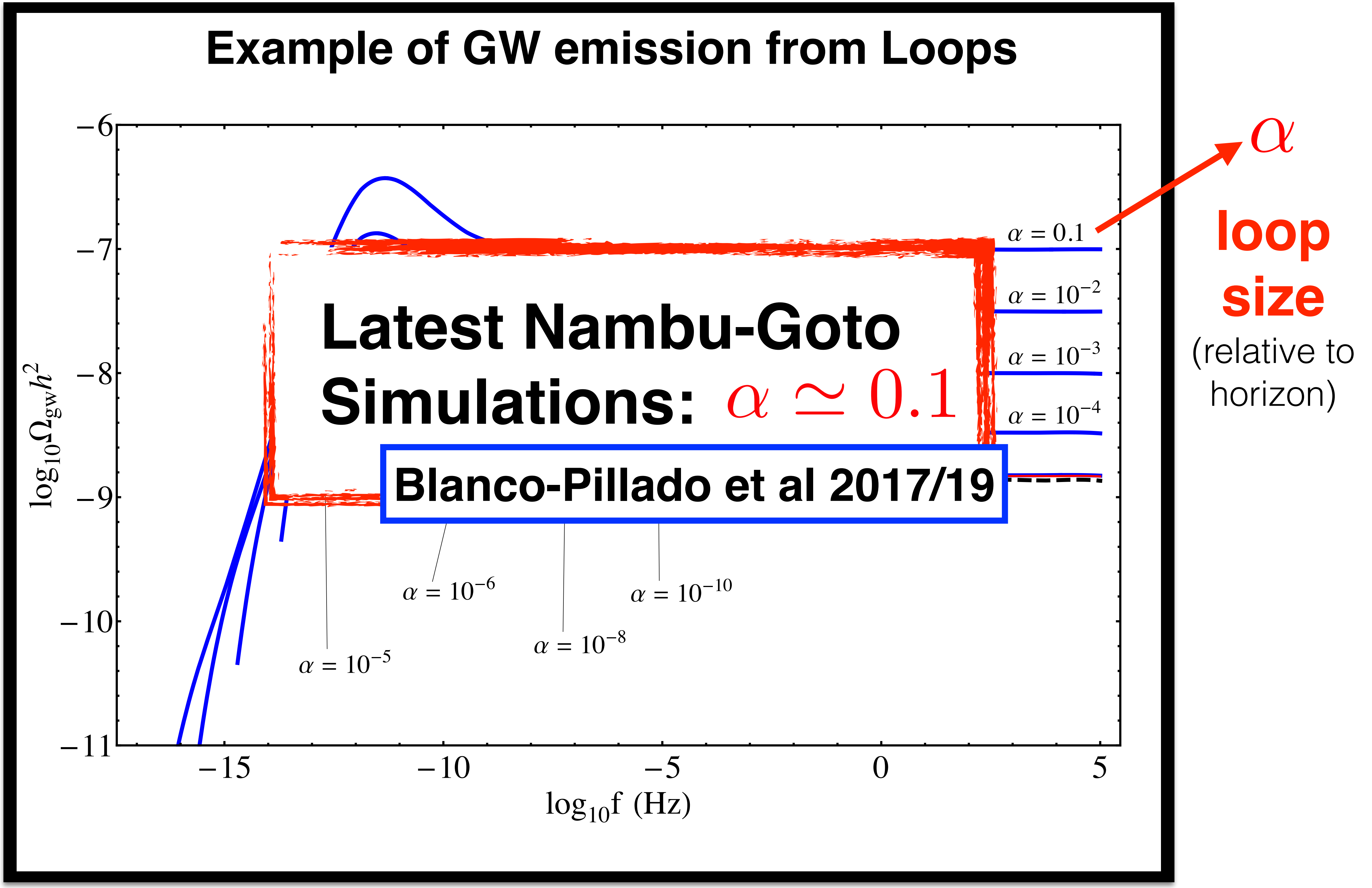
e.g. Sanidas et al 2012

Cosmic strings loops: GW background

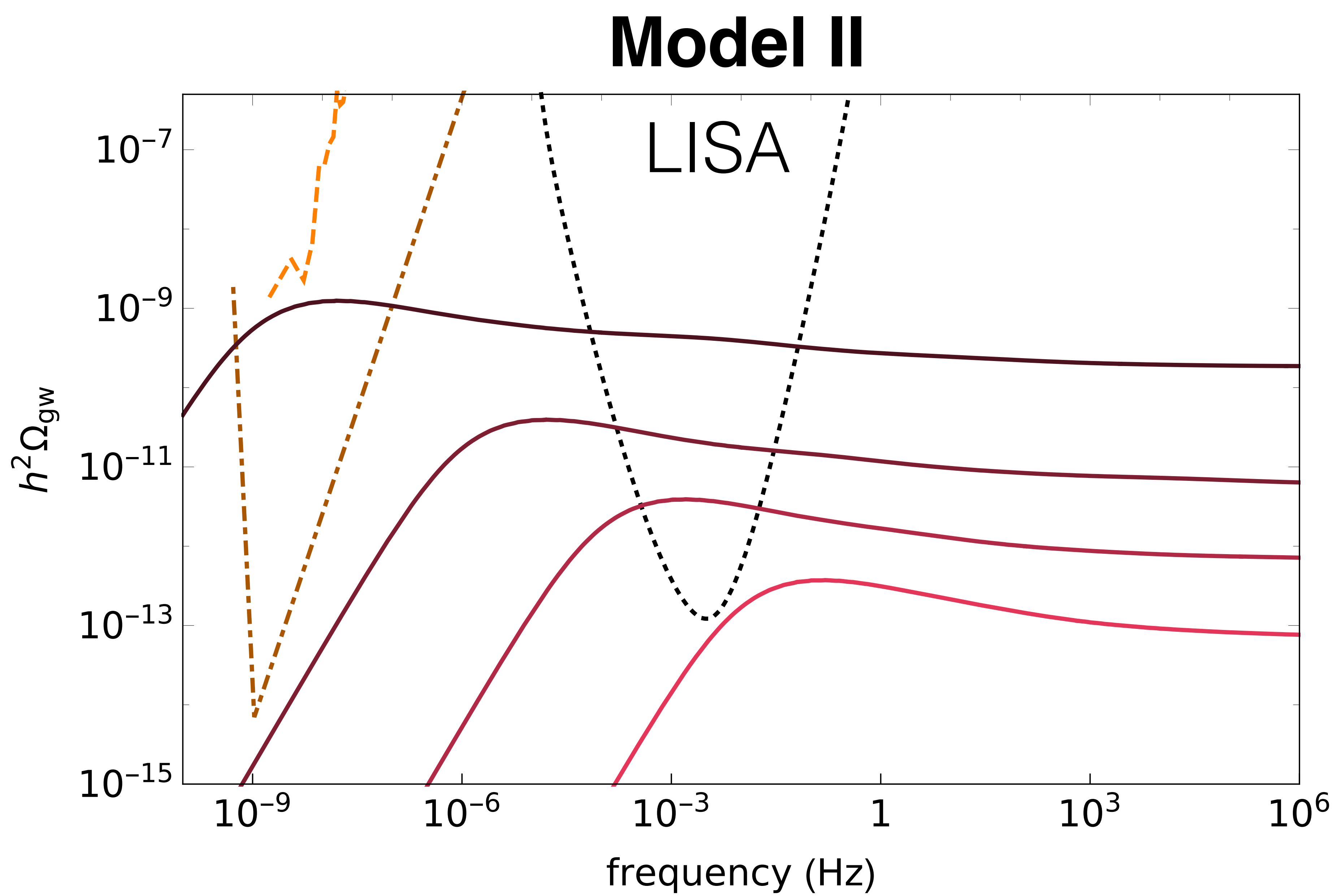


Sanidas et al 2012

Cosmic Strings Network: Loop configurations

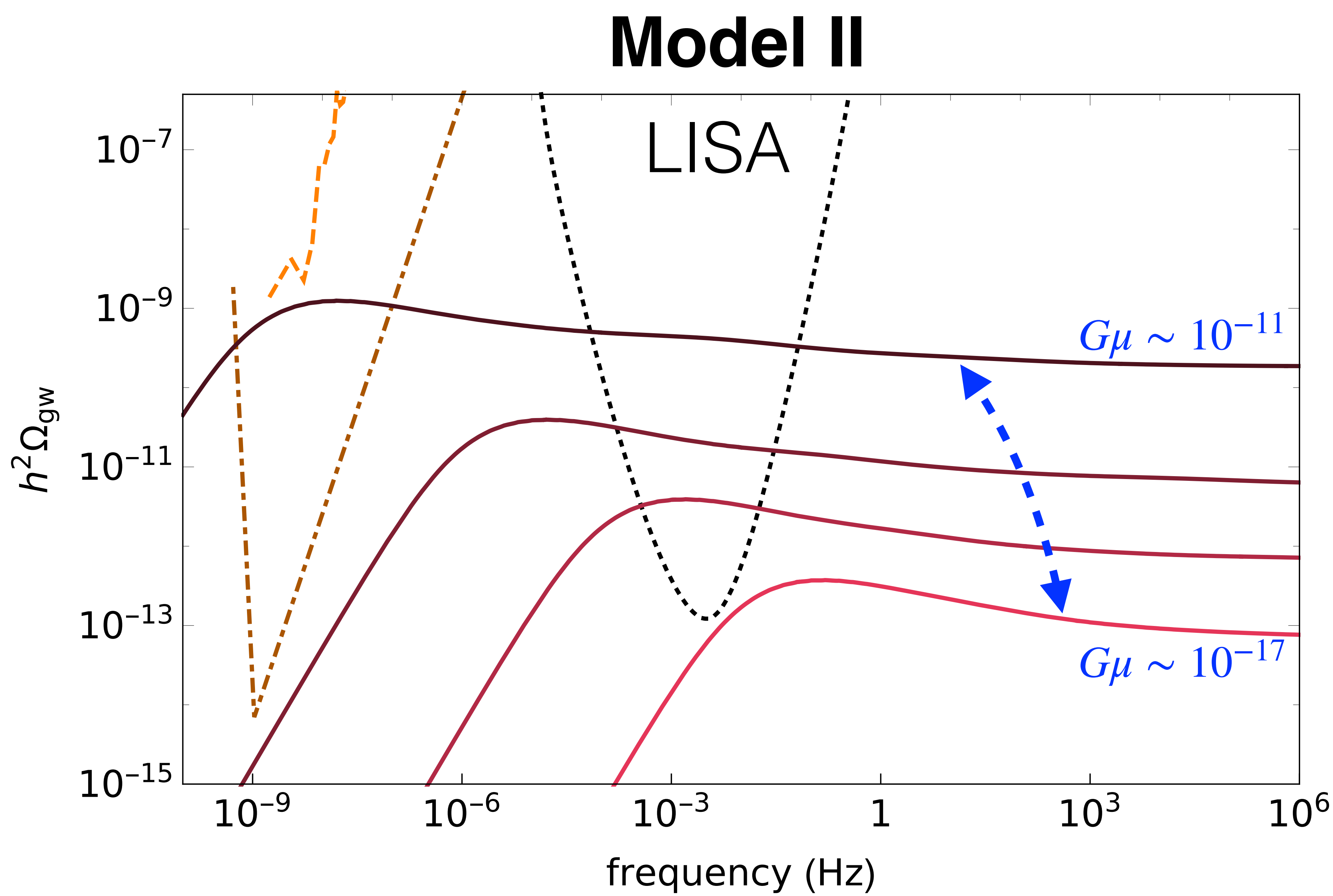


Model II (BOS) from LISA paper



LISA paper: **1909.00819 [astro-ph.CO]**

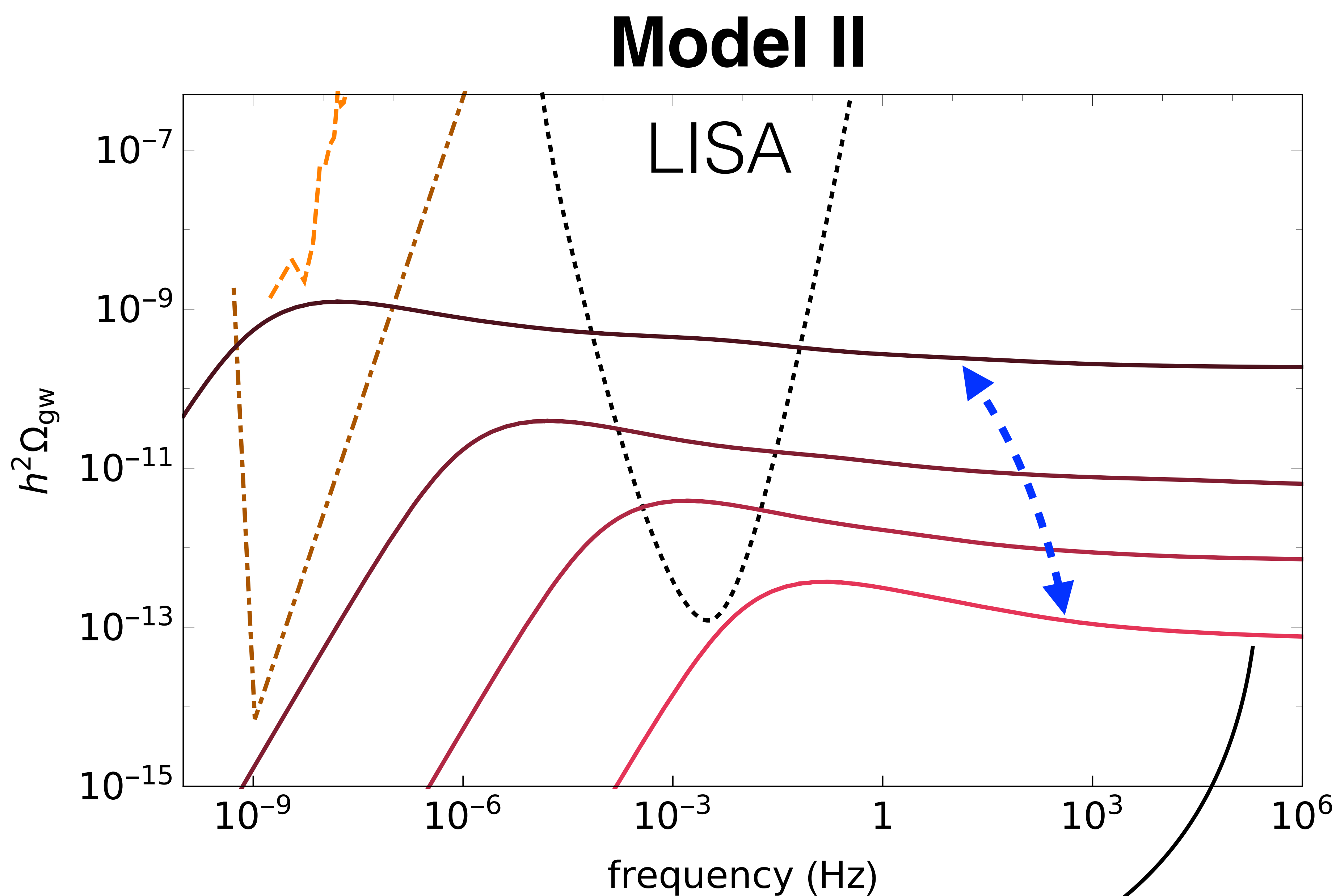
Model II (BOS) from LISA paper



$$G\mu \sim 10^{-11} - 10^{-17}$$

@ LISA: Very large parameter space !

Model II (BOS) from LISA paper



$$G\mu \gtrsim 10^{-17}$$

@ LISA: Very large parameter space !

GW background constrained by LISA

$$G\mu \gtrsim 10^{-17} \quad (v \gtrsim 10^{10}\ \mathrm{GeV})$$

CMB	PTA (today)	PTA (future)
$G\mu \sim 10^{-7}$	$G\mu \sim 10^{-11}$	$G\mu \sim 10^{-14}$

GW background constrained by LISA

$G\mu \gtrsim 10^{-17}$ $(v \gtrsim 10^{10}\ \text{GeV})$

	CMB	PTA (today)	PTA (future)
	$G\mu \sim 10^{-7}$	$G\mu \sim 10^{-11}$	$G\mu \sim 10^{-14}$
LISA improve:	$\mathcal{O}(10^{10})$	$\mathcal{O}(10^{6})$	$\mathcal{O}(10^{3})$

GW background constrained by LISA

$G\mu \gtrsim 10^{-17}$ $(v \gtrsim 10^{10}\ \mathrm{GeV})$

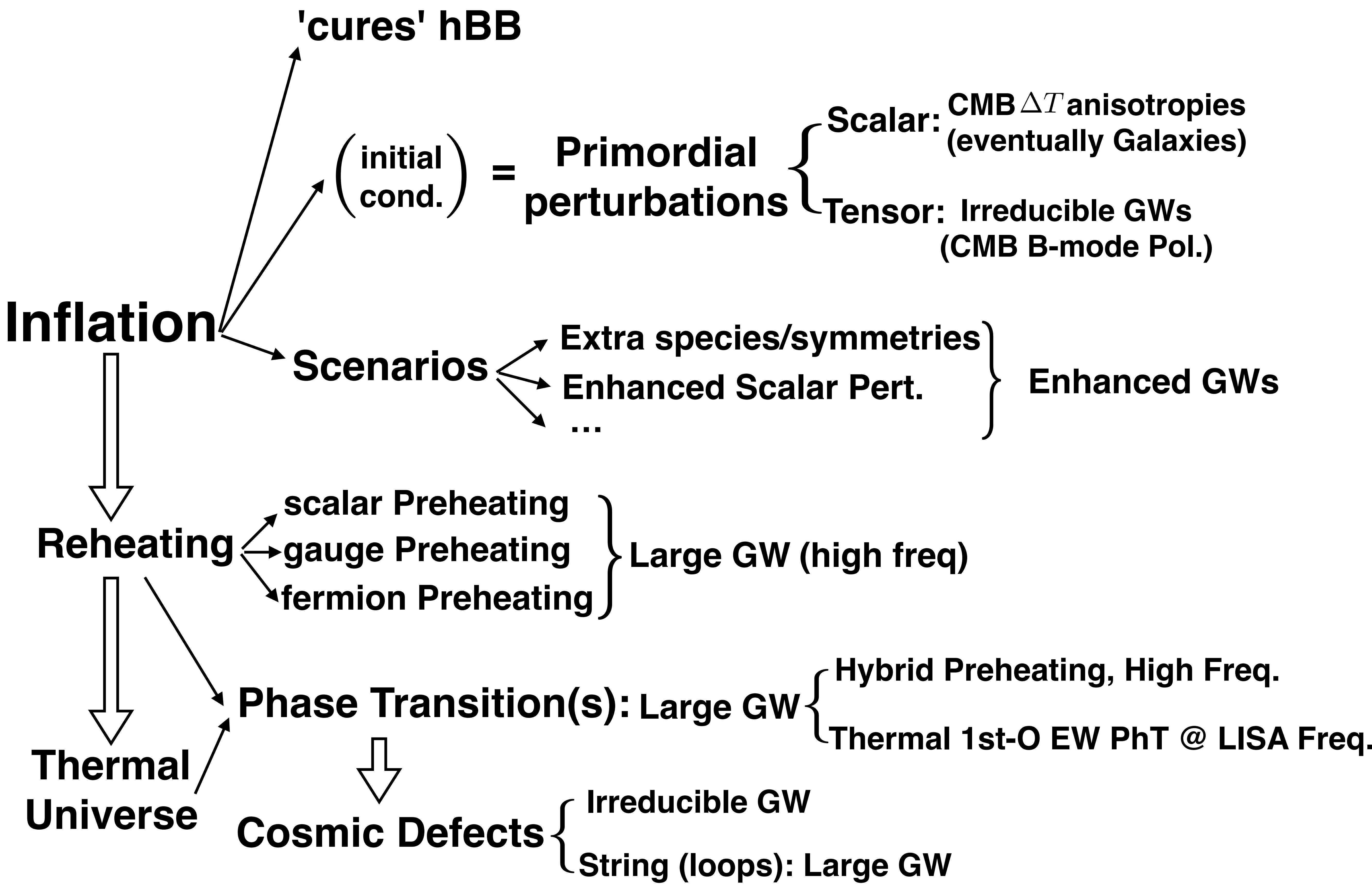
	CMB	PTA (today)	PTA (future)
	$G\mu \sim 10^{-7}$	$G\mu \sim 10^{-11}$	$G\mu \sim 10^{-14}$
LISA improve:	$\mathcal{O}(10^{10})$	$\mathcal{O}(10^{6})$	$\mathcal{O}(10^{3})$

(!)

LISA

- **Best constraints on Comic Strings**
- **(actually only way to obtain them)**
- **Discovery, or stringent constraints**

EARLY UNIVERSE in GWs ✓



EARLY UNIVERSE in GWs ✓

'cures' hBB

initial

Primordial

Seeds

otropies

alaxies)

Ws

Pol.)

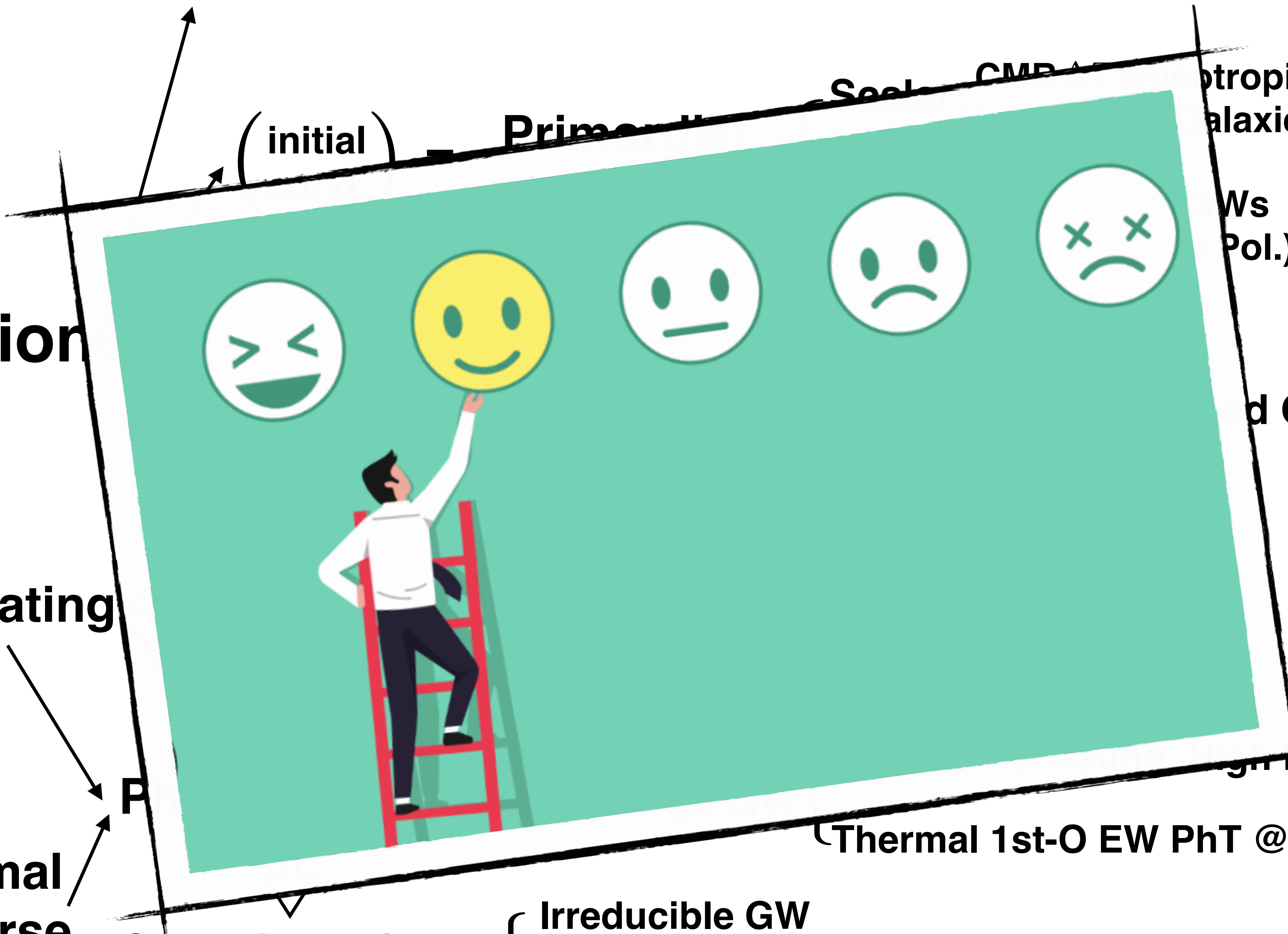
d GWs

Inflation

Reheating

Thermal Universe

P



Freq.

Thermal 1st-O EW PhT @ LISA Freq.

Cosmic Defects
- Irreducible GW
- String (loops): Large GW

Gravitational Wave Backgrounds

OUTLINE

1) Grav. Waves (GWs) ✓ — 1st Topic

Early Universe Sources:

2) GWs from Inflation ✓

3) GWs from Preheating ✓

4) GWs from Phase Transitions ✓

5) GWs from Cosmic Defects ✓

Core Topics

6) Astrophysical Background(s)

7) Observational Constraints/Prospects

(Briefly)

Gravitational Wave Backgrounds

OUTLINE

1) Grav. Waves (GWs) — 1st Topic

Early Universe Sources (Core Topics):

2) GWs from Inflation

3) GWs from Preheating

4) GWs from Phase Transitions

5) GWs from Cosmic Defects

Late Universe + Observations (Briefly):

6) Astrophysical Background(s)

7) Observational Constraints/Prospects

Gravitational Wave Backgrounds

OUTLINE

1) Grav. Waves (GWs) ✓ — 1st Topic

Early Universe Sources:

2) GWs from Inflation ✓
3) GWs from Preheating ✓
4) GWs from Phase Transitions ✓
5) GWs from Cosmic Defects ✓

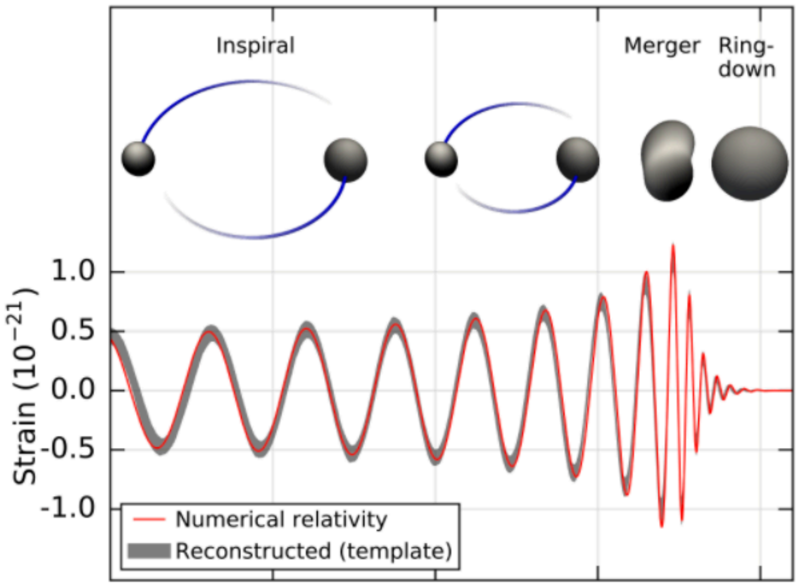
Core Topics

Late Universe + Observations:

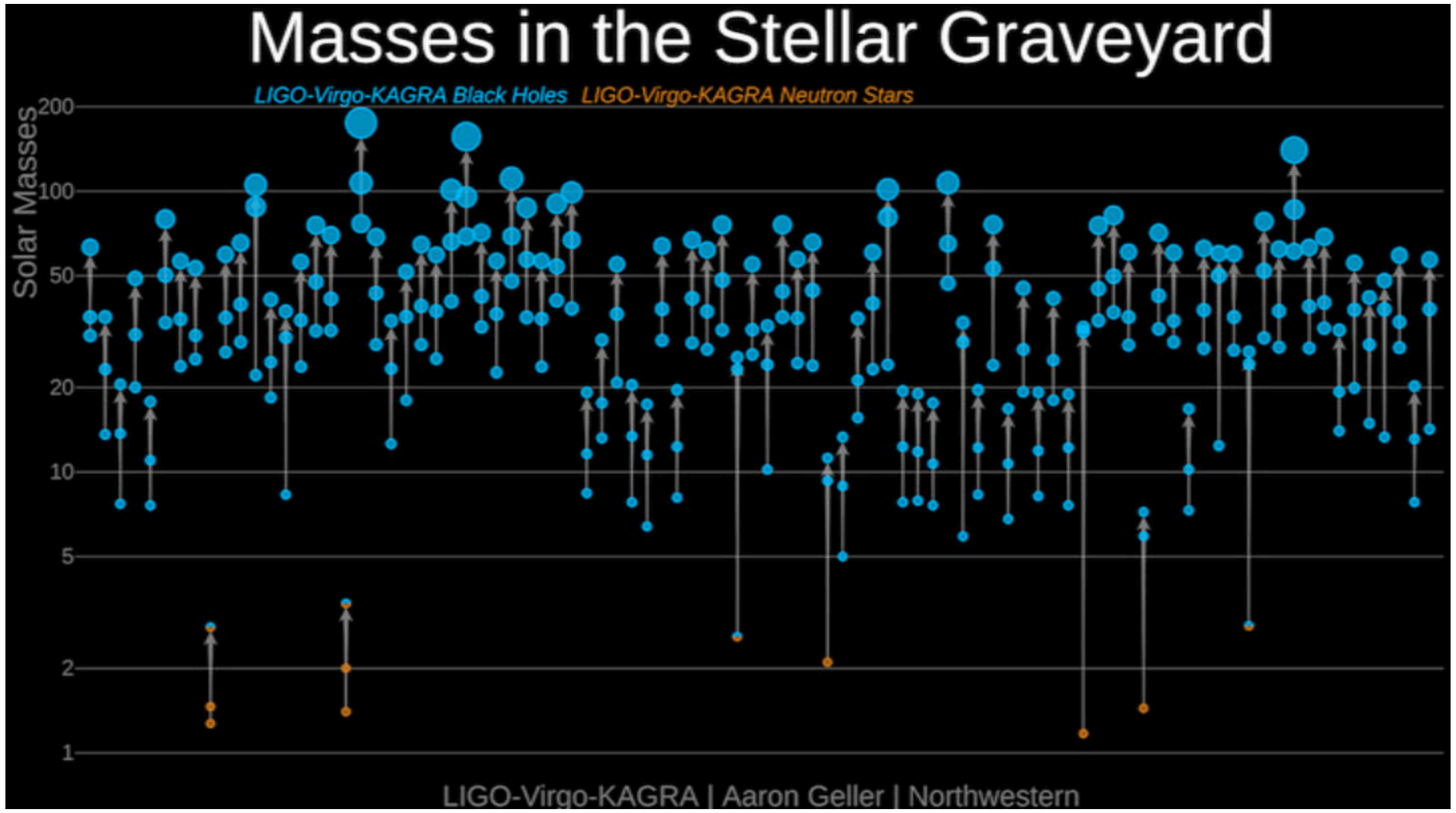
6) Astrophysical Background(s)
7) Observational Constraints/Prospects

(Briefly)

