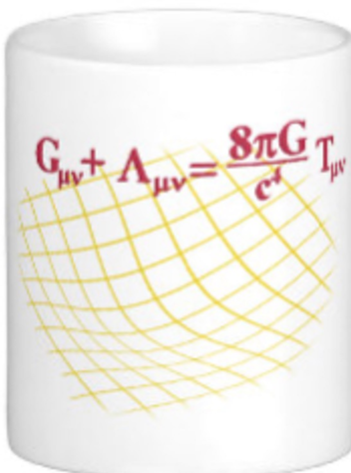
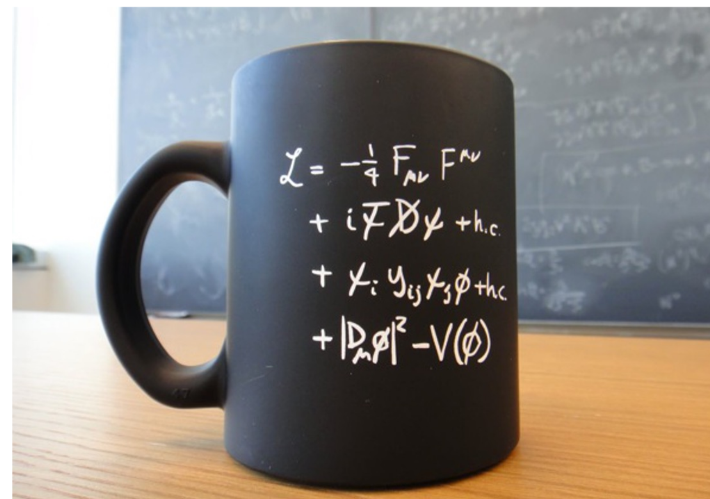
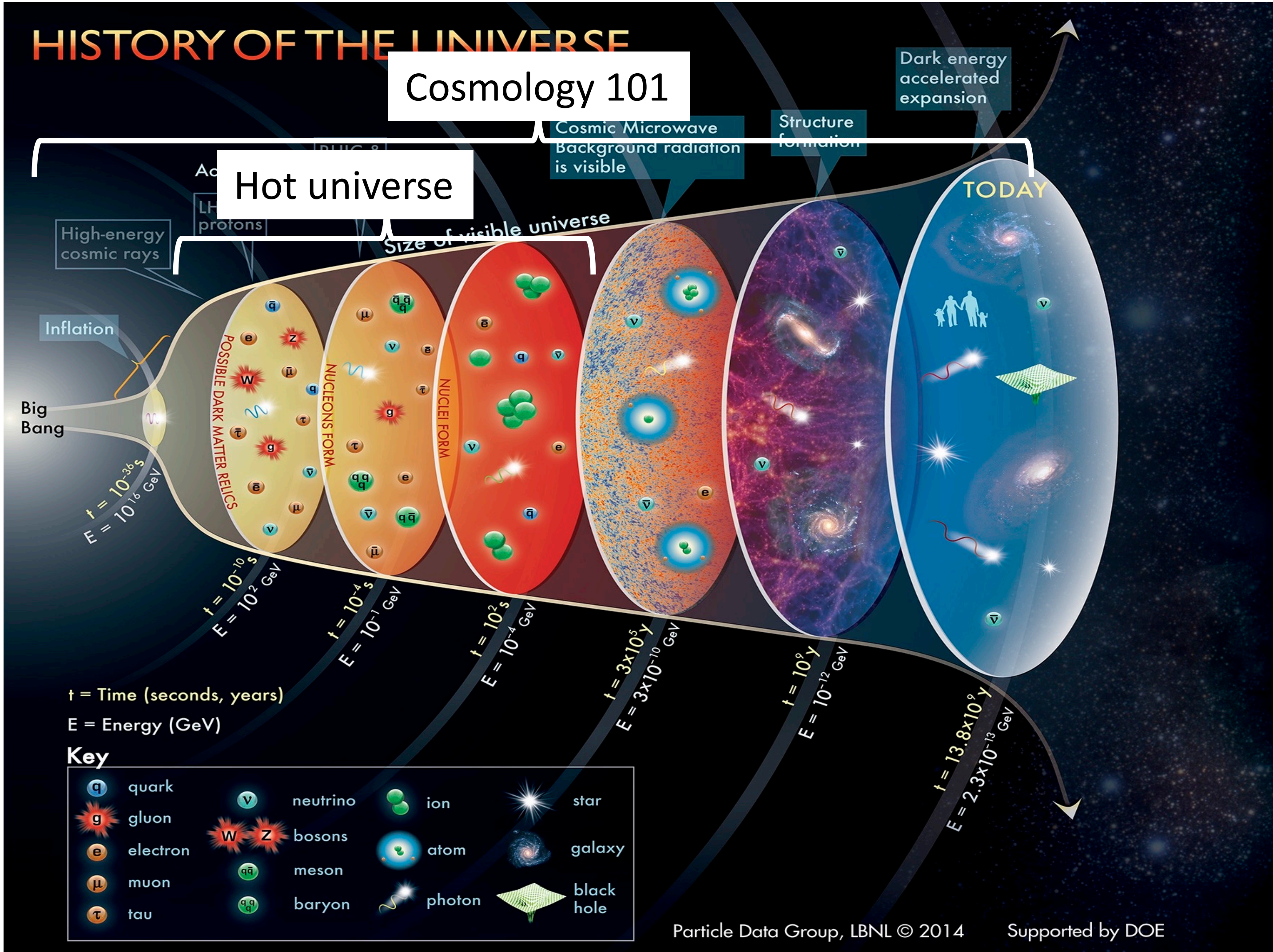
2. Hot universe…







HISTORY OF THE UNIVERSE
Cosmology 101
Hot universe
Dark energy accelerated expansion
Structure
Cosmic Microwave Background radiation is visible
High-energy cosmic rays
protons
Inflation
Big Bang
TODAY
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
$t = 10^{-36}$ s
$E = 10^{16}$ GeV
$t = 10^{-10}$ s
$E = 10^{2}$ GeV
$t = 10^{-4}$ s
$E = 10^{-1}$ GeV
$t = 10^{2}$ s
$E = 10^{-4}$ GeV
$t = 3\times10^{5}$ y
$E = 3\times10^{-10}$ GeV
$t = 10^{9}$ y
$E = 10^{-12}$ GeV
$t = 13.8\times10^{9}$ y
$E = 2.3\times10^{-13}$ GeV
t = Time (seconds, years)
E = Energy (GeV)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL © 2014
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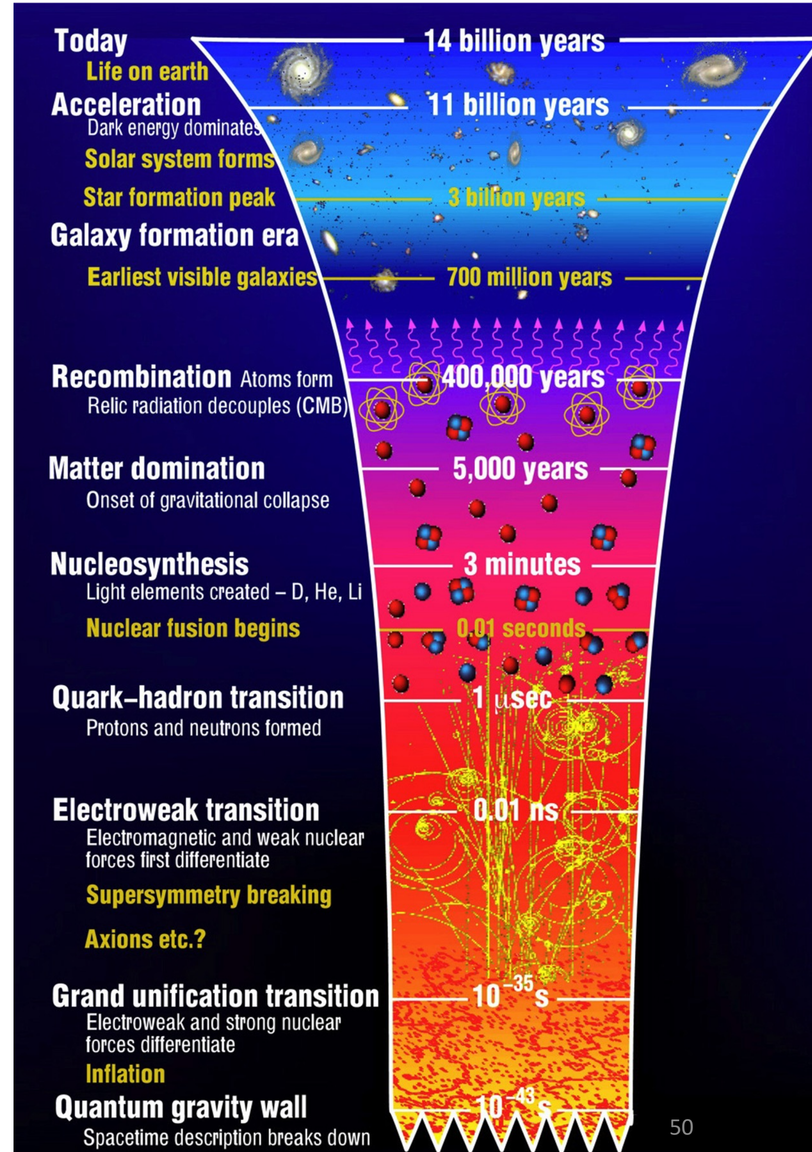
The hot universe...

