Fits of α_s using power corrections in the 3-jet region

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Determination of α_s

Strong coupling: least well-known coupling. Its uncertainty affects precision measurements. Determined by comparing accurate theory predictions with precise measurements for observables sensitive to α_s

Considerations that enter when determining whether an observable is suitable to be used to determine α_s:

- observable's sensitivity to α_s compared to experimental precision
- accuracy of the prediction (e.g. PDG imposes now at least NNLO accuracy)
- the size of non-perturbative effects
- the scale at which the measurement is performed

PDG average

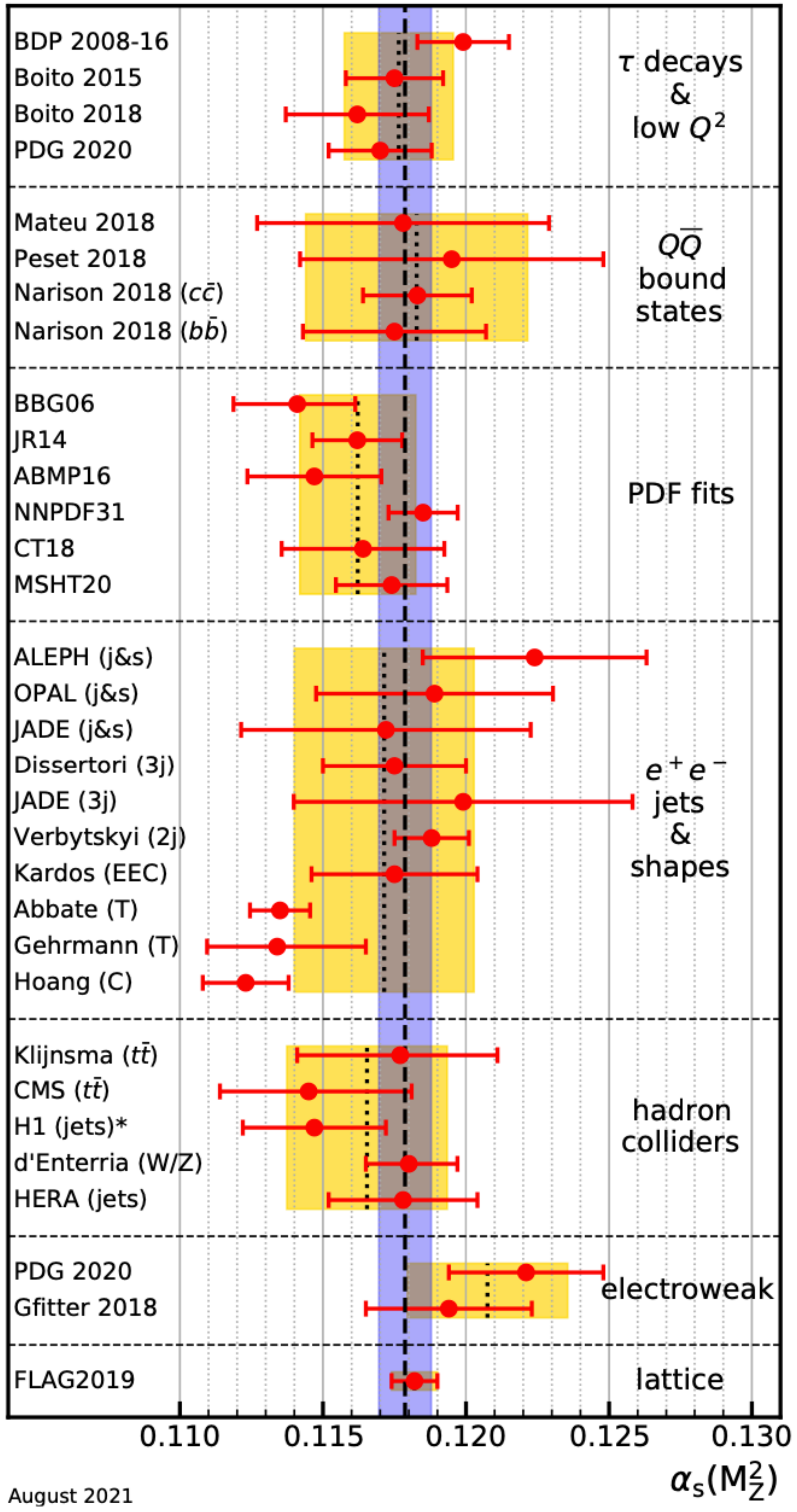
Procedure in the particle data group (PDG):

- decide which observables are included
- subdivide observables in categories
- provide an average for each category
- provide an average of all categories

⇒ the PDG average of α_s

$$\boxed{\alpha_s(M_Z^2) = 0.1179 \pm 0.0009}$$

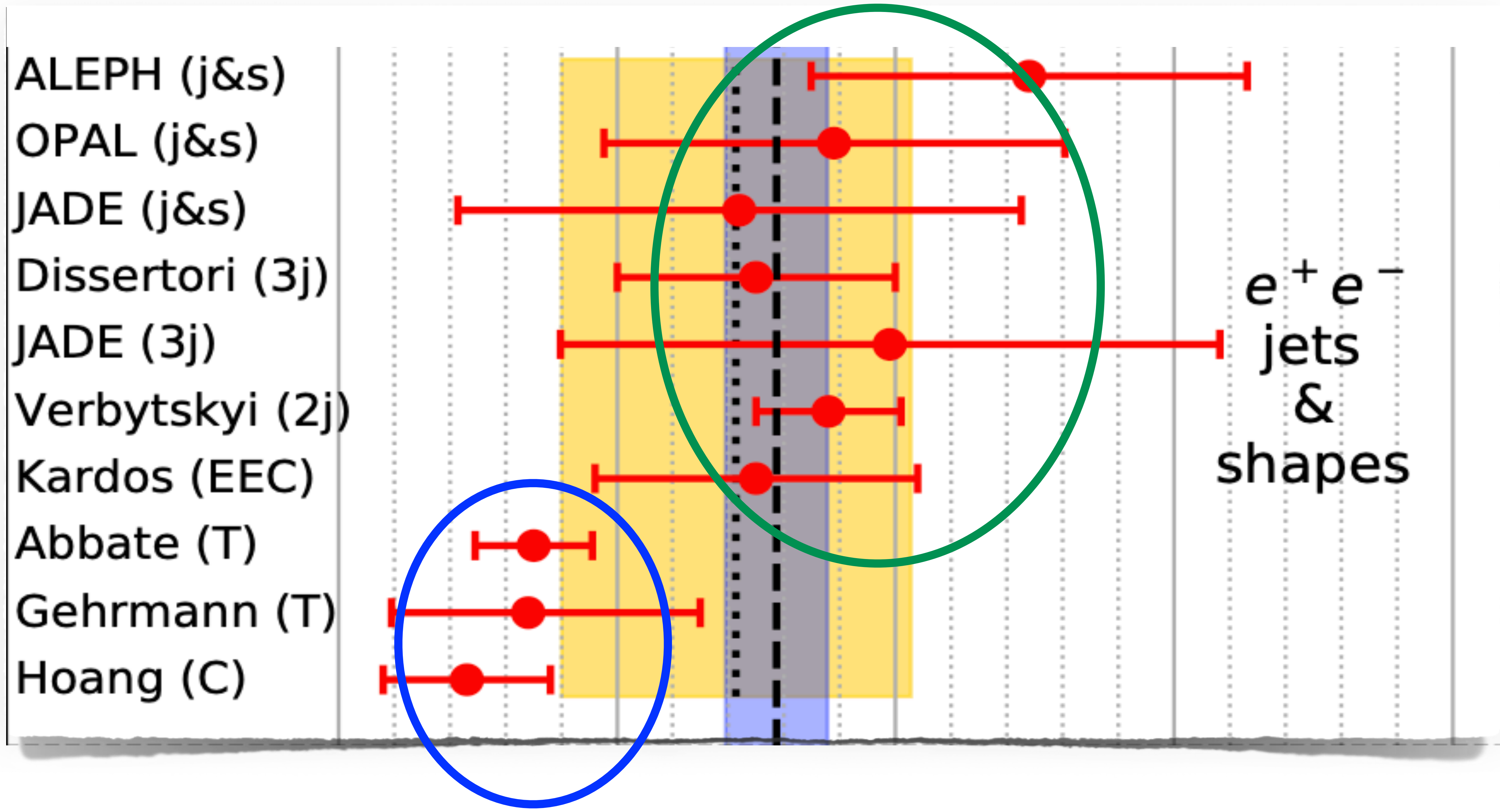
PDG, Huston, Rabbertz, GZ '21



Zooming-in on e^+e^- jet & shapes

<u>"e^+e^-: jet & shapes":</u>

longstanding discrepancy between α_s determinations based on non-perturbative corrections computed via Monte Carlos and those based on analytic approaches



Definition of the observable

Thrust:

$$T = \max_{\vec{n}_T} \left(\frac{\sum_i |\vec{p}_i \cdot \vec{n}_T|}{\sum_i |\vec{p}_i|} \right) \qquad \tau = 1 - T,$$

C-parameter:

$$\Theta^{\alpha\beta} = \frac{1}{\sum_i |p_i|} \sum_i \frac{p_i^\alpha p_i^\beta}{|\vec{p}_i|}, \qquad \alpha, \beta = 1, 2, 3$$

$$C = 3 \cdot (\lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3)$$

Durham y3:

$$y_{ij} = \frac{2 \min(E_i^2, E_j^2)(1 - \cos\theta_{ij})}{E_{\text{vis}}^2}$$

Hemishere masses:

$$M_j^2 = \frac{1}{E_{\text{vis}}^2} \left(\sum_{p_i \in \mathcal{H}_j} p_i \right)^2, \qquad j = 1, 2$$

$$M_{\text{H}}^2 = \max\left(M_1^2, M_2^2\right) \qquad M_{\text{D}}^2 = |M_1^2 - M_2^2|$$

