Assessing the freezeout stage with the method of moments

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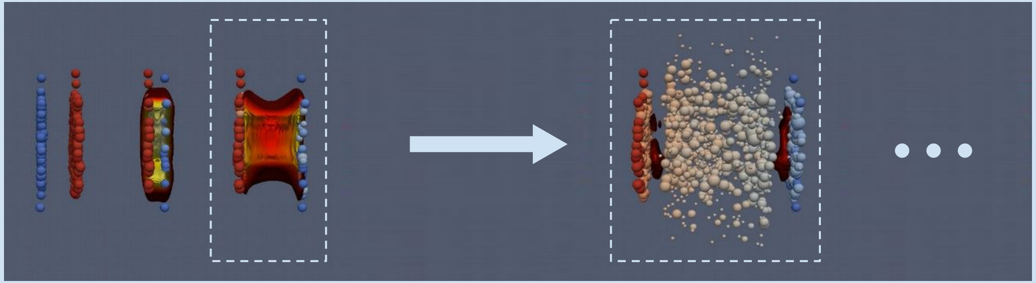
Motivation

- Relativistic heavy-ion collisions

H. Petersen & J. Bernhard, MADAI Collaboration (adapted)



Hydrodynamics | Transport

- QGP: dissipative effects play an important role
  P. Romatschke and U. Romatschke, Phys. Rev. Lett. ***99****, 172301 (2007)*
  J. Bernhard et al., Nat. Phys. ***15****.11 (2019)*
- Solve Israel-Stewart-like equations
  W. Israel and J. Stewart, Ann. Phys. ***118****, 341 (1979)*
  G. S. Denicol et al., Phys. Rev. D ***85****, 114047 (2012)*
  - relaxation equations for dissipative currents
- Derivation from an underlying microscopic theory

- A weakly interacting system
- Fluid description is no longer valid
- Boltzmann equation

$$k^\mu \partial_\mu f_{\mathbf{k}} = \frac{1}{2} \int dK' dP dP' \mathcal{W}_{\mathbf{k}\mathbf{k}' \leftrightarrow \mathbf{p}\mathbf{p}'} \left(f_{\mathbf{p}} f_{\mathbf{p}'} - f_{\mathbf{k}} f_{\mathbf{k}'} \right)$$

L. Boltzmann, Wiener Berichte ***66****, 275-370 (1872)*

Fluid → ? → Particles

Method of moments

<u>Irreducible moments</u> as fundamental fields

(of the non-equilibrium distribution function)

$$\rho_r^{\mu_1\cdots\mu_\ell} = \int dK\, E_{\mathbf{k}}^r k^{\langle\mu_1}\cdots k^{\mu_\ell\rangle}\delta f_{\mathbf{k}}$$

Non-equilibrium corrections (= "$\delta f_{\mathbf{k}}$")

Single-particle distribution function

$$f_{\mathbf{k}} = f_{0\mathbf{k}}\left(1 + \tilde{f}_{0\mathbf{k}}\sum_{\ell=0}^{\infty}\sum_{n=0}^{N_\ell}\mathcal{H}_{\mathbf{k}n}^{(\ell)}\,\rho_n^{\mu_1\cdots\mu_\ell}\,k_{\langle\mu_1}\cdots k_{\mu_\ell\rangle}\right)$$

- Exact solution obtained when the sums are taken to infinity
- Hydrodynamic limit: 14 degrees of freedom (moments up to rank 2)
  - derivation of relativistic dissipative fluid dynamics
    W. Israel and J. Stewart, Ann. Phys. ***118****, 341 (1979)*
    G. S. Denicol et al., Phys. Rev. D ***85****, 114047 (2012)*
    N. Weickgenannt et al. Phys. Rev. D ***106****, 096014 (2022)*
    G. S. Denicol et al, Phys. Rev. D ***98****, 076009 (2018)*
  - causal and stable fluid-dynamical theories
    T. S. Olson, Ann. Phys. ***199****, 18 (1990)*
    S. Pu et al., Phys. Rev. D ***81****, 114039 (2010)*
    CVPB and G. S. Denicol, Phys. Rev. D ***102****, 116009 (2020)*
    J. Sammet et al., Phys. Rev. D ***107****, 114028 (2023)*
- So far, moments have been calculated only up to rank 4
  CVPB and G. S. Denicol, Phys. Rev. D ***108****, 096020 (2023); G. S. Denicol et al., Phys. Rev. D* ***85****, 114047 (2012)*

Method of moments

A single equation!

