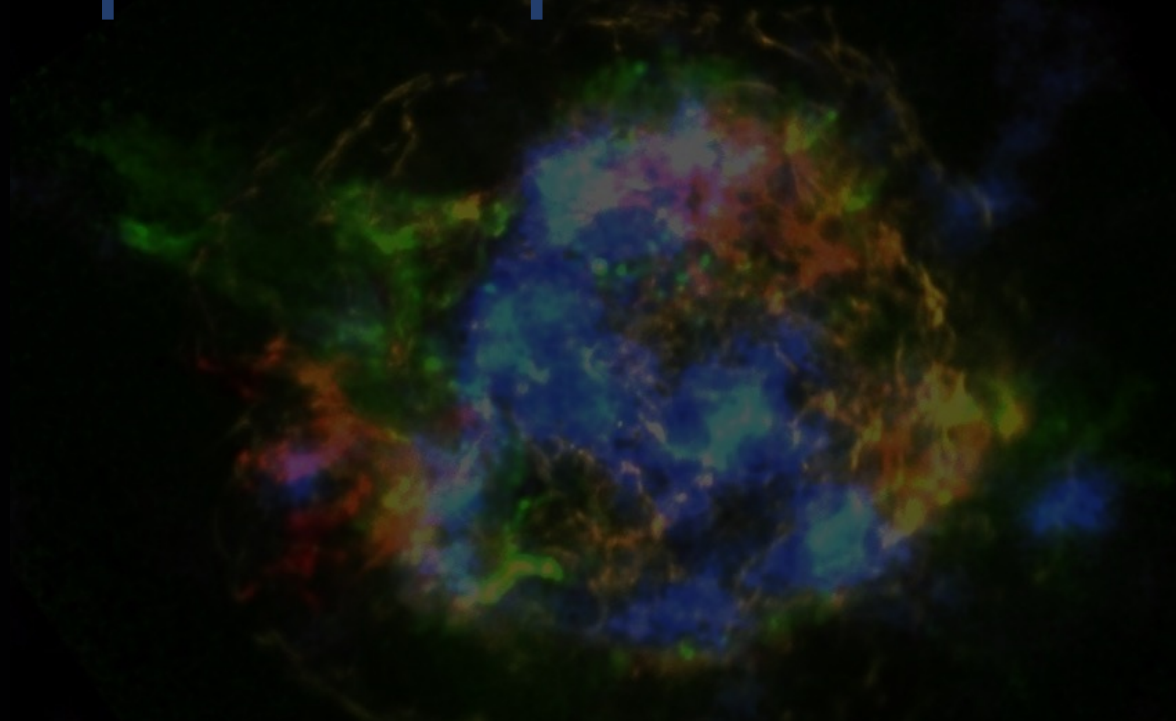


The nuclear equation of state and core-collapse supernova observables

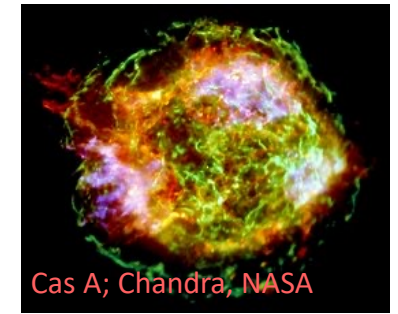
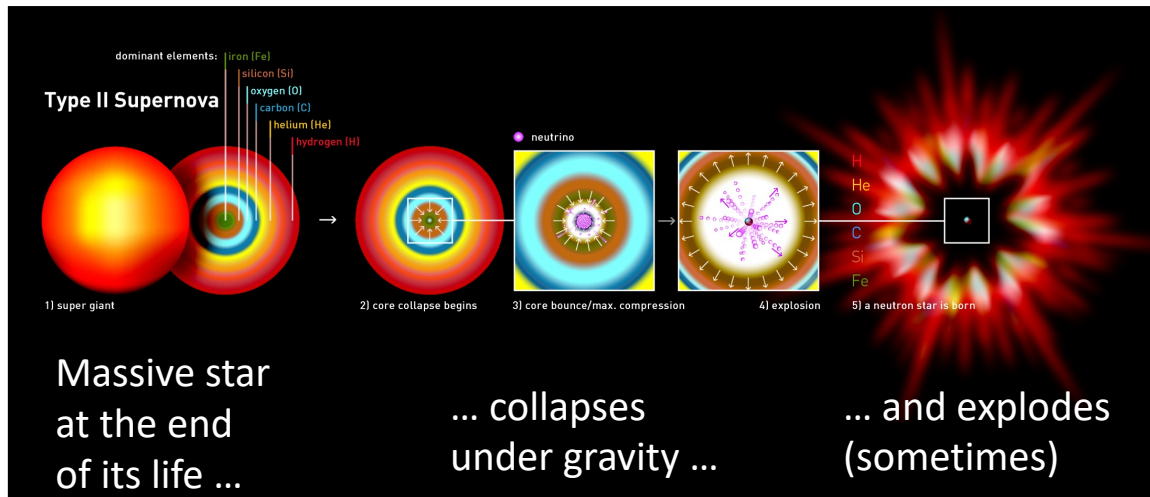


Carla Fröhlich

North Carolina State University



Core-collapse supernova simulations



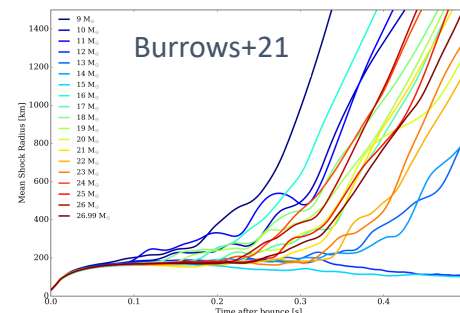
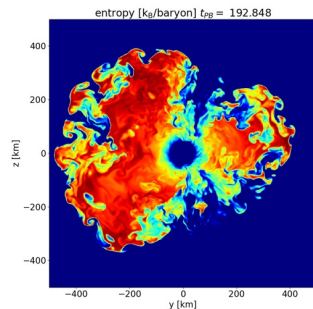
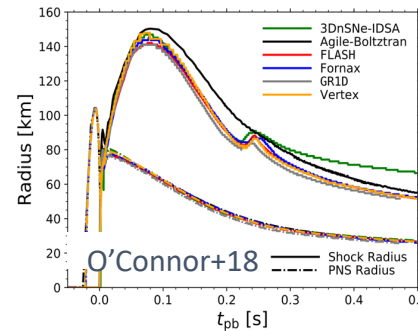
- Multi-dimensional problem
- Multi-physics problem:
 - General relativity
 - Nuclear physics of dense matter
 - Neutrino transport (trapped, diffusive, free-streaming regimes)
- Multi-scale problem:
 - shock formation at ~ 200 km vs entire star 10^8 km
 - collapse and shock formation ~ 1 s vs shock breakout ~ 1 day

Simulation Status:

1D: in general no self-consistent explosions
 ~ 10 CPUh/model

2D: models have converged

3D: mixed results
 \sim Mio CPUh/model

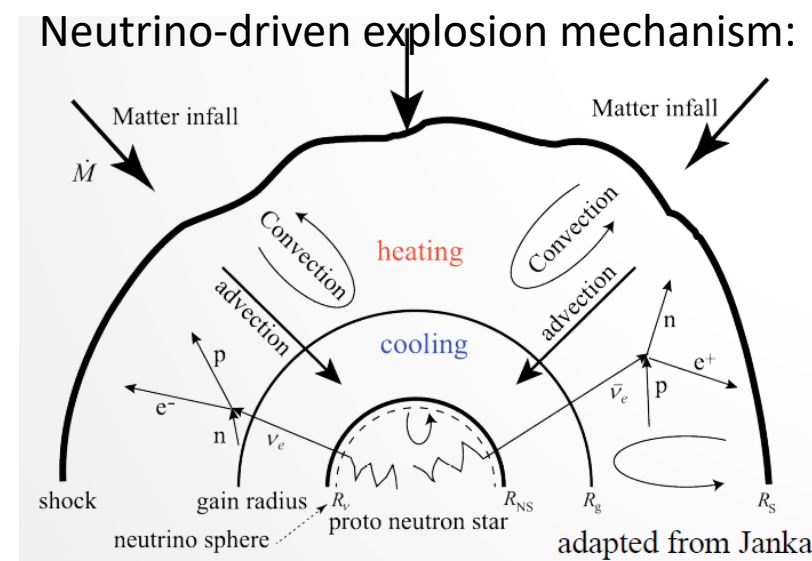


Paths forward?

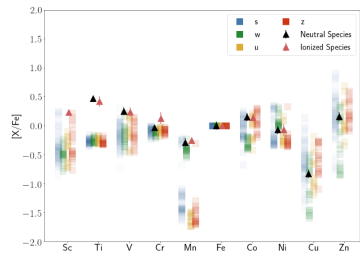
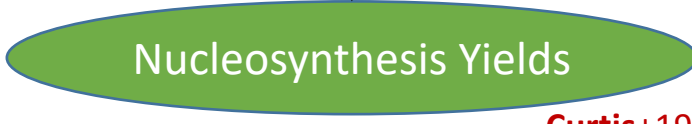
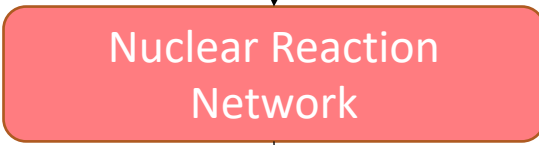
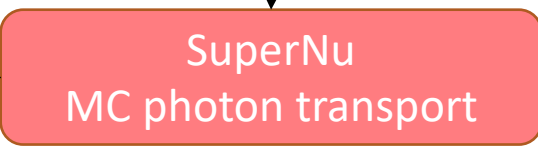
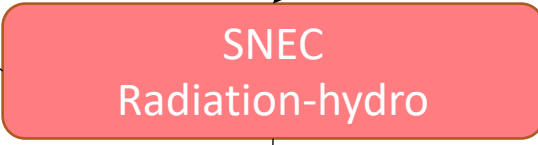
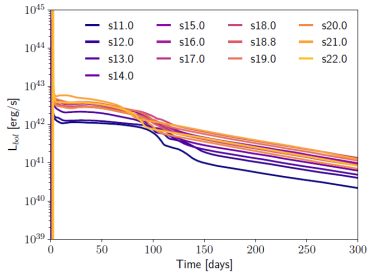
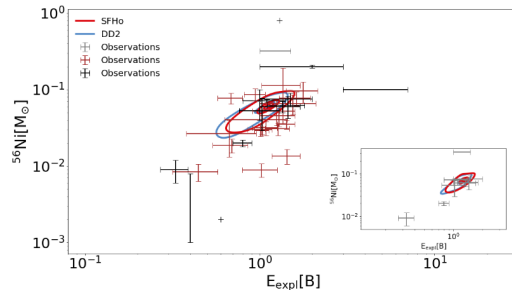
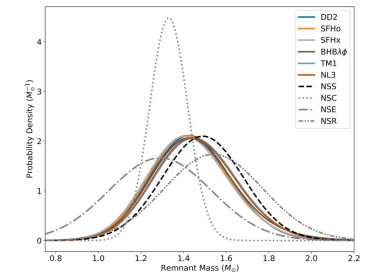
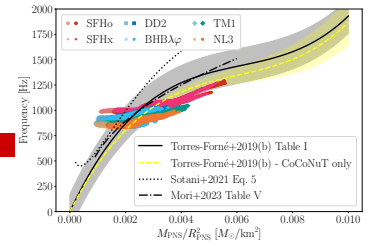
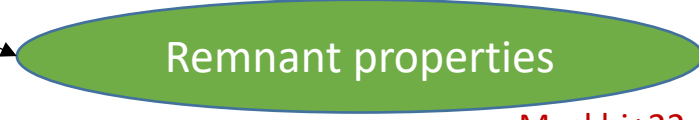
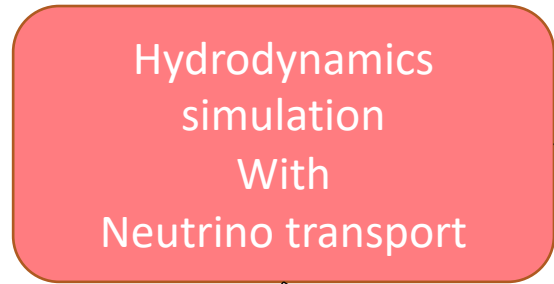
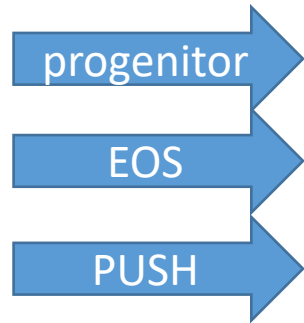
- Self-consistent 3D simulations (only a few)
- Effective models (many, $O(1000)$)

