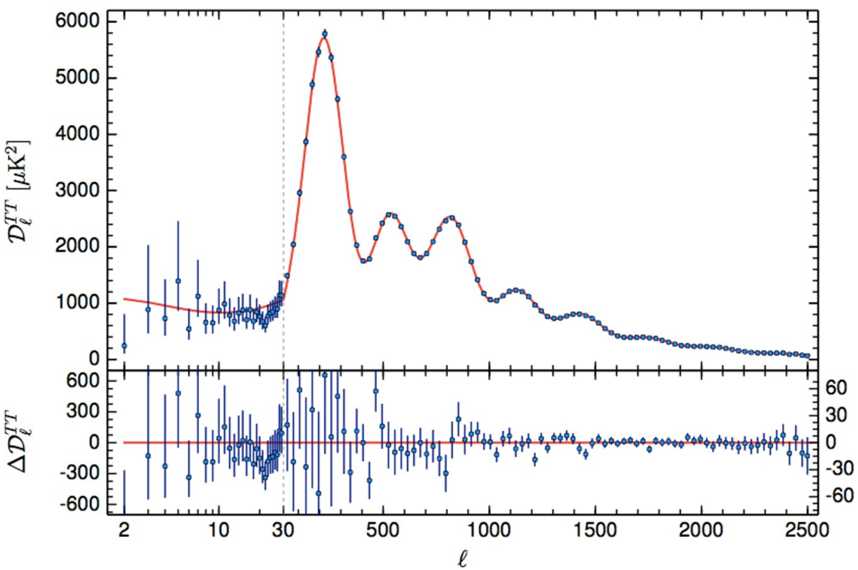
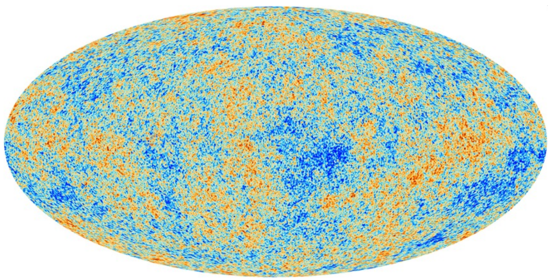
Deconstructing the CMB temperature power spectrum…



CMB temperature power spectrum…



CMB photon temperature fluctuations can be parameterised as:

$$T_\gamma(x^i, n^i, \eta) = \bar{T}_\gamma(\eta)[1 + \Theta(x^i, n^i, \eta)]$$

Position, direction, time

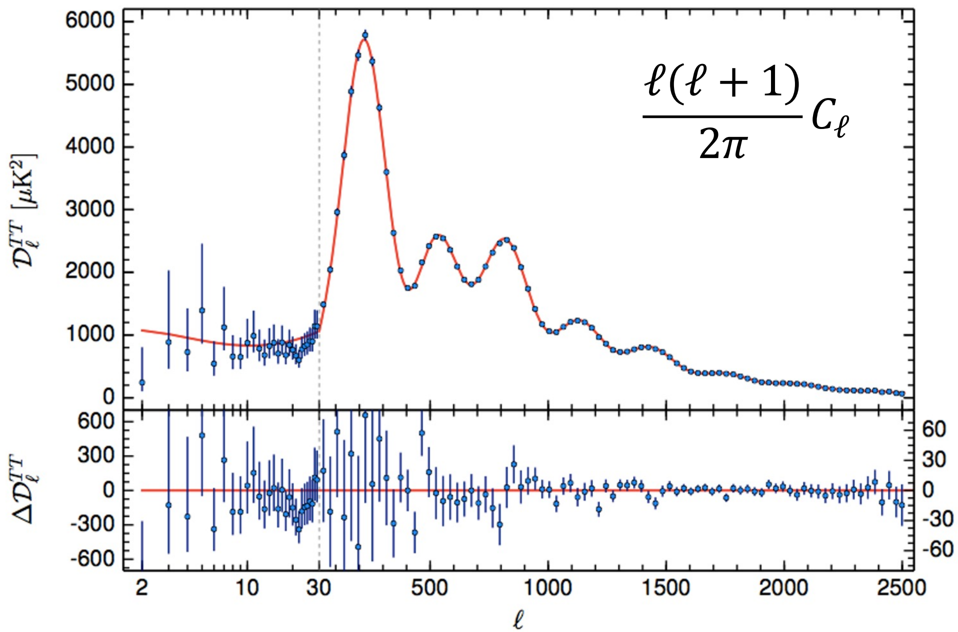
- We can only observe photons here and now (i.e., $\eta = \eta_0$) and map their temperatures on a 2D spherical map.

→ So it makes sense to decompose these fluctuations in terms of spherical harmonics $Y_{\ell m}(n^i)$.

$$\Theta(x^i, n^i, \eta_0) = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m}(x^i, \eta_0)\, Y_{\ell m}(n^i) \quad \Rightarrow \quad \langle a_{\ell m} a^*_{\ell m} \rangle = \delta_{\ell\ell'} \delta_{mm'} C_\ell$$

Temperature fluctuation power spectrum

CMB temperature power spectrum…



How do you **calculate C_ℓ from theory**?

k = Fourier wavenumber.

$$C_\ell = \frac{2}{\pi}\int d\,k\;k^2|\Theta_\ell(k,\eta_0)|^2$$

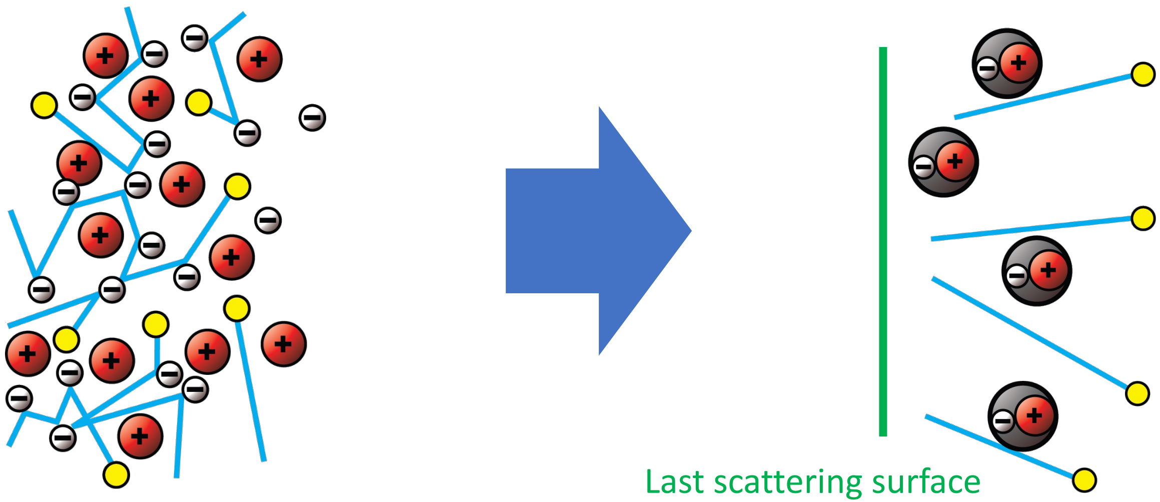
$\Theta(k,\eta_0)$ = Present-day Fourier-decomposed temperature fluctuation

… comes from solving numerically the Einstein-Boltzmann system.

But there is neat way to think about it that pretty much gets all the gross features of the power spectrum right.

Photon decoupling…

The most important event in the photon evolution history is decoupling ($T^* \sim 0.25$ eV, i.e., $z^* \sim 1100$ in most cosmological models).



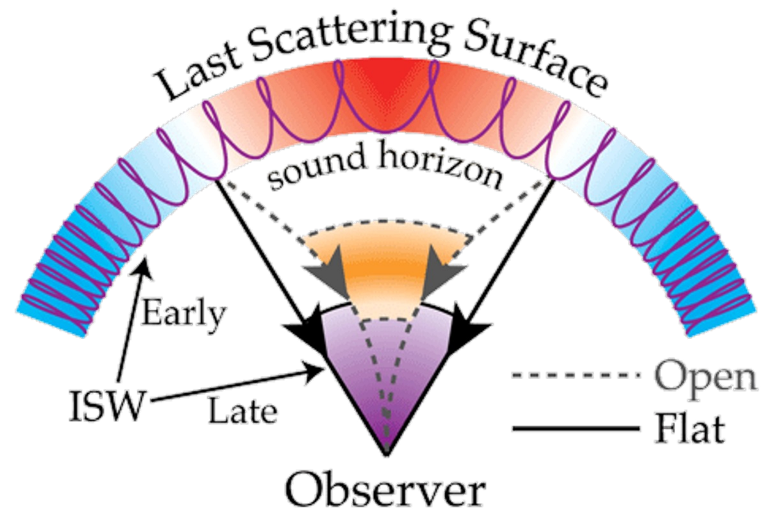
At $T > T^*$, photons scatter off free electrons efficiently, performing a random walk → the universe is opaque to photons

At $T < T^*$, electrons are bound in atoms; photons decouple and free-stream to infinity as if emanating from a last scattering surface.

CMB in two steps…

Relative to photon decoupling ($T^* \sim 0.25$ eV, $z^* \sim 1100$), CMB anisotropies can be understood in **two steps**:

- What happens **up to and at decoupling**?
  - Is the k mode superhorizon or subhorizon?
- What happens **after decoupling**?



5.1 Superhorizon up to the last scattering surface

From the Boltzmann hierarchy for photons, we have in the super-horizon limit ($k \ll \mathcal{H}$):

$$\dot{\delta}_\gamma = 4\dot{\Theta}_0^{(0)} \simeq 4\dot{\Phi}$$

$$\Rightarrow \dot{\Theta}_0^{(0)} = \dot{\Phi} \Rightarrow \Theta_0^{(0)}(k,\eta) = \Phi(k,\eta) + C$$

For adiabatic initial conditions:

$$\Theta_0^{(0)}(k, \eta \to 0) = -\frac{\Phi_p(k)}{2}$$

$$\Rightarrow \Theta_0^{(0)}(k,\eta) = \Phi(k,\eta) - \frac{3}{2}\Phi_p(k)$$

In ΛCDM-type cosmologies, photon decoupling happens during matter domination, i.e., $\eta_* > \eta_{eq}$. Thus,

$$\Phi(k,\eta_*) = \frac{9}{10}\Phi_p(k)$$

$$\Rightarrow \Theta_0^{(0)}(k,\eta_*) = \Phi(k,\eta_*) - \frac{\cancel{3}}{\cancel{2}}\left(\frac{\cancel{10}^{5}}{\cancel{9}_{3}}\right)\Phi(k,\eta_*)$$

$$= -\frac{2}{3}\Phi(k,\eta_*)$$

Using $\Psi \simeq \Phi$; we find

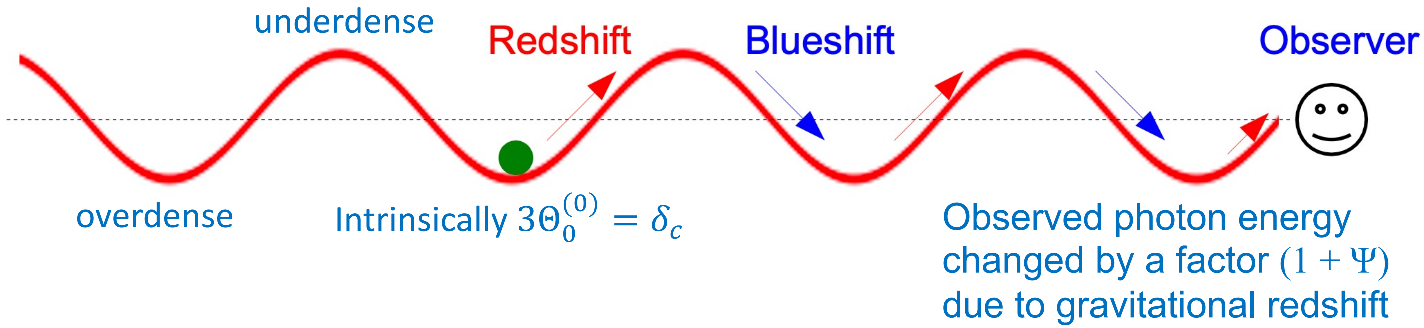
$$\boxed{\begin{aligned}(\Theta_0^{(0)} + \Psi)(k \ll \mathcal{H}_*, \eta_*) &= \frac{1}{3}\Psi(k,\eta_*) \\ \text{using Einstein's eqn} \longrightarrow\quad &= -\frac{1}{6}\delta_c(k,\eta_*)\end{aligned}}$$

Superhorizon up to the LSS…

At photon decoupling η_*, the **effective CMB temperature** perturbation on superhorizon scales is related to the CDM density perturbation via

$$\boxed{\left(\Theta_0^{(0)} + \Psi\right)(k \ll \mathcal{H}, \eta_*) \simeq -\frac{1}{6}\delta_c(k, \eta_*)}$$

→ An observed CMB hot spot corresponds to an **underdense region** (true only for adiabatic initial conditions).