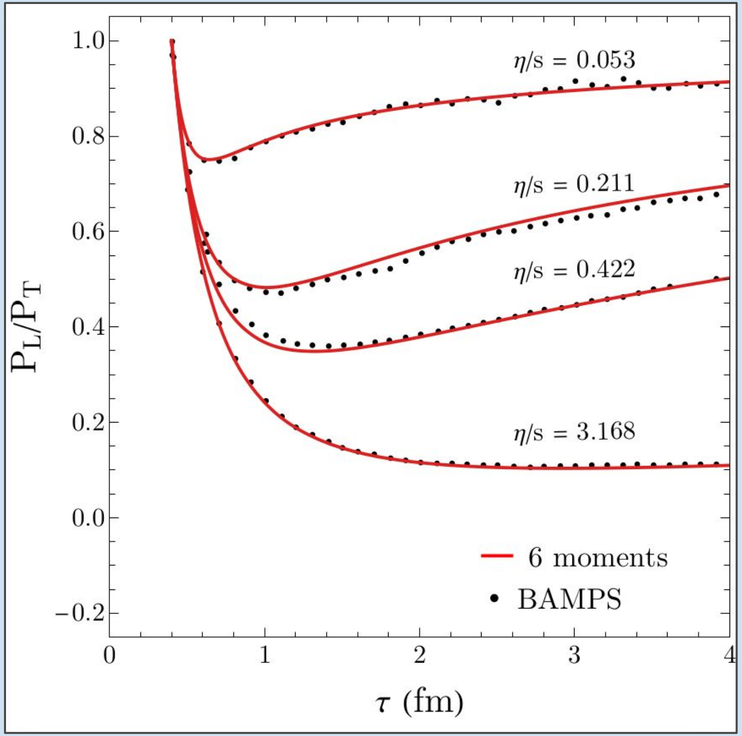
$$
\begin{aligned}
\dot{\varrho}_r^{\langle\mu_1\cdots\mu_\ell\rangle} = {} & \mathcal{C}_{r-1}^{\mu_1\cdots\mu_\ell} + r\varrho_{r-1}^{\mu_1\cdots\mu_{\ell+1}}\dot{u}_{\mu_{\ell+1}} - \Delta^{\mu_1\cdots\mu_\ell}_{\nu_1\cdots\nu_\ell}\nabla_{\nu_{\ell+1}}\varrho_{r-1}^{\nu_1\cdots\nu_{\ell+1}} + (r-1)\varrho_{r-2}^{\mu_1\cdots\mu_{\ell+2}}\sigma_{\mu_{\ell+1}\mu_{\ell+2}} + \ell\varrho_r^{\alpha\langle\mu_1\cdots\mu_{\ell-1}}\omega^{\mu_\ell\rangle}{}_\alpha + \\
& + \frac{\ell}{2\ell+1}\left[rm^2\varrho_{r-1}^{\langle\mu_1\cdots\mu_{\ell-1}} - (r+2\ell+1)\varrho_{r+1}^{\langle\mu_1\cdots\mu_{\ell-1}}\right]\dot{u}^{\mu_\ell\rangle} + \frac{1}{3}\left[(r-1)m^2\varrho_{r-2}^{\mu_1\cdots\mu_\ell} - (r+\ell+2)\varrho_r^{\mu_1\cdots\mu_\ell}\right]\theta + \\
& + \frac{\ell}{2\ell+3}\left[(2r-2)m^2\varrho_{r-2}^{\alpha\langle\mu_1\cdots\mu_{\ell-1}} - (2r+2\ell+1)\varrho_r^{\alpha\langle\mu_1\cdots\mu_{\ell-1}}\right]\sigma_\alpha^{\mu_\ell\rangle} - \frac{\ell}{2\ell+1}\nabla^{\langle\mu_1}\left(m^2\varrho_{r-1}^{\mu_2\cdots\mu_\ell\rangle} - \varrho_{r+1}^{\mu_2\cdots\mu_\ell\rangle}\right) + \\
& + \frac{\ell(\ell-1)}{4\ell^2-1}\left[(r-1)m^4\varrho_{r-2}^{\langle\mu_1\cdots\mu_{\ell-2}} - (2r+2\ell-1)m^2\varrho_r^{\langle\mu_1\cdots\mu_{\ell-2}} + (r+2\ell)\varrho_{r+2}^{\langle\mu_1\cdots\mu_{\ell-2}}\right]\sigma^{\mu_{\ell-1}\mu_\ell\rangle},
\end{aligned}
$$

CVPB and G. S. Denicol, arXiv:2401.10098 [nucl-th]

- Recover previously known equations
  G. S. Denicol et al., Phys. Rev. D ***85****, 114047 (2012); CVPB and G. S. Denicol, Phys. Rev. D* ***108****, 096020 (2023)*
- The dynamics of *all* moments can be assessed ➢ solve the Boltzmann equation
- But first: how to truncate these equations? Convergence of the solutions?
  - Massless classical gas in Bjorken flow *J. D. Bjorken, Phys. Rev. D* ***27****, 140 (1982)*

