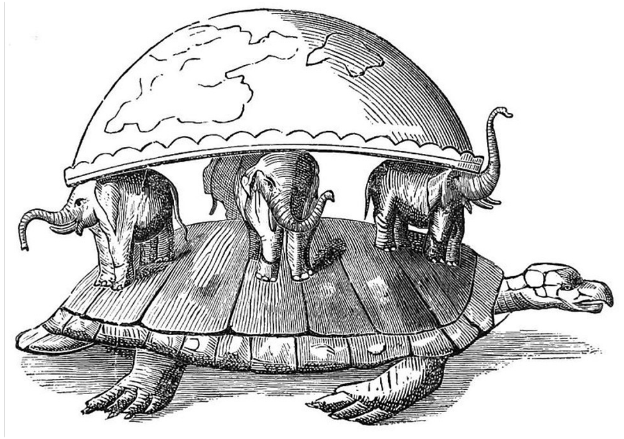
Cosmology

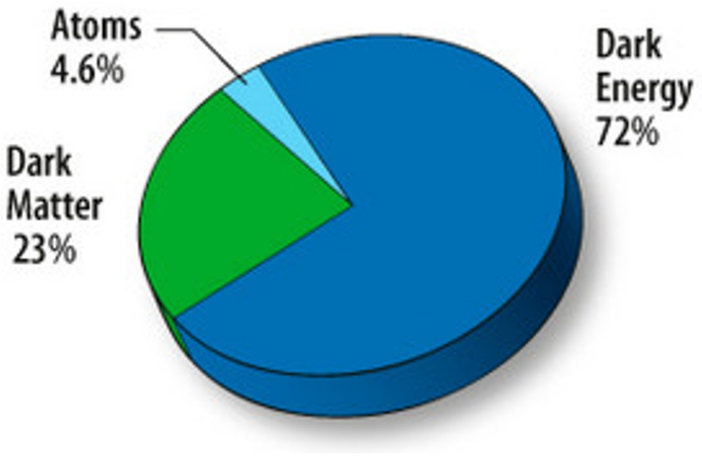


Yvonne Y. Y. Wong, UNSW Sydney

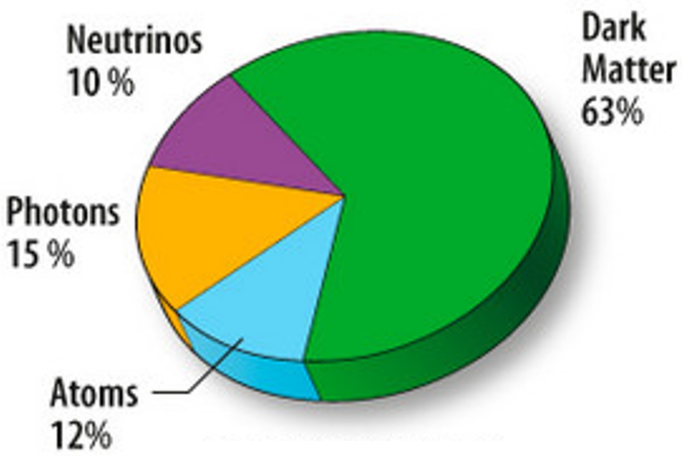
Crosslinks of Early Universe Cosmology, MITP Summer School,
July 15 – August 2, 2024

The concordance flat ΛCDM model...

The simplest model consistent with **present observations**.

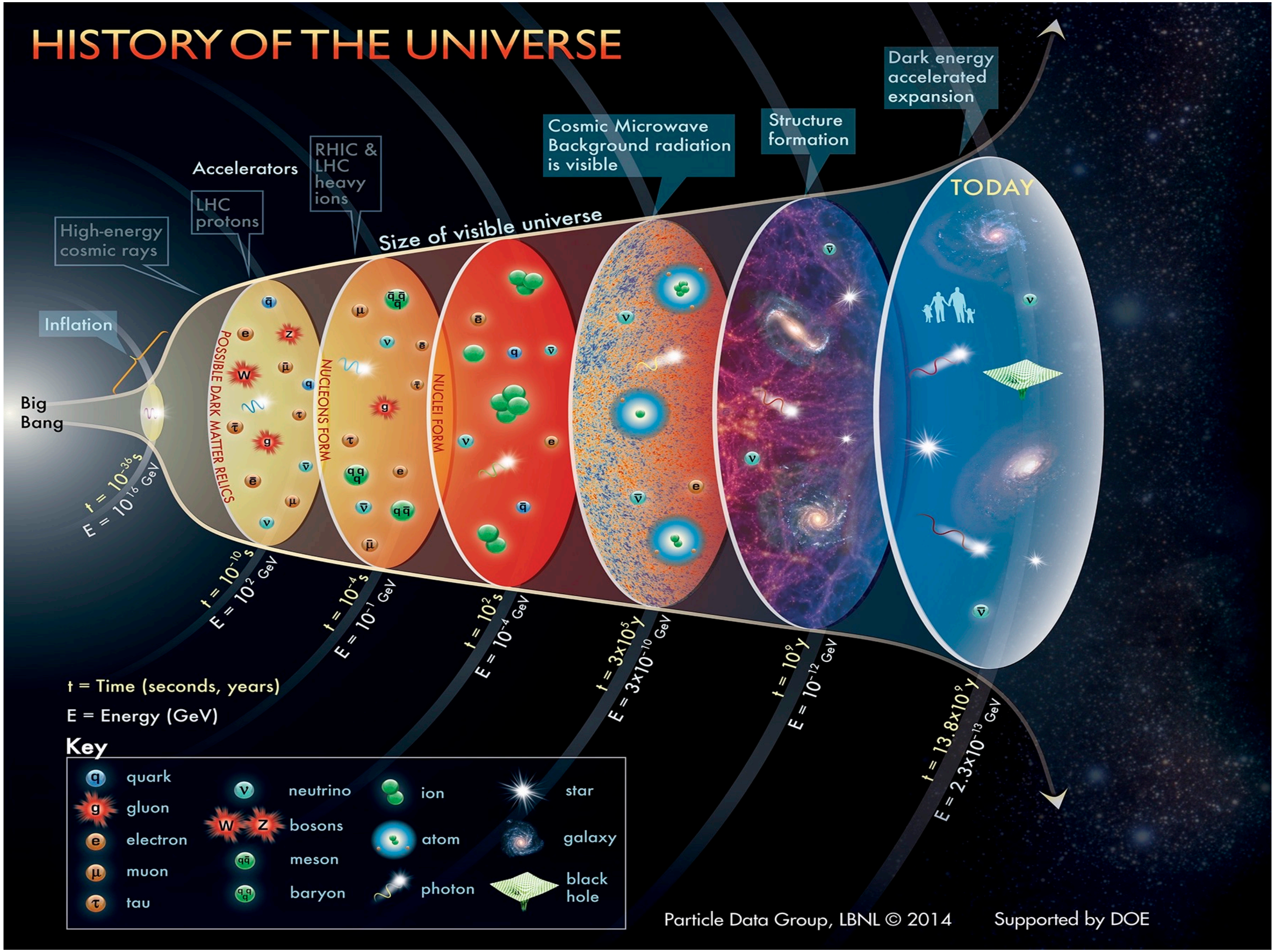


Composition today



13.4 billion years ago
(at photon decoupling)

Plus flat spatial geometry+initial conditions
from single-field inflation



Particle Data Group, LBNL © 2014 Supported by DOE

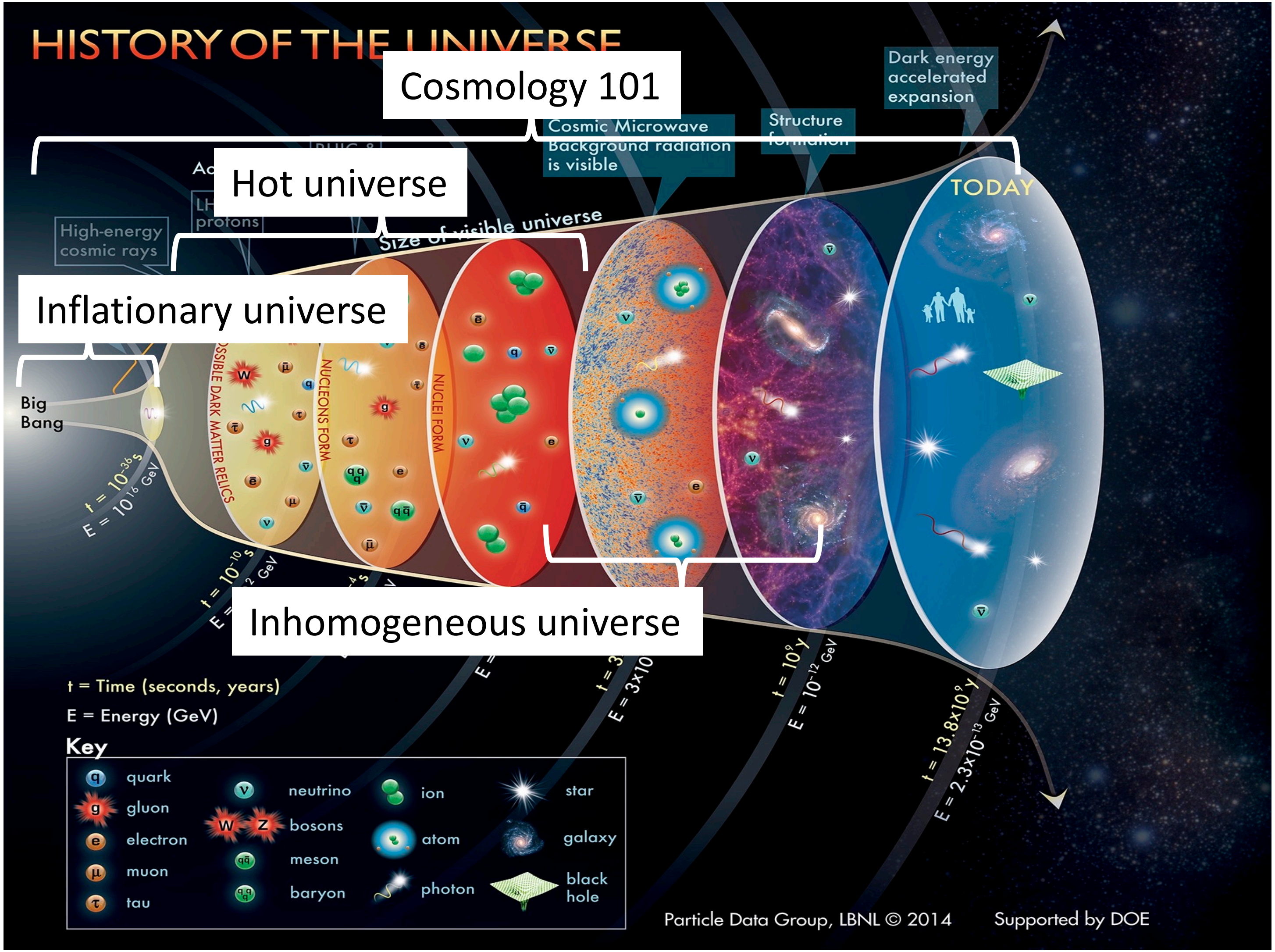
The grand lecture plan…

- **Lecture 1-2: Cosmology 101** (on slides, since I expect you all know this at some level already; I'm just filling in gaps)

    1. Homogeneous and isotropic universe
    2. Hot universe
    3. Inflationary universe

- **Lecture 3-7: Inhomogeneous universe** (mostly on the blackboard)

Cosmology 101



Inflationary universe

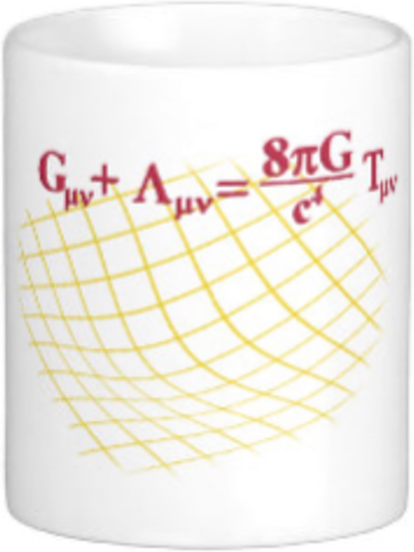
Hot universe

Inhomogeneous universe

Cosmology 101

1. Homogeneous and isotropic universe
2. Hot universe
3. Inflationary universe

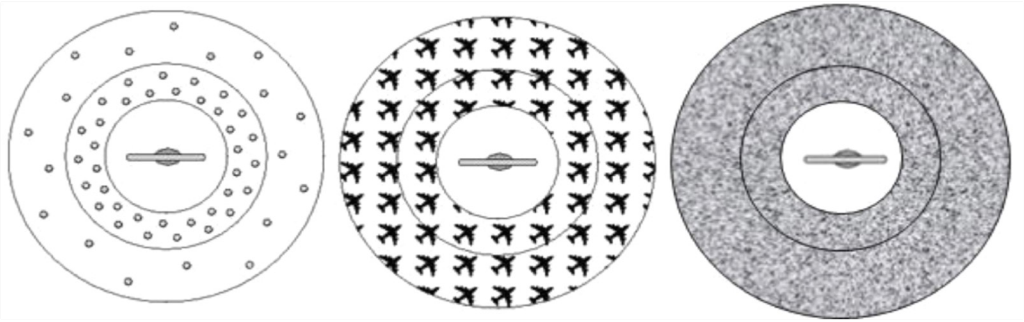
1. Homogeneous and isotropic universe...