Effective Supernova Models

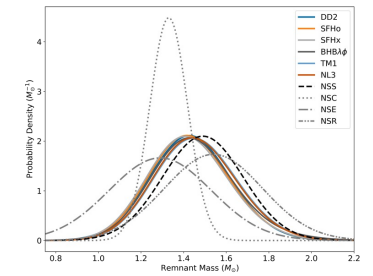
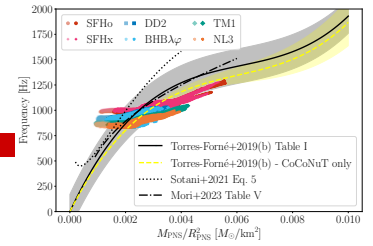
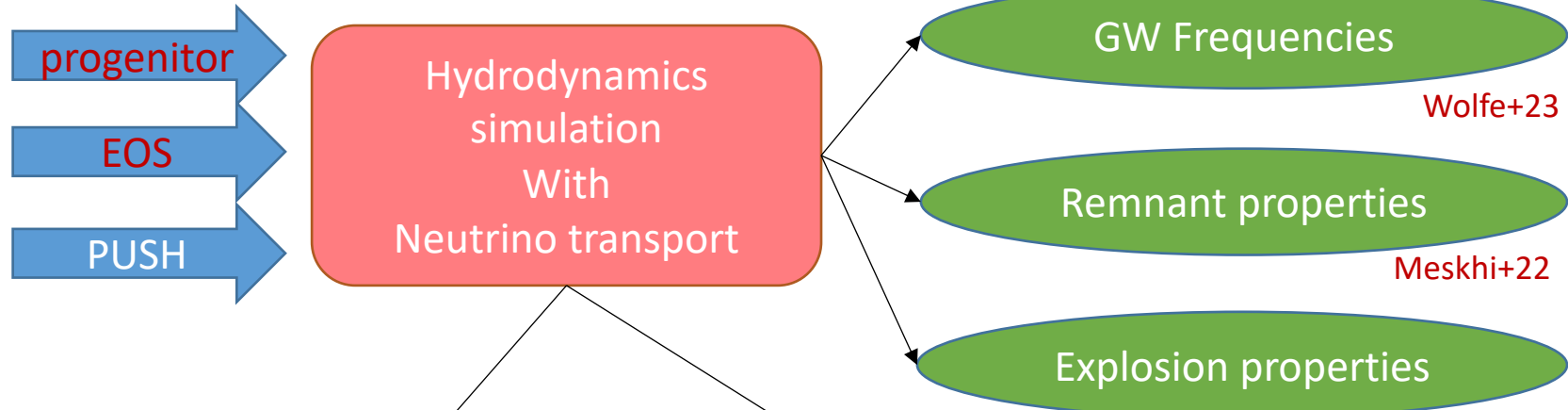
- Parametrize an aspect of multidimensional simulations in spherical symmetry
 - Pro: Computationally efficient → many simulations are possible
 - Con: Have parameters that require tuning
- Examples of effective models and semi-analytic models
 - PUSH Perego+15, Ebinger+19 Pejcha+15
 - PHOT-B Ertl+16, Ugliano+12 Muller+16
 - STIR Couch+20
- Self-consistent within the model framework → make predictions



From explosions to observables

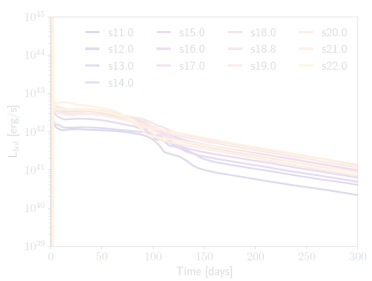


From explosions to observables



Bolometric lightcurves

Curtis+21



SNEC
Radiation-hydro

SuperNu
MC photon transport

Broadband LCs
Spectra

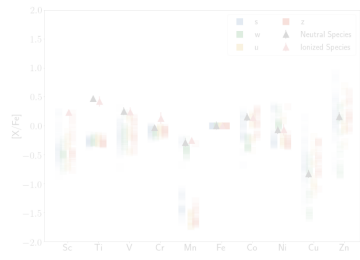
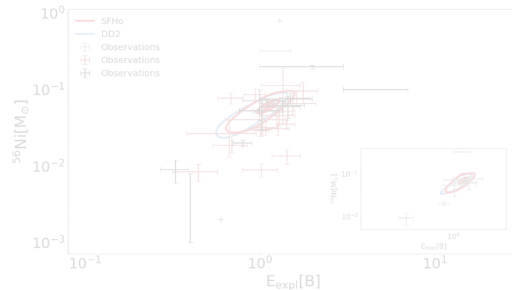
Curtis+21

