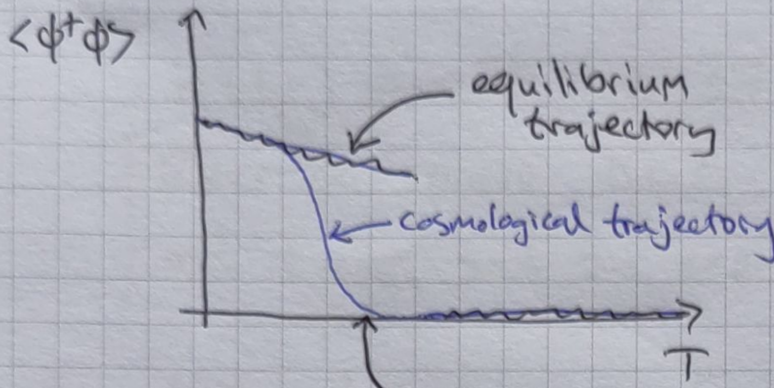
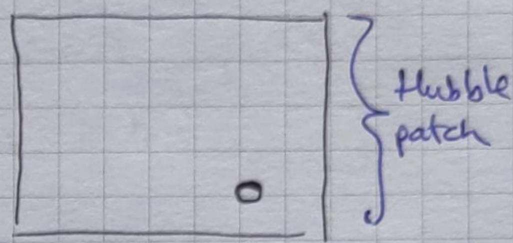
Phase transitions IV

Dynamics and gravitational waves





$\Gamma(t_0) \sim H^4(t_0) \Rightarrow$ one bubble per Hubble

Volume of growing bubble, assuming constant speed

$$V(t,t') = \frac{4\pi}{3}\left(R_0 + v_w|t-t'|\right)^2$$

growing until time t — nucleated at t' — $R_0 \to 0$, negligible (as we will find)

$h(t) \equiv$ fraction of universe in high-T phase

Near $t = t_0$, and before,

$$h(t) \simeq 1 - \int_{t_c}^{t} dt'\, \Gamma(t')\, V(t,t')$$

Accounting for nucleation only within high-T phase:

$$h(t) = \exp\left(-\int_{t_c}^{t} dt'\, \Gamma(t')\, V(t,t')\right)$$

Given that the nucleation rate depends on the exponential of a strongly varying function of T, we expect most bubbles are produced near $t' \simeq t$, and we can approximate $\Gamma(t') = Ae^{\beta t'}$.

$$\int_{-\infty}^{t} dt'\, Ae^{\beta t'} \frac{4\pi v_w^3}{3}(t-t')^3 = \frac{Ae^{\beta t}\, 8\pi v_w^3}{\beta^4}.$$

So that,

$$h(t) = \exp\left(-\frac{8\pi v_w^3}{\beta^4} A e^{\beta t}\right) \equiv \exp\left(-e^{\beta(t-t_f)}\right)$$

defines t_f

$$h(t_f) = e^{-1} \simeq 0.37.$$

[Exercise: prove that $h(t)$ varies most rapidly at $t=t_f$, and the expression for R_{bub} below.]

So, the duration and timescale of the transition are set by $1/\beta$.

$$n_{bub}(t) = \int_{t_c}^{t} dt' \Gamma(t') h(t')$$

$$= \frac{\beta^3}{8\pi v_w^3} \quad \text{at end of transition.}$$

$$R_{bub} = n_{bub}^{-1/3} = \frac{(8\pi)^{1/3} v_w}{\beta}.$$

average separation of bubble centres by end of transition.

Note, during radiation domination,

$$aT \simeq \text{const}$$

$$\frac{d}{dt} = -HT\frac{d}{dT}$$

So that,

$$\beta \equiv \frac{d}{dt}\log\Gamma = -HT\frac{d}{dT}\log\Gamma$$

$$\simeq HT\frac{d}{dT}S_{EFT}[\phi_b]$$

nucleation rate gives transition timescale.

Let's try putting in numbers for the electroweak scale:

$$\frac{1}{R_{bub}} \sim \beta \simeq \frac{T^2}{M_{pl}} \underbrace{T\frac{d}{dT}S_{EFT}[\phi_b]}_{\sim 10^2} \simeq \frac{10^4 \cdot 10^2}{10^{19}} \cdot \text{GeV} \sim \frac{1}{2\,\text{mm}}$$

$$H \sim \frac{1}{20\,\text{cm}}, \quad T \sim \frac{1}{10^{-2}\,\text{fm}}$$

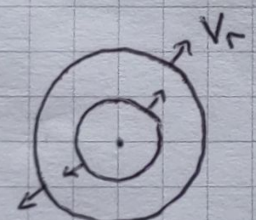
So, R_{bub} is really a macroscopic lengthscale, and R_0 (at nucleation) is many orders of magnitude smaller, as is the scalar field bubble wall profile.

