Phase transitions exercises II

1. For first-order phase transitions in the Abelian Higgs model, the Higgs becomes dynamically light compared to the gauge field near the transition. This is due to an approximate cancellation in, $m_3^2 = -\mu^2 + \left(\frac{\lambda}{3} + \frac{g^2}{4}\right)T^2 \ll \frac{g^2}{3}T^2 \equiv m_D^2$. The theory remains perturbative, suggesting the Higgs lies between the "soft" $\sim \sqrt{\lambda}\,T$ and "ultrasoft" $\sim \frac{\lambda}{\pi}T$ scales.

   Assuming the Higgs lies at the geometric midpoint, $m_3^2 \sim \frac{g^3 T^2}{\pi}$ "supersoft", and hence,

$$\lambda \sim \frac{g^3}{\pi},$$

   calculate the latent heat in the Abelian Higgs model to leading order in couplings. Evaluate L/T_c^4 for $\lambda = \frac{g^3}{6\pi}$, $g = 1$.

2. Consider the 3d EFT for the cubic anisotropy model, $\mathcal{L}_{EFT} = \frac{1}{2}\partial_i\phi_a\partial_i\phi_a + V(\phi)$, $a=1,2$

$$V(\phi) = \frac{1}{2}m_3^2\phi_a\phi_a + \frac{1}{4!}\lambda_3(\phi_1^4+\phi_2^4) + \frac{1}{4}\kappa_3\phi_1^2\phi_2^2 .$$

a) Show that when $m_3^2 = 0$, there is an apparent second order phase transition.

b) Deduce the expansion parameters in the symmetric phase, and in the broken phase. Is the prediction of a second order transition reliable?

c) Assuming $\kappa_3 >> \lambda_3$, show that there is a mass hierarchy in the broken phase, and integrate out the heavy modes to construct a new EFT.

d) Using this deduce:

- the order of the transition,
- the size of neglected loop contributions and where perturbation theory breaks down,
- the phase diagram for $\kappa_3 > \lambda_3$.