Phase transitions exercises I

1. The mass dimension (units) of fields and couplings depend on the space/spacetime dimension. Requiring canonical normalisation

$$\text{e.g. } \mathcal{L} \subset \frac{1}{2} \partial_A \phi \partial_A \phi$$

deduce the mass dimensions of the fields and couplings of the Abelian Higgs model in d dimensions. In each of $d = 3, 4, 5$ determine if the gauge coupling is:

superrenormalisable (relevant in IR, irrelevant in UV), renormalisable (marginal) or nonrenormalisable (irrelevant in IR, relevant in UV).

2. There is one sum-integral we have used a lot, so let's see how to compute it. Show that,

$$\sum\!\!\!\!\!\!\!\int_Q \frac{1}{Q^2} = \frac{T^2}{12} + O(\varepsilon), \quad \text{in } d = 3 - 2\varepsilon \text{ dimensions, } D = d+1.$$

Hints: $\frac{1}{A} = \int_0^\infty dt\, e^{-At}$, "Schwinger's proper time trick"

$$\Gamma(z) = \int_0^\infty dt\, t^{z-1} e^{-t}, \quad \Gamma(z+1) = z\Gamma(z), \quad \Gamma(\tfrac{1}{2}) = \sqrt{\pi}$$

$$1 + 2 + 3 + 4 + \ldots = \frac{-1}{12} + O(\varepsilon)$$

3. By matching the UV parts of the 1PI two-point functions for the gauge field, show that $m_D^2 = \frac{1}{3} e^2 T^2$. Also show that the spatial part of the gauge field does not get an effective mass.

4. a) Deduce the lowest order diagrams contributing to matching $c_6\phi^6$, and more generally $c_{2N}\phi^{2N}$ for $N \geq 3$. You may assume $g=0$ for simplicity.

b) Estimate the size of c_{2N} in powers of couplings and T.

5. At tree-level in the 3d EFT for the Abelian Higgs model,

$$V_{EFT}(\bar{\Phi}, A_0 = 0) = \frac{1}{2} m_3^2 \bar{\Phi}^2 + \frac{1}{4}\lambda_3 \bar{\Phi}^4,$$

where $m_3^2 = -\mu^2 + (\frac{\lambda}{3} + \frac{g^2}{4})T^2$, $\lambda_3 = \lambda T$. Using that

$$f = T V_{EFT}(\bar{\Phi}_{min}, 0) + (\text{field independent})$$

show that at $m_3^2 = 0$:

- $\frac{df}{dT}$ is continuous,
- $\frac{d^2f}{dT^2}$ is discontinuous.