most important (NB: $\mathbb{Z}_2 \Rightarrow$ DM is stable)

a) DM scattering ($3 =$ DM; $1,2 =$ SM / heat bath)

$\Rightarrow C_n[f] = 0$ $\rightsquigarrow$ exercise!

b) DM pair annihilation / creation

$1 =$ non-rel. DM ("χ") ; $2,3 =$ heat bath ("ψ") $\Rightarrow f_\psi = \dfrac{1}{e^{E/T} \pm 1}$ ($\mu \simeq 0$ in SM!)

- $|M|^2_{\leftarrow} = |M|^2_{\rightarrow}$ (CP invariance)
- detailed balance ("$\leftarrow = \rightarrow$ in EQ")
- $f_\chi \propto e^{-E/T}$ (NR DM in kinetic EQ)

$\rightsquigarrow$ exercise!

$$\boxed{\dot{n} + 3Hn = -\langle\sigma v\rangle (n^2 - n_{eq}^2)} = C_n[f]$$

"annihilation" "production"

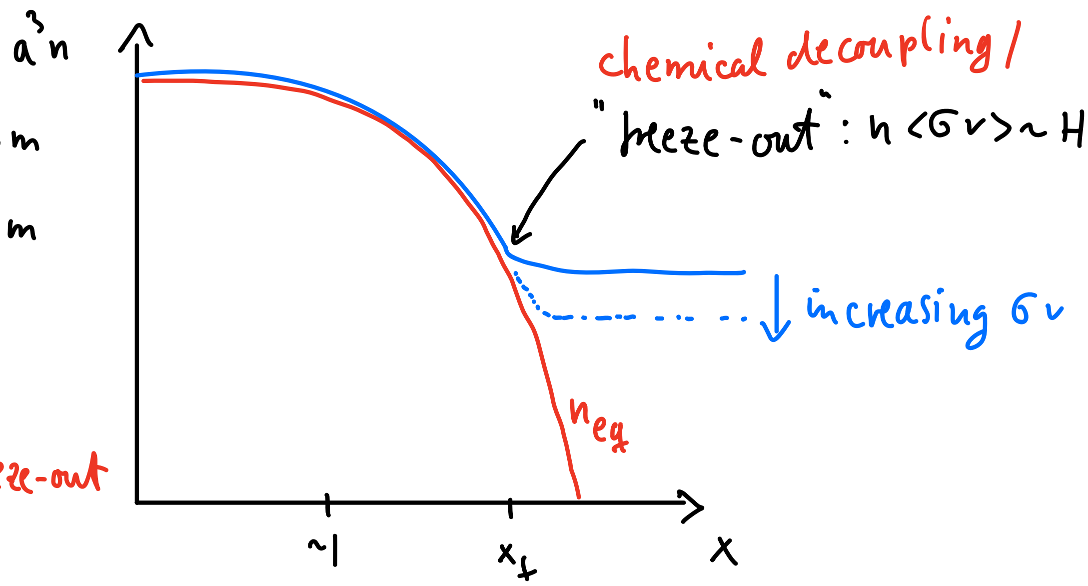
$$\underline{\underline{\langle \sigma v\rangle}} = \frac{\int d^3p \int d^3\tilde{p}\, f_\chi(E)\, f_\chi(\tilde{E})\, \sigma v_{Mol}}{\int d^3p \int d^3\tilde{p}\, f_\chi(E)\, f_\chi(\tilde{E})} = \ldots = \underline{\underline{\int_1^\infty d\tilde{s}\, \frac{4x\sqrt{\tilde{s}}\,(\tilde{s}-1)\, K_1(2\sqrt{\tilde{s}}\,x)}{K_2^2(x)}\, \sigma}}$$

Gondolo & Gelmini '91

$\left[x \equiv \frac{m}{T}\,;\ \tilde{s} \equiv \frac{s}{4m^2}\right]$

recall:

$$n_{eq} \propto \begin{cases} T^3 & \text{for } T \gg m \\ T^{3/2}\, e^{-m/T} & \text{for } T \ll m \end{cases}$$



