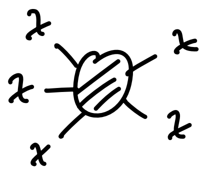
B) SIMP DM



$\rightsquigarrow$ DM can self-equilibrize!

⌜chemically $\leftrightarrow$ only kinetically for $\chi\chi \leftrightarrow \chi\chi$⌟

- rates in EQ: $\leftarrow \; = n_{eq}^2 \langle \sigma v \rangle$

$$\rightarrow \; = n_{eq}^3 \text{“}\langle \sigma v^2 \rangle\text{”}$$ "just a name", defined by $C[f]$

detailed balance ($\leftarrow = \rightarrow$)

$$\Rightarrow [\sigma v^2] = [n^{-1} \sigma] = \text{mass}^{-5}$$

$$\Rightarrow \langle \sigma v^2 \rangle \equiv \frac{\alpha_{eff}^3}{m_\chi^5}$$

- outside EQ: $n^2 \langle \sigma v \rangle = n^2 n_{eq} \langle \sigma v^2 \rangle$ ⌜keep scaling with n^2 + recover EQ case for $n \to n_{eq}$⌟

$$\Rightarrow \boxed{\boxed{\dot{n} + 3Hn = -\langle \sigma v^2 \rangle_{3\to 2} \left(n^3 - n^2 n_{eq} \right)}}$$

$\rightsquigarrow$ freeze-out at $Hn \sim \langle \sigma v^2 \rangle n^3 \sim \dfrac{\alpha_{eff}^3}{m_\chi^5} n^3$ | $H \sim \dfrac{T_f^2}{M_{pl}}$; $n \sim \dfrac{g_\chi(T_{eq})}{m_\chi} \dfrac{T_f^3}{T_{eq}^3}$

$$\sim \underbrace{\frac{T_f}{m_\chi}}_{\sim 1/15} T_{eq} T_f^2$$

⌜$M_{pl}^{-1} \sim \alpha_{eff}^3 m_\chi^{-3} T_{eq}^2 (T_f/m_\chi)^4$⌟

$$\Rightarrow \boxed{m_\chi \sim \alpha_{eff} \left(T_{eq}^2 M_{pl} \right)^{1/3}}$$ $\leftrightarrow$ WIMP: $m_\chi \sim \alpha_{eff} (T_{eq} M_{pl})^{1/2}$

e.g. for $\alpha_{eff} \sim 1$, $m_\chi \sim 100$ MeV : strong scale

$\rightsquigarrow$ a "SIMP miracle" ⌜~ same argument: in principle unrelated scales.

<u>BUT</u>: nothing like hierarchy problem that would independently suggest

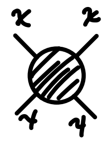
new physics at this scale!

generalization: not $3 \Leftrightarrow 2$ but $n \Leftrightarrow 2$

or this

$$\Rightarrow m_\chi \sim \alpha_{\rm eff} \left(T_{eq}^{n-1} M_{pl}\right)^{1/n}$$

NB: $3 \rightarrow 2$ heats up DM particles!

$\rightsquigarrow$ option a) additional  $\sigma \equiv \frac{\alpha_4^2}{m_\chi^2}$

cools DM to $T_\chi = T$ (as assumed in estimate above)

b) completely secluded dark sector:

DM "cannibalizes itself to keep warm"

$\rightsquigarrow$ λ_{FS} much larger than for WDM!

$\rightsquigarrow$ conserved entropy in dark sector:

$$s_\chi = \frac{\rho_\chi [+ p_\chi]}{T_\chi} \sim \frac{m_\chi}{T_\chi} (m_\chi T_\chi)^{3/2} e^{-m_\chi/T_\chi}$$