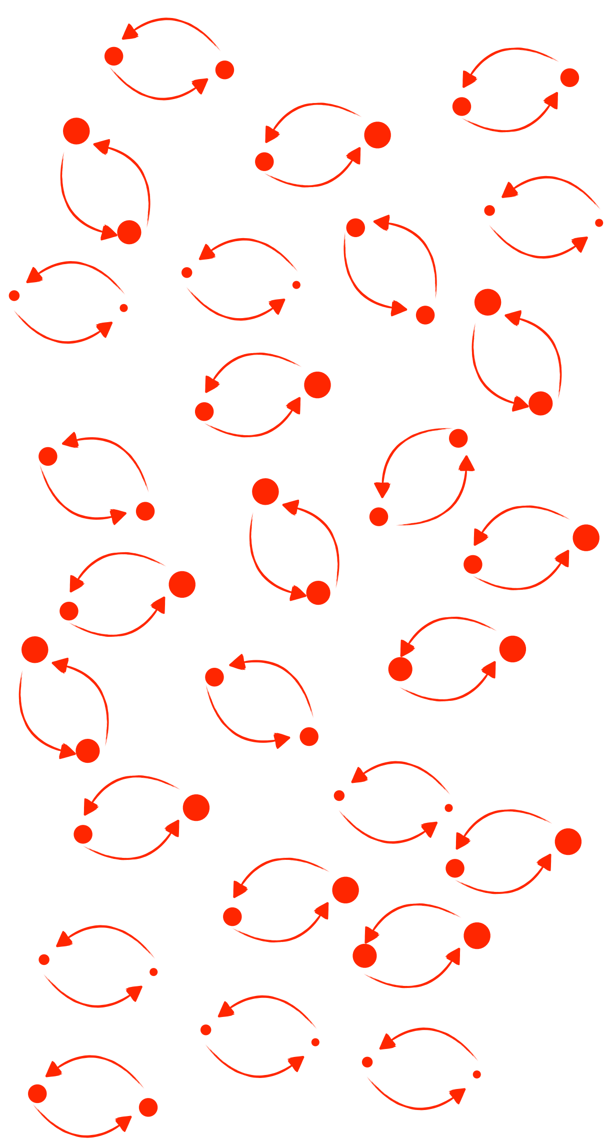
Late Universe $(0 \leq z \lesssim 10)$

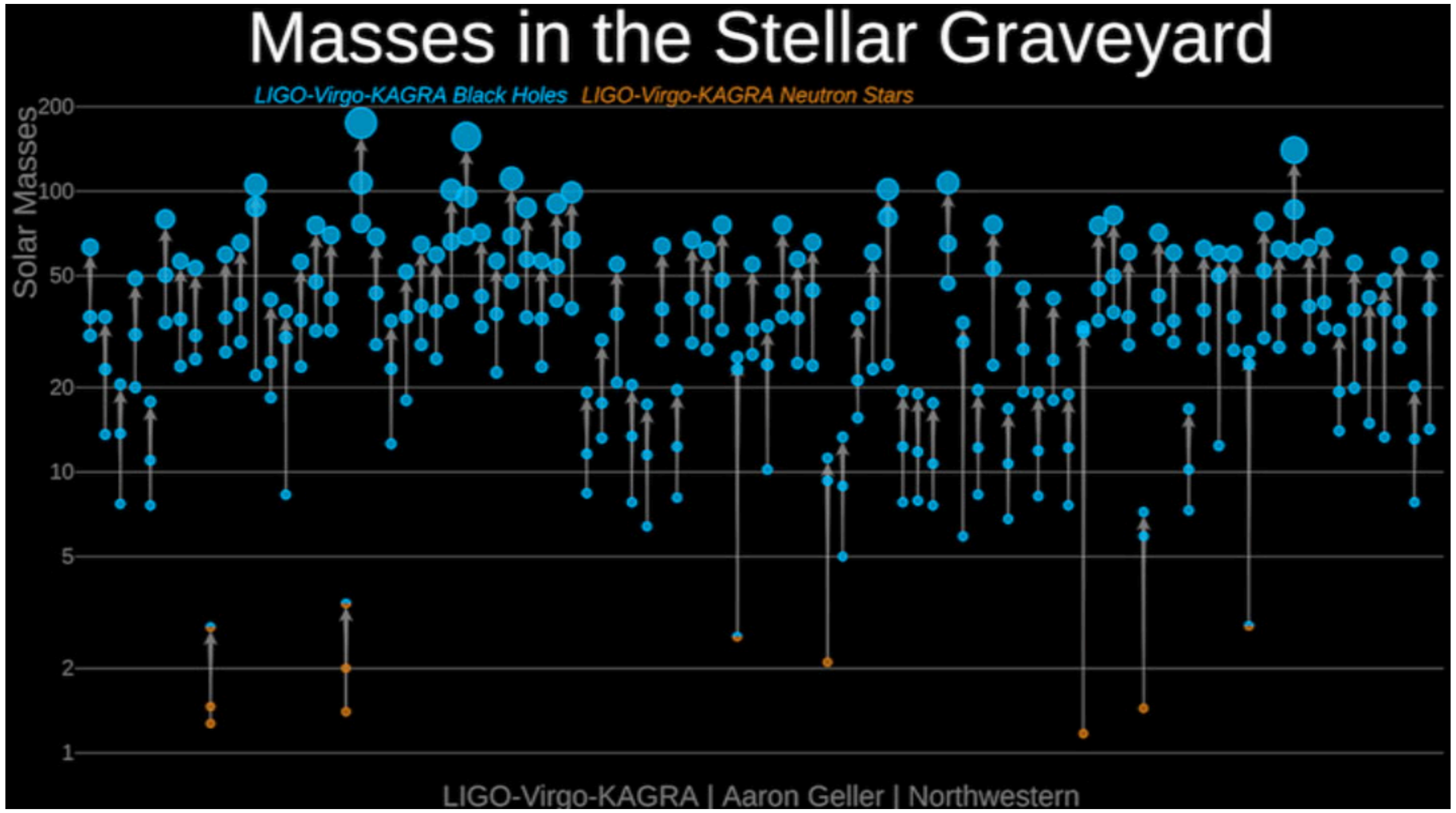


LIGO/VIRGO
2015-now

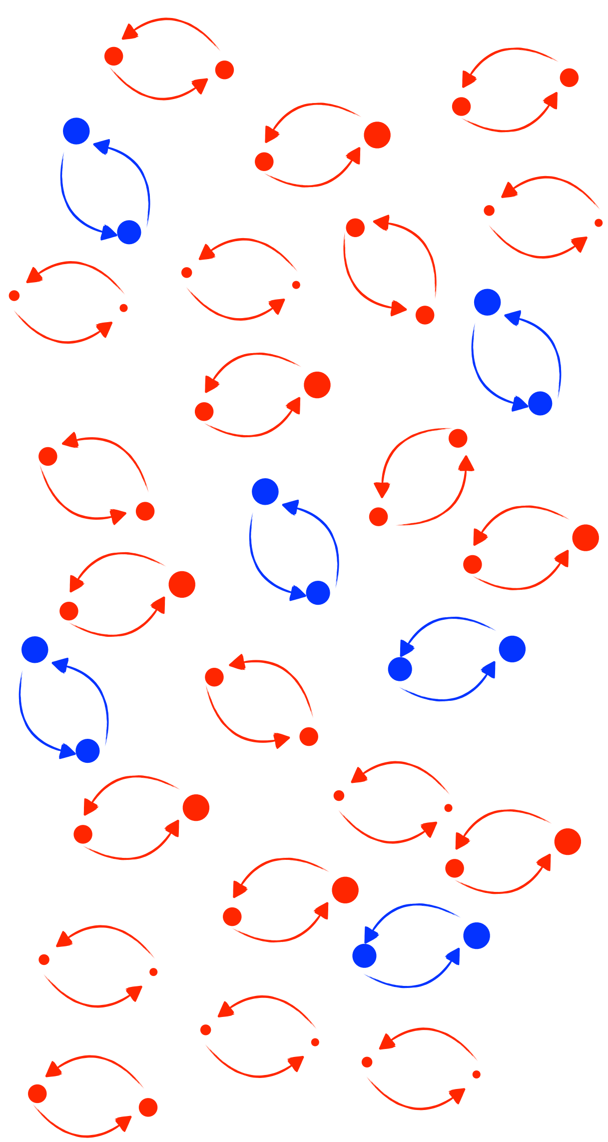


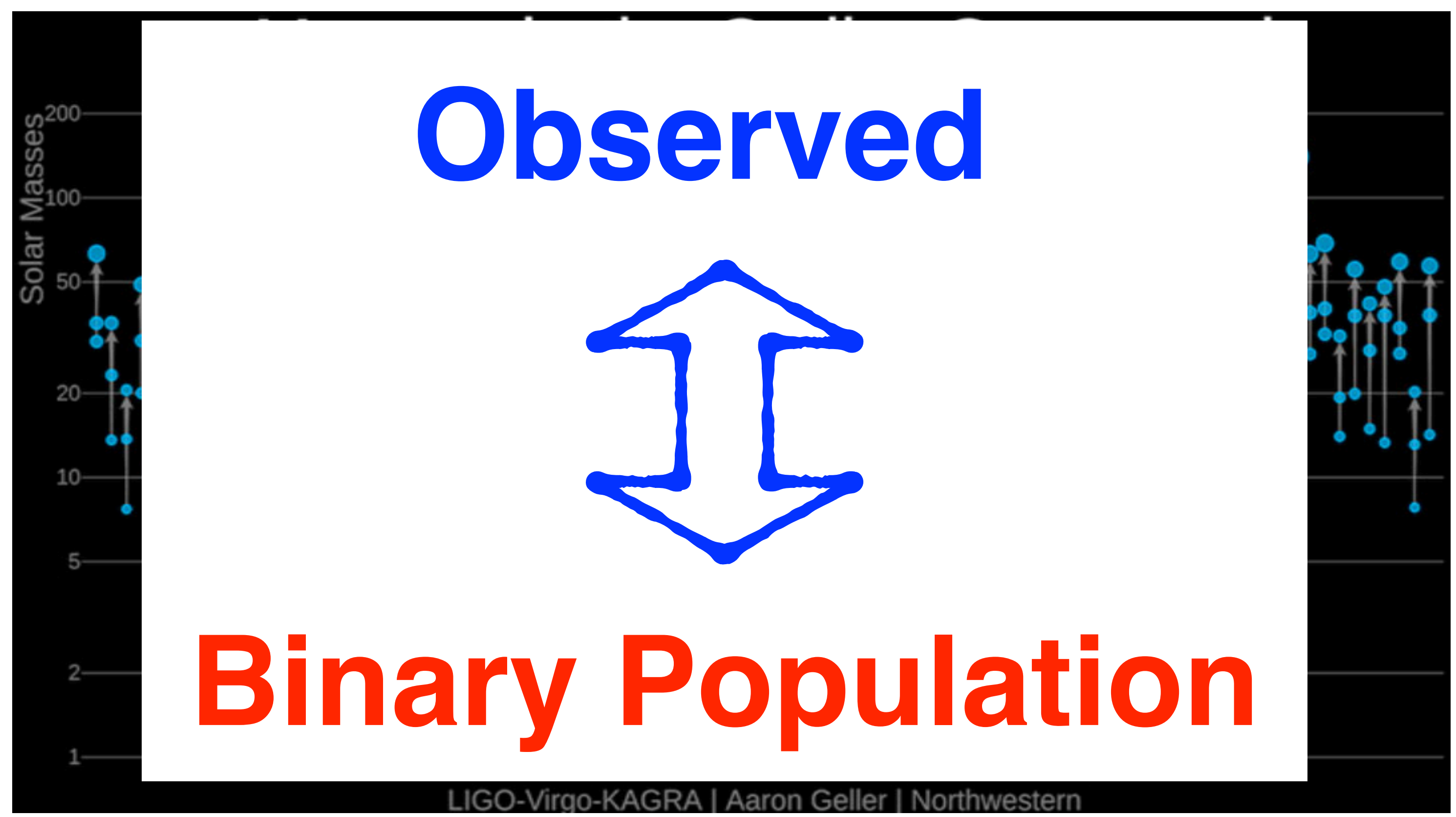
Late Universe $(0 \leq z \lesssim 10)$



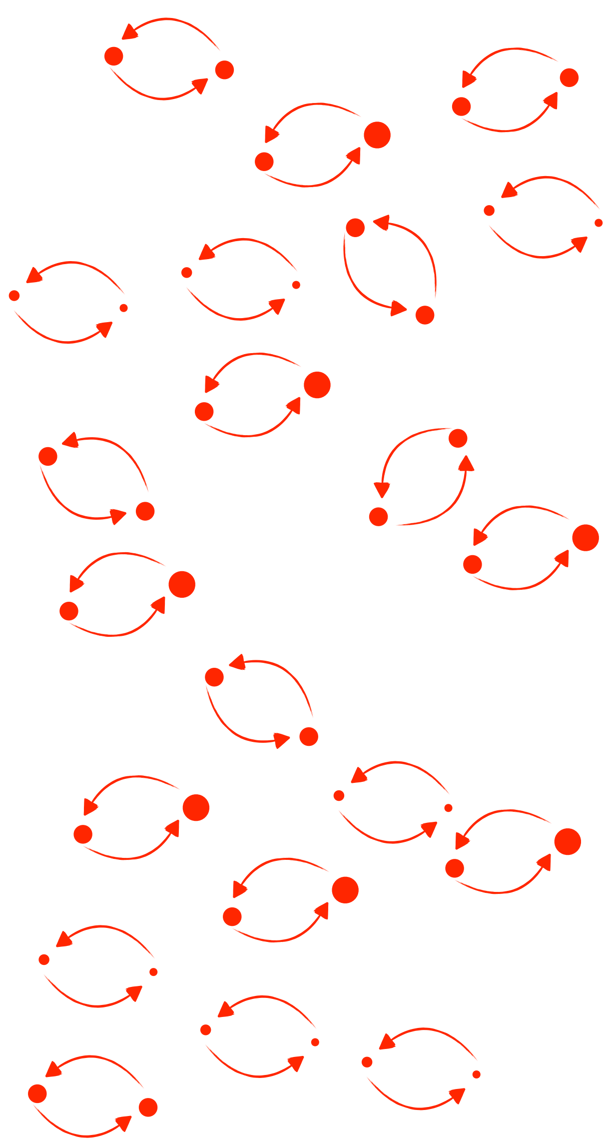


Late Universe $(0 \leq z \lesssim 10)$



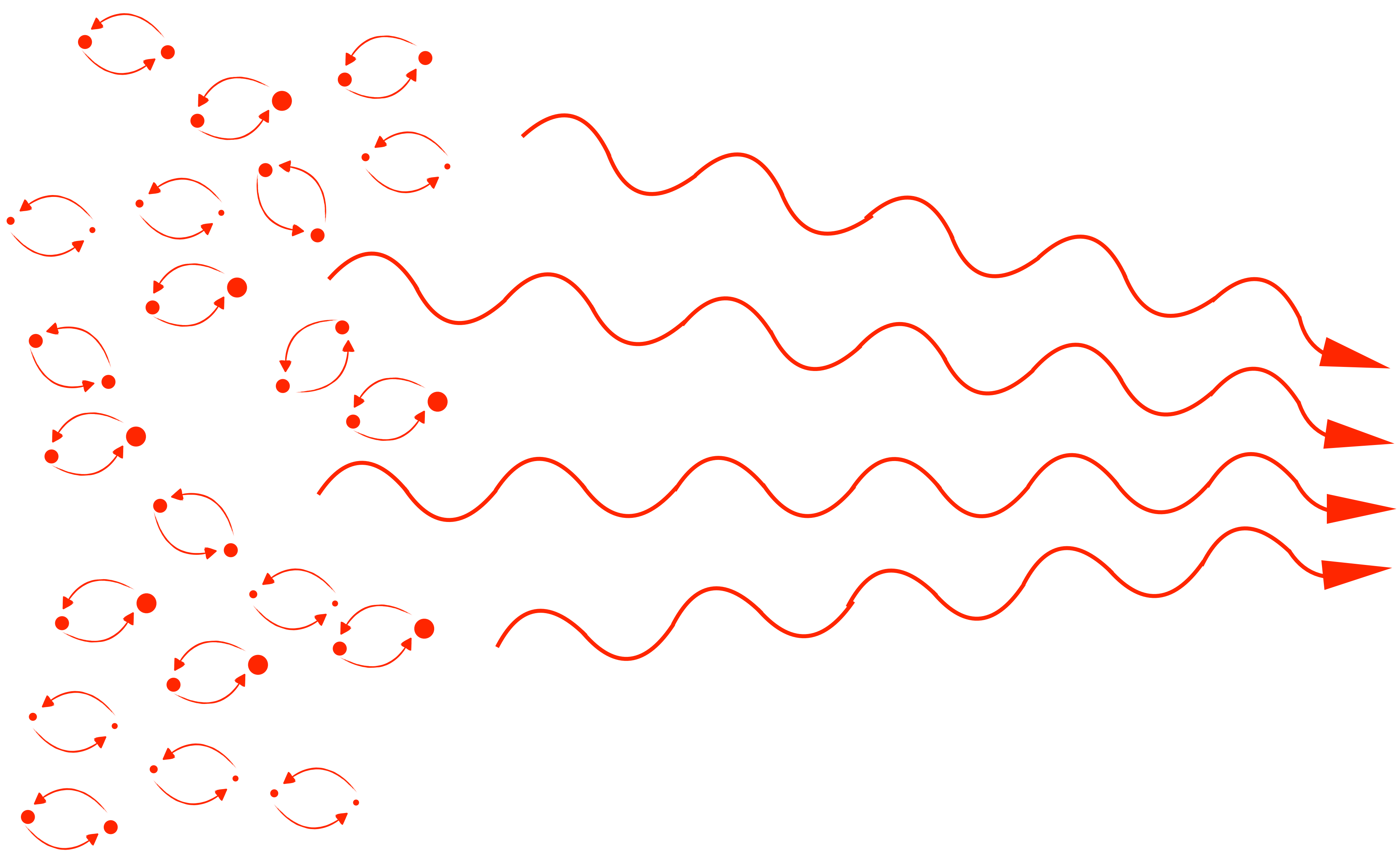


Late Universe $(0 \leq z \lesssim 10)$

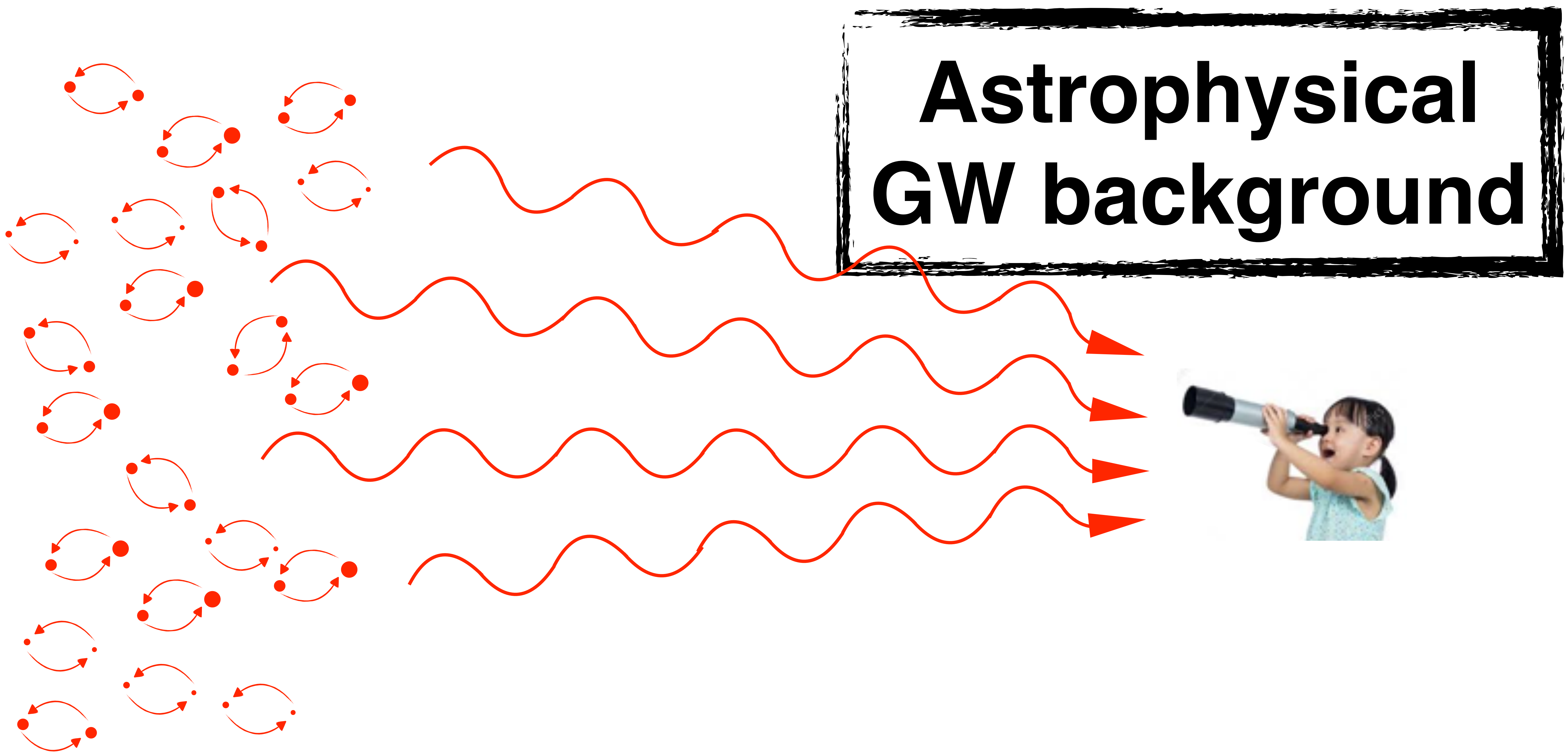


Mostly unresolved binaries !

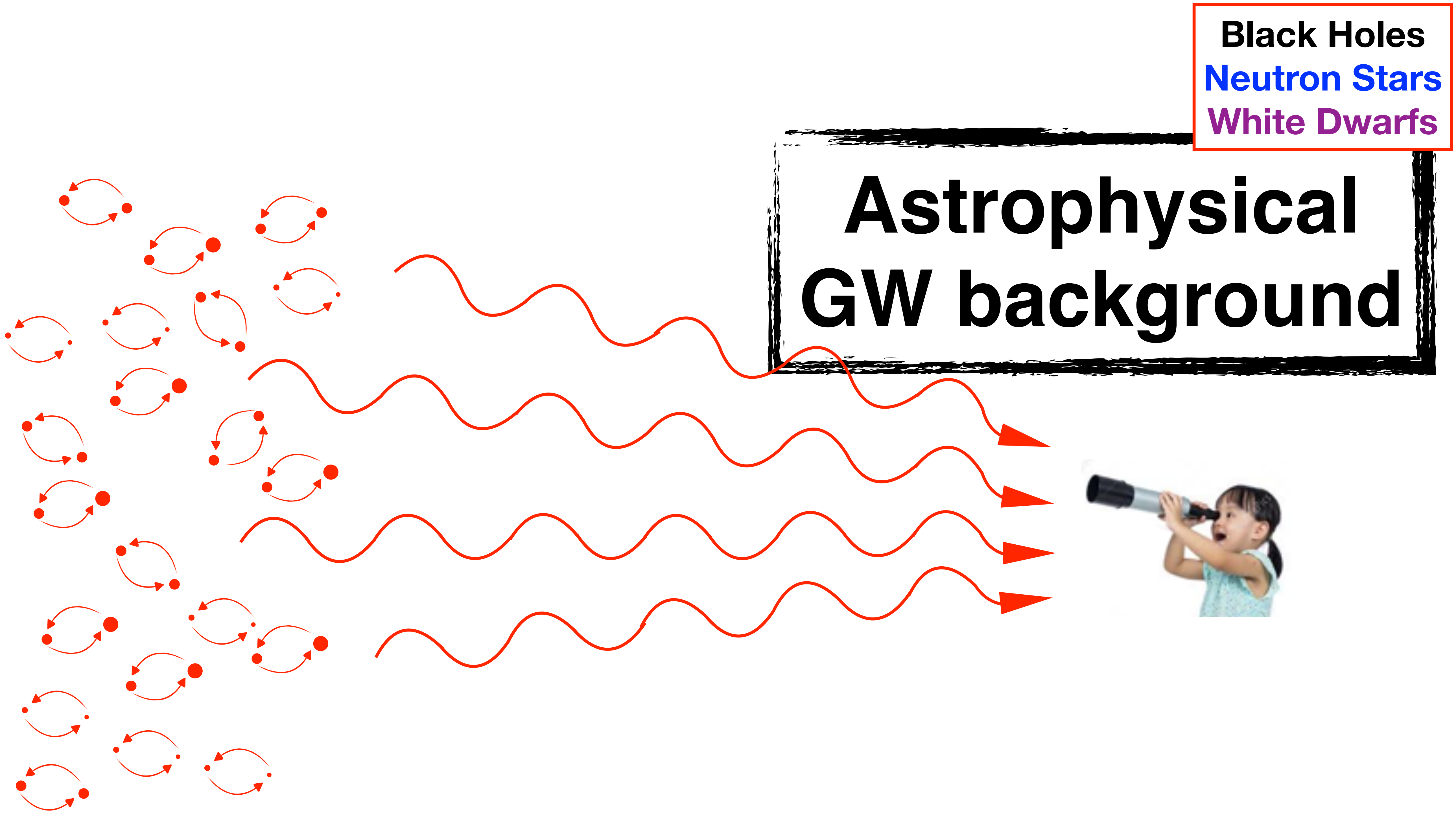
Late Universe $(0 \leq z \lesssim 10)$



Late Universe $(0 \leq z \lesssim 10)$

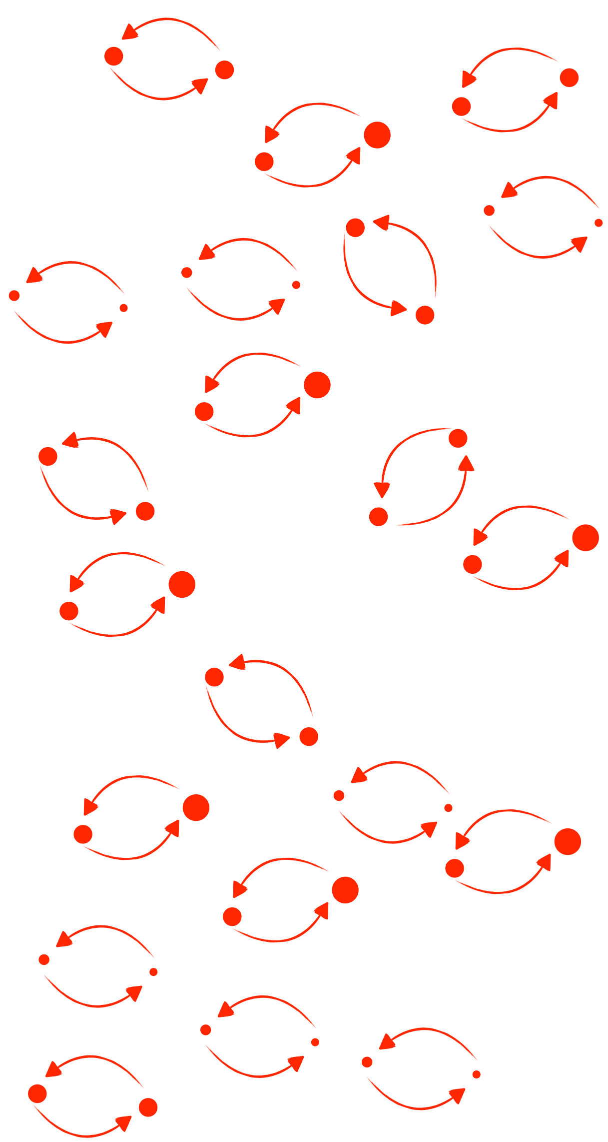


Late Universe $(0 \leq z \lesssim 10)$

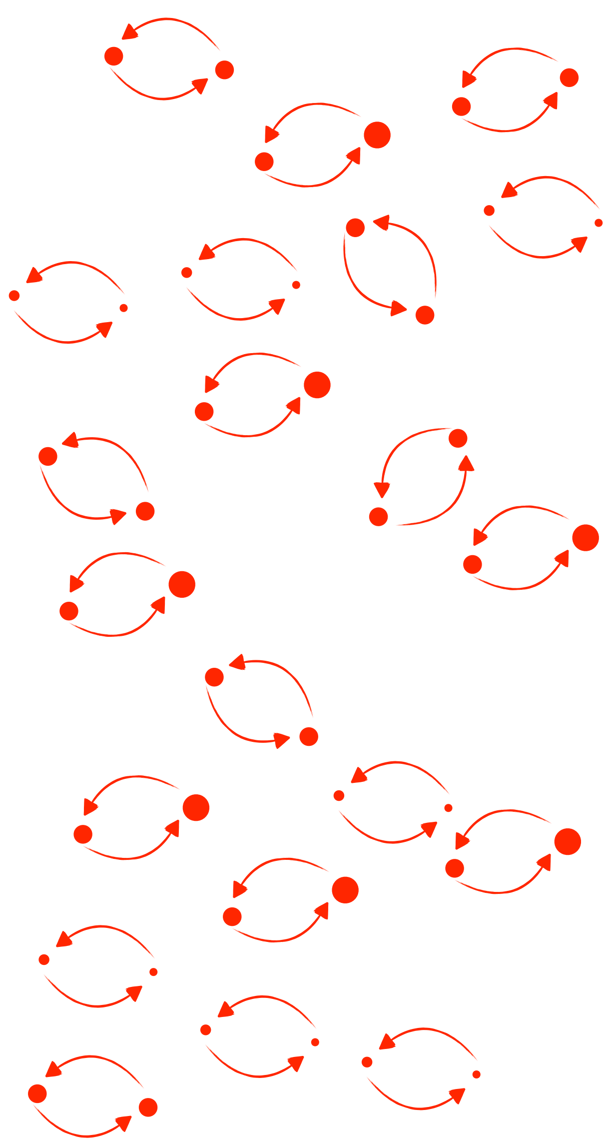


Late Universe $(0 \leq z \lesssim 10)$

$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\,\frac{d\rho_{\rm GW}}{d\log f}\,,$$



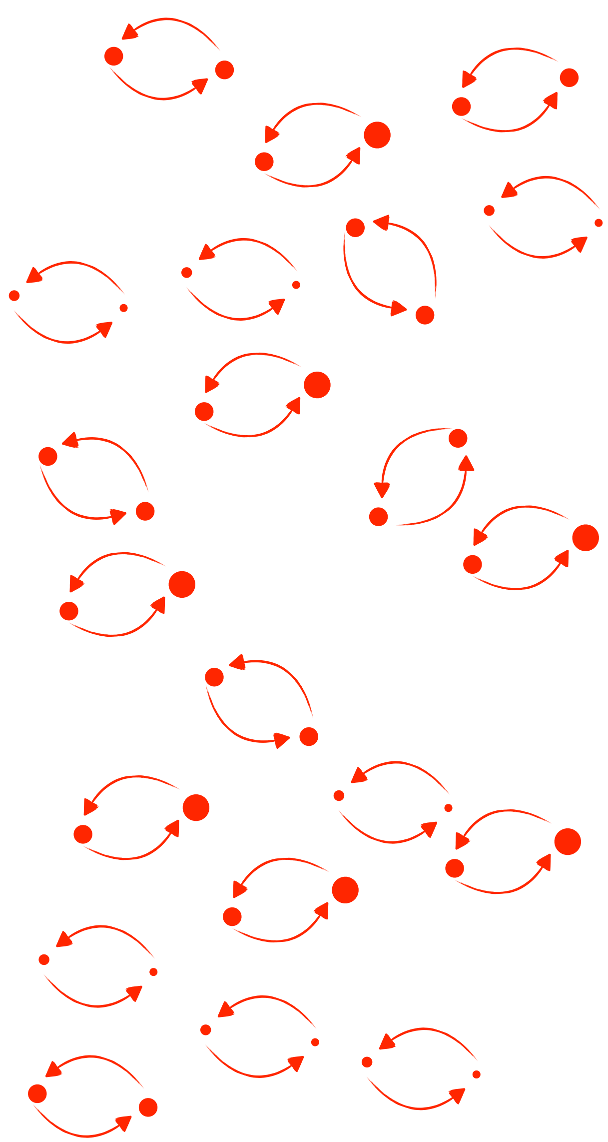
Late Universe $(0 \leq z \lesssim 10)$



$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\log f} = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f)\,,$$

↑ **Characteristic strain**

Late Universe $(0 \leq z \lesssim 10)$

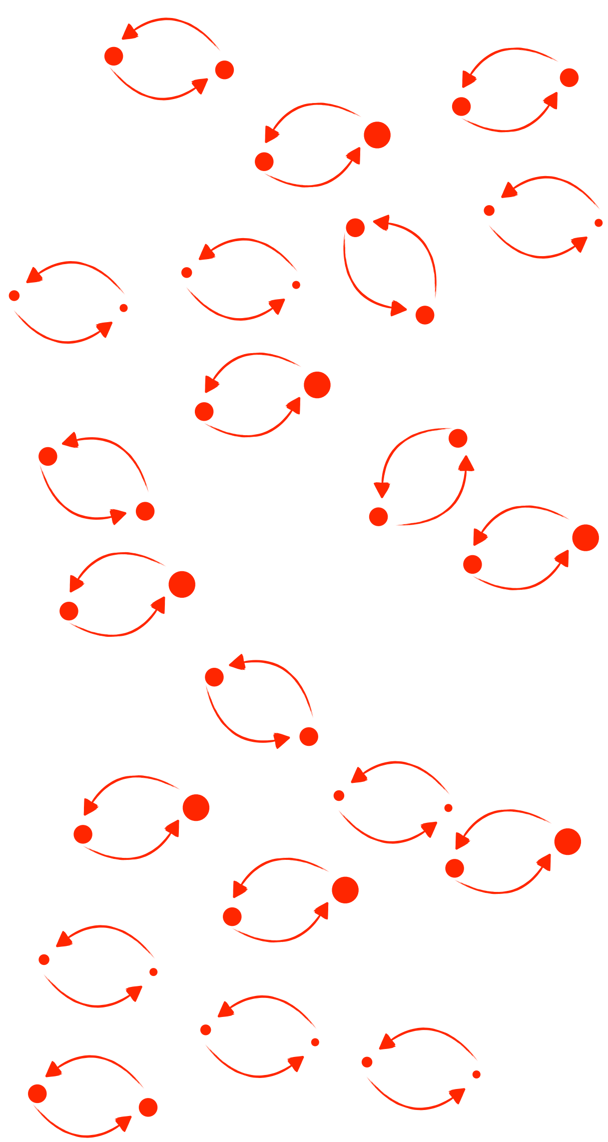


$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\log f} = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f)\,,$$

For binary population: $\dfrac{dn}{dz}$ $\left(\text{comoving number density}\right)$

Late Universe $\quad (0 \leq z \lesssim 10)$

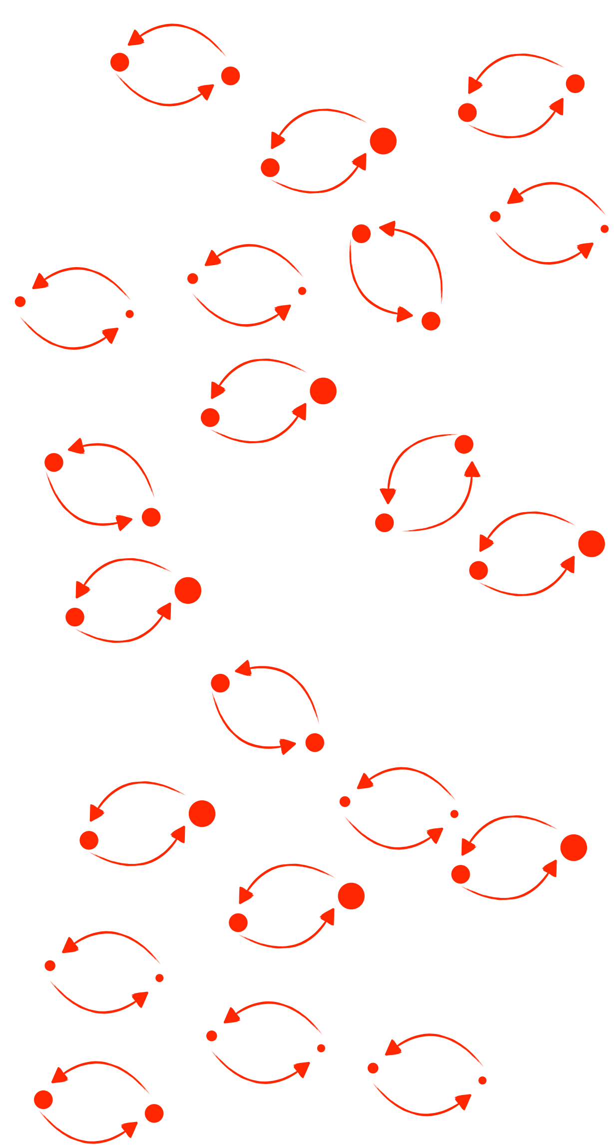
$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\log f} = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f)\,,$$



