Linear stability analysis of IS-Theory with nonzero background charge

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I. Abstract

- Brito and Denicol [Phys. Rev. D 102 (2020) 116009]: Linear stability of Israel-Stewart theory with net-charge diffusion for massless, classical gas of noninteracting particles.
- Goal of this work: Extend theory to nonvanishing background charge.
- Main Results:
  - Change of charge-diffusion coefficient: becomes at most four times its value for zero background charge.
  - Systematic study to identify parameter regions where solutions remain stable and causal.

II. Second-order dissipative fluid dynamics

- Fluid dynamics: conservation of charge and energy-momentum:

$$\partial_\mu N^\mu = 0 \,, \tag{1}$$
$$\partial_\mu T^{\mu\nu} = 0 \,. \tag{2}$$

- In Landau frame, fluid four-velocity u^μ follows flow of energy, such that tensor decomposition of charge current and energy-momentum tensor reads

$$N^\mu = n u^\mu + n^\mu \,, \tag{3}$$
$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu} \,. \tag{4}$$

Here, n and ε are charge density and energy density in fluid rest frame, P is pressure, Π is bulk viscous pressure, n^μ is charge-diffusion current, and $\pi^{\mu\nu}$ is shear-stress tensor. Furthermore, $\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu$ is projector onto three-space orthogonal to u^μ.

- Eqs. (1)–(2) are 5 eqs. for 14 unknowns, need to provide 9 additional eqs. to close the system of eqs. of motion.
In second-order dissipative fluid dynamics, these are relaxation-type equations for the dissipative currents Π, n^μ, and $\pi^{\mu\nu}$, which can be derived from underlying microscopic theory, e.g., Boltzmann equation

$$\begin{aligned} \tau_\Pi D\Pi + \Pi &= -\zeta\theta - \ell_{\Pi n}\nabla_\mu n^\mu - \tau_{\Pi n} n^\mu \nabla_\mu P \\ &\quad - \delta_{\Pi\Pi}\Pi\theta - \lambda_{\Pi n} n^\mu \nabla_\mu \alpha + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} \,, \end{aligned} \tag{5}$$

$$\begin{aligned} \tau_n D n^{\langle\mu\rangle} + n^\mu &= \varkappa \nabla^\mu \alpha - \tau_n n_\nu \omega^{\nu\mu} \\ &\quad - \delta_{nn} n^\mu \theta - \ell_{n\Pi}\nabla^\mu \Pi + \ell_{n\pi}\Delta^{\mu\nu}\nabla_\alpha \pi^\alpha_\nu \\ &\quad + \tau_{n\Pi}\Pi\nabla^\mu P - \tau_{n\pi}\pi^{\mu\nu}\nabla_\nu P - \lambda_{nn} n_\nu \sigma^{\mu\nu} \\ &\quad + \lambda_{n\Pi}\Pi\nabla^\mu\alpha - \lambda_{n\pi}\pi^{\mu\nu}\nabla_\nu\alpha \,, \end{aligned} \tag{6}$$

$$\begin{aligned} \tau_\pi D\pi^{\langle\mu\nu\rangle} + \pi^{\mu\nu} &= 2\eta\sigma^{\mu\nu} + 2\tau_\pi \pi_\lambda^{\langle\mu}\omega^{\nu\rangle\lambda} - \delta_{\pi\pi}\pi^{\mu\nu}\theta \\ &\quad -\tau_{\pi\pi}\pi^{\lambda\langle\mu}\sigma^{\nu\rangle}_\lambda + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} - \tau_{\pi n} n^{\langle\mu}\nabla^{\nu\rangle}P \\ &\quad +\ell_{\pi n}\nabla^{\langle\mu}n^{\nu\rangle} + \lambda_{\pi n} n^{\langle\mu}\nabla^{\nu\rangle}\alpha \,, \end{aligned} \tag{7}$$

with $\alpha \equiv \beta\mu$, $\beta = 1/T$, and $\omega_{\mu\nu} \equiv (\nabla_\mu u_\nu - \nabla_\nu u_\mu)/2$ being the fluid vorticity.