Mass tracers

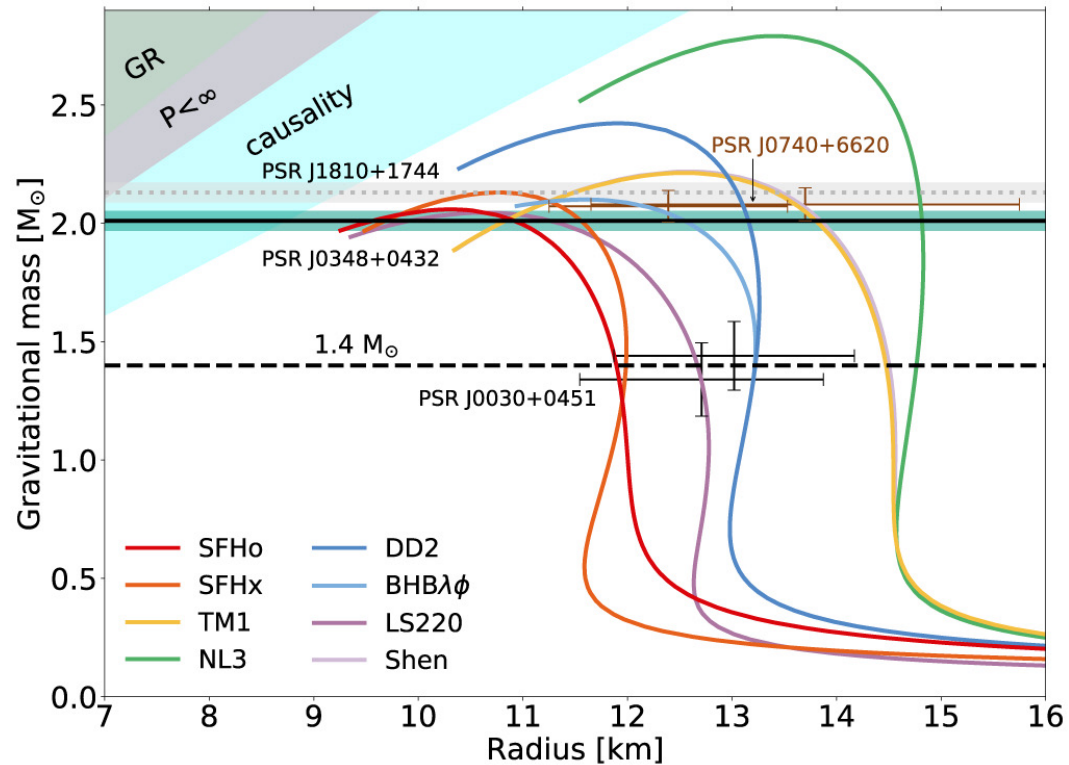
Nuclear Reaction
Network

Nucleosynthesis Yields

Curtis+19



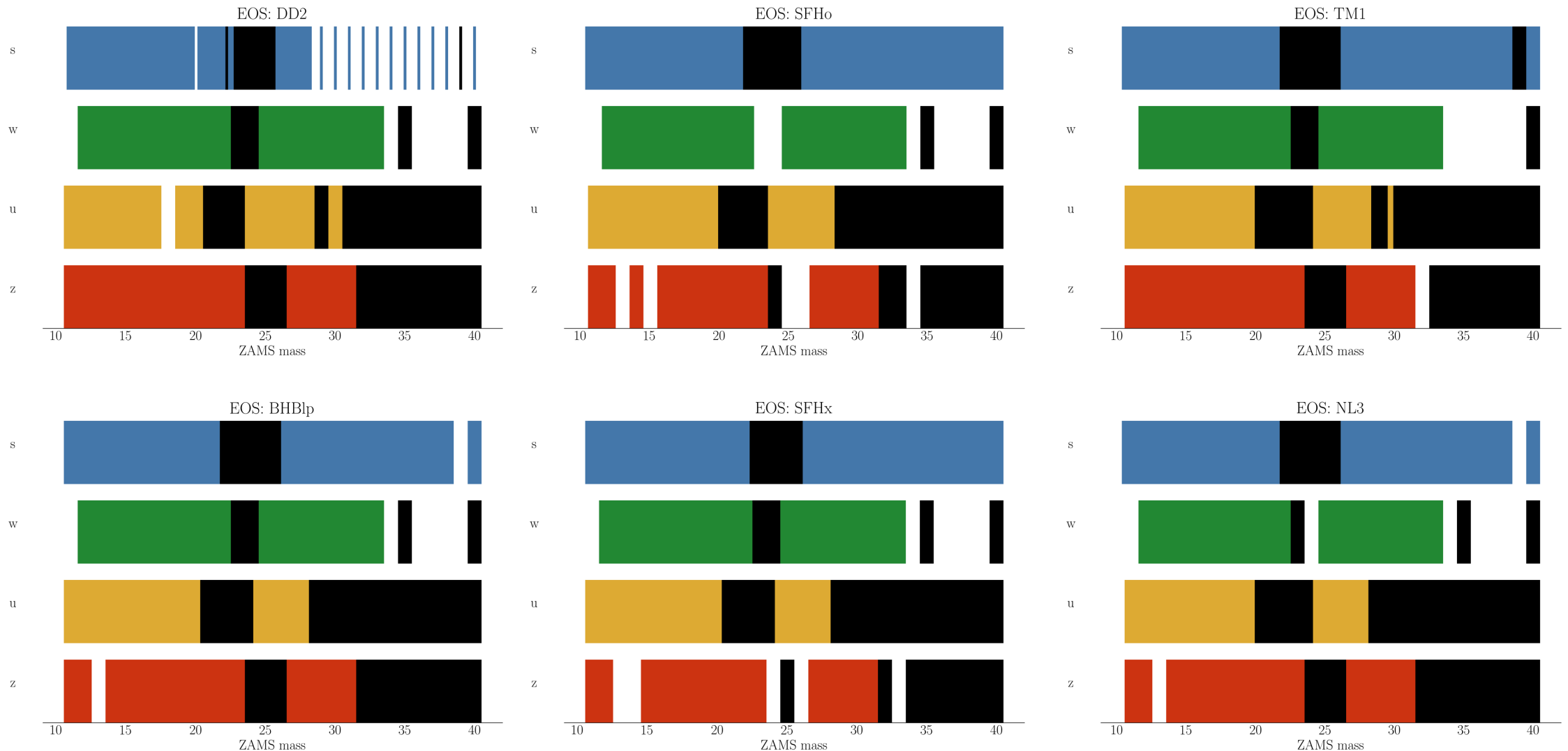
Inputs: progenitors and nuclear EOSs



Progenitors:

- 11-40 M_{sun} (mostly 1 M_{sun} increments)
- Three metallicities: solar, low, zero

1500 Supernova Simulations: What can we learn about the EOS?

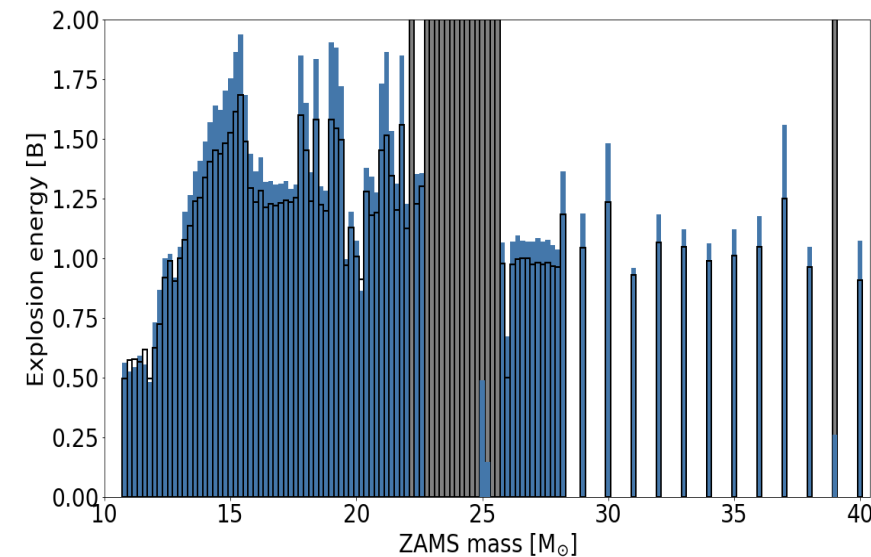
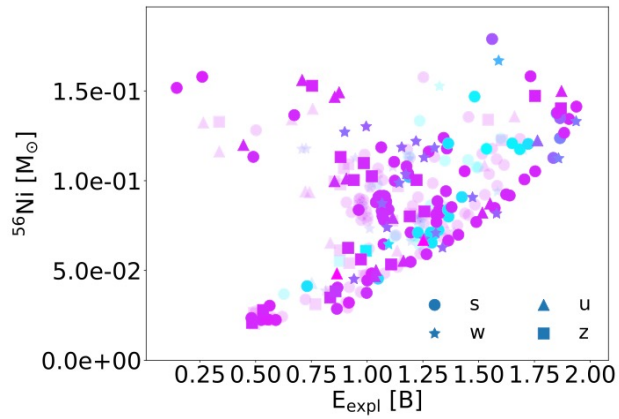


1500 Supernova Simulations: What can we learn about the EOS?

- "Obvious" supernova observables: ^{56}Ni and explosion energy
- Statistical analysis of remnant mass distributions
- GW eigenfrequency analysis for neutron stars

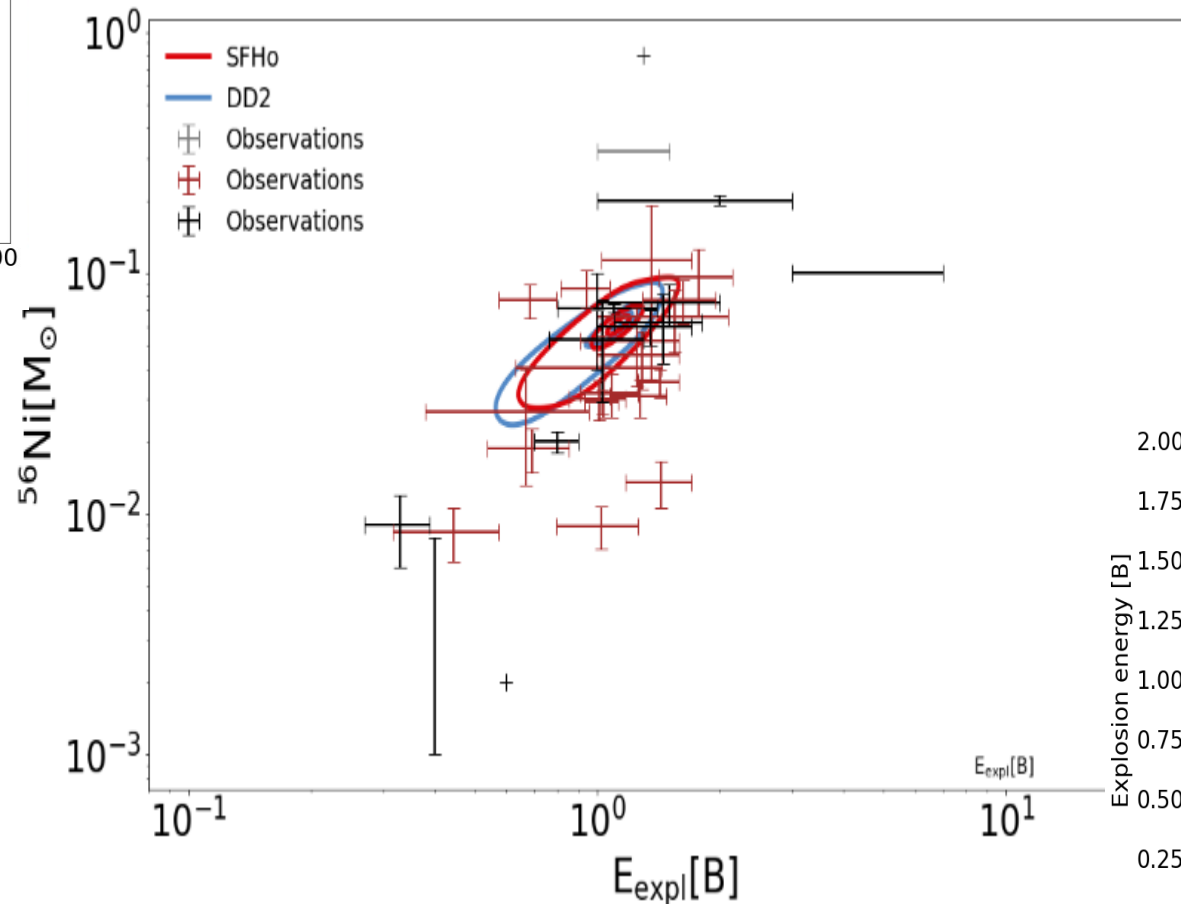
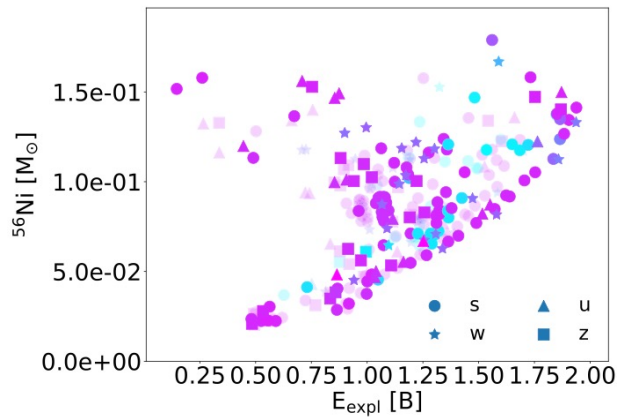
Explosion energy and ^{56}Ni yields

→ In PUSH: Explosion energy and ^{56}Ni mass emerge self-consistently with each other

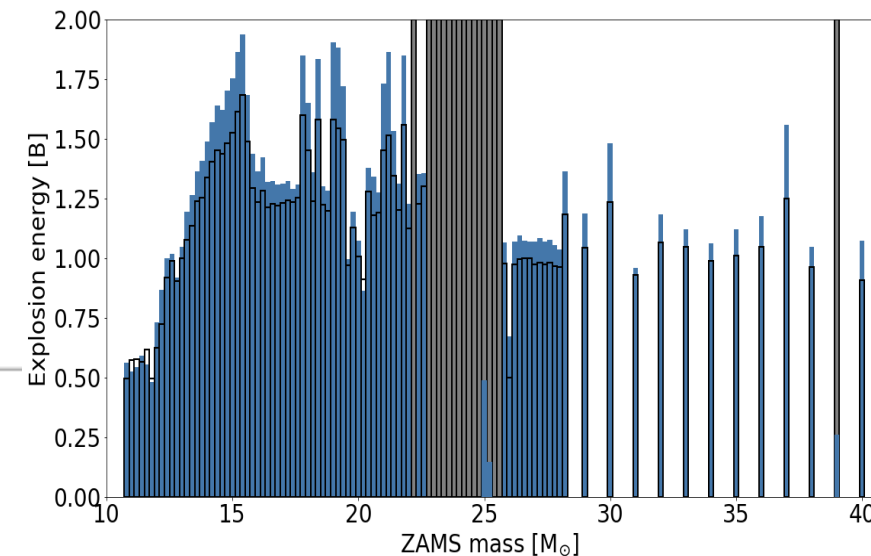


Explosion energy and ^{56}Ni yields

→ In PUSH: Explosion energy and ^{56}Ni mass emerge self-consistently with each other