Subhorizon up to the LSS…

Here the photons and baryons are tightly coupled because of Compton scattering, i.e., $\dot{\kappa} \gg \mathcal{H}$, $\rightarrow$

- Leads to **acoustic oscillations**.
- To get acoustic oscillations with baryon loading, the relevant equations are:

$$\dot{\delta}_b + k v_b^{(0)} - 3\dot{\Phi} = 0$$

$$\dot{v}_b^{(0)} + \mathcal{H} v_b^{(0)} - k\Phi = \frac{\dot{\kappa}}{R}\left(v_\gamma^{(0)} - v_b^{(0)}\right)$$

$$\frac{1}{R} \equiv \frac{4}{3}\frac{\bar{\rho}_\gamma}{\bar{\rho}_b} = \left[\frac{3}{4}\frac{\Omega_b h^2}{\Omega_\gamma h^2} a\right]^{-1}$$

baryons

$$\dot{\delta}_\gamma + \frac{4}{3} k v_\gamma^{(0)} - 4\dot{\Phi} = 0$$

Photons

$$\dot{v}_\gamma^{(0)} - k\left[\frac{1}{4}\delta_\gamma + \Phi\right] = -\dot{\kappa}\left(v_\gamma^{(0)} - v_b^{(0)}\right)$$

5.2 Subhorizon up to decoupling

For those k modes that are subhorizon before photon decoupling, the tightly-coupled limit applies before $\dot{\kappa} \gg \mathcal{H}$. As we approach decoupling, however, we also need to consider $\mathcal{H}/\dot{\kappa}$ effects, in order to see how baryons affect the photon perturbations.

The relevant equations are:

photons
$$\begin{cases} \dot{\delta}_\gamma = -\frac{4}{3} k v_\gamma^{(0)} + 4\dot{\Phi} \\ \dot{v}_\gamma^{(0)} = k\left[\frac{1}{4}\delta_\gamma + \Psi\right] - \dot{\kappa}\left(v_\gamma^{(0)} - v_b^{(0)}\right) \end{cases}$$

baryons
$$\begin{cases} \dot{\delta}_b + k v_b^{(0)} - 3\dot{\Phi} = 0 \\ \dot{v}_b^{(0)} + \mathcal{H} v_b^{(0)} - k\Psi = -\frac{\dot{\kappa}}{R}\left(v_b^{(0)} - v_\gamma^{(0)}\right) \quad (*) \end{cases}$$

where

$$\frac{1}{R} \equiv \frac{4}{3}\frac{\bar{\rho}_\gamma}{\bar{\rho}_b}$$

|| $R =$ baryon-to-photon energy density ratio

$$= \left[\frac{3}{4}\frac{\Omega_b h^2}{\Omega_\gamma h^2} a\right]^{-1} \sim 10^7 a^{-1}$$

First rewrite $(*)$ as

$$v_b^{(0)} = v_\gamma^{(0)} - \frac{R}{\dot{\kappa}}\left[\dot{v}_b^{(0)} - k\Psi + \mathcal{H} v_b^{(0)}\right]$$

$$= v_\gamma^{(0)} - R\frac{\mathcal{H}}{\dot{\kappa}}\left[\frac{1}{\mathcal{H}}\dot{v}_b^{(0)} - \frac{k}{\mathcal{H}}\Psi + v_b^{(0)}\right]$$

In the tightly-coupled limit:

$$v_b^{(0)} = v_\gamma^{(0)} + O\left(\frac{\mathcal{H}}{\dot{\kappa}}\right)$$

Therefore:

$$v_b^{(0)} \simeq v_\gamma^{(0)} - R\frac{\mathcal{H}}{\dot{\kappa}}\left[\frac{1}{\mathcal{H}}\dot{v}_\gamma^{(0)} - \frac{k}{\mathcal{H}}\Psi + v_\gamma^{(0)}\right] + O\left(\frac{\mathcal{H}^2}{\dot{\kappa}^2}\right) \quad (A)$$

Then, (A) can now be inserted into the v_γ equation:

$$\dot{v}_\gamma^{(0)} = k\left[\frac{1}{4}\delta_\gamma + \Psi\right] - R\left[\dot{v}_\gamma^{(0)} - k\Psi + \mathcal{H}v_\gamma^{(0)}\right]$$

$$\Rightarrow (1+R)\dot{v}_\gamma^{(0)} = \frac{k}{4}\delta_\gamma - R\mathcal{H}v_\gamma^{(0)} + (1+R)k\Psi$$

$$= \frac{k}{4}\delta_\gamma - 3R\frac{\mathcal{H}}{k}\left[\dot{\Phi} - \frac{1}{4}\dot{\delta}_\gamma\right] + (1+R)k\Psi \quad (*)$$

Furthermore:

$$\dot{\delta}_\gamma = -\frac{4}{3}kv_\gamma^{(0)} + 4\dot{\Phi}$$

We can substitute $\dot{v}_\gamma^{(0)}$ from (*) into this to get

$$\ddot{\delta}_\gamma + \frac{R}{1+R}\mathcal{H}\dot{\delta}_\gamma + \frac{k^2}{3}\frac{1}{1+R}\delta_\gamma$$

$$= 4\ddot{\Phi} + \frac{4R}{1+R}\mathcal{H}\dot{\Phi} - \frac{4}{3}k^2\Psi$$

Lastly, since $R \propto a$, we must also have

$$\frac{\dot{R}}{R} = \frac{\dot{a}}{a} = \mathcal{H}$$

$$\Rightarrow \quad R\mathcal{H} = \dot{R}$$

Thus, the final equation for δ_γ is:

$$\boxed{\ddot{\delta}_\gamma + \frac{\dot{R}}{1+R}\dot{\delta}_\gamma + \frac{k^2}{3}\frac{1}{1+R}\delta_\gamma = 4\ddot{\Phi} + 4\frac{\dot{R}}{1+R}\dot{\Phi} - \frac{4}{3}k^2\Psi}$$

or in terms of the monopole temperature:

$$\delta_\gamma = 4\Theta_0^{(0)}$$

$$\boxed{\ddot{\Theta}_0^{(0)} + \frac{\dot{R}}{1+R}\dot{\Theta}_0^{(0)} + \frac{k^2}{3}\frac{1}{1+R}\Theta_0^{(0)} = \ddot{\Phi} + \frac{\dot{R}}{1+R}\dot{\Phi} - \frac{k^2}{3}\Psi}$$

This is a driven and damped harmonic oscillator with angular frequency $\omega = k c_s$, where

$$\boxed{c_s \equiv \sqrt{\frac{1}{3(1+R)}}}$$

sound speed of the tightly-coupled photon-baryon fluid.

Subhorizon up to the LSS…

Before decoupling, Compton scattering ensures that photon and baryons system form a tightly-coupled fluid.

There is a similar equation for the dipole.

- Equation of motion for the monopole in this limit:

$$\ddot{\Theta}_0^{(0)} + \frac{\dot{R}}{1+R}\dot{\Theta}_0^{(0)} + k^2 c_s^2 \Theta_0^{(0)} = \ddot{\Phi} + \frac{\dot{R}}{1+R}\dot{\Phi} - \frac{k^2}{3}\Psi$$

- A **damped** and **driven** harmonic oscillator with sound speed:

$$c_s^2 \equiv \frac{1}{3(1+R)} \qquad R \equiv \frac{3}{4}\frac{\Omega_b h^2}{\Omega_\gamma h^2} a$$

Baryon-to-photon ratio

→ The presence of baryons lowers the fluid sound speed.

Approximate solution

Suppose Ψ and Φ are constant in time (can be justified during matter domination), and consider the limit

$$k^2 c_s^2 \gg \left(\frac{\dot{R}}{1+R}\right)^2 \quad (*)$$

or equivalently:

$$\frac{k^2}{3(1+R)} \gg \left(\frac{R}{1+R}\right)^2 \mathcal{H}^2$$

easy to satisfy on subhorizon scales

Then, the DE simplifies to

$$\ddot{\Theta}_0^{(0)} + k^2 c_s^2 \Theta_0^{(0)} = -\frac{k^2}{3}\Psi$$

The same condition (*) also allows for a WKB solution:

$$(\Theta_0 + \Psi)(k,\eta) = C_1 \sin(k r_s) + C_2 \cos(k r_s) - R\Psi$$

for the monopole, and using $k\Theta_1^{(0)} = \dot{\Phi} - \dot{\Theta}_0^{(0)}$, we can construct a solution for the dipole:

$$\Theta_1^{(0)}(k,\eta) = -c_s C_1 \cos(k r_s) + c_s C_2 \sin(k r_s).$$

Here, we have defined

$$\boxed{r_s(\eta) \equiv \int_0^\eta d\eta' \, c_s(\eta')}$$

comoving sound horizon

i.e., the coordinate distance travelled by a sound wave since $\eta = 0$. The comoving sound horizon is perhaps the most important quantity in CMB physics!

We saw earlier that during MD, a superhorizon k mode has solution

$$(\Theta_0^{(0)} + \Psi)(k \ll \mathcal{H}, \eta) = \frac{1}{3}\Phi(k,\eta) = \text{constant}$$

Our WKB solution must also respect this finding in the limit $kr_s \to 0$. This immediately means

$$C_1 = 0$$
$$C_2 \, [\Theta_0^{(0)} + (1+R)\Psi](k, \eta \to 0)$$

Thus, the final solution is

$$\boxed{\begin{aligned} [\Theta_0^{(0)} + \Psi](k,\eta) &= [\Theta_0^{(0)} + (1+R)\Psi](k,0)\cos(kr_s) \\ &\quad - R\Psi \\ \Theta_1^{(0)}(k,\eta) &= c_s[\Theta_0^{(0)} + (1+R)\Psi](k,0)\sin(kr_s) \end{aligned}}$$

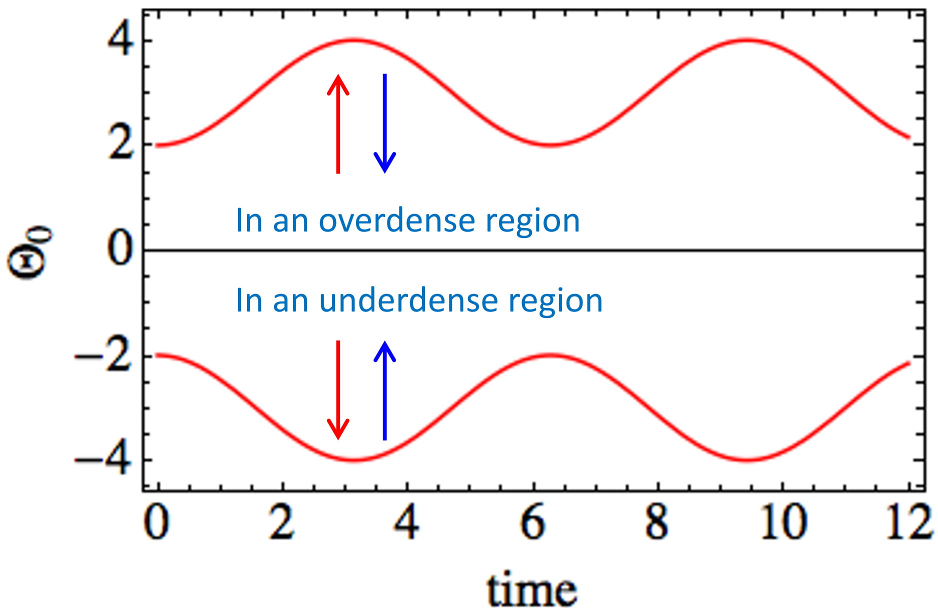
Only the cosine mode is excited in the monopole solution, while the dipole solution contains only the sine mode. This is a consequence of the adiabatic initial conditions. If the initial conditions had been a mixture of adiabatic and isocurvature modes, then in general the sine mode would have been excited too in the monopole solution.

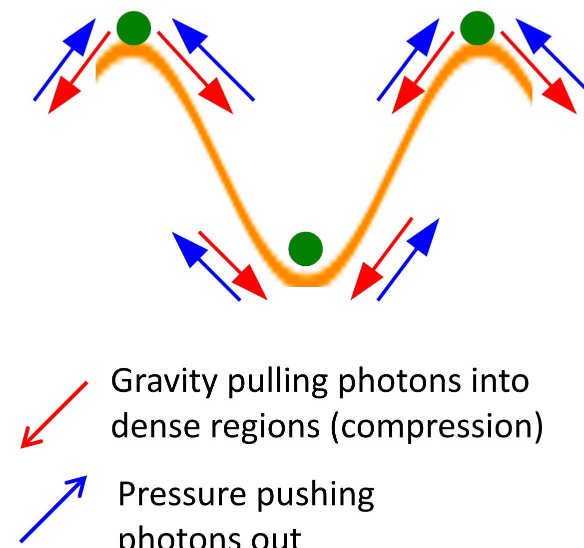
Acoustic oscillations: monopole…

Suppose for now baryons are negligible: $R = 0$.