FLRW universe…

Cosmological principle: our universe is spatially homogeneous and isotropic on sufficiently large length scales (i.e., we are not special).

- Homogeneous → same everywhere
- Isotropic → same in all directions
- Sufficiently large scales → $> O(100) Mpc$



Isotropic but not homogeneous

Homogeneous but not isotropic

Homogeneous and isotropic

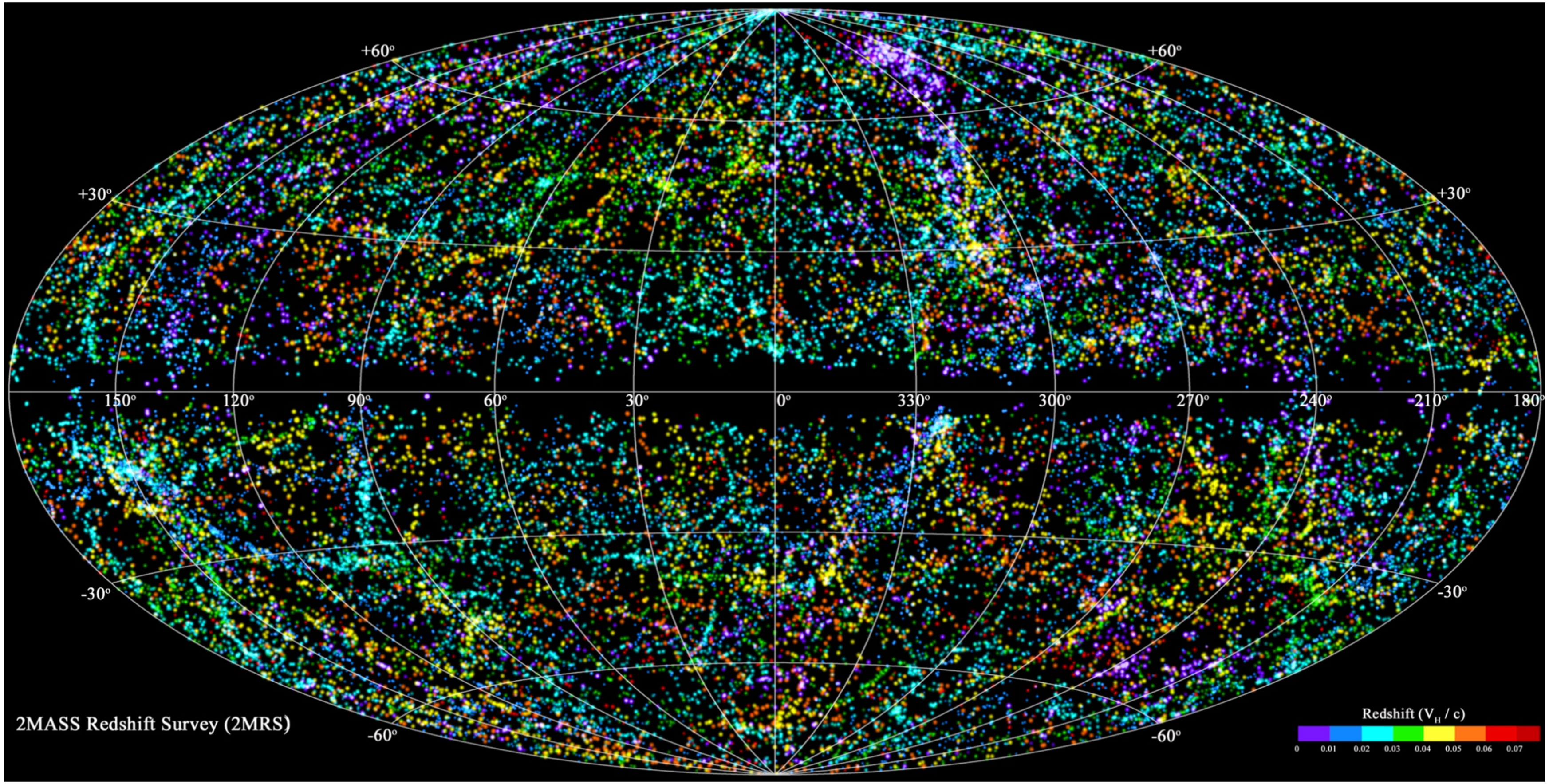
FLRW universe…

Cosmological principle: our universe is spatially homogeneous and isotropic on sufficiently large length scales (i.e., we are not special).

- Homogeneous → same everywhere
- Isotropic → same in all directions
- Sufficiently large scales → $> O(100)Mpc$

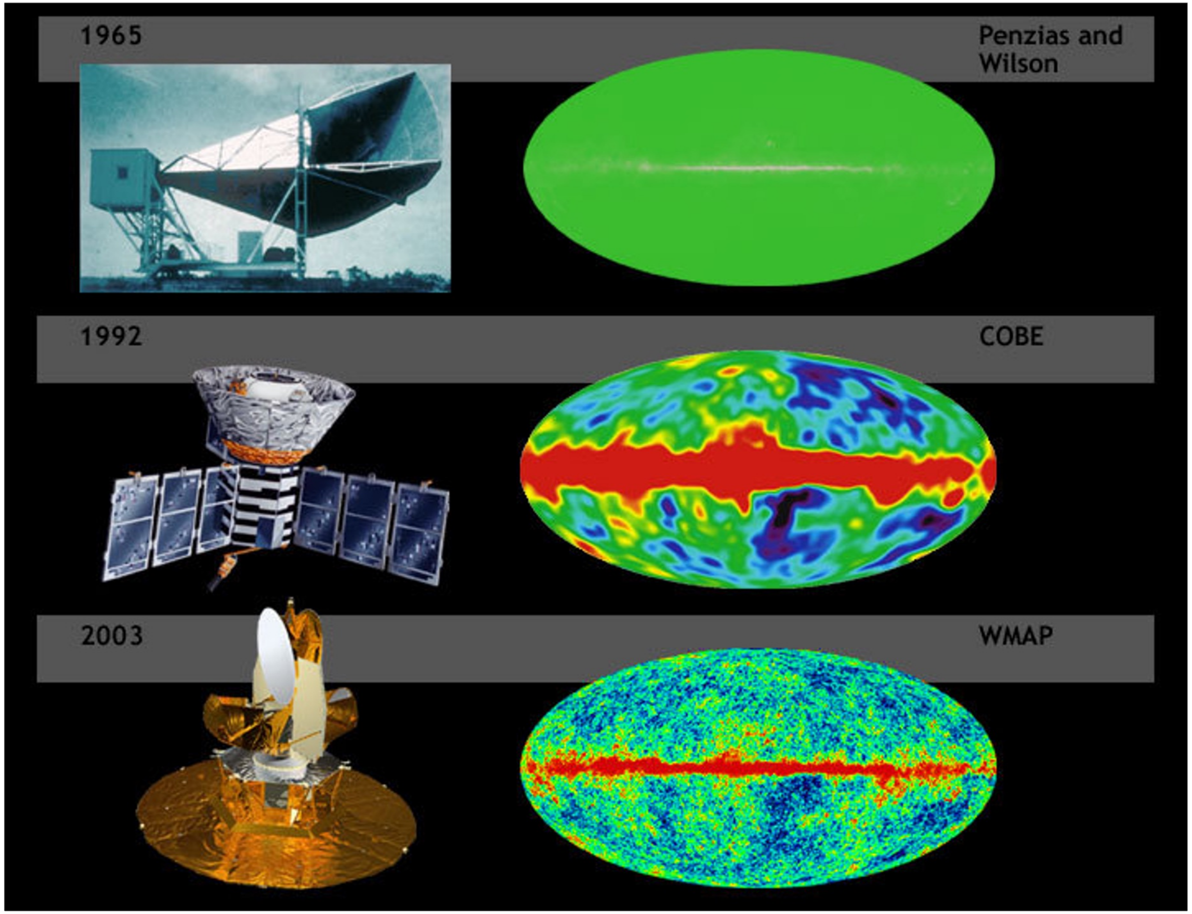
- $1\ \mathrm{pc} = 1\ \mathrm{parsec} = 3.0856\times10^{18}\ \mathrm{cm}$
  - Distance from Sun to Galactic centre $\sim 10\ \mathrm{kpc}$
  - Distance to the Virgo cluster $\sim 20\ \mathrm{Mpc}$
  - Size of the visible universe $\sim O(10\ \mathrm{Gpc})$

Evidence for large-scale homogeneity and isotropy:



Local galaxy distribution as measured by the 2Mass Redshift Survey

Evidence for large-scale homogeneity and isotropy:



Cosmic microwave background (temperature)

2.73 K background

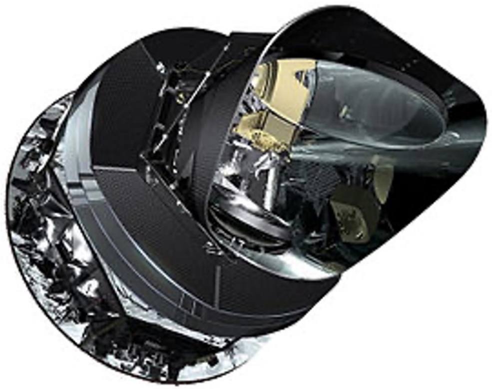
(1978)

$\sim 10^{-5}$ temperature fluctuations
($\sim 7°$ resolution)

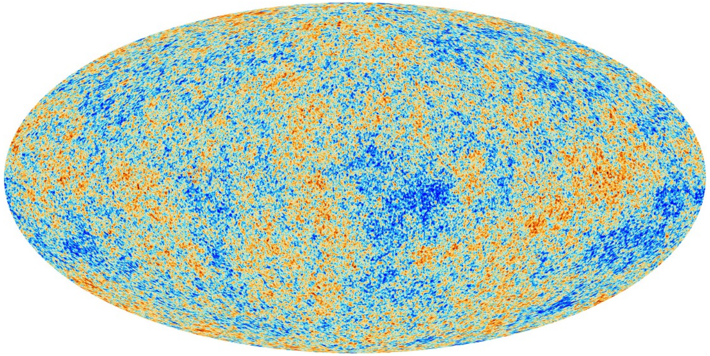
(2006)

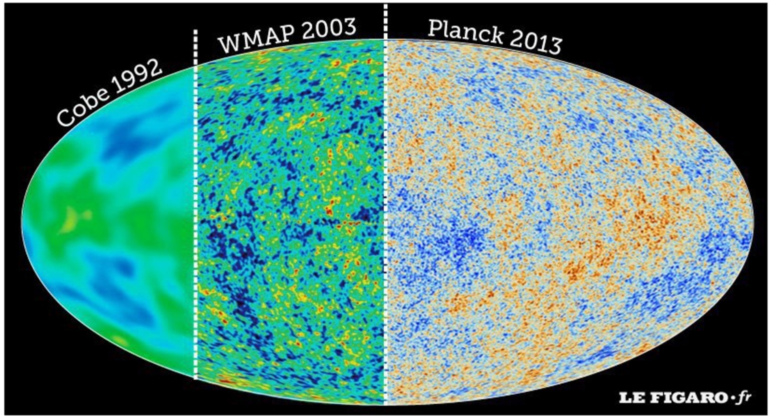
$\sim 0.2°$ resolution

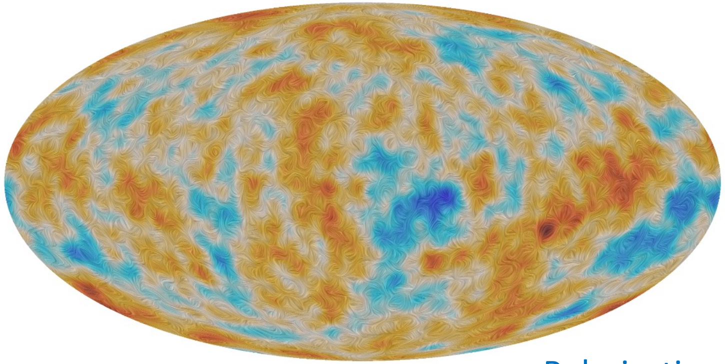
State-of-the-art: Temperature and polarisation fluctuations in the **cosmic microwave background** as seen by Planck. (Latest results 2018)



Temperature







Polarisation

FLRW universe…

Homogeneity and isotropy imply maximally symmetric 3-spaces (3 translational and 3 rotational symmetries).