The early universe was a **very hot and dense place**.

- Particle interactions (e.g., scattering) can happen very frequently.
- What interactions are available depends on the particle physics theory.
- But if an interaction rate (per particle) far exceeds the Hubble expansion rate,

$$\Gamma_{\text{int}} \gg H$$

the interaction can be taken to be in a **state of equilibrium**.



Classic example: weak interaction…

Say you have a gas of ultra-relativistic particles with **temperature** T.

- The Weak interaction rate per particle is estimated to be

$$\Gamma_{\text{int}} = n\langle\sigma v\rangle \sim G_F^2 T^5$$

Fermi constant

Number density of scattering centres $n \sim T^3$

Cross-section $\sigma \sim G_F^2 T^2$

Relative velocity $v \sim 1$

- The Hubble expansion rate is

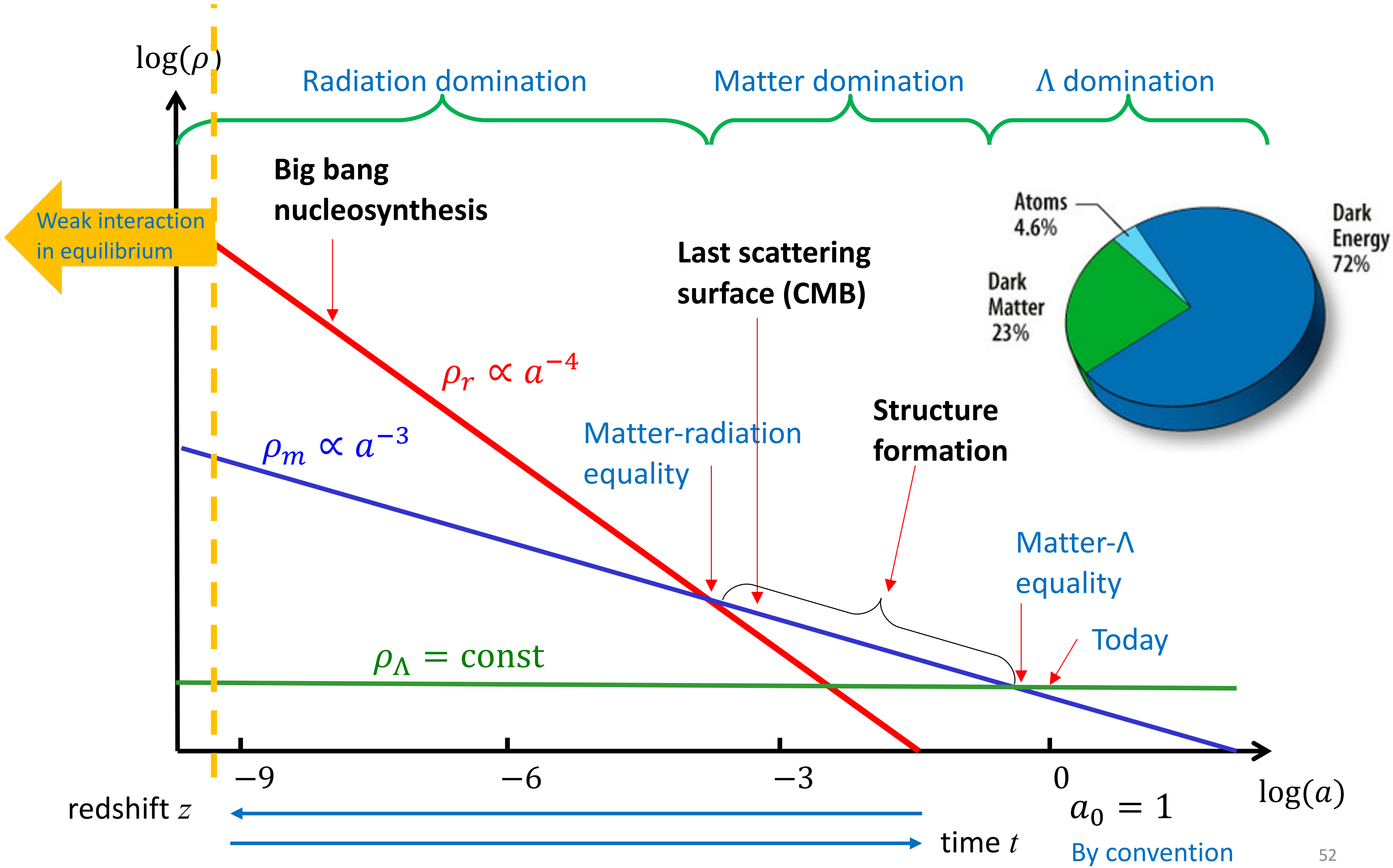
$$H = \sqrt{\frac{8\pi G}{3}\sum_\alpha \rho_\alpha} \sim \frac{T^2}{m_{\text{planck}}}$$

Planck mass

$$G_F \sim 10^{-5}\ \text{GeV}^{-2}$$
$$m_{\text{pl}} \sim 10^{19}\ \text{GeV}$$

$$\frac{\Gamma_{\text{int}}}{H} \sim m_{\text{planck}} G_F^2 T^3 \sim \left(\frac{T}{1\ \text{MeV}}\right)^3$$

Weak interactions are in equilibrium at $T \gg 1$ MeV.



log(ρ)
Radiation domination
Matter domination
Λ domination
Weak interaction in equilibrium
Big bang nucleosynthesis
Last scattering surface (CMB)
Atoms 4.6%
Dark Energy 72%
Dark Matter 23%
ρ_r ∝ a^−4
ρ_m ∝ a^−3
Matter-radiation equality
Structure formation
Matter-Λ equality
Today
ρ_Λ = const
−9
−6
−3
0
log(a)
redshift z
a_0 = 1
time t
By convention

Equilibrium thermodynamics...

In the ideal gas limit, when an interaction is in equilibrium, all participating particles have phase space distributions described by one of the equilibrium forms:

$f(p)$ = Phase space distribution

$$f_{\mathrm{eq}}(p) = \frac{1}{\exp[(E(p) - \mu)/T] \pm 1}$$

$+$ Fermi-Dirac
$-$ Bose-Einstein

μ = Chemical potential

T = Temperature

- All particles participating in that interaction have the same temperature T.
- Their chemical potentials satisfy $\sum_{\mathrm{initial}} \mu_i = \sum_{\mathrm{final}} \mu_f$.

Equilibrium thermodynamics…

The chemical potentials of all particles participating in an interaction in equilibrium satisfy:

$$\sum_{\text{initial}} \mu_i = \sum_{\text{final}} \mu_f$$

- Trivial condition for **elastic scattering**, e.g., $i + j \to i + j$
- For **inelastic processes**, e.g, $i + j \to k + l$, you can also call it a chemical equilibrium.
- If a chain of interactions like $\bar{\phi}\phi \leftrightarrow \cdots \leftrightarrow \gamma\gamma$ is **all in equilibrium**, then

Antiparticle ($\bar{\phi}$) Particle (ϕ)

$$\mu_\phi + \mu_{\bar{\phi}} = \cdots = 2\mu_\gamma = 0 \quad \Rightarrow \quad \boxed{\mu_{\bar{\phi}} = -\mu_\phi}$$

Because the number of photons in the universe at $z \gtrsim 10^6$ is not conserved.

Equilibrium thermodynamics…

When $\mu_{\bar{\phi}} = -\mu_\phi$ is satisfied, the **particle-antiparticle asymmetry** is related to the chemical potential via:

$$n_\phi - n_{\bar{\phi}} \propto \int d^3p \left[f_\phi(p) - f_{\bar{\phi}}(p)\right]$$

Particle number density – antiparticle number density

$$\propto \frac{\mu_\phi}{T} \qquad \text{for } \frac{\mu_\phi}{T} \ll 1$$

- In standard cosmology (i.e., standard model of particle physics in FLRW universe), any such asymmetry should be of the same order of magnitude as the matter-antimatter asymmetry $\sim 10^{-10}$.

→ Thus, unless your goal is to compute the particle-antiparticle asymmetry, it suffices for most applications to set $\mu = 0$.

Equilibrium thermodynamics...

Given its phase space distribution $f(p)$, it is straightforward to find a particle species' bulk properties:

Number density: $$n_\alpha = \frac{g_\alpha}{(2\pi)^3}\int d^3p\, f_\alpha(\vec{p})$$

Internal d.o.f.