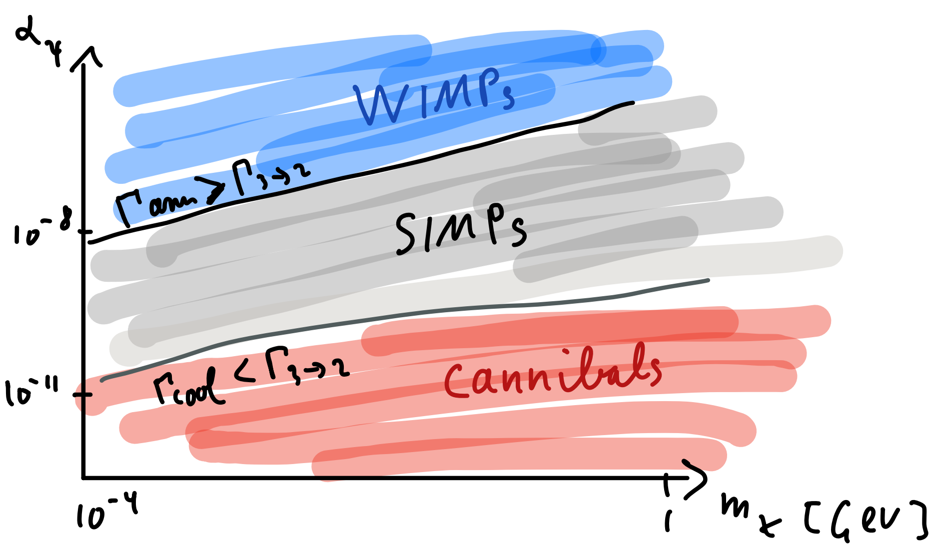
$$a^3 s_\chi = \text{const.} \underset{a \propto \frac{1}{T}}{\Rightarrow} T_\chi \sim \frac{1}{\log(1/T)}$$

$\rightarrow$ fast growth of $\frac{T_\chi}{T}$ during cannibal phase!

$\Rightarrow$ e.g. for $m_\chi \sim 10$ MeV (!)

must start w/ $s_\chi \lesssim 3 \cdot 10^{-5} s_{SM}$ [1602.4219]

$\Leftrightarrow (T_\chi / T)_{T \to \infty} \sim \mathcal{O}(10^{-2} - 10^{-1})$



$\alpha_4 \ll 0.01$ because $m_\chi \ll 100$ GeV

III (Fully) non-thermal production

How light can DM be?

- $m \lesssim$ few MeV : excluded by BBN for fully thermalized DM (too large ΔN_{eff})
- $m \lesssim$ few keV : ~ ~ structure formation for DM [directly] produced from SM thermal bath (~> FIMPs..)
- $m \lesssim 1\,(0.1)$ keV : ~ ~ phase-space degeneracy in dSphs for fermionic DM (Tremaine-Gunn, completely independent of production)

~> bosonic light DM ~ unconstrained* for non-thermal production

NB: very high number density ~> more "field" than "particle"!

e.g. $n_\gamma^{CMB} \sim T_\gamma^3 \sim 400\ cm^{-3}$

$$\Omega h^2 \sim 0.1 \;\hat{=}\; n_{DM} \sim \left(\frac{m}{keV}\right)^{-1} cm^{-3}$$

[$\times \sim 10^6$ @ Galactic scales / $r \sim R_\odot$]

---

* $m_{DM} < 10^{-22}$ eV $\Rightarrow \lambda_B = \frac{1}{m \cdot v} >$ size of dwarf galaxies

~> excluded!

III.1 Misalignment mechanism

Scalar field ϕ: $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)$

$$\Rightarrow 0 = \Box\phi + V' = \ddot{\phi} + 3H\dot{\phi} \underbrace{- a^{-2}\nabla^2\phi}_{\to \frac{k^2}{a^2}\phi} + V'$$

$\to$ redshifts quickly away $\to$ neglect

⌜but important to study, e.g., axion miniclusters⌟

1. "slow roll" at early times ($T \gtrsim T_c$): ⌜like inflation; difference: $\rho_\phi \ll \rho$!⌟

$$3H\dot{\phi} \gg m^2\phi \simeq 0 \Rightarrow 0 \simeq \ddot{\phi} + 3H\dot{\phi} \Rightarrow \dot{\phi} \propto a^{-3}$$

$$\Rightarrow \text{(very soon)} \quad \phi(t) \longrightarrow \phi = \phi_I = \text{const.}$$

2. late times ($T \lesssim T_c$):

   $V(\phi)$ becomes relevant ⌜e.g. phase transition⌟

   assume slow change in $V'(\phi)$

   $\rightsquigarrow$ ϕ starts to oscillate around minimum

   $\downarrow$ WKB ansatz: $\phi = A(t)\, e^{i\Theta(t)}$

   ⌜A = slow, Θ = fast⌟

   ρ_ϕ, P_ϕ behave like CDM!