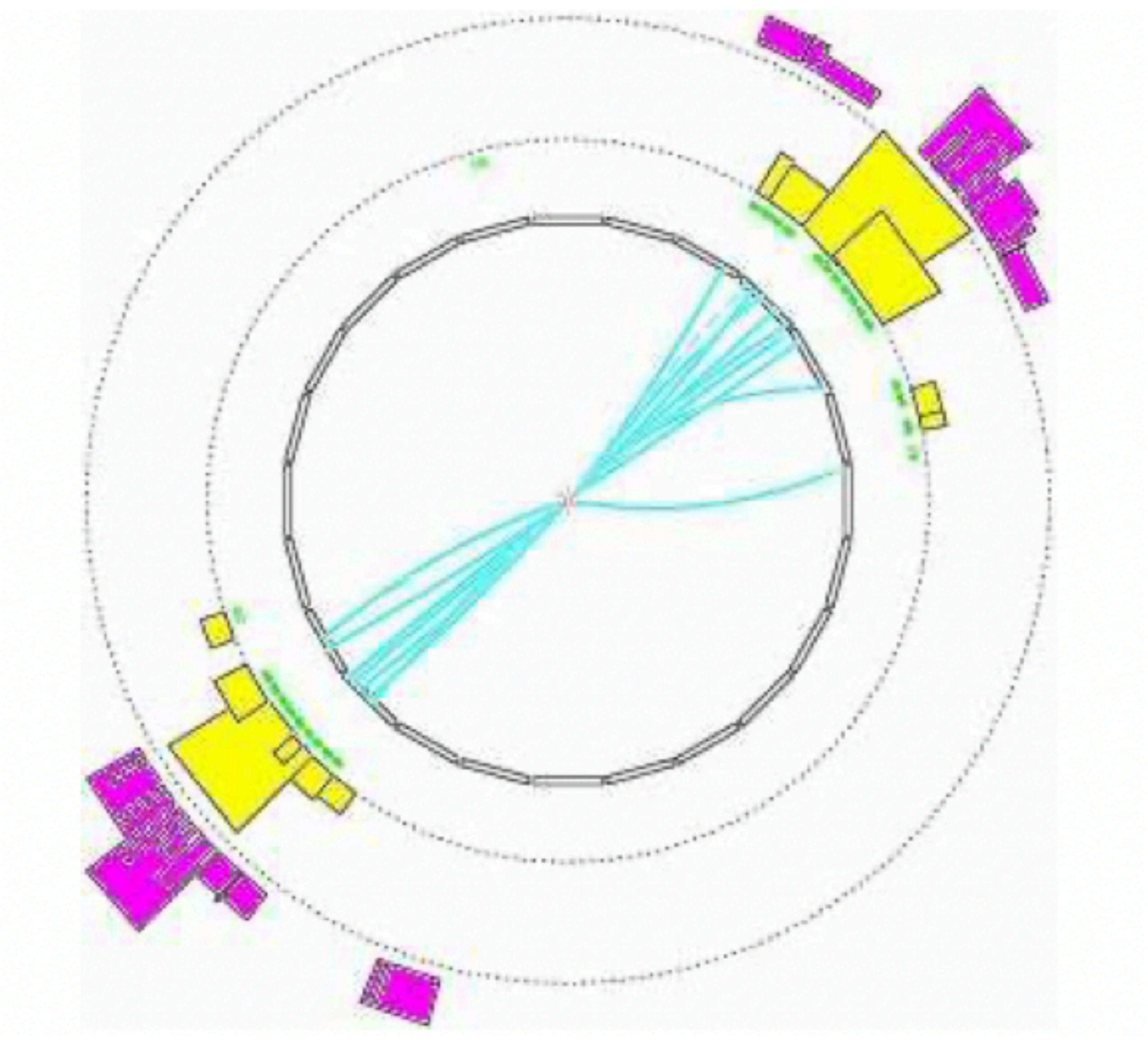
Wide broadening:

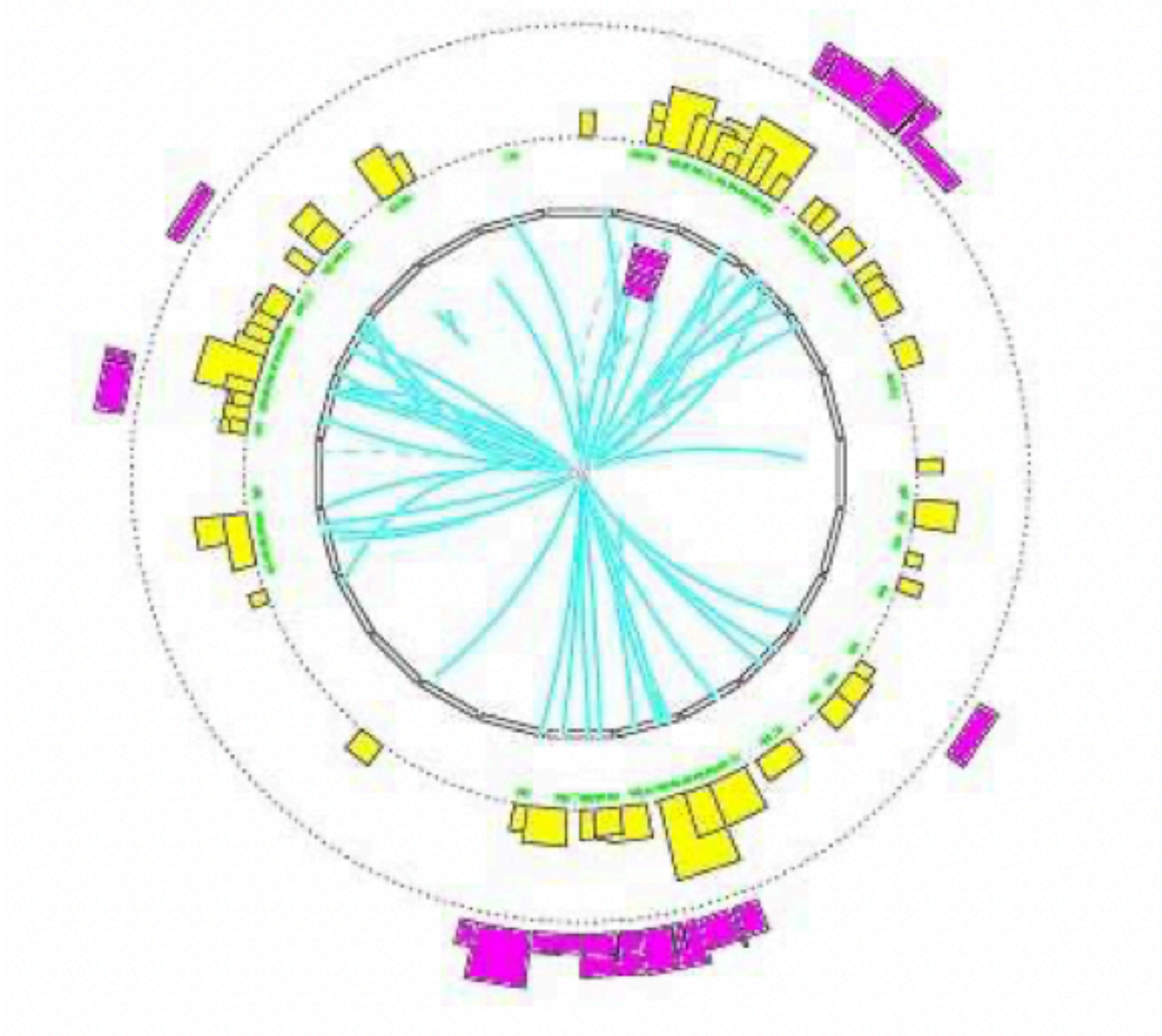
$$B_j = \frac{\sum_{p_i \in H_j} |\vec{p}_i \times \vec{n}_T|}{2 \sum_i |\vec{p}_i|}, \qquad j = 1, 2$$

$$B_W = \max\left(B_1, B_2\right)$$

Why event shapes?

- Perhaps the most basic class of final-state observables in QCD
- Event shapes provide a continue measure of deviation from Born-like 2-jet events





<u>2-jet event:</u> event-shape v << 1

<u>3-jet event:</u> event-shape v ~ 1

e^+e^- jet & shapes

1. Linear sensitivity to α_s in the 3-jet region 😀

2. NNLO + NNLL (at least) perturbative accuracy through standard resummation techniques or SCET based 😀

Criteria

1. observable's sensitivity to α_s wrt experimental precision
2. accuracy of the prediction
3. the size of non-perturbative effects
4. the scale at which the measurement is performed

4. Measurements performed in a large range of energy scales, from about 35-206 GeV, most precise data at 91.2 GeV 😀

3. Relatively large, Λ/Q linear power corrections 🙁

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e^+e^- jet & shapes

Two approaches to non-perturbative corrections:

- use Shower Monte Carlos hadronization models. Often criticised as this does not bear a clean relation to field-theoretical calculations, furthermore MCs are tuned using low-level perturbative (MC) calculations and used then with higher-order calculations
- analytic approaches
  - dispersive-like approaches: based on the emission of a very-soft, non-perturbative gluon with an associated non-perturbative coupling α_0

    Dokshitzer, Marchesini, Webber, Salam
  - factorisation based-approach to split perturbative and non-perturbative. Often used in combination with Soft-Collinear-Effective-Theroy (SCET) based predictions

    Collins, Soper, Korchemsky, Sterman; Abbate, Bauer, Hoang, Mateu, Schwartz, Stuart, Thaler…

e^+e^- jet & shapes

Two approaches to non-perturbative corrections:

- use Shower Monte Carlos hadronization models. Often criticised as this does not bear a clean relation to field-theoretical calculations, furthermore

Both analytic approaches calculate non-perturbative corrections in the 2-jet region and apply them also to the 3-jet region

Dokshitzer, Marchesini, Webber, Salam

  - factorisation based-approach to split perturbative and non-perturbative. Often used in combination with Soft-Collinear-Effective-Theroy (SCET) based predictions