$\rightsquigarrow$ relevant for $\left\{ \begin{matrix} \text{HDM}, \gamma, \ldots \\ \text{CDM} \end{matrix} \right\}$ freeze-out rough estimate: $\bullet\ n_\chi \cdot \langle\sigma v\rangle \overset{!}{\sim} H_f \sim \frac{T_f^2}{m_{pl}}$

$\bullet\ n_f \sim \frac{g_\chi^{eq}}{m_\chi} \frac{T_f^3}{T_{eq}^3} \overset{!}{\sim} T_{eq}\, T_f^2 \underbrace{\frac{T_f}{m_\chi}}_{\text{"}\mathcal{O}(1)\text{"}}$ with $g_\chi^{eq} \sim g_\gamma^{eq} \sim T_{eq}^4$

$$\boxed{\langle\sigma v\rangle \sim \frac{1}{T_{eq} \cdot m_{pl}}}$$

for correct relic density

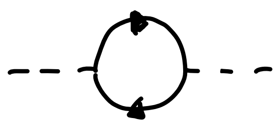
$T_{eq} \sim 3\,\mathrm{eV}$ $\quad m_{pl} \sim 2 \cdot 10^{18}\,\mathrm{GeV}$

numerically: $\Omega_\chi h^2 \sim \frac{3 \cdot 10^{-27}\,\mathrm{cm^3/s}}{\langle\sigma v\rangle} \sim \frac{3 \cdot 10^{-10}\,\mathrm{GeV}^{-2}}{\langle\sigma v\rangle}$ $\quad \Big| \quad \sigma v \sim \frac{\alpha^2}{m_\chi^2}$

~ 0.1 e.g. for $m_\chi \sim \mathrm{TeV}$ & $\alpha \sim 10^{-2}$

$\Rightarrow$ Weakly Interacting Massive Particles are generically produced with correct relic density!

$\rightsquigarrow$ a "miracle"? $\bullet$ cf. SM hierarchy problem

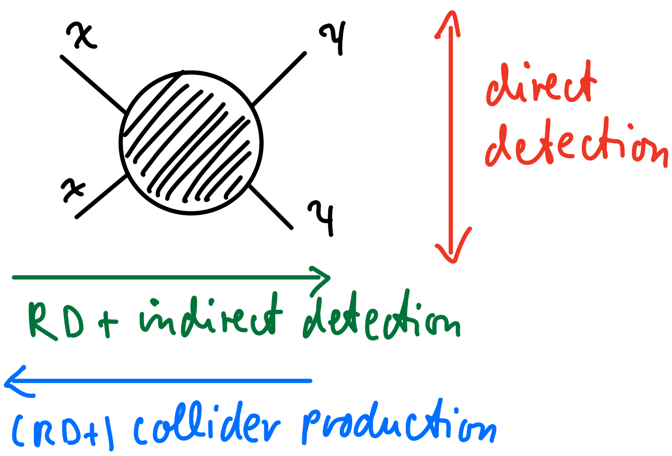


$\Rightarrow \delta m_h^2 \propto \Lambda^2$

$\bullet$ many good candidate BSM theories, e.g. SUSY

= independent motivation from particle physics (+ correct RD)

$\bullet$ WIMP DM is highly predictive:



$\rightsquigarrow$ vanilla WIMPs are <u>ruled out</u>

(:⟺ ⊘ ~ 1 number ; very close to EW scale)

"exceptions": A) $\Psi_{RD} \neq \Psi_{\text{detection}}$

(very common!) B) $\langle\sigma v\rangle_{RD} \gg \langle\sigma v\rangle_0$

<u>examples</u> for B:

i) $\sigma v = a + b\, v_{rel}^2 + \dots$

$\Rightarrow \langle\sigma v\rangle \overset{v=v_{lab}}{=} a + 6 b\, x^{-1} + \dots$

NB: s-wave $= a + a_2 v^2 + \dots$ ($\to b$)

p-wave $= b_2 v^2 + b_4 v^4 + \dots$

today : $v \sim 10^{-3}$

freez-out : $T \sim \frac{1}{3}\frac{p^2}{m} \sim \frac{m}{x_f}$

$\Rightarrow v \sim \sqrt{\frac{3}{20}} \sim 0.4$

$\Rightarrow$ can be solved <u>analytically</u>:

Kolb & Turner

$$\boxed{\Omega h^2 \simeq 8.77 \cdot 10^{-11}\,\text{GeV}^{-2}\, \frac{x_f}{g_{eff}^{1/2}} \times \left[a + 3 b\, x_f^{-1} + \dots\right]^{-1}}$$

ii) strong v-dependence (other than i)

- (kinematic) thresholds: $\chi\chi \to \Psi\Psi$; $m_\Psi \gtrsim m_\chi$
- (s-channel) resonances: $\chi\chi \to \phi^* \to \Psi\Psi$; $m_\phi \sim 2 m_\chi$
- non-perturbative effects: "Sommerfeld enhancement", bound states, ...

iii) co-annihilations

$\rightarrow$ network of "dark" particles w/ $m_{\chi_n} > \dots > m_{\chi_1} \equiv m_\chi$, charged under same Z_2 (e.g. R parity) $\rightsquigarrow$ will eventually decay to χ

$\Rightarrow n \equiv n_{\chi_1} + n_{\chi_2} + \dots + n_{\chi}$ satisfies same Boltzmann eq., with

$\langle\sigma v\rangle_{\text{eff.}} = \sum_{ij} c_{ij} \langle\sigma v\rangle_{ij}$ [hep-ph/9704361]

iv) asymmetric DM : $n_\chi \neq n_{\bar\chi}$ $\rightarrow$ cf. standard model...

motivation : $\Omega_\chi h^2 \sim \Omega_b h^2$ $\rightsquigarrow$ ~ no indirect detection

v) freeze-out in secluded / "dark" sector (= A&B)

⋮

$\Rightarrow$ need <u>numerical</u> treatment!

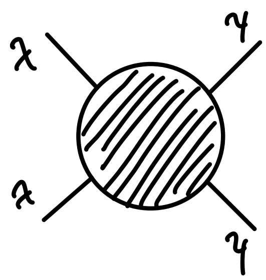
$\rightsquigarrow$ highly accurate <u>public</u> tools : "DarkSUSY" $\rightsquigarrow$ exercises!

micrOMEGAs

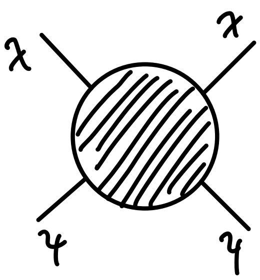
madDM

⋮

<u>Kinetic decoupling</u> review: 0903.0189



$\Rightarrow$ <u>chemical</u> EQ until $x_f \sim 20$ $\rightsquigarrow$ $\boxed{\Omega_\chi h^2}$



$\Rightarrow$ <u>kinetic</u> EQ until $x_{kd} \gg x_f$ [generically, since $n_\psi \gg n_\chi$]

$\rightsquigarrow$ minimal protohalo mass $\boxed{M_{cut}}$

two independent effects:

a) no growth of density perturbations

on scales $\lambda < r_s = \int_0^{t_{kd}} dt \, \frac{c_s}{a}$ $\leftarrow$ $= \sqrt{\frac{dp}{d\rho}}$ = sound speed

around decoupling : (dark) acoustic oscillations

$\rightsquigarrow$ additional cutoff, may imprint features on $P(k)$

b) wash-out of density perturbations

on scales $\lambda < \lambda_{FS} = \int_{t_{kd}}^{t_{eq}} dt \, \frac{\langle v \rangle}{a}$

sufficient to consider temperature parameter : / "velocity dispersion"
/ "2nd moment of f"

$$T_\chi \equiv \left\langle \frac{p^2}{3E} \right\rangle = \frac{g}{n} \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E} f(E) = \begin{cases} T & \text{for } T \gg T_{kd} \\ \frac{c}{a^2} & \text{for } T \ll T_{kd} \; [\ll m] \end{cases}$$

transition typically very fast

$$\overset{T_{kd}}{\simeq} \begin{cases} T & \text{for } T \gtrsim T_{kd} \\ T_{kd} \left(\frac{a_{kd}}{a}\right)^2 & \text{for } T \lesssim T_{kd} \end{cases}$$

