# EXPLORING the MATTER in the CENTER of NEUTRON STARS

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- Dense Matter in Neutron Stars: Speed of Sound and Equation of State
  - Observational constraints from heavy neutron stars and binary mergers
  - Bayesian inference results and constraints on phase transitions
- Phenomenology and Models
  - Low-energy nucleon structure and a two-scales scenario
  - Quark-hadron continuity and crossover
  - Chiral symmetry restoration : from first-order phase transition to crossover
  - Dense baryonic matter as a (relativistic) Fermi liquid

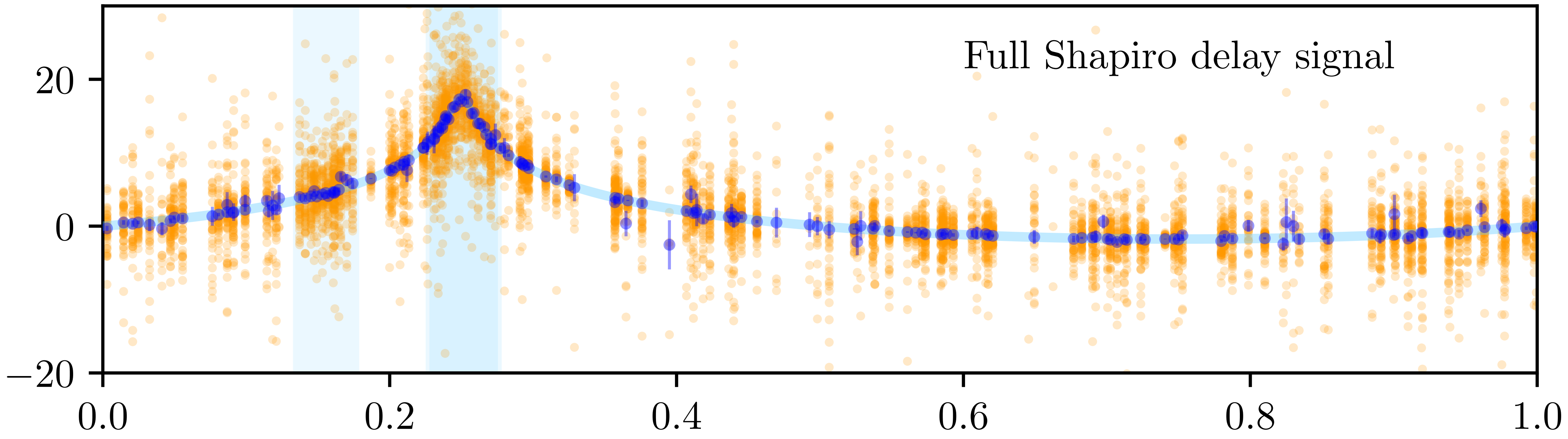
# Part One

## Equation-of-State of Dense Baryonic Matter :

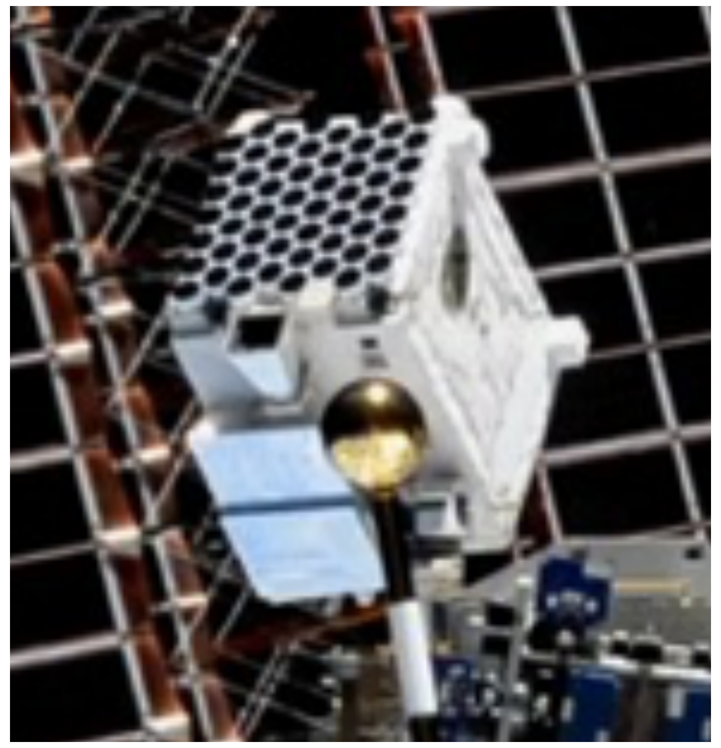
## Empirical Constraints from Neutron Stars