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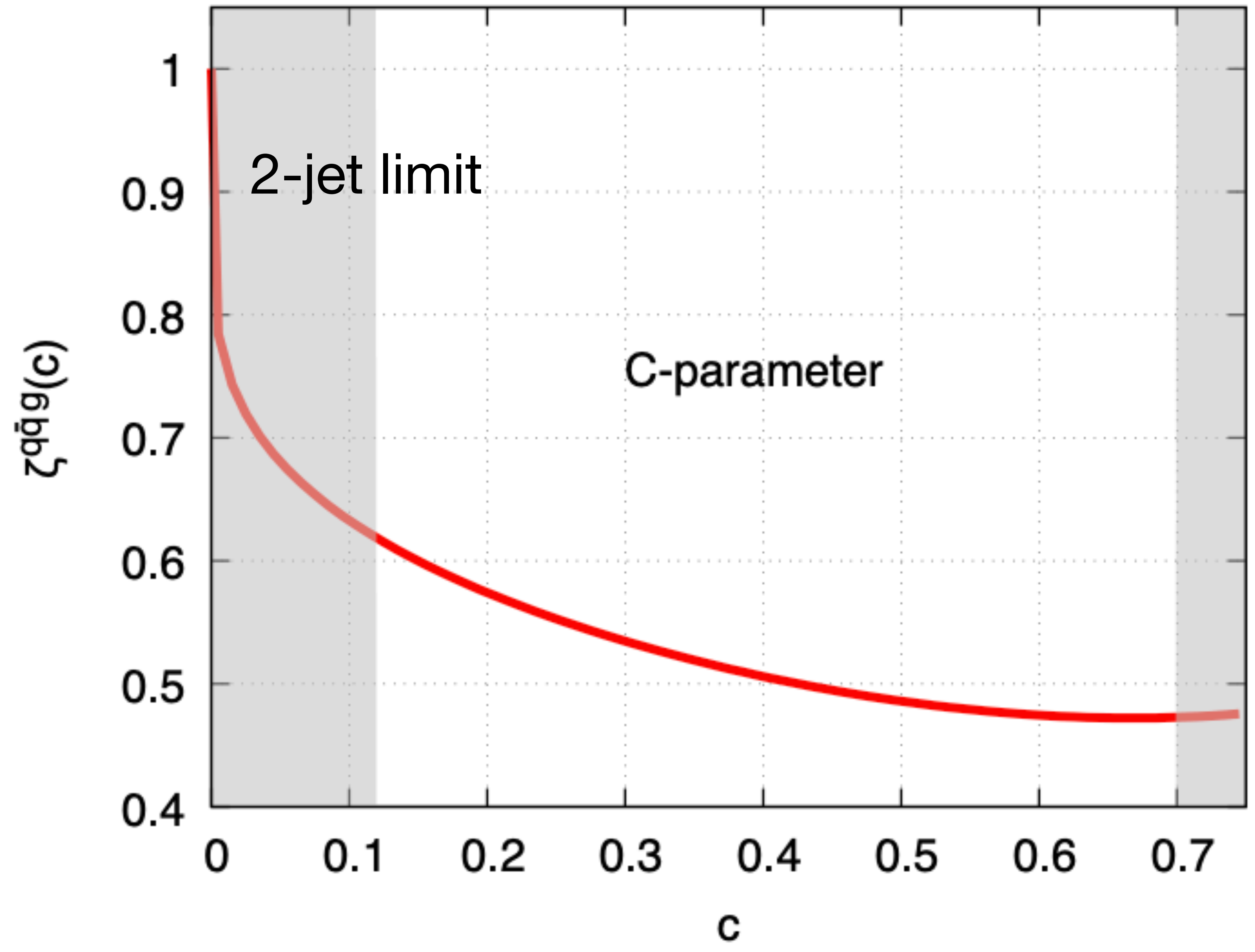
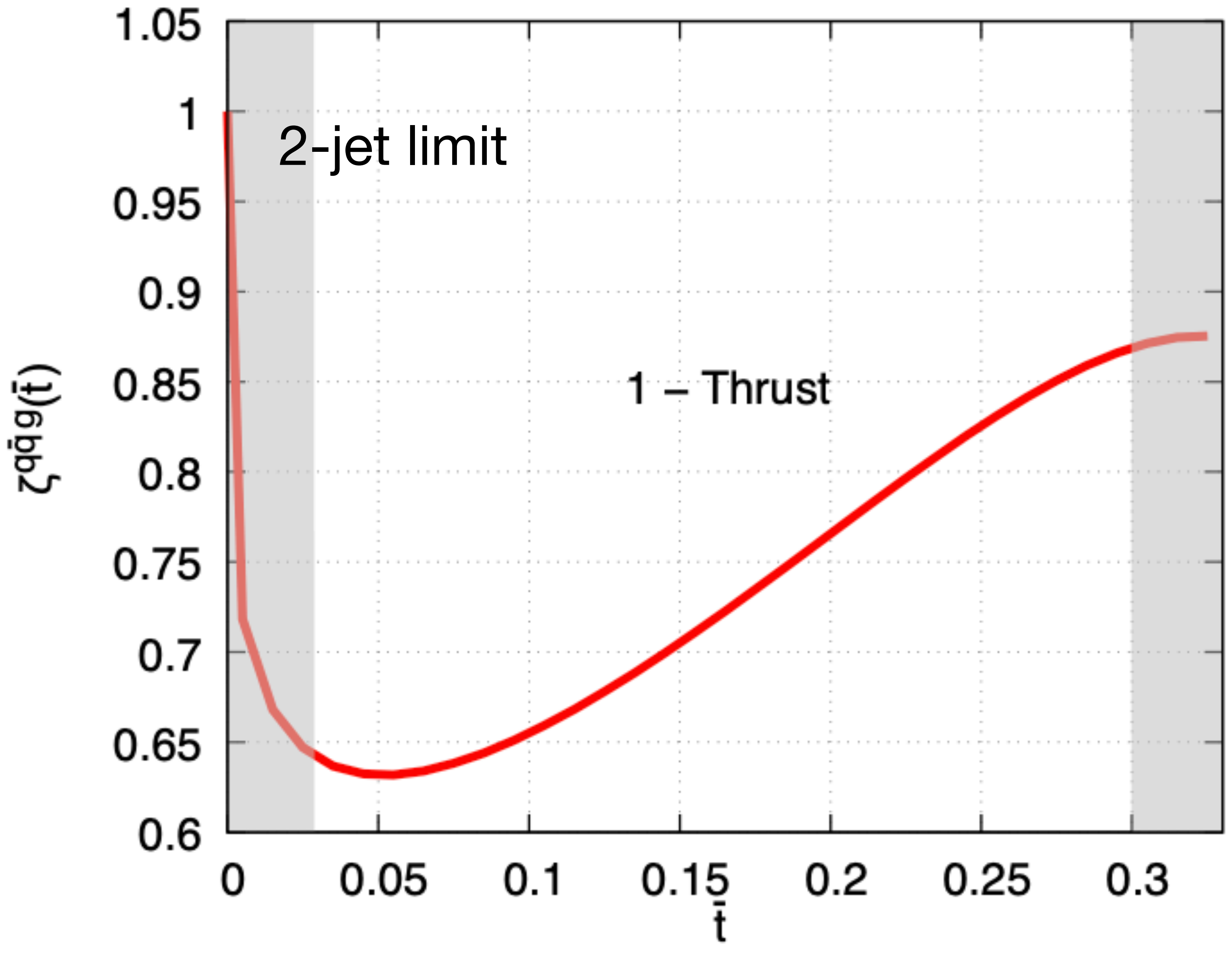
Non-perturbative corrections

Recently, non-perturbative corrections to 3-jet region have been computed for C-parameter and thrust in the context of the renormalon framework

Caola et al. 2204.02247 (see also Luisoni, Monni, Salam 2012.00622; Caola et al. 2108.00622)