Energy density: $$\rho_\alpha = \frac{g_\alpha}{(2\pi)^3}\int d^3p\, E f_\alpha(\vec{p})$$

$\sim T^4$ Ultra-relativistic $T \gg m$

$\sim m(mT)^{3/2}e^{-m/T}$ Non-relativistic $T \ll m$

The energy density of a non-relativistic particle species is highly suppressed!

Pressure: $$P_\alpha = \frac{g_\alpha}{(2\pi)^3}\int d^3p\, \frac{|\vec{p}|^2}{3E} f_\alpha(\vec{p})$$

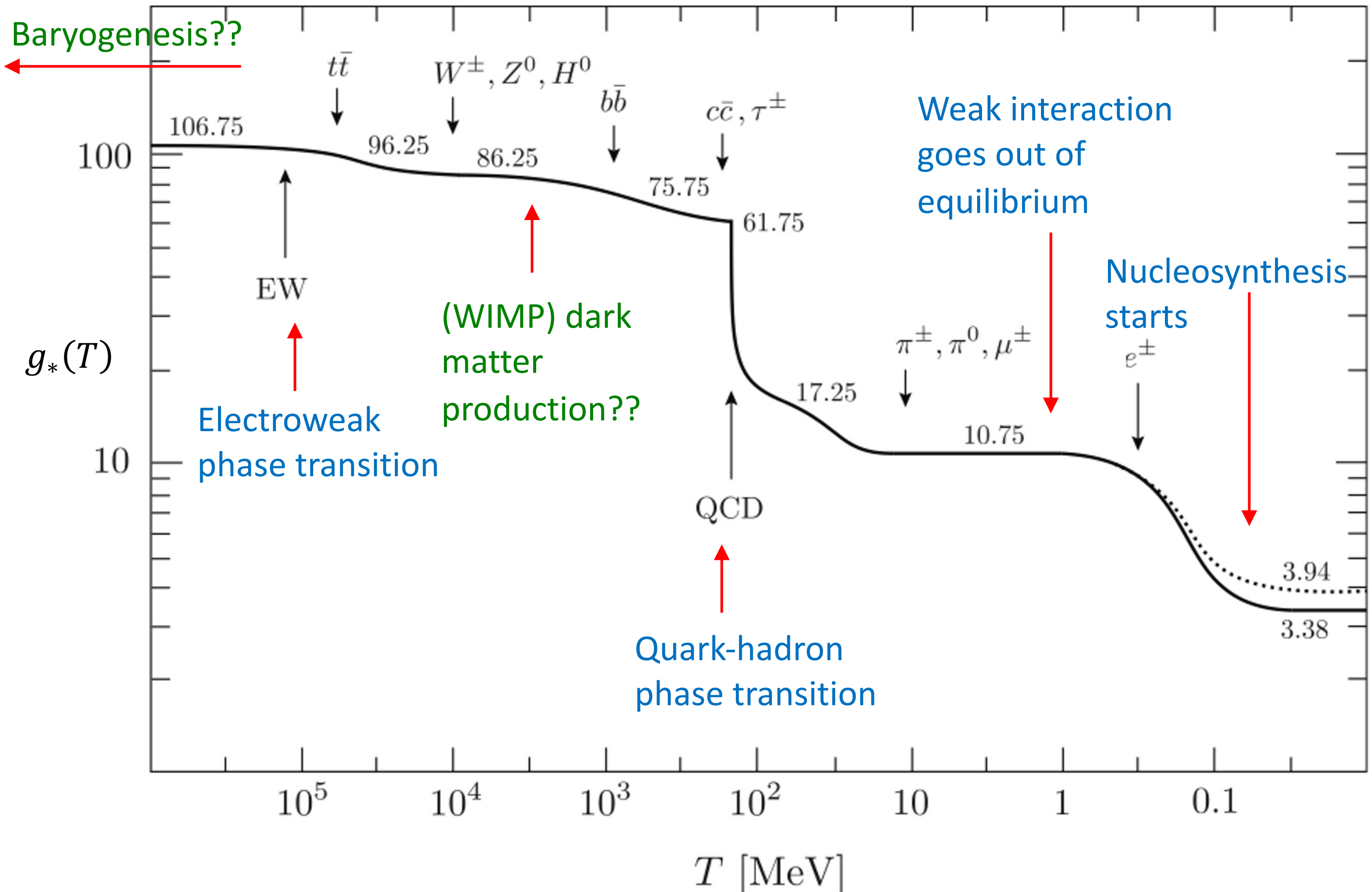
- We can therefore express the Hubble expansion rate in the early universe as:

$$H^2(t) = \frac{8\pi G}{3}\sum_\alpha \rho_\alpha \equiv \frac{8\pi G}{3}\frac{\pi^2}{30} g_*(T_\gamma)T_\gamma^4$$

Photon temperature

g_* is a temperature-dependent function, dominated by relativistic species, specific to a particle physics theory.

g_* of the standard model of particle physics:



Photon temperature

Time

What's left?

Mainly
- Photons
- Neutrinos

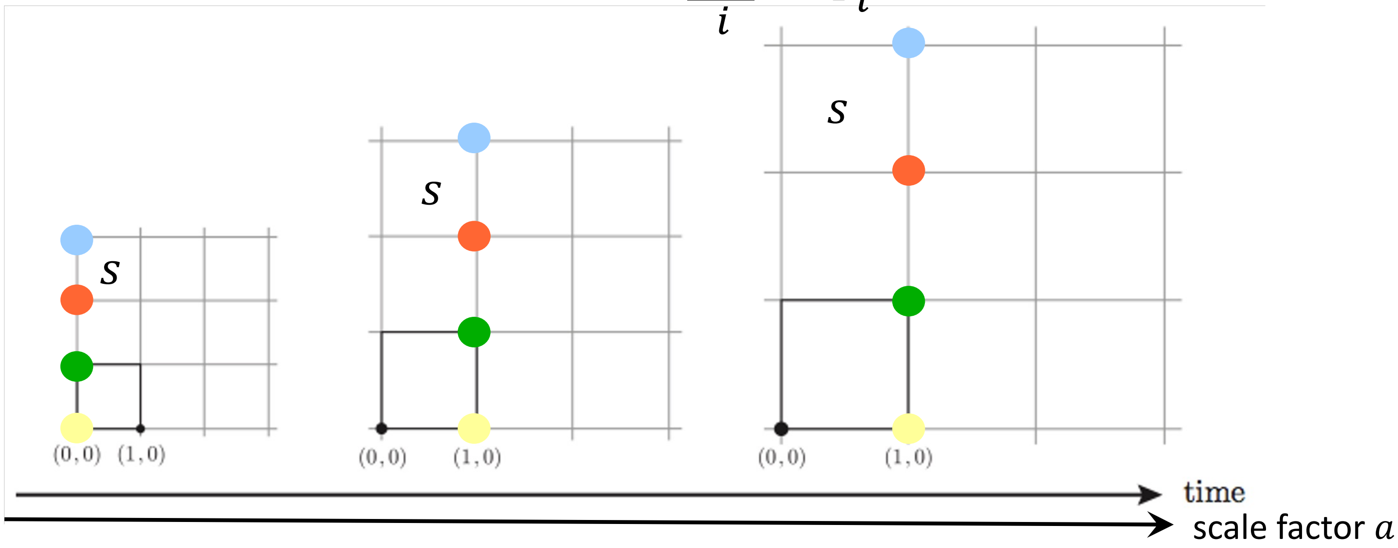
Small amounts* of
- Electrons
- Nucleons
- Nuclei

* Small means $< 10^{-9} n_\gamma$

Comoving entropy density & conservation...

Where expansion is **quasi-static** so that equilibrium is maintained, the comoving entropy density s is approximately conserved.

Comoving entropy density $$s = a^3 S \equiv a^3 \sum_i \frac{\rho_i + P_i}{T_i}$$



Comoving entropy and evolution of T_γ…

As for the energy density, we can define the **entropy degrees of freedom**, g_{*s}, via

$$s_{\text{total}} = a^3 \sum_i \frac{\rho_i + P_i}{T_i} \equiv a^3 \frac{2\pi^2}{45} g_{*s}(T_\gamma) T_\gamma^3$$

g_{*s} is a temperature-dependent function, dominated by relativistic species, specific to a particle physics theory. In general, $g_{*s} \neq g_*$, except when $T_i = T_\gamma$ for all particle species i.