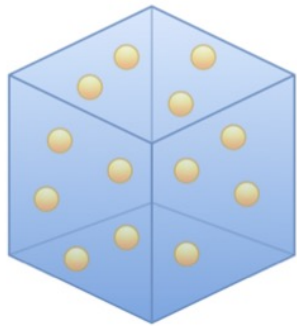
Cannot distinguish between DD2 and SFHo EOS



Comparing (marginal) distributions

Model Assumptions

Equation of State

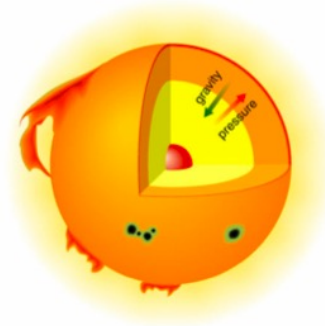


Equation of State



Simulations

PUSH method

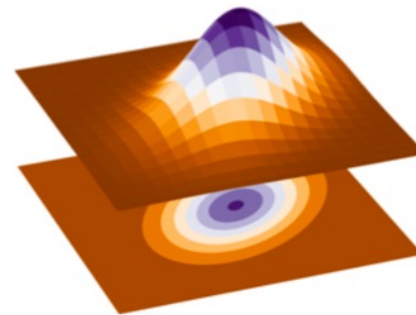
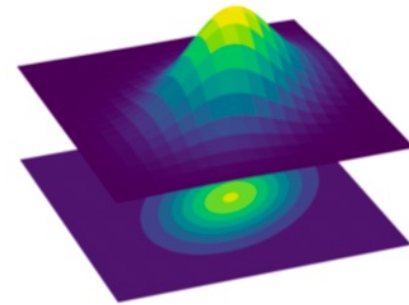


Stellar Core-Collapse



Data

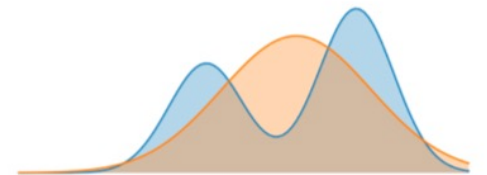
Real Data



Simulated Data



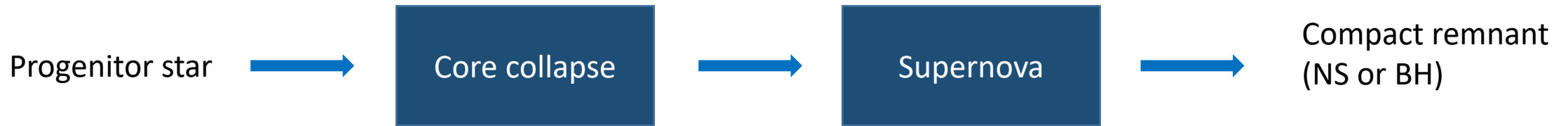
Statistical Analysis



Distance Between Distributions

Nature vs simulations

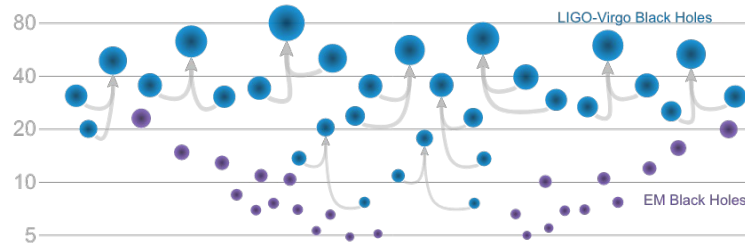
Simulation:



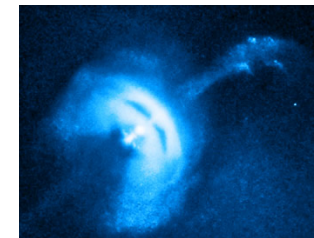
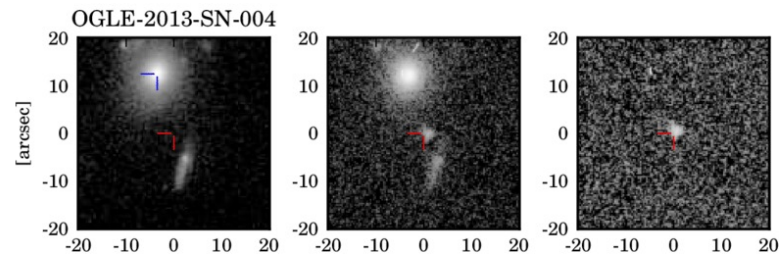
→ compare marginal distributions since we do not have (input,output) pairs from nature

Nature:

