Dark matter

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⚠ Ask & interrupt me at any time ⚠

outline

I. Intro: basic evidence & properties $\lesssim$ 1 lecture

⇝ main thing we know: $\Omega h^2 \sim 0.12 \pm 1\%$ ⇝ focus on production

II. Production from the thermal bath $\gtrsim$ 2 lectures

II.1: Freeze-out mechanism

("standard", [co-annihilation, aDM?, dark sectors] [impact of] kinetic decoupling)

II.2: Freeze-in mechanism

(following an approach that stresses analogy w/ II.1)

II.3: Alternatives

(SIMP, Pandemic, ...)

III. (Fully) non-thermal production $\lesssim$ 1 lecture

[III.1: Misalignment] → see lecture by J. Jaeckel

(axions, dark photons)

III.2: Gravitational production

(Bogoliubov vs. decay !?)

III.3: Primordial black holes

⇝ quite a few other options [e.g. DM from decay];
personal selection, with focus on what is most commonly discussed

+ only sporadic bibliographic references

⇝ contact/ask me for more pointers to literature!
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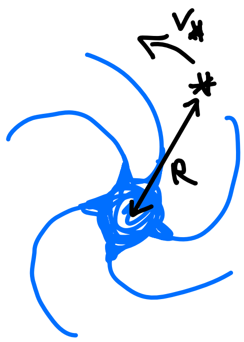
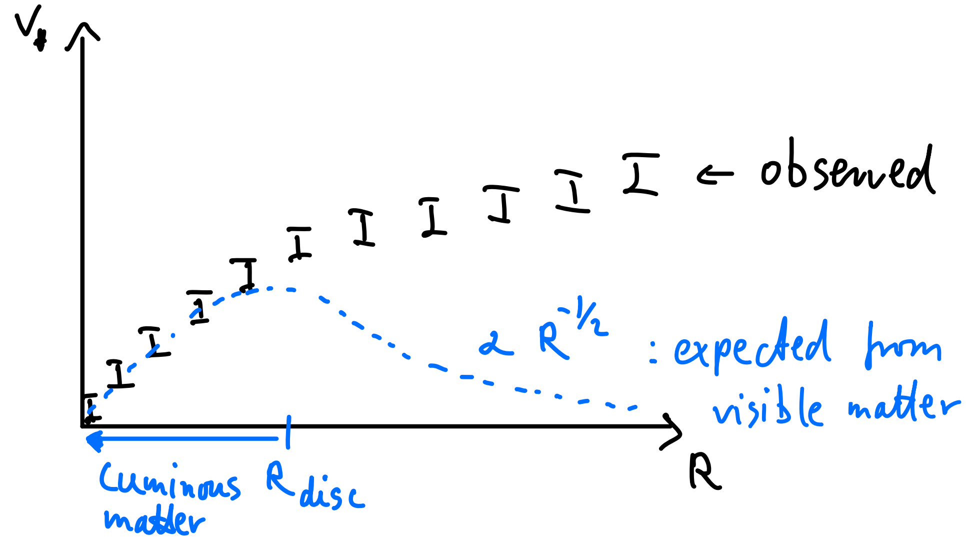
I. (see also very extensive recent review: 2406.01705)

Direct

Evidence on galactic scales

most often quoted: rotation curves ⌈mostly since precision measurements of Rubin & Ford in 1970⌋





Newton: $G_N m_* \frac{M(r<R)}{R^2} = m_* \frac{v_*^2}{R}$ ⇒ v_* directly traces enclosed mass

⇒ invisible / "dark" matter (DM) must contribute to M for $r \gtrsim R_{disc}$! ⇝ $M/L = \mathcal{O}(10)$ in MW, M31, ...

⌈statistical version: Jeans analysis ⇝ dSphs : $M/L = \mathcal{O}(1000)$⌋

⇝ very intuitive, hence often used in popular context

BUT no longer main argument for existence of DM!

- rotation curves are rather diverse
- " " alone (!) can be "explained" differently ⇝ MOND...