For binary population: $\dfrac{dn}{dz}$ $\left(\text{comoving number density}\right)$

$$h_c^2(f) = \frac{4}{\pi}\frac{G}{c^2} f^{-2}\int_0^\infty dz \frac{dn}{dz}\frac{1}{1+z} f_r \frac{dE_{\rm GW}}{df_r}\bigg|_{f_r = f(1+z)}$$

Late Universe $(0 \leq z \lesssim 10)$



$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\log f} = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f)\,,$$

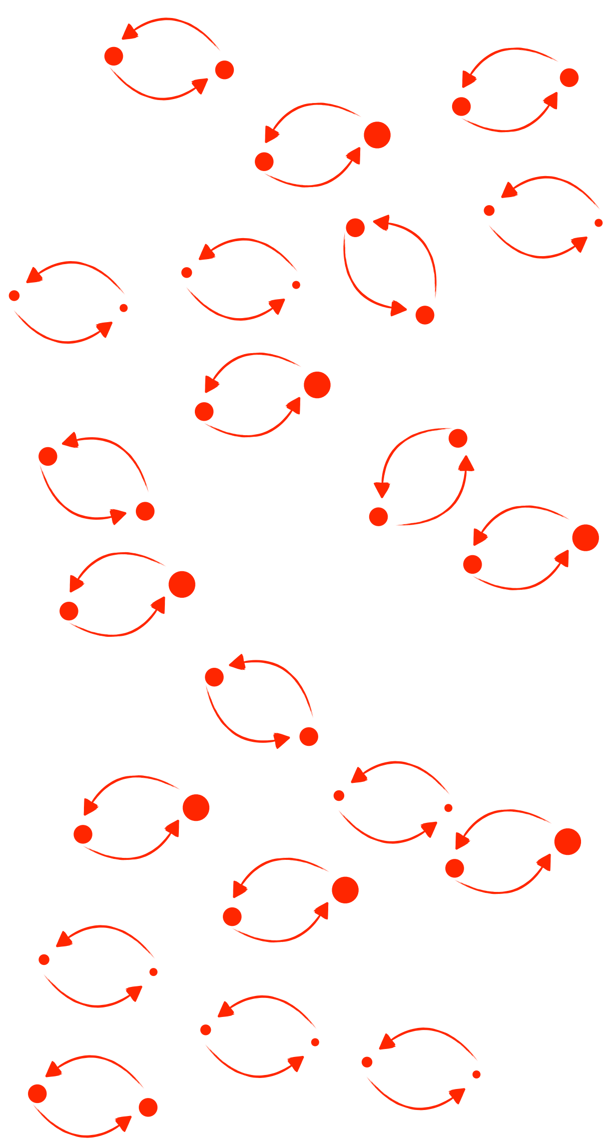
For binary population: $\dfrac{dn}{dz}$ (**comoving number density**)

$$h_c^2(f) = \frac{4}{\pi}\frac{G}{c^2} f^{-2}\int_0^\infty dz \frac{dn}{dz}\frac{1}{1+z} f_r \frac{dE_{\rm GW}}{df_r}\bigg|_{f_r = f(1+z)}$$

E.S. Phinney
astro-ph/0108028

source-frame energy spectrum

Late Universe $(0 \leq z \lesssim 10)$

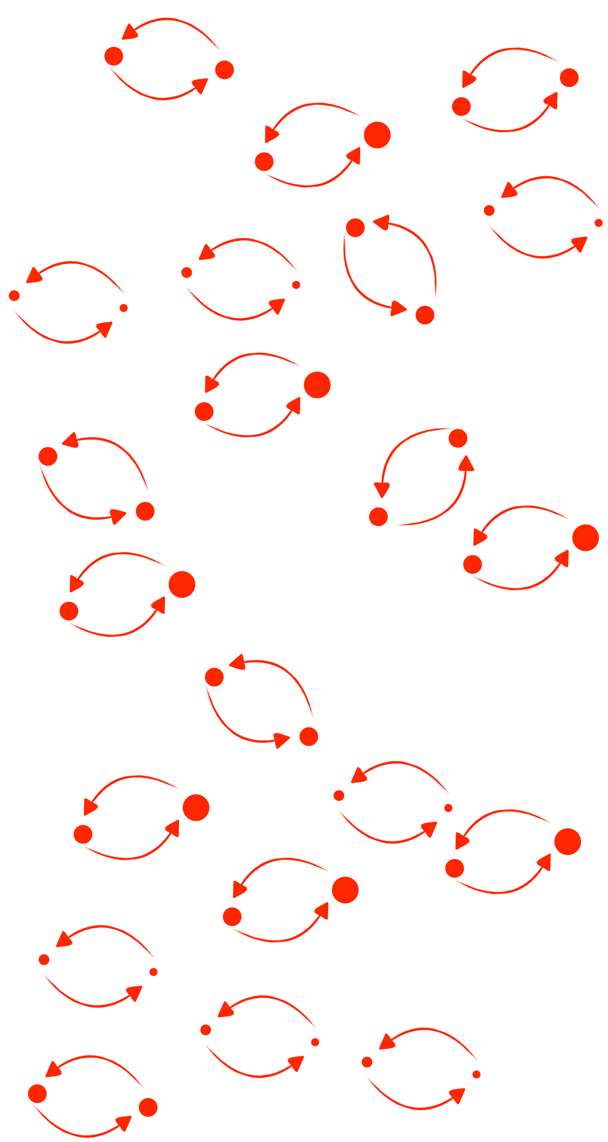


$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\log f} = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f)\,,$$

$$h_c^2(f) = \frac{4}{\pi}\frac{G}{c^2} f^{-2}\int_0^\infty dz \frac{dn}{dz}\frac{1}{1+z} f_r \frac{dE_{\rm GW}}{df_r}\bigg|_{f_r = f(1+z)}$$

Source frame frequency

Late Universe $(0 \leq z \lesssim 10)$



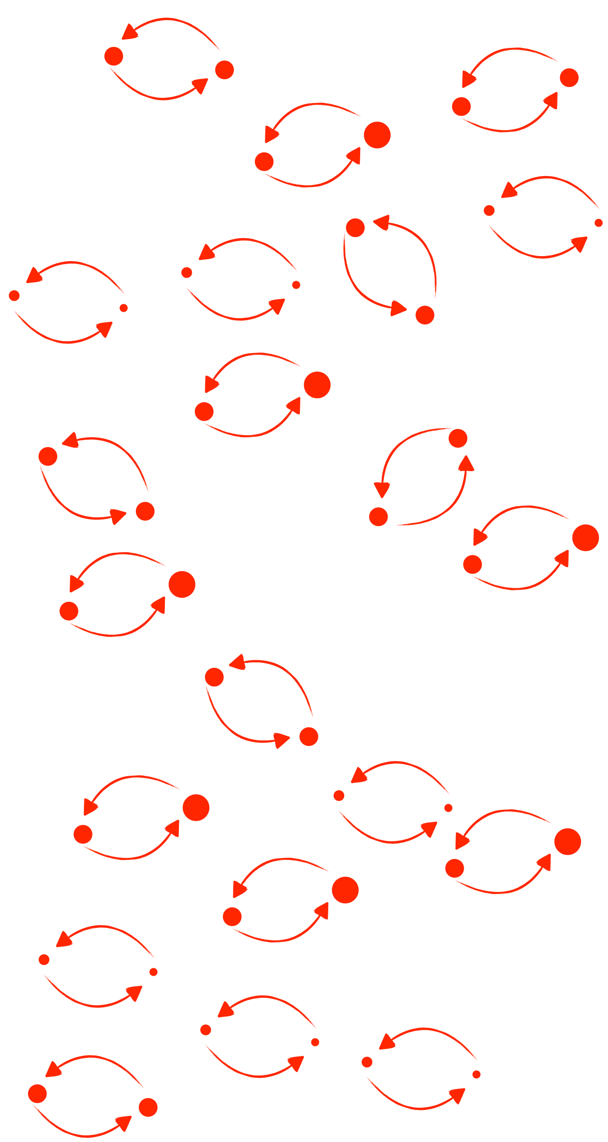
$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\log f} = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f)\,,$$

$$h_c^2(f) = \frac{4}{\pi}\frac{G}{c^2} f^{-2} \int_0^\infty dz \frac{dn}{dz}\frac{1}{1+z} f_r \frac{dE_{\rm GW}}{df_r}\bigg|_{f_r = f(1+z)}$$

$$\frac{dE_{\rm GW}}{df_r} = \frac{\pi}{3}\frac{1}{G}\frac{(G\mathcal{M})^{5/3}}{\pi^{1/3} f_r^{1/3}}\bigg|_{f_r = f(1+z)}$$

Chirp mass $\mathcal{M} = (m_1 + m_2)^{2/5} \left(\frac{m_1 m_2}{m_1 + m_2}\right)^{3/5}$

Late Universe $(0 \leq z \lesssim 10)$



$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\log f} = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f)\,,$$

$$h_c^2(f) = \frac{4}{\pi}\frac{G}{c^2} f^{-2}\int_0^\infty dz \frac{dn}{dz}\frac{1}{1+z} f_r \frac{dE_{\rm GW}}{df_r}\bigg|_{f_r = f(1+z)}$$

Using:

$$\frac{dn}{dz} \equiv \frac{d\mathcal{N}}{dz dm_1 dm_1 dV} \equiv R(z)\; p(m_1, m_2)\; \frac{dt_r}{dz}$$

Merging Rate

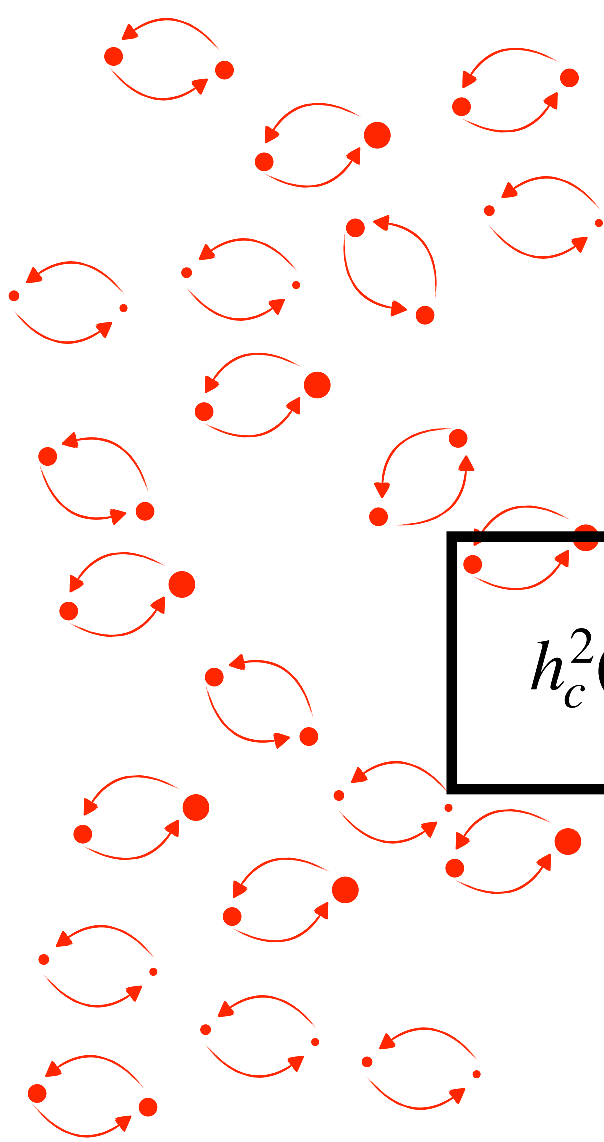
Mass function (distribution)

$$\frac{1}{(1+z)H(z)}$$

Late Universe $(0 \leq z \lesssim 10)$

$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\log f} = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f)\,,$$

$$h_c^2(f) = \frac{4}{\pi}\frac{G}{c^2} f^{-2}\int_0^\infty dz \frac{dn}{dz}\frac{1}{1+z} f_r \frac{dE_{\rm GW}}{df_r}\bigg|_{f_r = f(1+z)}$$



$$h_c^2(f) = \frac{4G^{5/3} f^{-2/3}}{3\pi^{1/2}c^2}\int dm_1 dm_2\, p(m_1, m_2)\int_0^{z_{\rm max}} dz' R(z') \frac{\mathcal{M}^{5/3}(m_1, m_2)}{(1+z)^{1/3}}\frac{dt_r}{dz}$$

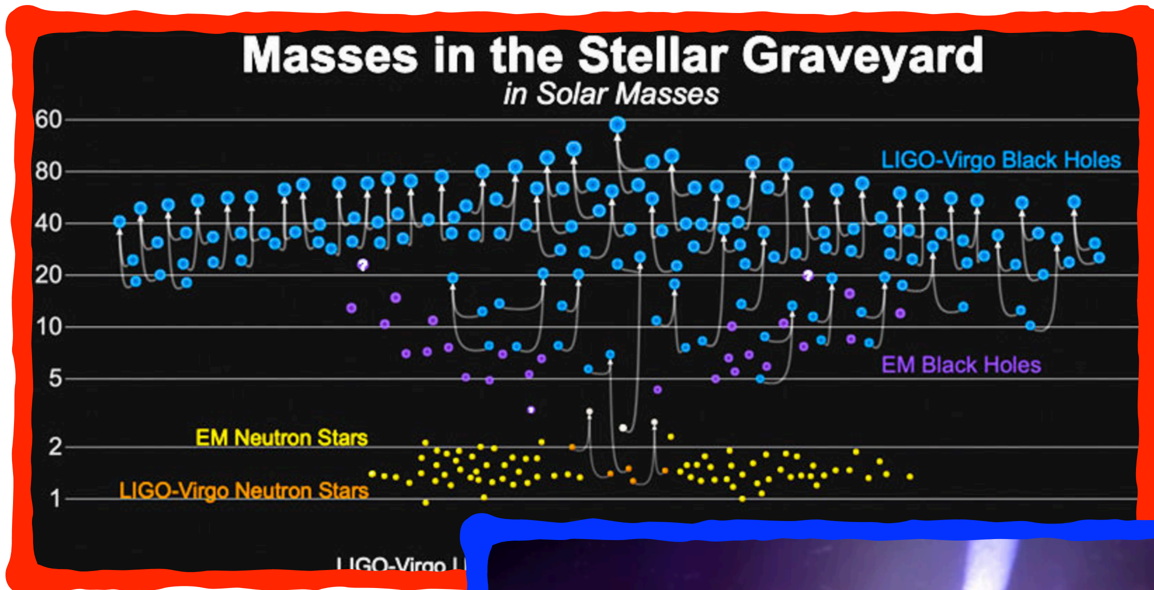
Late Universe $(0 \leq z \lesssim 10)$

Mass Function

Spin distribution

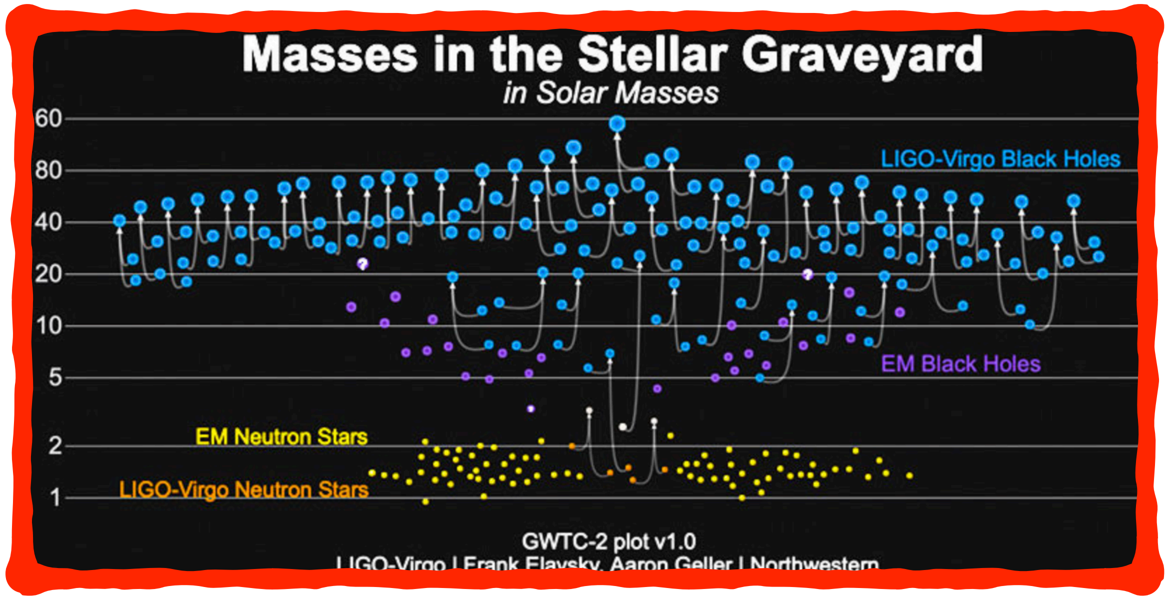
(Black Holes, Neutron Stars & White Dwarfs)

Late Universe $(0 \leq z \lesssim 10)$



Calibrated
Mass Function
Spin distribution
(Black Holes, Neutron Stars & White Dwarfs)

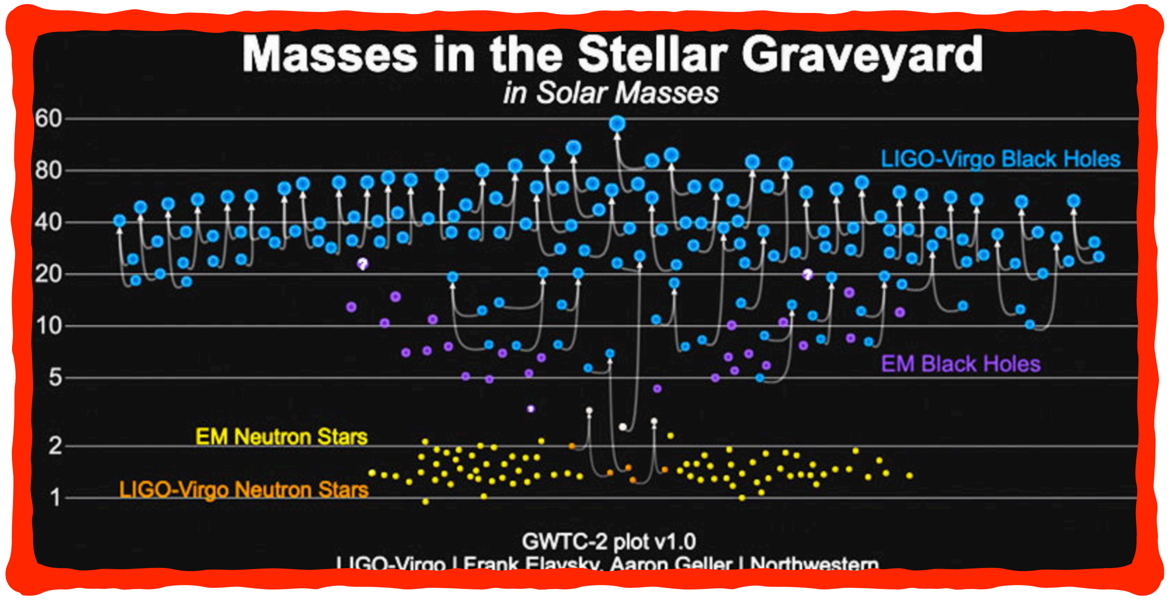
Late Universe $(0 \leq z \lesssim 10)$



→

Calibrated
Mass Function
Spin distribution
(Black Holes, Neutron Stars & White Dwarfs)

Late Universe $(0 \leq z \lesssim 10)$



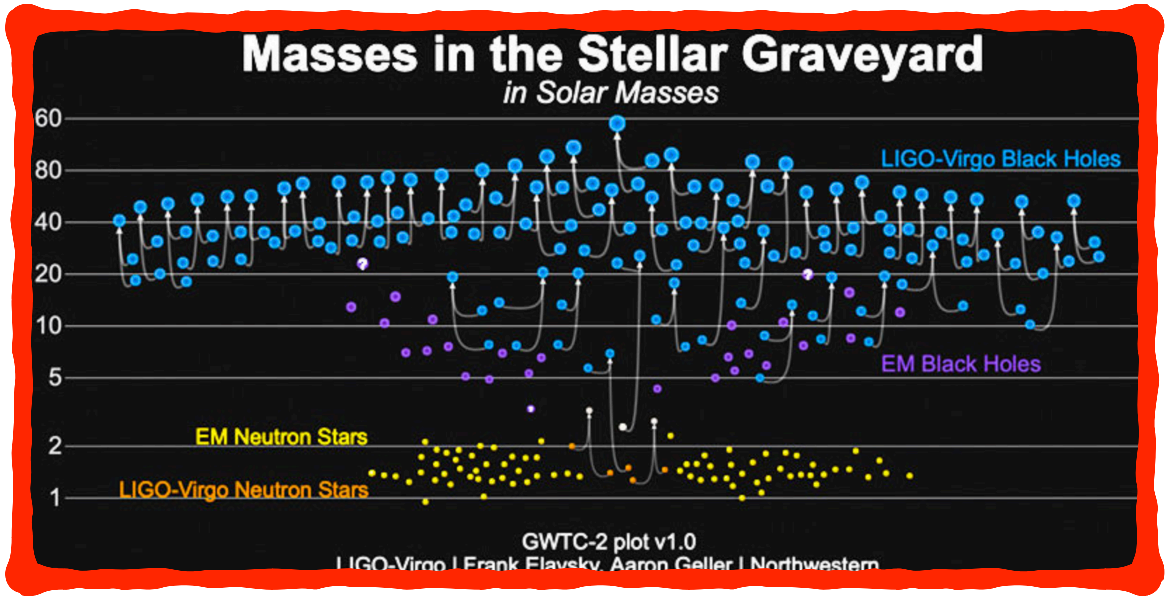
→ **Calibrated** **Mass Function** **Spin distribution** (**Black Holes**, Neutron Stars & White Dwarfs)

Computation of Solar-Mass Binary BH Grav. Wave Background

(LISA Collaboration)

with *S. Babak, C. Caprini, N. Karnesis, P. Marcoccia, M. Pieroni A. Ricciardone, A. Sesana, J. Torrado*

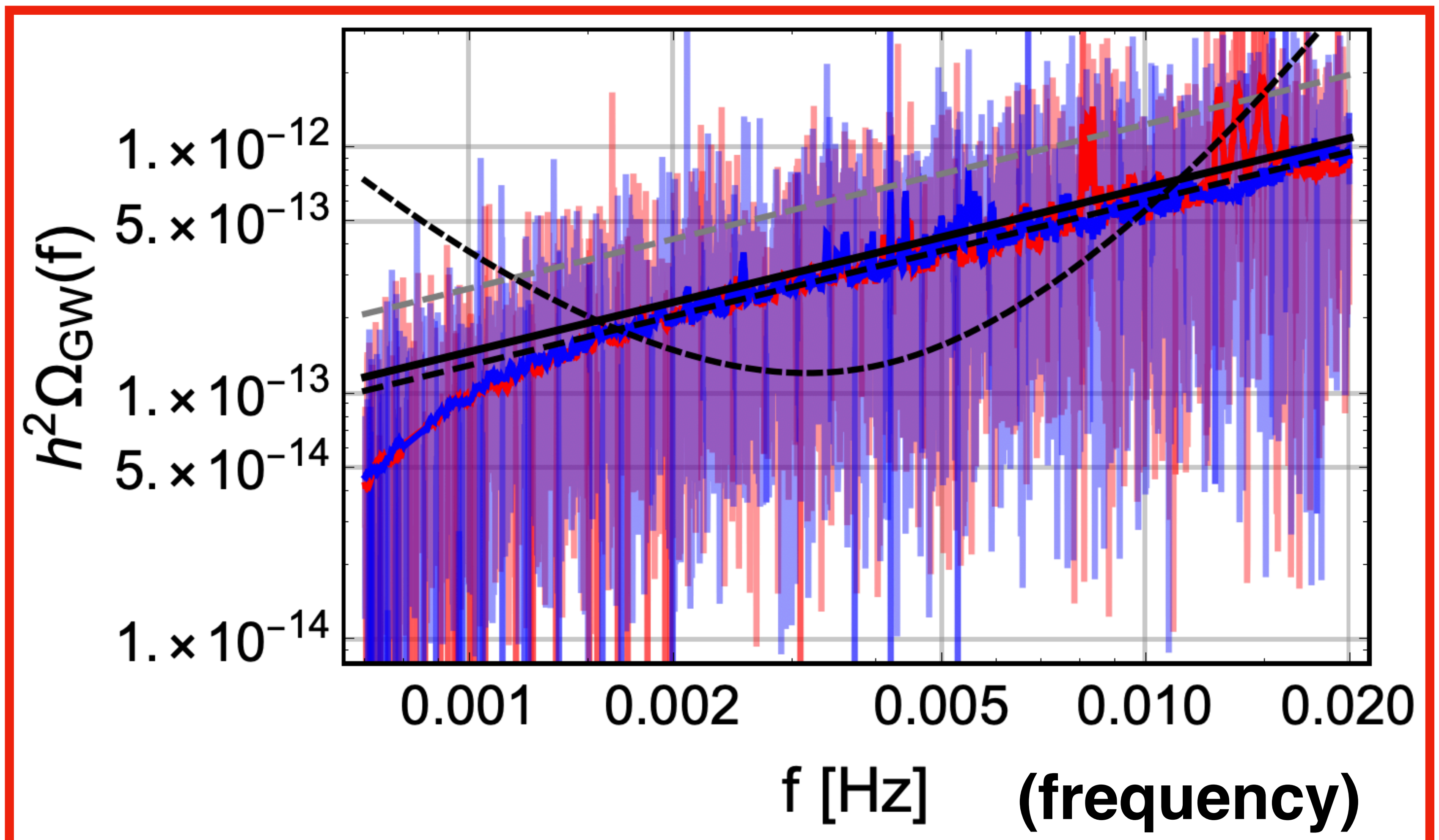
Late Universe $(0 \leq z \lesssim 10)$



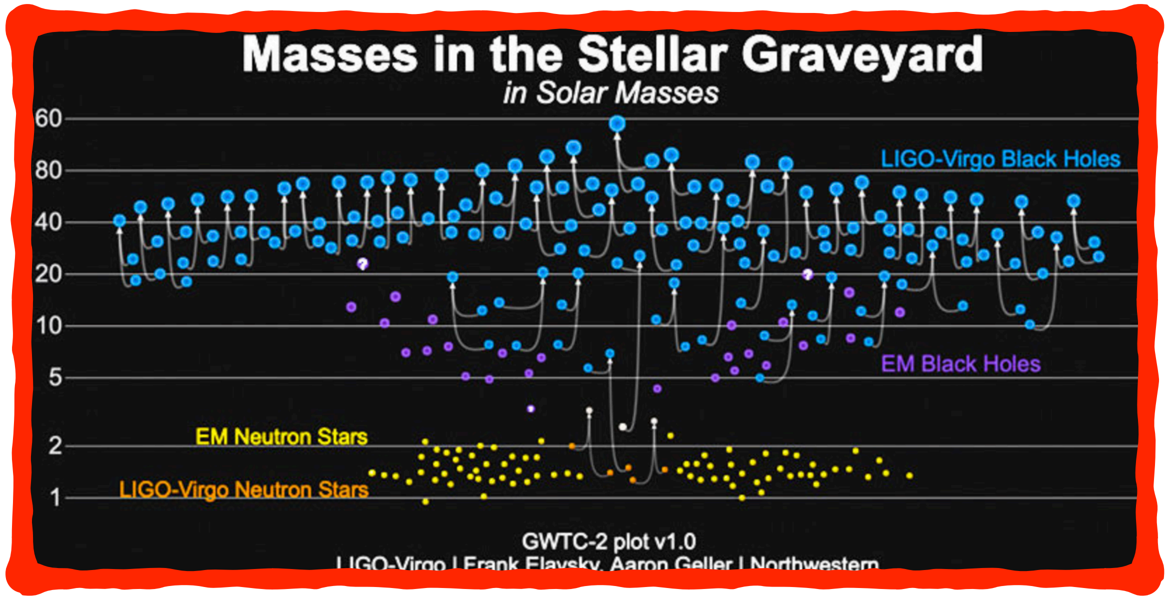
→ **Calibrated**
Mass Function
Spin distribution
(Black Holes, Neutron Stars & White Dwarfs)

GWB Spectrum

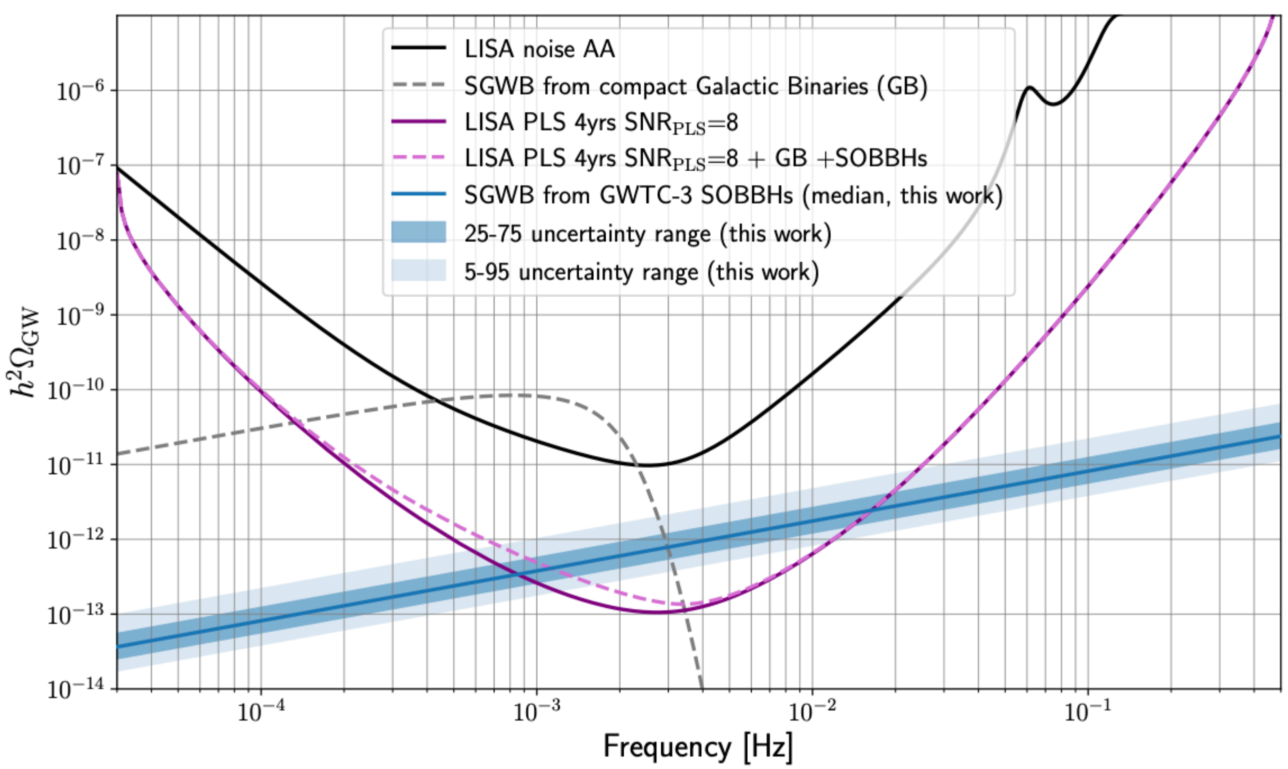
$\Omega_{\rm GW} \propto f^{2/3}$



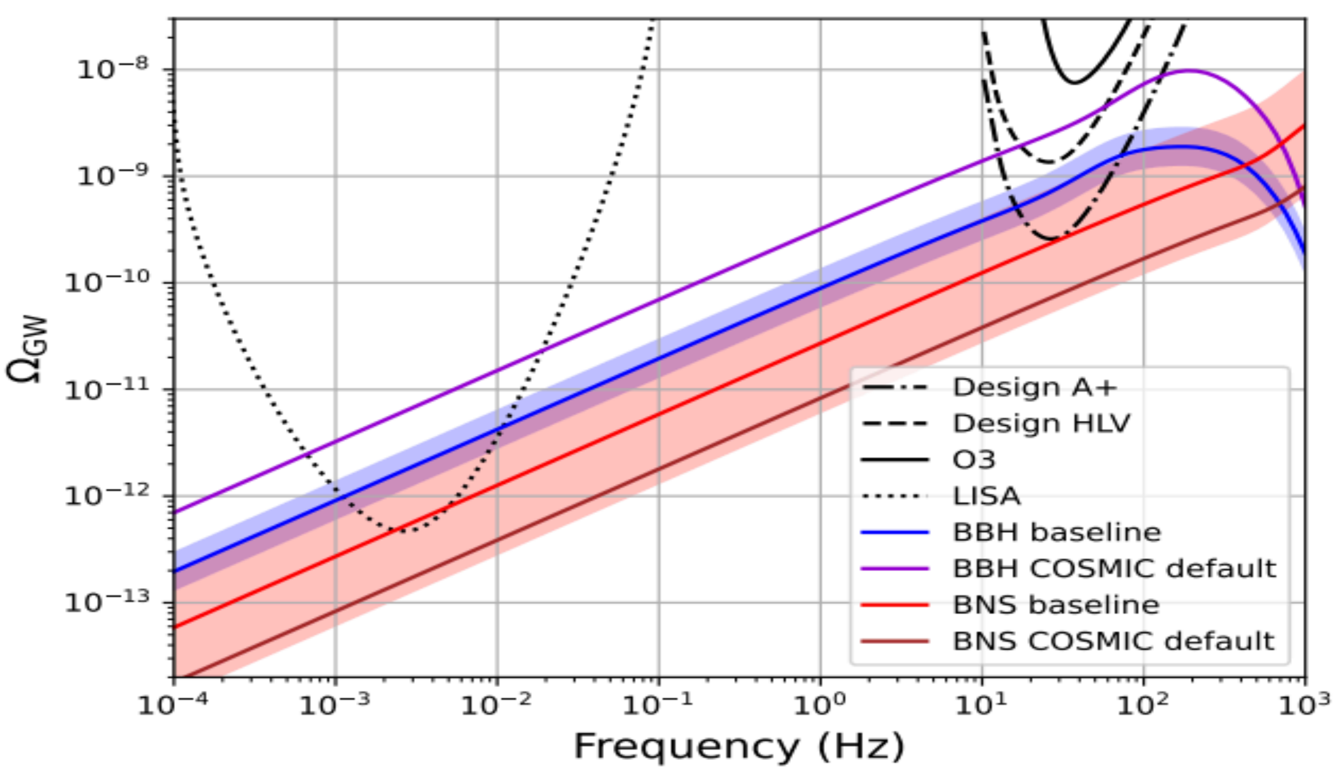
Late Universe $(0 \leq z \lesssim 10)$



Calibrated
Mass Function
Spin distribution
(Black Holes, Neutron Stars & White Dwarfs)