Assuming constant Φ and Ψ.

- Take a fixed Fourier k-mode and see how it evolves in time → acoustic oscillations





Acoustic oscillations: monopole & dipole...

Assuming constant Φ and Ψ.

Suppose for now baryons are negligible: $R = 0$.

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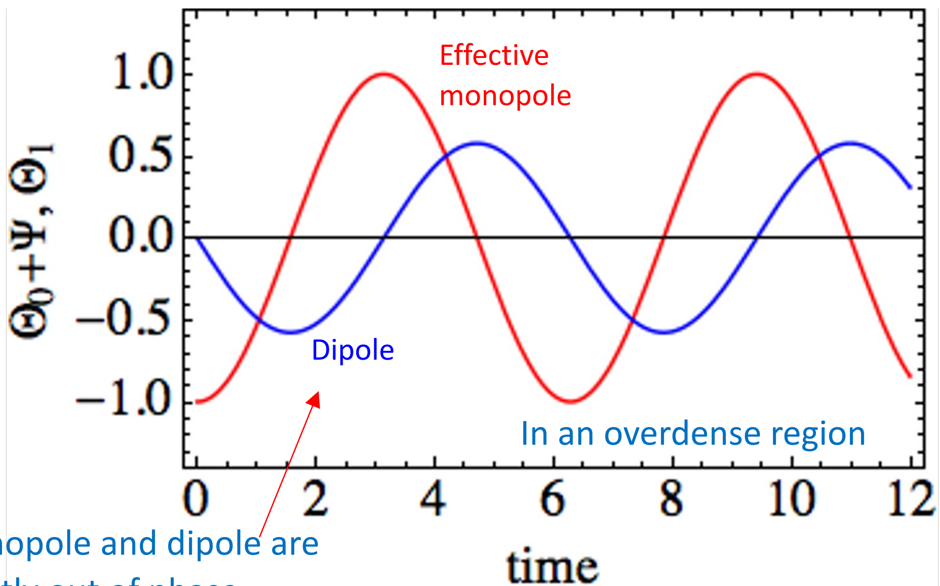
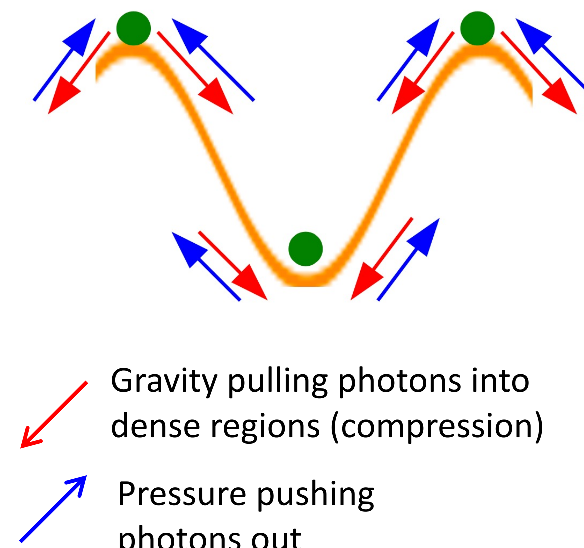


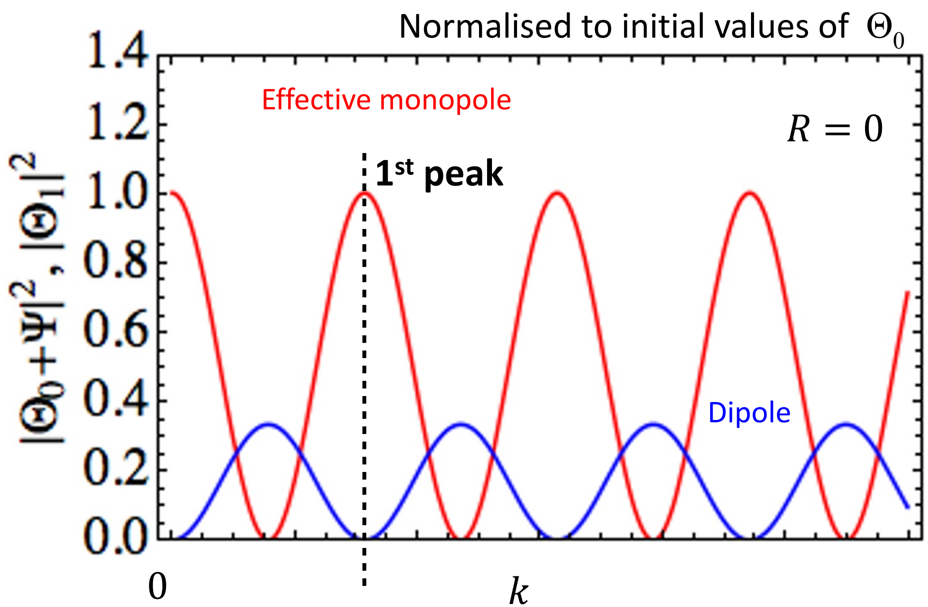
Monopole and dipole are exactly out of phase.



Acoustic oscillations: monopole & dipole…

Monopole and dipole spectra at decoupling η_*:

Assuming constant Φ and Ψ.



Peak positions:

$$k_{\text{peak}} = \frac{n\pi}{r_s(\eta_*)}, n = 1,2,3,\ldots$$

$$r_s(\eta_*) \equiv \int_0^{\eta_*} d\eta' c_s(\eta')$$

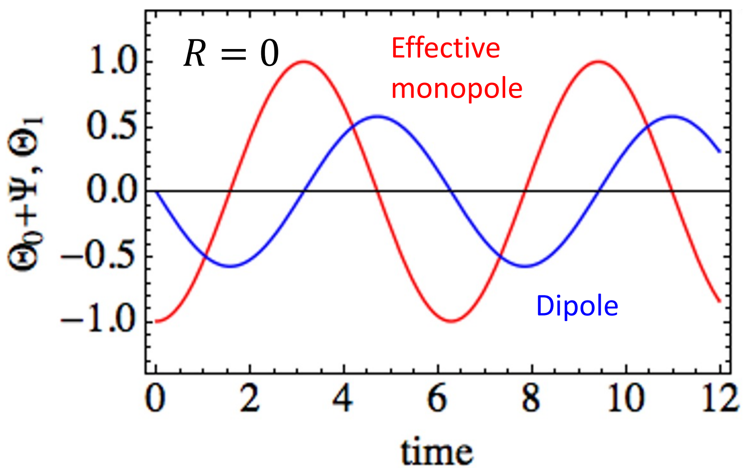
Sound horizon at decoupling
= Coordinate distance travelled
by a sound wave since time $\eta = 0$.

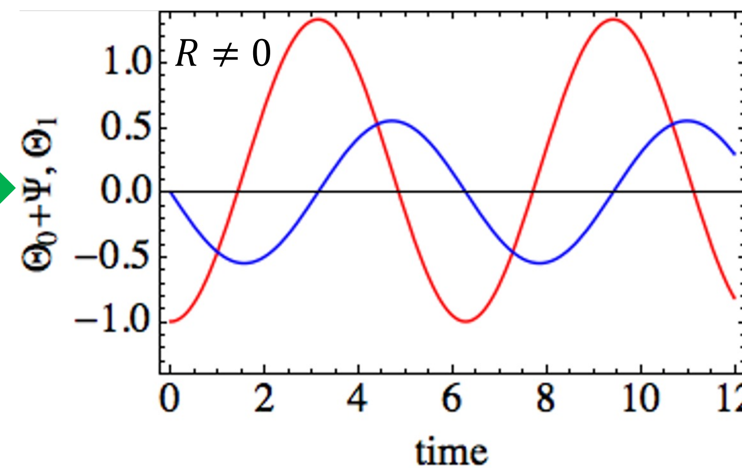
- Position of 1st peak corresponds to the k mode that has completed **exactly one compression** at photon decoupling.

Acoustic oscillations: add baryons...

Now let's **put the baryons back** in, i.e., $R \neq 0$.

Assuming constant Φ and Ψ.



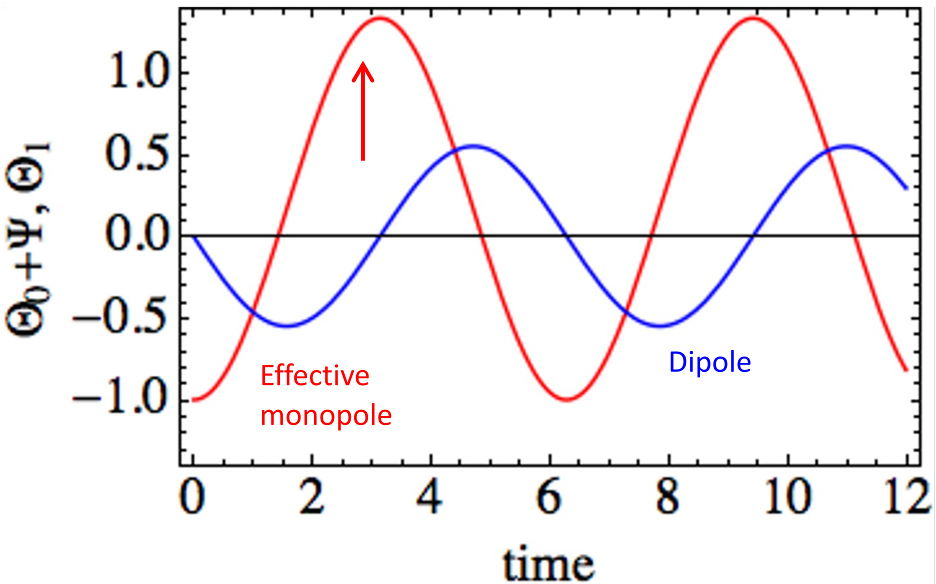


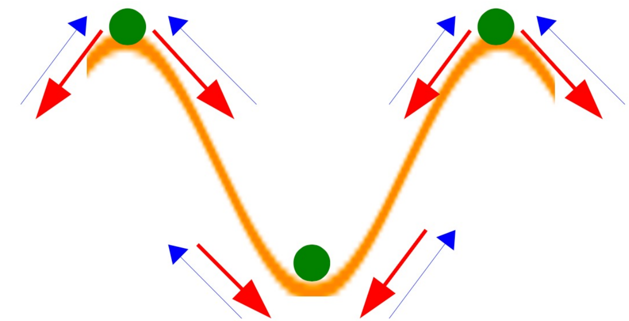
- The presence of baryons offsets the midpoint of acoustic oscillations for the **effective monopole**, reduces the **sound horizon**, and alters the oscillation **amplitudes** (monopole and dipole).

Acoustic oscillations: add baryons...

Assuming constant Φ and Ψ.

The **physical reason** is straightforward:





Gravity pulling photons into dense regions (compression)

Pressure pushing photons out

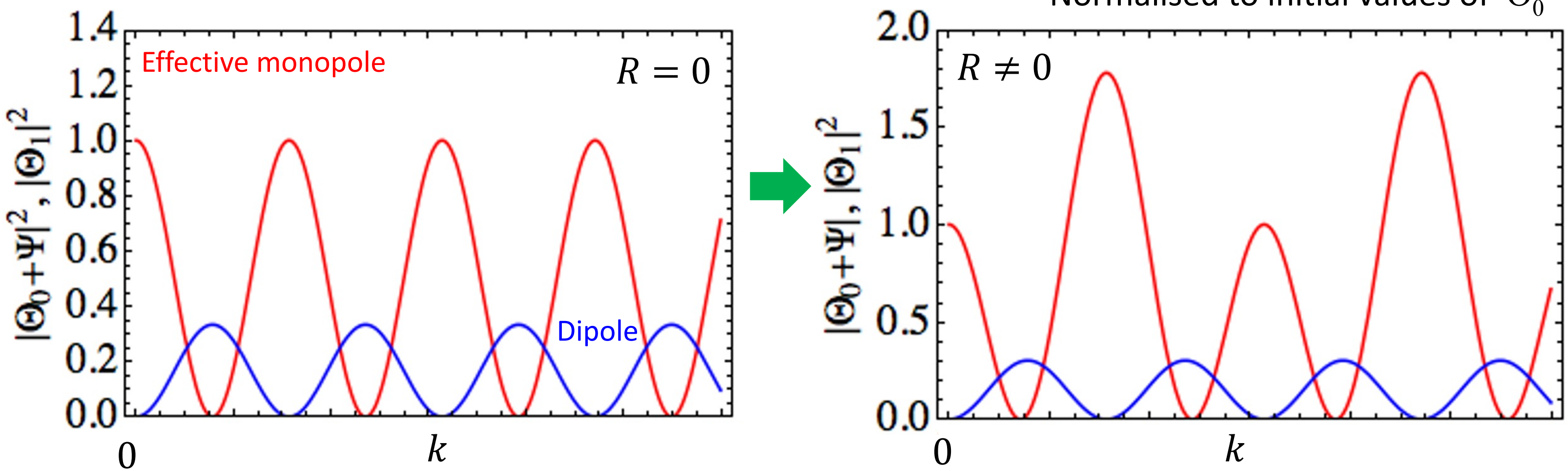
- A **reduced sound speed** due to baryon inertia leads to less pressure resistance → the photon are compressed more and become hotter.