- Comoving entropy conservation, $s_{\text{total}} = \text{const.}$, implies

$$a^3 g_{*s}(T_\gamma) T_\gamma^3 = \text{const.} \Rightarrow T_\gamma \propto g_{*s}^{-1/3} a^{-1}$$

ν-to-γ temperature ratio from entropy…

Decoupling

1 MeV

$T_\gamma = m_e$

Time

QED plasma

3 families of neutrinos + antineutrinos

$e^\pm, \gamma$ ← Same temperature → $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$

$$g_{*s,1} = g_\gamma + (g_e + 3\times g_\nu)\,\frac{7}{8}$$

$$T_{\gamma,2} = T_{\gamma,1}\left(\frac{g_{*s,1}}{g_{*s,2}}\right)^{1/3}\frac{a_1}{a_2}$$

Pre-e^+e^--annihilation

Post-e^+e^--annihilation

γ ← Different temperatures → $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$

$$T_{\nu,2} = T_{\nu,1}\,\frac{a_1}{a_2}$$

$$g_{*s,2} = g_\gamma + 3\times g_\nu\,\frac{7}{8}\left(\frac{T_{\nu,2}}{T_{\gamma,2}}\right)^3$$

$$\frac{T_{\nu,2}}{T_{\gamma,2}} = \left(\frac{4}{11}\right)^{1/3}$$

Out of equilibrium…

An interaction goes **out of equilibrium** when the interaction rate per particle drops significantly below the expansion rate with time, i.e.,

$$\Gamma_{\text{int}} \ll H$$

Interaction rate per particle (Γ_{int}); Hubble expansion rate (H)

- When this happens, the interaction can no longer change the energy density and/or the number density of the participating particles.

Out of equilibrium…

You may also have heard of these terms:

- **Decoupling**: a particle species decouples when all its interactions with other particle species are out of equilibrium.

- **Freeze-out**: when all number-changing interactions of a particle species (e.g., annihilation/pair production) go out of equilibrium.

Out of equilibrium…

In some BSM models, an interaction may never even reach equilibrium, i.e., $\Gamma_{\mathrm{int}} \gg H$ is never satisfied.

- **Freeze-in** of a particle species is when there are just sufficient interactions to make the particles in some abundance, but the interaction does not reach equilibrium rates.

Tracking out-of-equilibrium processes…

The effect of an out-of-equilibrium interaction on particle species 1 can be tracked using the **Boltzmann equation**:

$$P^{\alpha}\frac{\partial f_1}{\partial x^{\alpha}} - \Gamma^{i}_{\alpha\beta}P^{\alpha}P^{\beta}\frac{\partial f_1}{\partial P^{i}} = C[f_1]$$

Collision term (Lorentz-invariant); "short range" interactions

Gravity goes in here; "long range" interactions

- $f_1(x^{\alpha}, P^{i})$ is the 1-particle phase density defined such that the number of particles 1 in a phase space volume $dx^1 dx^2 dx^3 dP_1 dP_2 dP_3$ is

$$dN = f_1(x^{\alpha}, P^{i})dx^1 dx^2 dx^3 dP_1 dP_2 dP_3$$

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Collision term (Lorentz-invariant); “short range” interactions

Gravity goes in here; “long range” interactions

- In a homogeneous and isotropic FLRW universe, gravity causes only expansion.

$f_1(x^\alpha, P^i) \rightarrow f_1(t, p)$

Physical momentum measured by a comoving observer

$$\frac{\partial f_1}{\partial t} - Hp \frac{\partial f_1}{\partial p} = \frac{1}{E_1} C[f_1]$$

$\frac{dp}{dt} = Hp$ i.e., redshifting momentum

Tracking out-of-equilibrium processes…

The effect of an out-of-equilibrium interaction on particle species 1 can be tracked using the **Boltzmann equation**:

$$P^{\alpha}\frac{\partial f_1}{\partial x^{\alpha}} - \Gamma^{i}_{\alpha\beta}P^{\alpha}P^{\beta}\frac{\partial f_1}{\partial P^{i}} = C[f_1]$$

Collision term (Lorentz-invariant); “short range” interactions

- **The collision term** for e.g., $1 + 2 \rightarrow 3 + 4$

9D phase space integral

Energy-momentum conservation

Matrix element

$$C[f_1] = \frac{1}{2}\int \prod_{i=2}^{4} \frac{d^3 p_i}{(2\pi)^3 2E_i}(2\pi)^4 \delta^4(P_1 + P_2 - P_3 - P_4)|M|^2$$
$$\times[f_3 f_4(1 \pm f_1)(1 \pm f_2) - f_1 f_2(1 \pm f_3)(1 \pm f_4)]$$

Quantum statistical factors

How phase space density evolves when C[f]=0.

When there are **no collisions**, then the only thing about a particle that can change is its physical momentum due to redshift.

$$p \propto a^{-1}$$

- It is thus useful to define a non-redshifting momentum that is constant in time.

$$\boxed{q \equiv ap}$$

- Then, $f_1(t,p) \rightarrow f_1(t,q)$, and Boltzmann equation becomes:

$$\frac{\partial f_1}{\partial t} = 0 \quad \Rightarrow$$

The phase space density $f_1(t,q)$ remains constant in time when there are no collisions.

Full-momentum or not?

Depending on the problem at hand, you may or may not need to solve the **full momentum-dependent** Boltzmann equation in its full glory.

- Problems that require it include:
  - Precision neutrino decoupling/$N_{\rm eff}$ (density matrix)
  - Light sterile neutrino thermalisation (density matrix)
  - Freeze-in.
- Problems for which it is perhaps not really necessary include:
  - WIMP freeze-out
  - Recombination and photon decoupling
  - Standard thermal leptogenesis
  - …

Momentum-integrated Boltzmann equation…

Where full-momentum dependence is unnecessary, we can integrate the Boltzmann equation in momentum to form an equation of motion for the **number density**:

$$\frac{g_1}{(2\pi)^3}\int d^3p\left[\frac{\partial f_1}{\partial t}-Hp\frac{\partial f_1}{\partial p}\right]=\frac{g_1}{(2\pi)^3}\int d^3p\frac{1}{E_1}C[f_1]$$

$$\frac{dn_1}{dt}+3Hn_1=\frac{g_1}{(2\pi)^3}\int d^3p\frac{1}{E_1}C[f_1]$$

Still very ugly

Simplifications…

We can often make several simplifying assumptions to make the collision integral easier to compute.

- Ignore quantum statistics factors and assume Maxwell-Boltzmann statistics, i.e., assume the equilibrium distribution takes the form:

$$f_{\mathrm{eq}} = e^{-\frac{E_i}{T}}$$

- Assume a common temperature T for the participating species (justified by elastic scattering processes being in equilibrium).
- Assume a phase space density of the form:

$$f_i = \frac{n_i}{n_i^{\mathrm{eq}}} e^{-\frac{E_i}{T}}$$

Simplifications...

Here's the **simplified collision integral**:

$$\frac{g_1}{(2\pi)^3}\int d^3p\,\frac{1}{E_1}C[f_1] = -n_1^{\text{eq}}n_2^{\text{eq}}\langle\sigma v\rangle\left[\frac{n_1 n_2}{n_1^{\text{eq}}n_2^{\text{eq}}} - \frac{n_3 n_4}{n_4^{\text{eq}}n_4^{\text{eq}}}\right]$$

Thermally-averaged cross section

$$\langle\sigma v\rangle \equiv \frac{1}{n_1^{\text{eq}}n_2^{\text{eq}}}\int\prod_{i=1}^{4}\frac{d^3p_i}{(2\pi)^3 2E_i}(2\pi)^4\delta^4(P_1+P_2-P_3-P_4)|M|^2 e^{\frac{-E_1-E_2}{T}}$$

- Its advantage is that the thermally-averaged cross section is now just a function of temperature, rather than a function time.
- It can be pre-computed.

Application 1: WIMP freeze-out…

WIMP = **weakly-interacting massive particle**, a generic cold dark matter candidate.

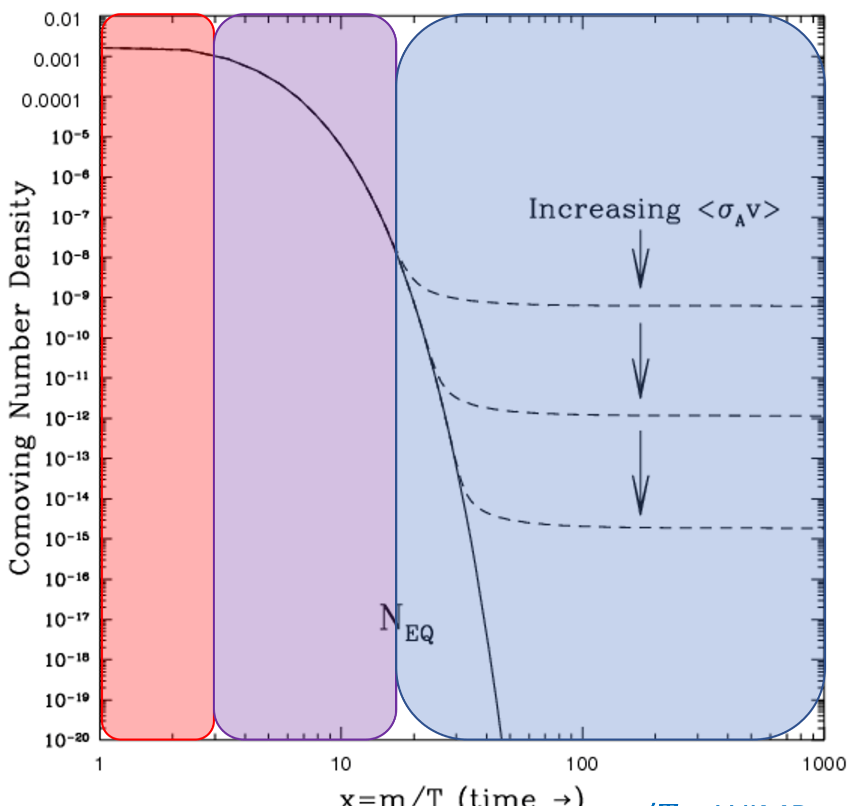
- Generic production process: $X\bar{X} \to \psi\bar{\psi}$