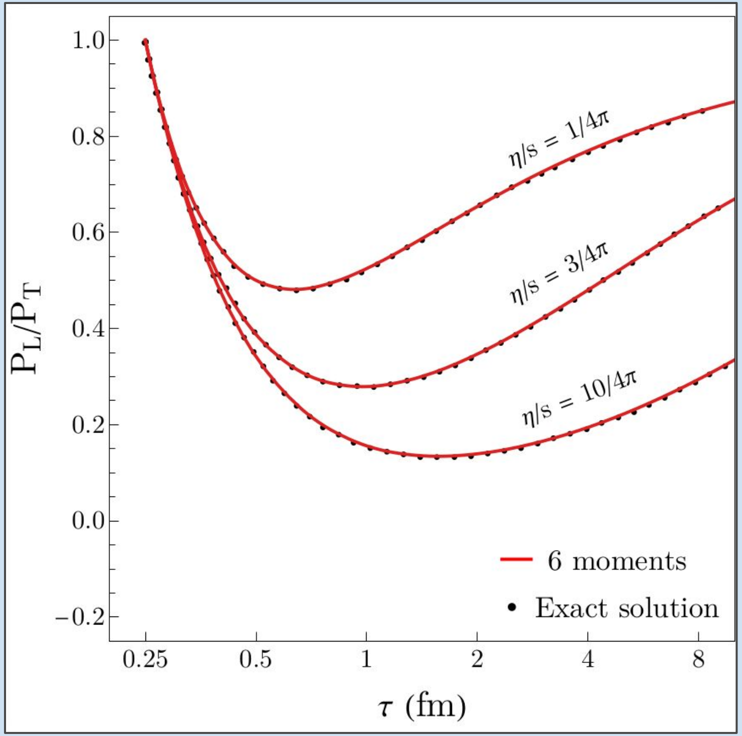
Pressure anisotropy

τ_0 = 0.4 fm, T(τ_0) = 500 MeV



Z. Xu and C. Greiner, Phys. Rev. C **71** (2005) 064901; Phys. Rev. C **76** (2007) 024911.

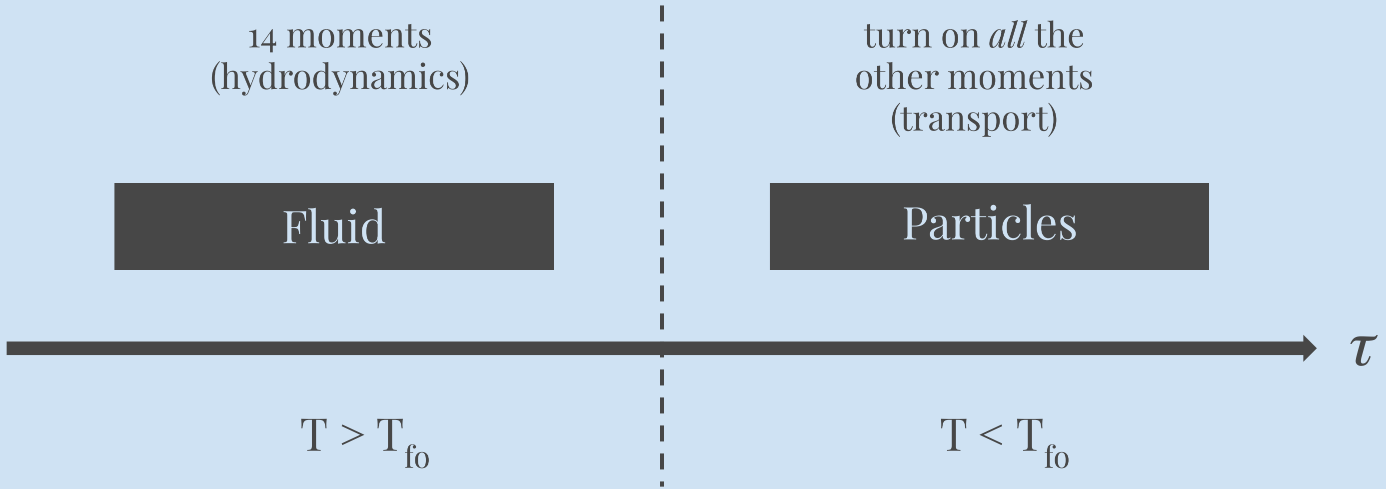
τ_0 = 0.25 fm, T(τ_0) = 300 MeV



W. Florkowski et al., Nucl. Phys. A **916**, 245-259 (2013)

Future perspective: the freezeout stage

- At this stage, we have a dilute system: Boltzmann equation
- Our proposal: include more fields rather than using particlization
- Solve hydrodynamics and transport simultaneously



[work in preparation]