Acoustic oscillations on the LSS…

Assuming constant Φ and Ψ.

$\boldsymbol{R \neq 0}$ **monopole and dipole spectra** at decoupling η_* :

Normalised to initial values of Θ_0

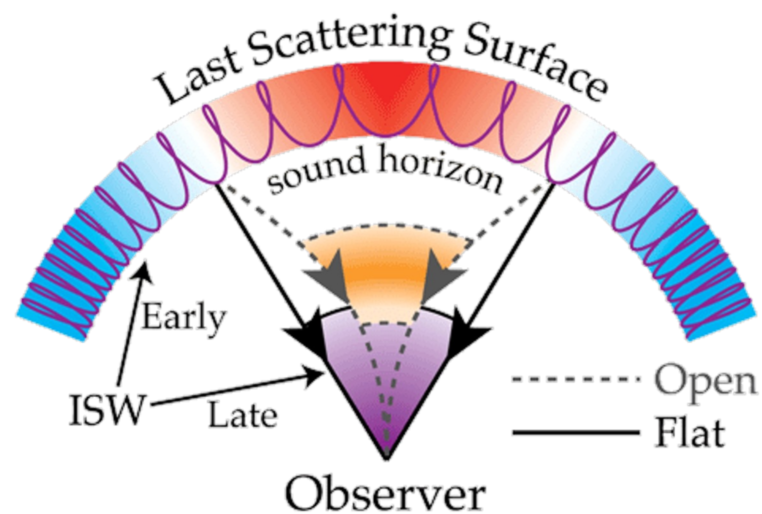


- Odd and even peaks how have different heights, where the **height ratio** depends on the baron-to-photon ratio R.
- Essential features remain even for time-dependent Φ and Ψ.

CMB in two steps…

Relative to photon decoupling ($T^* \sim 0.25$ eV, $z^* \sim 1100$), CMB anisotropies can be understood in **two steps**:

- What happens **up to and at decoupling**?
  - Is the k mode superhorizon or subhorizon?
- What happens **after decoupling**?



5.3 After decoupling

After decoupling, photons free stream. We saw earlier that the Boltzmann equation for the photon temperature fluctuation is:

$$\frac{\partial \Theta^{(0)}}{\partial \eta} + n^i \partial_i \Theta^{(0)} + \text{gravitational source} = \text{Collisions}$$

In k-space, this is equivalently:

$$\frac{\partial \Theta^{(0)}}{\partial \eta} + i k_i \overset{\text{photon direction}}{n^i} \Theta^{(0)} + \ldots = \ldots$$

Thus, free-streaming corresponds to a solution

$$\Theta^{(0)}(k^i, \eta, n^i) \sim \Theta^{(0)}(\eta_*) \exp\left[-i k_i n^i (\eta - \eta_*)\right]$$

monopole + dipole solution on the last scattering surface.

Decomposing to multipoles:

$$\Theta^{(0)}(k^i, \eta, n^i) = \sum_{l=0}^{\infty} (2l+1) i^l \Theta_l(k, \eta) \underbrace{P_l(\hat{k} \cdot \hat{n})}$$

Legendre polynomials $\sim m=0$ spherical harmonics

Then, the free-streaming solution for $\Theta_\ell(k,\eta)$ is equivalently

$$\boxed{\Theta_\ell^{(0)}(k,\eta) \sim \Theta^{(0)}(\eta_*)\, j_\ell[k(\eta-\eta_*)]}$$

where $j_\ell(x)$ is a spherical Bessel function of degree ℓ, arising from the plane wave expansion

$$e^{i\underline{k}\cdot\underline{x}} = \sum_{\ell=0}^{\infty} (2\ell+1)\, i^\ell j_\ell(kx)\, P_\ell(\hat{k}\cdot\hat{x})$$

That was a schematic solution. A proper calculation actually yields

$$\Theta_\ell^{(0)}(k,\eta) \simeq \left[\Theta_0^{(0)}(k,\eta_*) + \Psi_0^{(0)}(k,\eta_*)\right] j_\ell[k(\eta-\eta_*)] - \frac{3}{k}\Theta_1^{(0)}(k,\eta_*)\frac{d}{d\eta} j_\ell[k(\eta-\eta_*)]$$

After decoupling…

Photon free-streaming spreads the monopole and dipole solutions on the last scattering surface to all multipoles ℓ.

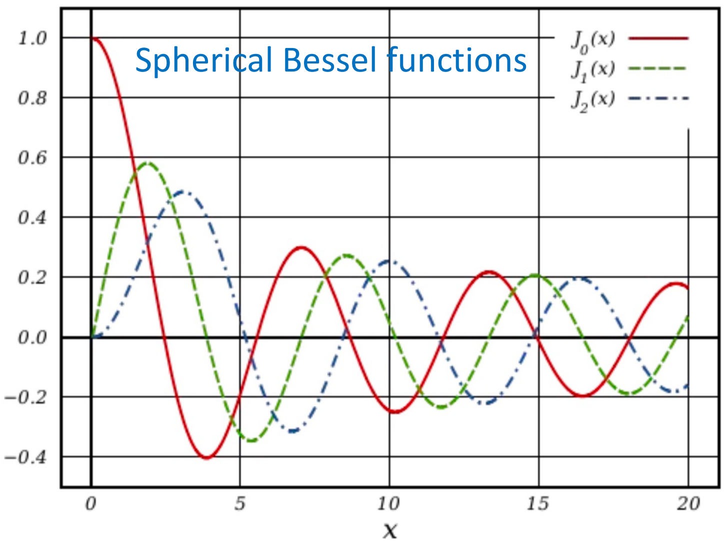
$$\Theta_\ell^{(0)}(k,\eta_0) \simeq \left[\Theta_0^{(0)}(k,\eta_*) + \Psi_0^{(0)}(k,\eta_*)\right] j_\ell[k(\eta_0-\eta_*)] \quad \text{monopole}$$

$$-\frac{3}{k}\Theta_1^{(0)}(k,\eta_*)\frac{d}{d\eta} j_\ell[k(\eta_0-\eta_*)] \quad \text{dipole}$$

After decoupling…

Photon free-streaming spreads the monopole and dipole solutions on the last scattering surface to all multipoles ℓ.

$$\Theta_\ell^{(0)}(k,\eta_0) \simeq \left[\Theta_0^{(0)}(k,\eta_*) + \Psi_0^{(0)}(k,\eta_*)\right] j_\ell[k(\eta_0-\eta_*)]$$ monopole



- $j_\ell(x)$ peaks at $x \sim \ell$ (not exactly though)

→ $\Theta_\ell^{(0)}(k,\eta_0)$ gets most contribution from k modes satisfying

$$\boxed{k \sim \frac{\ell}{\eta_0 - \eta_*} = \frac{\ell}{\chi_*}}$$

χ_* = Comoving distance to the LSS

After decoupling…

Photon free-streaming spreads the monopole and dipole solutions on the last scattering surface to all multipoles ℓ.

$$\Theta_\ell^{(0)}(k,\eta_0) \simeq \left[\Theta_0^{(0)}(k,\eta_*) + \Psi_0^{(0)}(k,\eta_*)\right] j_\ell[k(\eta_0-\eta_*)]$$ monopole

Where should we expect to find the acoustic peaks?

$$\ell_{\rm peak} = k_{\rm peak}\,\chi_*$$

$$= \frac{n\pi\chi_*}{r_s(\eta_*)}$$

Comoving sound horizon up to the LSS

- $j_\ell(x)$ peaks at $x \sim \ell$ (not exactly though)

→ $\Theta_\ell^{(0)}(k,\eta_0)$ gets most contribution from k modes satisfying

$$\boxed{k \sim \frac{\ell}{\eta_0-\eta_*} = \frac{\ell}{\chi_*}}$$

χ_* = Comoving distance to the LSS

But there is more: the ISW effect…

The **Integrated Sachs-Wolfe effect** is an additional contribution from time-dependent potentials.

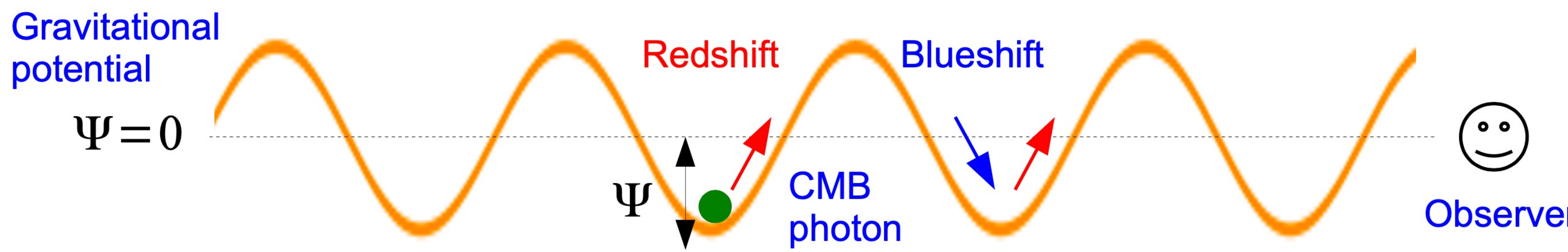
$$\Theta_\ell^{(0)}(k,\eta_0) \simeq \left[\Theta_0^{(0)}(k,\eta_*) + \Psi_0^{(0)}(k,\eta_*)\right] j_\ell[k(\eta_0-\eta_*)] \quad \text{monopole}$$

$$-\frac{3}{k}\Theta_1^{(0)}(k,\eta_*)\frac{d}{d\eta} j_\ell[k(\eta_0-\eta_*)] \quad \text{dipole}$$

$$+\int_0^{\eta_0} d\,\eta e^{-|\kappa|(\eta)}\left[\dot{\Psi}(k,\eta)+\dot{\Phi}(k,\eta)\right] j_\ell[k(\eta_0-\eta)] \quad \text{ISW}$$

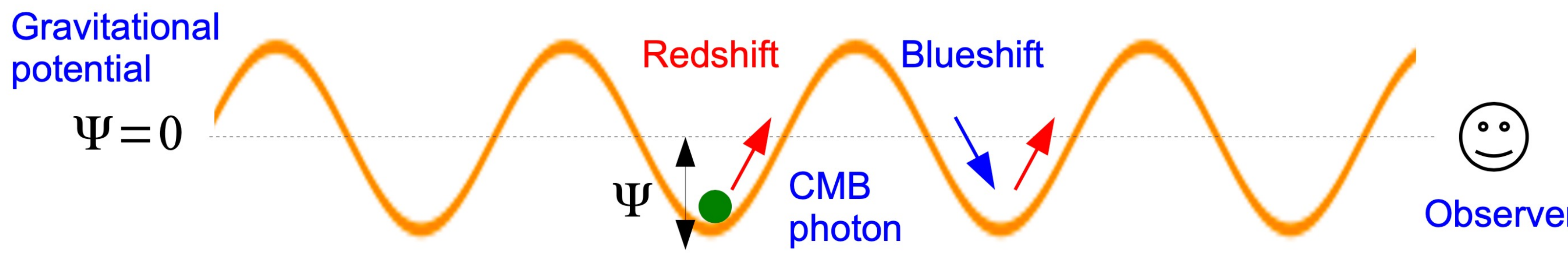
Sachs-Wolfe effect:

$$\frac{\Delta T}{T}_{\text{observed}} = \frac{\Delta T}{T}_{\text{intrinsic}} + \Psi$$