Black holes



Supernovae



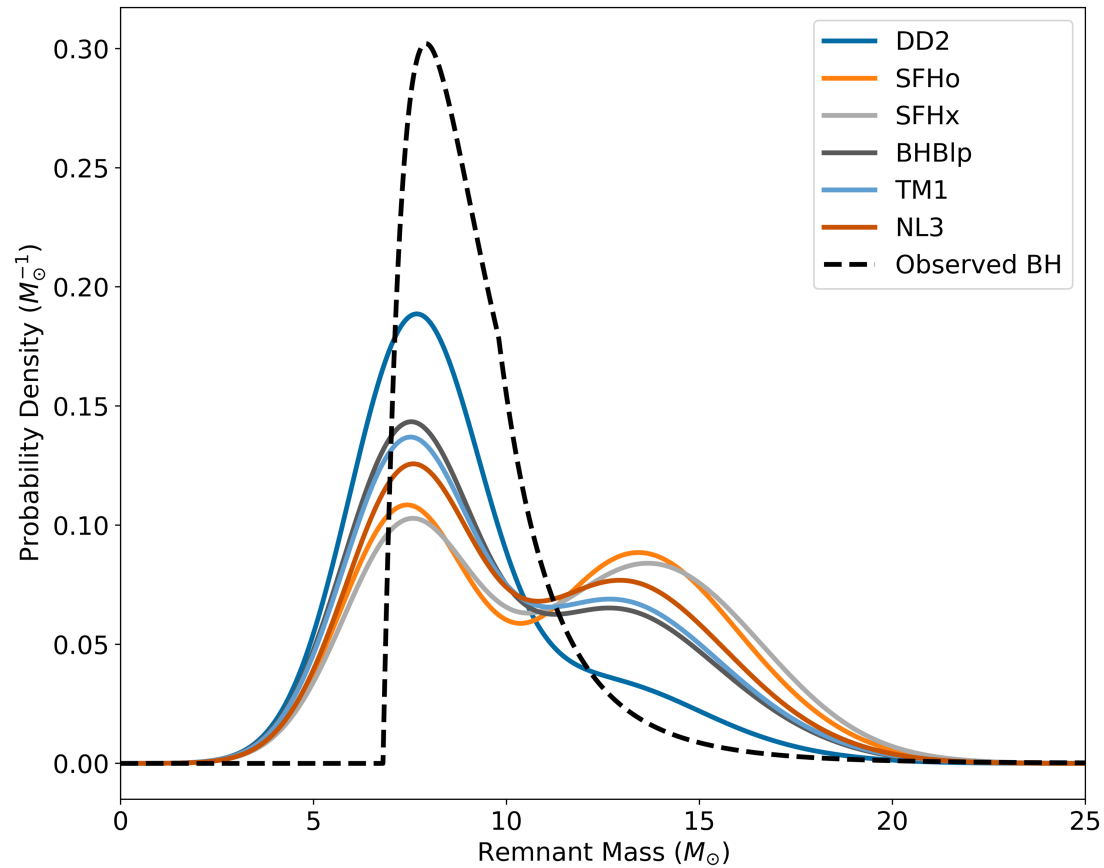
Neutron stars

Stars

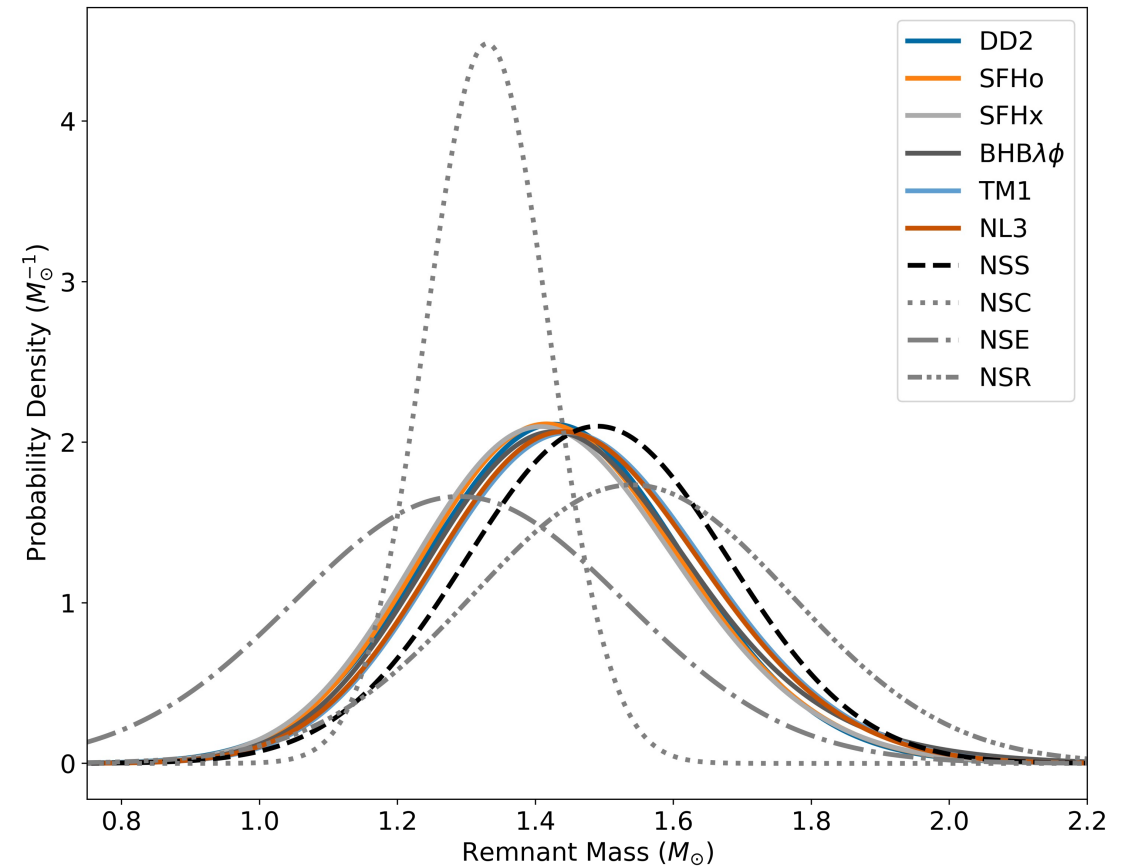


Probability Density Functions

Black holes



Neutron stars



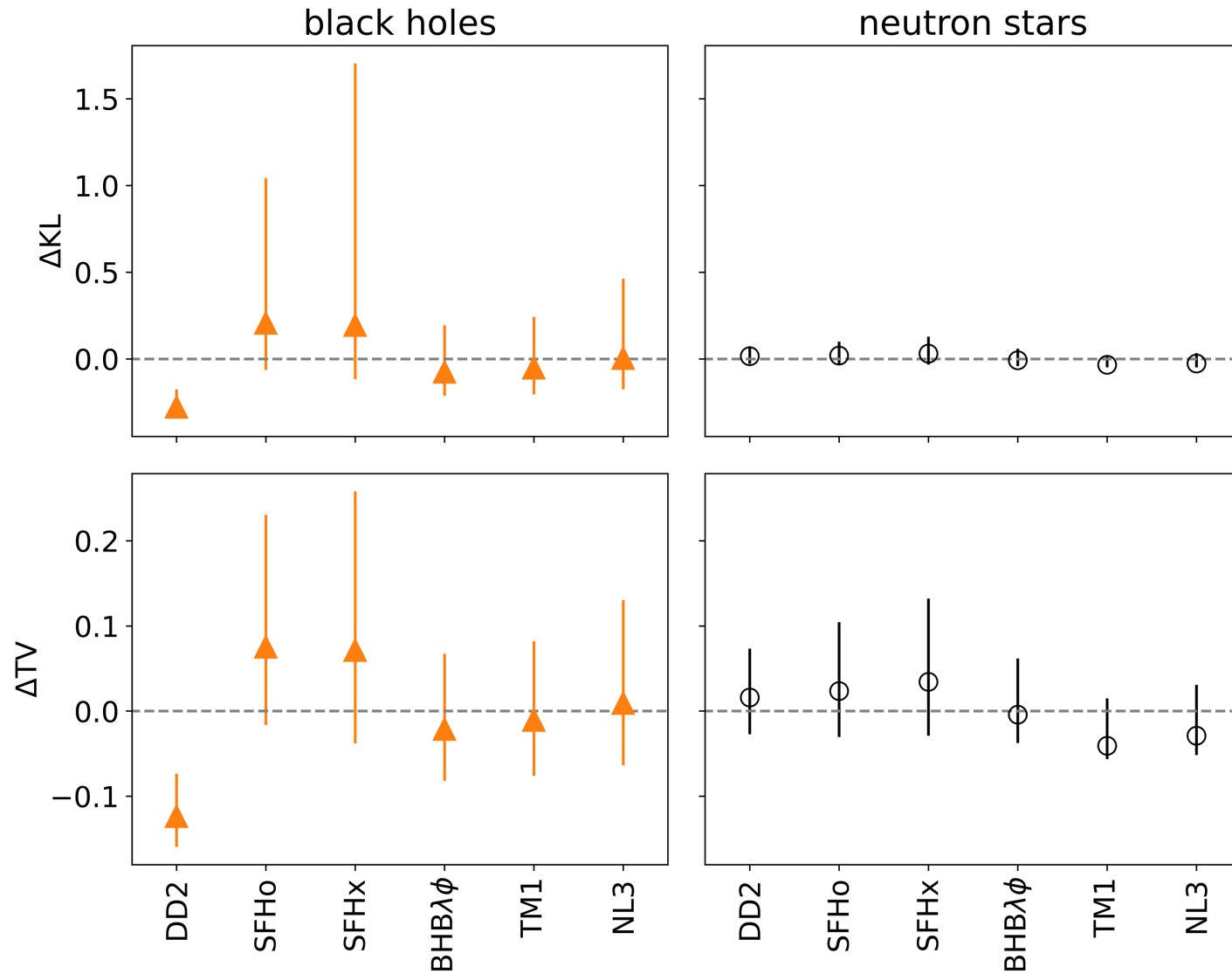
Results: nuclear EOS

$$\Delta = D_{EOS} - \bar{D}$$

$\Delta < 0 \rightarrow$ more favored

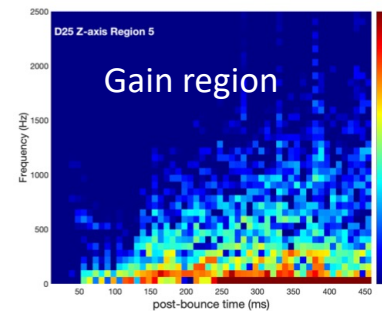
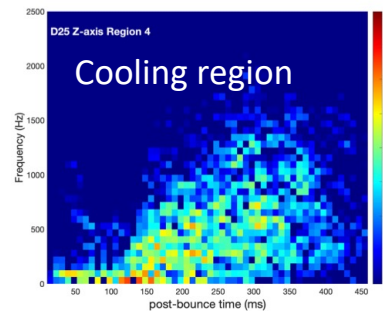
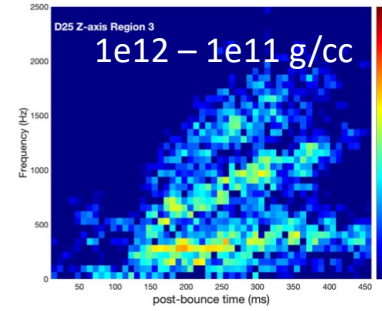
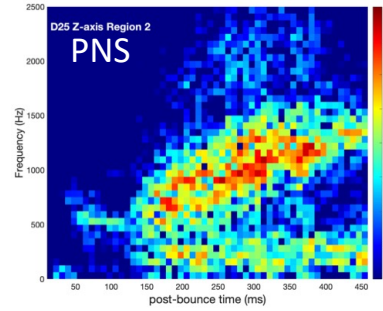
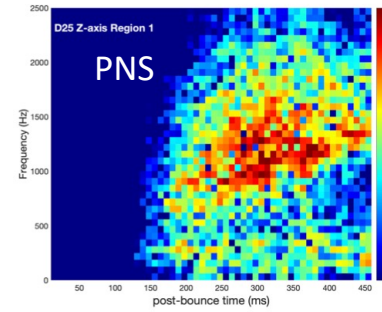
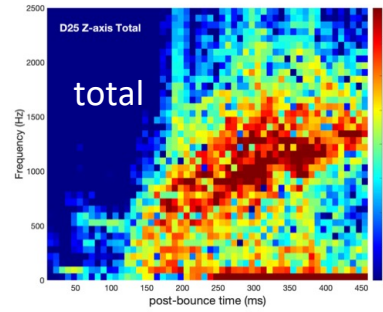
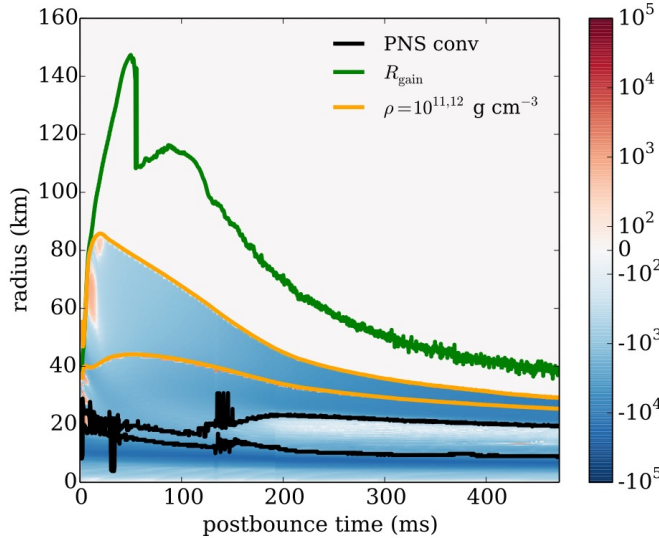
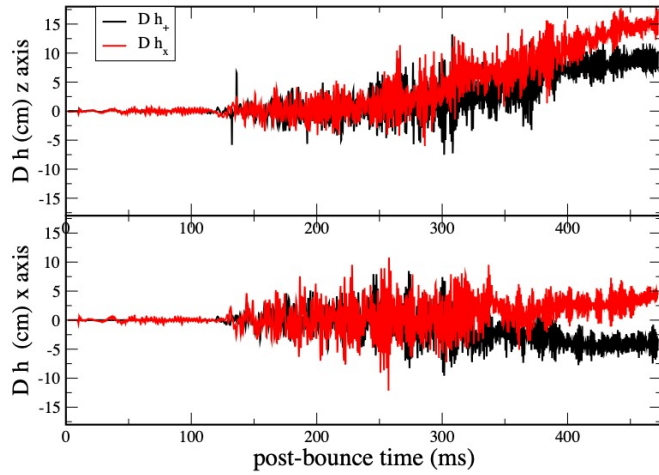
$\Delta > 0 \rightarrow$ less favored

- Caveats:
 - No progenitors of 8-11Msun
 \rightarrow lowest-mass NSs are missing
 - Based on single stars
 \rightarrow binarity not considered
- Next steps:
 - Include other observables

