1. If the chemical potential of a particle is μ, the chemical potential of its antiparticle is $-\mu$. Show that, for $T \gg m, \mu$, the difference in number densities of a particle and its antiparticle is
$$\Delta n_X = n_X - n_{\bar{X}} \simeq C \cdot \mu \frac{T^2}{3}\,,$$
where $C = 1$ for bosons and $C = 1/2$ for fermions.

2. In thermal equilibrium, the chemical potential of a particle of type A is
$$\mu_A = \sum_i \mu_i Q_A^{(i)}\,,$$
where μ_i are chemical potentials associated with the *conserved* quantum charges $Q_A^{(i)}$ carried by the particle.

   Consider temperatures 10^{12} GeV $< T \ll T_{\text{EW}}$. Assume a chemical potential $\tilde{\mu}$ for a given $(B-L)$ asymmetry. (Assume the three lepton number densities are equal to each other.) For this calculation, it is enough to work with the conserved charges $(B-L)$ and weak hypercharge Y. So, a particle of type I has the chemical potential
$$\mu_I = \tilde{\mu}(B_I - L_I) + \mu_Y \frac{Y_I}{2}\,.$$
   Write the chemical potentials for one generation of leptons and quarks and the Higgs doublet components h^0 and h^+.

3. The universe is neutral under weak hypercharge, which means
$$\sum_I Y_I \cdot \Delta n_I = 0\,.$$
   Use this constraint together with the results of question 1 and 2 to show:
$$B = \frac{8N_f + 4N_s}{22N_f + 13N_s}(B-L)\,,$$
   where N_f is the number of fermion generations and N_s is the number of Higgs doublets.

4. For leptogenesis we consider right-handed neutrino interactions of the form $\mathcal{L} \supset y_{i\alpha}\bar{N}_i^c \tilde{H}^\dagger \ell_\alpha$+h.c. **(i)** Calculate the decay widths $\Gamma(N_1 \to h\ell_\alpha), \Gamma(N_1 \to h\bar{\ell}_\alpha)$ at tree level and show that they are equal. **(ii)** By including one-loop diagrams, calculate the CP violation in this decay, defined as
$$\epsilon = \frac{\Gamma(N_1 \to h\ell_\alpha) - \Gamma(N_1 \to h\bar{\ell}_\alpha)}{\Gamma_{\text{tot}}}$$

5. Think about what would change for different models of baryogenesis if the universe was matter dominated during the asymmetry production.