- A spacetime geometry that satisfies these requirements is the Friedmann-Lemaître-Robertson Walker (FLRW) geometry:

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta \, d\phi^2)\right]$$

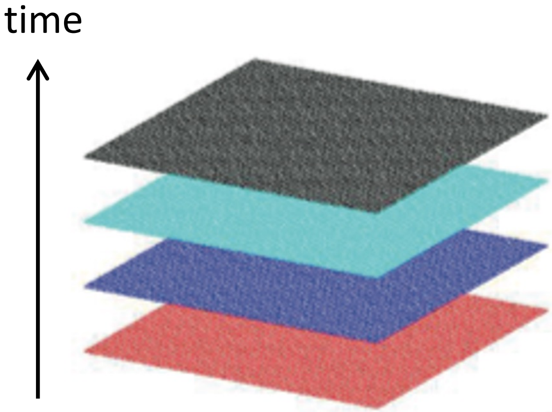
FLRW metric

$a(t)$ = scale factor

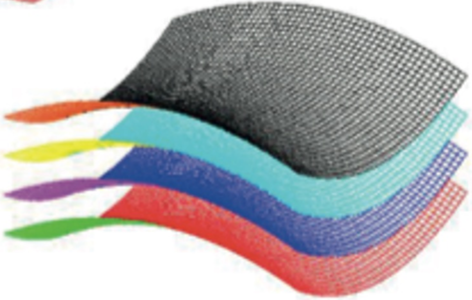
Spatial geometry
$K \;=\; -1$ (hyperbolic), 0 (flat), $+1$ (spherical)

- $\frac{a(t_2)}{a(t_1)} =$ factor by which a physical length scale increases between time t_1 and t_2.

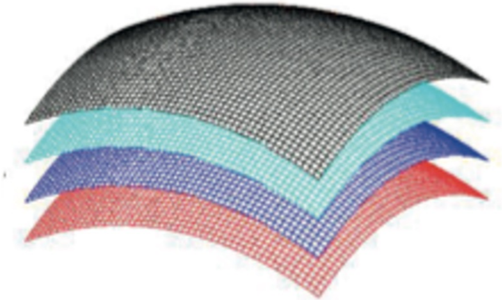
$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$



flat $K = 0$



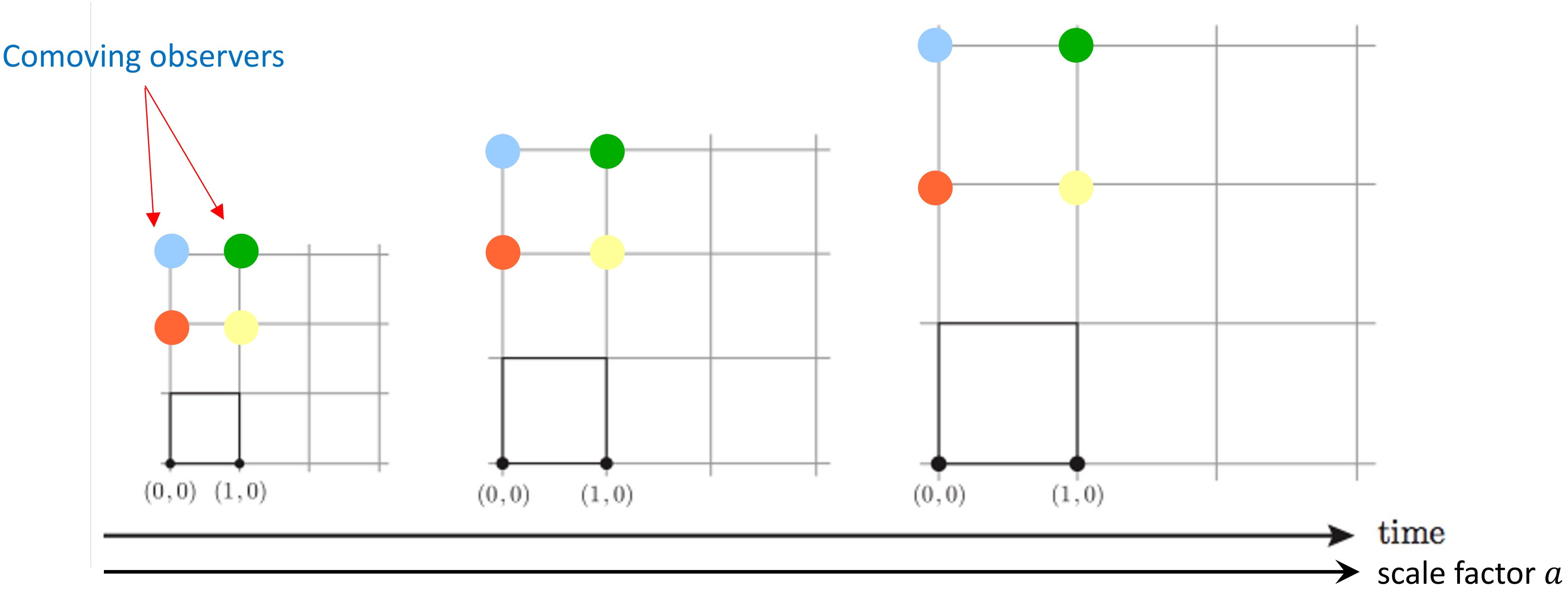
negatively curved $K = -1$



positively curved $K = +1$

An observer at rest with the FLRW spatial coordinates is a **comoving observer**.

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

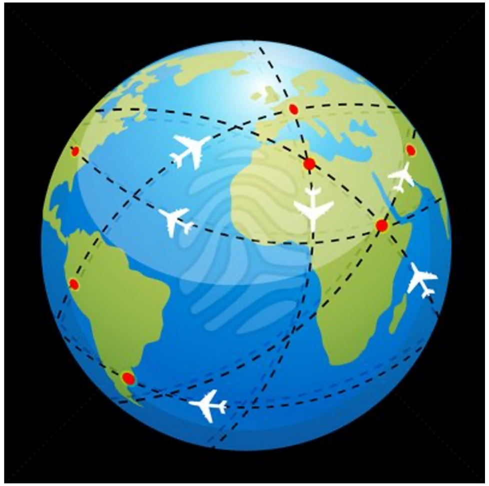


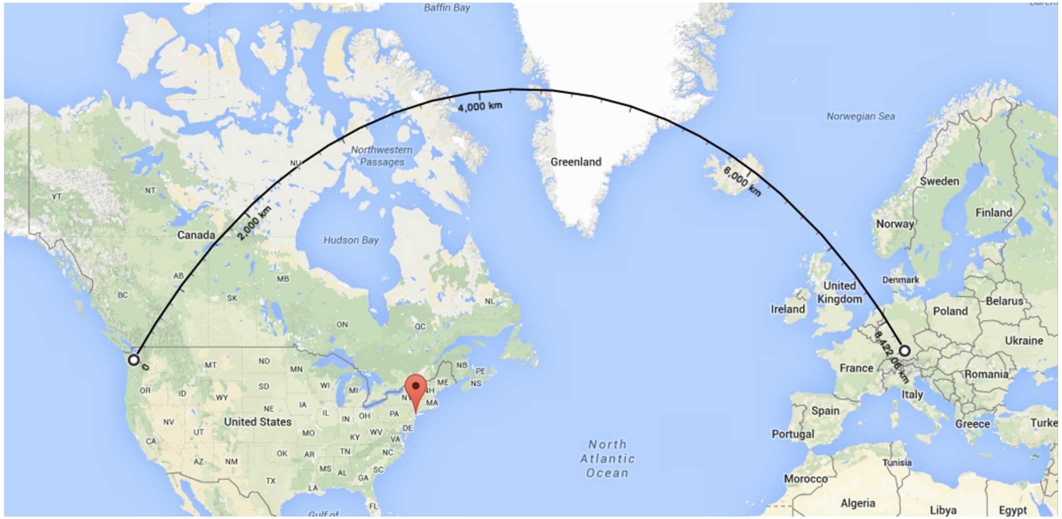
→ The **physical distance** between two comoving observers increases with time, but the coordinate distance between them remains unchanged.

Geodesics…

In the **absence of other forces**, test particles move on the geodesics of a spacetime geometry, i.e., the “straight lines” of a curved spacetime.

- It's like flight paths, which follow (more or less) the geodesics on the surface of the Earth.





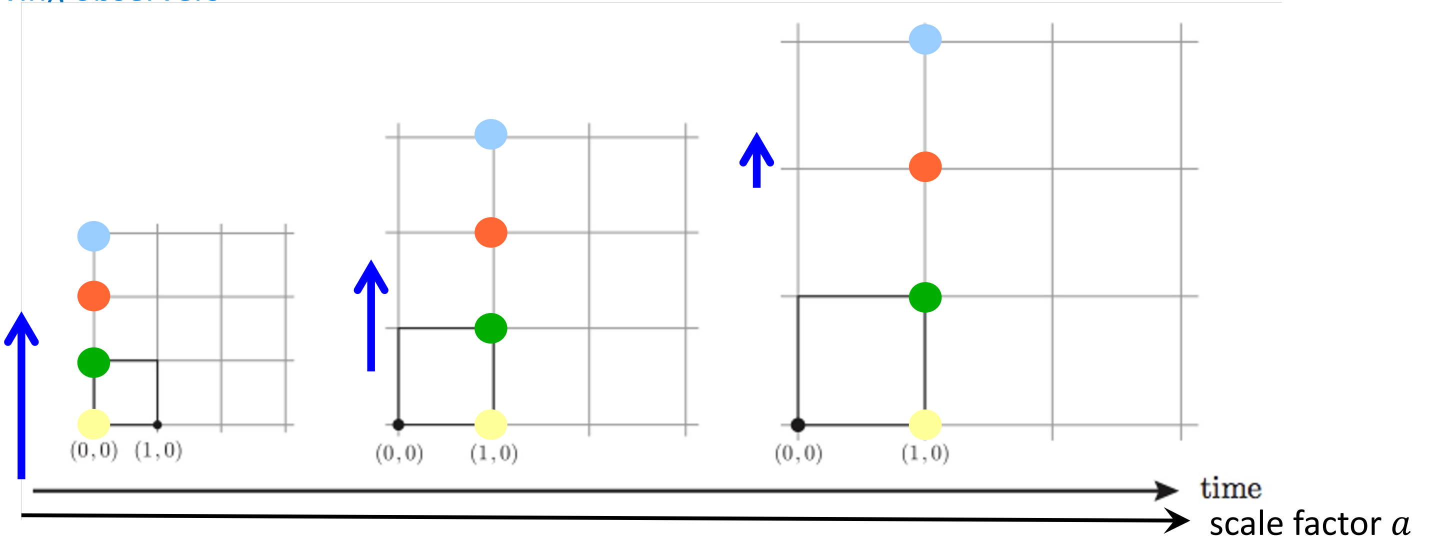
Geodesics and cosmological redshift...