$\Rightarrow$ Dynamics of the scale $\sim R_{bub}$ governed by hydrodynamics, coupled to the order parameter. Hydro. is universal in IR.

$$T^{\mu\nu}_{fluid} = (e+p)u^\mu u^\nu + p g^{\mu\nu} + \text{(viscous terms)}$$
$$T^{\mu\nu}_{\phi} = \partial^\mu\phi\,\partial^\nu\phi - g^{\mu\nu}\cdot\tfrac{1}{2}(\partial\phi)^2$$
$$\left.\begin{aligned} &\partial_\mu\left(T^{\mu\nu}_{fluid} + T^{\mu\nu}_{\phi}\right) = 0 \\ &\partial_\mu\partial^\mu\phi + \underbrace{\eta u^\mu \partial_\mu \phi}_{\substack{\text{dissipation} \\ \text{of scalar field energy} \\ \text{into fluid}}} - V'_{eff}(\phi) = \underbrace{\xi}_{\substack{\text{fluctuation} \\ \text{(often dropped)}}} \end{aligned}\right\} \text{Coupled fluid scalar equations}$$

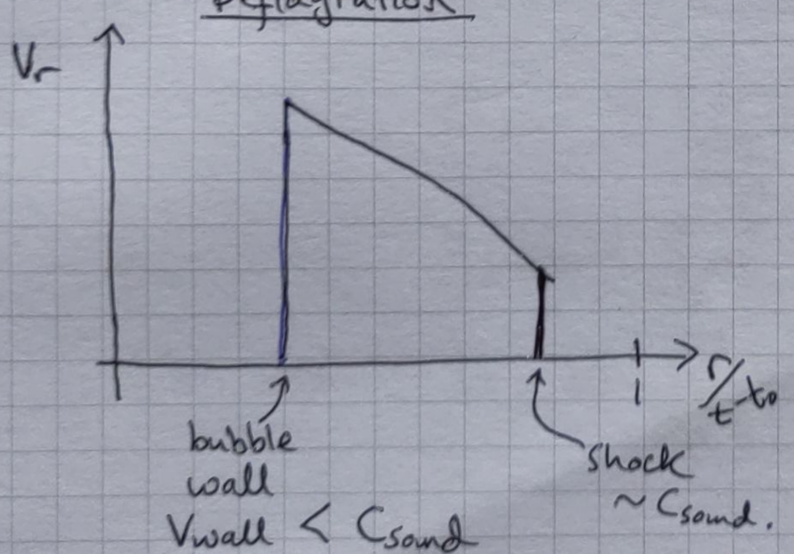
Numerical simulations:

- single growing bubble $\longrightarrow$ scaling solutions at late times as only one scale

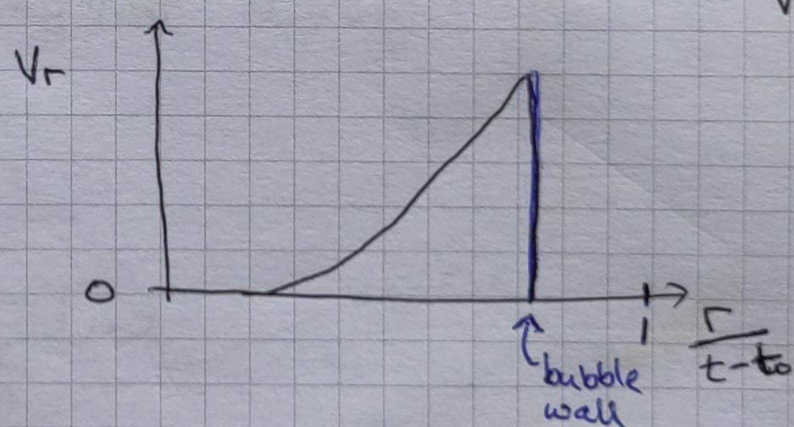


Two types of solutions.