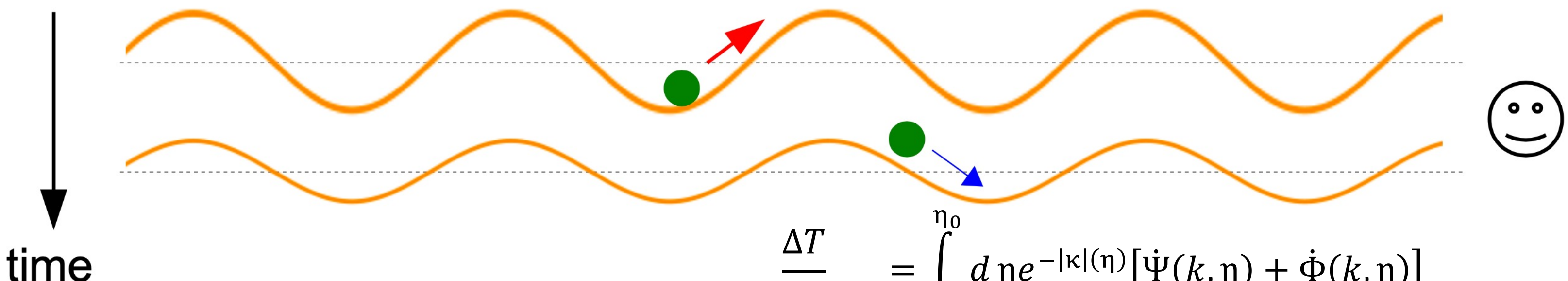


Sachs-Wolfe effect:

$$\frac{\Delta T}{T}_{\text{observed}} = \frac{\Delta T}{T}_{\text{intrinsic}} + \Psi$$



Integrated Sachs-Wolfe effect (potential decay after decoupling):



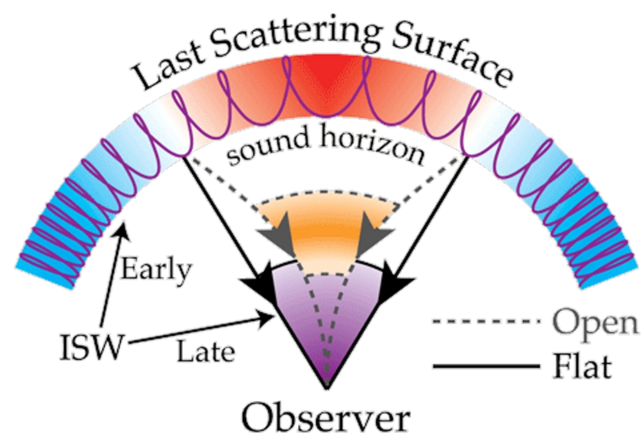
$$\frac{\Delta T}{T}_{\text{ISW}} = \int_0^{\eta_0} d\,\eta e^{-|\kappa|(\eta)} \left[\dot{\Psi}(k,\eta) + \dot{\Phi}(k,\eta)\right]$$

- In the time-dependent case, photons suffer less gravitational redshift than in the case of constant Φ and Ψ → **Larger** observed temperature fluctuation

Early and late ISW…

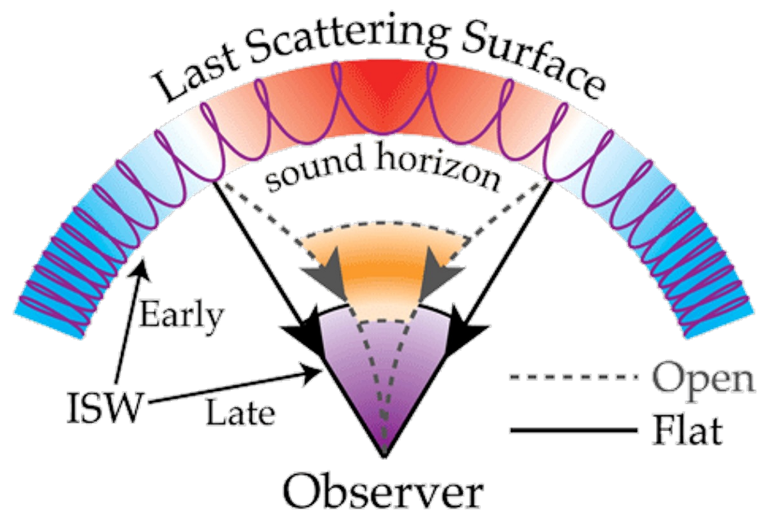
Except deep in matter domination, the ISW effect is always present.

- **Early ISW effect**: due to transition from radiation to matter domination
  - Effects mainly around the first acoustic peak
- **Late ISW effect**: due to transition from matter to dark energy domination.
  - Contributions mainly left of first peak

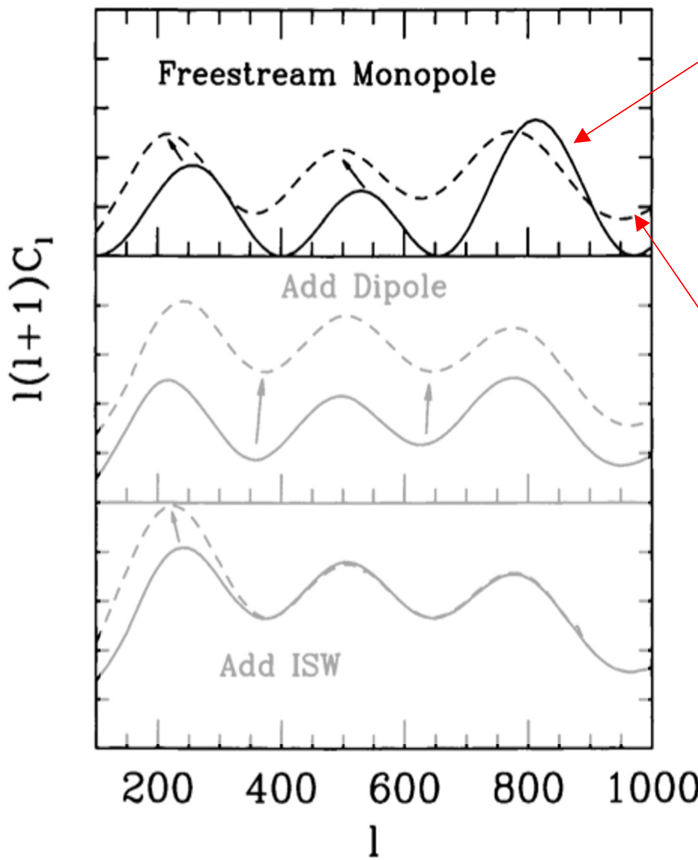


Let’s put it back together…

$$\Theta_\ell^{(0)}(k,\eta_0) \simeq \left[\Theta_0^{(0)}(k,\eta_*) + \Psi_0^{(0)}(k,\eta_*)\right] j_\ell[k(\eta_0-\eta_*)]$$
$$-\frac{3}{k}\Theta_1^{(0)}(k,\eta_*)\frac{d}{d\eta} j_\ell[k(\eta_0-\eta_*)]$$
$$+\int_0^{\eta_0} d\,\eta e^{-|\kappa|(\eta)}\left[\dot{\Psi}(k,\eta)+\dot{\Phi}(k,\eta)\right] j_\ell[k(\eta_0-\eta)]$$



CMB TT power spectrum deconstructed…



Naïve projection (peak of the spherical Bessel function)

$$\left[\Theta_0 + \Psi\right](k = \ell/(\eta_0 - \eta_*), \eta_*)$$

Position of the first peak

$$\ell_{\text{peak}} \sim \frac{\pi \chi_*}{r_s}$$

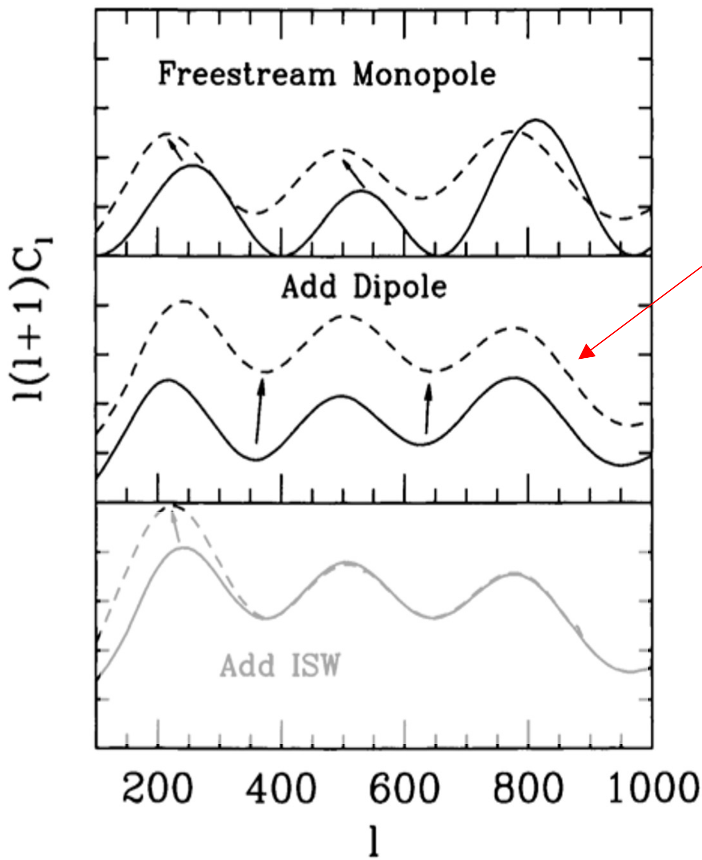
Comoving distance to the LSS

Comoving sound horizon up to the LSS

Proper **free-streaming** (full spherical Bessel function) in fact shifts peaks a little from their naïve positions.

$$\Theta_\ell^{(0)}(k,\eta_0) \simeq \left[\Theta_0^{(0)}(k,\eta_*) + \Psi_0^{(0)}(k,\eta_*)\right] j_\ell[k(\eta_0-\eta_*)]$$
$$-\frac{3}{k}\Theta_1^{(0)}(k,\eta_*)\frac{d}{d\eta} j_\ell[k(\eta_0-\eta_*)]$$
$$+\int_0^{\eta_0} d\eta\, e^{-|\kappa|(\eta)}\left[\dot{\Psi}(k,\eta)+\dot{\Phi}(k,\eta)\right] j_\ell[k(\eta_0-\eta)]$$

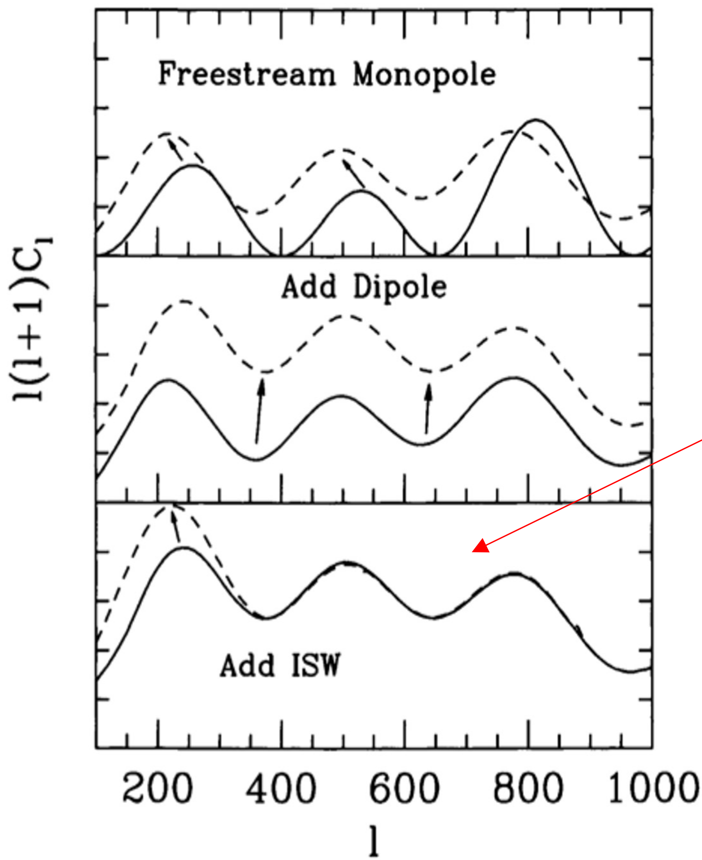
CMB TT power spectrum deconstructed…



Monopole and dipole add incoherently (because of property of spherical Bessel function); adding dipole makes the troughs less prominent.

$$\Theta_\ell^{(0)}(k,\eta_0) \simeq \left[\Theta_0^{(0)}(k,\eta_*) + \Psi_0^{(0)}(k,\eta_*)\right] j_\ell[k(\eta_0-\eta_*)]$$
$$-\frac{3}{k}\Theta_1^{(0)}(k,\eta_*)\frac{d}{d\eta}j_\ell[k(\eta_0-\eta_*)]$$
$$+\int_0^{\eta_0} d\,\eta e^{-|\kappa|(\eta)}[\dot{\Psi}(k,\eta)+\dot{\Phi}(k,\eta)]j_\ell[k(\eta_0-\eta)]$$