All test particles (massive or massless) moving on geodesics of an FLRW universe suffer cosmological redshift:

Momentum of a point particle measured by comoving observers

$$|\vec{p}| \propto a^{-1}$$

⇒ Momentum of a particle decreases with expanding space.



Or in terms of wavelength: $\lambda \propto 1/|\vec{p}|$

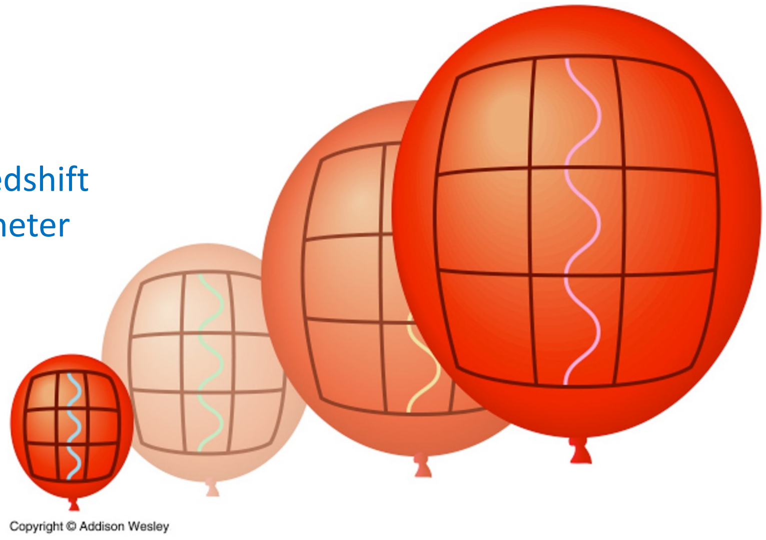
Wavelength measured
by comoving observer

t_0= today

z = Redshift
parameter

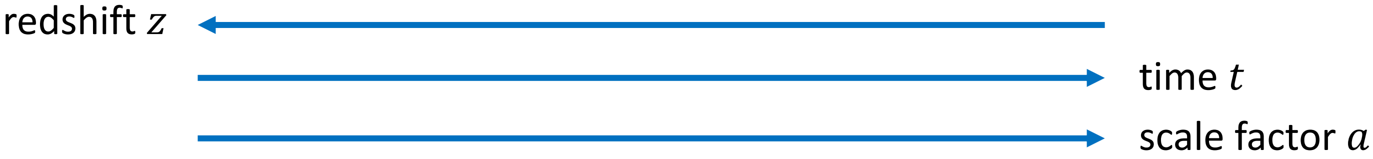
$$\frac{\lambda_0}{\lambda_e} = \frac{a(t_0)}{a(t_e)} \equiv 1 + z$$

Wavelength of particle
(usually photon) emitted
by comoving emitter



- A particle emitted at a very early time t when the scale factor a was very small would be observed today with a very large redshift z

→ There is a one-to-one correspondence between t, a, and z:


redshift z

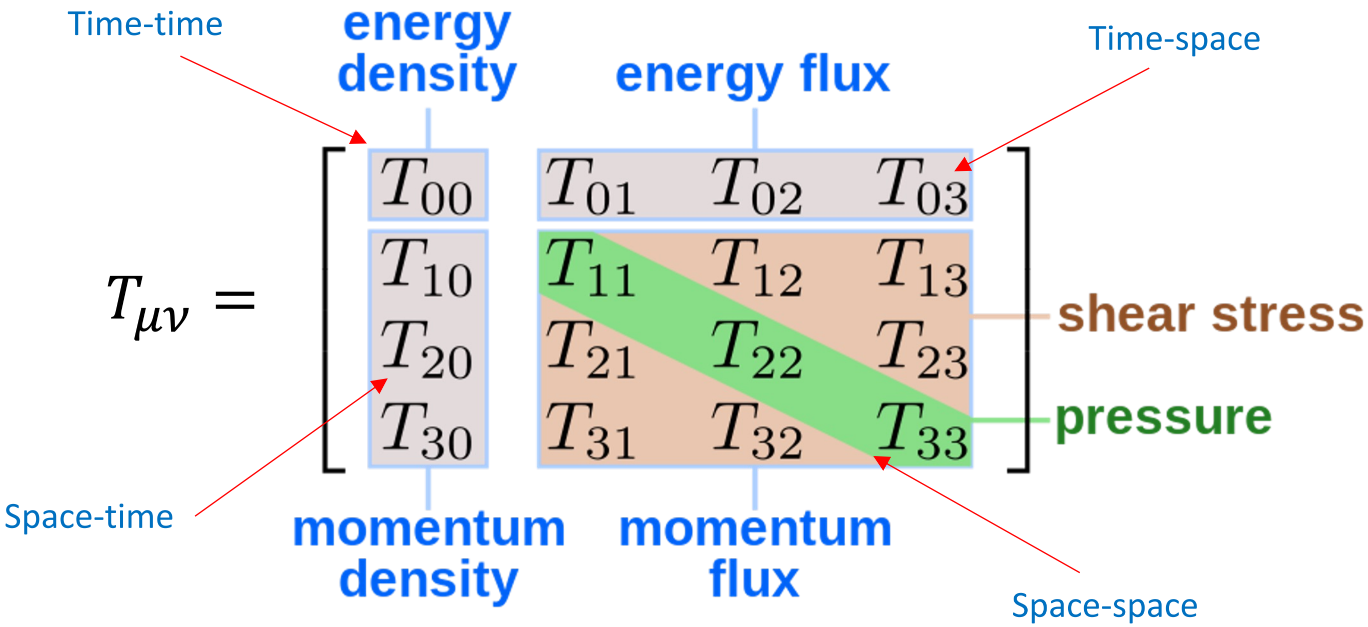
time t

scale factor a

→ We use them interchangeably as a measure of time.

Matter/energy content (stuff in the universe).

In GR, the stress-energy tensor $T_{\mu\nu}$ encodes the matter/energy content.



Time-time

energy density

energy flux

Time-space

$$T_{\mu\nu} = \begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

shear stress

pressure

Space-time

momentum density

momentum flux

Space-space

Matter/energy content (stuff in the universe).

In GR, the stress-energy tensor $T_{\mu\nu}$ encodes the matter/energy content.

- Homogeneity and isotropy imply **only one viable form**:

ρ_α = Energy density (energy per unit volume) of substance α in its rest frame

$$T^{\mu}{}_{\nu(\alpha)} = \begin{pmatrix} -\rho_\alpha(t) & 0 & 0 & 0 \\ 0 & P_\alpha(t) & 0 & 0 \\ 0 & 0 & P_\alpha(t) & 0 \\ 0 & 0 & 0 & P_\alpha(t) \end{pmatrix}$$

P_α = Pressure of substance α in its rest frame

Metric convention $(-,+,+,+)$

- $\rho(t)$ and $P(t)$ can depend on time, but **not** on the spatial coordinates.

→ How do they evolve with time?

Matter/energy content: conservation law…

Local conservation of energy-momentum in an FLRW universe implies:

Energy density

Pressure

$$\frac{d\rho_\alpha}{dt} + 3\frac{\dot{a}}{a}(\rho_\alpha + P_\alpha) = 0$$

Continuity equation (from $\nabla_\mu T^{\mu\nu}_{(\alpha)} = 0$)

- There is **one such continuity equation for each substance $\boldsymbol{\alpha}$**.
- We need in addition to specify a relation between $\rho(t)$ and $P(t)$, i.e., the **equation of state** of the substance α, which is a property of the substance.
    - It's common to use an equation of state parameter w: $w_\alpha(t) \equiv \dfrac{P_\alpha(t)}{\rho_\alpha(t)}$
    - Assuming a constant w: $\rho_\alpha(t) \propto a^{-3(1+w_\alpha)}$ ➡ How energy density evolves with the scale factor.

Matter/energy content: what's there?

$\rho_\alpha(t) \propto a^{-3(1+w_\alpha)}$

- **Non-relativistic** matter
  - Atoms (or constituents thereof)
  - Dark matter (does not emit light but feels gravity); GR people call it "dust"

$w_m \simeq 0$

$\Rightarrow \rho_m \propto a^{-3}$

Volume expansion

- **Ultra-relativistic** radiation
  - Photons (main the CMB)
  - Relic neutrinos
  - Gravitational waves

$w_r = 1/3$

$\Rightarrow \rho_r \propto a^{-4}$

Volume expansion
+ momentum redshift

- **Other** funny things
  - Cosmological constant/vacuum energy
  - ??

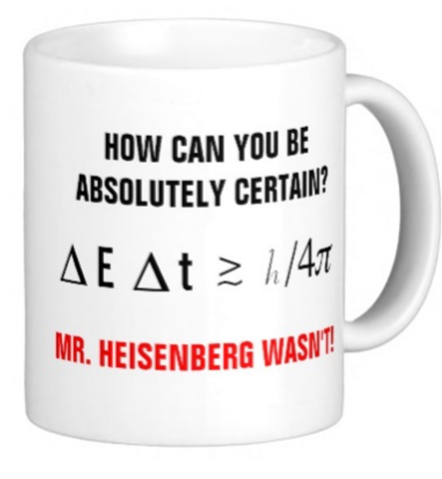
$w_\Lambda = -1$

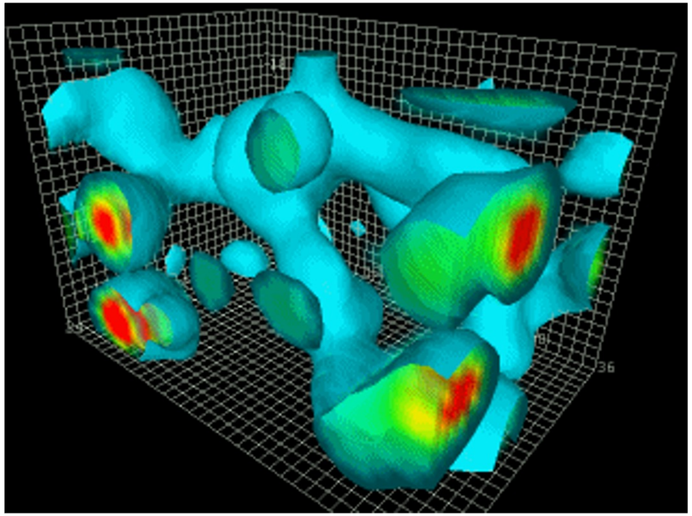
$\Rightarrow \rho_\Lambda \propto \text{constant}$

More space,
more energy

Vacuum energy as the cosmological constant.

Heisenberg's uncertainty principle permits the temporary appearance of virtual particles in otherwise empty space.





D. Leinweber,
U. Adelaide,
simulation of
the QCD vacuum

- Good candidate for the cosmological constant, but we will probably need a theory of quantum gravity to see if the numbers really work out.

Matter/energy content: what’s there?

- **Non-relativistic** matter
  - Atoms (or constituents thereof)
  - Dark matter (does not emit light but feels gravity); GR people call it “dust”

$w_m \simeq 0$

$\Rightarrow \rho_m \propto a^{-3}$

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  - Photons (main the CMB)
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$w_r = 1/3$

$\Rightarrow \rho_r \propto a^{-4}$

- **Other** funny things
  - Cosmological constant/vacuum energy
  - ??