Deflagration

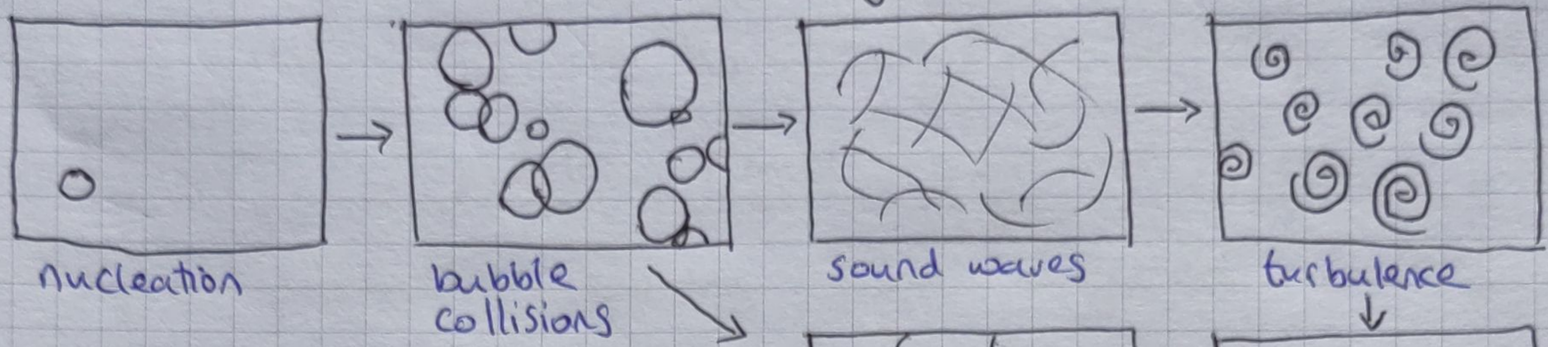


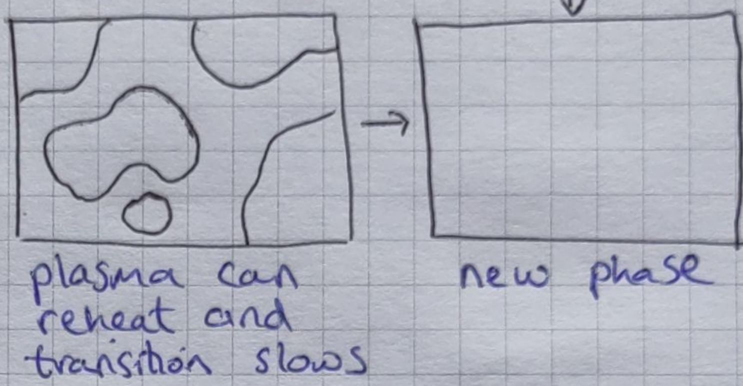
Detonation : $v_{wall} \gtrsim c_{sound} \simeq \frac{1}{\sqrt{3}}$



Evolution of phase transition depends strongly on solution type. N.B. if $\nexists$ solution which is static, the bubble may keep accelerating (runaway).

Large-scale simulations of many bubbles:





N.B. not all scales can be resolved, so this is a patchwork and work in progress.

Also, depending on the pattern of symmetry breaking, relics may form which can survive until today:
- baryons
- topological defects: cosmic strings, domain walls, monopoles
- magnetic fields
- some dark matter production mechanisms involve a phase transition.

All this can lead to different dynamics. More in later lectures.

~~The~~ Gravitational waves

$$G_{\mu\nu} = \frac{8\pi}{M_{Pl}^2} T_{\mu\nu}$$

e.g. FLRW — e.g. homogeneous average

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \quad T_{\mu\nu} = \bar{T}_{\mu\nu} + \delta T_{\mu\nu}$$