<u>Ratio of full non-perturbative corrections to the 2-jet limit</u>



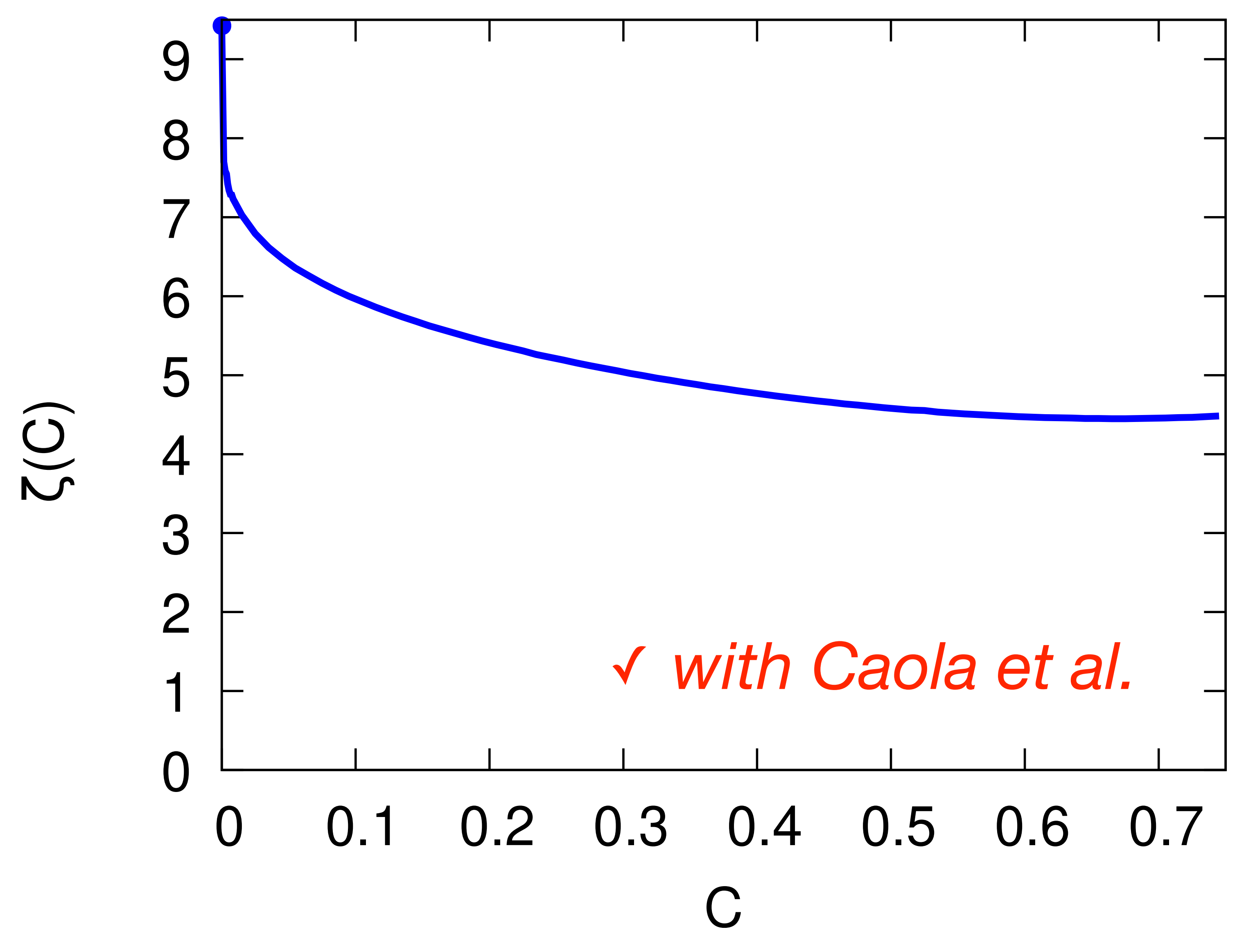


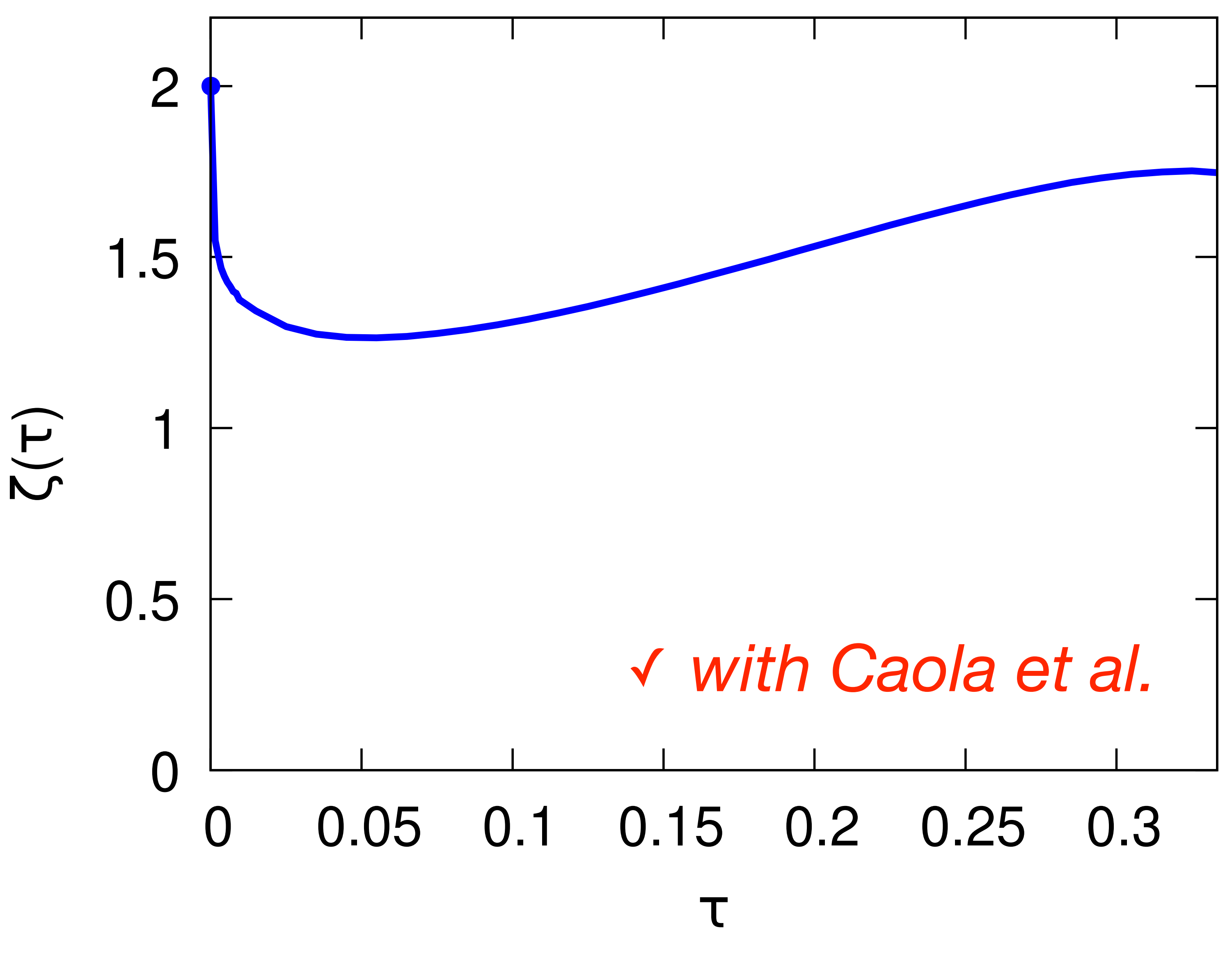
Does the new calculation of the non-perturbative corrections lift the tension in the determination of α_s from C-parameter and thrust … ?

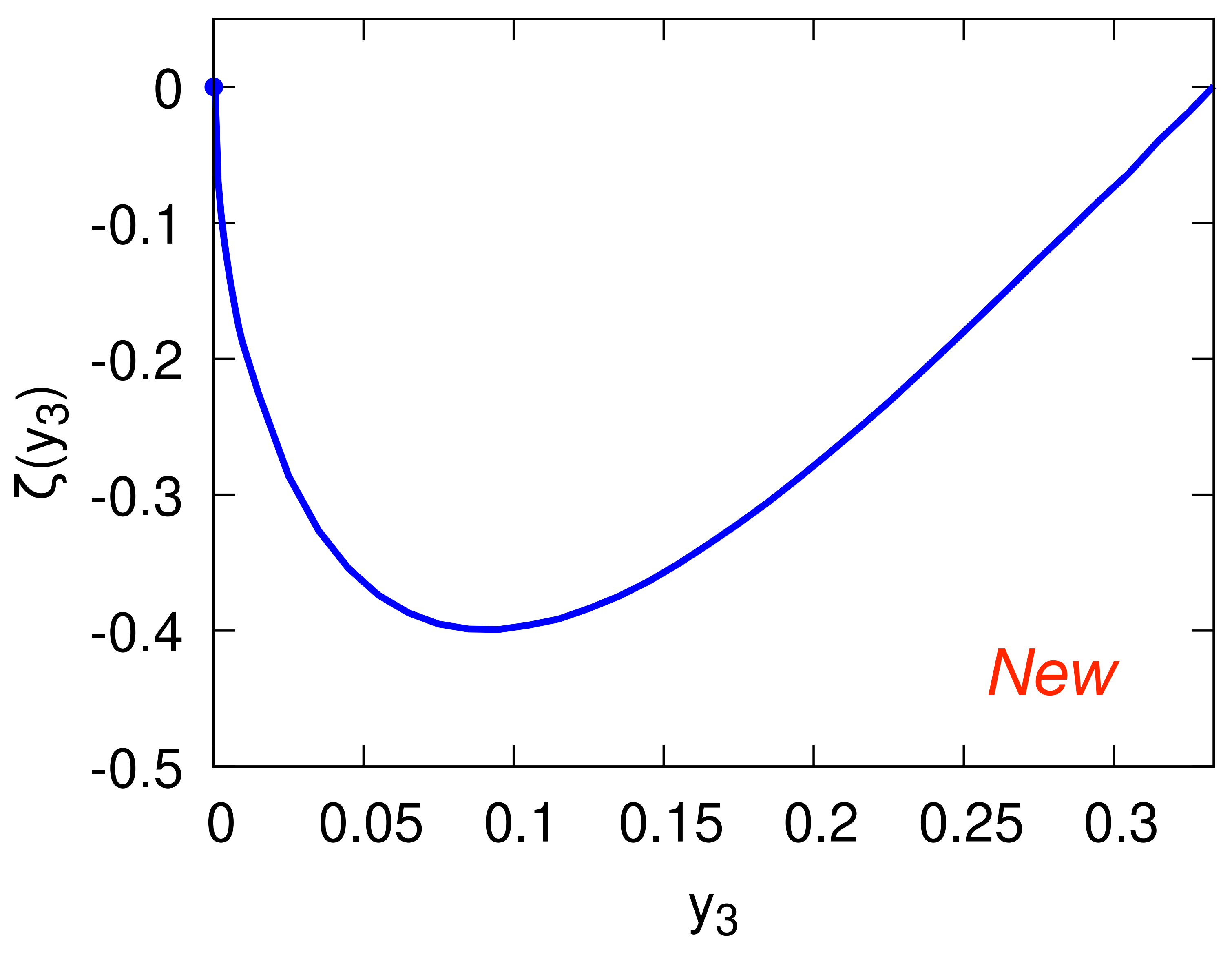
Are the newly computed corrections preferred by data …?

Correction with respect to 2-jet limit

We found a way to compute these non-perturbative corrections numerically, in a rather observable-independent way.