Standard model particles ($X\bar{X}$) — WIMP ($\psi\bar{\psi}$)

- Standard model particles X can be assumed to be in thermal equilibrium.
- Some equilibirum elastic scattering, e.g., $\psi X \to \psi X$, is also likely present.
- The **integrated Boltzmann equation**:

$$\frac{dn_{\mathrm{DM}}}{dt} + 3Hn_{\mathrm{DM}} = -\langle\sigma v\rangle(n_{\mathrm{DM}}^2 - n_{\mathrm{eq}}^2)$$

Application 1: WIMP freeze-out…



m/T = WIMP mass/bath temperature

Solution of the integrated Boltzmann equation for the **comoving WIMP number density**:

$$N_{\mathrm{DM}} \equiv a^3 n_{\mathrm{DM}}$$

- For 10 GeV to 1 TeV mass WIMPs, the solution translates roughly to:

$$\Omega_{\mathrm{DM}} \sim 0.2 \left(\frac{0.7}{h}\right)^2 \left(\frac{3\times10^{-26}\ \mathrm{cm^3 s^{-1}}}{\langle\sigma v\rangle}\right)$$

Application 2: Recombination...

At $T > O(1)$ eV, **Compton scattering** $\gamma e^- \to \gamma e^-$ keeps photons and free electrons (i.e., not in an atom) in equilibrium.

→ At these times, the universe is opaque to photons.

- But the free electron density n_e is governed by:

Recombination

$$e + p \leftrightarrow H + \gamma$$

Neutral hydrogen

$$\frac{dn_e}{dt} + 3Hn_e = -n_e^{\mathrm{eq}} n_p^{\mathrm{eq}} \langle \sigma v \rangle \left[\frac{n_e n_p}{n_e^{\mathrm{eq}} n_p^{\mathrm{eq}}} - \frac{n_H}{n_H^{\mathrm{eq}}} \right]$$

Boltzmann equation for n_e

- When n_e is so low that the photon scattering rate drops below the Hubble rate, the universe becomes transparent to photons → Cosmic microwave background

Take-home message…

In the early universe, an interaction is:

- **In equilibrium** when the interaction rate per particle far exceeds the expansion rate:

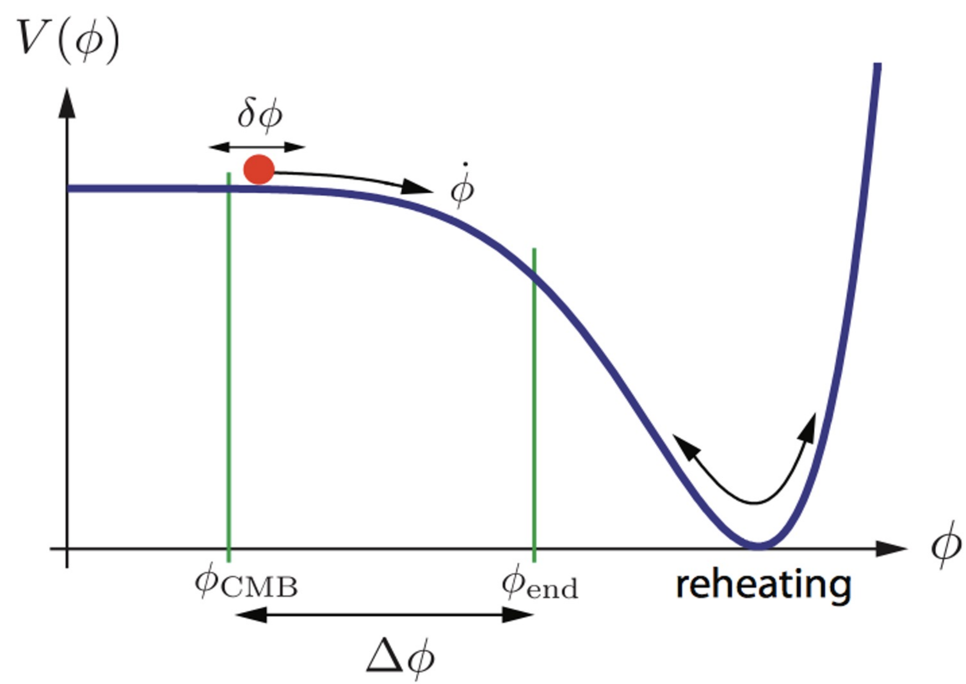
$$\Gamma_{\mathrm{int}} \gg H$$

- **Totally out of equilibrium** when the opposite condition ensues:

$$\Gamma_{\mathrm{int}} \ll H$$

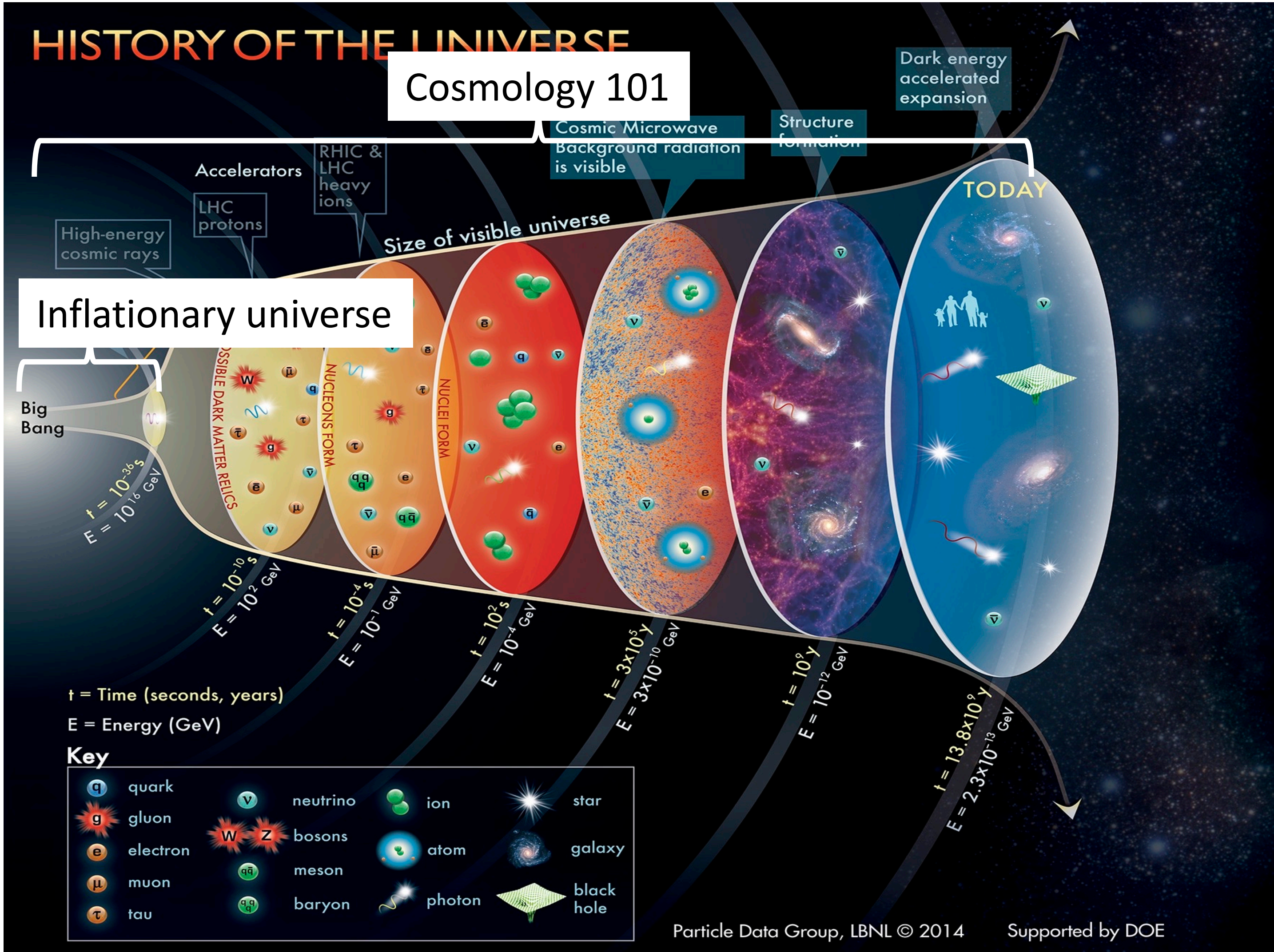
- We use the **Boltzmann equation** to determine how interactions that may not be in equilibrium at all times affect the abundances (i.e., number densities) of the participating particle species.