Direct

Evidence on cluster scales

= largest gravitationally bound systems

$M \sim 10^{15} M_\odot$, mostly hot gas

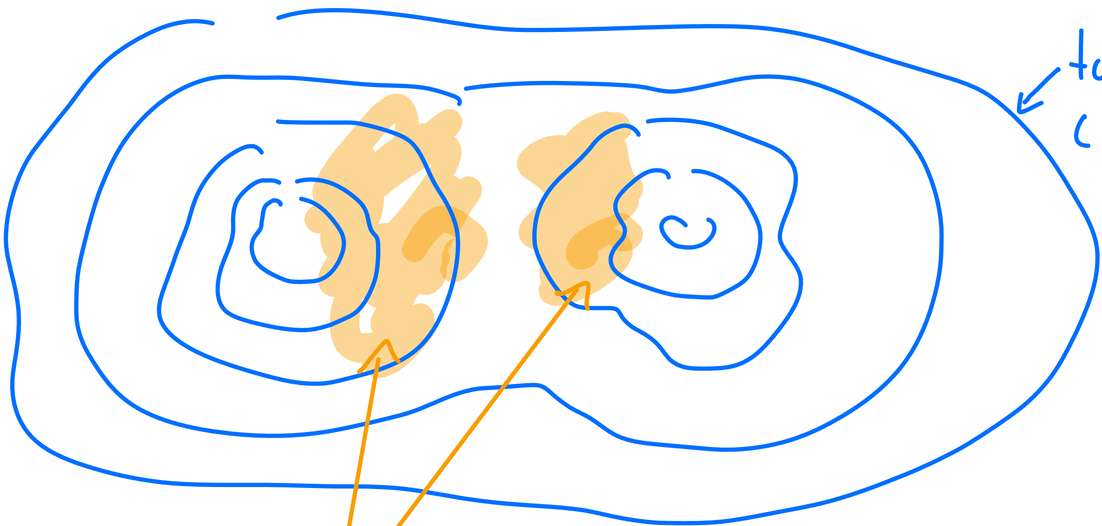
• historical (Zwicky, 1933): use virial theorem
+ earlier!

$$\langle T \rangle = -\frac{1}{2}\langle V \rangle \quad \Big| \; M \sim N \cdot m \;;\; N \gg 1$$ (highly simplistic model)

$$N \frac{m v^2}{2} \sim \frac{1}{2} \frac{G m^2}{R} \cdot \frac{N(N-1)}{2} \Rightarrow N \cdot m \sim M \sim \frac{R v^2}{G}$$ > observed!

$\rightsquigarrow \Omega_{DM} \sim 0.2 \ll \frac{\rho_{DM,0}}{\rho_c}$ ⌈clusters are only "slightly" non-linear $\rightsquigarrow$ more reasonable tracers of entire universe⌋

• direct observation of colliding clusters:



$\Rightarrow$ – DM dominates

– DM is "collision-less": $\sigma_{DM-DM} \lesssim 1\,cm^2 \left(\frac{m_{DM}}{g}\right)$

⌈$\rightsquigarrow$ quite different from ordinary matter⌋ $\sim mb \left(\frac{m_{DM}}{GeV}\right)$

Evidence from cosmology

cf. lectures by Yvonne: ΛCDM

• needed to describe the inhomogeneous universe

- precision cosmology : plethora of data

$\Rightarrow \boxed{\Omega_{DM} h^2 \simeq 0.12} \pm 1\%$

- absolutely crucial : one of only 6 independent parameters
- 1% → ~10% for much more general case (7ΛCDM), allowing DM to invisibly disappear to dark radiation [1803.03644]

⌜NB : $\Omega_{DM} \neq 0$ already follows from linear gravity (= Einstein or Newton/Poisson on expanding background)

$$\rho_i(t, \vec{x}) = \bar{\rho}_i(t) \{ 1 + \delta_i(t, \vec{x}) \} \quad ; \ \delta_i \ll 1$$

↑ BG FLRW

⇒ inside horizon, during matter domination:

$p_i \simeq 0$ ("matter") ⇒ $\delta_i(t) \propto a(t)$

⌜⟷ $p_i = \frac{1}{3}\rho_i$ ("radiation") ⇒ $\delta_i \sim$ const. / osc... ⌟

observation 1) $T_{CMB} = 2.7\,K \sim 10^{-3} \cdot (13.6\,eV)$

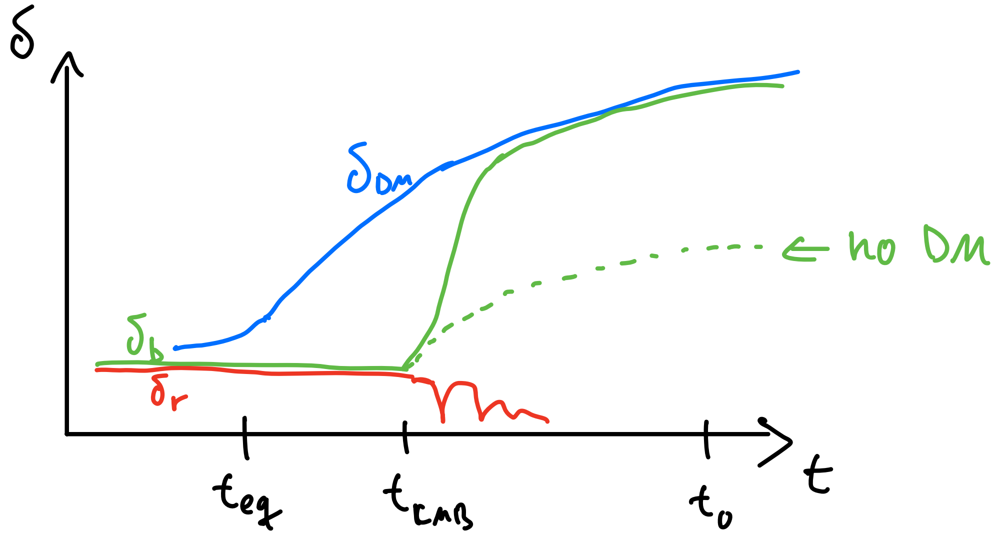
10^{-3} = redshift factor (a_{CMB})

2) $4 \frac{\Delta T}{T}\Big|_{CMB} \sim 10^{-5} \overset{\rho_\gamma \propto T^4}{\simeq} \delta_\gamma \simeq \delta_b$

⇒ without DM : $\delta_b(t_0) = \delta_b(t_{CMB}) \cdot \frac{a_0}{a_{CMB}} \sim 10^{-2}$

⇝ the visible universe would still be homogeneous!

⇒ there must be sizeable "invisible" gravitational potentials already at CMB times



Bottom line

- main <u>evidence</u> for DM from cosmology
  - precision measurement of $\Omega_{DM} h^2$
  - <u>very</u> hard for any alternative explanations
    [gold standard for MOND-like theories today: P_{lin} and $P_{non\text{-}lin}$, <u>not</u> just rotation curves!]
- many <u>constraints</u> on
  - how cold DM must be
    (structure formation, BBN)
  - interactions with visible matter
    (direct & indirect searches, early universe production)
  - interactions with itself or other "dark sector" particles
    (structure formation, production)

⇝ **DM = BSM** (but vanilla CDM works depressingly well!)

= vanilla CDM ?

- Any <u>theory</u> of DM should at least allow to predict $\Omega_{DM} h^2$, ideally with matching accuracy!