More details: lecture by J. Jaeckel next week!

III.2 Gravitational production

⇝ In particular at end of inflation. One option: inflaton decay ⇝ Boltzmann
Here: what happens if there are "no" couplings? Well, always gravity...

QFT in curved space

recall minimal coupling : $\partial_\mu \to \nabla_\mu$; $\eta_{\mu\nu} \to g_{\mu\nu}$

QFT : "particles" are representations of the Poincaré group

$\Rightarrow$?

e.g. free scalar field : $(\Box_\eta + m^2)\phi = 0$

$$\Rightarrow \phi \sim e^{-ik\cdot x} a_{\vec{k}} + e^{ik\cdot x} a^\dagger_{\vec{k}}$$

= operator-valued solution of KG equation, "particle-wave duality"

⇝ Def. vacuum : $a_k |0\rangle = 0$ ⇝ Lorentz invariant!

1-particle states : $\sim a_k^\dagger |0\rangle$

NB: need time-like Killing vector to distinguish positive/negative frequency modes!

$$\partial_t u_{\vec{k}} = \mp i\omega f_{\vec{k}} \quad ; \omega > 0$$

in general : $(\Box_g + m^2)\phi = 0$ $\quad \Big| \; \phi(x) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\vec{x}} \phi_{\vec{k}}(t)$

$$⇝ \phi_{\vec{k}}^{(t)} = u_{\vec{k}}^{(t)} a_{\vec{k}} + u_{\vec{k}}^{*(t)} a^\dagger_{-\vec{k}} \quad \text{[for real } \phi(x)\text{]}$$

would-be positive frequency mode

$$= v_{\vec{k}} b_{\vec{k}} + v^*_{\vec{k}} b^\dagger_{-\vec{k}}$$

$u, v =$ $\underbrace{\text{complete \& orthonormal}}_{(\#)}$ set of solutions to KG eq.

$\left[(u_i, u_j) = -i \int_\Sigma d\Sigma^\mu \sqrt{-g_\Sigma}\, u_i \overleftrightarrow{\partial_\mu} u_j^* = \delta_{ij} \right]$

⇝ choice no longer unique, even if locally of (flat space =) harmonic oscillator type!

$$(\#) \Rightarrow v_k = \alpha_k^{(t)} u_k + \beta_k^{(t)} u_k^* \quad \Leftrightarrow \quad \begin{pmatrix} b_k \\ b_k^+ \end{pmatrix} = \begin{pmatrix} \alpha_k^* & -\beta_k^* \\ -\beta_k & \alpha_k \end{pmatrix} \begin{pmatrix} a_k \\ a_k^+ \end{pmatrix}$$

"Bogoliubov coefficients"

NB: mix of "positive" and "negative" frequencies! (wrt u_k)

e.g. vacuum w.r.t. u_k : $a_k |0\rangle_u = 0$

⇒ particle number w.r.t. v_k:

$$n_v = {}_u\langle 0 | b_k^+ b_k | 0 \rangle_u = |\beta_k|^2 \neq 0 \qquad \text{"particle production"}$$

In general: u_k = some initial mode choice at $t = t_0$, typically in asymptotically flat region / free-fall coordinates

⇝ $u_k \sim \frac{1}{\sqrt{\omega_k}} e^{-i\omega_k t} [e^{i\vec{k}\vec{x}}]$

v_k = general solution to KG eq. at $t > t_0$

⇒ $\beta_k(t)$ ⇒ continuous particle production

⇝ e.g. inflation

toy model example [2206.10929]