3. Inflationary universe...



Cosmology 101

Inflationary universe

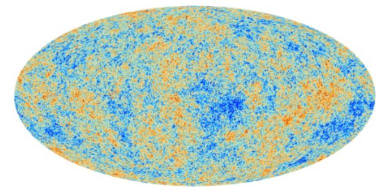


Motivation…

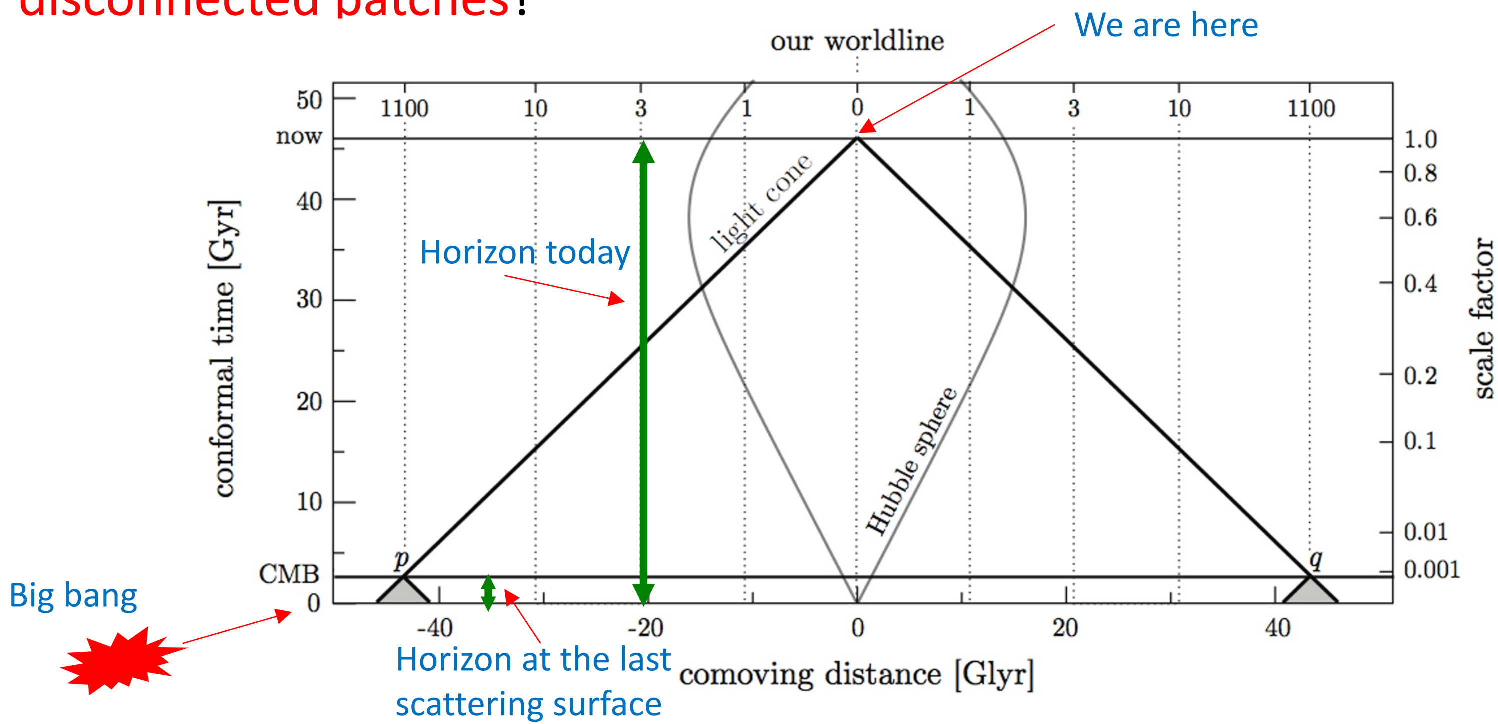
Standard hot big bang (what we've discussed so far) is consistent with observational data.

- Nonetheless, three (arguably philosophical) puzzles motivate the introduction of inflation in cosmology:
    - The **horizon** problem
    - The **flatness** problem
    - The **relic** problem

Motivation 1: the horizon problem…



Why is the CMB **so uniform** even though it is made up of many causally disconnected patches?

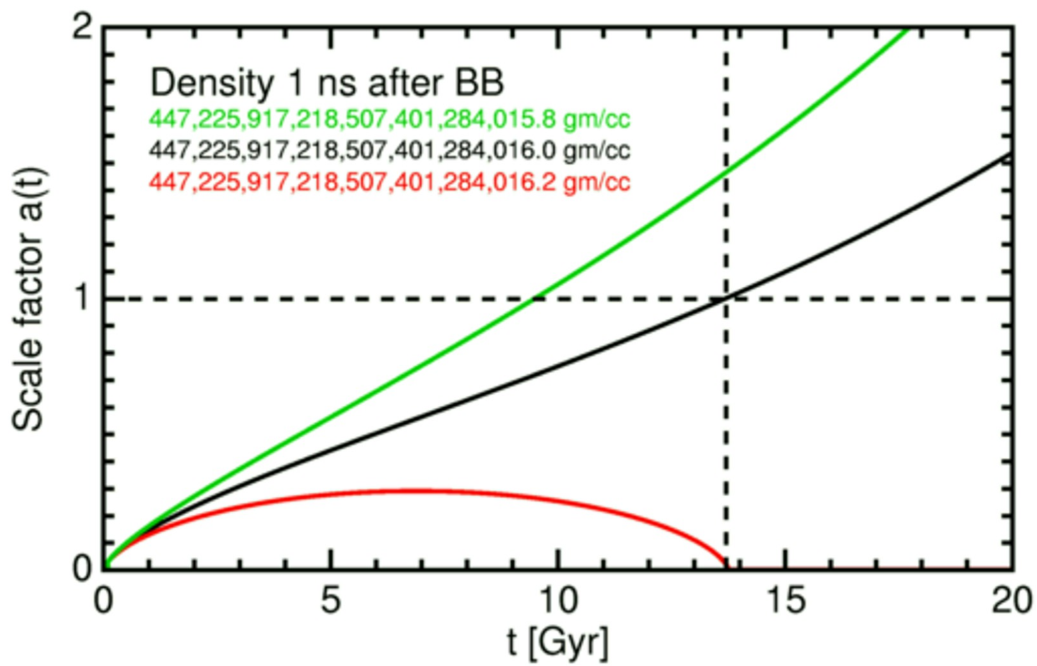


Motivation 2: the flatness problem…

The universe appears to have a flat spatial geometry today; $|\Omega_K| < 0.01$ from observations.

- But in order to appear flat today, the amount of fine-tuning required at Planck time (i.e., at $T \sim M_{\text{planck}}$; $t \sim 10^{-44}$ s) is one part in 10^{60}.

→ **How did that happen**?

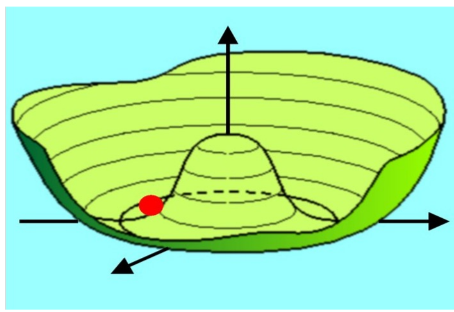


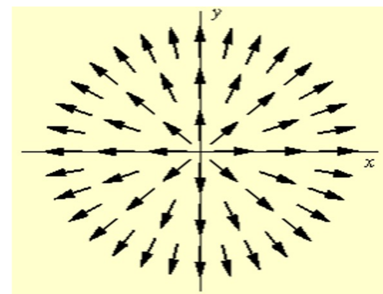
One part in 10^{24} fine-tuning already at 1 ns post BB….

Motivation 3: the relic problem…

Sometimes called the **monopole problem**: many BSM theories (GUTs in particular) predict topological defects from symmetry breaking (monopoles, strings, domain walls, etc.)