Babak et al 2023



Lehoucq et al 2023

Gravitational Wave Backgrounds

OUTLINE

1) Grav. Waves (GWs) — 1st Topic

Early Universe Sources (Core Topics):

2) GWs from Inflation
3) GWs from Preheating
4) GWs from Phase Transitions
5) GWs from Cosmic Defects

Late Universe + Observations (Briefly):

6) Astrophysical Background(s)
7) Observational Constraints/Prospects

Gravitational Wave Backgrounds

OUTLINE

1) Grav. Waves (GWs) ✓ — 1st Topic

Early Universe Sources — Core Topics

2) GWs from Inflation ✓

3) GWs from Preheating ✓

4) GWs from Phase Transitions ✓

5) GWs from Cosmic Defects ✓

Late Universe + Observations

6) Astrophysical Background(s) ✓

7) Observational Constraints/Prospects

← (Briefly)

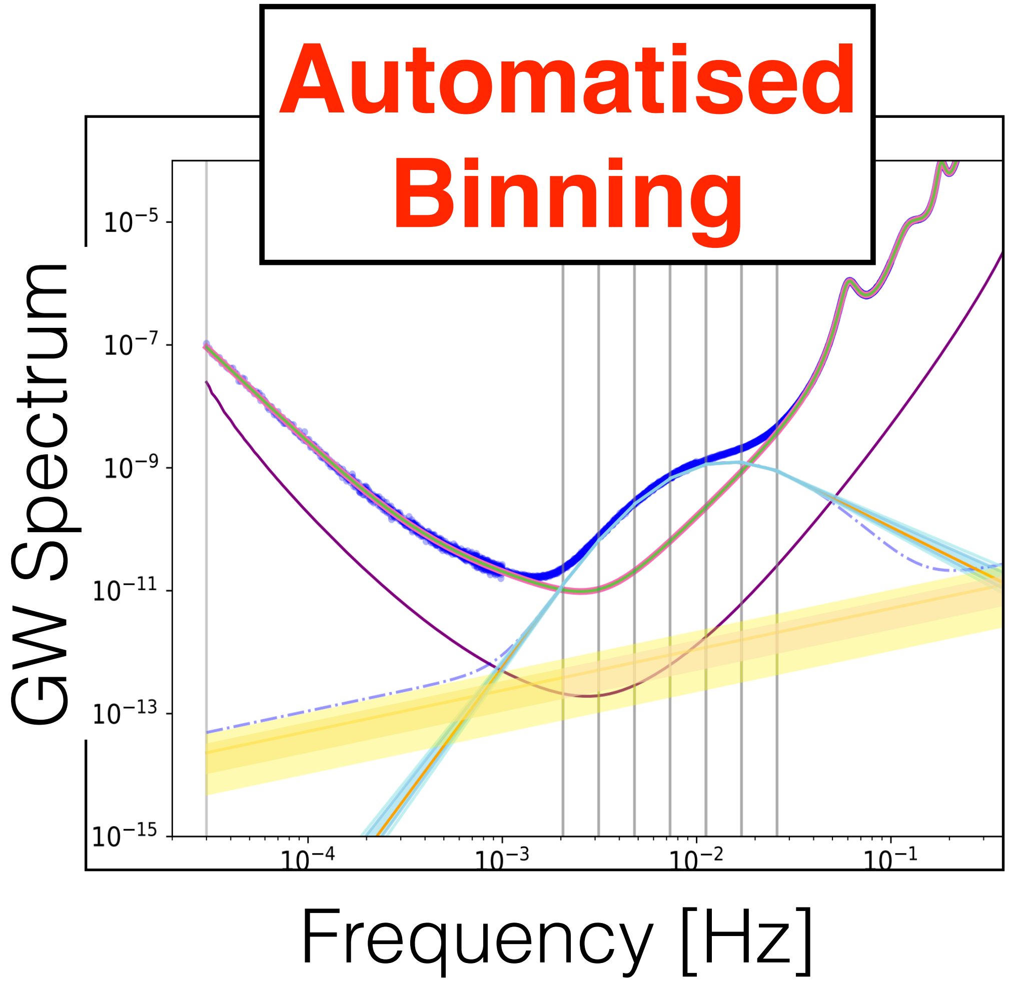
GWB Observations

O1. Data Analysis
O2. Constraints
O3. Evidence (PTA)

O1. Data Analysis

O1. Data Analysis

POWER SPECTRUM RECONSTRUCTION



Code `SGWBinner`

(Caprini et al [1906.09244](https://arxiv.org/abs/1906.09244))

O1. Data Analysis

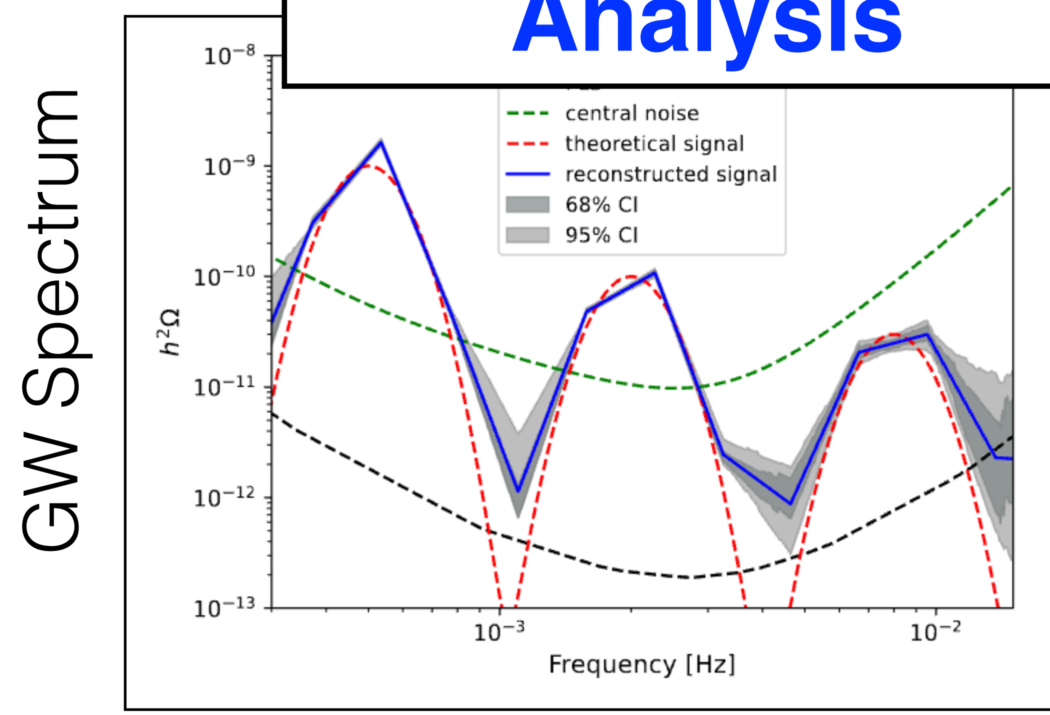
POWER SPECTRUM RECONSTRUCTION

Automatised Binning

Code `SGWBinner`

(Caprini et al 1906.09244)

Machine Learning Analysis



Code `GWBackFinder`

(Dimitriou et al 2309.08430)

Code `saqqara`

(Alvey et al 2309.07954)

O1. Data Analysis

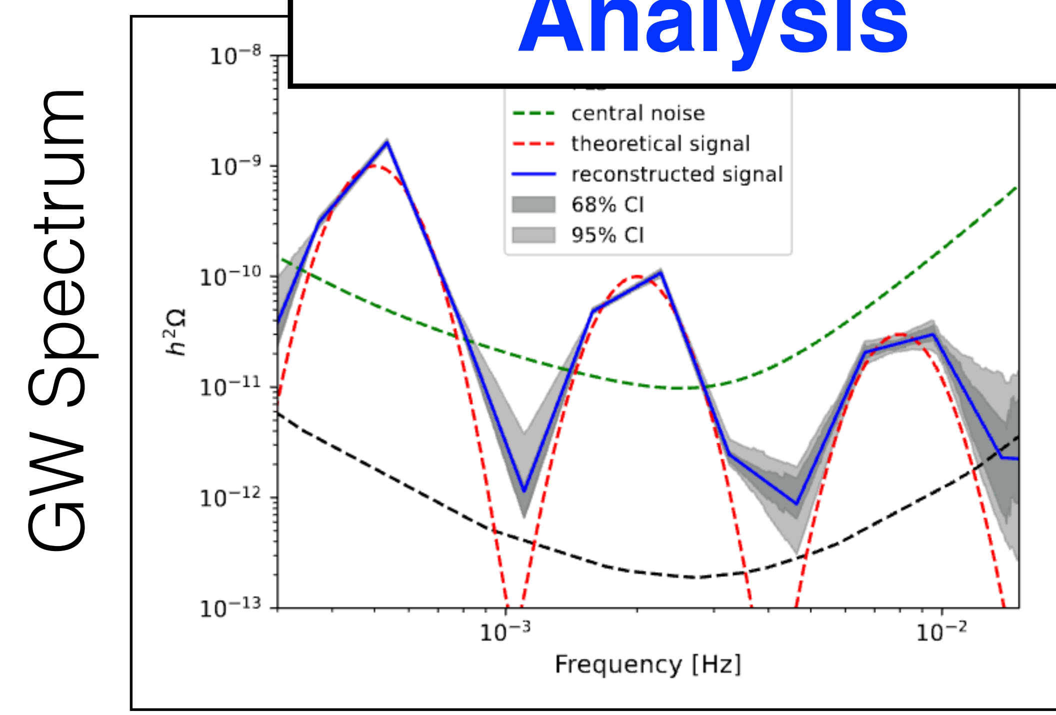
POWER SPECTRUM RECONSTRUCTION

Automatised Binning

Code SGWBinner

(Caprini et al 1906.09244)

Machine Learning Analysis



Code GWBackFinder

(Dimitriou et al 2309.08430)

Code saqqara

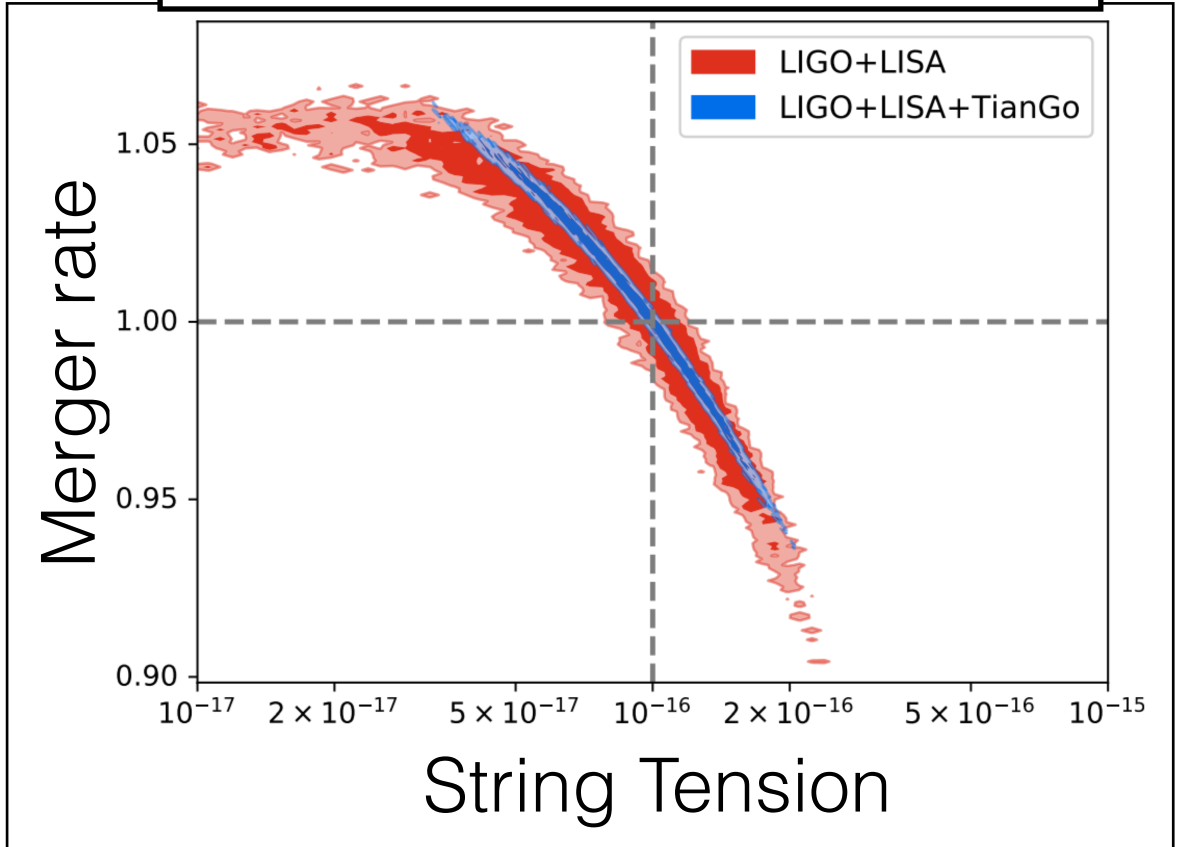
(Alvey et a

Can be adapted to any experimental Noise (parametrised)

O1. Data Analysis

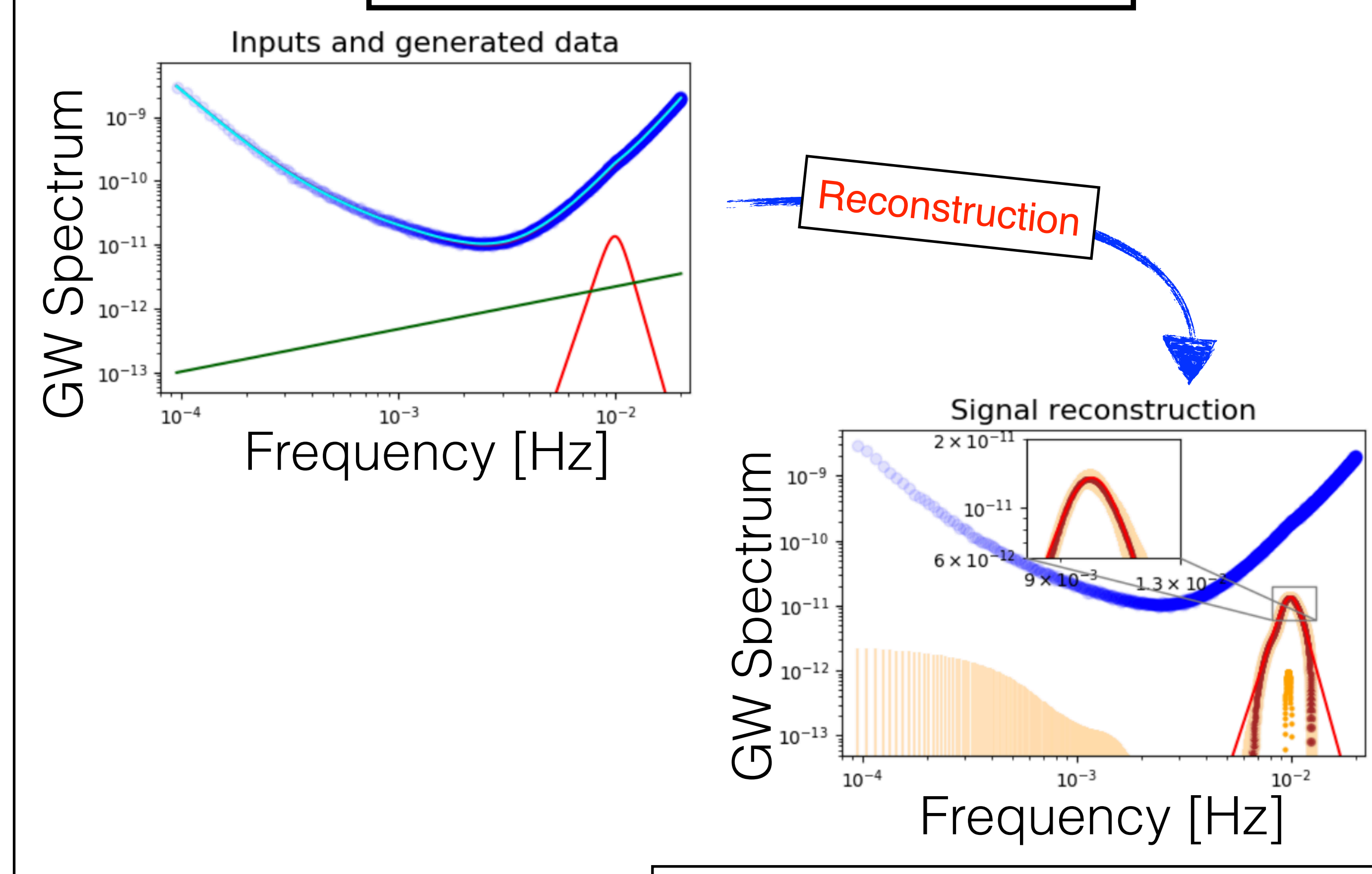
SIGNAL SEPARATION

Combining experiments



(e.g. Cui et al 2012.07874)

Reconstruction over foreground

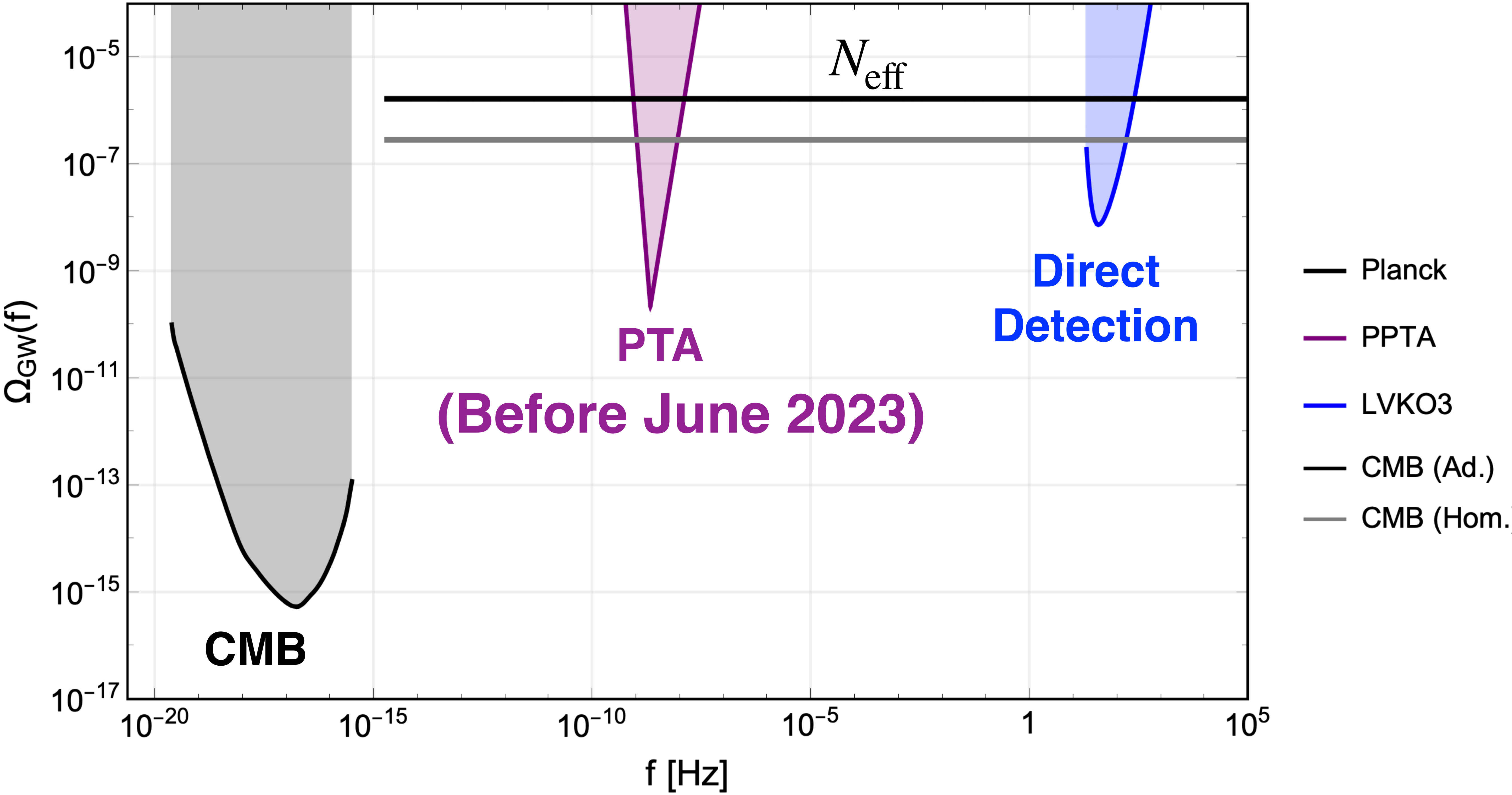


(e.g. Pieroni et al 2004.01135)

O2. Constraints

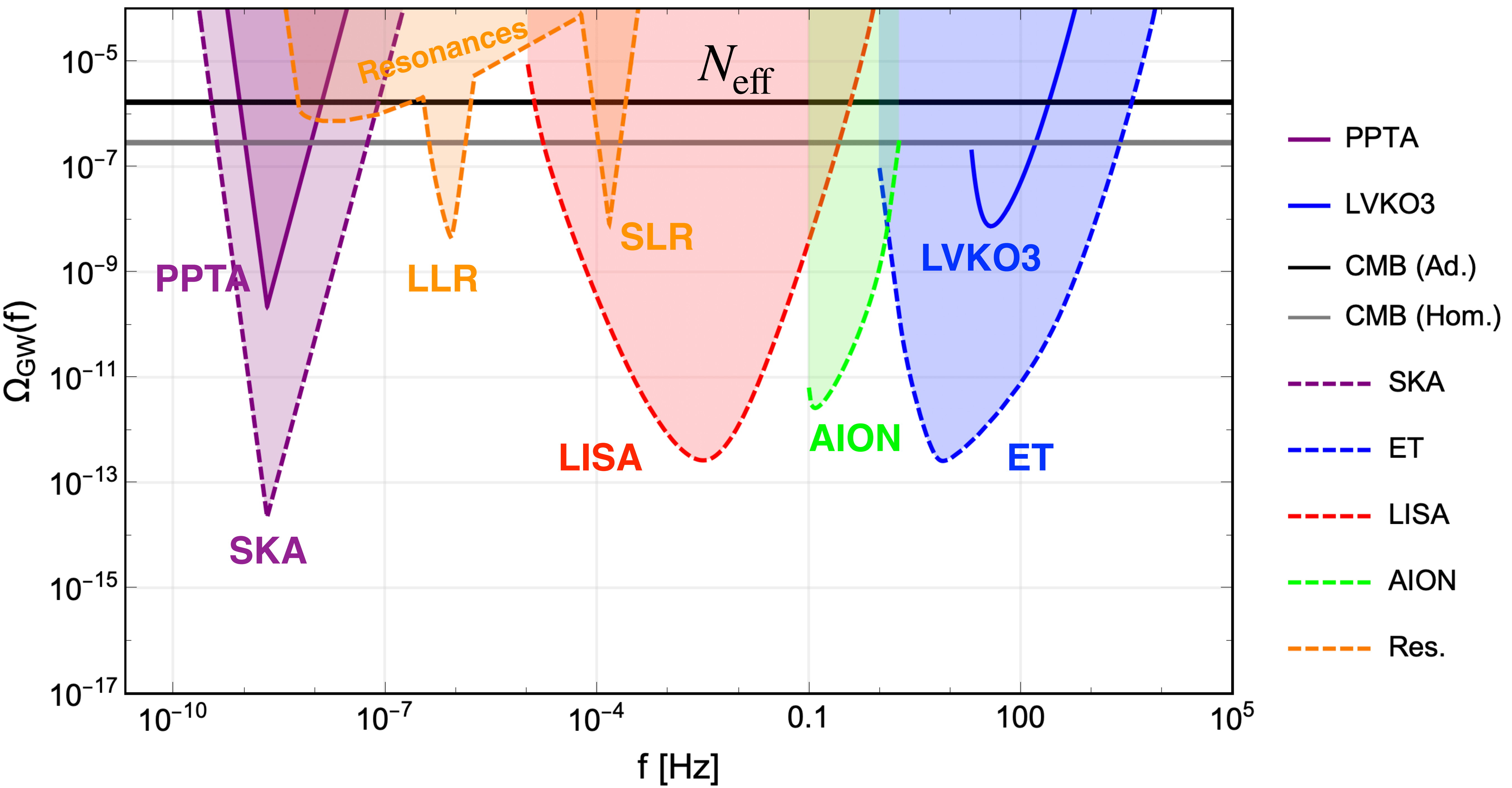
O2. Constraints

(Before June 2023)



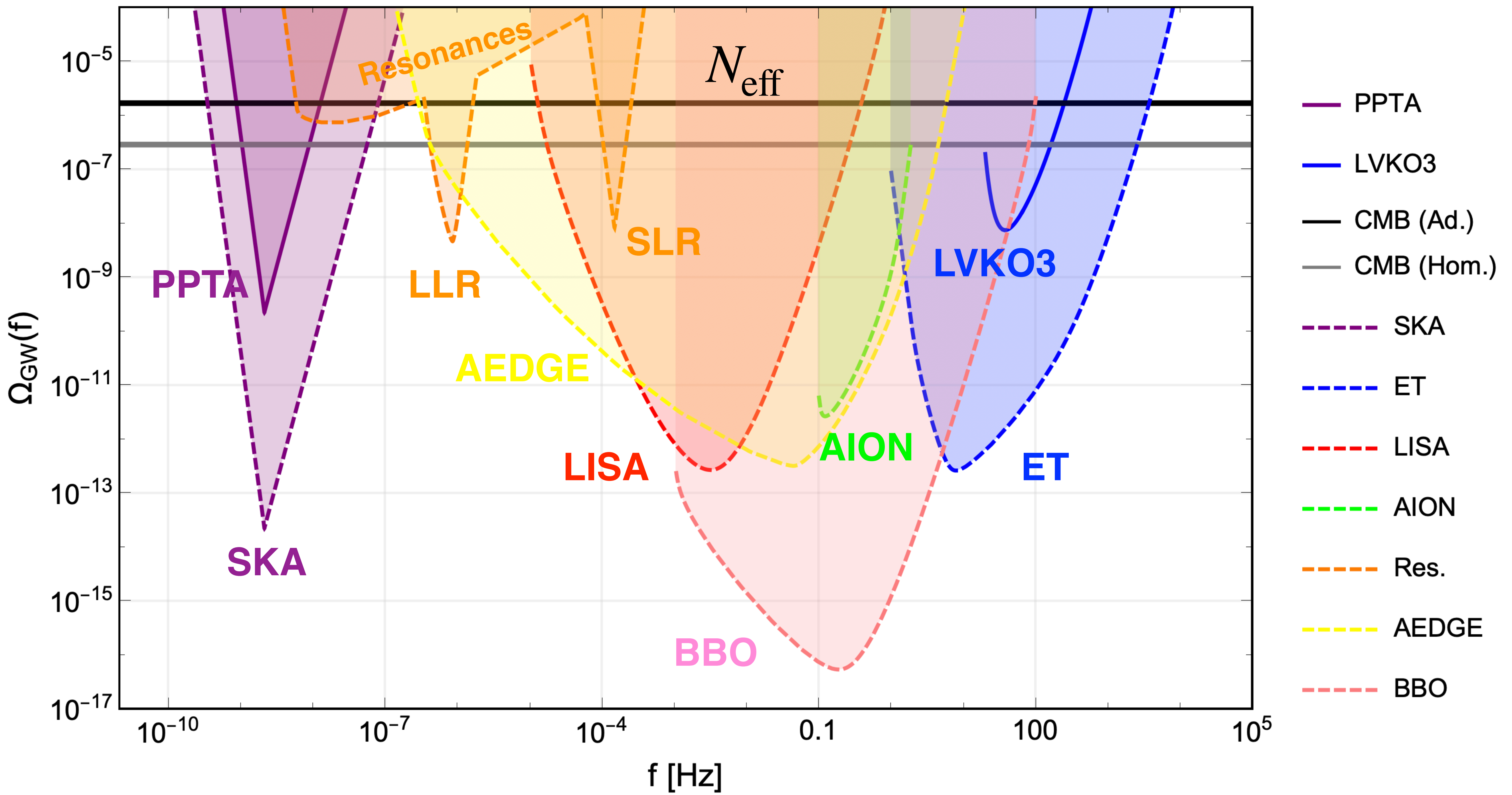
O2. Constraints

Future ~ 15-20 yrs

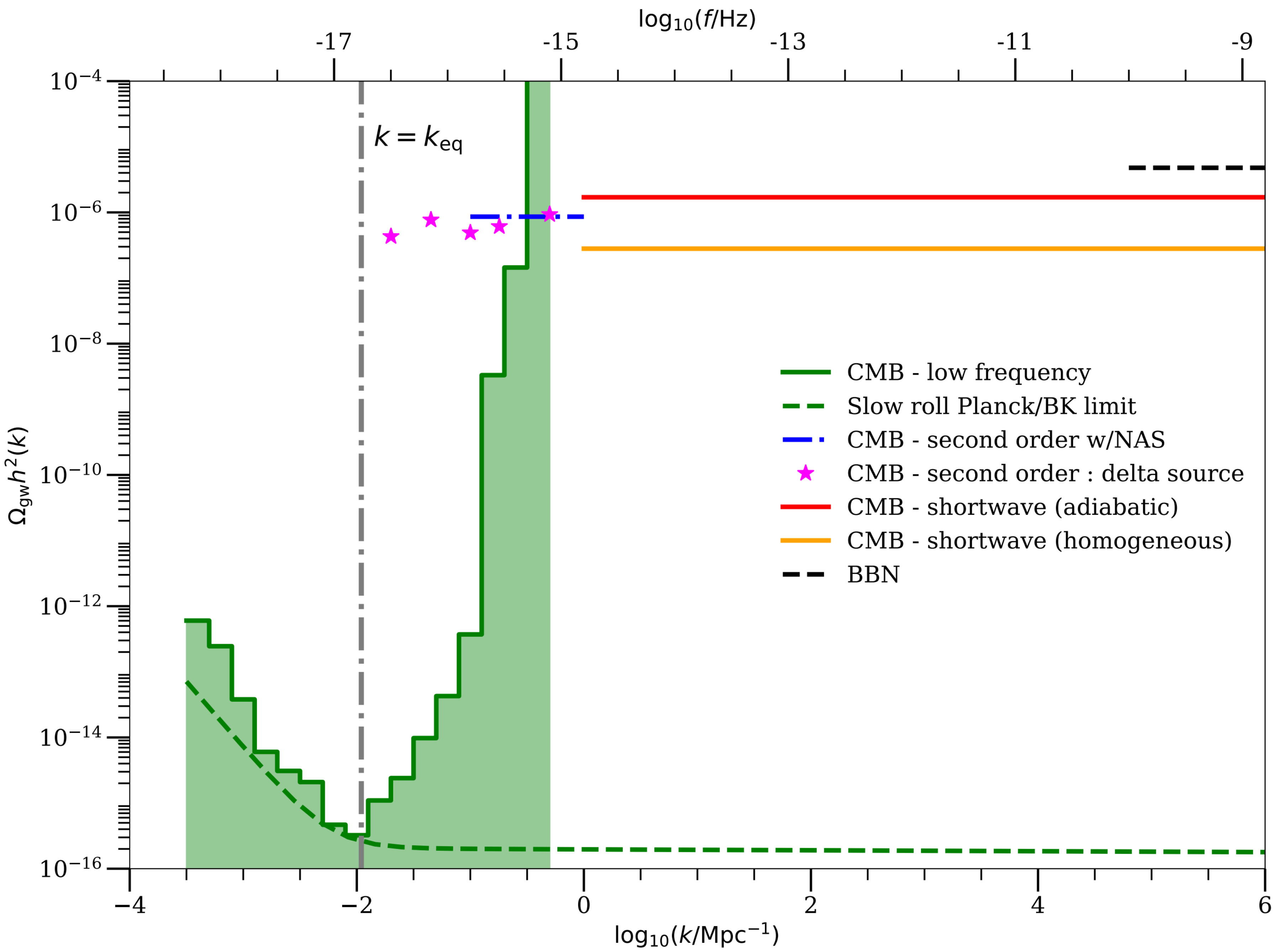


O2. Constraints

Future ~ 30-40 yrs



CMB Latest Analysis

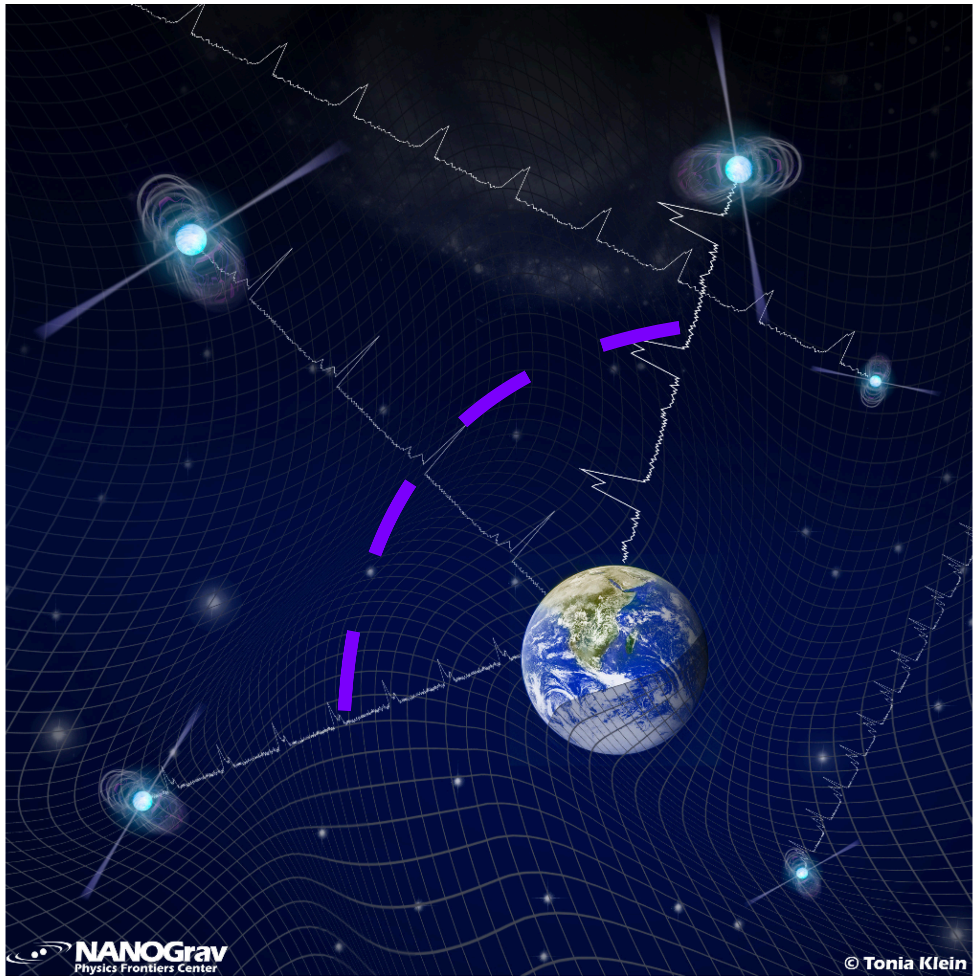


(from Copeland et al 2004.11396)