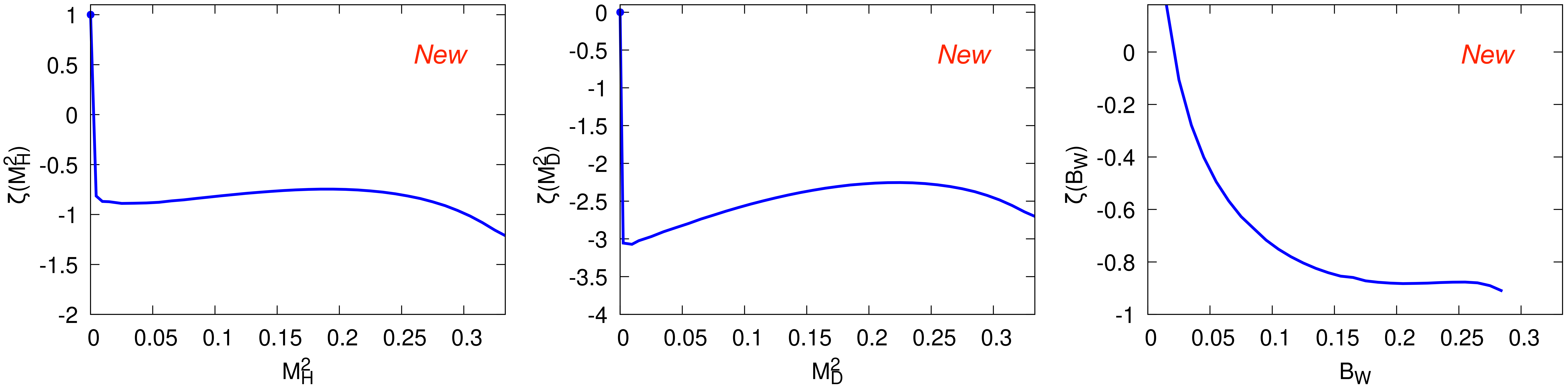




➡y_3 has no linear power corrections in the 2-jet limit

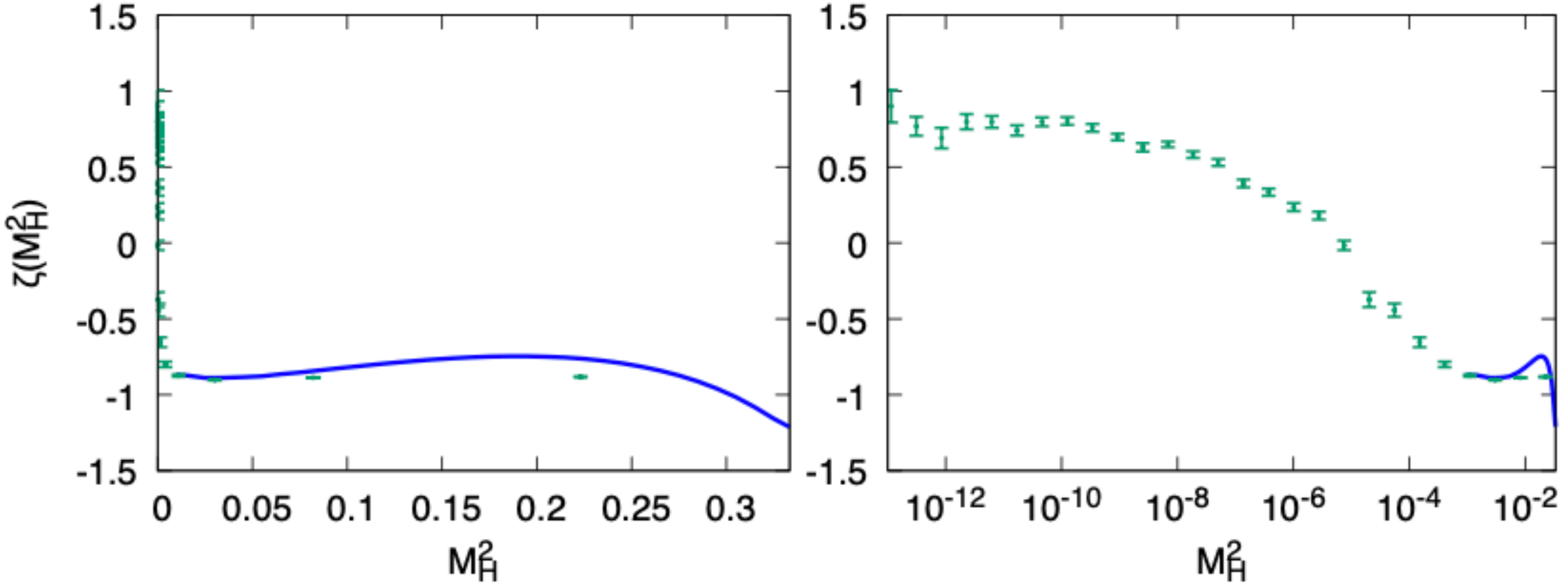
Correction with respect to 2-jet limit

For other observables, the two-jet limit is numerically very difficult to reach since there is an abrupt transition from the 2-jet to the 3-jet



Correction with respect 2-jet limit

We had to resort to quadruple precision to see the transition, for instance of the heavy-jet mass we obtain:



The 2-jet limit must be reached up to single-logs and constant terms, but these are, for some observables, numerically very important

Remarks

- there are clear indications that the 2-jet calculation is not a good approximation the in the 3-jet region, where α_s is fitted (at best it is wrong by a factor of order 1)
- for some observables there is a very abrupt transition from the 2-jet to the 3-jet region. This is an indication that sub-leading logarithms are numerically very important

➡We perform fits of α_s limiting ourselves to the three well-behaved observables C, 1-T and y_3 as measured by ALEPH at 91.2 GeV

ALEPH Eur. Phys. J. C 35 (2004) 457–486

Fit details

Fit is performed by minimising

$$\chi^2 = \sum_{i,j} \left(\frac{1}{\sigma_{\rm exp}} \frac{d\sigma_{\rm exp}(v_i)}{dv_i} - \frac{1}{\sigma_{\rm th}} \frac{d\sigma_{\rm th}(v_i)}{dv_i} \right) V_{ij}^{-1} \left(\frac{1}{\sigma_{\rm exp}} \frac{d\sigma_{\rm exp}(v_j)}{dv_j} - \frac{1}{\sigma_{\rm th}} \frac{d\sigma_{\rm th}(v_j)}{dv_j} \right)$$

with

$$V_{ij} = \delta_{ij}(R_i^2 + T_i^2) + (1 - \delta_{ij}) C_{ij} R_i R_j + \mathrm{Cov}_{ij}^{(\mathrm{Sys})}$$

R_i^2 statistical error

T_i^2 theory error $T_{\rm i} = \dfrac{\max(O(Q), O(Q/2), O(Q/4)) - \min(O(Q), O(Q/2), O(Q/4))}{2}$

C_{ij} statistical correlation matrix $C_{ij} = \dfrac{\frac{N_{ij}}{N} - \frac{N_i N_j}{N^2}}{\sqrt{\frac{N_i}{N} - \frac{N_i^2}{N^2}} \sqrt{\frac{N_j}{N} - \frac{N_j^2}{N^2}}}$

$\mathrm{Cov}_{ij}^{(\mathrm{Sys})}$ covariance matrix of systematic errors

Fit results

Variation	$\alpha_s(M_Z)$	α_0	χ^2	χ^2/N_{deg}
Default setup	**0.1174**	**0.64**	**6.8**	**0.15**
Renormalization scale $Q/4$	0.1180	0.60	6.1	0.14
Renormalization scale Q	0.1182	0.68	7.9	0.18
NP scheme (B)	0.1186	0.79	6.4	0.15
NP scheme (C)	0.1194	0.84	4.7	0.11
NP scheme (D)	0.1184	0.66	5.2	0.12
P-scheme	0.1150	0.63	9.5	0.22
D-scheme	0.1188	0.79	5.1	0.12
no scheme	0.1168	0.58	8.1	0.18
No heavy to light correction	0.1176	0.68	6.2	0.14
`Herwig6`	0.1174	0.60	14.7	0.33
`Herwig7`	0.1174	0.60	10.9	0.25
Ranges (2)	0.1166	0.62	12.3	0.22
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Replica method (around average)	0.1180	0.62	5.8	0.13
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y_3 clustered	0.1166	0.67	7.6	0.17
C	0.1252	0.47	0.9	0.06
τ	0.1188	0.64	0.7	0.03
y_3	0.1196	1.90	0.02	0.002
C, τ	0.1230	0.51	2.0	0.05

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➡ We use NNLO predictions at scale μ_R = Q/2 and vary the scale up and down by a factor of two

- Antenna-based calculation: A. Gehrmann-De Ridder, T. Gehrmann and E. W. N. Glover, hep-ph/0505111
- ColorFull Subtraction: Del Duca et al 1603.08927, 1606.03453

➡ For the NNLO prediction we rely on the public code EERAD3

Gehrmann-De Ridder, Gehrmann, Glover, Heinrich
0710.0346, 0711.4711, 0802.0813

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Non-perturbative corrections can be included as

➡ a shift of the NNLO integrated distribution (scheme “a”)

➡ a shift of the LO distribution only (scheme “b”)

➡ a shift of the differential distribution (scheme “c”)

➡ as in scheme “a” without any estimate of quadratic corrections included in other schemes (scheme “d”)

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Explicitly:

$$\Sigma^{(a)}(v) = \Sigma_{\rm NNLO}(v - \delta v)$$

$$\Sigma^{(b)}(v) = \Sigma_B(v - \delta v) + \Sigma_{\rm NNLO}(v) - \Sigma_B(v)$$

$$\Sigma^{(c)}(v) = \Sigma_{\rm NNLO}(v) - \delta v \frac{d\Sigma_B(v)}{dv}$$

with

$$\delta(v) = \zeta(v) H_{\rm NP} + \boxed{\left(\tilde{\zeta}(v) \times \frac{Q_0}{\lambda_0} - \zeta(v)\right) \times \frac{Q_0}{\lambda_0} \times H_{\rm NP}^2}$$

Estimate of quadratic corrections

$$\Sigma^{(d)}(v) = \Sigma_{\rm NNLO}(v - \delta v) \quad \text{with} \quad \delta(v) = \zeta(v) H_{\rm NP}$$

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Ambiguity in the event-shape definitions when applied to massive particles. Correct to different schemes using Monte Carlos:

- ➡ E-sheme (our default): make particle massless conserving the energies
- ➡ P-scheme: make particle massless conserving the three-momentum
- ➡ Decay-scheme: decay each massive particle isotropically in its CM frame into two massless particles
- ➡ Standard: do not correct

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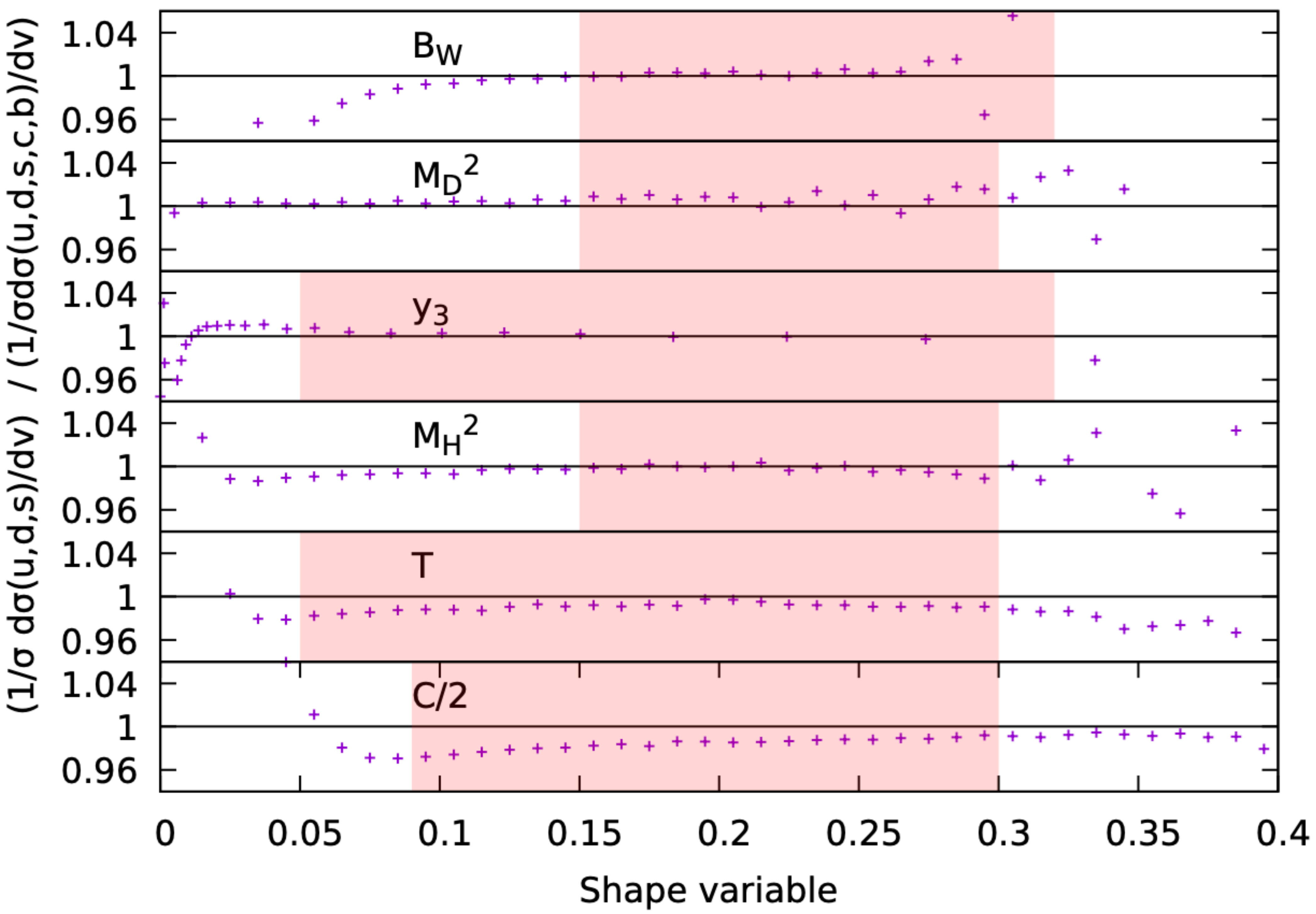
NNLO deals with massless quarks. Use Monte Carlo to correct for massive charm and bottom

$$v_i^{(\text{corr})} = v_i \times \frac{v_i^{\text{MC},uds}}{v_i^{\text{MC},udscb}}$$



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As default Monte Carlo for the calculation of the migration matrix for the mass-schemes and heavy-to-light correction we used `Pythia` 8.