Boltzmann equation:

$$C_T \equiv g \int \frac{d^3p}{(2\pi)^3} \frac{C[f]}{E} \frac{p^2}{3E}$$

$$\overset{!}{=} g \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E} \left(\partial_t - H p \cdot \nabla_p\right) f$$

$$= \partial_t (n T_\chi) + H g \int \frac{d^3p}{(2\pi)^3} f \underbrace{\frac{\partial}{\partial p^i}\left(p^i \frac{p^2}{3E}\right)}_{= 3\frac{p^2}{3E} + 2\frac{p^2}{3E} - \frac{1}{3}\frac{p^4}{E^2}}$$

$E = \sqrt{p^2 + m^2}$

$$= n \left\{ \dot{T}_\chi + \underbrace{\frac{\dot{n}}{n}}_{= -3H \text{ if } C_n = 0} T_\chi + 5 H T_\chi \left[- \frac{1}{3} H \left\langle \frac{p^4}{E^2} \right\rangle \right] \right\}$$

$p^2/E^2 \ll 1$

2→2 elastic scattering:

$$C = \frac{1}{2g} \int \frac{d^3k}{(2\pi)^3 2\omega} \int \frac{d^3\tilde{k}}{(2\pi)^3 2\tilde{\omega}} \int \frac{d^3\tilde{p}}{(2\pi)^3 \tilde{E}} (2\pi)^4 \delta^{(4)}(\tilde{p} + \tilde{k} - p - k)$$

$$|M|^2_{\chi\psi \leftrightarrow \chi\psi} \left\{ [1 \mp f_\psi(\omega)] f_\psi(\tilde{\omega}) f(\tilde{E}) - [1 \mp f_\psi(\tilde{\omega})] f_\psi(\omega) f(E) \right\}$$

- $t \ll m^2$
- $T \lesssim m$

↓

$$\ldots\ C_T = \frac{3}{2}\gamma(T)\, n\,(T-T_\chi) \times \left\{1+\mathcal{O}\left(\frac{p^2}{E^2}\right)\right\}$$

momentum transfer rate [$= \frac{1}{48\pi^3 g\, m^3}\int d\omega\, f_\nu\, \partial_\omega\left(k^4 \langle|M|^2\rangle_t\right)$

$\neq$ scattering rate $\sim \frac{m}{T}\cdot\gamma$

[reason: "random walk" in momentum space]

$$\langle|M|^2\rangle_t = \frac{1}{8k^4}\int_{-4k^2_{cm}}^{0} dt\,(-t)\,|M|^2$$

$$= 16\pi\, m_\chi^2\, \sigma_T$$

"transfer cross section"]

$$\Rightarrow \boxed{\boxed{\dot{T}_\chi + 2HT_\chi = \frac{3}{2}\gamma\,(T-T_\chi)}}$$

$\rightsquigarrow$ independent eq. for T_χ

$\downarrow$

T_{kd}

$\downarrow$

$M_{cut} \sim (10^{-11} - 10^{-3})M_\odot$ for neutralinos ("WIMPs")

[$\neq$ "earth mass" $\simeq 10^{-6}M_\odot$!] $\longrightarrow$ difficult to observe directly

$\lesssim 10^{10} M_\odot$ e.g. for $\begin{cases} \sim\text{TeV DM w/} \sim\text{MeV mediators} \\ \sim\text{keV (warm) DM} \end{cases}$

$\rightarrow$ constrained by Ly-α, dwarf galaxy count

important exceptions :

$x_{kd} \not\gg x_{cd} \Rightarrow$ direct impact on $\Omega_\chi h^2$

$\rightsquigarrow$ can not use standard $\dot{n} + 3Hn = -\langle\sigma v\rangle\,(n^2 - n_{eq}^2)$!

$\rightsquigarrow$ must consider coupled eqs. for n, T_χ

(or solve BE at phase-space level)