source fluctuation

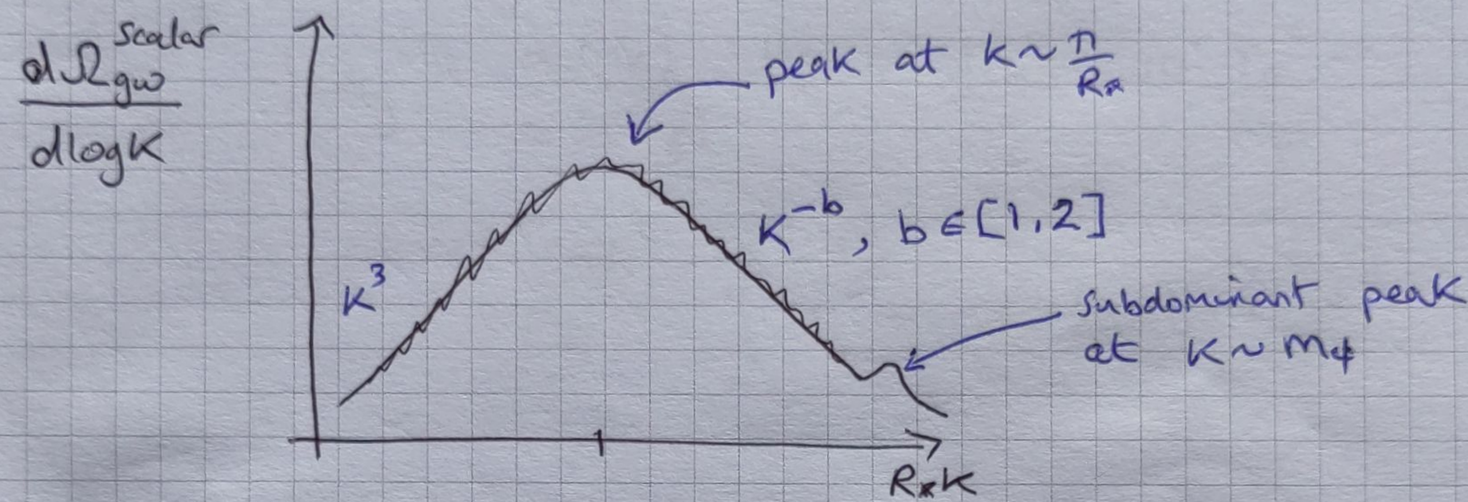
$$\Rightarrow \quad h_{\mu\nu}(x) = \frac{8\pi}{M_{Pl}^2} \int_y \underbrace{(G_R)_{\mu\nu\rho\sigma}(x,y)}_{\text{retarded Green function}} \overbrace{\delta T^{\rho\sigma}(y)}^{\text{source fluctuation}}$$

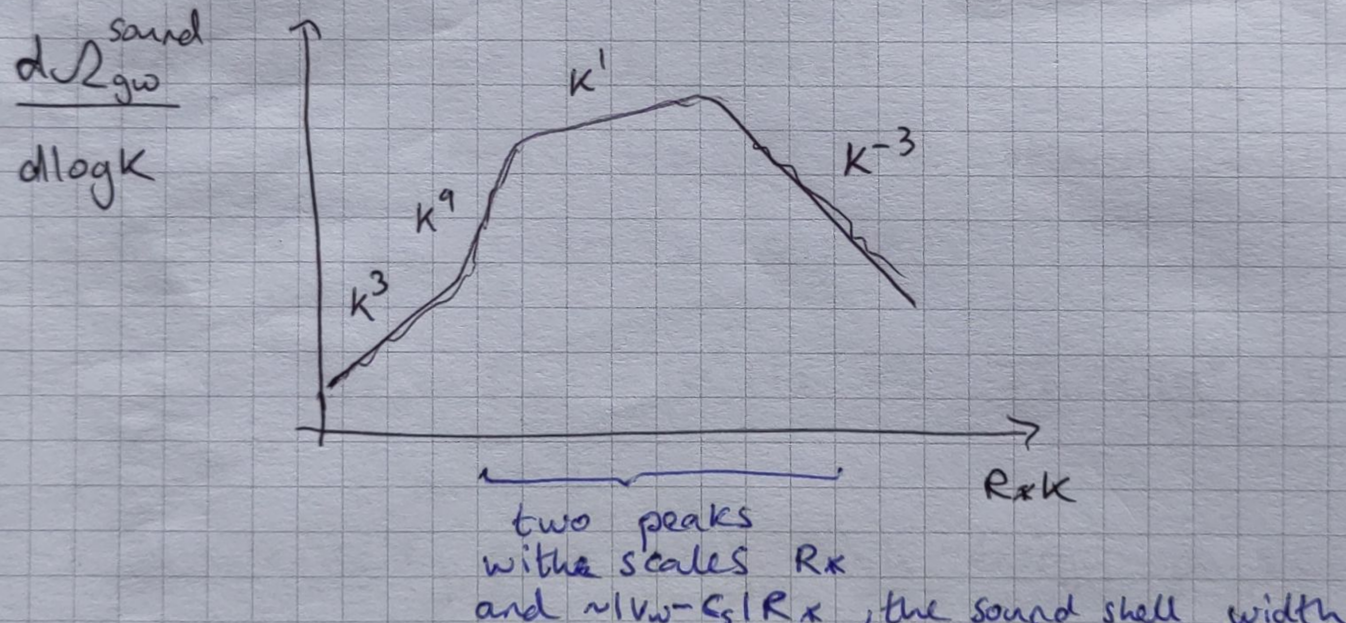
$$\rho_{gw} = \frac{M_{Pl}^2}{32\pi} \langle \dot{h}_{ij} \dot{h}_{ij} \rangle$$ } energy density in gravitational waves