II: Production from the thermal bath

Boltzmann equation : $L[f_i] = C[f_i]$

⇝ formally describes evolution of $f_i(\vec{x}, \vec{p}, t)$ for any particle species i

Liouville operator : ⇝ evolution in absence of collisions

[cf. Liouville's theorem in classical mechanics
$$\frac{\partial f}{\partial t} + \{f, H\} = \frac{df}{dt} = 0$$]

$$L[f] \equiv \frac{df}{d\lambda} = \frac{\partial x^\mu}{\partial \lambda}\frac{\partial f}{\partial x^\mu} + \frac{dp^i}{\partial \lambda}\frac{\partial f}{\partial p^i}$$

($p^0 = p^0(p^i)$)

standard choice:
p^i = "physical momentum" $= m\frac{dx^i}{d\tau}$ ← free-fall coordinates
= measured by freely-falling observer

$\Leftrightarrow \bar{p}^i \equiv m\frac{d\bar{x}^i}{d\tau} = \frac{\partial \bar{x}^i}{\partial x^\nu}p^\nu$ = standard momentum in GR (= 4-vector)
→ geodesic eqn. in free fall

$$g^{\mu\nu} = \eta^{\rho\sigma}\frac{\partial \bar{x}^\mu}{\partial x^\rho}\frac{\partial \bar{x}^\nu}{\partial x^\sigma} \underset{\text{FRW}}{\Rightarrow} \frac{\partial \bar{x}^0}{\partial x^0} = 1, \quad \frac{\partial \bar{x}^i}{\partial x^j} = a^{-1}\delta^i_j$$

$$\Rightarrow \boxed{\bar{p}^i = a^{-1}p^i}$$ ($\bar{p}^0 = p^0$)

NB: not comoving $= a p^i$!

$$\Rightarrow L[f] = p^0\partial_t f + \frac{\partial f}{\partial p^i}\left\{\frac{da}{d\tau}\bar{p}^i + a\frac{d\bar{p}^i}{d\tau}\right\}$$

$\lambda = \tau$; $f = f(t, \vec{p})$ for FRW

$\frac{da}{d\tau}\bar{p}^i = \dot{a}p^0\bar{p}^i = Hp^0p^i$

$a\frac{d\bar{p}^i}{d\tau}$: $= -\Gamma^i_{\mu\nu}\bar{p}^\mu\bar{p}^\nu \quad | \; \Gamma^i_{0j} = \Gamma^i_{j0} = H\delta^i_j$
$= -2H\bar{p}^i\bar{p}^0$

→ exercises

$$\Rightarrow \boxed{L = p^0(\partial_t - H\,\vec{p}\cdot\nabla_p)\, f(t,\vec{p})}$$

↑ physical momentum

$$\ulcorner = p^0 \partial_t f(t, \vec{p}\cdot a)$$

↑ comoving momentum ⌟

$\Rightarrow$ for <u>number density</u> $n(t)$:

$$C_n[f] \equiv g\int \frac{d^3p}{(2\pi)^3}\,\frac{C[f]}{E}$$

$$\overset{!}{=} \partial_t \Big[\underbrace{g\int\frac{d^3p}{(2\pi)^3} f}_{\equiv n(t)}\Big] - Hg\int\frac{d^3p}{(2\pi)^3}\underbrace{\vec{p}\cdot\nabla_p f}_{\to -f(\nabla_p\vec{p}) = -3}$$

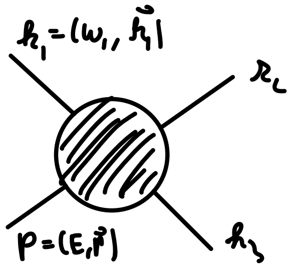
$$= \dot{n} + 3Hn \qquad \Big|\ H = \frac{\dot{a}}{a}$$

$$= a^{-3}\partial_t(a^3 n)$$

$\Rightarrow$ " C_n = change of comoving number density "

II.1 : Freeze-out mechanism

Collision operator for $2 \leftrightarrow 2$ processes :



$$C[f] = \frac{1}{2g}\int \prod_i \left(\frac{d^3k_i}{(2\pi)^3 2\omega_i}\right)(2\pi)^4\,\delta^{(4)}(p + k_1 - k_2 - k_3)$$

$$\times \Big\{ |M|^2_{\leftarrow}\, f_2 f_3 (1 \mp f_1)(1 \mp f) \qquad \text{Fermi suppression / Bose enhancement}$$

↑ summed over all d.o.f. g_i!

$$- |M|^2_{\rightarrow}\, f f_1 (1 \mp f_2)(1 \mp f_3) \Big\}$$