O3. Evidence (PTA)

O3. Evidence (PTA)

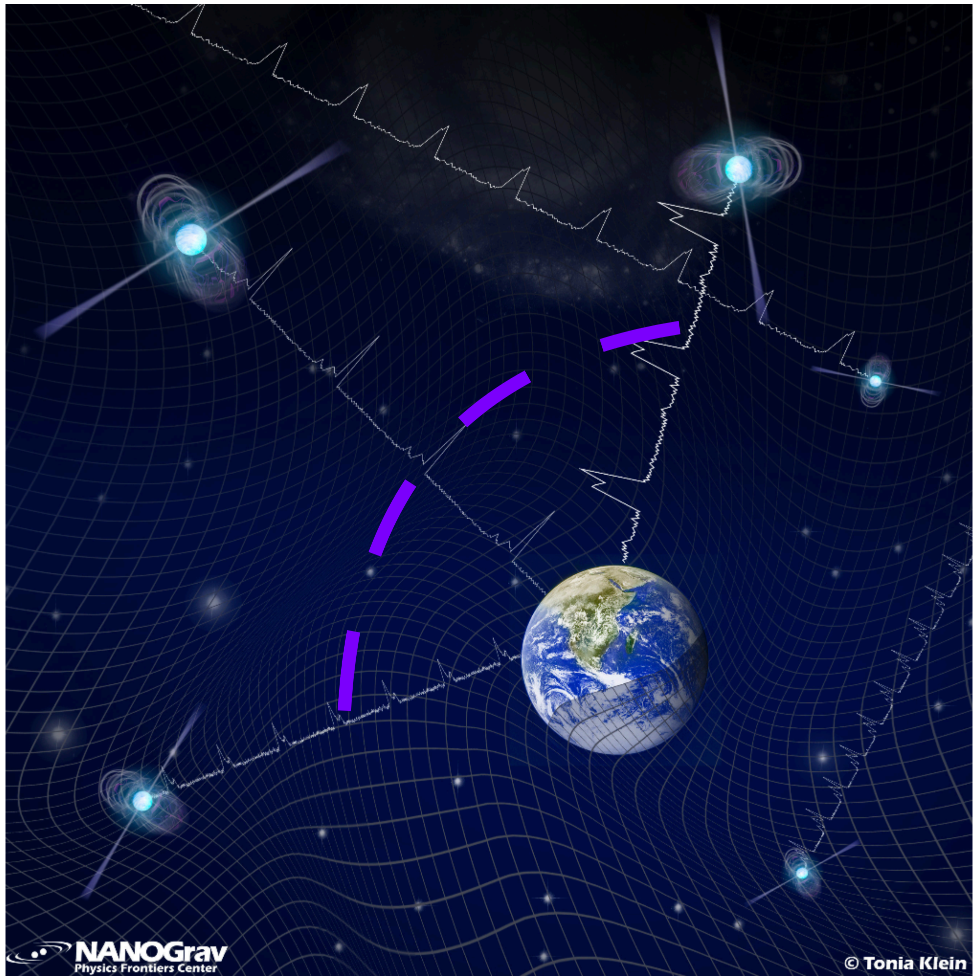
NanoGrav, EPTA+IPTA, PPTA, CPTA
(Pulsar Timing Array Collaborations)



(June 2023)

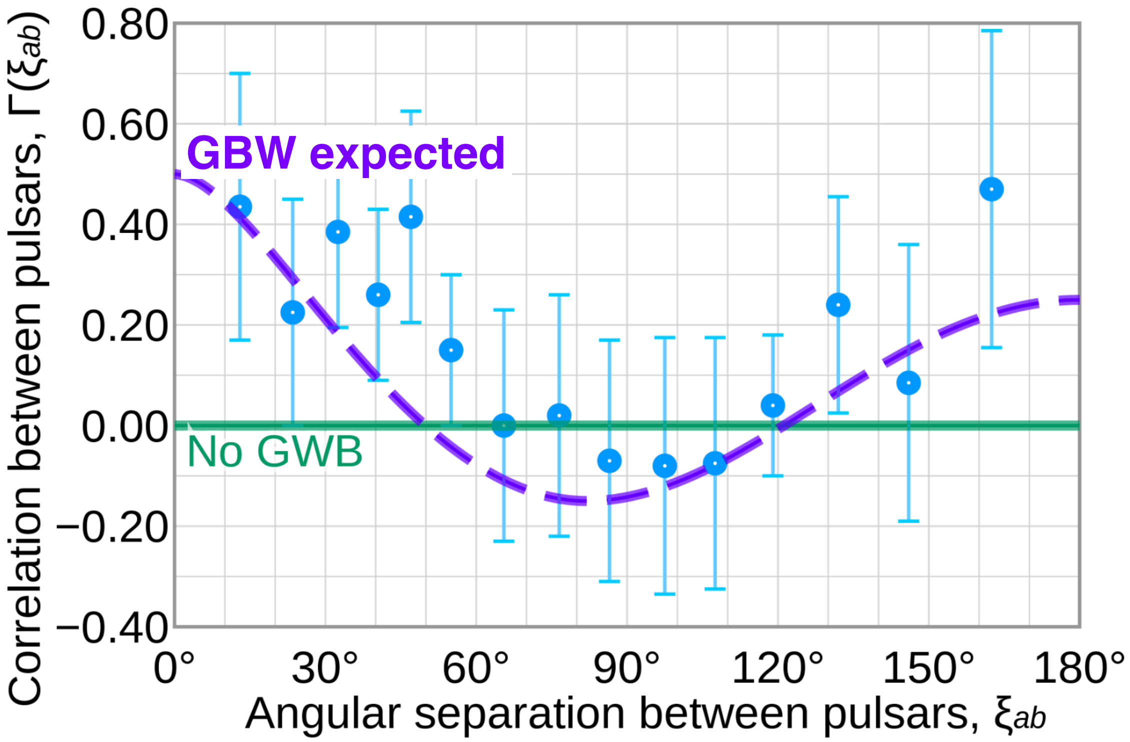
O3. Evidence (PTA)

NanoGrav, EPTA+IPTA, PPTA, CPTA
(Pulsar Timing Array Collaborations)



[Hellings, Downs: Astrophys. J. 265 (1983) L39]

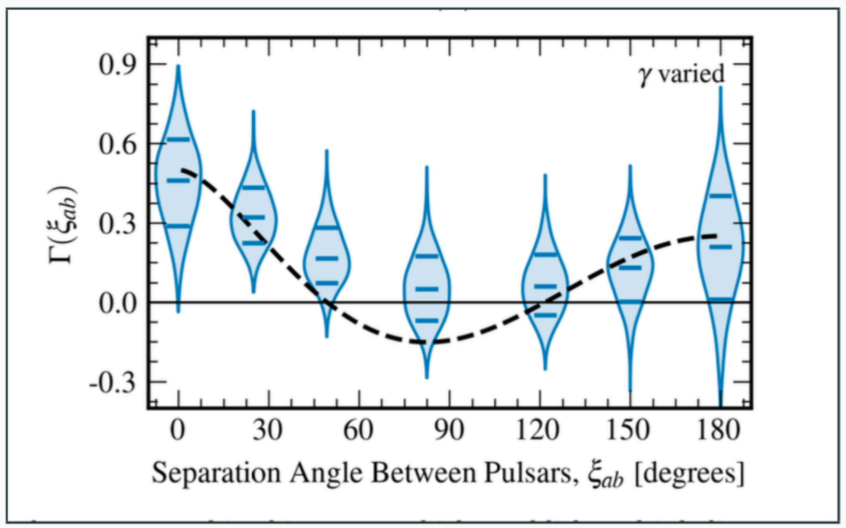
Hellings - Downs



O3. Evidence (PTA)

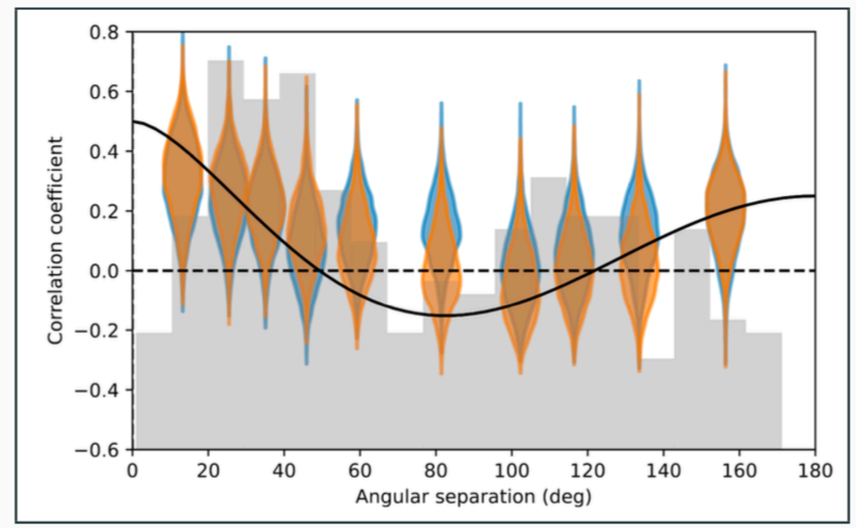
"Strong" Evidence

2306.16213: NANOGrav



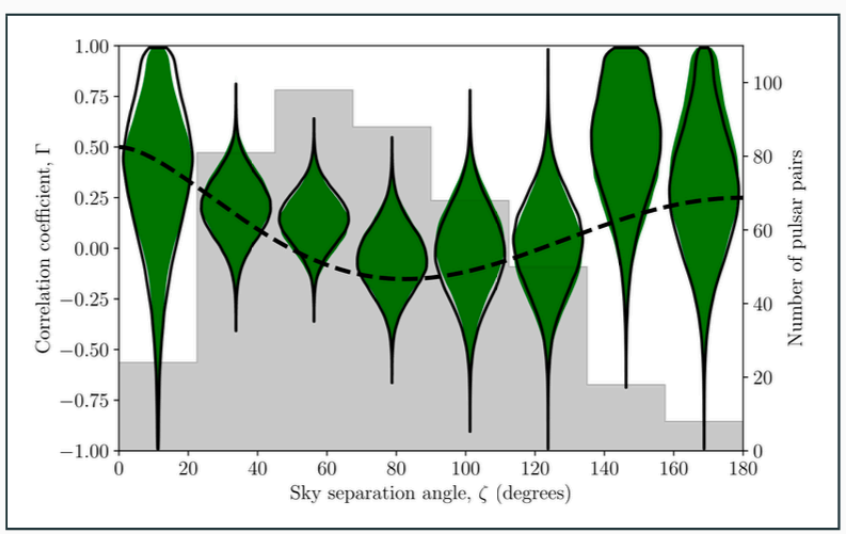
68 pulsars, 16 yr of data, HD at $\sim 3 \cdots 4\,\sigma$

2306.16214: EPTA+InPTA



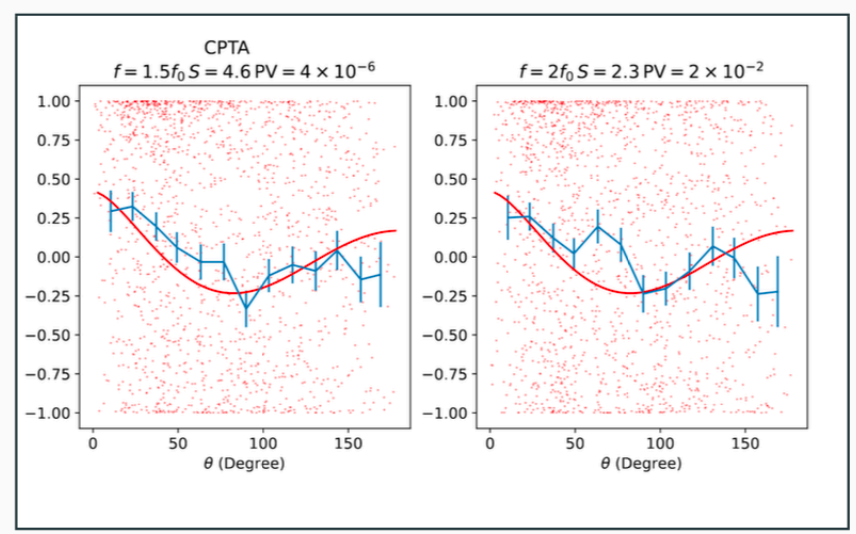
25 pulsars, 25 yr of data, HD at $\sim 3\,\sigma$

2306.16215: PPTA



32 pulsars, 18 yr of data, HD at $\sim 2\,\sigma$

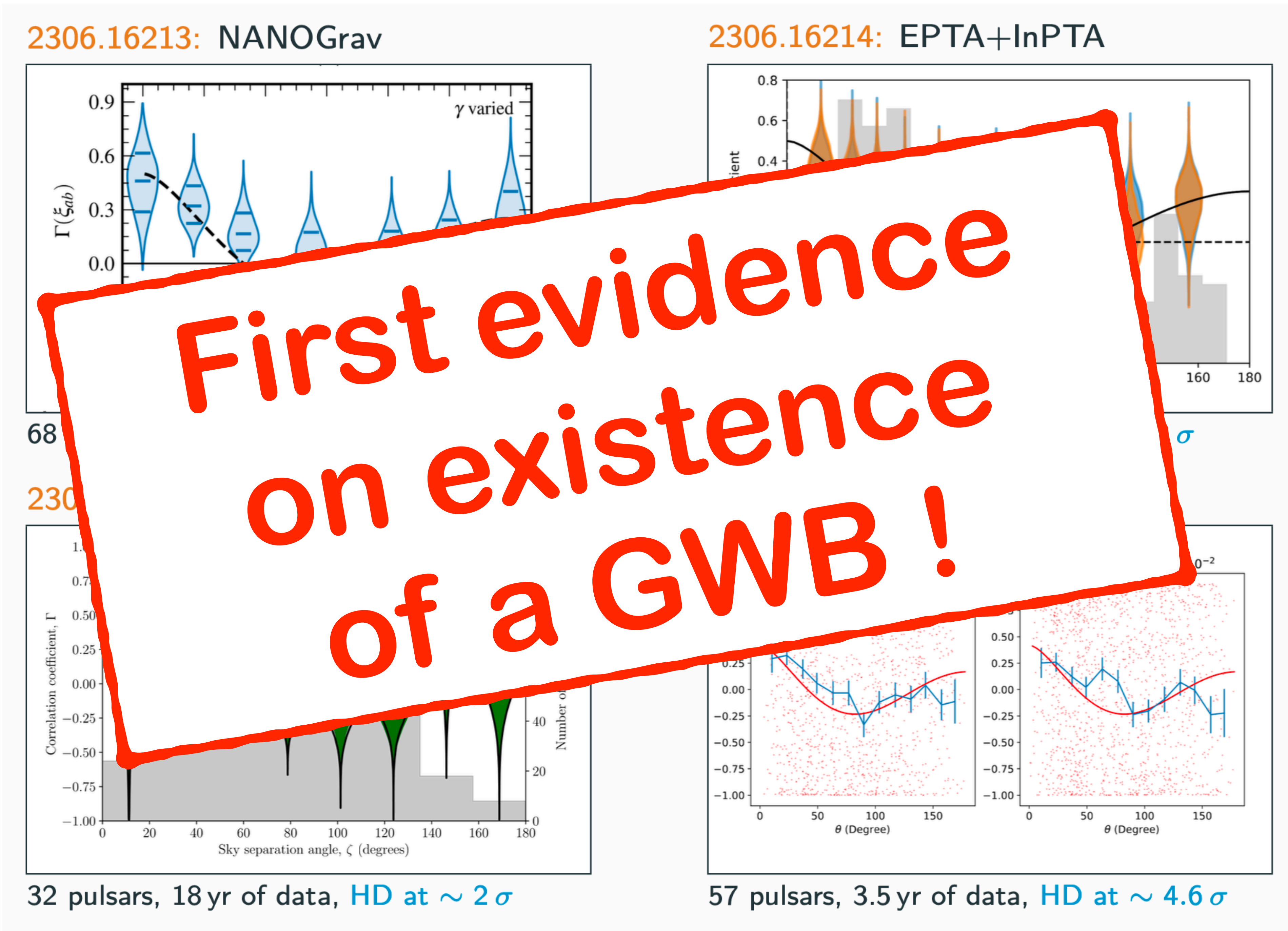
2306.16216: CPTA



57 pulsars, 3.5 yr of data, HD at $\sim 4.6\,\sigma$

O3. Evidence (PTA)

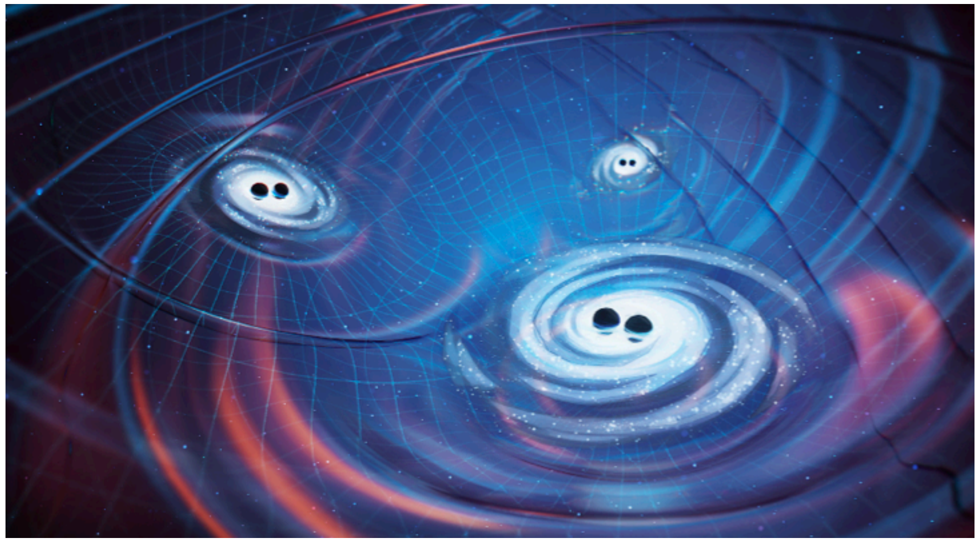
"Strong" Evidence



O3. Evidence (PTA)

Interpretation

Super Massive BHB (SMBHB, expected)

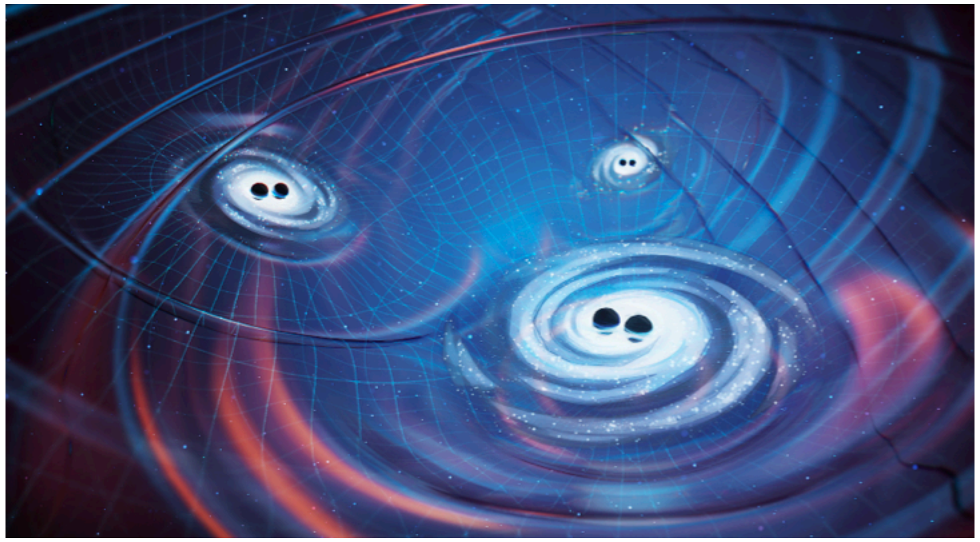


NanoG: 2306.16220 [astro-ph.HE]
EPTA: 2306.16227 [astro-ph.CO]

O3. Evidence (PTA)

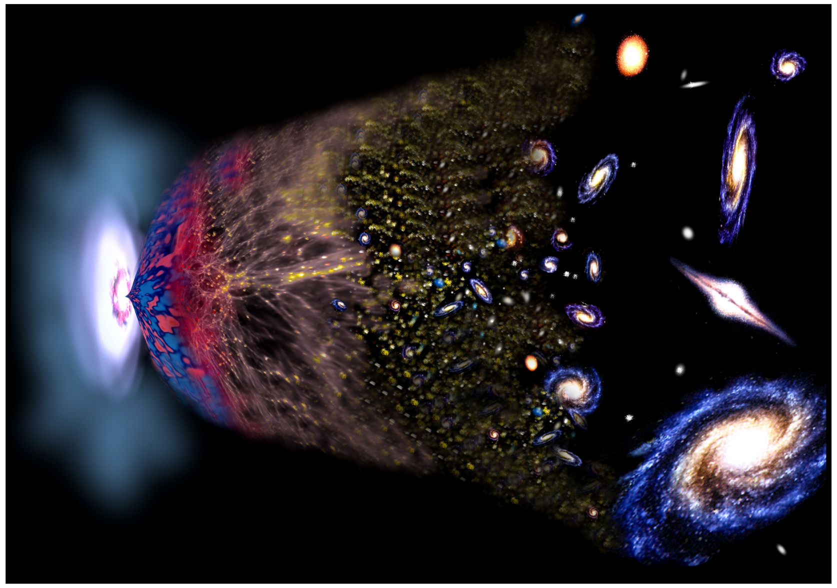
Interpretation

Super Massive BHB (SMBHB, expected)



or

Cosmological GWB (more speculative)



NanoG: 2306.16220 [astro-ph.HE]
EPTA: 2306.16227 [astro-ph.CO]

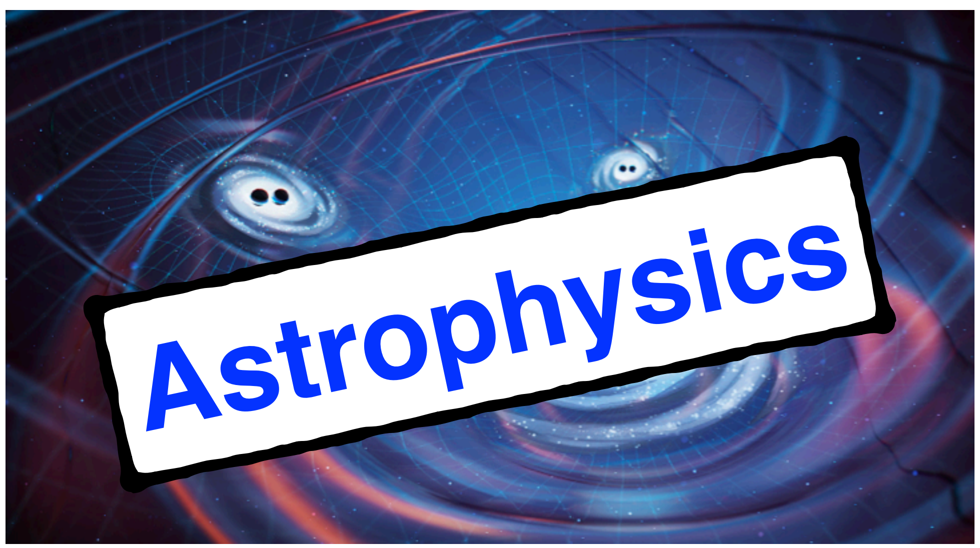
EPTA: 2306.16227 [astro-ph.CO]
NanoG: 2306.16219 [astro-ph.HE]

Franciolini+, Vagnozzi, Kitajima+, Cai+, Ellis+, Han+, Wang+, Liu+, DGF+, Inomata+, Bai+, Fujikura+, …

O3. Evidence (PTA)

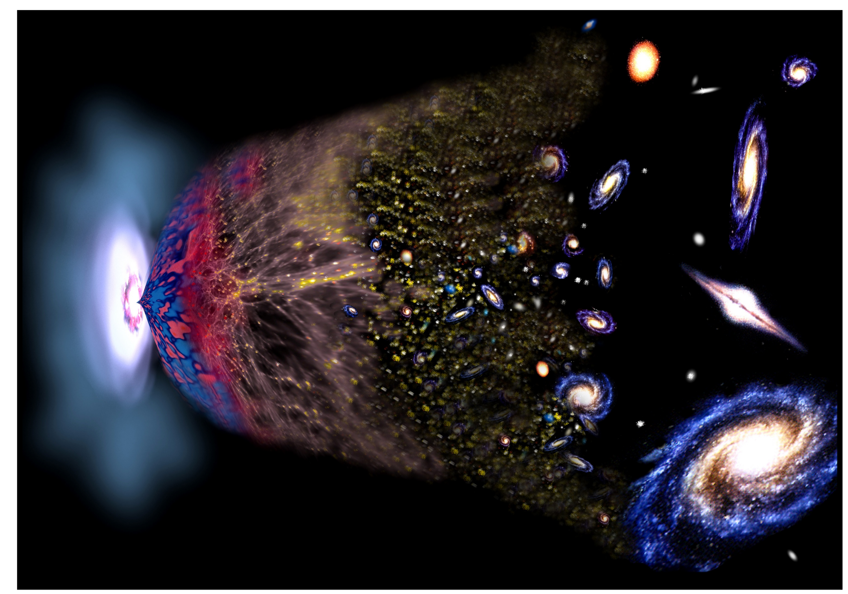
Interpretation

Super Massive BHB (SMBHB, expected)



or

Cosmological GWB (more speculative)



NanoG: 2306.16220 [astro-ph.HE]
EPTA: 2306.16227 [astro-ph.CO]

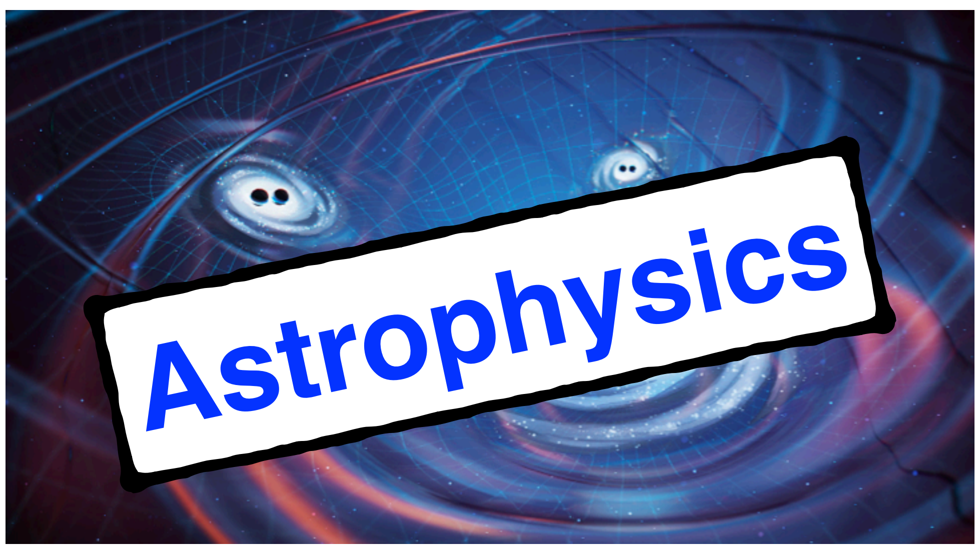
EPTA: 2306.16227 [astro-ph.CO]
NanoG: 2306.16219 [astro-ph.HE]

Franciolini+, Vagnozzi, Kitajima+, Cai+, Ellis+, Han+, Wang+, Liu+, DGF+, Inomata+, Bai+, Fujikura+, …

O3. Evidence (PTA)

Interpretation

Super Massive BHB (SMBHB, expected)



or

Cosmological GWB (more speculative)

NanoG: 2306.16220 [astro-ph.HE]
EPTA: 2306.16227 [astro-ph.CO]

EPTA: 2306.16227 [astro-ph.CO]
NanoG: 2306.16219 [astro-ph.HE]

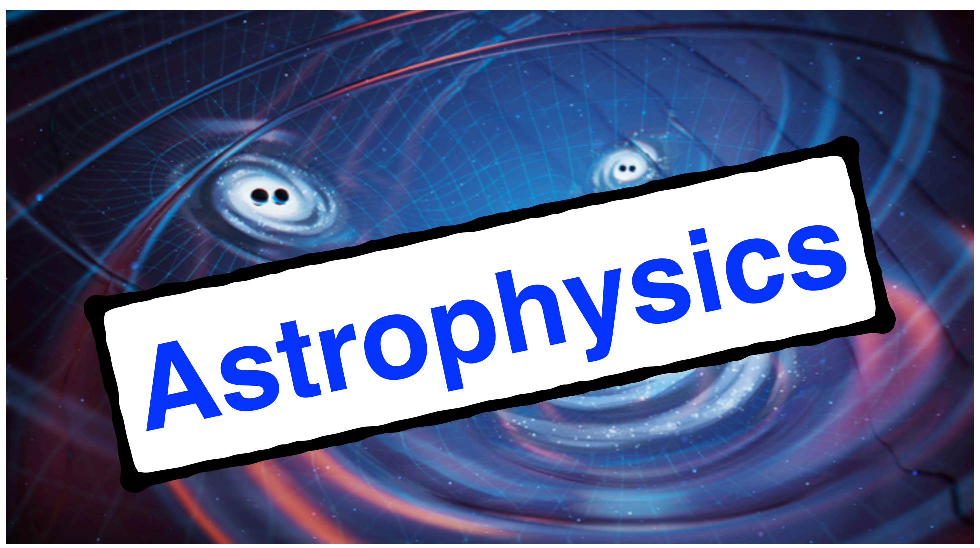
Franciolini+, Vagnozzi, Kitajima+, Cai+, Ellis+, Han+, Wang+, Liu+, DGF+, Inomata+, Bai+, Fujikura+, …

O3. Evidence (PTA)

Interpretation

Super Massive BHB (SMBHB, expected)

Cosmological GWB (more speculative)



or

NanoG: 2306.16220 [astro-ph.HE]
EPTA: 2306.16227 [astro-ph.CO]

EPTA: 2306.16227 [astro-ph.CO]
NanoG: 2306.16219 [astro-ph.HE]

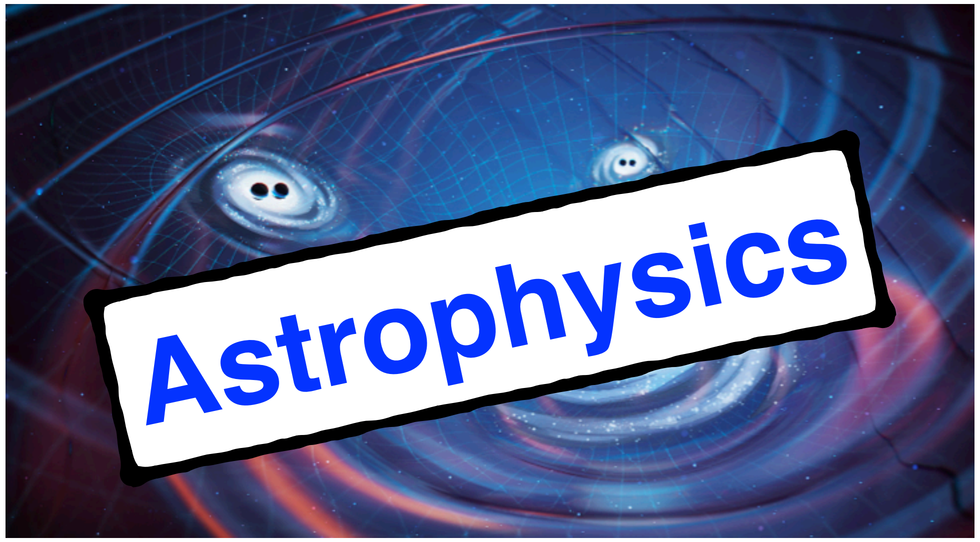
Franciolini+, Vagnozzi, Kitajima+, Cai+, Ellis+, Han+, Wang+, Liu+, DGF+, Inomata+, Bai+, Fujikura+, …

O3. Evidence (PTA)

Interpretation

Super Massive BHB (SMBHB, expected)

Cosmological GWB (more speculative)



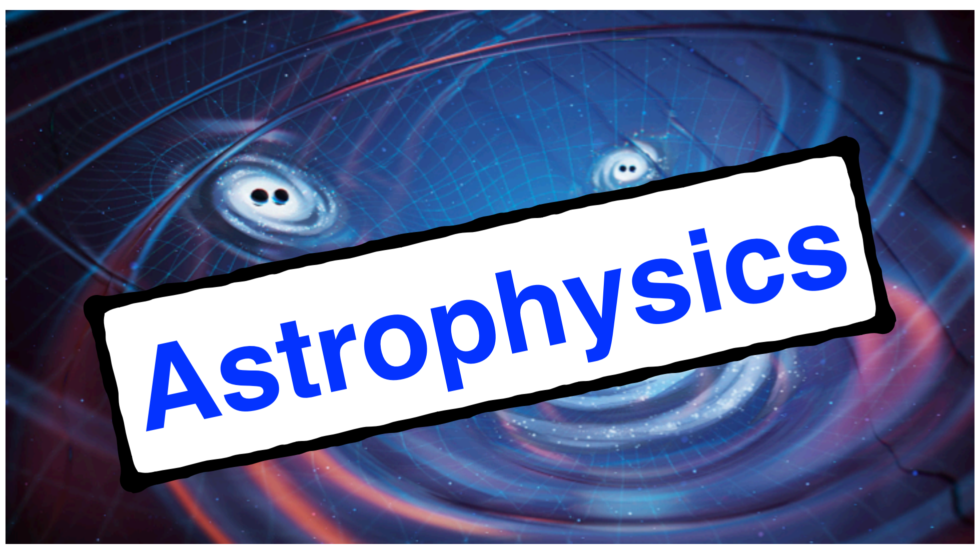
or

Key: Anisotropies ?