$w_\Lambda = -1$

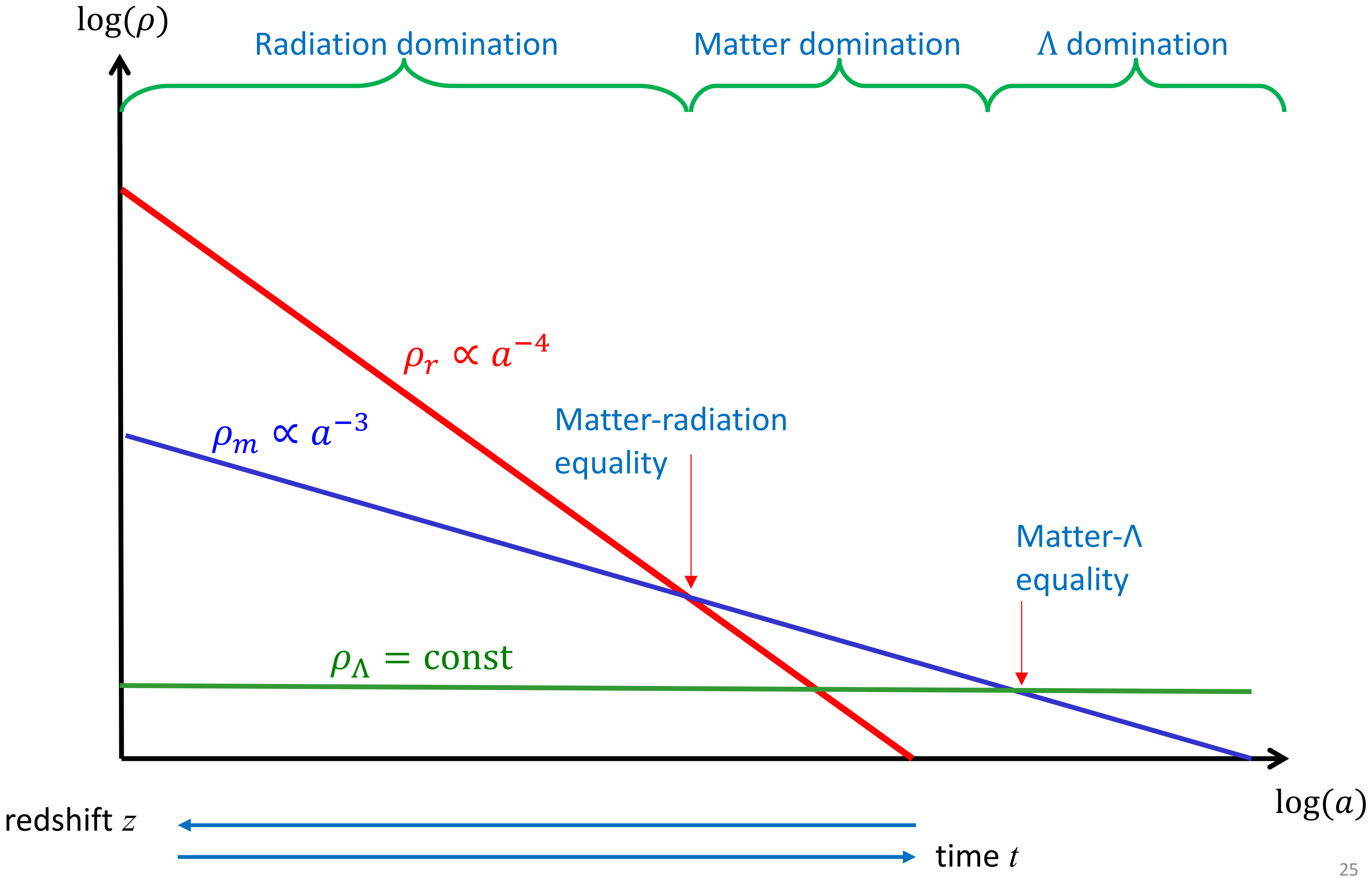
$\Rightarrow \rho_\Lambda \propto \text{constant}$

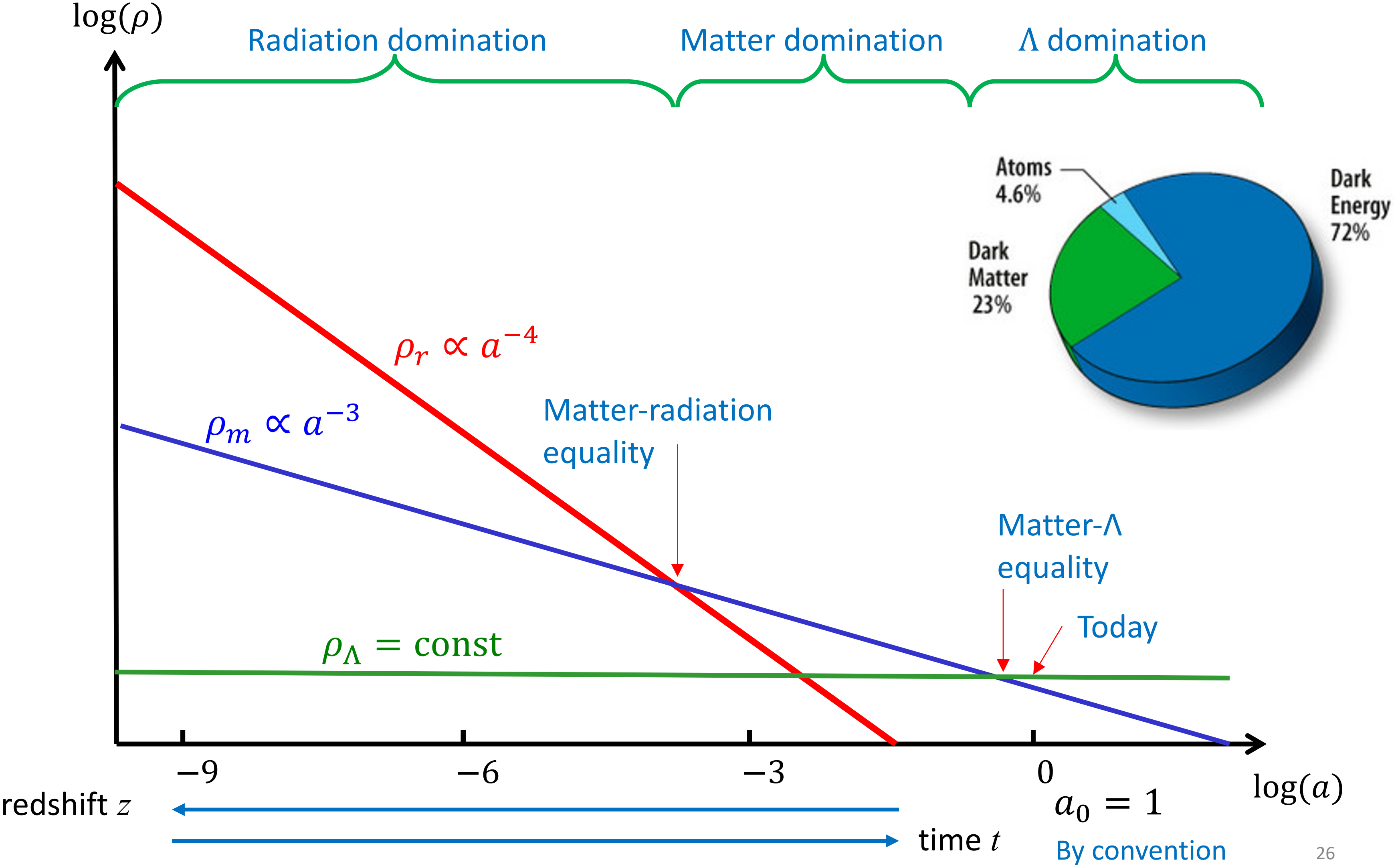
As $a \longrightarrow 0$:

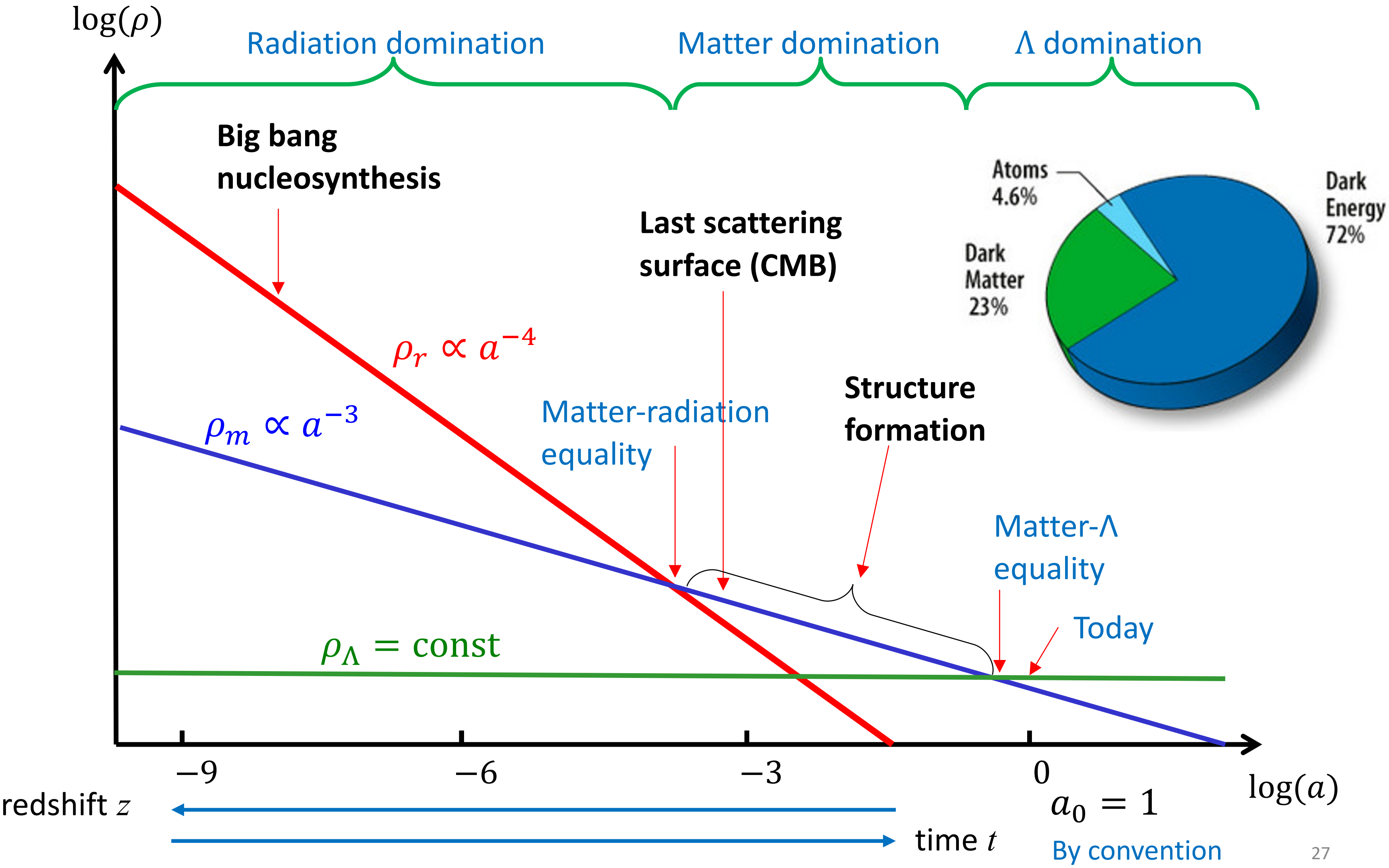
$\bar{\rho}_r \gg \bar{\rho}_m > \bar{\rho}_\Lambda$

As $a \longrightarrow \infty$:

$\bar{\rho}_\Lambda \gg \bar{\rho}_m > \bar{\rho}_r$


$\log(\rho)$
Radiation domination
Matter domination
Λ domination
$\rho_r \propto a^{-4}$
$\rho_m \propto a^{-3}$
Matter-radiation equality
Matter-Λ equality
$\rho_\Lambda = \text{const}$
$\log(a)$
redshift z
time t


$\log(\rho)$
Radiation domination
Matter domination
Λ domination
Atoms 4.6%
Dark Energy 72%
Dark Matter 23%
$\rho_r \propto a^{-4}$
$\rho_m \propto a^{-3}$
Matter-radiation equality
Matter-Λ equality
Today
$\rho_\Lambda = \text{const}$
-9
-6
-3
0
$\log(a)$
redshift z
$a_0 = 1$
time t
By convention