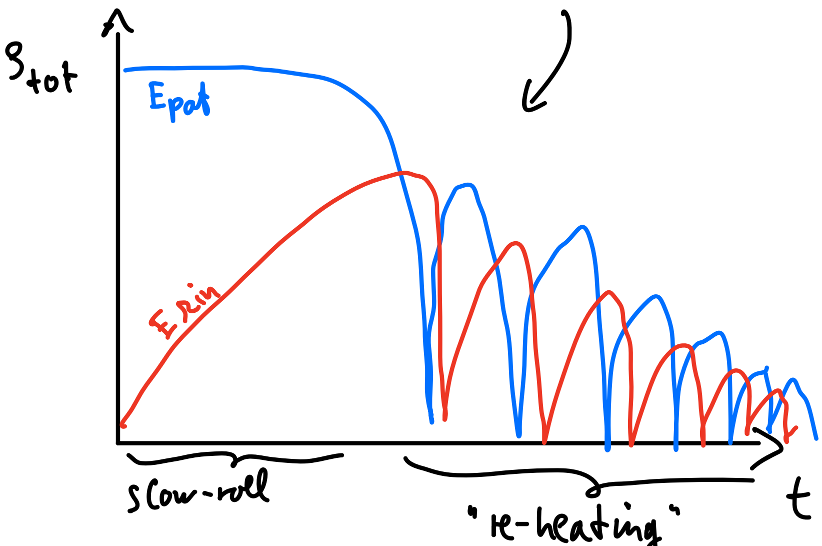
$$S = \int d^4x \sqrt{|g|} \left\{ \frac{M_{Pl}^2}{2} R + \mathcal{L}_\phi + \mathcal{L}_\chi \right\}$$

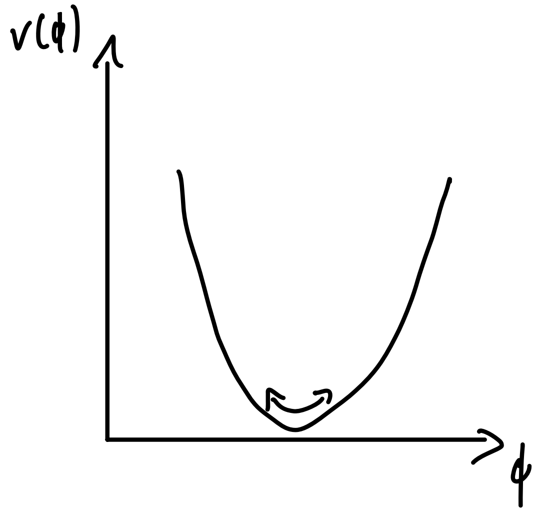
inflaton ; $\rho \sim \rho_\phi$

$\frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - V(\phi)$

$= \frac{1}{2} g^{\mu\nu} (\partial_\mu \chi)(\partial_\nu \chi) - \frac{1}{2} m^2 \chi^2$

$\rho_\chi \ll \rho_\phi$





$\rightsquigarrow$ rapid oscillations, essential for production of χ

$d\eta \equiv a^{-1}(t)\, dt$ "conformal time" $\Rightarrow g_{\mu\nu} = a(\eta)\, \eta_{\mu\nu}$

$\tilde{\chi} \equiv a^{-1} \chi$

$$\Rightarrow \sqrt{|g|}\, \mathcal{L}_\chi = \frac{1}{2} (\tilde{\chi}')^2 - \frac{1}{2} \tilde{\chi}\, \omega^2 \tilde{\chi} \quad ; \quad \omega^2 \equiv \underbrace{-\nabla^2}_{\to +k^2} + a^2 m_\chi^2 \underbrace{- \frac{a''}{a}}_{>0} > 0$$

$$\tilde{\pi} = \frac{\partial \mathcal{L}_\chi}{\partial \tilde{\chi}'} = \tilde{\chi}' \Rightarrow H = \int d^3x \left\{ \tilde{\chi}' \tilde{\pi} - \mathcal{L}_\chi \right\} = \int d^3x \left\{ \frac{1}{2} \tilde{\pi}^2 + \frac{1}{2} \tilde{\chi}\, \omega^2 \tilde{\chi} \right\}$$

$\omega = \omega(\eta) \Rightarrow H = H(\eta) \Rightarrow$ particle production !

$$\hat{x}(x) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\vec{x}} \tilde{x}_k(t) \; ; \; \tilde{x}_k = f_k a_{\vec{k}} + f_k^* a^\dagger_{-\vec{k}}$$ $f \,\hat{=}\, u$ above

$$\tilde{\pi}(x) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\vec{x}} \tilde{\pi}_k(t) \; ; \; \tilde{\pi}_k = g_k a_{\vec{k}} + g_k^* a^\dagger_{-\vec{k}}$$ $g \neq v$ above!