O3. Evidence (PTA)

Interpretation

Super Massive BHB (SMBHB, expected)



Key: Anisotropies ?

or

Cosmological GWB (more speculative)

Key: Spectrum

Inflation, Domain Walls, Cosmic Strings
1stO-PhT, audible Axions, 2ndOI-GWB, …

Summary
& Outlook

Gravitational Wave Backgrounds

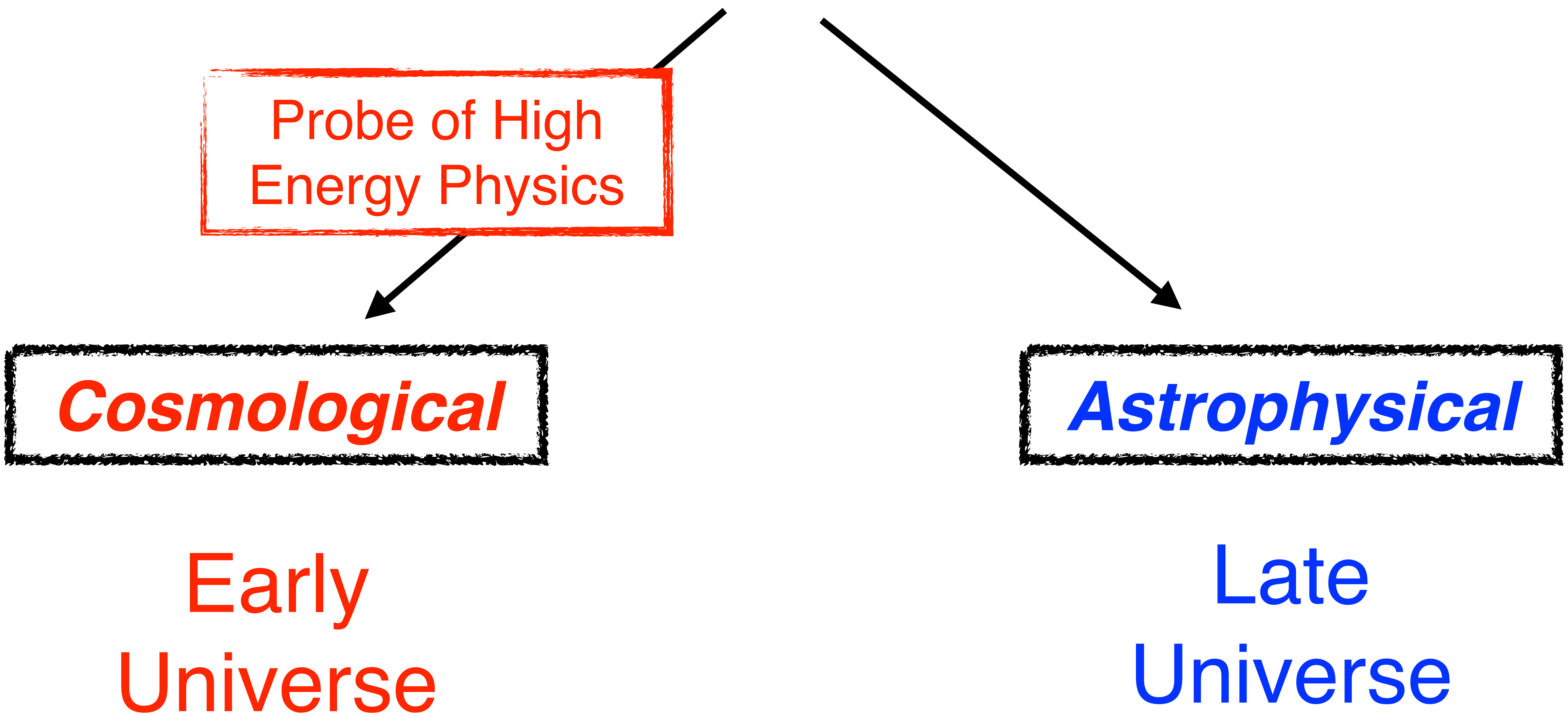
Cosmological

Early Universe

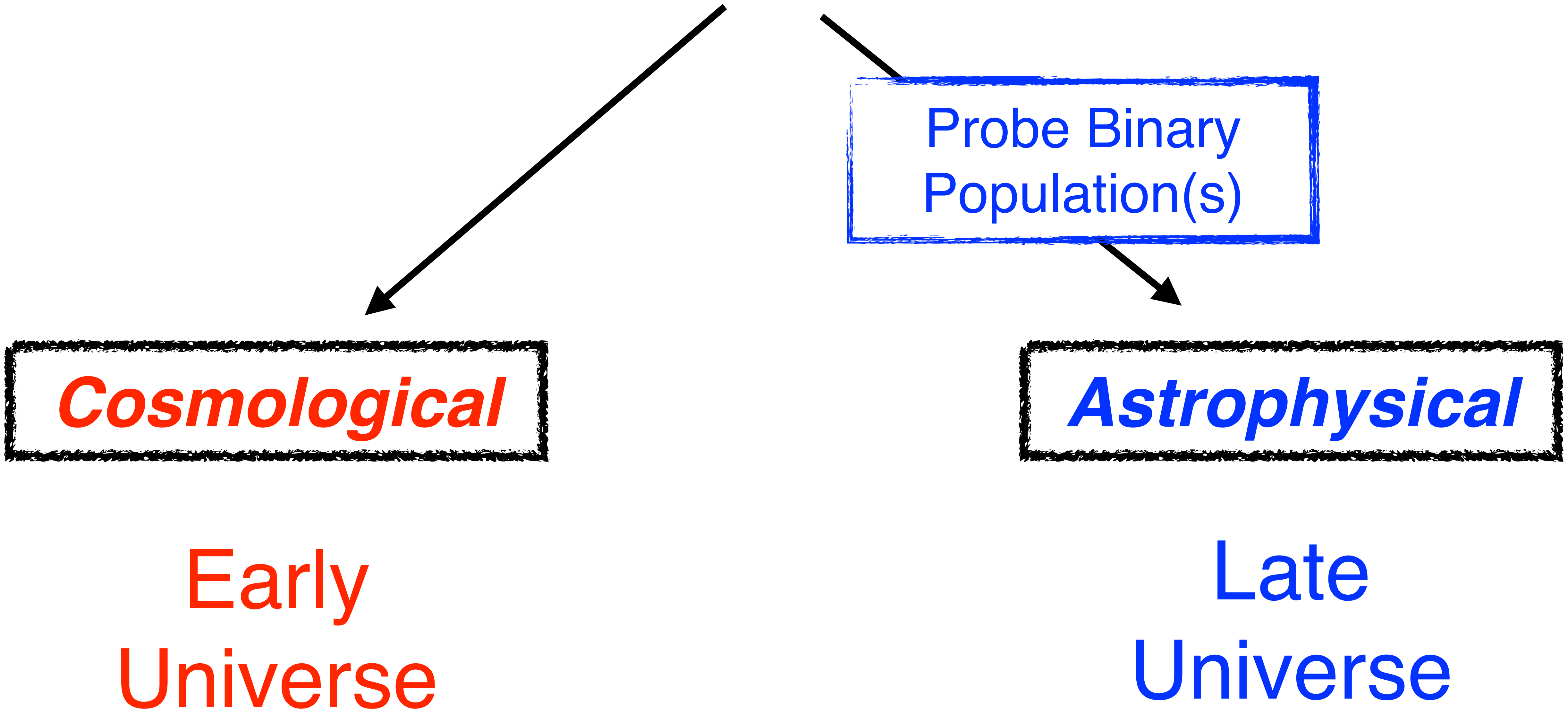
Astrophysical

Late Universe

Gravitational Wave Backgrounds



Gravitational Wave Backgrounds



Gravitational Wave Backgrounds

HOLY GRAIL

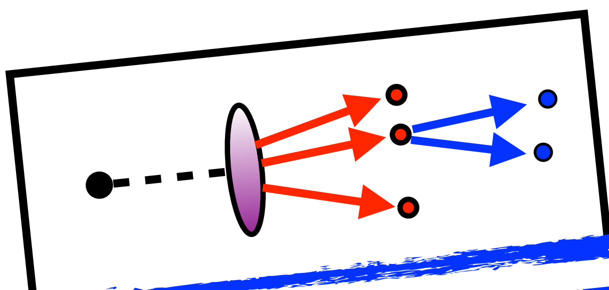
Cosmological

Early Universe

Astrophysical

Late Universe

Gravitational Wave Backgrounds



HOLY GRAIL

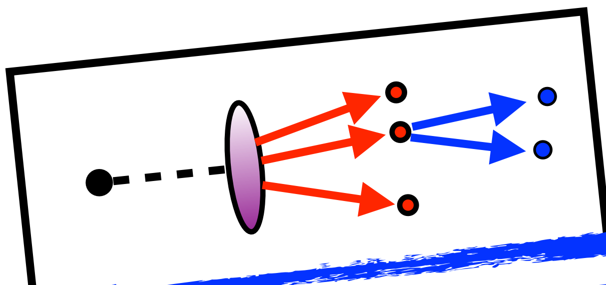
Cosmological

Early Universe

Astrophysical

Late Universe

Gravitational Wave Backgrounds



HOLY GRAIL

Cosmological

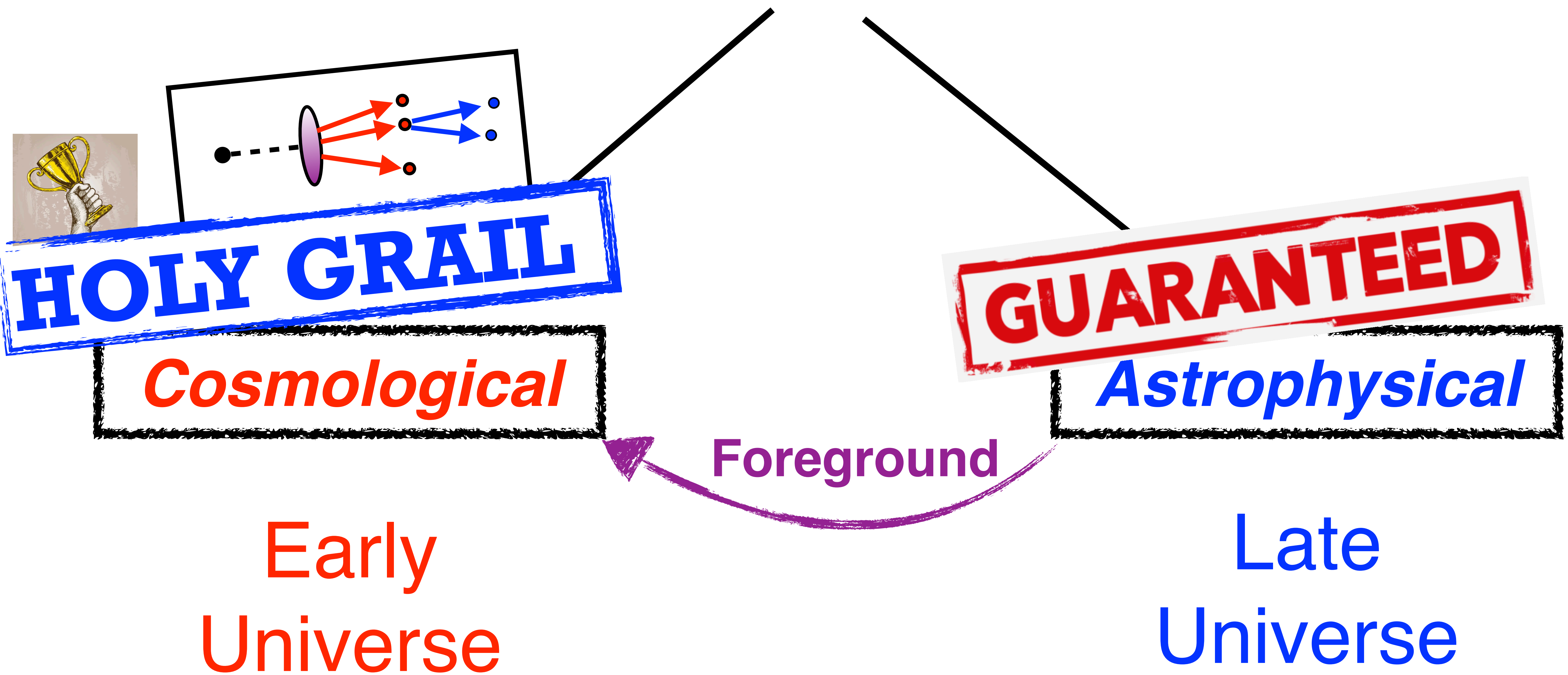
Early Universe

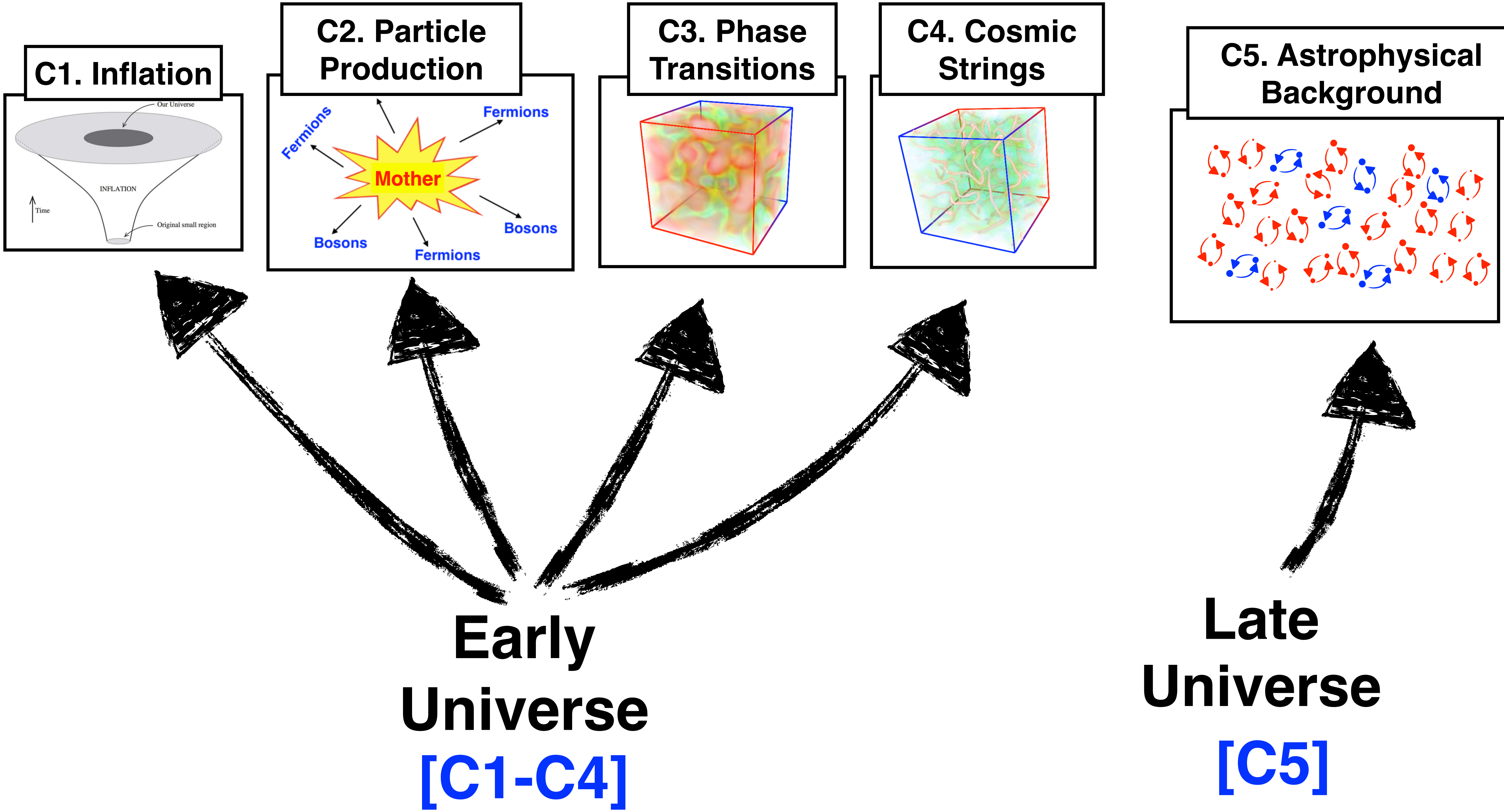
GUARANTEED

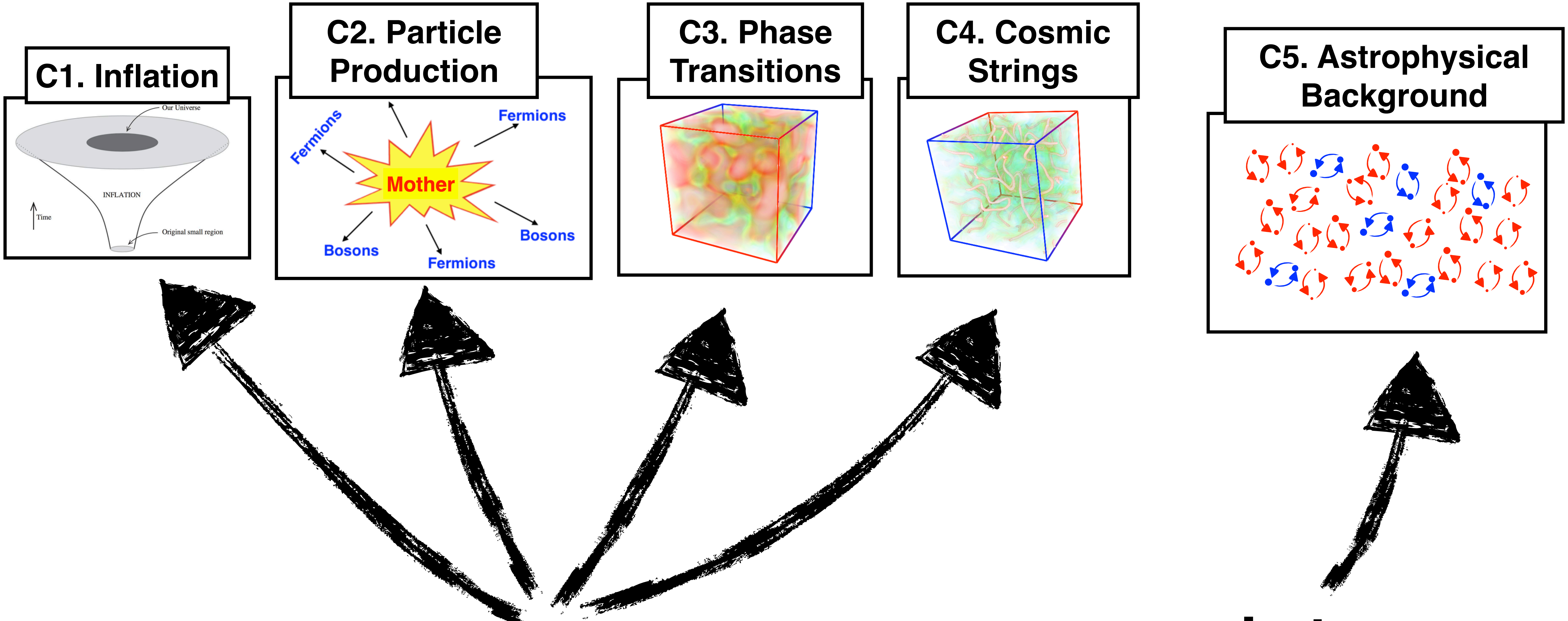
Astrophysical

Late Universe

Gravitational Wave Backgrounds




C1. Inflation
Our Universe
INFLATION
Time
Original small region
C2. Particle Production
Fermions
Fermions
Mother
Bosons
Bosons
Fermions
C3. Phase Transitions
C4. Cosmic Strings
C5. Astrophysical Background
Early Universe
[C1-C4]
Late Universe
[C5]



Late Universe

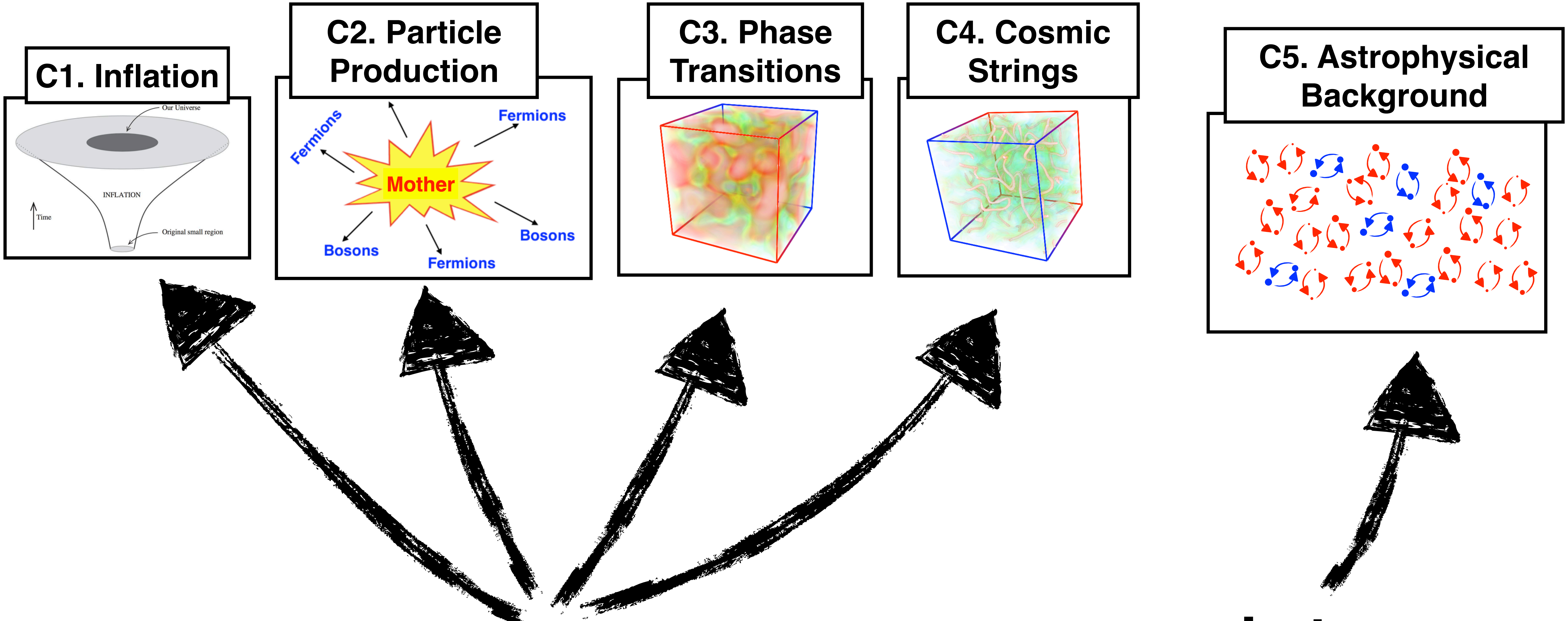
[C5]

UNIQUE OPPORTUNITY

Probe high energy physics

(otherwise inaccessible)

Complementary/Beyond Particle Colliders !



Nature/observability of Inflation

Eq. of State of the early universe

Primordial perturbations at all scales

Viability of Axion species

Did phase transitions take place?

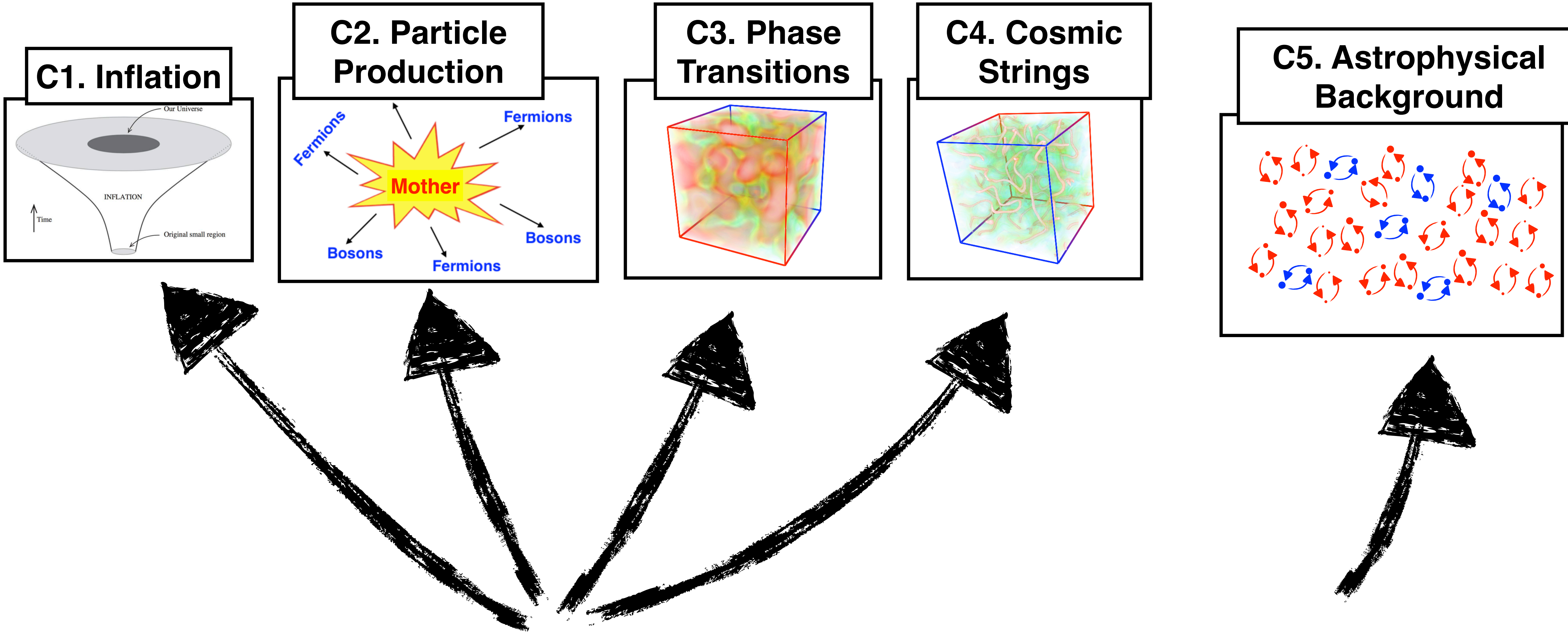
Are there primordial black holes ?

Are cosmic strings in the Universe ?

....

Late Universe

[C5]



Nature/observability of Inflation

Eq. of State of the early universe

Primordial perturbations at all scales

Viability of Axion species

Did phase transitions take place?

Are there primordial black holes ?

Are cosmic strings in the Universe ?

....

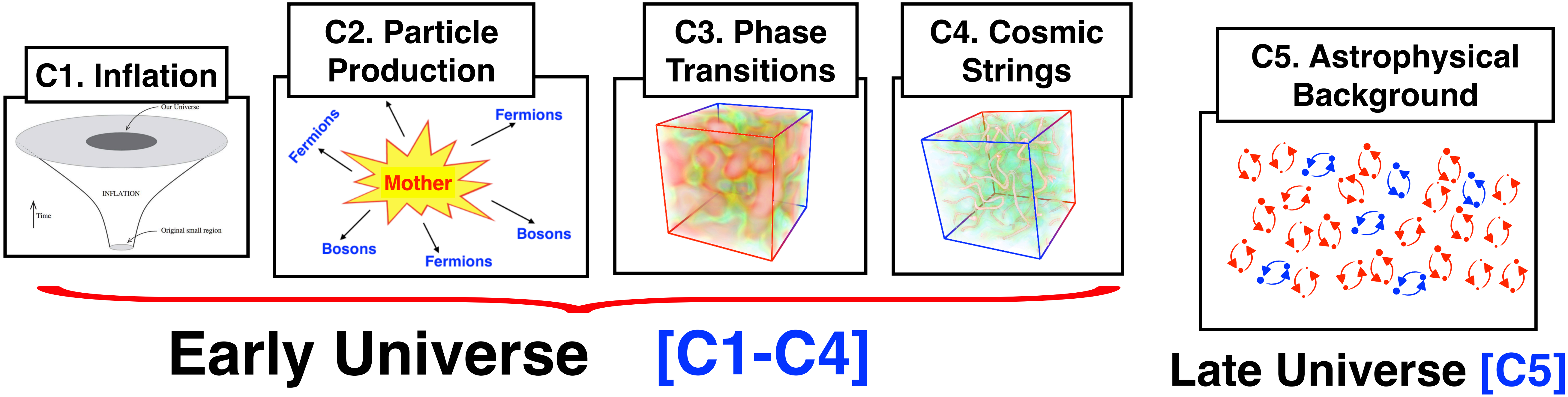
Foreground role

Population properties

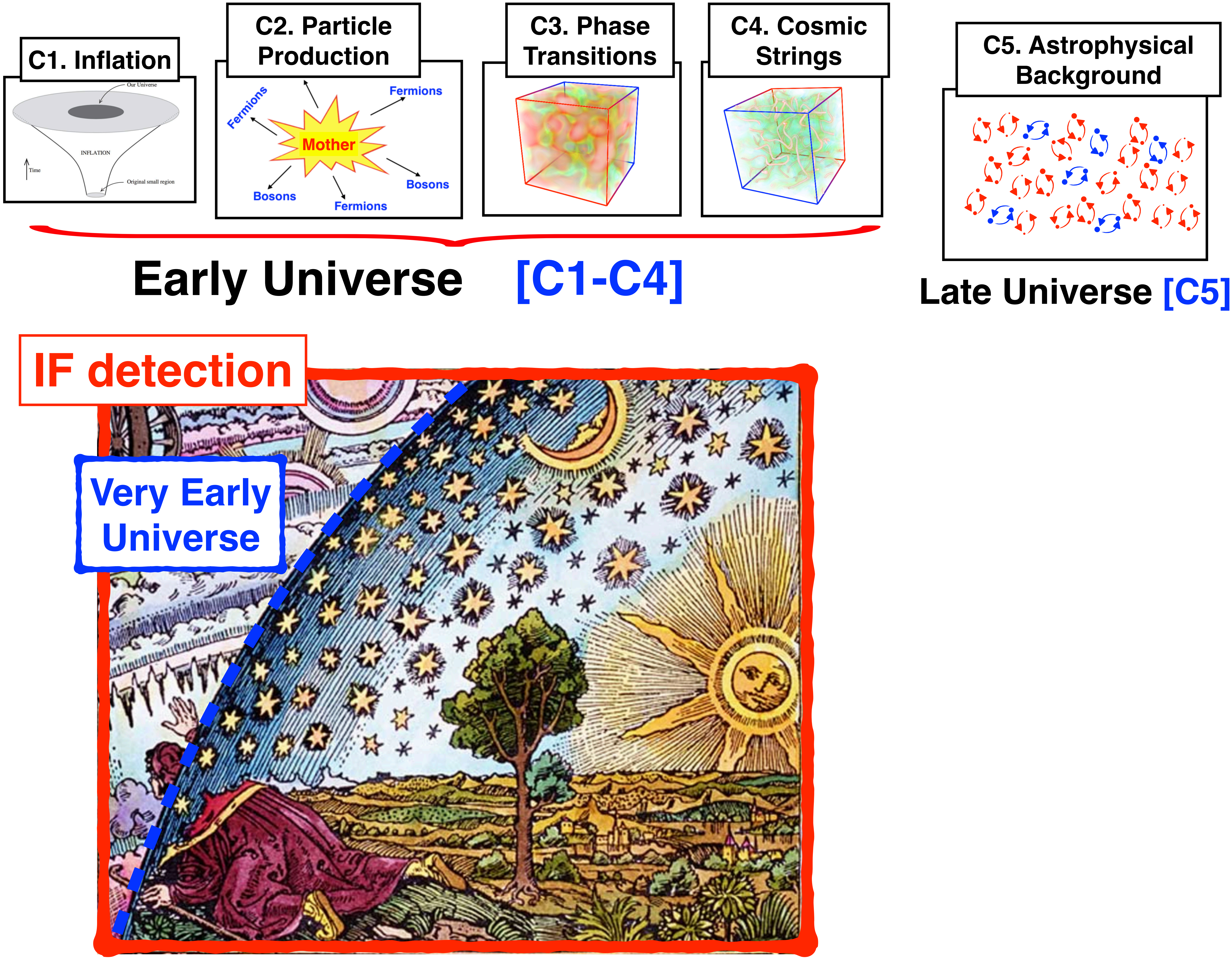
Anisotropies

Cross-correlations

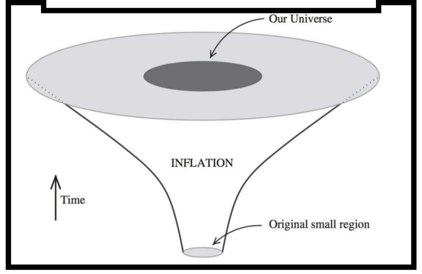
Gravity properties



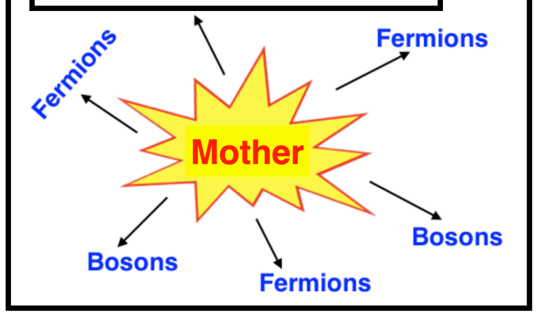
C1. Inflation
Our Universe
INFLATION
Time
Original small region
C2. Particle Production
Fermions
Mother
Fermions
Bosons
Bosons
Fermions
C3. Phase Transitions
C4. Cosmic Strings
C5. Astrophysical Background
Early Universe [C1-C4]
Late Universe [C5]