To assess the sensitivity to the Monte Carlo used we also use `Herwig` 6 and `Herwig` 7.

Fit results

Variation	$\alpha_s(M_Z)$	α_0	χ^2	χ^2/N_{deg}
Default setup	**0.1174**	**0.64**	**6.8**	**0.15**
Renormalization scale $Q/4$	0.1180	0.60	6.1	0.14
Renormalization scale Q	0.1182	0.68	7.9	0.18
NP scheme (B)	0.1186	0.79	6.4	0.15
NP scheme (C)	0.1194	0.84	4.7	0.11
NP scheme (D)	0.1184	0.66	5.2	0.12
P-scheme	0.1150	0.63	9.5	0.22
D-scheme	0.1188	0.79	5.1	0.12
no scheme	0.1168	0.58	8.1	0.18
No heavy to light correction	0.1176	0.68	6.2	0.14
`Herwig6`	0.1174	0.60	14.7	0.33
`Herwig7`	0.1174	0.60	10.9	0.25
Ranges (2)	0.1166	0.62	12.3	0.22
Ranges (3)	0.1178	0.69	2.4	0.07
Replica method (around average)	0.1180	0.62	5.8	0.13
Replica method (around default)	0.1180	0.62	5.8	0.13
y_3 clustered	0.1166	0.67	7.6	0.17
C	0.1252	0.47	0.9	0.06
τ	0.1188	0.64	0.7	0.03
y_3	0.1196	1.90	0.02	0.002
C, τ	0.1230	0.51	2.0	0.05

Default range fixed to the left of where resummation effects are important.

To assess sensitivity to range by varying the lower edge by a factor 2/3 and 3/2

observable	default	Fit ranges (2)	Fit ranges (3)
C	$[0.25:0.6]$	$[0.17:0.6]$	$[0.375:0.6]$
τ	$[0.1:0.3]$	$[0.067:0.3]$	$[0.15:0.3]$
y_3	$[0.05:0.3]$	$[0.033:0.3]$	$[0.075:0.3]$

Fit results

Variation	$\alpha_s(M_Z)$	α_0	χ^2	χ^2/N_{deg}
Default setup	**0.1174**	**0.64**	**6.8**	**0.15**
Renormalization scale $Q/4$	0.1180	0.60	6.1	0.14
Renormalization scale Q	0.1182	0.68	7.9	0.18
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y_3 clustered	0.1166	0.67	7.6	0.17
C	0.1252	0.47	0.9	0.06
τ	0.1188	0.64	0.7	0.03
y_3	0.1196	1.90	0.02	0.002
C, τ	0.1230	0.51	2.0	0.05

We implement correlations using a minimum overlap method

$$\text{Cov}_{ij}^{(\text{Sys})} = \delta_{ij} S_i^2 + (1-\delta_{ij}) \min(S_i^2, S_j^2)$$

To assess sensitivity to this, we also use replicas provided to us privately by Hasko Stenzel either around the default central value, or around the average of the replicas

$$\text{Cov}_{ij}^{(\text{Sys})} = \sum_r \left(v_i^{(r)} - \bar{v}_i\right)\left(v_j^{(r)} - \bar{v}_j\right) = N_r\left(\bar{v}_{ij} - \bar{v}_i \bar{v}_j\right)$$

Fit results

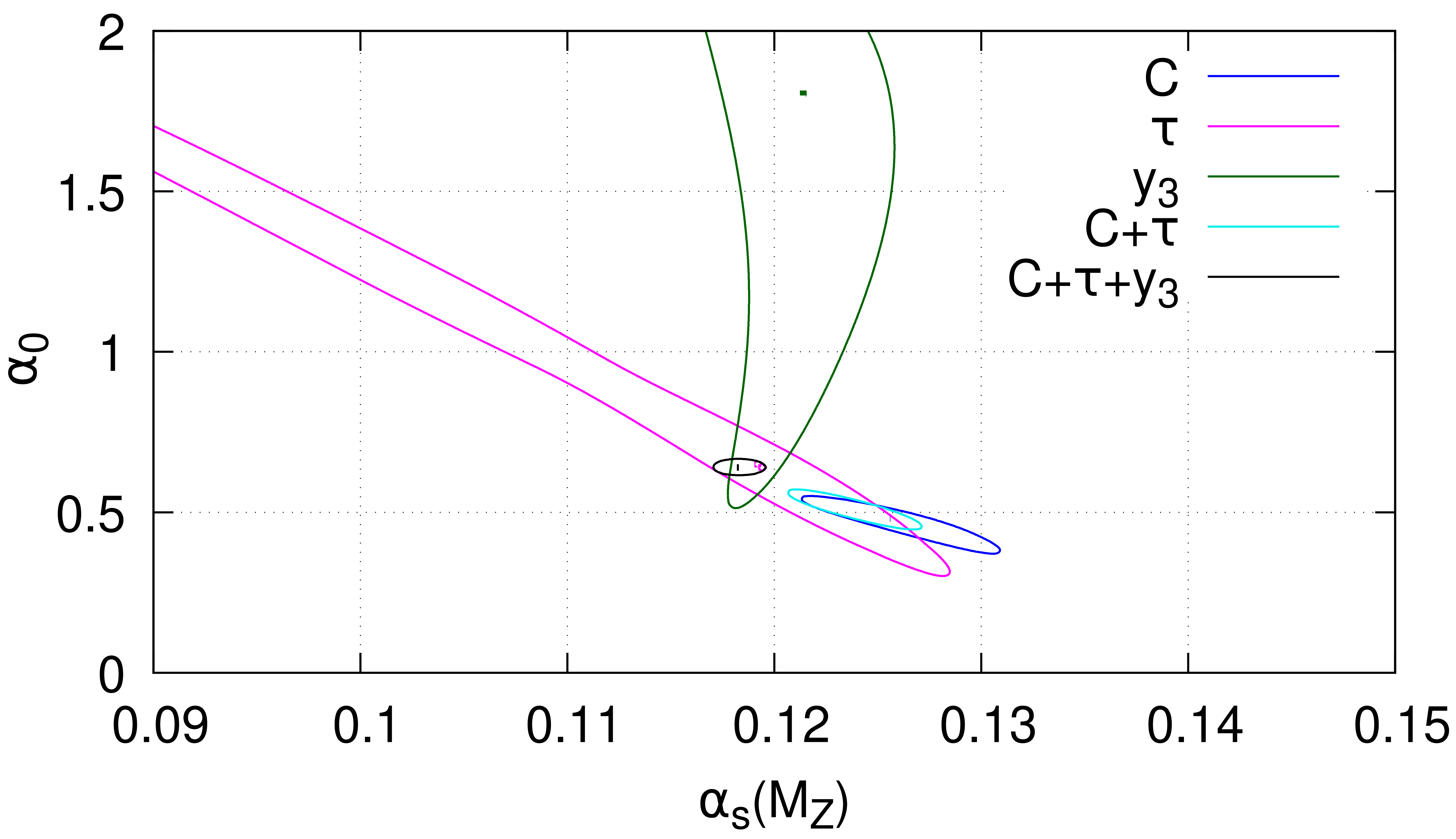
Variation	$\alpha_s(M_Z)$	α_0	χ^2	χ^2/N_{deg}
Default setup	**0.1174**	**0.64**	**6.8**	**0.15**
Renormalization scale $Q/4$	0.1180	0.60	6.1	0.14
Renormalization scale Q	0.1182	0.68	7.9	0.18
NP scheme (B)	0.1186	0.79	6.4	0.15
NP scheme (C)	0.1194	0.84	4.7	0.11
NP scheme (D)	0.1184	0.66	5.2	0.12
P-scheme	0.1150	0.63	9.5	0.22
D-scheme	0.1188	0.79	5.1	0.12
no scheme	0.1168	0.58	8.1	0.18
No heavy to light correction	0.1176	0.68	6.2	0.14
`Herwig6`	0.1174	0.60	14.7	0.33
`Herwig7`	0.1174	0.60	10.9	0.25
Ranges (2)	0.1166	0.62	12.3	0.22
Ranges (3)	0.1178	0.69	2.4	0.07
Replica method (around average)	0.1180	0.62	5.8	0.13
Replica method (around default)	0.1180	0.62	5.8	0.13
y_3 clustered	0.1166	0.67	7.6	0.17
C	0.1252	0.47	0.9	0.06
τ	0.1188	0.64	0.7	0.03
y_3	0.1196	1.90	0.02	0.002
C, τ	0.1230	0.51	2.0	0.05

Even in the 3-jet region y_3 is only additive if one assumes *no clustering* among the two soft partons from gluon splitting. We have computed the non-perturbative correction under this assumption.