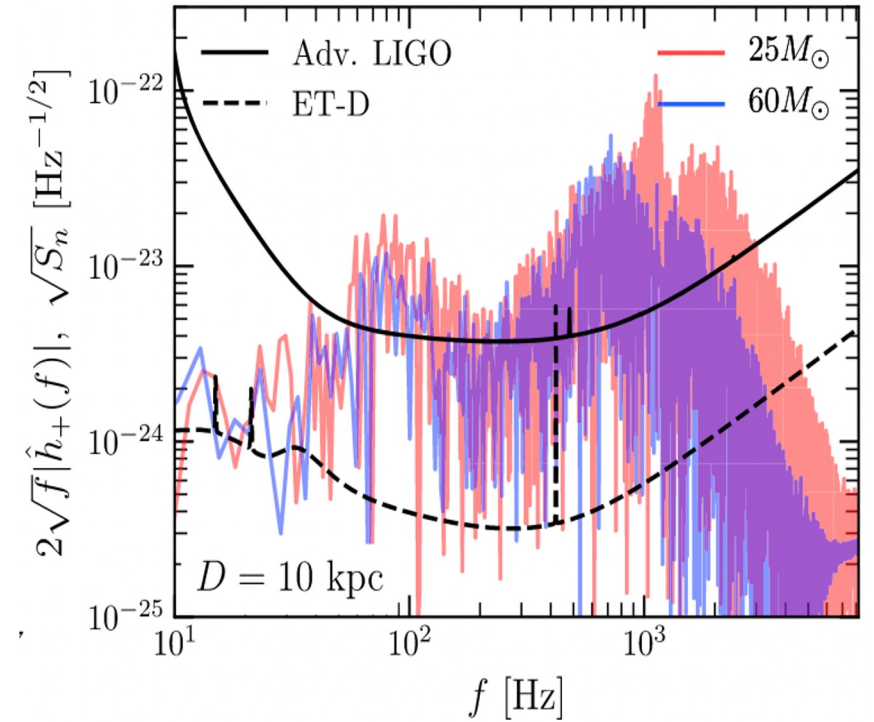


Gravitational Waves from CCSNe

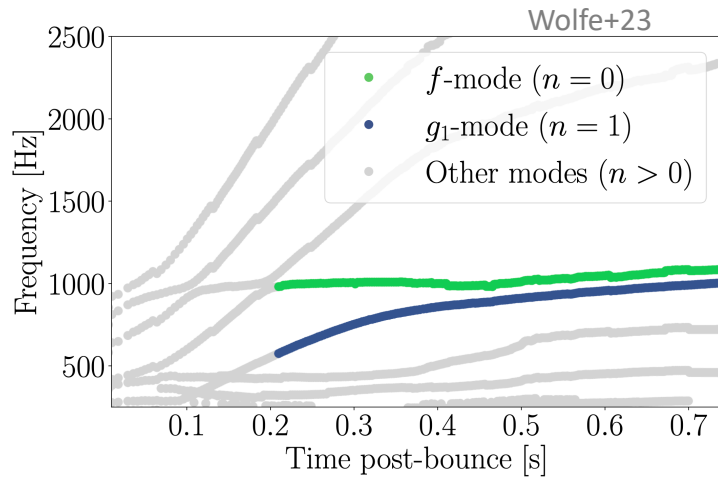
Mezzacappa+23



Radice+19



Eigenfrequencies from perturbation analysis



Identification of:

- f-mode
- g-mode

(Cowling scheme)

- Linear perturbation analysis of general-relativistic hydrodynamics background

Torres-Forné+18, Morozova+18, Torres-Forné+19, Sotani+20

- Calculate time-frequency evolution (without amplitudes) from spherically-symmetric proto-neutron star background

- Identify frequencies that characterize astrophysical properties of proto-neutron star

- Universal relations for PNS surface gravity

Torres-Forne+19

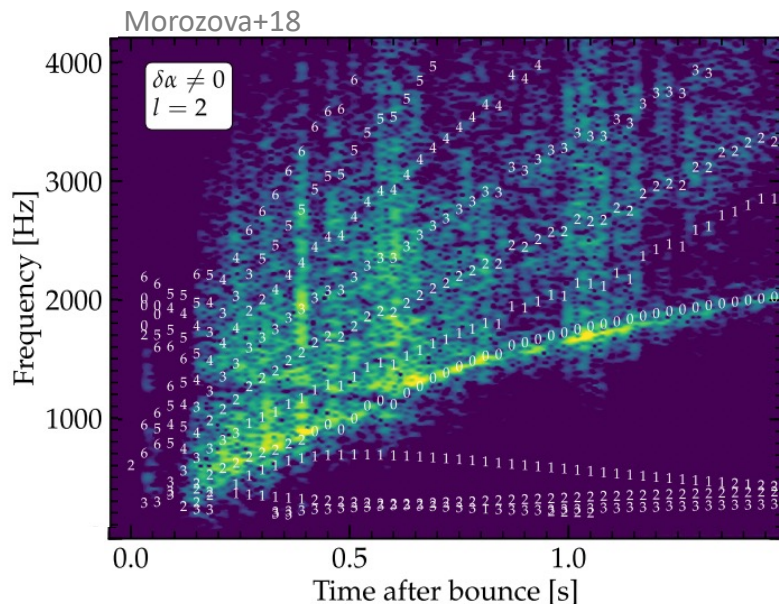
- Multi-messenger observations of core-collapse

Warren+20, Nakamura+22

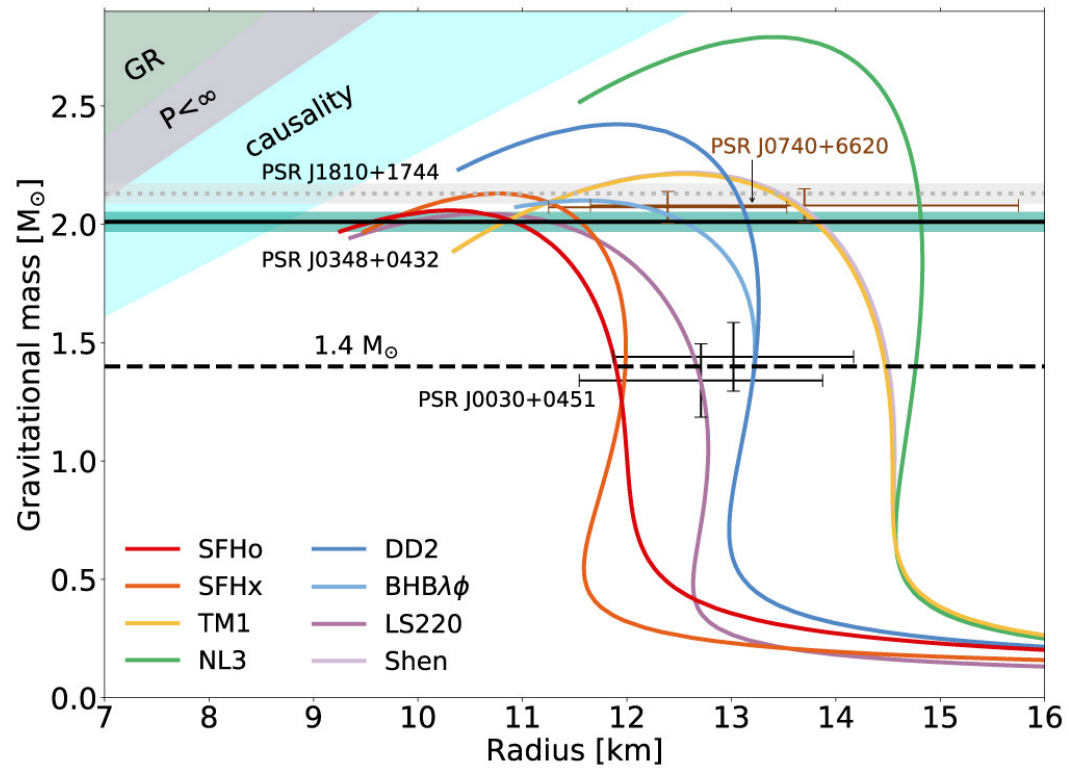
- Parameter estimation

Bizouard+21, Powell+22

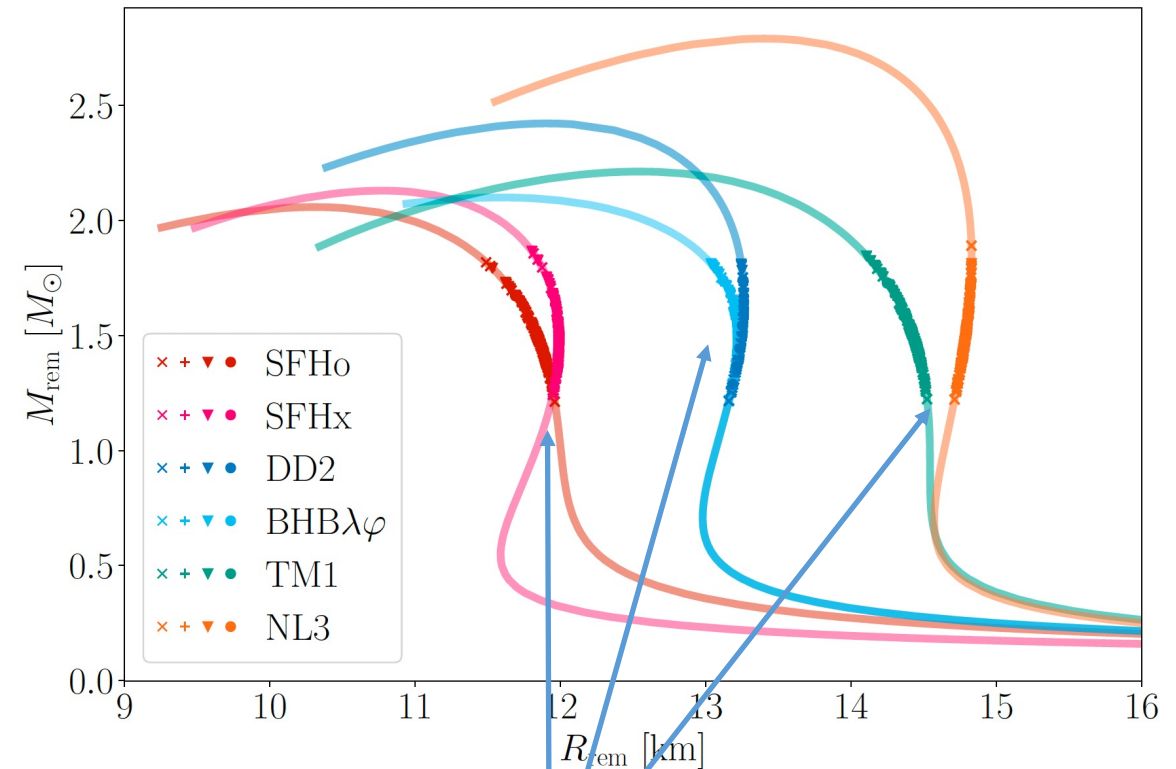
Comparison between eigenfrequency analysis and full simulation