C1. Inflation
Our Universe
INFLATION
Time
Original small region
C2. Particle Production
Mother
Fermions
Fermions
Fermions
Bosons
Bosons
C3. Phase Transitions
C4. Cosmic Strings
C5. Astrophysical Background
Early Universe [C1-C4]
Late Universe [C5]
IF detection
Very Early Universe

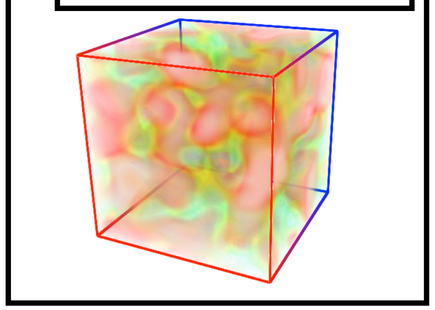
C1. Inflation



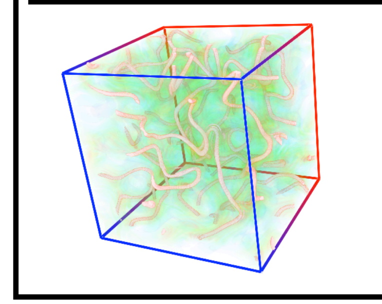
C2. Particle Production



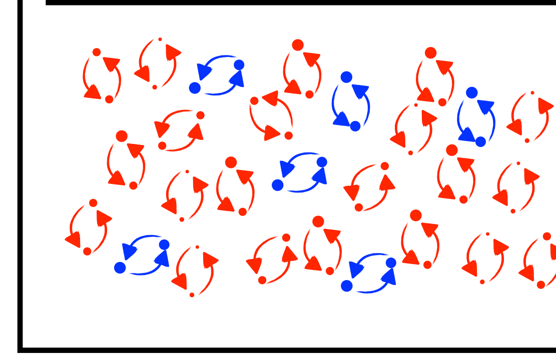
C3. Phase Transitions



C4. Cosmic Strings



C5. Astrophysical Background



Early Universe [C1-C4]

Late Universe [C5]

IF detection

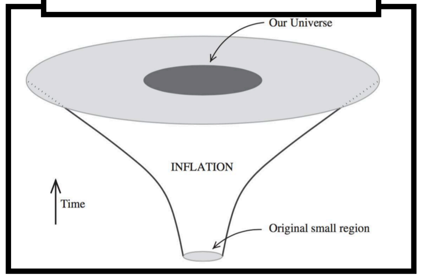
* Energies above terrestrial means

* Fundamental Physics

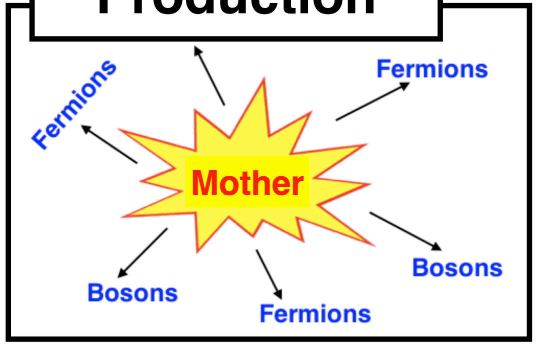
* Beyond the Standard Model (BSM)

* ~ Origin of the Universe

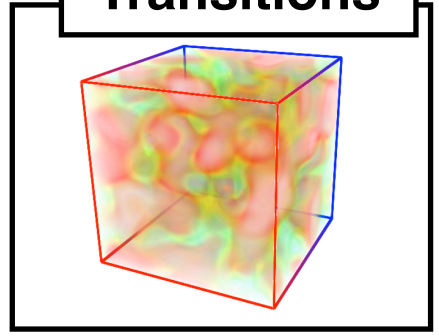
C1. Inflation



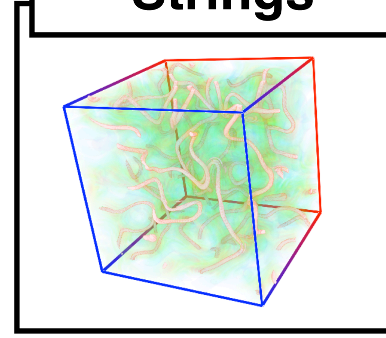
C2. Particle Production



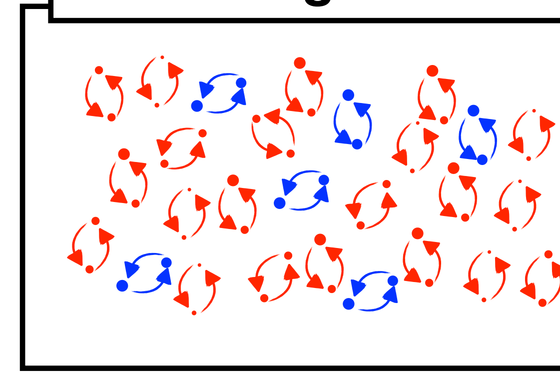
C3. Phase Transitions



C4. Cosmic Strings



C5. Astrophysical Background



Early Universe [C1-C4]

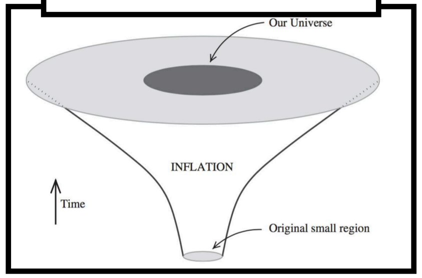
Late Universe [C5]

IF detection

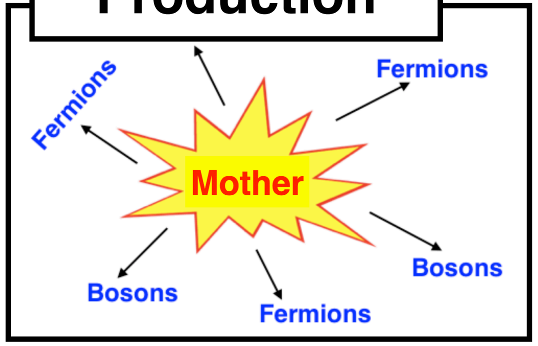
* Energies above terrestrial means
* Fundamental Physics
* Beyond the Standard Model (BSM)
* ~ Origin of the Universe

Otherwise inaccessible !

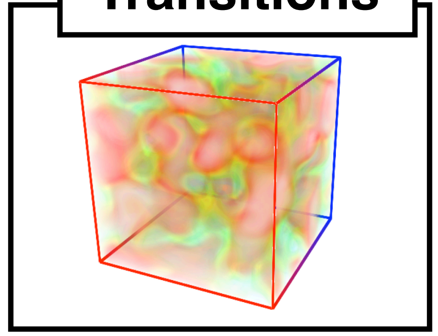
C1. Inflation



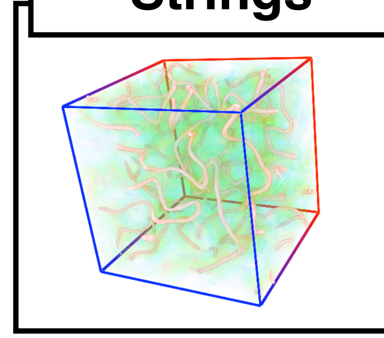
C2. Particle Production



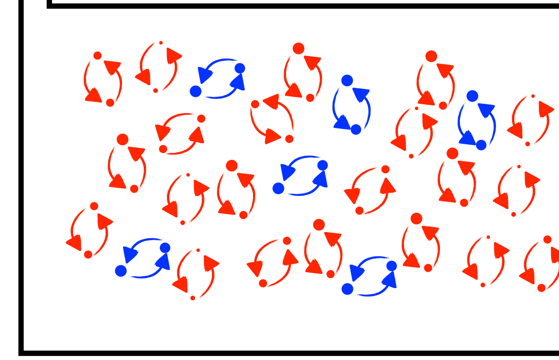
C3. Phase Transitions



C4. Cosmic Strings



C5. Astrophysical Background



Early Universe [C1-C4]

Late Universe [C5]

IF detection

* Energies above terrestrial means

* Fundamental Physics

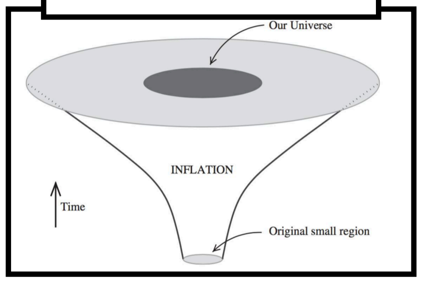
* Beyond the Standard Model (BSM)

* ~ Origin of the Universe

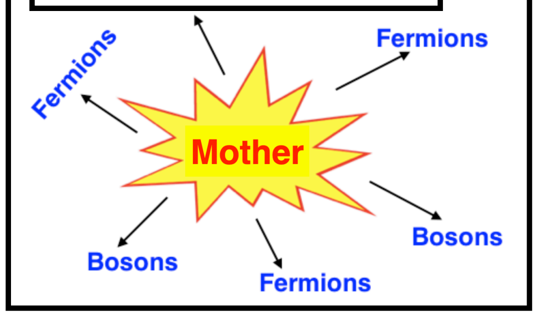
New window into Universe

As revolutionary as CMB discovery !

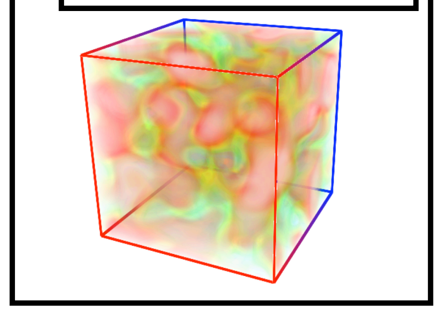
C1. Inflation



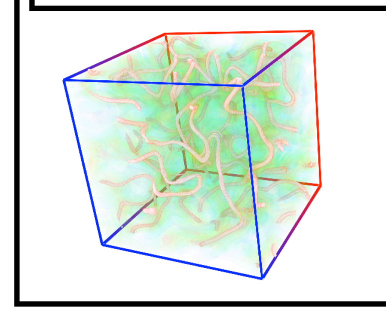
C2. Particle Production



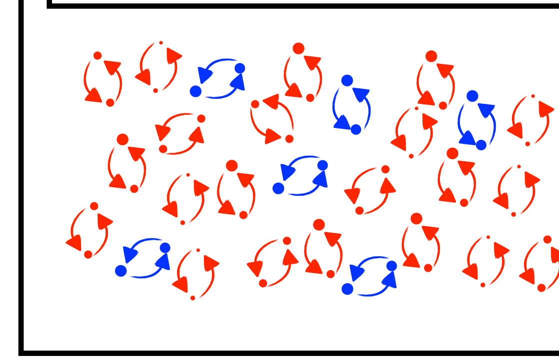
C3. Phase Transitions



C4. Cosmic Strings



C5. Astrophysical Background



Early Universe [C1-C4]

Late Universe [C5]

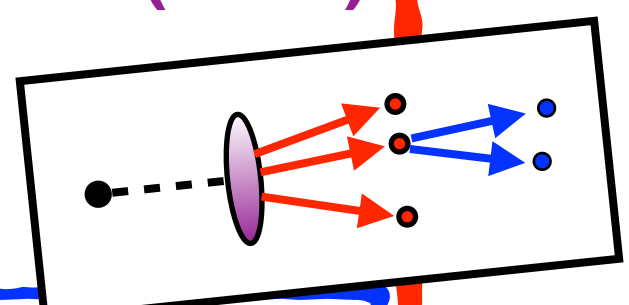
IF detection

* Energies above terrestrial means

* Fundamental Physics

* Beyond the Standard Model (BSM)

* ~ Origin of the Universe

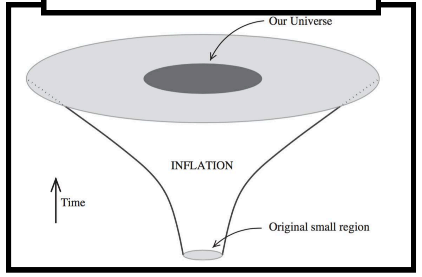


New window into Universe

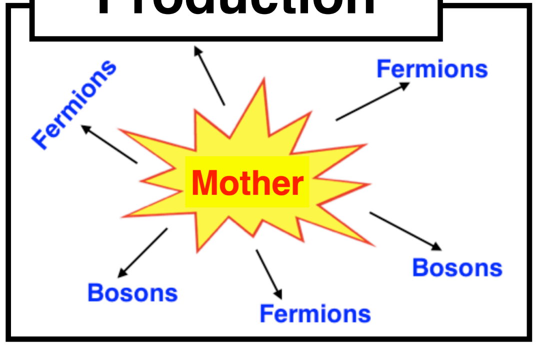
As revolutionary as CMB discovery !

Complementary & Beyond Colliders → Only way !

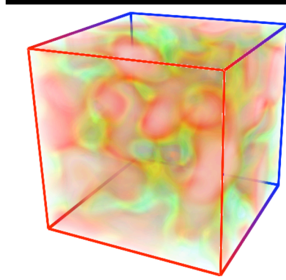
C1. Inflation



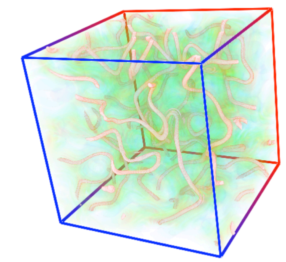
C2. Particle Production



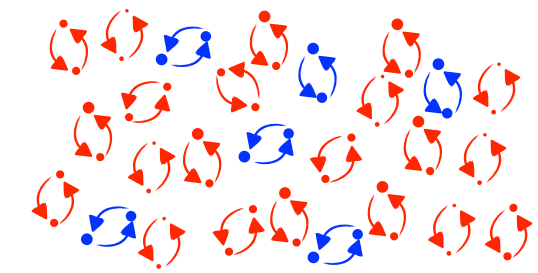
C3. Phase Transitions



C4. Cosmic Strings



C5. Astrophysical Background



Early Universe [C1-C4]

Late Universe [C5]

Otherwise

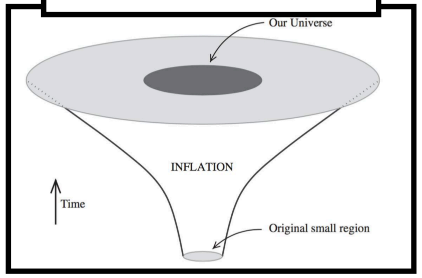
* Astrophysical Back's guaranteed !

* Stringent constraints on BSM

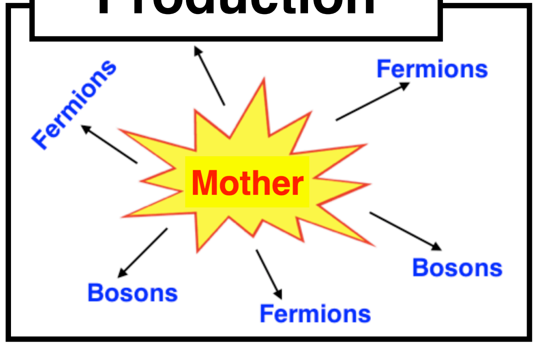
* Ruling out Early Univ. Scenarios

* Scalar Perturbation at all Scales

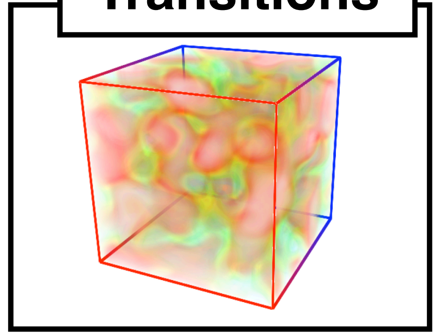
C1. Inflation



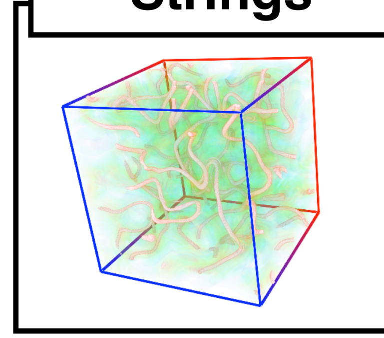
C2. Particle Production



C3. Phase Transitions

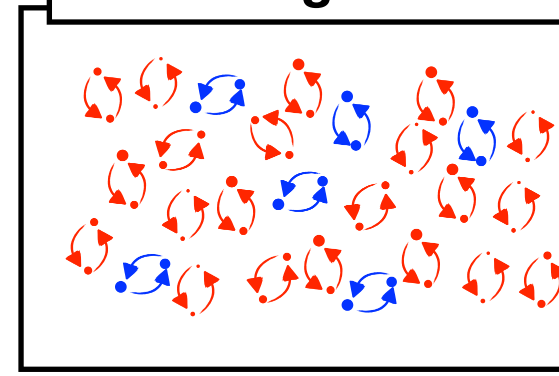


C4. Cosmic Strings



Early Universe [C1-C4]

C5. Astrophysical Background



Late Universe [C5]

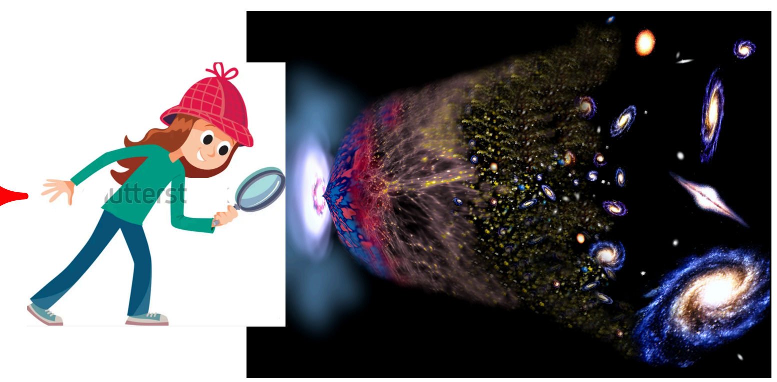
Otherwise

* Astrophysical Back's guaranteed !

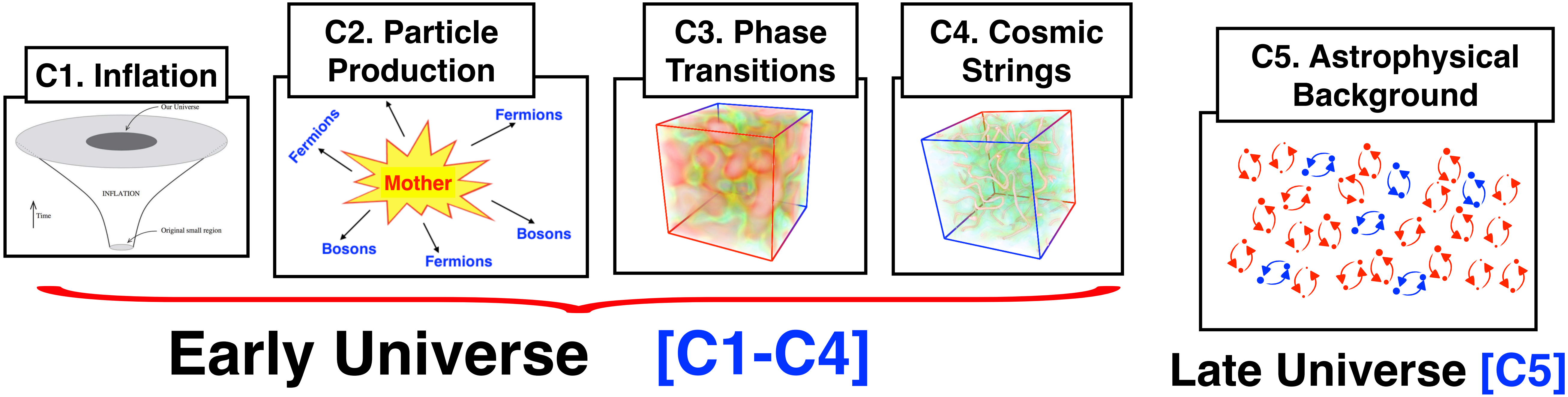
* Stringent constraints on BSM

* Ruling out Early Univ. Scenarios

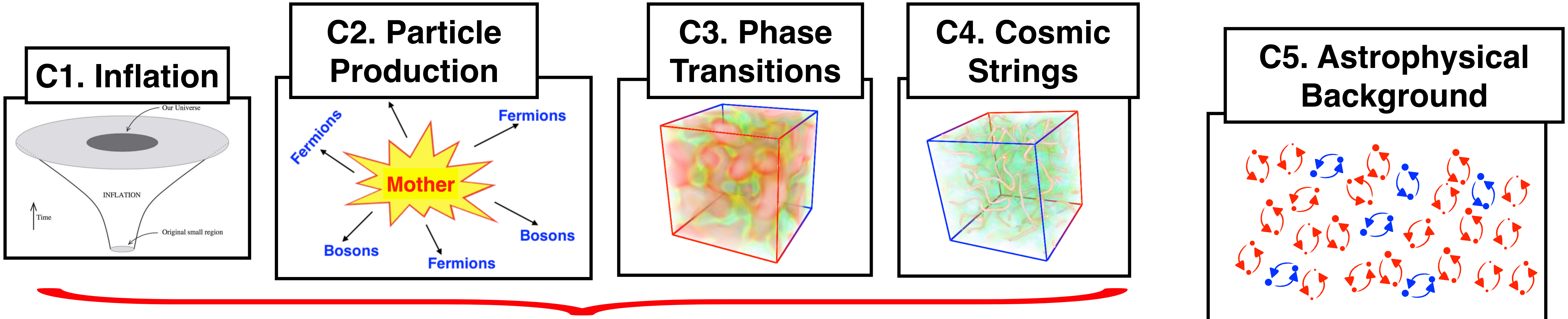
* Scalar Perturbation at all Scales



Dawn of Early Universe GW Cosmology



Early Universe GWB Program



Early Universe [C1-C4]

Late Universe [C5]

Early Universe GWB Program

But before we conclude …

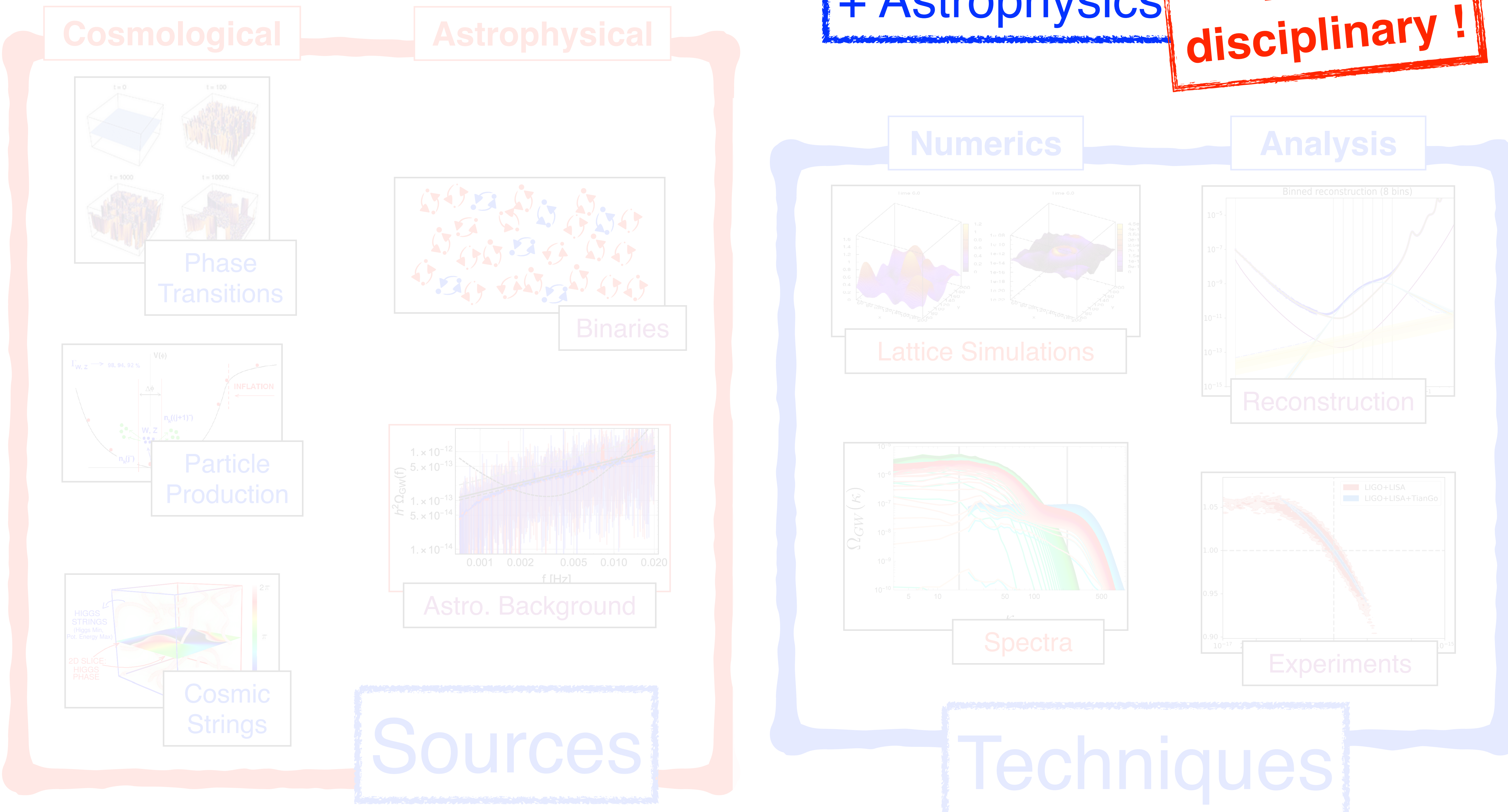
Final Remarks

Expertise

Particle Physics + Cosmology

+ Astrophysics

Multi-disciplinary !



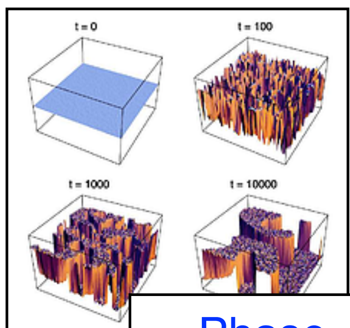
Expertise

Particle Physics + Cosmology

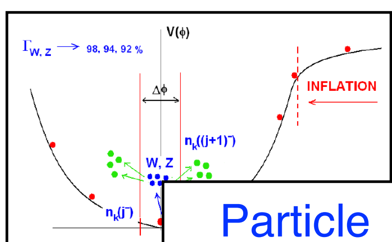
+ Astrophysics

Multi-disciplinary !

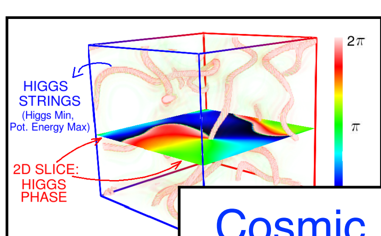
Cosmological



Phase Transitions

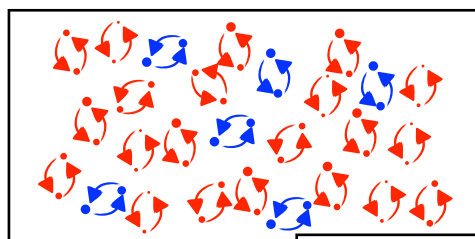


Particle Production

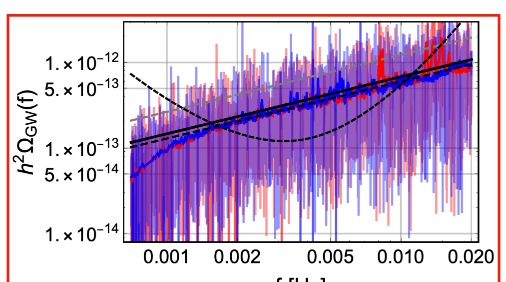


Cosmic Strings

Astrophysical



Binaries



Astro. Background

Sources

Numerics

Lattice Simulations

Analysis

Reconstruction

Spectra

Experiments

Techniques

Expertise

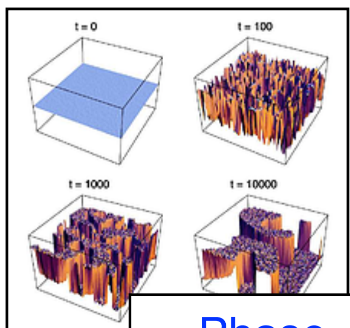
Particle Physics + Cosmology

+ Astrophysics

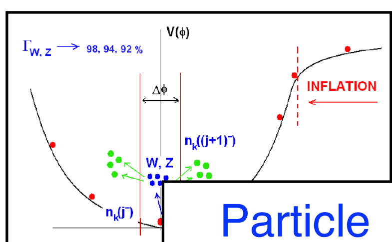
Multi-disciplinary !

Sources

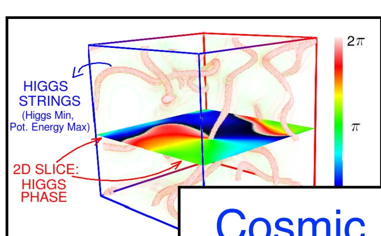
Cosmological



Phase Transitions

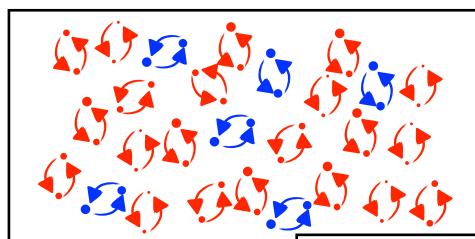


Particle Production

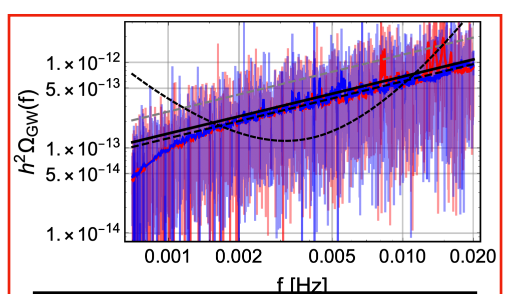


Cosmic Strings

Astrophysical



Binaries

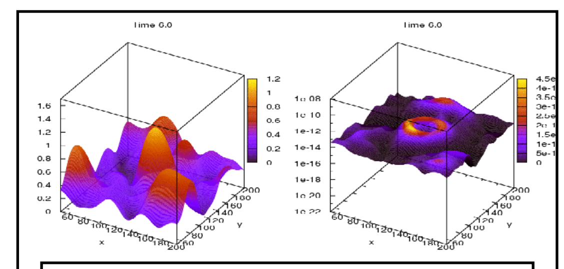


Astro. Background

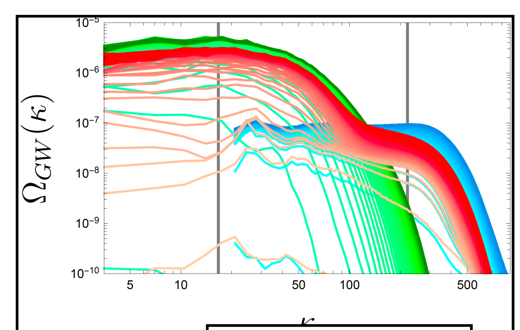
Sources

Techniques

Numerics

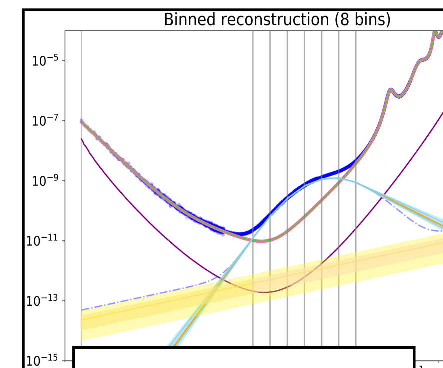


Lattice Simulations

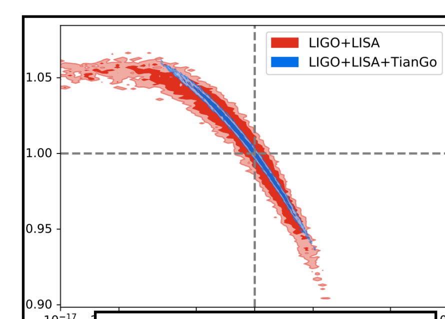


Spectra

Analysis



Reconstruction



Experiments

Techniques

Propaganda, Part I

Review on Gravitational Waves from the Early Universe

Caprini & Figueroa

arXiv:1801.04268

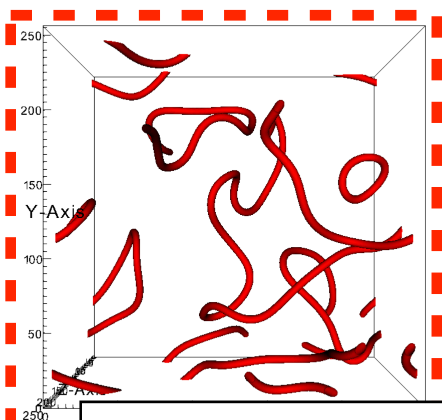
Propaganda, Part II

For you early universe numerics …

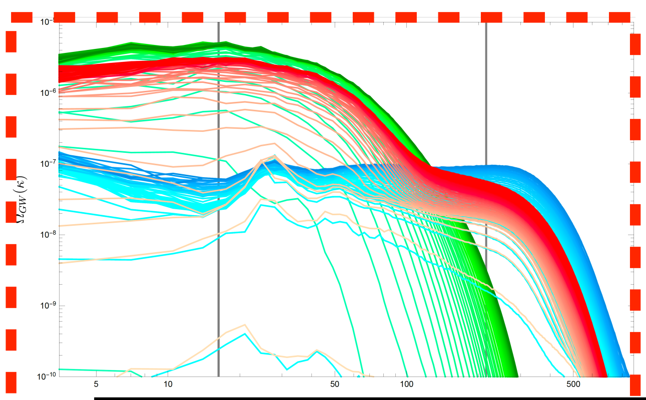
CosmoLattice

Figueroa, Florio, Torrenti, Valkenburg, arXiv: [2102.01031]()

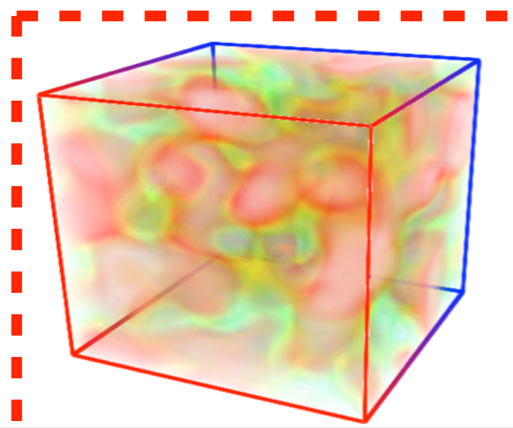
(Grav. Wave module already available!)