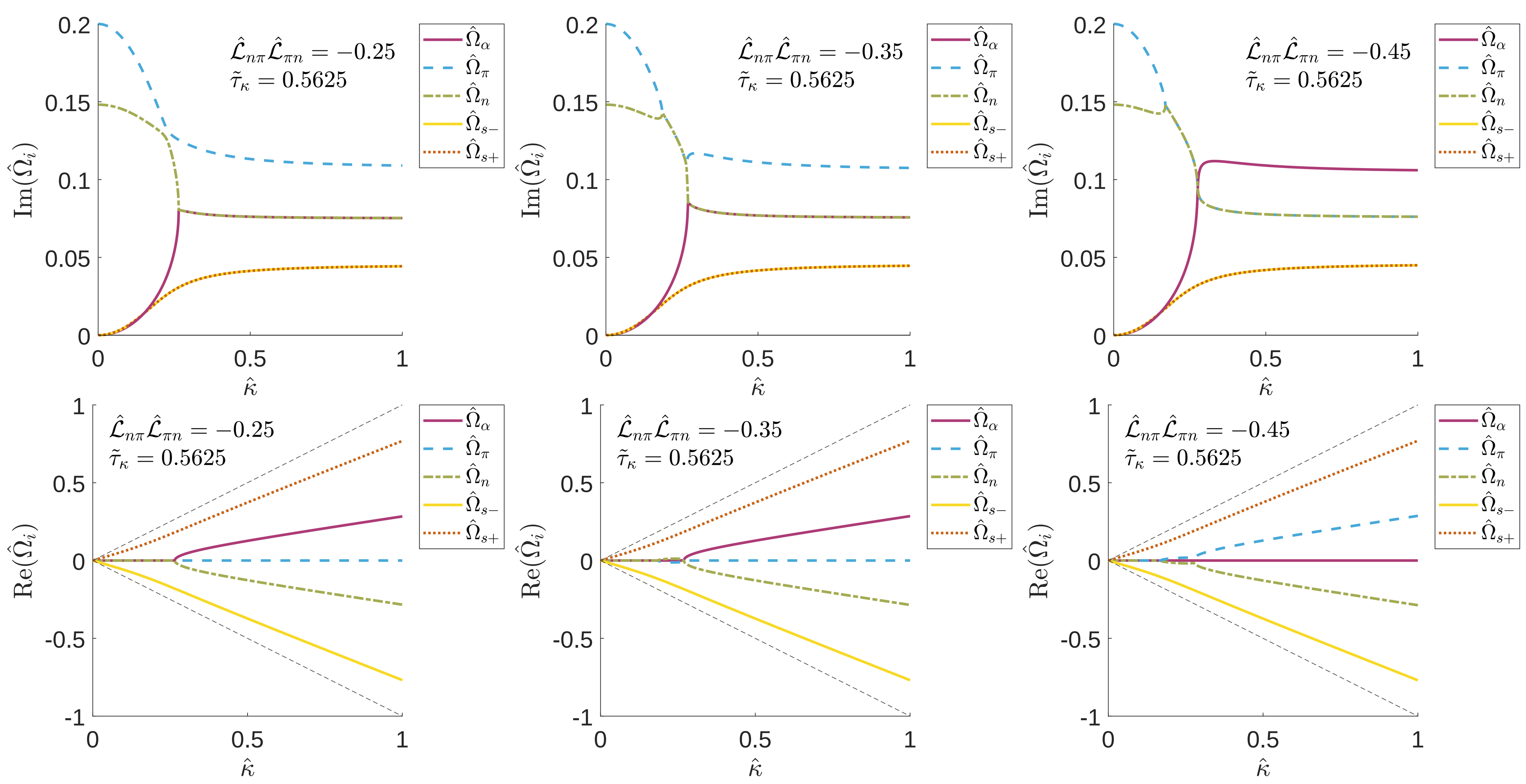
III. Linear stability analysis

- Eqs. (3)–(7) are linearized, Fourier-transformed, made dimensionless, and their modes are analyzed for causality and stabilty.
- Explicit calculation: We consider ideal gas of classical, massless particles, i.e., velocity of sound (squared) $c_s^2 = 1/3$ and bulk viscous pressure vanishes. Furthermore, $w_0 = 4P_0$, $\bar{n}_0 = P_0\beta_0$.
- In this case our system reduces to:

$$\begin{pmatrix} \hat\Omega & -3\frac{n_0}{\bar n_0}\hat\Omega & -\frac{n_0}{\bar n_0}\hat\kappa & -\hat\kappa & 0 \\ \frac{3n_0}{4\bar n_0}\hat\Omega & -3\hat\Omega & -\hat\kappa & 0 & 0 \\ -\frac{n_0}{4\bar n_0}\hat\kappa & \hat\kappa & \hat\Omega & 0 & -\hat\kappa \\ -i\hat\tau_\kappa\hat\kappa & 0 & 0 & 1 + i\hat\tau_n\hat\Omega & i\hat{\mathcal{L}}_{n\pi}\hat\kappa \\ 0 & 0 & -\frac{4}{3}i\hat\kappa & -\frac{2}{3}i\hat{\mathcal{L}}_{\pi n}\hat\kappa & 1 + i\hat\tau_\pi\hat\Omega \end{pmatrix} \begin{pmatrix} \delta\tilde\alpha \\ \delta\tilde\beta/\beta_0 \\ \delta\tilde u_\parallel \\ \delta\tilde\xi_\parallel \\ \delta\tilde\chi_\parallel \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} . \tag{8}$$