Breaking of global $U(1)$





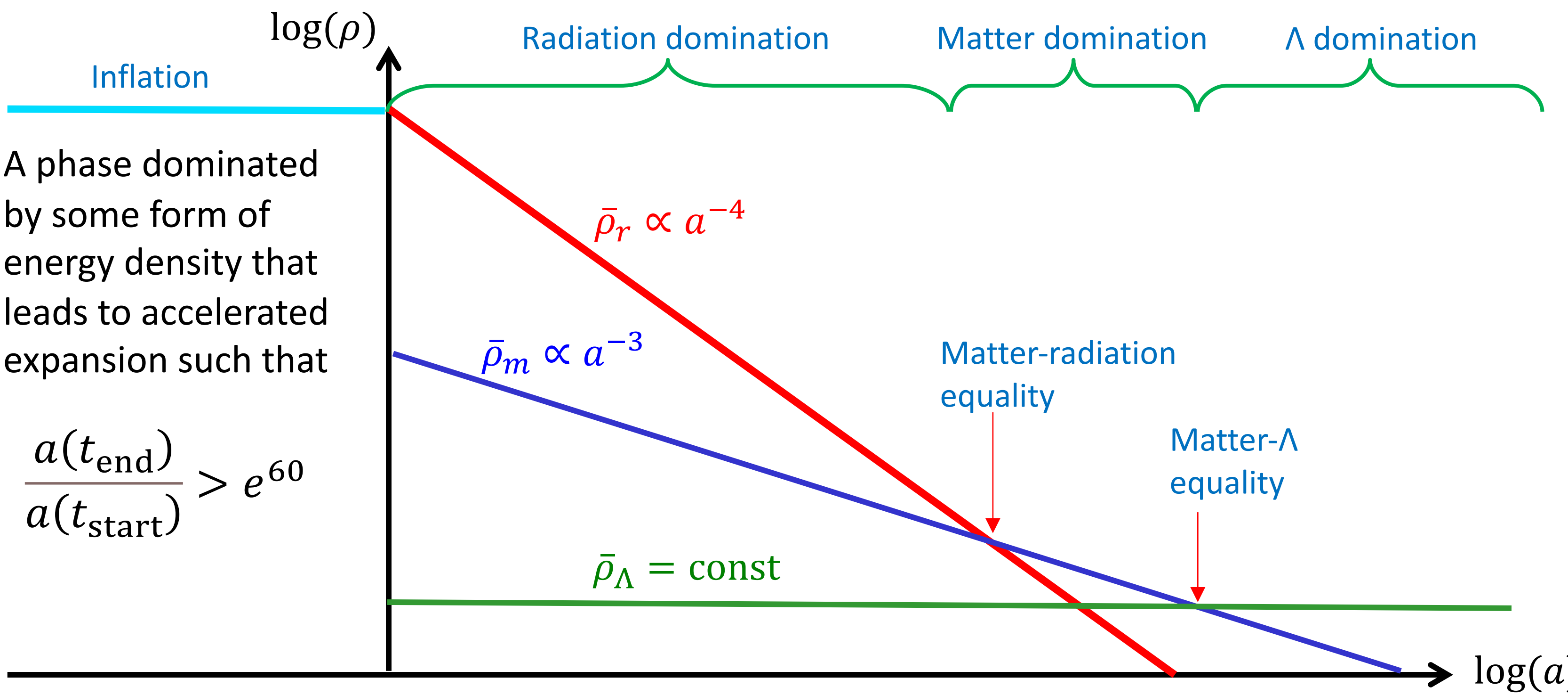
"Hole"

The minimum energy state (the rim of the Mexican hat) has a "hole" when you go around $0 \to 2\pi$: a string

- We generally expect one such defect per causally-connected region at the time of symmetry breaking → must be **many of such defects** in the visible universe today.
- The problem: Why haven't we seen these defects?

Inflation is the solution…

Introduce a phase of accelerated expansion before radiation domination.



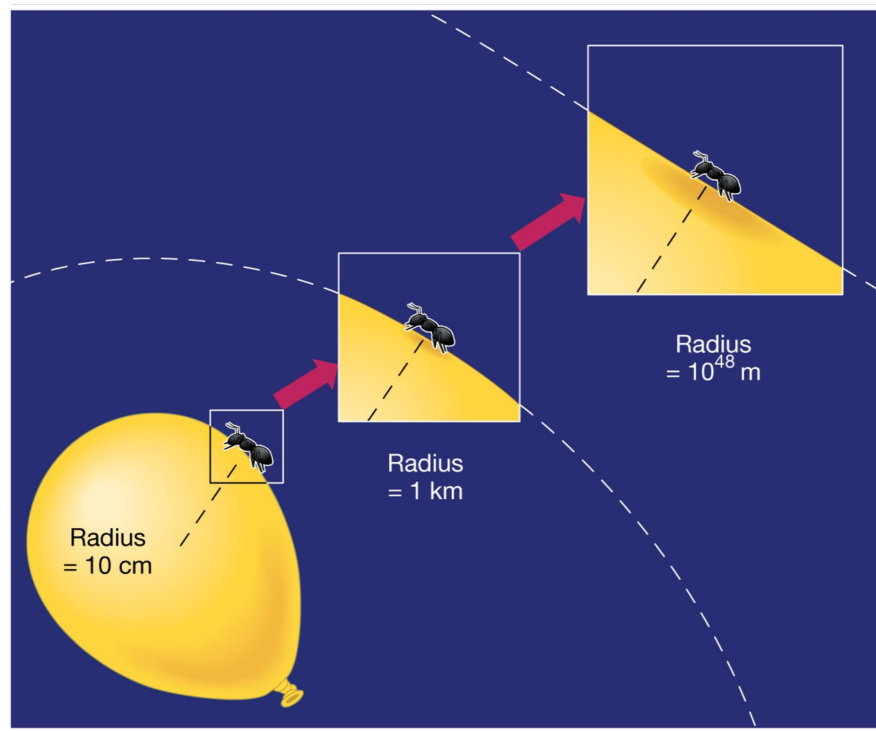
A phase dominated by some form of energy density that leads to accelerated expansion such that

$$\frac{a(t_{\text{end}})}{a(t_{\text{start}})} > e^{60}$$

Inflation is the solution…

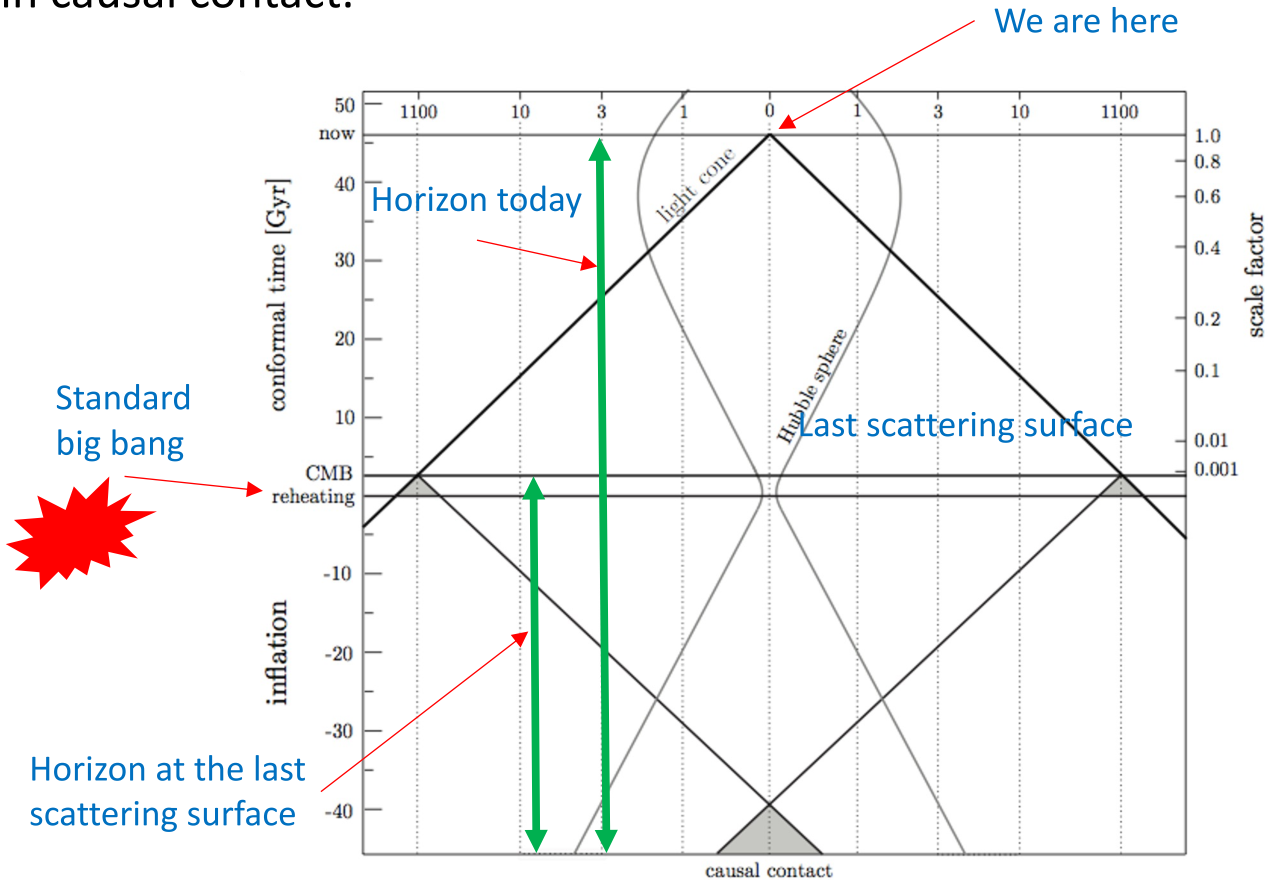
- The **flatness problem** is solved because exponential expansion stretches the radius of curvature of any initial non-flat geometry, so that locally space looks flat.