Cosmic Strings



Grav. Wave emission



Non-linear dynamics

?

Whatever you want !

Propaganda, Part II

For you early universe numerics …

CosmoLattice

1st CL School 2022: Sept 5-8

@Valencia:

Propaganda, Part II

For you early universe numerics …

CosmoLattice

2nd CL School 2023: Sept 25-29

@Valencia:

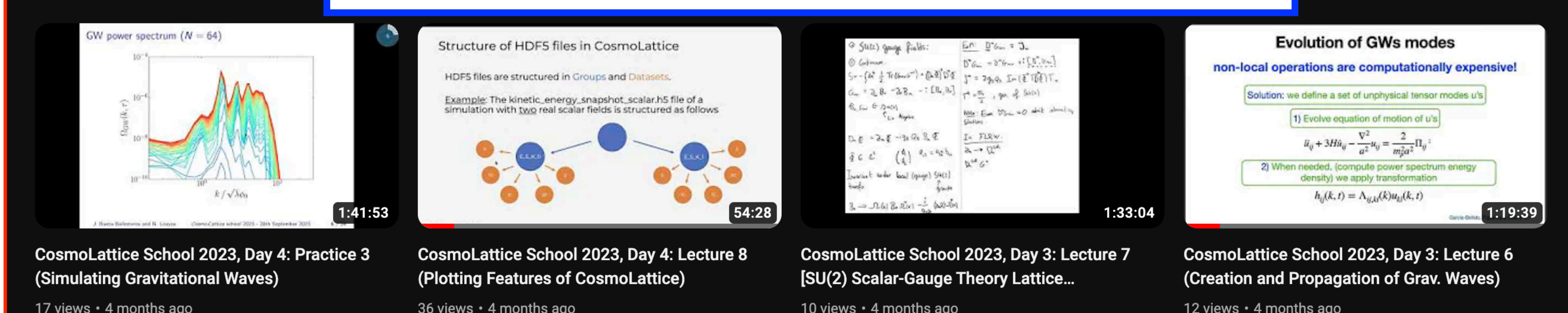
Propaganda, Part II

For you early universe numerics …

CosmoLattice

2nd CL School 2023: Sept 25-29

https://www.youtube.com/@CosmoLattice/videos



Propaganda, Part II

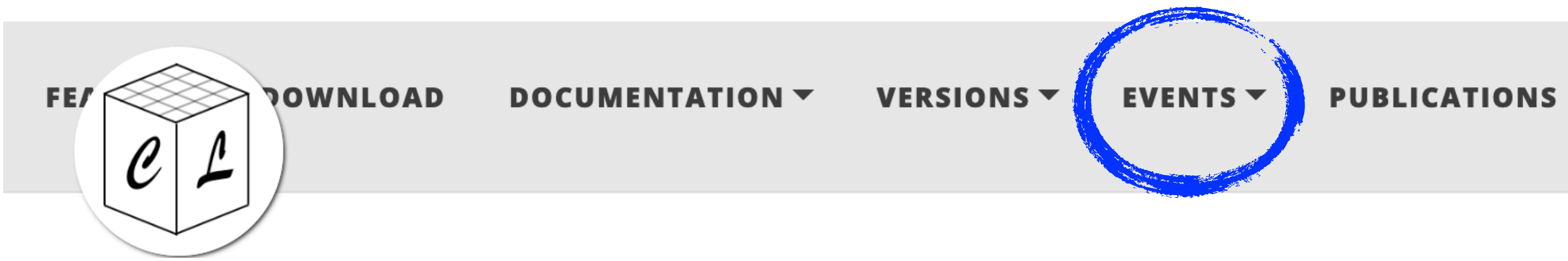
For you early universe numerics …

CosmoLattice

3rd CL School … 2024 or 2025

Details for 3rd CL School TBA at:

https://cosmolattice.net



Thanks for your attention

Vielen Dank für Ihre Aufmerksamkeit

Back Slides

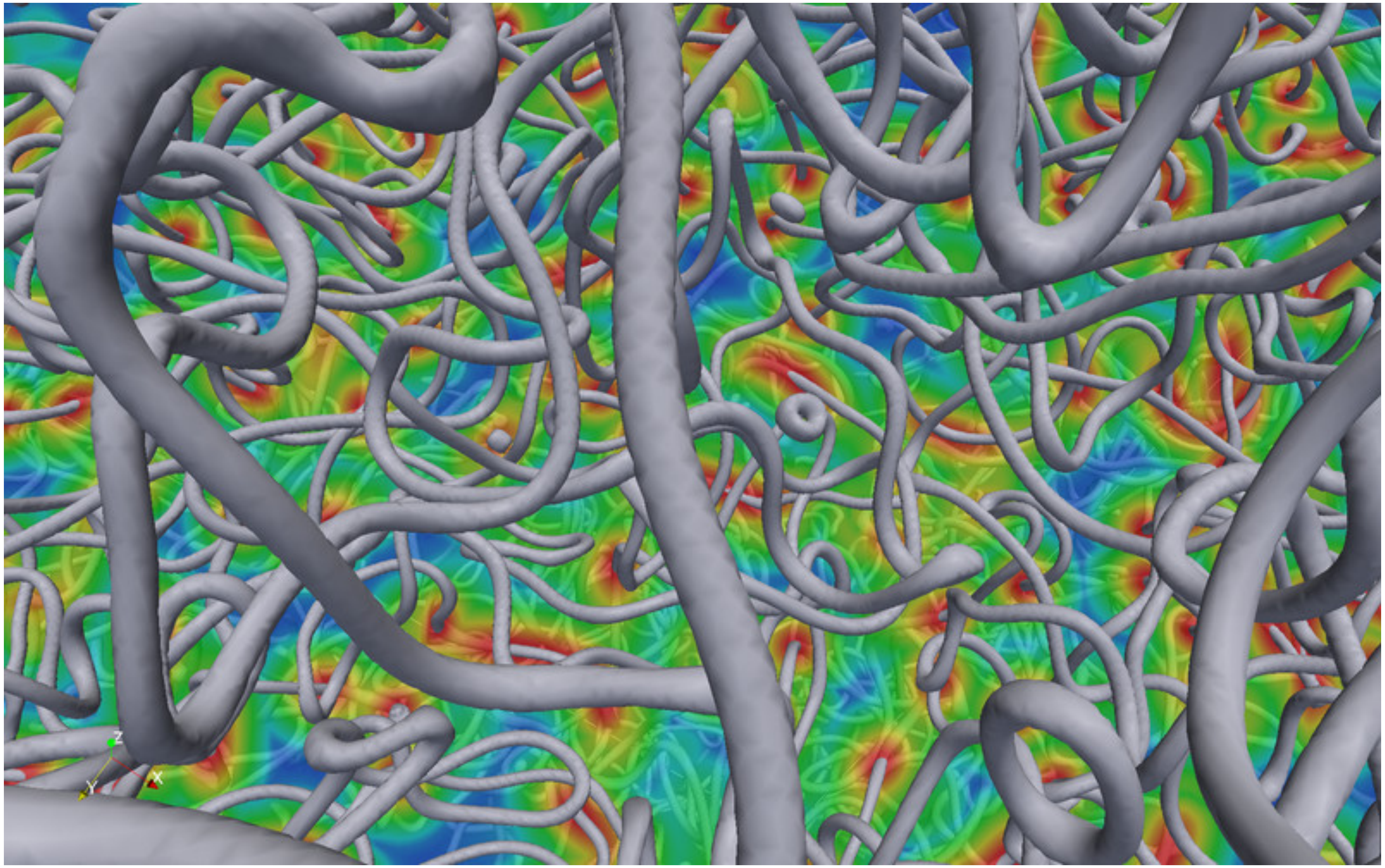
CMB defects

Kibble Mechanism

Lattice Strings

Modeling

What about lattice simulations ?



(Image: David Daverio)

Modeling

What about lattice simulations ?

Abelian-Higgs Simulations

* Loops formed ! … but decay into scalar/gauge fields

* If loops disappear… then no GW ?

* There is an **irreducible GW emission** from the long string network, but negligible vs NG loop GW emission

Modeling

What about lattice simulations ?

Abelian-Higgs Simulations

* Loops formed ! … but decay into scalar/gauge fields

* If loops disappear… then no GW ?

* There is an **irreducible GW emission** from the long string network, but negligible vs NG loop GW emission

Open debate in the ArXiv !

Vachaspati et al 2019/20

Copeland et al 2023/24

Modeling

What about lattice simulations ?

Abelian-Higgs Simulations

* Loops formed ! … but decay into scalar/gauge fields

* If loops disappear… then no GW ?

* There is an **irreducible GW emission** from the long string network, but negligible vs NG loop GW emission

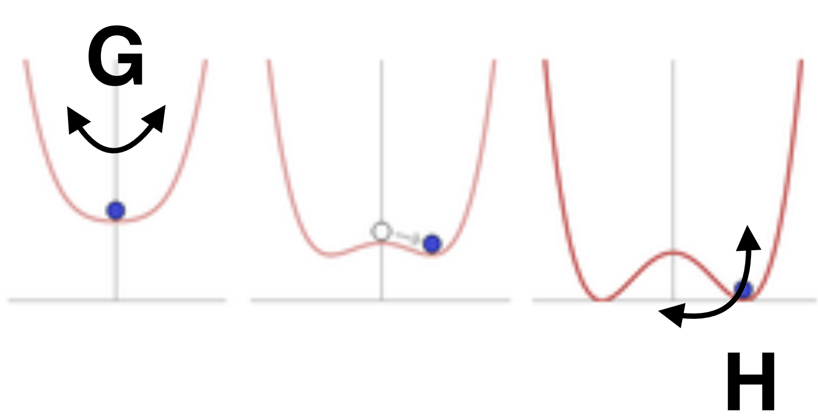
So … LISA results based on Nambu-Goto strings !

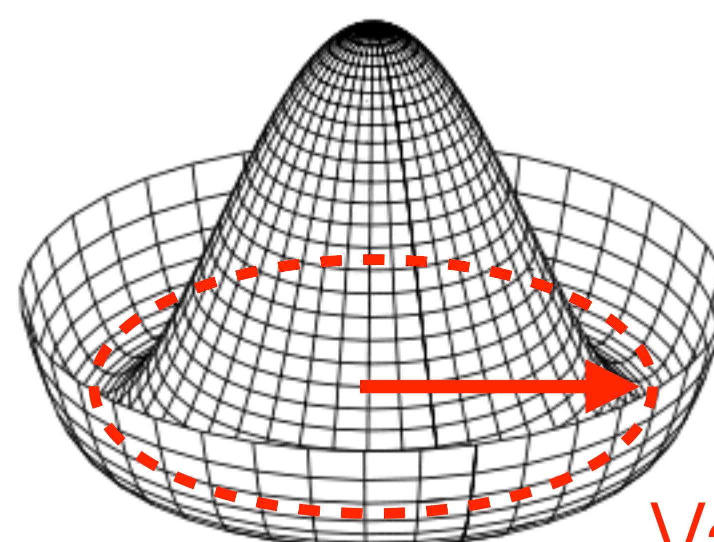
Kibble Mechanism

Introduction to Cosmic Defects

Kibble'76

us recall the more general situation. In a model with symmetry group G, the vacuum expectation value $\langle \phi \rangle$ will be restricted to lie on some orbit of G. If H is the isotropy subgroup of G at one point $\langle \phi \rangle$, i.e. the subgroup of transformations leaving $\langle \phi \rangle$ unaltered, then the orbit may be identified with the coset space $M = G/H$. Physically H is the subgroup of unbroken symmetries, and M is the manifold of degenerate vacua. As we shall see, the topological properties of M (specifically its homotopy groups) largely determine the geometry of possible domain structures.





M = G/H

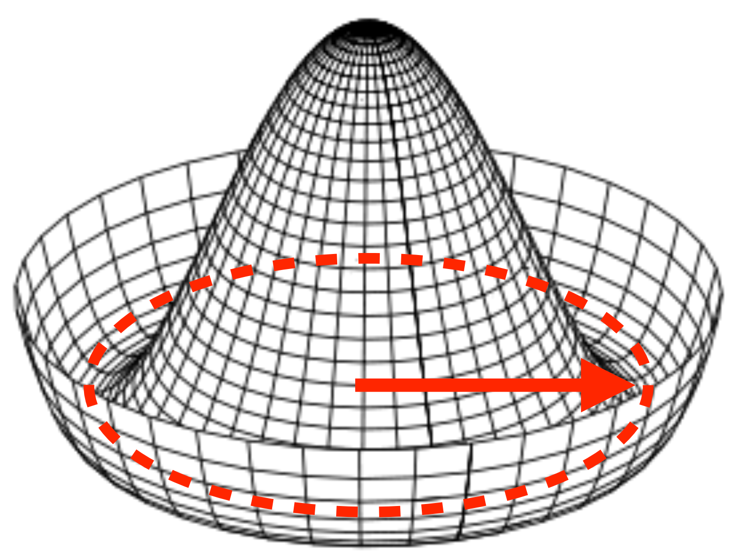
Vacuum
Manifold

Introduction to Cosmic Defects

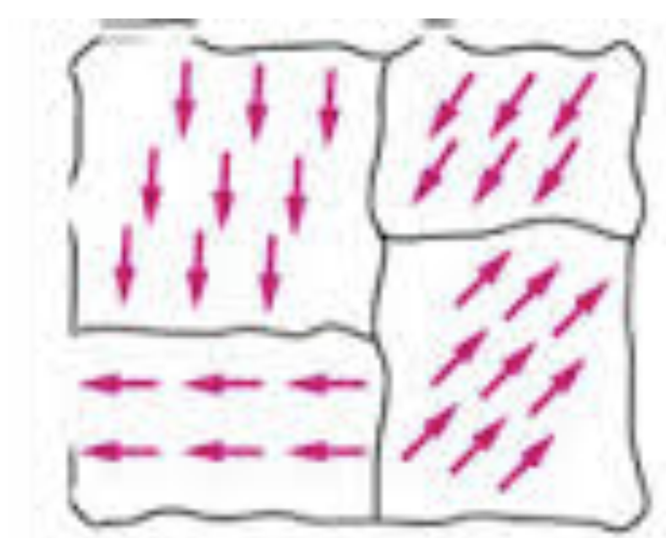
6. Conclusions and discussion

On this basis we showed that a domain structure can be expected to arise. The topological character of this structure depends on the homotopy groups $\pi_k(M)$ of the manifold M of degenerate vacua. Domain walls can form if $\pi_0(M)$ is nontrivial, i.e. if M is non-connected. If it has n connected components we find an n-phase emulsion. The formation of cosmic strings requires that $\pi_1(M)$ be nontrivial, i.e. that M is not formed of simply connected components. Finally, 'monopoles' can form if $\pi_2(M)$ is nontrivial.

Kibble'76



M = G/H



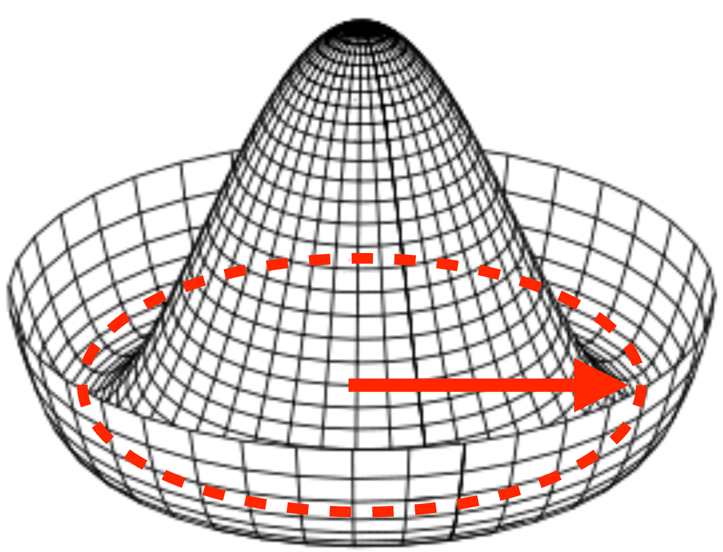
Introduction to Cosmic Defects

6. Conclusions and discussion

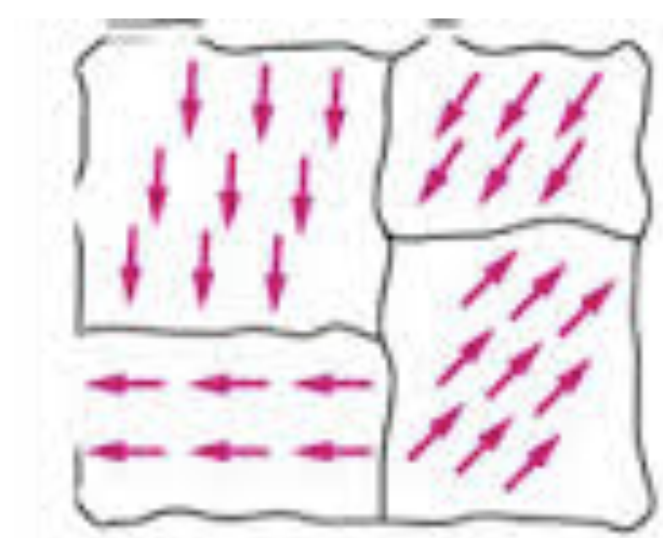
On this basis we showed that a domain struct… be expected to arise. The topological character of this structure … otopy groups $\pi_k(M)$ of the manifold M of degenerate … $\pi_0(M)$ is nontrivial, i.e. if M is non-connected. If … an n-phase emulsion. The formation of cosmic s… (M) be nontrivial, i.e. that M is not formed of simply connected co… …nally, 'monopoles' can form if $\pi_2(M)$ is nontrivial.

homotopy groups

Kibble'76

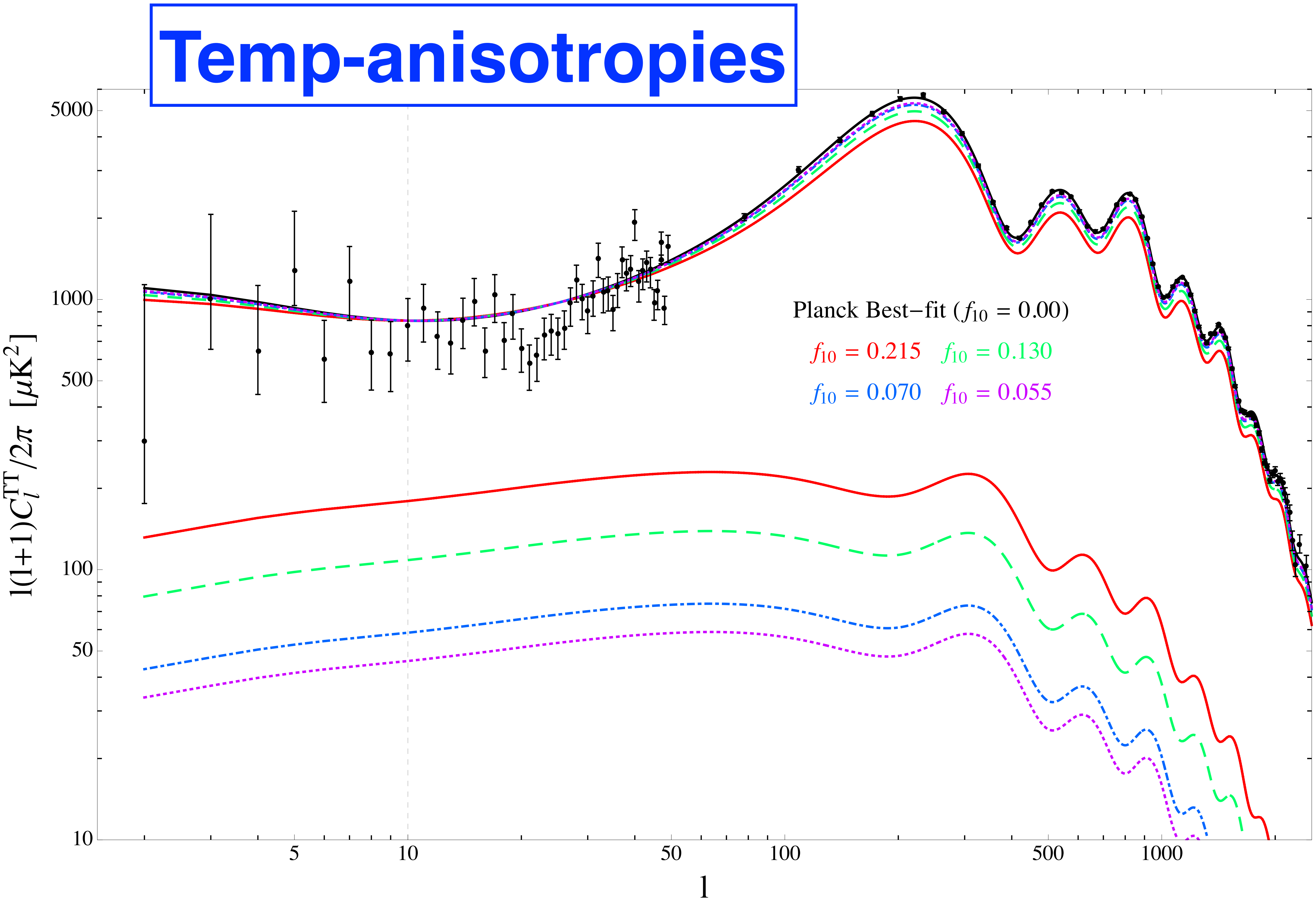


M = G/H



CMB Defects (Back) SLIDES

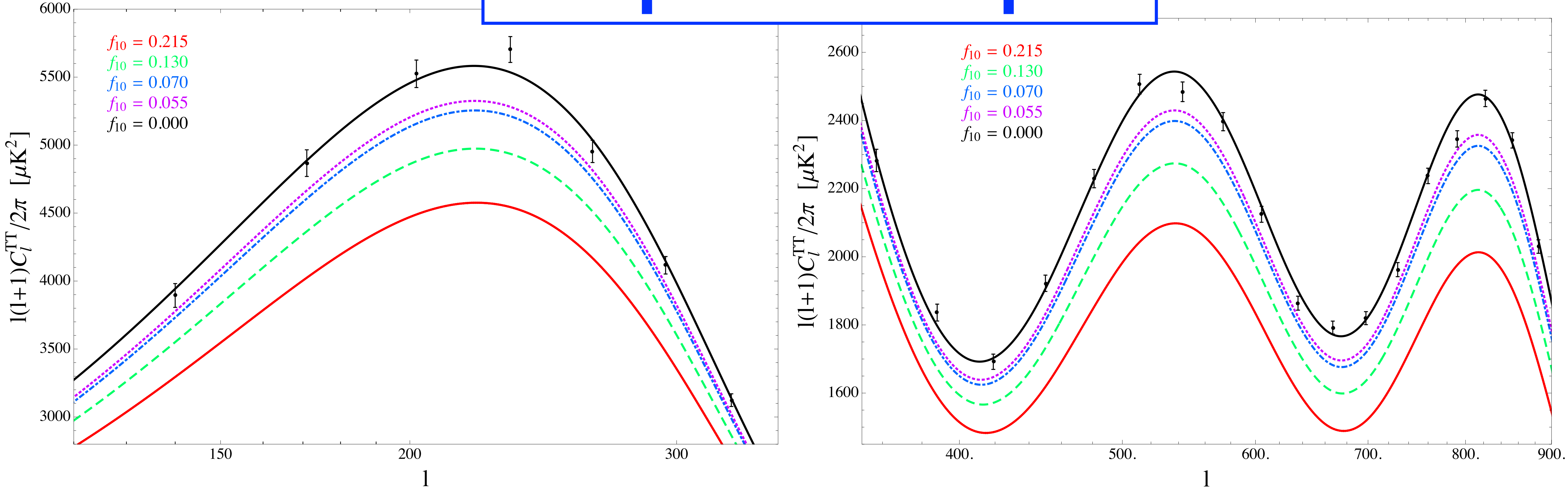
Cosmic Microwave Background



Durrer et al, JCAP 2014

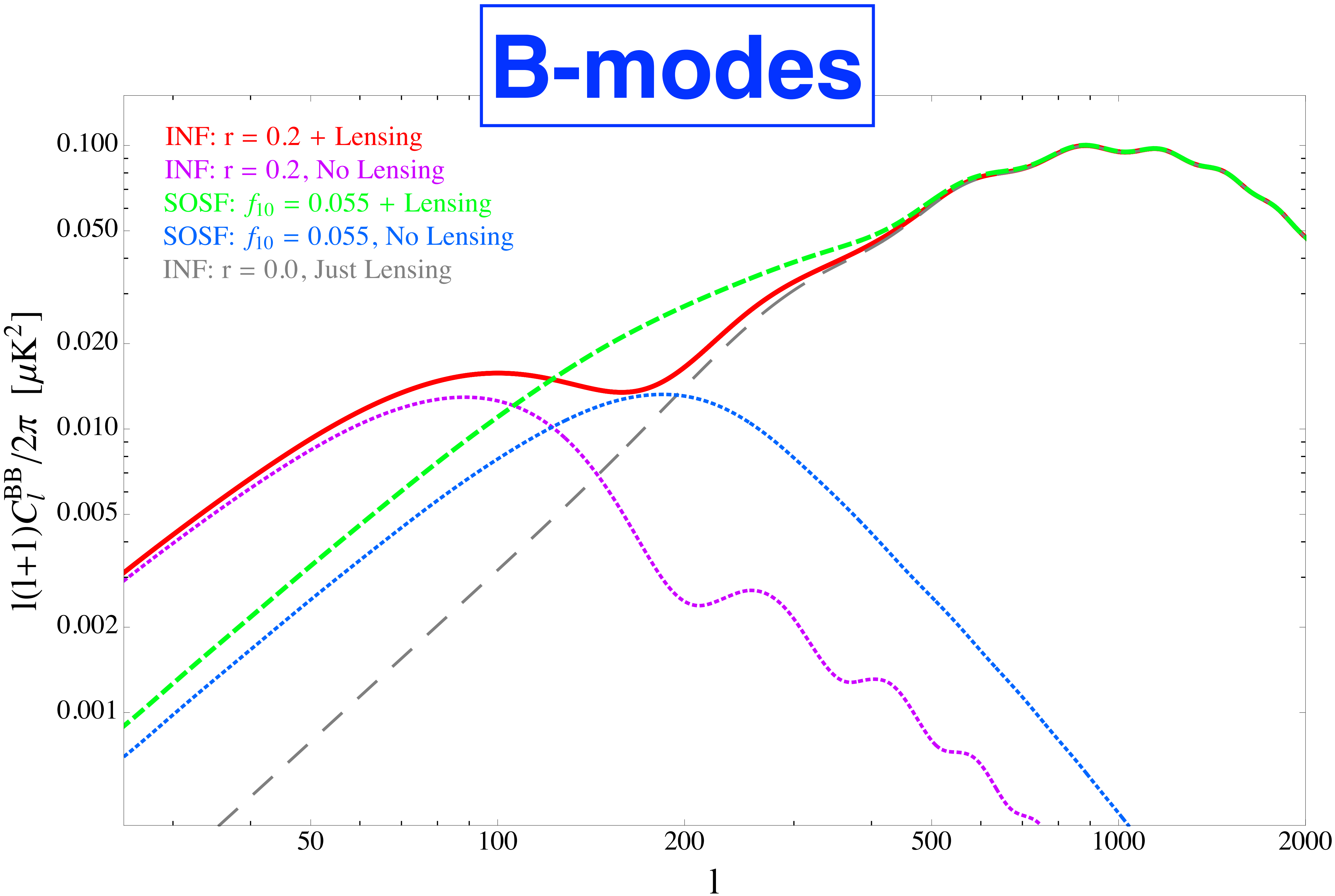
Cosmic Microwave Background

Temp-anisotropies



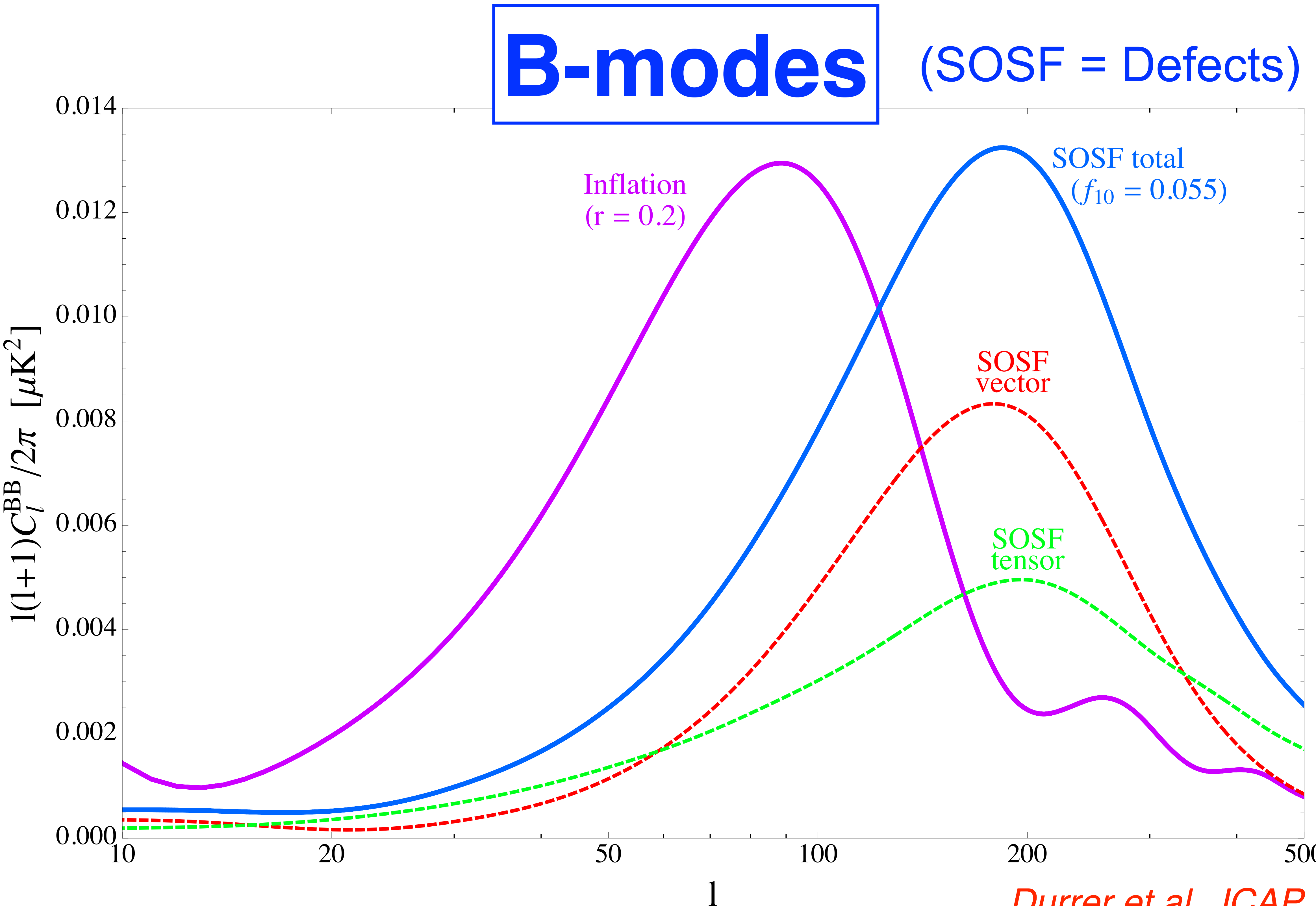
Durrer et al, JCAP 2014

Cosmic Microwave Background



Durrer et al, JCAP 2014

Cosmic Microwave Background



Durrer et al, JCAP 2014