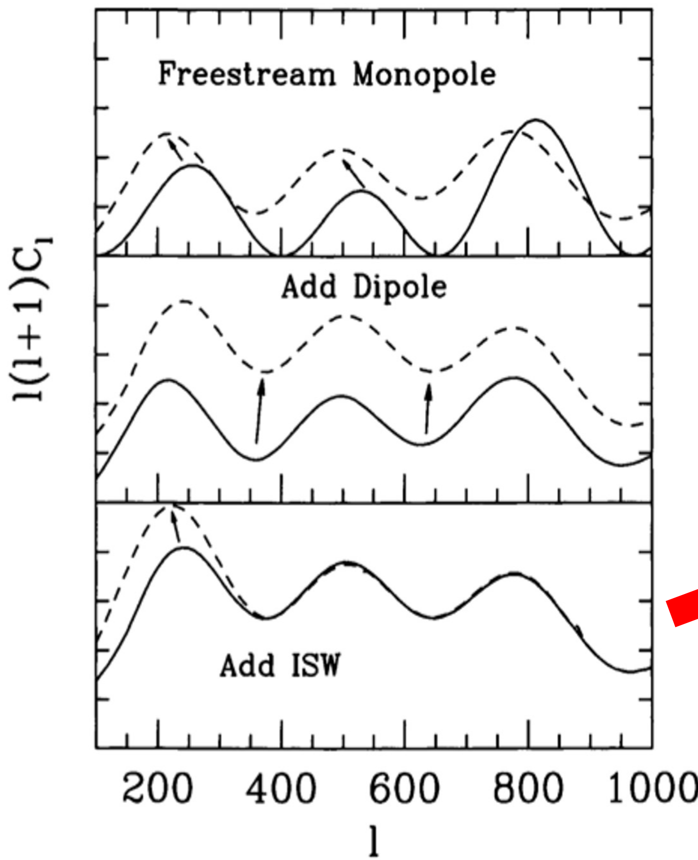
CMB TT power spectrum deconstructed…



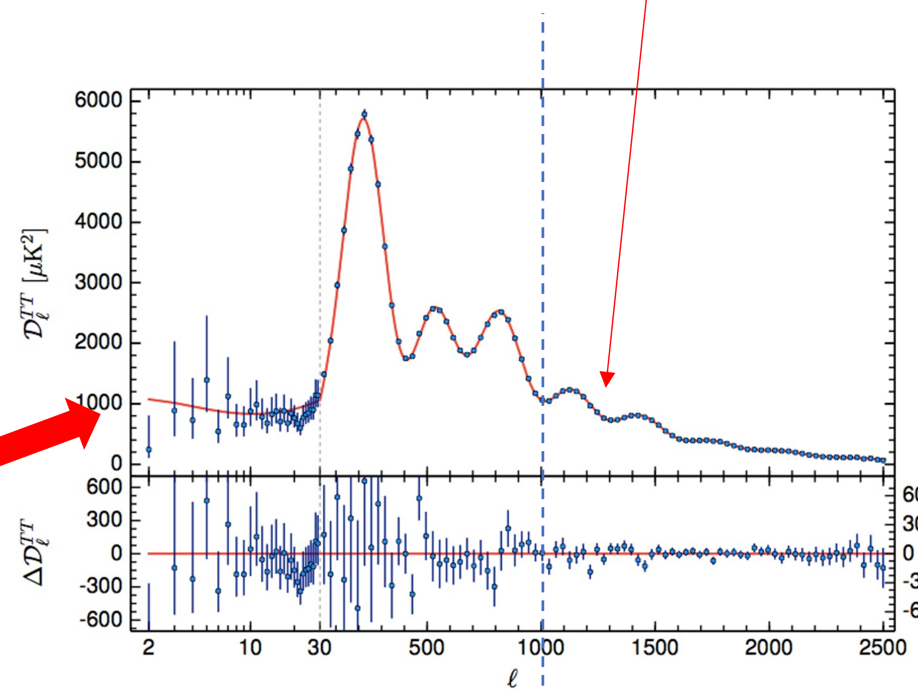
ISW effect adds in phase with the monopole

$$\Theta_\ell^{(0)}(k,\eta_0) \simeq \left[\Theta_0^{(0)}(k,\eta_*) + \Psi_0^{(0)}(k,\eta_*)\right] j_\ell[k(\eta_0-\eta_*)]$$
$$-\frac{3}{k}\Theta_1^{(0)}(k,\eta_*)\frac{d}{d\eta} j_\ell[k(\eta_0-\eta_*)]$$
$$+\int_0^{\eta_0} d\,\eta e^{-|\kappa|(\eta)}\left[\dot{\Psi}(k,\eta)+\dot{\Phi}(k,\eta)\right] j_\ell[k(\eta_0-\eta)]$$

CMB TT power spectrum deconstructed…



One more thing: Diffusion damping



Diffusion damping...

Previously, we invoked the tightly-coupled limit, which assumes Compton scattering keeps photons and baryons moving as one fluid.

- In reality this is **never exactly true**.
- Photons random walk between scattering, leading to diffusion.
- Diffusion washes out temperature differences on scales smaller than the **diffusion length**:

$$\lambda_D = \sqrt{N_{\text{scatter}}}\lambda_{\text{MFP}} \sim \frac{1}{\sqrt{n_e \sigma_T H}}$$

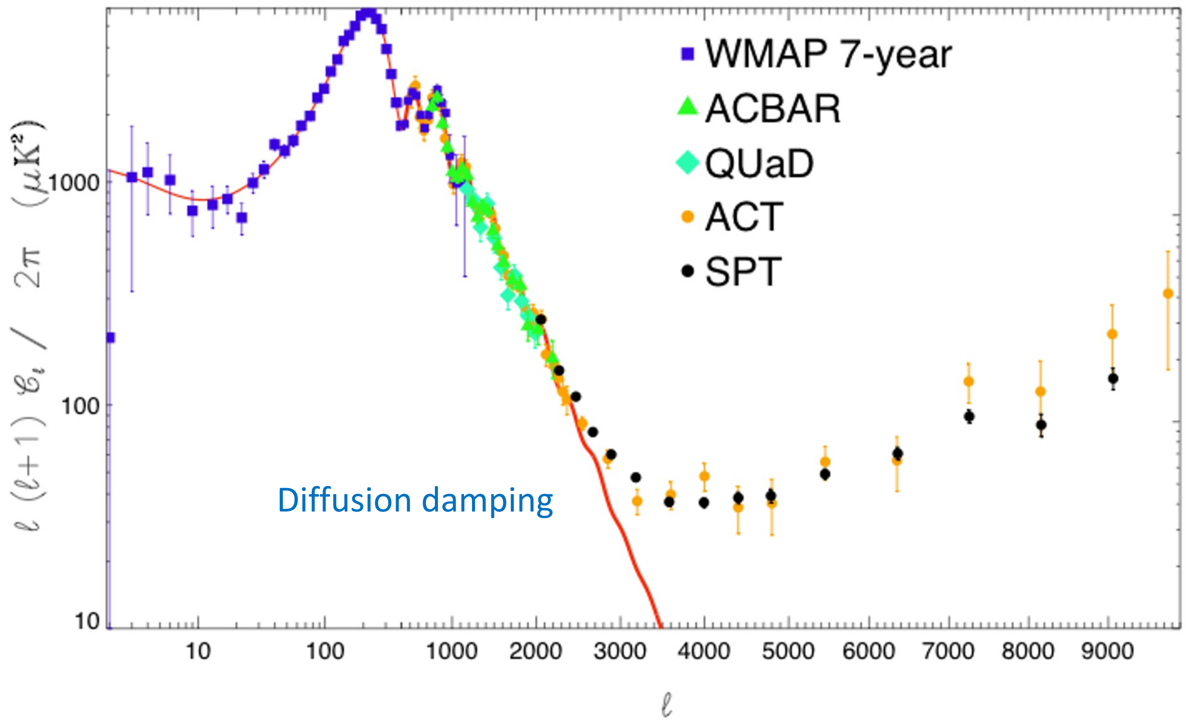
Hubble rate

~ a few Mpc at decoupling

Photon mean free path

Free-electron density

Thomson scattering cross-section



CMB temperature measurements here are foreground-dominated.

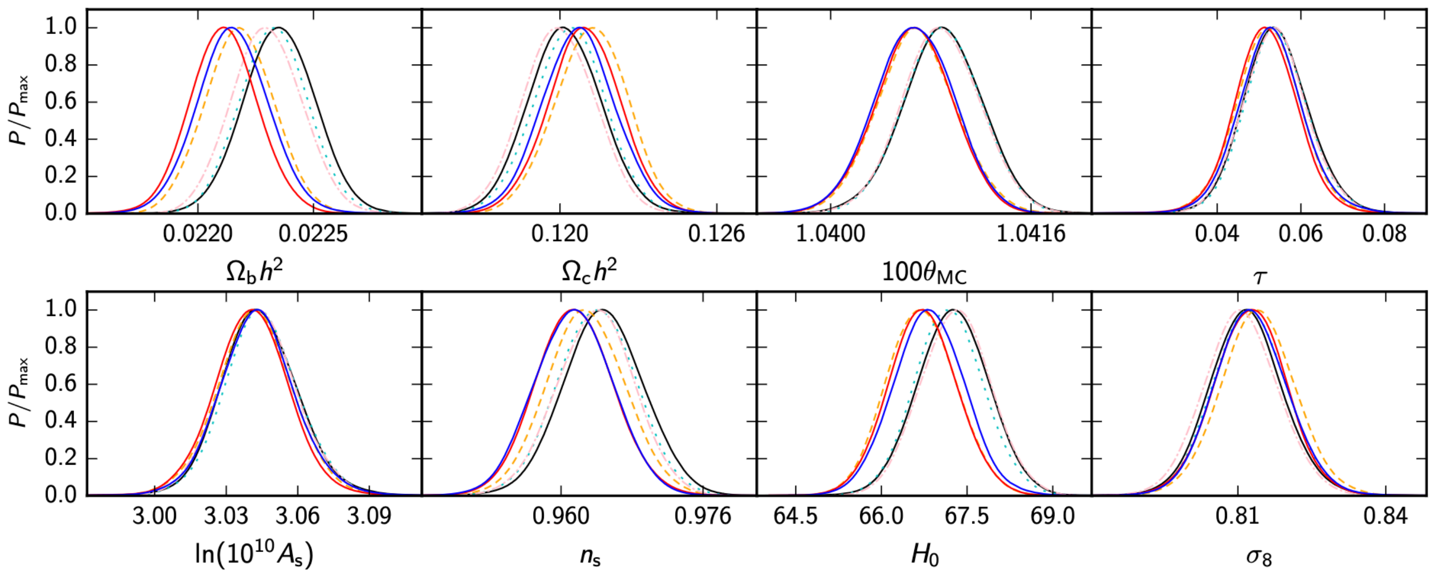
Baryon-to-photon ratio

Damping scale: $$r_d^2(\eta_*) \equiv \int_0^{\eta_*} \frac{d\eta'}{a(\eta')\bar{n}_e\sigma_T}\left[\frac{R^2 + (16/15)(1+R)}{6(1+R)^2}\right]$$

Free electron number density

Thomson scattering cross-section

Where cosmological parameter constraints come from…



Cosmological parameters…

Some standard parameters of interest:

- **Matter density** (including dark matter): $\omega_m = \Omega_m h^2$
- **Baryon density**: $\omega_b = \Omega_b h^2$
- **Hubble parameter**, **spatial curvature**, **dark energy**: $h, \Omega_K, \Omega_\Lambda$
- **Inflation parameters**: scalar fluctuation amplitude A_s, spectral index n_s

- **Others**: number of neutrino families N_{eff}, neutrino mass sum $\sum m_\nu$

- The CMB temperature anisotropies do **not** measure these parameters *per se*, rather some combinations thereof.
  - Let's see how that works.

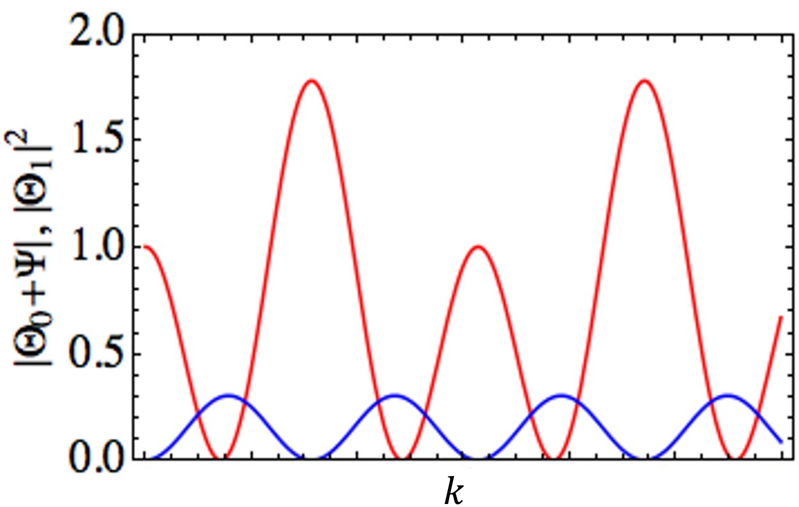
Odd-even peak heights: baryon-photon ratio…

We have seen earlier that the baryon-to-photon ratio *R* causes uneven fluctuation peak heights in the CMB TT spectrum.