Neutron star masses from our simulations



Ghosh+22

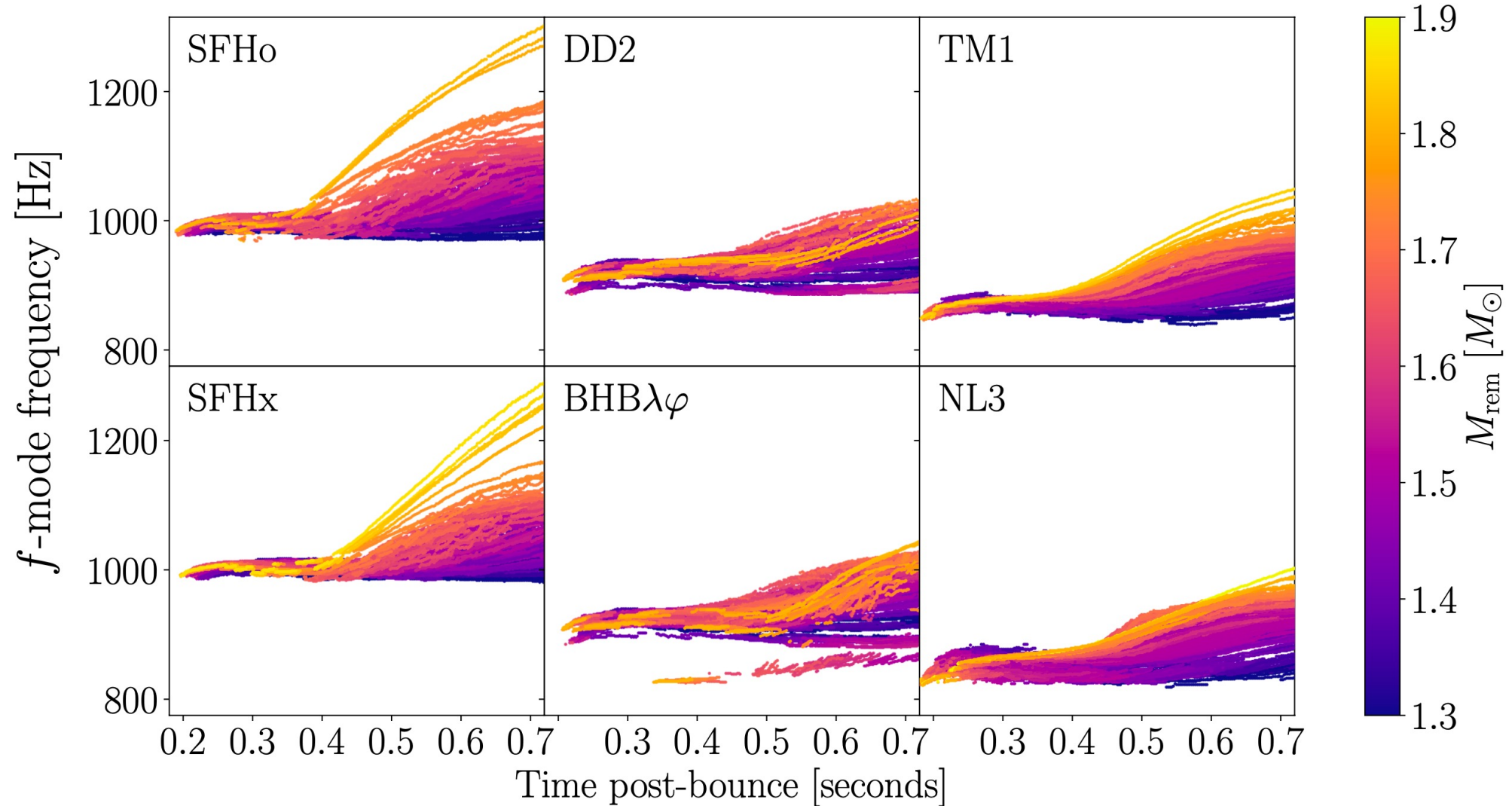


Wolfe+23

>1000 simulations

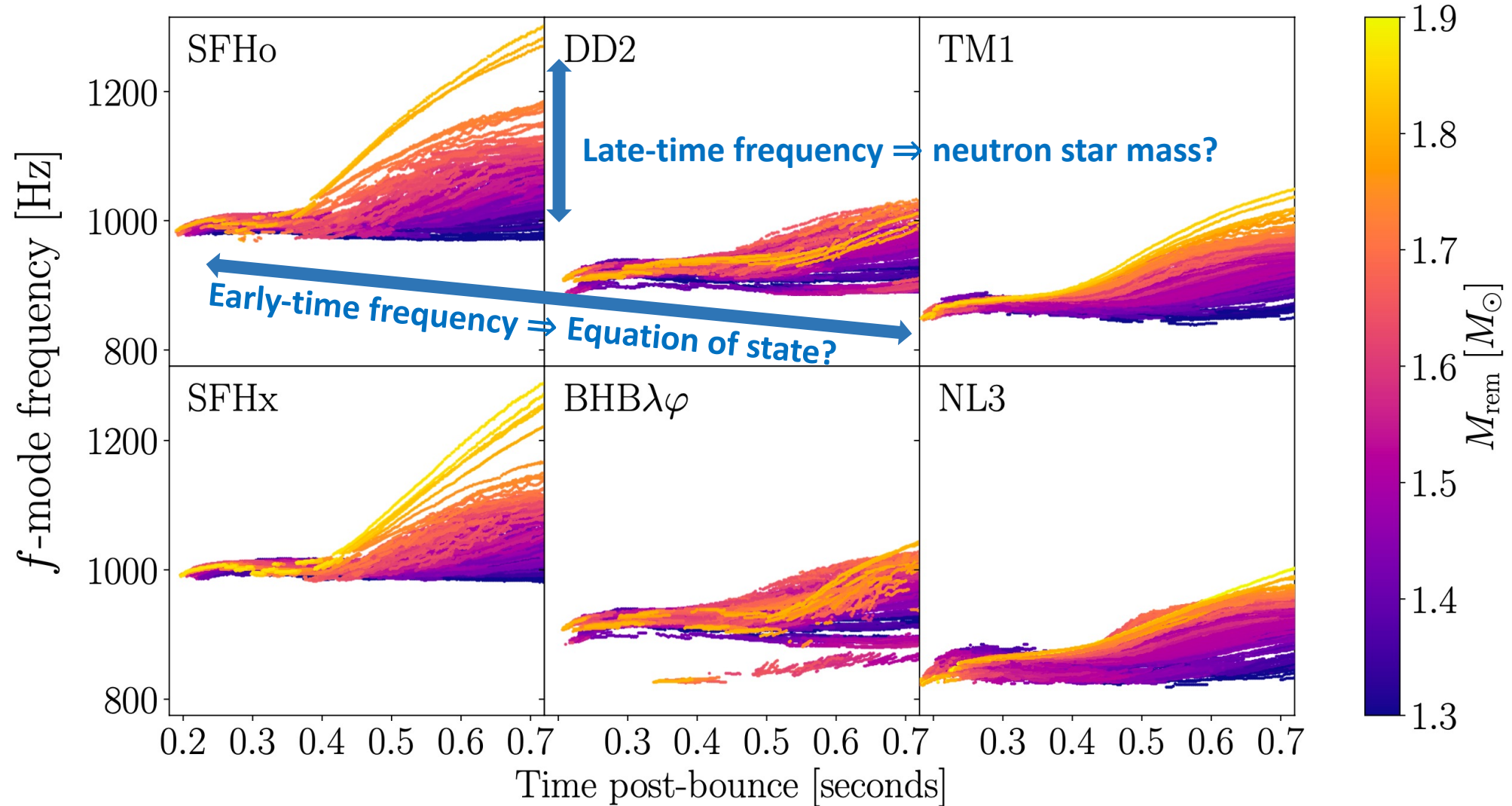
Frequencies depend on nuclear EOS

Wolfe+2023



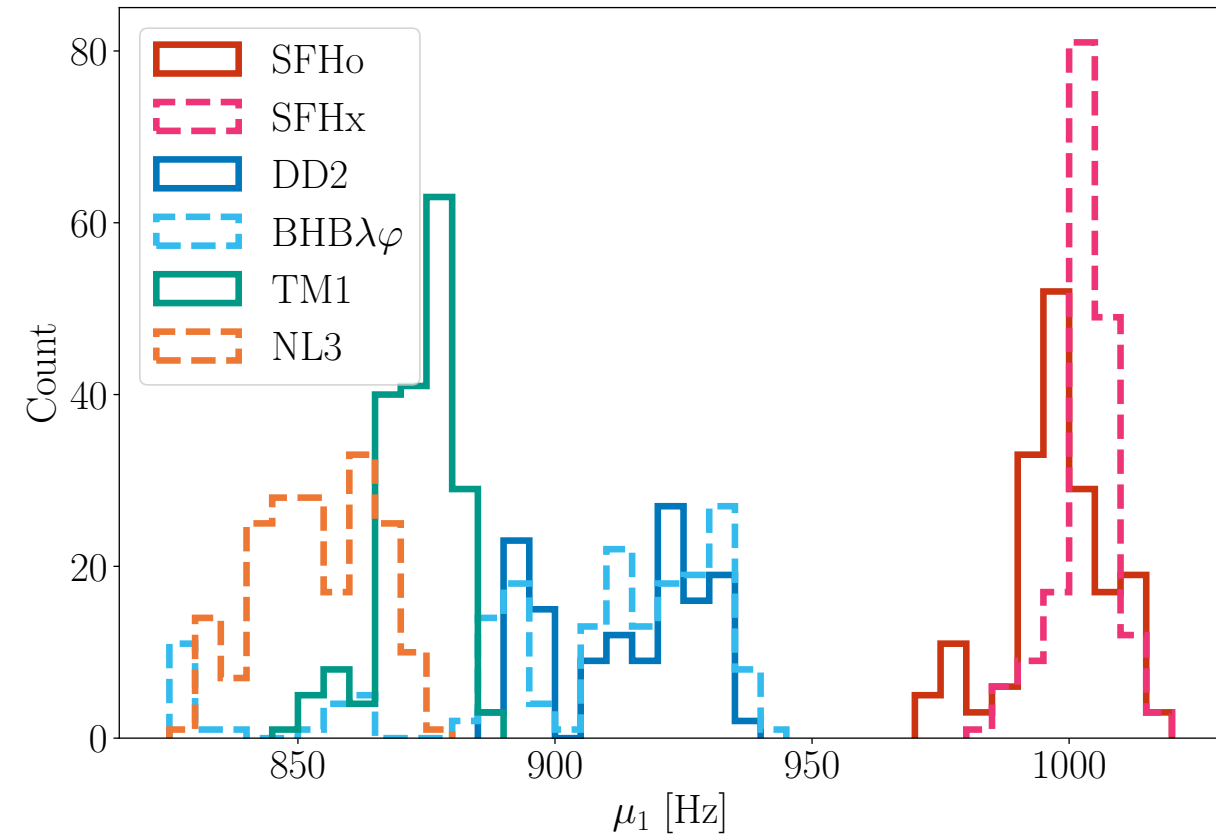
Frequencies depend on nuclear EOS

Wolfe+2023

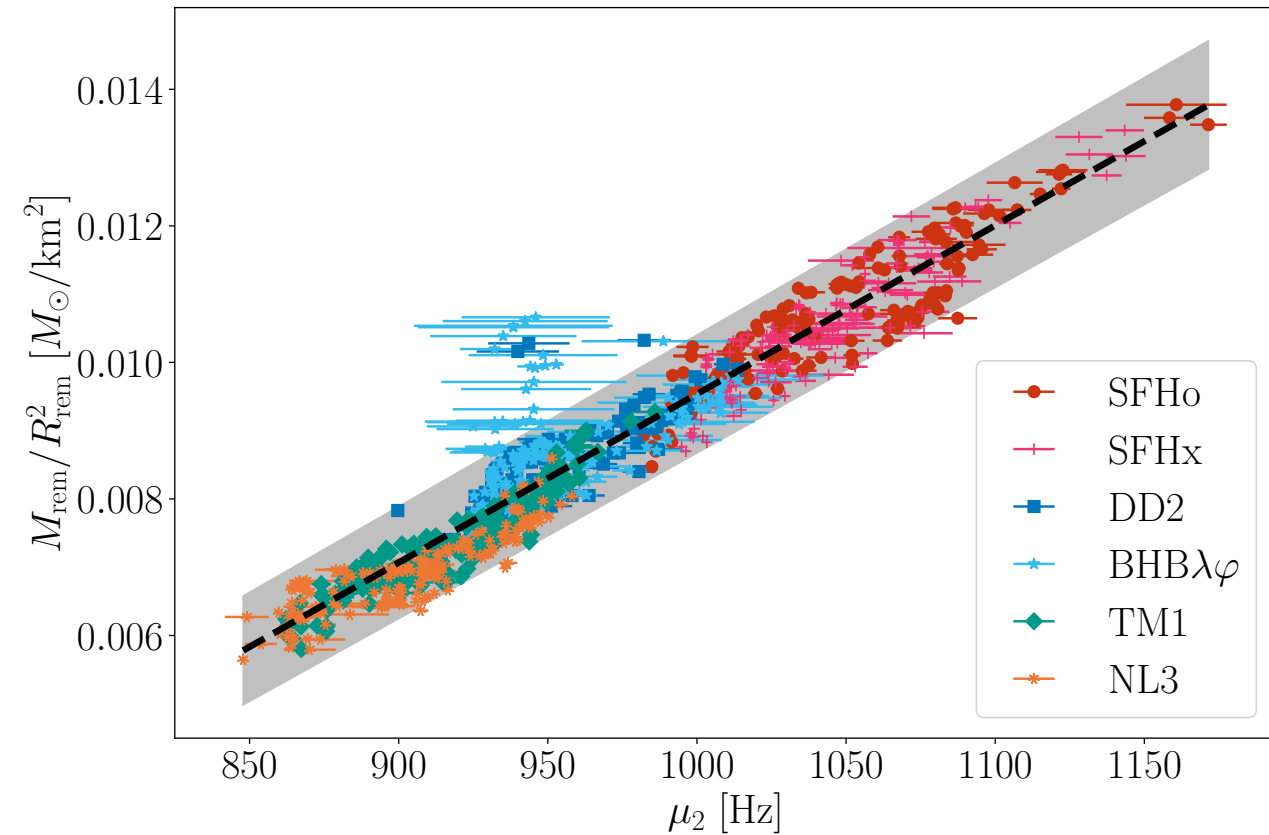


Characteristic frequencies

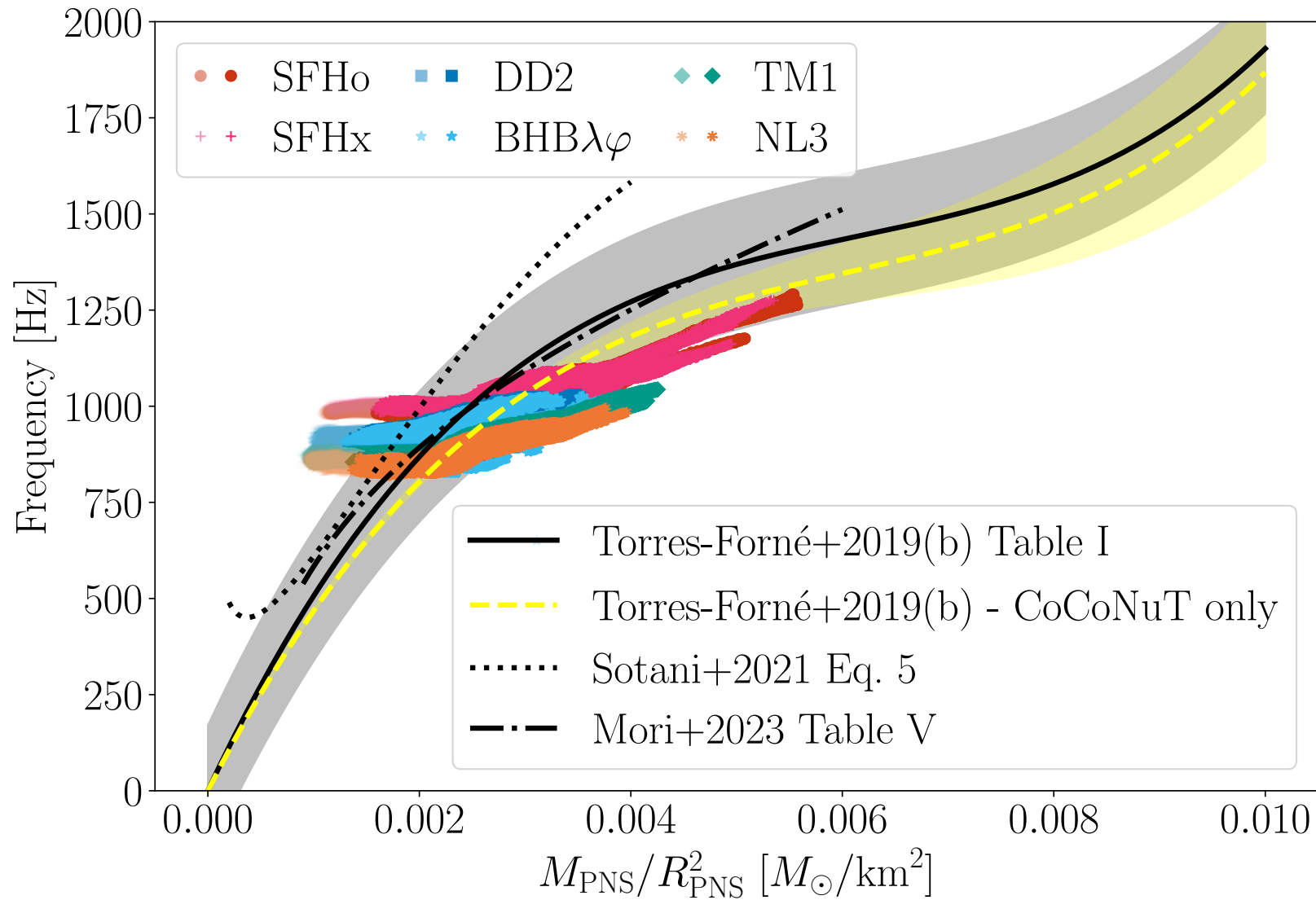
Wolfe+2023



Characteristic early-time frequency



Characteristic late-time frequency



Summary

- How to harness supernova simulations to learn about the nuclear EOS
 - Perform many (~ 1000) simulations without prescribing the outcome
 - \rightarrow NS and BH masses
 - Statistical analysis: Distance between distributions
 - GW eigenfrequency analysis: time-evolution of f-mode

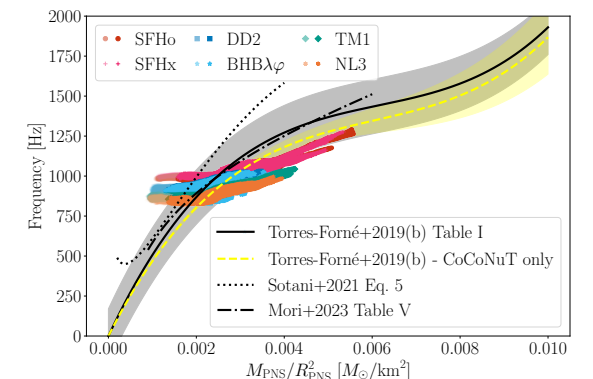
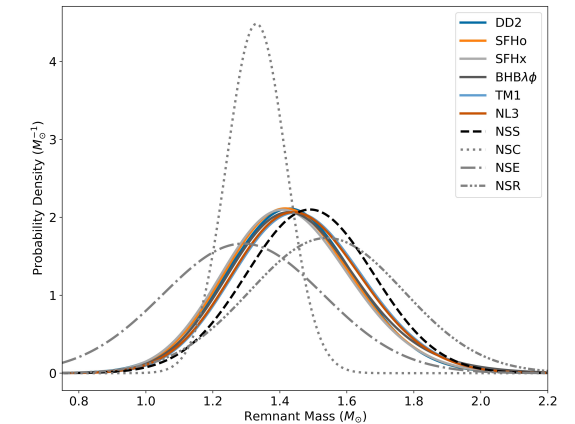


Table 1. EOS models used in this study.

Label	EOS	Model for uniform nuclear matter	Nuclei	n_B^0 (fm^{-3})	K (MeV)	J (MeV)	L (MeV fm^{-3})	m_n^*/m_n	m_p^*/m_p	M_{max} (M_{\odot})	Ref.
SFHo	SFHo	RMF, SFHo	NSE	0.1583	245.4	31.57	47.10	0.7609	0.7606	2.06	1