Radiation domination

Matter domination

Λ domination

Big bang nucleosynthesis

$\rho_r \propto a^{-4}$

$\rho_m \propto a^{-3}$

$\rho_\Lambda = \text{const}$

Last scattering surface (CMB)

Matter-radiation equality

Structure formation

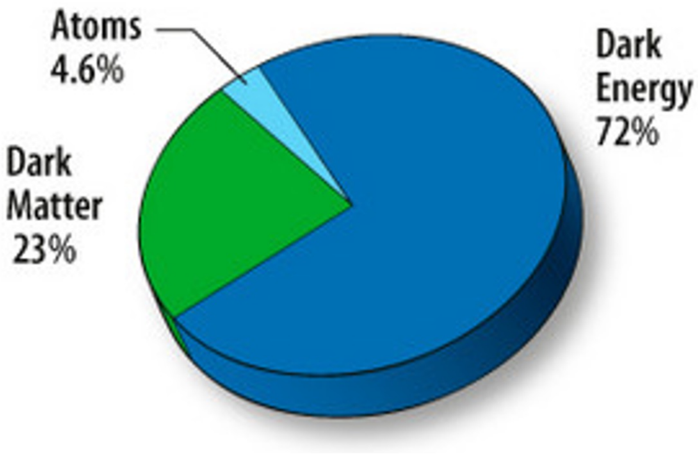
Matter-Λ equality

Today

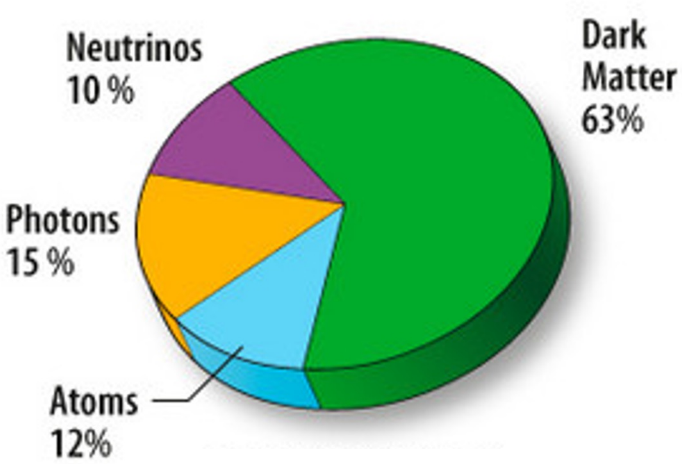
$a_0 = 1$

By convention

Different evolution for different forms of energy densities means that radiation dominated in the early universe, while dark energy was unimportant.



Composition today



13.4 billion years ago
(at photon decoupling)

Friedmann equation…

The Friedmann equation describes the evolution of the scale factor $a(t)$.

G = Gravitational constant

$$H^2(t) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\sum_{\alpha}\rho_\alpha - \frac{K}{a^2}$$

$H(t)$ = Hubble parameter

Spatial curvature: $K = 0, +1, -1$

Some over all forms of energy density

- The Friedman equation is itself derived from Einstein's equation:

R = Ricci scalar and tensor (nonlinear functions of the 2nd derivative of the spacetime metric)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$$

Stress-energy tensor

Friedmann equation…

We may also have seen the Friedmann equation in this form:

$$H^2(t) = H^2(t_0)[\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_\Lambda + \Omega_K a^{-2}]$$

Present-day reduced energy density

$$\Omega_\alpha = \frac{\bar{\rho}_\alpha(t_0)}{\rho_{\text{crit}}(t_0)}, \qquad \rho_{\text{crit}}(t) \equiv \frac{3H^2(t)}{8\pi G}, \qquad \Omega_K \equiv -\frac{K}{H^2(t_0)}$$

Critical density

- A **flat universe** means

$$\Omega_K = 0 \quad \Rightarrow \quad \Omega_m + \Omega_r + \Omega_\Lambda \simeq \Omega_m + \Omega_\Lambda = 1$$

Radiation energy density is negligibly small today:

From measuring the CMB temperature a and energy spectrum:

$$\Omega_r \sim 10^{-5}$$

Friedmann equation…

We may also have seen the Friedmann equation in this form:

$$H^2(t) = H^2(t_0)[\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_\Lambda + \Omega_K a^{-2}]$$

Present-day reduced energy density

$$\Omega_\alpha = \frac{\bar{\rho}_\alpha(t_0)}{\rho_{\rm crit}(t_0)}, \qquad \rho_{\rm crit}(t) \equiv \frac{3H^2(t)}{8\pi G}, \qquad \Omega_K \equiv -\frac{K}{H^2(t_0)}$$

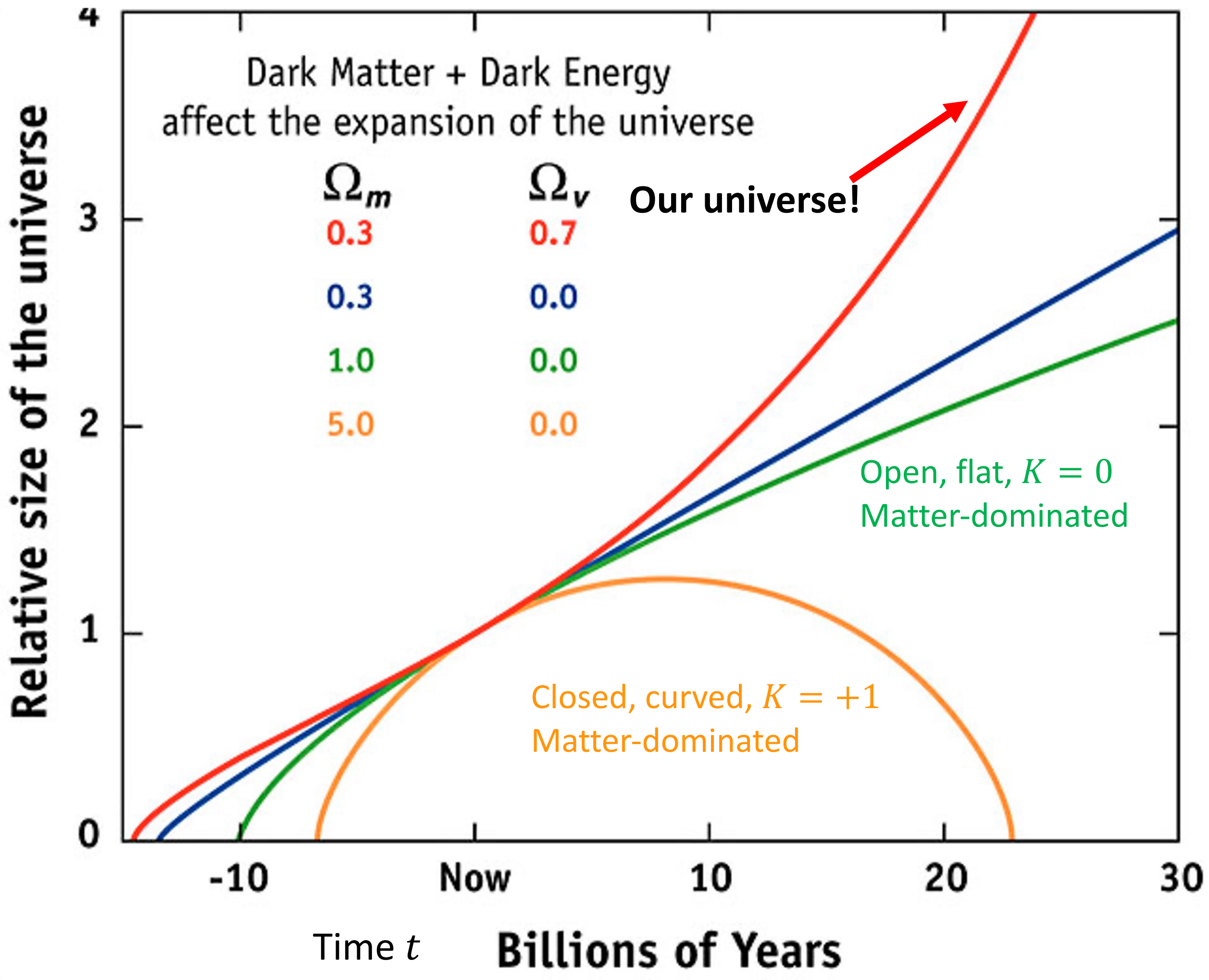
Critical density

- From **current observations**:

$$\Omega_m \sim 0.3, \qquad \Omega_\Lambda \sim 0.7, \qquad |\Omega_K| < 0.01$$
$$H_0 \equiv H(t_0) \sim 70\ \mathrm{kms^{-1}Mpc^{-1}}$$

e.g., Aghanim et al. [Planck collaboration] (2019)

Scale factor $a(t)$



Our universe!

Open, flat, $K = 0$
Matter-dominated

Closed, curved, $K = +1$
Matter-dominated

Time t

Friedmann equation: accelerated expansion...

Yet another form of the Friedmann equation:

Obtained by combining the usual Friedmann equation for $H(t)$ and the continuity equation.

Acceleration of the scale factor

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\sum_{\alpha}(\rho_\alpha + 3P_\alpha)$$

Compare with

$$H^2(t) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\sum_{\alpha}\rho_\alpha - \frac{K}{a^2}$$

- **Accelerated or decelerated expansion** happens when:

Acceleration $\sum_{\alpha}(\rho_\alpha + 3P_\alpha) < 0$ $\Longrightarrow$ $w_{\text{eff}} = \frac{\sum_\alpha P_\alpha}{\sum_\alpha \rho_\alpha} < -\frac{1}{3}$

Deceleration $\sum_{\alpha}(\rho_\alpha + 3P_\alpha) > 0$ $\Longrightarrow$ $w_{\text{eff}} = \frac{\sum_\alpha P_\alpha}{\sum_\alpha \rho_\alpha} > -\frac{1}{3}$

Distances, horizons, etc.

The FLRW metric written slightly differently…

Here’s the FLRW metric again: $ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2 d\Omega\right]$

- Define:

$$d\chi^2 \equiv \frac{dr^2}{1-Kr^2} \Rightarrow \begin{cases} \chi = \operatorname{arcsinh} r & K = -1 \\ \chi = r & K = 0 \\ \chi = \arcsin r & K = +1 \end{cases}$$

$$d\eta \equiv \frac{dt}{a}$$

$\eta =$ conformal time

- Then, the FLRW metric can also be written as:

$$\boxed{ds^2 = a^2(\eta)\left[-d\eta^2 + d\chi^2 + \begin{pmatrix} \sinh^2\chi \\ \chi^2 \\ \sin^2\chi \end{pmatrix} d\Omega\right]} \qquad \begin{matrix} K = -1 \\ K = 0 \\ K = +1 \end{matrix}$$

Comoving distance…

Take a radial light ray emitted at η_e and observed at η_0, i.e., today:

$$ds^2 = 0 = a^2(\eta)[-d\eta^2 + d\chi^2]$$

- The **comoving distance** is the coordinate distance covered by the light ray between η_e and η_0:

Friedmann eqn

$$\chi(z_e) \equiv \eta_0 - \eta_e = \int_{t_e}^{t_0} \frac{c\,dt}{a} = \int_{z_0=0}^{z_e} \frac{dz}{H(z)}$$