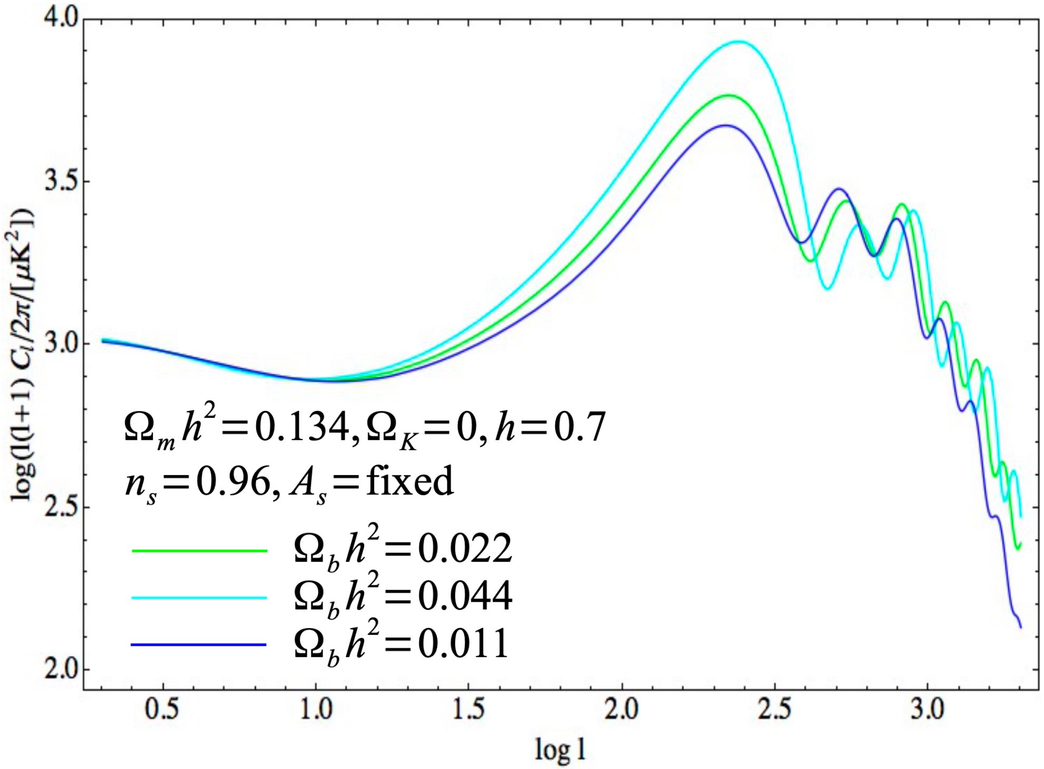
$$R \equiv \frac{3}{4}\frac{\bar{\rho}_b}{\bar{\rho}_\gamma} = \frac{3}{4}\frac{\Omega_b h^2}{\Omega_\gamma h^2} a$$

Photon energy density

- Since $\Omega_\gamma h^2$ is known, measuring the odd-to-even peak ratio gives $\Omega_b h^2$.
- Probably the most robust (i.e., model-independent) parameter measurement from the CMB.



Odd-even peak heights: baryon-photon ratio…



Increasing the baryon density enhances the uneven odd and even peak heights (note especially the first two peaks)

→ Measures the baryon-photon ratio R.

Early ISW effect: matter-radiation equality…

Decaying potentials during transition from radiation to matter domination enhance the 1st peak.

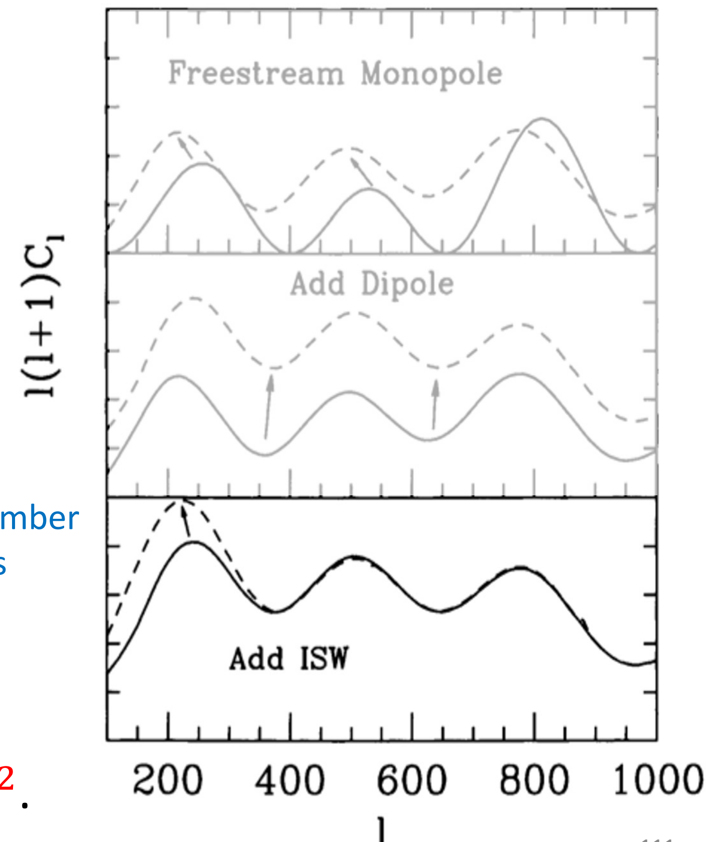
- The **ratio of the 1st to 3rd peak** probes the early ISW effect.
- The parameter that controls this transition is the **redshift of matter radiation equality**, $z_{\rm eq}$.

$$1 + z_{\rm eq} = \frac{\Omega_m h^2}{\Omega_r h^2} \simeq 2.4\times 10^5 \frac{\Omega_m h^2}{1 + 0.2271 N_{\rm eff}}$$

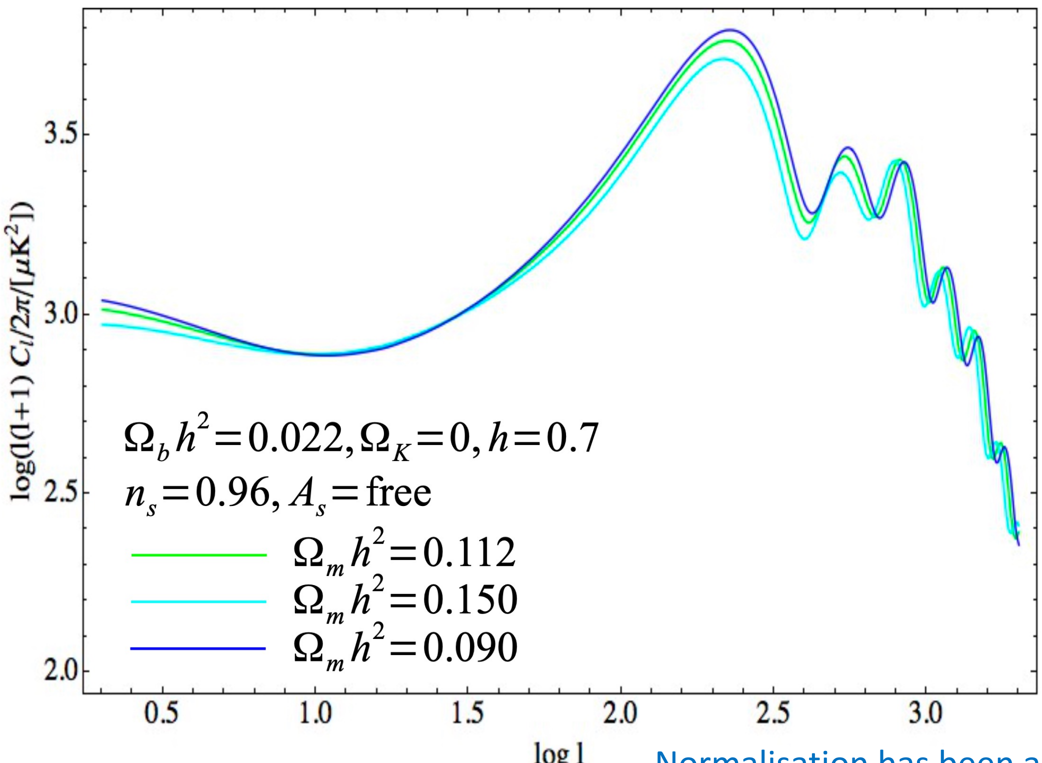
Effective number of neutrinos

Photons + massless neutrinos

→ If $N_{\rm eff}$ is known, then early ISW yields $\Omega_m h^2$.



Early ISW effect: matter-radiation equality…



Normalisation has been adjusted for easy comparison.

Changing the matter density modifies the **early ISW effect**.

- Keeps 1st to 2nd peak ratio largely unchanged but alters the 1st to 3rd peak ratio.
- Good for measuring the redshift of MR equality.
- (Upturn at low ℓ is due to the late ISW effect.)

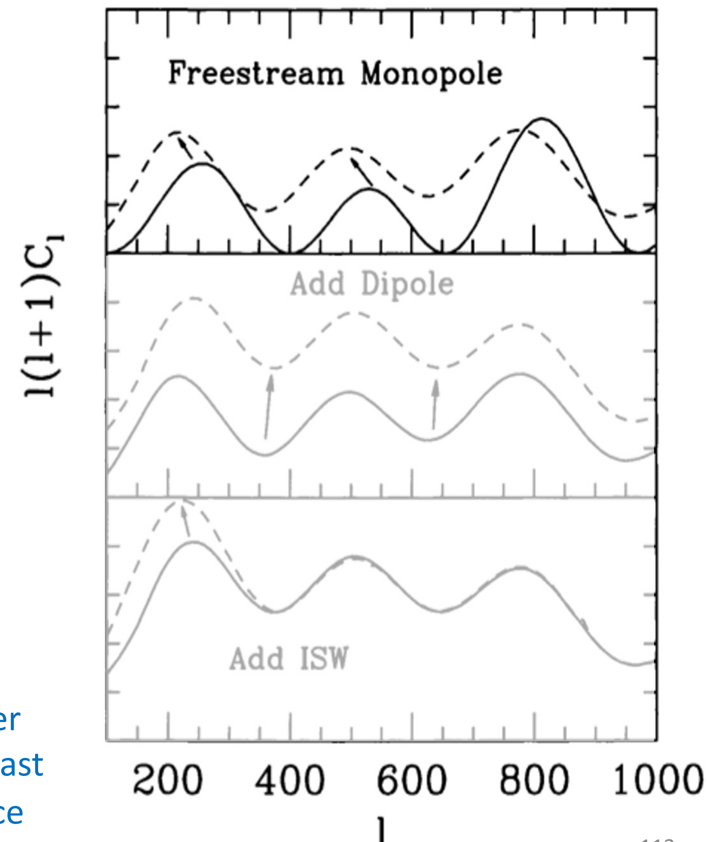
Angular sound horizon…

We have seen that the position of the 1st acoustic peak is given roughly by

Comoving distance to the last scattering surface

Sound horizon at decoupling

$$\ell_{\mathrm{peak}} \sim \frac{\pi(\eta_0 - \eta_*)}{r_s(\eta_*)} \qquad \eta_0 - \eta_* = \chi(\eta_*)$$



- Had we allowed for spatial curvature:

$$\chi(\eta_*) \to \begin{matrix} \sin[\chi(\eta_*)] & K = +1 \\ \sinh[\chi(\eta_*)] & K = -1 \end{matrix}$$

- More generally, the 1st peak position is described by the **angular sound horizon**:

$$\theta_s \equiv \frac{\pi}{\ell_{1^{\mathrm{st}}\,\mathrm{peak}}} = \frac{a(\eta_*) r_s(\eta_*)}{d_A(\eta_*)}$$