SFHx	SFHx	RMF, SFHo	NSE	0.1602	238.8	28.67	23.18	0.7179	0.7174	2.13	1
TM1	HS(TM1)	RMF, TM1	NSE	0.1455	281.6	36.95	110.99	0.6343	0.6338	2.21	2, 3
NL3	HS(NL3)	RMF, NL3	NSE	0.1482	271.5	37.39	118.49	0.5954	0.5949	2.79	2, 3
DD2	HS(DD2)	RMF, DD2	NSE	0.1491	242.7	31.67	55.03	0.5628	0.5622	2.42	2, 3
BHB $\lambda\phi$	BHB $\lambda\phi$	RMF, DD2, hyperons	NSE	0.1491	242.7	31.67	55.03	0.5628	0.5622	2.10	4
LS220	LS-EOS	Skyrme	CLD, SNA	0.155	220	29.6	73.7	1.0	1.0	2.06	5
Shen	Shen-EOS	RMF, TM1	RMF, TFA, SNA	0.145	281	36.9	110.8	0.634		2.18	6,7

NOTE— Saturation density n_B^0 , incompressibility K , symmetry energy J , symmetry energy slope coefficient L , effective neutron mass m_n^* , effective proton mass m_p^* , and maximum mass M_{max} of a cold neutron star. RMF: relativistic mean field. NSE: nuclear statistical equilibrium. CLD: compressible liquid drop. SNA: single nucleus approximation. TFA: Thomas-Fermi approximation.

References— (1) Steiner et al. (2013) (2) Hempel & Schaffner-Bielich (2010a) (3) Hempel et al. (2012) (4) Banik et al. (2014) (5) Lattimer & Douglas Swesty (1991) (6) Shen et al. (1998) (7) Shen et al. (1998)

Table 1 from Ghosh, Wolfe, Fröhlich (2022)