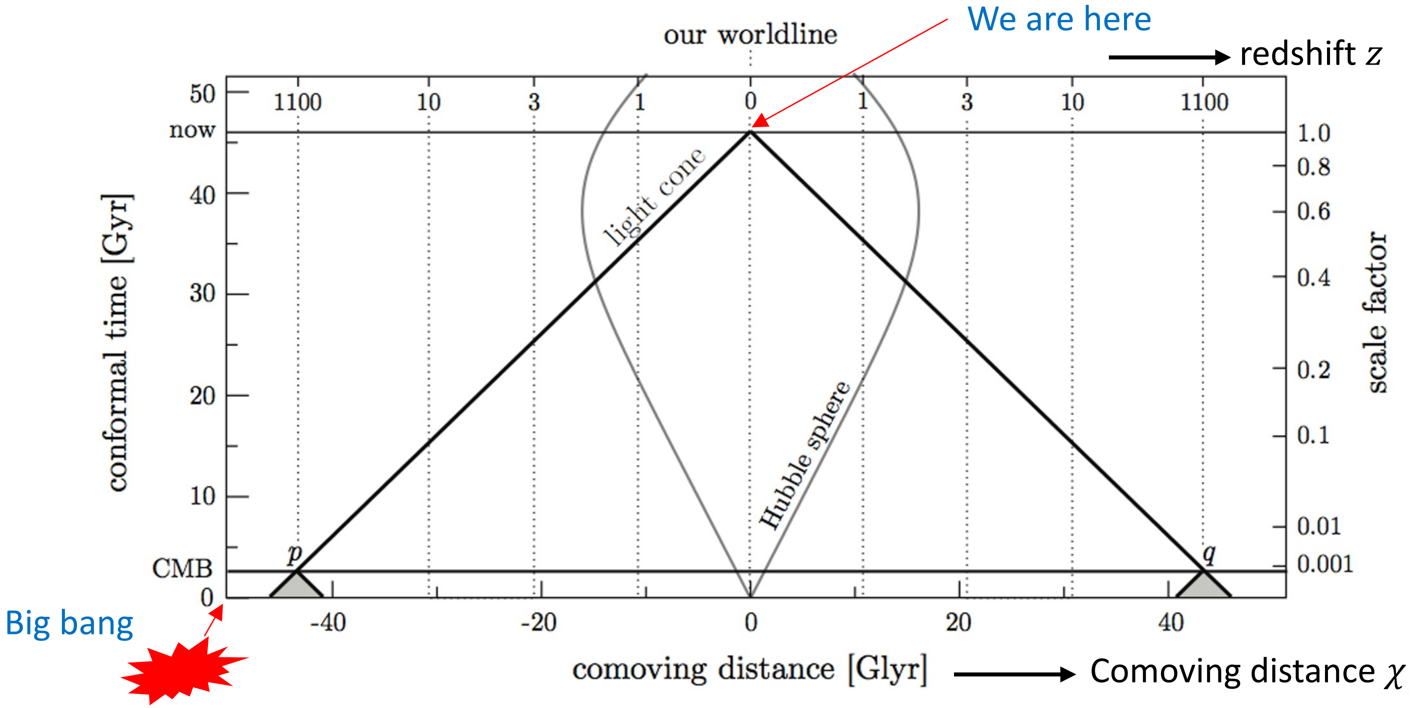
Emission redshift

Hubble rate

Comoving distance…

$$\chi(z_e) \equiv \int_{z_0=0}^{z_e} \frac{dz}{H(z)}$$

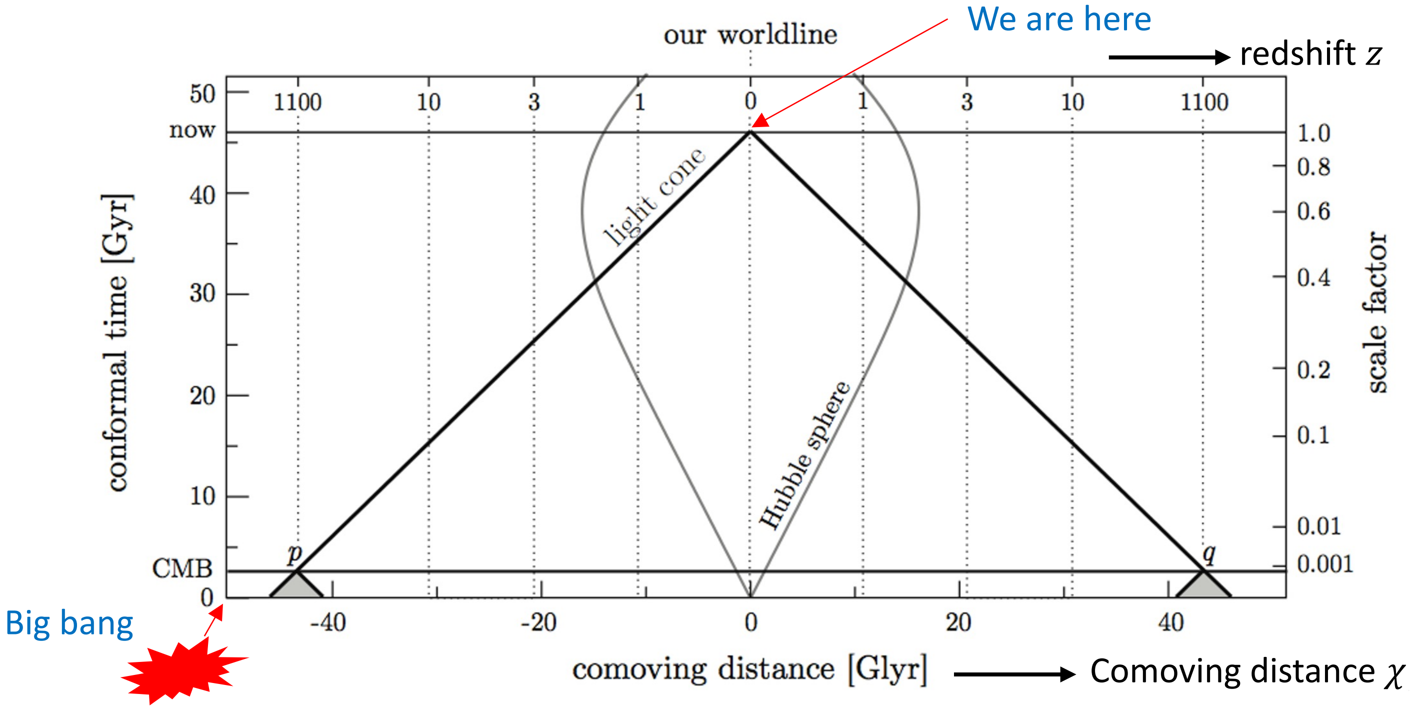
The **comoving distance** χ is the coordinate distance travelled by a light ray between emission at z_e and observation at $z_0 = 0$.



Comoving distance…

$$\chi(z_e) \equiv \int_{z_0=0}^{z_e} \frac{dz}{H(z)}$$

The **comoving distance** χ is the coordinate distance travelled by a light ray between emission at z_e and observation at $z_0 = 0$.



We are here

redshift z

Comoving distance χ

Big bang

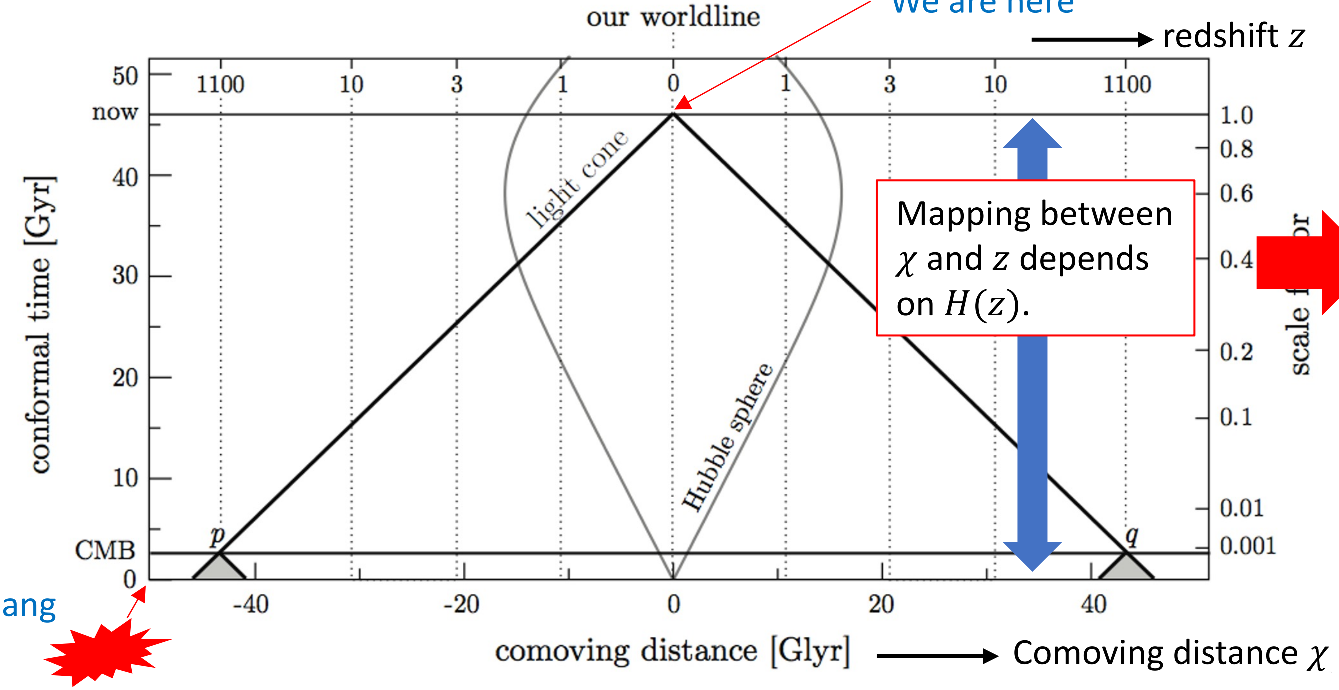
Comoving particle horizon

$\chi(z_e \to \infty)$

Comoving distance...

$$\chi(z_e) \equiv \int_{z_0=0}^{z_e} \frac{dz}{H(z)}$$

The **comoving distance** χ is the coordinate distance travelled by a light ray between emission at z_e and observation at $z_0 = 0$.



We are here

→ redshift z

Mapping between χ and z depends on $H(z)$.

Big bang

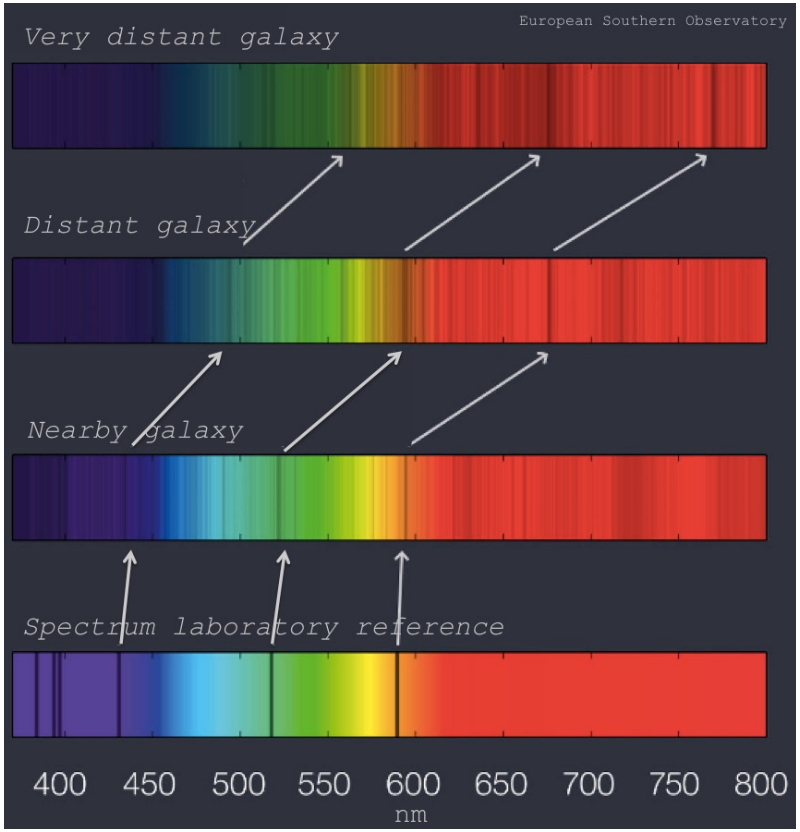
→ Comoving distance χ

If we measure both χ and z, then we also get an idea about $H(z)$ between emission and observation.

