Exercise Sheet

Question 1: What is the power per area, i.e. total energy per area and per time, of dark matter at Earth?
How much of this power can be harvested (in theory) by typical detection strategies for WIMPs and Axions?

The total power can be simply estimated from

$$\rho_{DM} v_{DM} \sim 0.3 \frac{\text{GeV}}{\text{cm}^3} 200 \frac{\text{km}}{\text{s}} \sim 2.4 \times 10^{-6} \text{eV}^4 \, 200 \frac{\text{km}}{\text{s}} \sim 10\text{W} \tag{1}$$

Since WIMP detection experiments conserve the number of WIMPs, only their kinetic energy can be obtained. At best $\sim v^2/2 \sim 10^{-6}$ of the total energy. Axions on the other hand are completely converted. Therefore up to 100% can be collected (with space bending magnetic field strengths ;-)).

Question 2: Estimate today's velocity of dark matter axions far outside from galaxies in a scenario (similar to the postinflationary one) where the relevant fluctuations are of the order of the Hubble scale at the time when the field starts oscillating. Do so for a constant mass. And neglect the change in the number of degrees of freedom.

At the time the field starts to oscillate we have,

$$3H_1 \sim m_1. \tag{2}$$

Accordingly we have a momentum

$$p_1 \sim H_1 \sim m_1. \tag{3}$$

For a constant mass we have,

$$\begin{aligned} v_0 \sim \frac{p_0}{m} \quad &\sim \quad \frac{p_1}{m}\frac{T_0}{T_1} \sim \frac{T_0}{T_1} \sim \frac{T_0}{\sqrt{M_p m}} \sim 10^{-17} \left(\frac{\text{eV}}{m}\right)^{1/2} \\ &\sim \quad 2\,\text{nm/s} \times \left(\frac{\text{eV}}{m}\right)^{1/2}. \end{aligned} \tag{4}$$

This is certainly small enough ;-).

Question 3: In the lecture we noted that,

$$\Omega_a h^2 = \kappa_a \left(\frac{f_a}{10^{12}\,\text{GeV}}\right)^{7/6} \theta_i^2, \tag{5}$$

This is actually the behaviour for sufficiently small values of f_a. At what scale of f_a do you expect a change in the dependence on f_a?

A change in the behavior is expected when the field only starts oscillating at temperatures below the QCD scale. In this case the mass can be taken as constant and the $\sqrt{m}$ behavior of a constant mass ALP is recovered. The relevant axion mass is estimated from

$$m \sim H \sim 0.33\sqrt{g_\star}T_c^2/M_{\rm Pl} \sim 3 \times 10^{-10}\,\text{eV} \tag{6}$$

for $g_\star \sim 30$ and $T_c \sim 300\,\text{MeV}$. This gives,

$$f_a \sim 2 \times 10^{16}\,\text{GeV}. \tag{7}$$

There is a bit of a range to that but it's not too far off.