To assess the error we also compute it under the assumption that they *always* cluster (corresponding to a massive gluon)

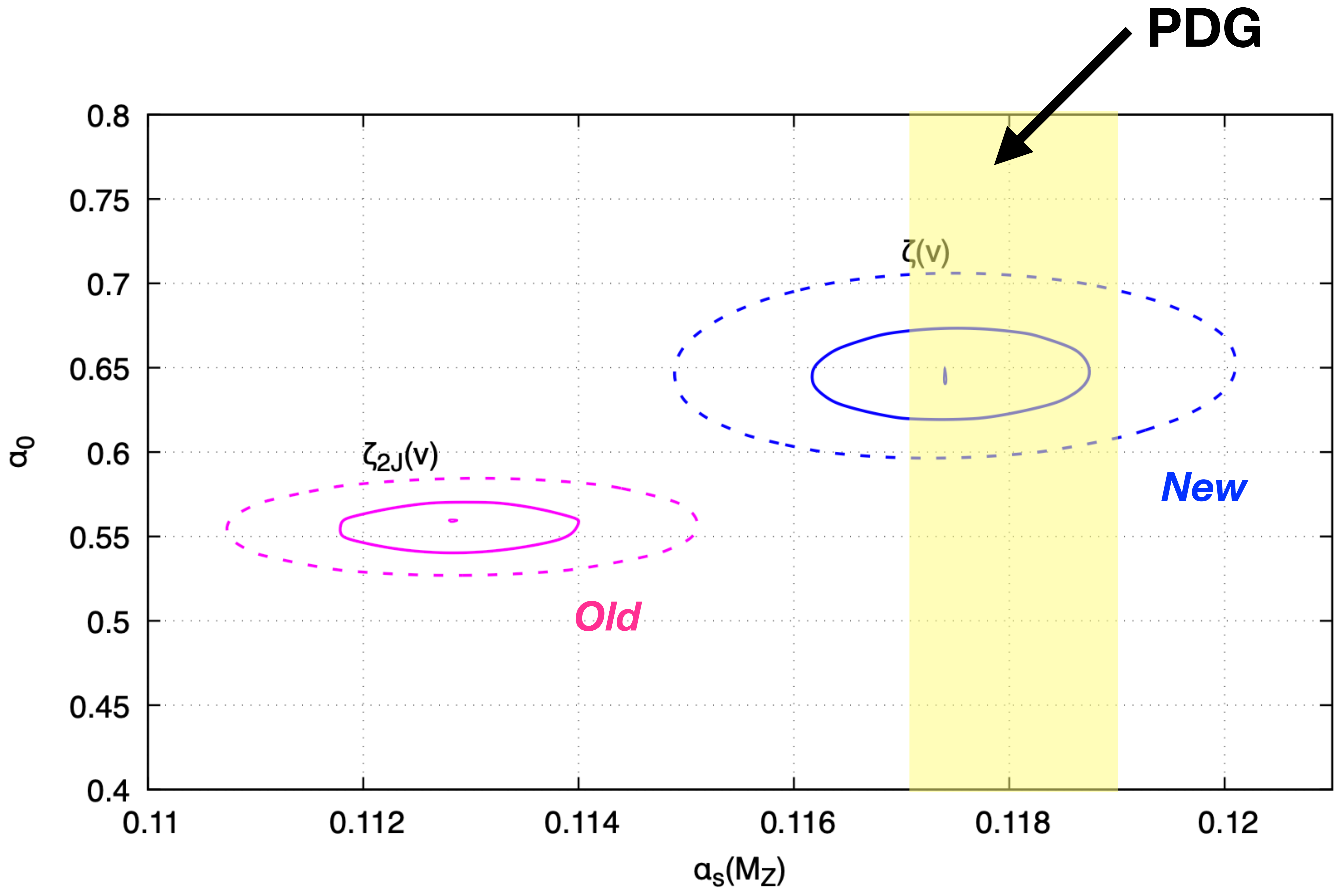
Fit results

Variation	$\alpha_s(M_Z)$	α_0	χ^2	χ^2/N_{deg}
Default setup	**0.1174**	**0.64**	**6.8**	**0.15**
Renormalization scale $Q/4$	0.1180	0.60	6.1	0.14
Renormalization scale Q	0.1182	0.68	7.9	0.18
NP scheme (B)	0.1186	0.79	6.4	0.15
NP scheme (C)	0.1194	0.84	4.7	0.11
NP scheme (D)	0.1184	0.66	5.2	0.12
P-scheme	0.1150	0.63	9.5	0.22
D-scheme	0.1188	0.79	5.1	0.12
no scheme	0.1168	0.58	8.1	0.18
No heavy to light correction	0.1176	0.68	6.2	0.14
`Herwig6`	0.1174	0.60	14.7	0.33
`Herwig7`	0.1174	0.60	10.9	0.25
Ranges (2)	0.1166	0.62	12.3	0.22
Ranges (3)	0.1178	0.69	2.4	0.07
Replica method (around average)	0.1180	0.62	5.8	0.13
Replica method (around default)	0.1180	0.62	5.8	0.13
y_3 clustered	0.1166	0.67	7.6	0.17
C	0.1252	0.47	0.9	0.06
τ	0.1188	0.64	0.7	0.03
y_3	0.1196	1.90	0.02	0.002
C, τ	0.1230	0.51	2.0	0.05



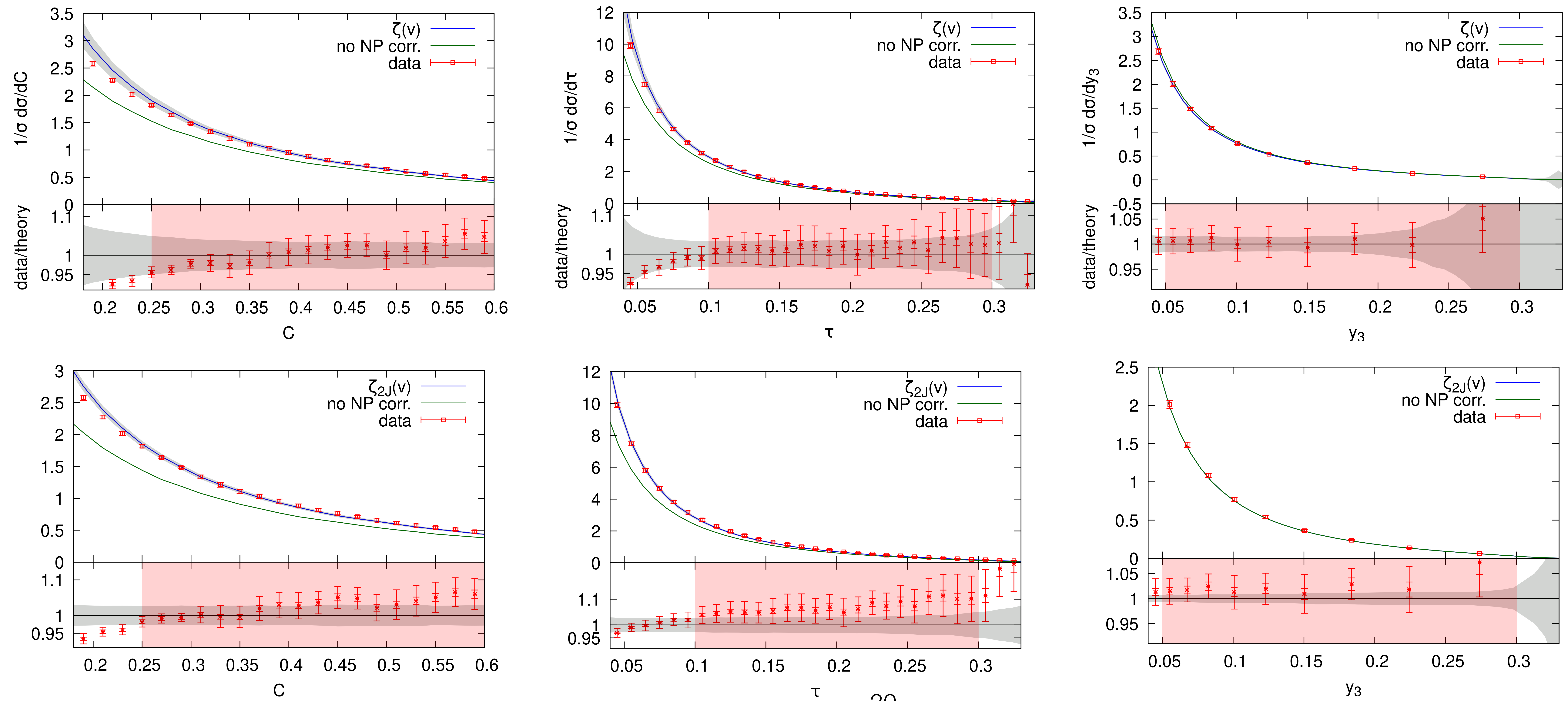
Fit result using 2-jet NP corrections

Variation	$\alpha_s(M_Z)$	α_0	χ^2	χ^2/N_{deg}
Default setup	**0.1128**	**0.56**	**16.1**	**0.37**
Renormalization scale $Q/4$	0.1144	0.53	9.7	0.22
Renormalization scale Q	0.1132	0.56	18.9	0.43
NP scheme (B)	0.1124	0.64	26.5	0.60
NP scheme (C)	0.1128	0.74	17.0	0.39
NP scheme (D)	0.1128	0.56	16.1	0.37
P-scheme	0.1104	0.54	20.7	0.47
D-scheme	0.1120	0.66	16.4	0.37
no scheme	0.1130	0.51	15.2	0.35
No heavy to light correction	0.1126	0.58	16.1	0.37
`Herwig6`	0.1134	0.52	30.3	0.69
`Herwig7`	0.1132	0.52	21.6	0.49
Ranges (2)	0.1116	0.54	30.1	0.55
Ranges (3)	0.1130	0.60	9.3	0.29
Replica method (around average)	0.1144	0.54	12.9	0.29
Replica method (around default)	0.1144	0.54	12.9	0.29
y_3 clustered	0.1128	0.56	16.1	0.37
C	0.1236	0.44	0.8	0.05
τ	0.1194	0.51	1.0	0.05
y_3	0.1152	–	1.7	0.22
C, τ	0.1220	0.45	2.1	0.06



Quality of the fits

Theory prediction compared to data for **observables entering the fits**:

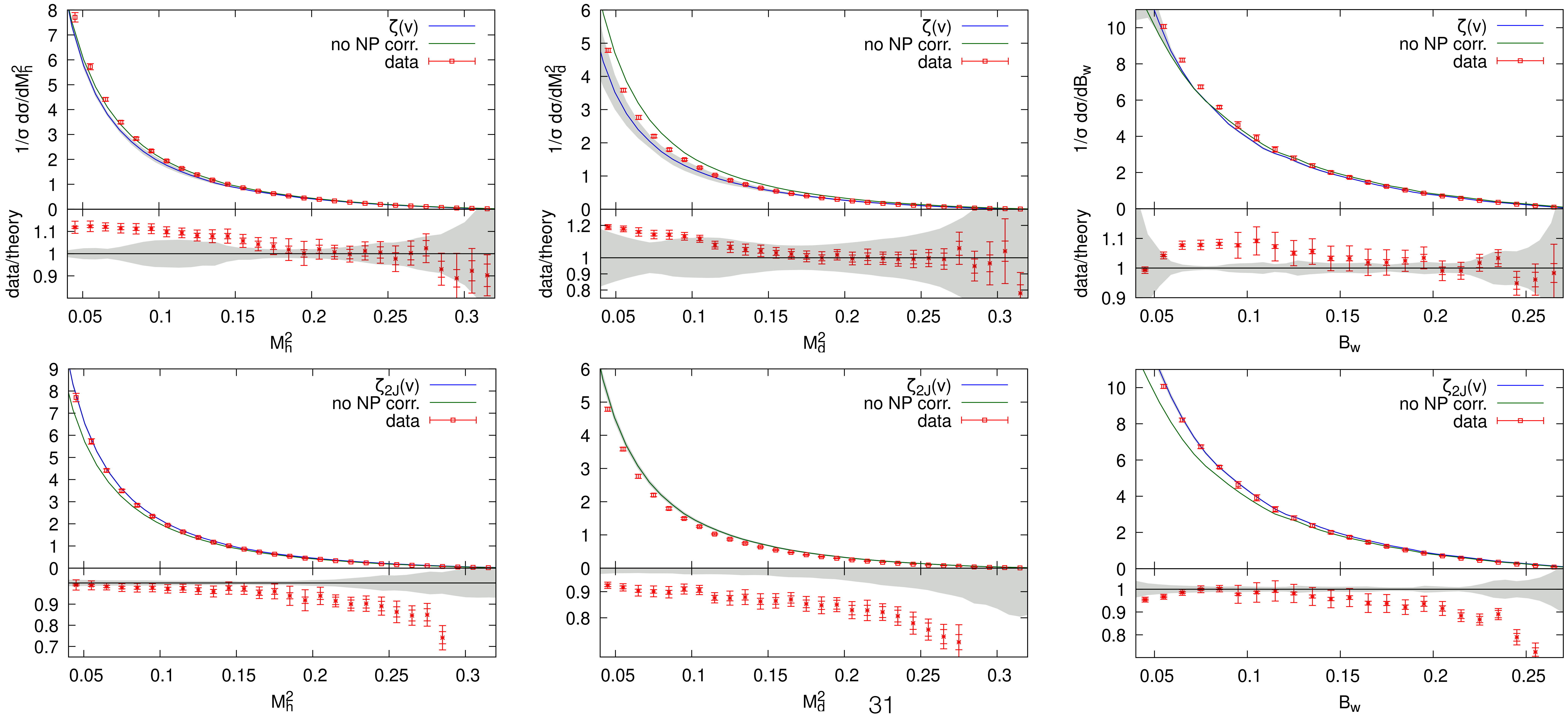


New
3-jet power corrections 😀

Old
2-jet power corrections 😀

Description of other observables

Description of other **observables not entering the fits**:

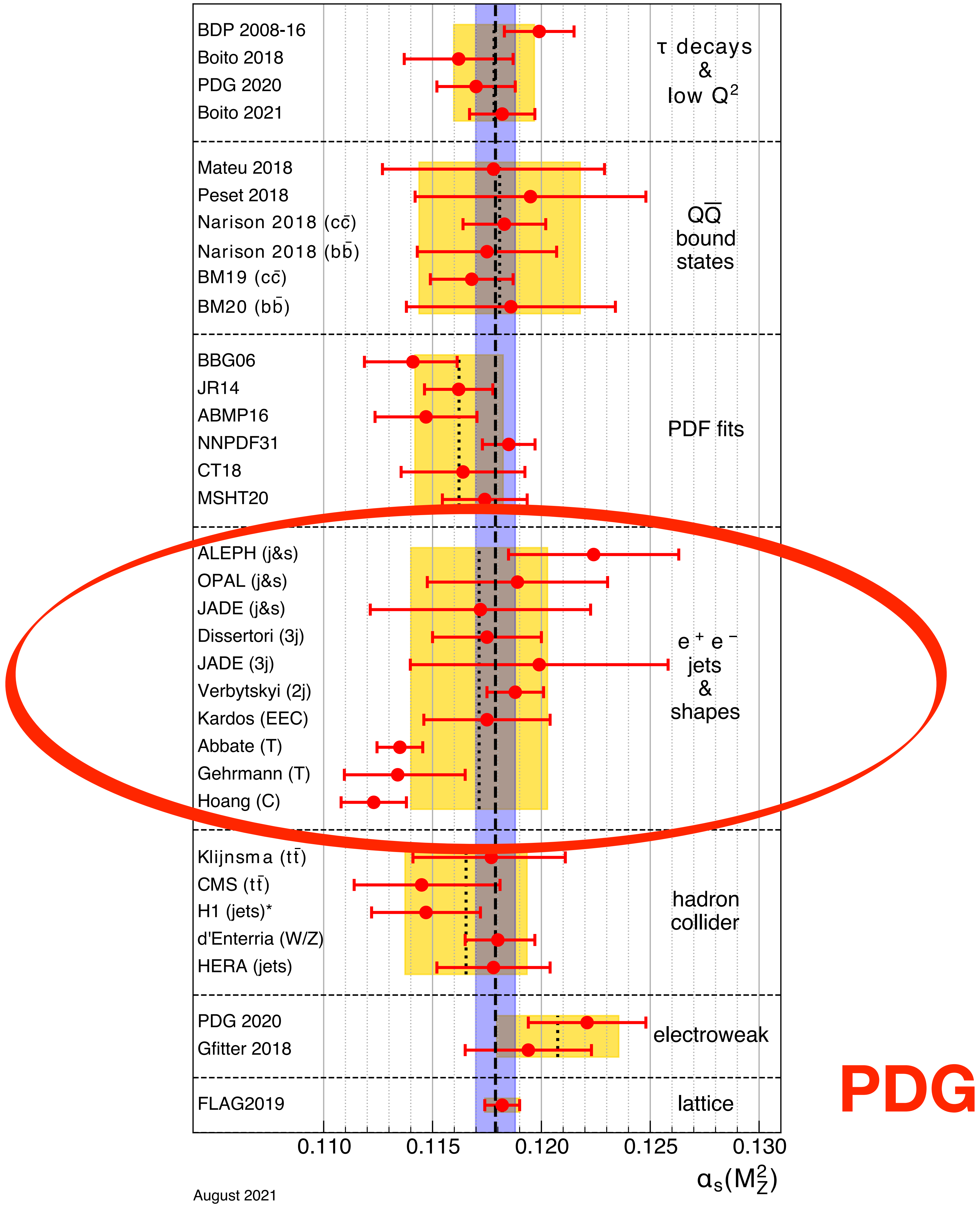


New* 3-jet *power corrections 😐

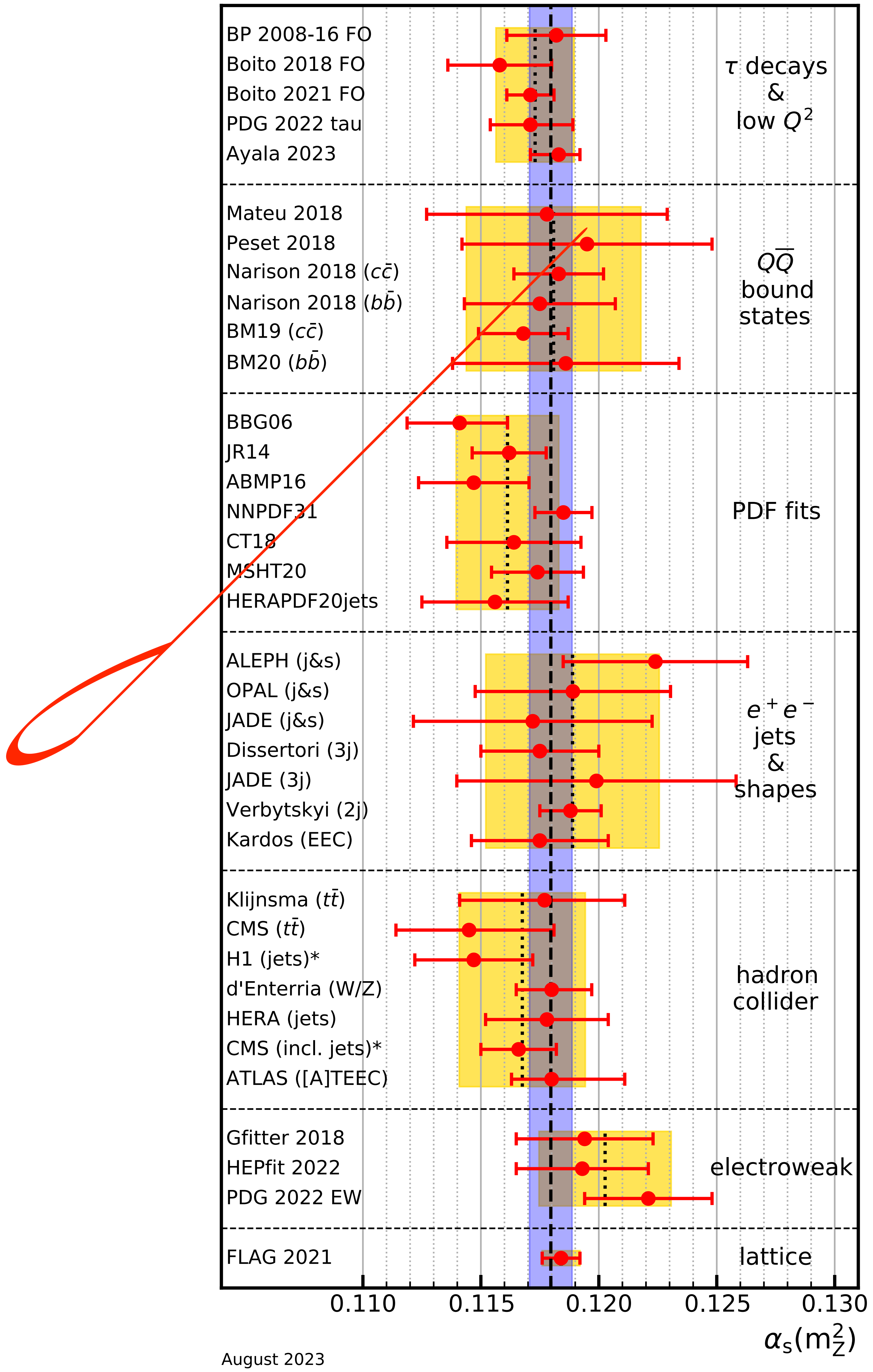
Old* 2-jet *power corrections 🙁

Impact on PDG fit

PDG, Huston, Rabbertz, GZ



PDG 2021



PDG 2023