Angular diameter distance to the last scattering surface

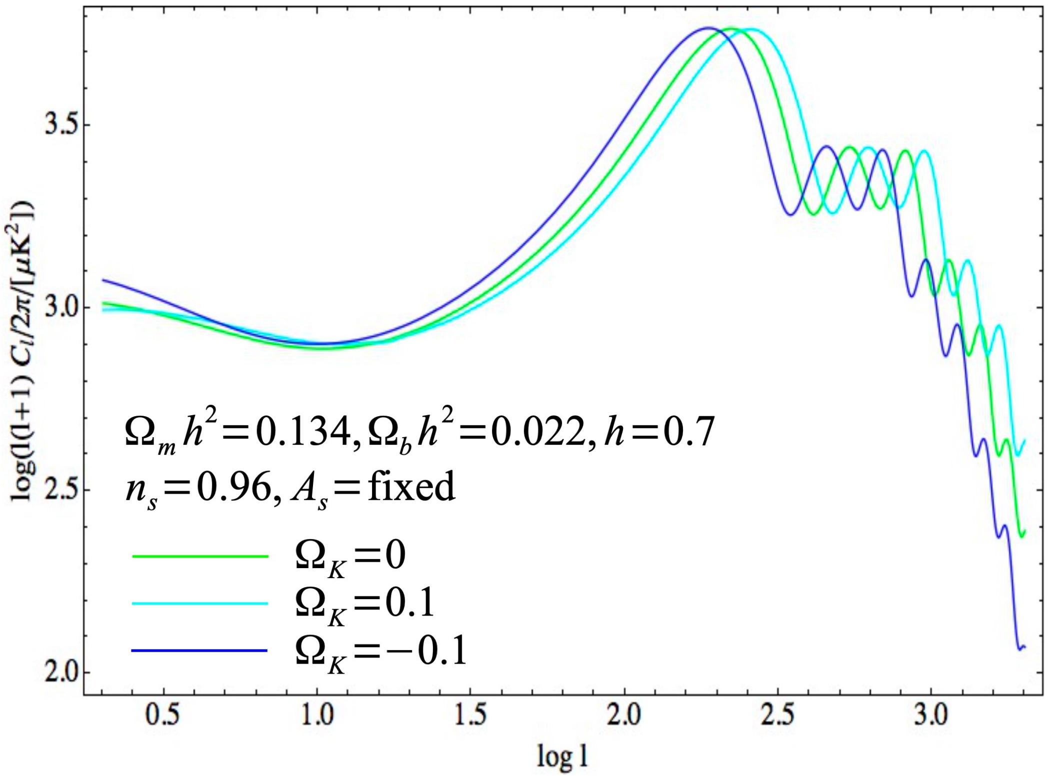
Angular sound horizon…

For **fixed** $\Omega_b h^2$ (from odd-even peak ratios) and $z_{\rm eq}$ (from early ISW), the main parameter dependence of θ_s goes something like this:

$$\theta_s = \frac{a(\eta_*) r_s(\eta_*)}{d_A(\eta_*)} \propto \frac{(\Omega_m h^2)^{-1/2}}{\int_0^{z_*} \frac{dz}{\sqrt{\Omega_m h^2 a^{-3} + \Omega_K h^2 a^{-2} + (h^2 - \Omega_m h^2 - \Omega_K h^2)}}}$$

- Thus, if $\Omega_m h^2$ is known (because $N_{\rm eff}$ is known), then the remaining unknown parameters in θ_s are Ω_k and h, which are **degenerate**.
- If $\Omega_m h^2$ is **not** known (because $N_{\rm eff}$ is **not** known), then there is a 3-way degeneracy and there's still more work to do. More on this in a bit!

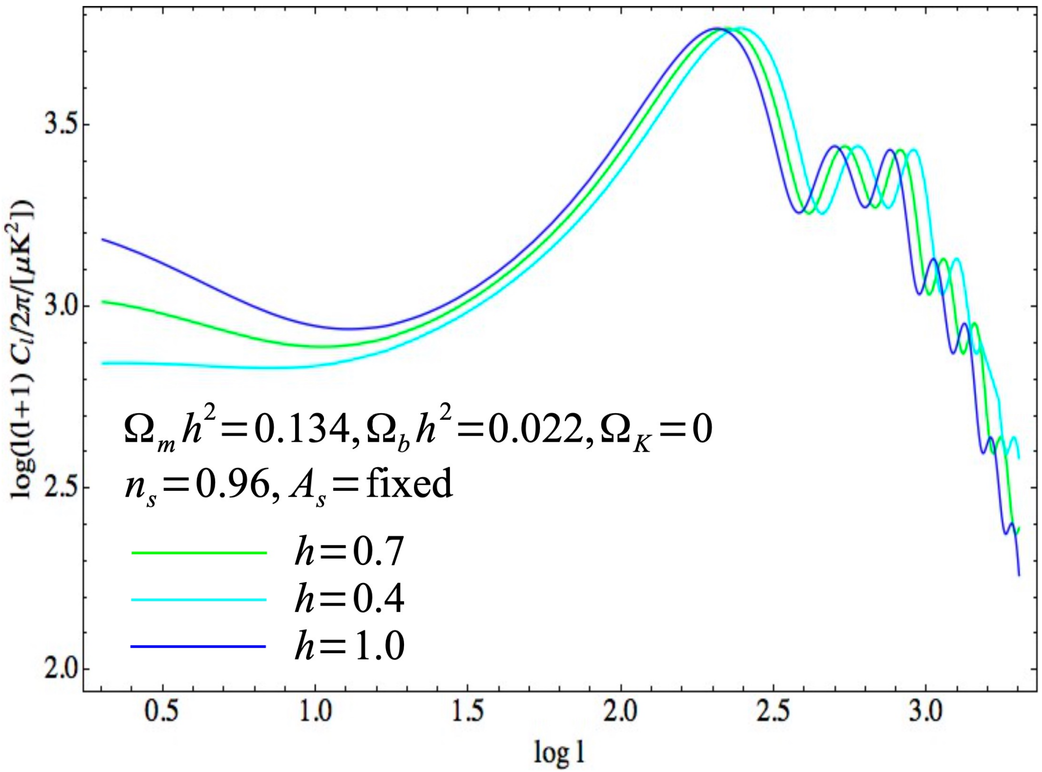
Angular sound horizon…



Changing the spatial geometry alters the way the acoustic peaks on the LSS are projected onto ℓ space.

- Shifts the positions of the peaks.

Angular sound horizon…

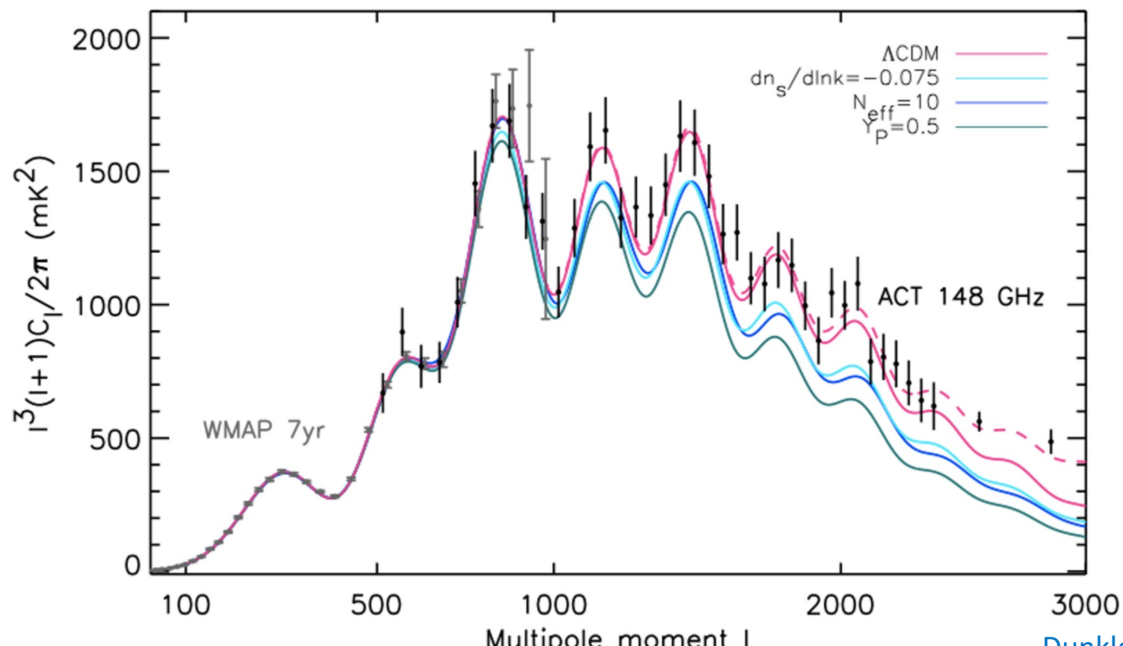


Same effect if we keep the geometry flat but vary the Hubble parameter h.

- This means it is not possible to pin down both h and Ω_K at the same time using θ_s alone (parameter degeneracy).
- However, h and Ω_K have very different late ISW effects, and so can be distinguished using CMB temperature data.

Angular damping scale…

First measured by ACT and SPT; now also measured by Planck and ground-based successors to ACT/SPT.



Dunkley et al. [ACT] 2010

Angular damping scale…

Like the angular sound horizon, for **fixed** $\Omega_b h^2$ (from odd-even peak ratios) and z_{eq} (from early ISW), the main parameter dependence of θ_d is:

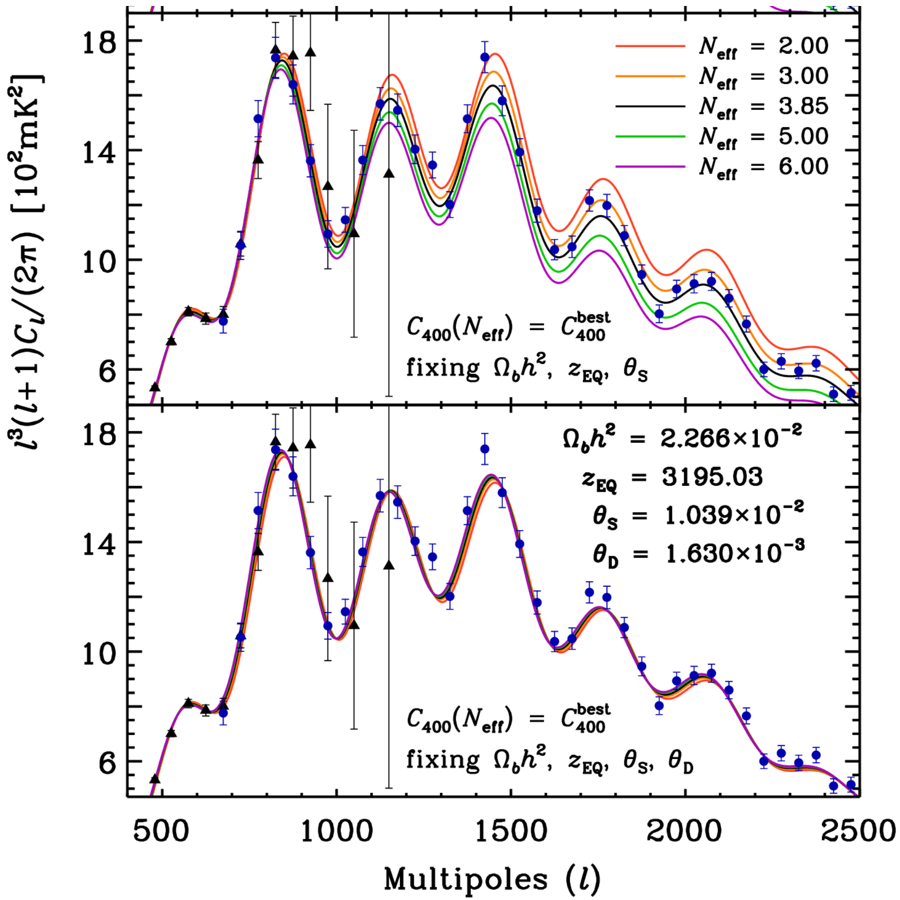
$$\theta_d = \frac{a(\eta_*) r_d(\eta_*)}{d_A(\eta_*)} \propto \frac{(\Omega_m h^2)^{-1/4}}{\int_0^{z_*} \frac{dz}{\sqrt{\Omega_m h^2 a^{-3} + \Omega_K h^2 a^{-2} + (h^2 - \Omega_m h^2 - \Omega_K h^2)}}}$$

$$\propto (\Omega_m h^2)^{1/4} \theta_s \longleftarrow \text{Angular sound horizon}$$

→ The **ratio** of θ_d and θ_s measures $\Omega_m h^2$ independently of dark energy, spatial curvature, Hubble rate, etc.

- If $\Omega_m h^2$ is **not** already known from early ISW, (because N_{eff} is **not** known), this θ_d / θ_s measurement also allows us to measure the effective number of neutrinos N_{eff}!

Irreducible signature of N_{eff}...



Hou, Keisler, Knox et al. 2011

Take-home message…

Uniformity is good, but fluctuations are even better.

- Statistical properties of the CMB fluctuations are strongly dependent on
  - The redshift of matter-radiation equality** (1^{st} to 3^{rd} peak heights)
  - The **baryon-to-photon ratio** (odd-to-even peak heights)
  - The **sound horizon** at decoupling (peak positions)
  - The **distance to the last scattering surface** (peak positions)
  - The **damping scale** at decoupling (damping tail)
  - The **late ISW effect** (low-ℓ multipoles)

- Understanding how various cosmological parameters affect these physical quantities enables us to constrain them.