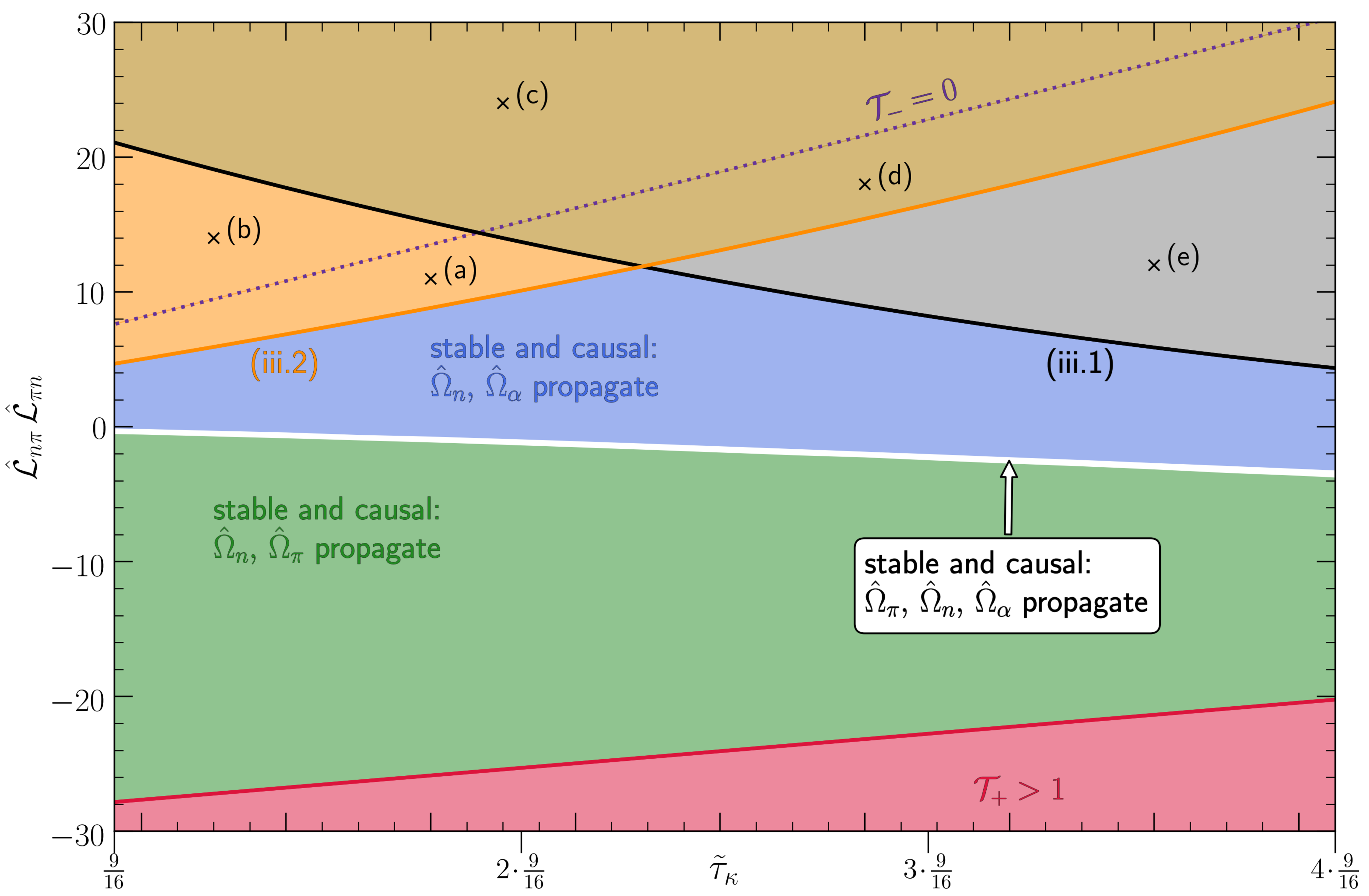
- Determinant yields a polynomial of fifth order, we obtain five different modes.

IV. Results: Analysis of modes



- Increasing $\tilde\tau\kappa$ leads to earlier propagation of the shear and charge-diffusion modes.
- Product $\hat{\mathcal{L}}_{n\pi}\hat{\mathcal{L}}_{\pi n}$ is responsible for attraction of shear and charge-diffusion modes.
- Summary: Non-zero coupling $\hat{\mathcal{L}}_{n\pi}\hat{\mathcal{L}}_{\pi n}$ leads to two critical wave numbers. At first critical wave number: shear mode and one charge mode start to propagate while the other charge mode remains non-propagating. At second critical wave number: shear mode becomes non-propagating mode, while the two charge modes propagate.
- These plots represent just one example of the white, blue, and green regions in the causality-stability plot below. Analyzing modes for every pair of parameter values determines causality and stability regions.

IV. Results: Causality and stabilty regions



- For very large negative values of $\hat{\mathcal{L}}_{n\pi}\hat{\mathcal{L}}_{\pi n}$, system becomes acausal, which implies instability (red area).
- Increasing $\hat{\mathcal{L}}_{n\pi}\hat{\mathcal{L}}_{\pi n}$, we find a stable and causal region, which is further subdivided in three parts.
- We also investigated the region of positive values for $\hat{\mathcal{L}}_{n\pi}\hat{\mathcal{L}}_{\pi n}$, where system remains causal, but exhibits unstable modes. This region can again be subdivided into five parts characterized by qualitatively different behavior of the various dispersion relations. We have studied these analytically in our Paper [2].

References

- [1] C. Brito, G. Denicol [Phys. Rev. D 102 (2020) 116009]
- [2] **J.S.**, M. Mayer, D.H. Rischke [Phys. Rev. D 107 (2023) 114028]

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