- The **relic problem** is solved, because the exponential expansion of space dilutes the abundance of defects, provided inflation occurs after their production



cation, Inc.

- The **horizon problem** is solved because the horizon can be made arbitrarily large, so that the part of the last scattering we observer was in some distant past in causal contact.



Implementing inflation using a scalar field…

How do you implement inflation? We can't use vacuum energy, because once Λ dominates, you cannot ever get back to radiation/matter domination.

- Whatever drive inflation must be dynamical: a **scalar field** works.
  - A spatially homogeneous real scalar field ϕ has energy density and pressure given by:

$$\rho_\phi = \frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 + V(\phi) \qquad P_\phi = \frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 - V(\phi)$$

From

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\mathcal{L})}{\partial g^{\mu\nu}}$$

- If the potential exceeds the kinetic term, i.e., $V(\phi) \gg (\partial\phi/\partial t)^2$, then

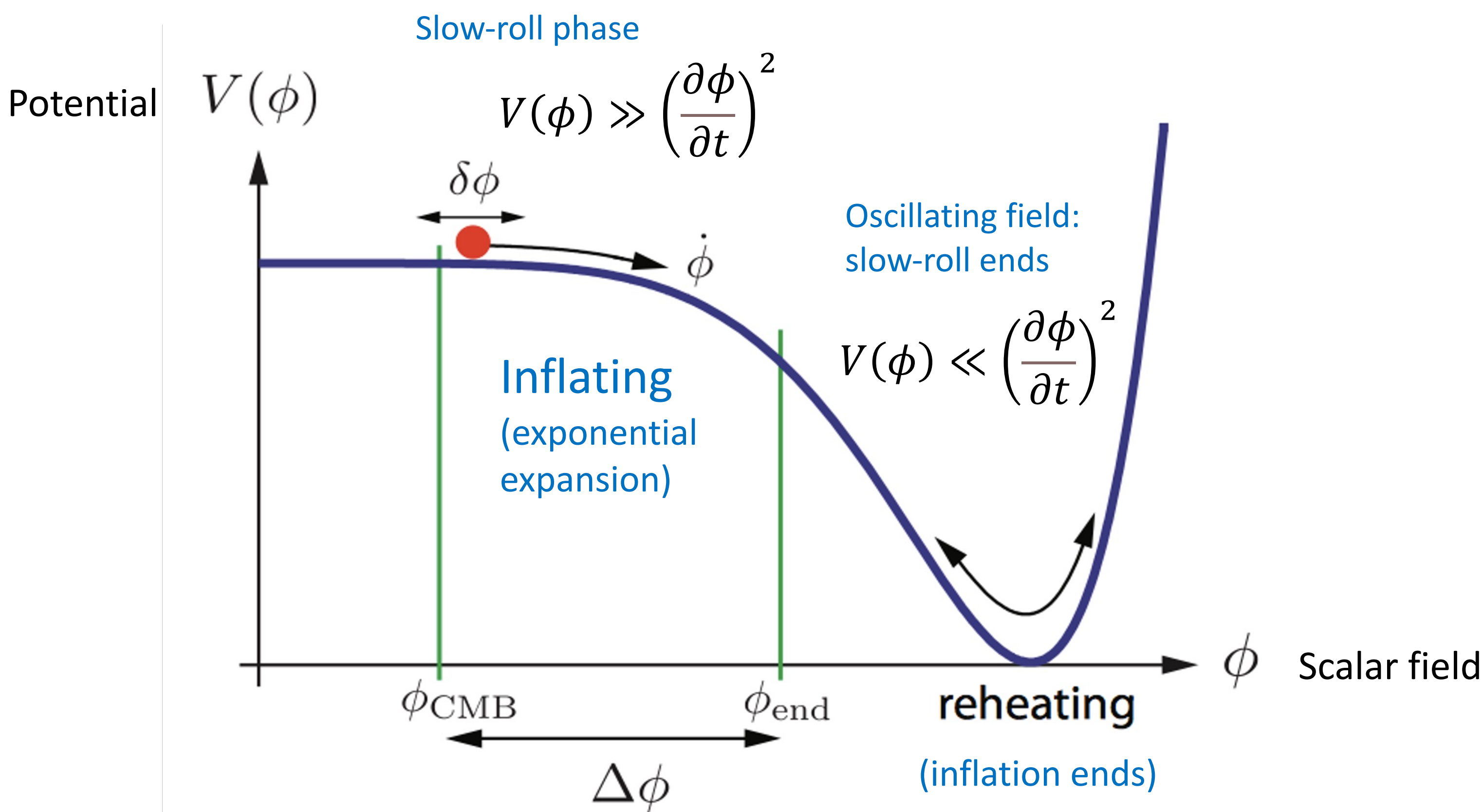
Equation of state parameter for the scalar field.

$$w_\phi = \frac{P_\phi}{\rho_\phi} \simeq -1$$

⇒

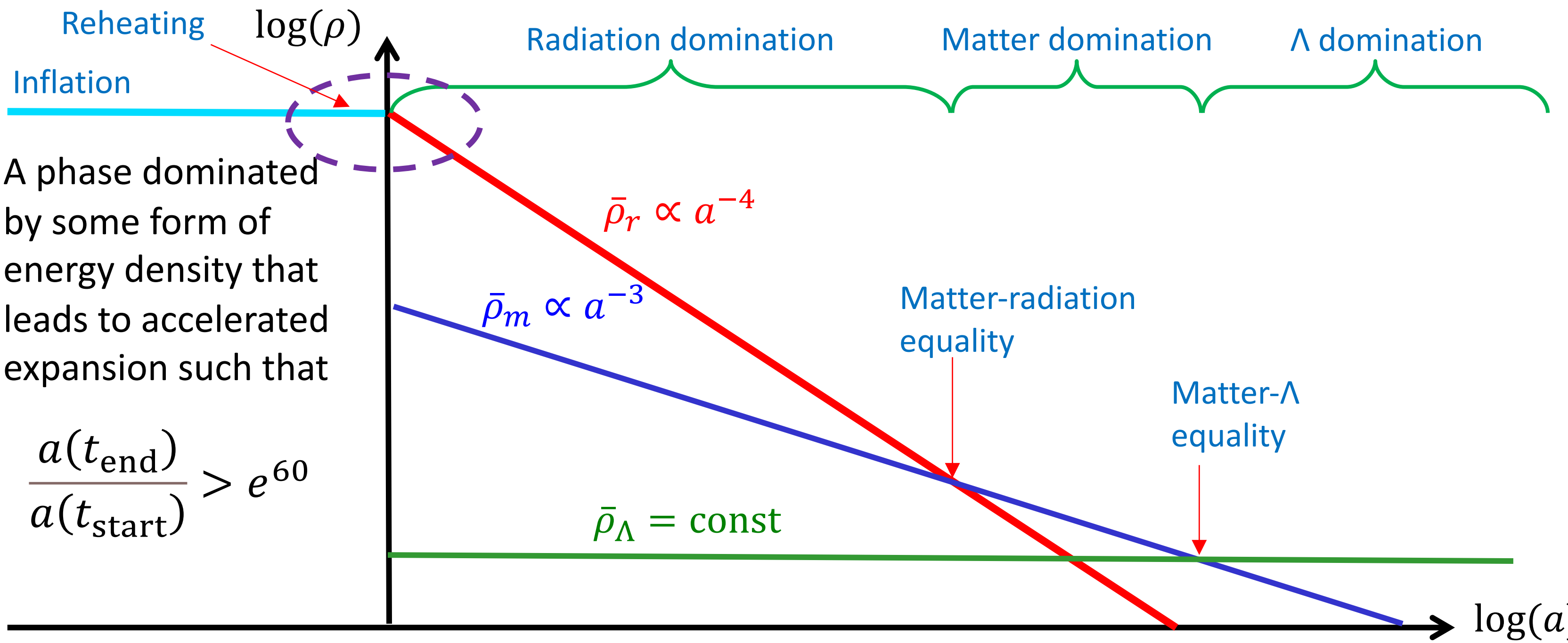
The scalar field will drive a phase of **exponential expansion** if its energy density dominates over everything else.

The basic picture of slow-roll inflation…



After inflation: reheating…

Not well-understood, but the idea is to have the scalar field decay and convert its energy into relativistic (standard model) particles.



A phase dominated by some form of energy density that leads to accelerated expansion such that

$$\frac{a(t_{\text{end}})}{a(t_{\text{start}})} > e^{60}$$

Take-home message…

- Inflation solves a number of problems by postulating a phase of **exponential expansion** before radiation domination.

- The simplest way to implement this idea is to use a scalar field "slowly rolling" down its potential.

    - Inflation ends when the scalar field reaches the bottom of the potential well and starts to oscillate.

    - When all of the energy in the scalar field has been converted into relativistic (standard model) particles somehow (via the process of reheating), radiation domination can begin.