From numerical simulations of phase transitions one gets $T_{\mu\nu} \longrightarrow h_{\mu\nu} \longrightarrow \rho_{gw}$, or $\Omega_{gw} = \dfrac{\rho_{gw}}{\rho_{tot}}$.

$$\Omega_{gw} \simeq \Omega_{gw}^{scalar} + \Omega_{gw}^{sound} + \Omega_{gw}^{turbulence} + \Omega_{gw}^{other}$$

After production spectra from different sources linearly superpose.





Still some uncertainty about these power laws, but the broad picture is expected to hold good.

Typical electroweak-like phase transitions have

$$\frac{d\Omega_{gw}^{sound}}{d\log k} \gg \frac{d\Omega_{gw}^{scalar}}{d\log k}$$

partly because the scalar field dynamics is relatively short lived.

Scalings and order of magnitude estimates.

Frequency

We denote by $*$ the time of production.

$$f_* \sim \frac{1}{R_*} \sim \underset{\substack{\uparrow \\ \text{ball park (really model dependent)}}}{10^2} H_* \sim 10^2 \frac{T_*^2}{M_{PL}}$$

Redshifting to today:

$$f_{today} \simeq \frac{a(T_*)}{a(T_{CMB})} f_* \sim \frac{T_{CMB}}{T_*} \cdot 10^2 \frac{T_*^2}{M_{pl}}$$

$$\sim \underbrace{\left(\frac{T_{CMB}}{M_{pl}}\right)}_{\text{generic small number}} \cdot \underset{\substack{\uparrow \\ \text{nucleation dependent}}}{10^2} \cdot \underset{\substack{\uparrow \\ \text{scale of transition}}}{T_*}$$

For an electroweak scale transition,

$$f_{today} \sim 10^{-2}\,\text{Hz} \Rightarrow \text{LISA optimum sensitivity!}$$

Note that $f_{today} \propto T_*$ so,

$$\underbrace{f_{LIGO} \sim f_{ET} \sim 10^2\,\text{Hz}}_{\text{sensitivity}} \Rightarrow T_* \sim 10^6\,\text{GeV} \sim 1\,\text{PeV}$$

so can potentially probe very high energy physics.

Amplitude

The retarded Green function for the Einstein operator can be split into a transverse-traceless projector and a scalar function.

The projection gives the shear stress,

$$\Pi_{ij} \equiv \Lambda_{ijkl} T_{kl}.$$

In a radiation dominated FLRW universe the scalar $f^{\underline{n}}$ is

$$\frac{\cos(k(t_1-t_2))}{t_1 t_2} ,$$ leading to

$$\frac{d\Omega_{gw}}{d\log k} \propto \int \frac{dt_1}{t_1} \int \frac{dt_2}{t_2} \cos(k(t_1-t_2)) \langle \Pi_{ij}(k,t_1)\Pi_{ij}(k,t_2)\rangle$$

$$\propto \left(\frac{L}{e}\right)^2 \cdot \left(t_{\text{lifetime}} H_*\right)\left(t_{\text{correlation}} H_*\right)$$

For a phase transition, ~~often~~ one typically expects

$$t_{\text{lifetime}} \sim t_{\text{correlation}} \sim 1/\beta .$$

Putting in rough numbers, from simulations, one finds,

$$\frac{d\Omega_{gw}^{\text{sound}}}{d\log k} \sim 10^{-5}\left(\frac{L}{e}\right)^2\left(\frac{H_*}{\beta}\right)^2$$

$$\sim 10^{-5}\cdot(10^{-2})^2\cdot(10^{-2})^2 \sim 10^{-13}$$

LISA is sensitive down to $\sim 10^{-12-13}$, Einstein Telescope too. More speculative detectors propose sensitivities $\sim 10^{-14-17}$ for AEDGE+, DECIGO, BBO.