ansatz : $f_k = \frac{\tilde{\alpha}_k(t)}{\sqrt{2\omega_k}} + \frac{\tilde{\beta}_k(t)}{\sqrt{2\omega_k}}$

$$g_k = -i\sqrt{\frac{\omega_k}{2}}\,\tilde{\alpha}_k + i\sqrt{\frac{\omega_k}{2}}\,\tilde{\beta}_k$$

[why?
⇝ correct flat space normalization
for $\tilde{\alpha}_{\vec{k}} = e^{-i\omega_k \eta}$; $\tilde{\beta}_{\vec{k}} = 0$]

two conditions:

$$\boxed{\begin{aligned} &i)\;\; \tilde{\pi}_k = \tilde{x}_k' \\ &ii)\;\; \tilde{\pi}_k' = -\frac{\partial \tilde{\pi}}{\partial \tilde{x}_k} = \omega_k^2 \tilde{x}_k \end{aligned}} \Rightarrow \boxed{\begin{aligned} \tilde{\alpha}_k' &= -i\omega_k \tilde{\alpha}_k + \frac{\omega_k'}{2\omega_k}\tilde{\beta}_k \\ \tilde{\beta}_k' &= i\omega_k \tilde{\beta}_k + \frac{\omega_k'}{2\omega_k}\tilde{\alpha}_k \end{aligned}}$$

[e.o.m.]

(fully general, no assumptions)

initial condition: $\tilde{\alpha}_k(\eta=0) = 1$
$\tilde{\beta}_k(\eta=0) = 0$

- "zero order" $\omega_k' \simeq 0 \Rightarrow \tilde{\alpha}_k = e^{-i\omega_k\eta}$, $\tilde{\beta}_k = 0$ ✓

(ordinary plane waves)

⇝ $(\tilde{\alpha},\tilde{\beta}) \equiv \underbrace{e^{-i\omega_k\eta}}_{u_k\cdot\sqrt{2\omega_k}}(\alpha,\beta)$ in notation of intro

- "full" $|\tilde{\beta}_k|^2 \ll 1$ (occupation number remains small) $\Rightarrow \boxed{\tilde{\beta}_k \simeq \int_0^\eta d\eta' \frac{\omega_k'}{2\omega_k} e^{-2i\int_0^{\eta'} d\eta''\omega_k(\eta'')}}$

$\downarrow$ $\omega_k \leftarrow a, H$; use $\phi(t) = \bar{\Phi}\sin(m_\phi t)$
WKB ansatz, stationary phase...

$\frac{k}{a} \gtrsim H_e \sim m_\phi$: $\quad |\beta_k|^2 \propto k^{-9/2}$ [fully analytical result, including prefactors]

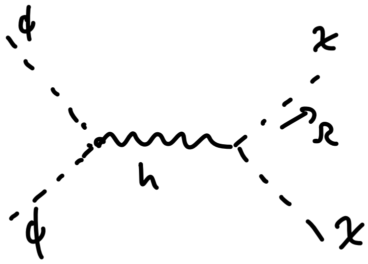
high energy / sub-horizon @ end of inflation

$$\frac{k}{a} \lesssim H_e \sim m_\phi: \quad |\beta_k|^2 \propto \begin{cases} k^{-6} & \text{for } m_\chi = 0 \\ k^{-3} & \text{for } m_\chi \sim 0.1\, m_\phi \end{cases}$$

Alternative picture:

$$g_{\mu\nu} \simeq \eta_{\mu\nu} + 2 h_{\mu\nu} \quad \rightsquigarrow \quad S \simeq \int d^4x \left\{ \mathcal{L}_{h,kin} + \overbrace{\mathcal{L}_\phi + \mathcal{L}_\chi}^{\text{Minkowski!}} - h^{\mu\nu}\left(T^\phi_{\mu\nu} + T^\chi_{\mu\nu}\right) \right\}$$

=> production through



$$\xrightarrow{L[f]=C[f]} \quad f_\chi(k) \propto k^{-9/2}$$

$$\equiv |\beta_k|^2 \text{ for } E_k > m_\phi$$

!

⇝ take-home:

- very complementary viewpoints
- perturbative regime – can use both approaches
  ($k > m_\phi$) [but Boltzmann typically much easier in practice!]
- non- " " – must use Bogoliubov