Measuring redshift…

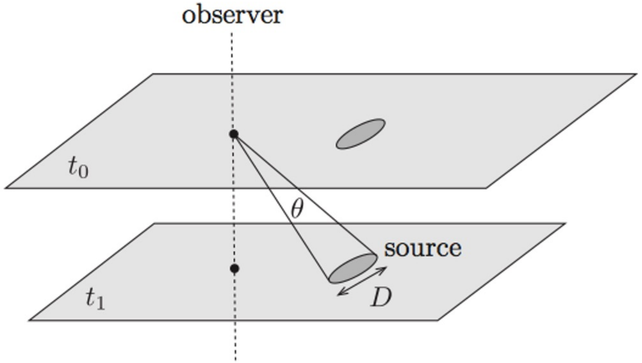
Astronomical redshifts are most accurately measured using spectroscopy.

- Look for **frequency shifts** of absorption/emission lines of known atomic spectra from distant objects.
- Much less accurate is photometry, where redshift of an object is determined from the brightness of an object viewed through different colour filters.



Measuring distance…

Standard ruler: source of a known physical size D; we measure its **angular size** θ



→ **Angular diameter distance,** $d_A = D/\theta$

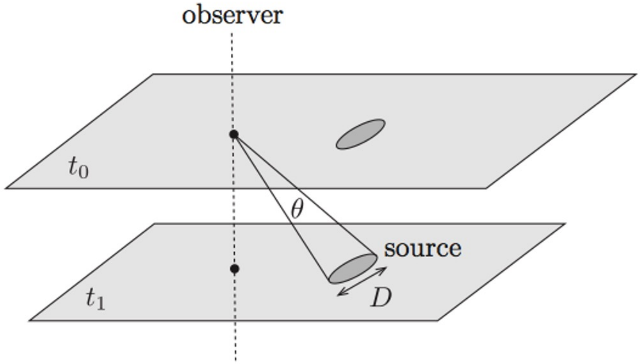
In an FLRW universe:

$$d_A\big(a(z)\big) \equiv a \begin{pmatrix} \sinh\chi(a) \\ \chi(a) \\ \sin\chi(a) \end{pmatrix} \quad \begin{matrix} K=-1 \\ K=0 \\ K=+1 \end{matrix}$$

Comoving distance

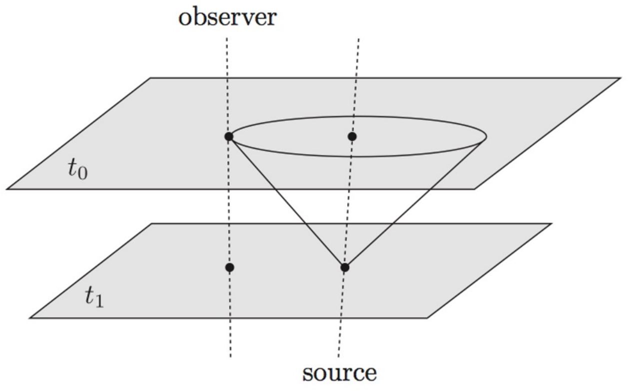
Measuring distance…

Standard ruler: source of a known physical size D; we measure its **angular size** θ



→ **Angular diameter distance,** $d_A = D/\theta$

Standard candle: source of a known luminosity L; we measure its **flux** F.



→ **Luminosity distance,** $d_L = \sqrt{L/(4\pi F)}$

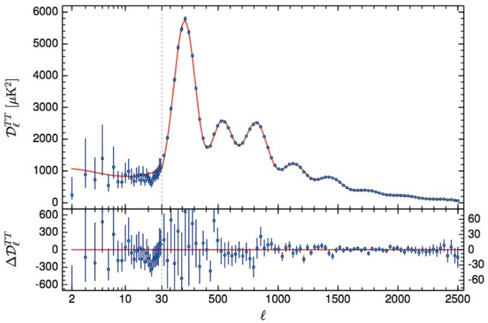
In an FLRW universe:

$$d_A\big(a(z)\big) \equiv a \begin{pmatrix} \sinh\chi(a) \\ \chi(a) \\ \sin\chi(a) \end{pmatrix} \qquad \begin{matrix} K = -1 \\ K = 0 \\ K = +1 \end{matrix} \qquad d_L\big(a(z)\big) \equiv \frac{1}{a} \begin{pmatrix} \sinh\chi(a) \\ \chi(a) \\ \sin\chi(a) \end{pmatrix}$$

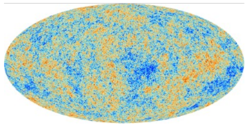
Comoving distance

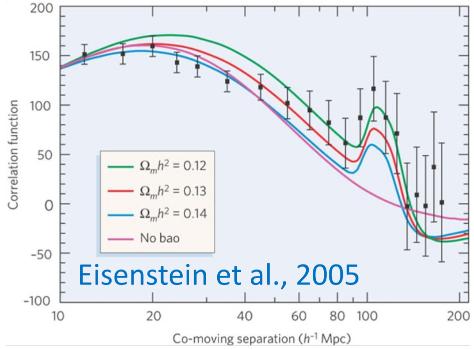
Measuring distance…

Standard ruler: source of a known physical size D; we measure its **angular size** θ



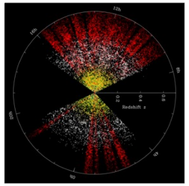
Acoustic oscillation in the CMB at $z \sim 1100$



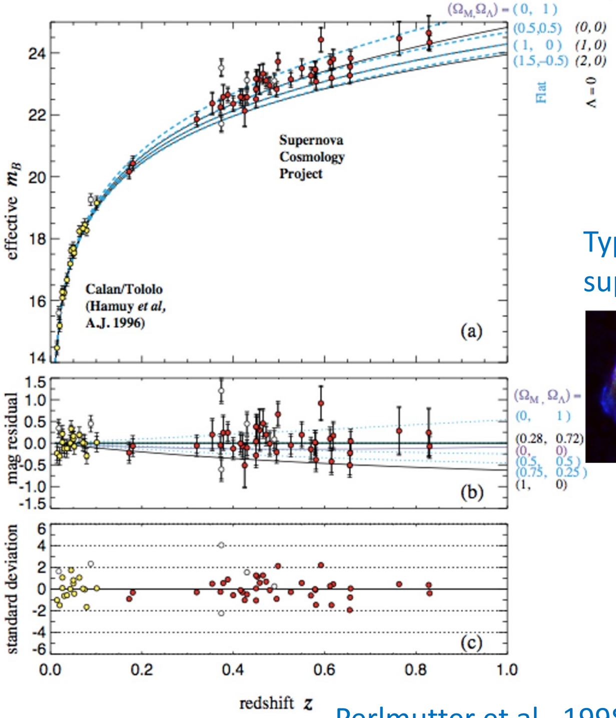


Eisenstein et al., 2005

Baryon acoustic oscillation (BAO) peak at $z = 0.35$ measured by SDSS.

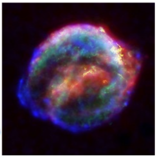


Standard candle: source of a known luminosity L; we measure its **flux** F.



Perlmutter et al., 1998

Type Ia supernovae



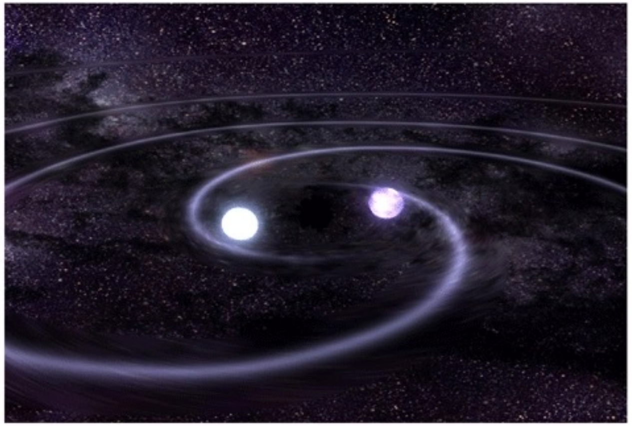
Measuring distance: a newer idea…

Standard siren:

- Use the frequency shift in the gravitational waves emitted during the in-spiral phase of a compact binary system to determine the chirp mass:

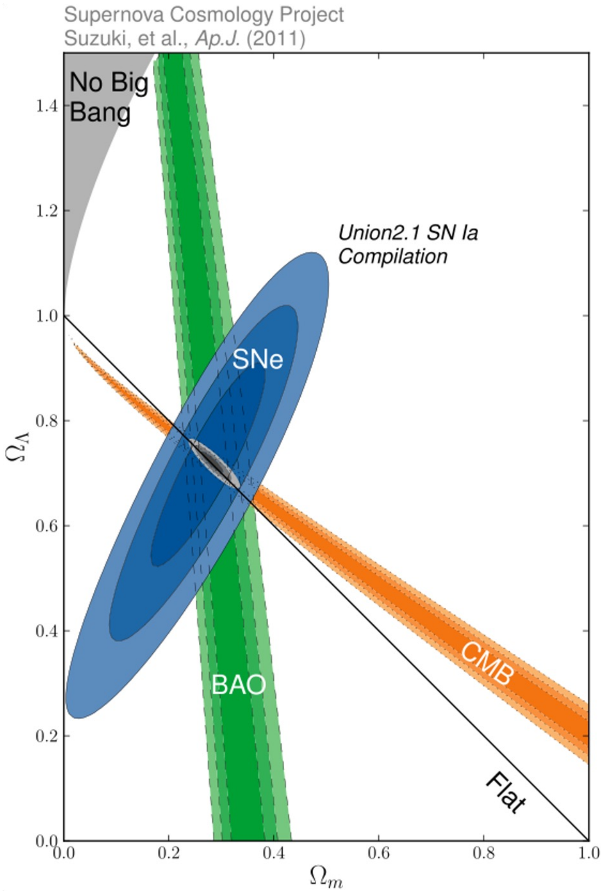
$$\mathcal{M} = \frac{(m_1\ m_2\ )^{3/5}}{(m_1\ + m_2\ )^{1/5}}$$

- Use the chirp mass to calculate the power of the GW emission.
- Power/flux gives luminosity distance.



It's essentially like a standard candle, except that the luminosity is calculable.

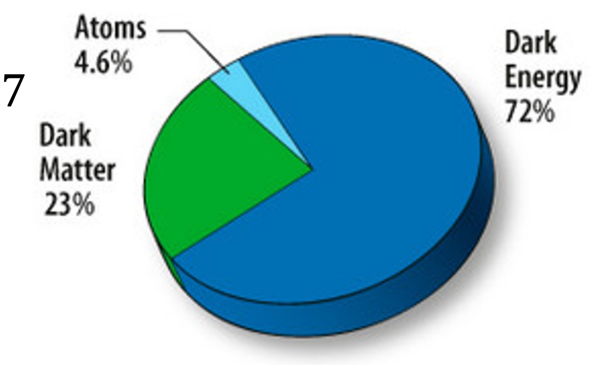
Combining distance vs redshift…



Combining standard ruler (CMB, BAO) and standard candle (SNIa) measurements yields a best fit that corresponds to the flat ΛCDM model:

$$\Omega_m \sim 0.3, \Omega_\Lambda \sim 0.7$$

$$\Omega_m + \Omega_\Lambda \sim 1$$



- In fact, combining any two will give this result.

Hubble length…

Another length scale of interest in the **comoving Hubble length**, defined as

$$\ell_H \equiv c\mathcal{H}^{-1} = c(aH)^{-1}$$

- The Hubble time H^{-1} is the time scale over which the universe expands by a factor of $\sim$2.
- Roughly speaking, events at a time t separated by $\ll \ell_H$ in space can influence each other instantaneously at that time t.
- We often call ℓ_H the “Hubble horizon”. But strictly speaking it is **not** a horizon.

Take-home message…

- Assumption of homogeneous and isotropic on large scales → FLRW universe
- + simple models of possible matter/energy content (radiation, non-relativistic matter, vacuum energy)

→ Friedmann equation: $H^2(t) = H^2(t_0)[\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_\Lambda + \Omega_K a^{-2}]$

- Distance measurements enable us to reconstruct the energy density parameters of $H(t)$, and hence to a large degree the energy content in the universe today.