# NEUTRON STARS : DATA

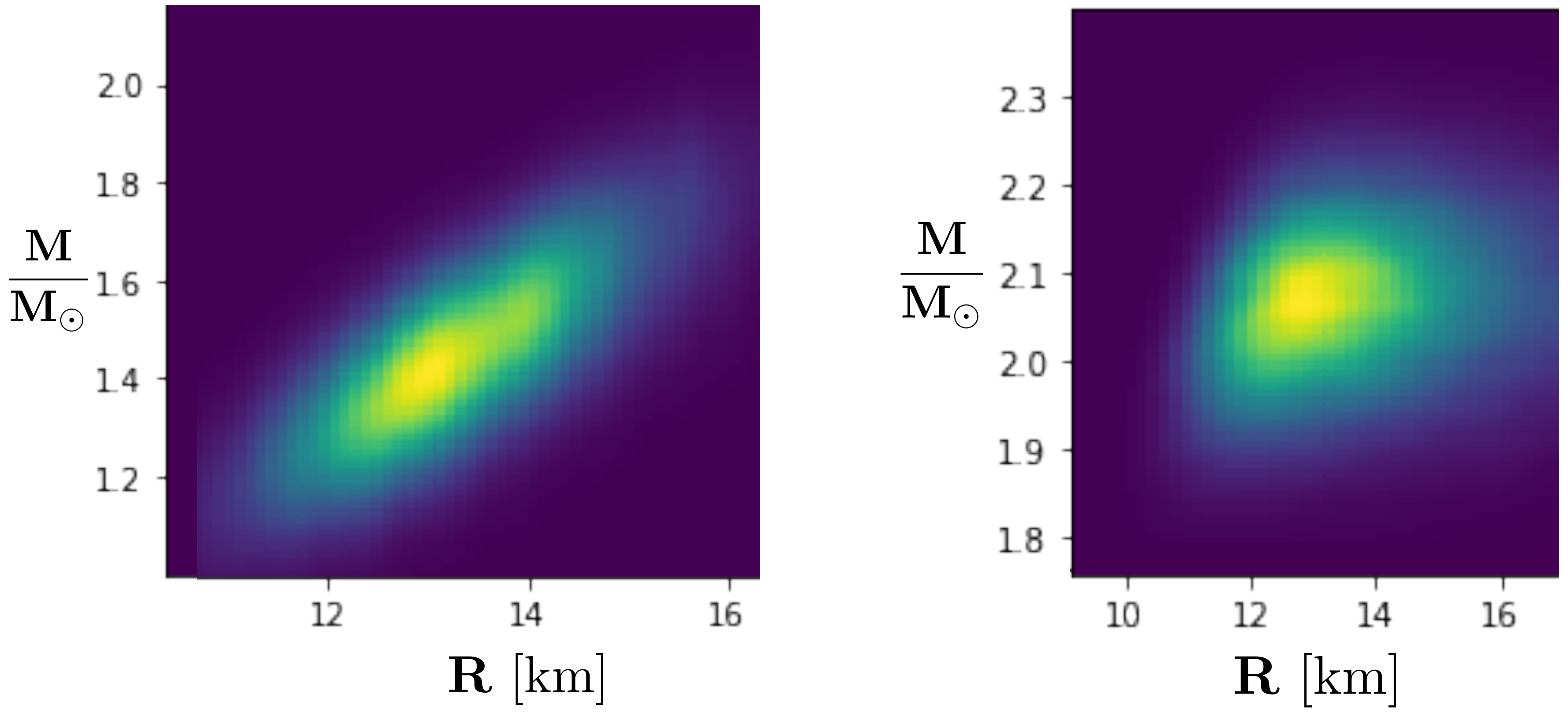
- Database for **inference of Equation-of-State** and other properties of neutron stars

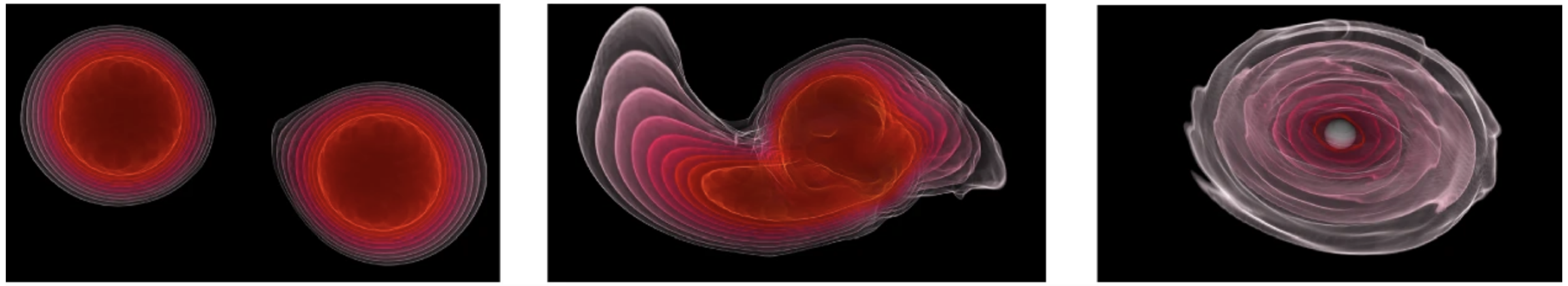


- **Neutron star masses**
  Shapiro delay measurements
  (Green Bank Telescope)



- **Masses and radii**
  X rays from hot spots on the surface of rotating neutron stars
  (NICER Telescope @ ISS)





- **Tidal deformabilities**
  Gravitational wave signals of neutron star mergers
  (LIGO and Virgo Collab.)

# NEUTRON STARS : DATA

- **Masses of $2\,\mathrm{M}_\odot$ stars** (Shapiro delay measurements)

PSR J0348+0432

$$\boxed{\mathrm{M} = 2.01 \pm 0.04\ \mathrm{M}_\odot}$$

J. Antoniadis et al. : Science 340 (2013) 1233232

PSR J1614-2230

$$\boxed{\mathrm{M} = 1.908 \pm 0.016\ \mathrm{M}_\odot}$$

Z. Arzoumanian et al. , Astrophys. J. Suppl. 235 (2018) 37

PSR J0740+6620

$$\boxed{\mathrm{M} = 2.08 \pm 0.07\ \mathrm{M}_\odot}$$

E. Fonseca et al. , Astrophys. J. Lett. 915 (2021) L12

- **Masses and Radii** (NICER)

PSR J0030+0451

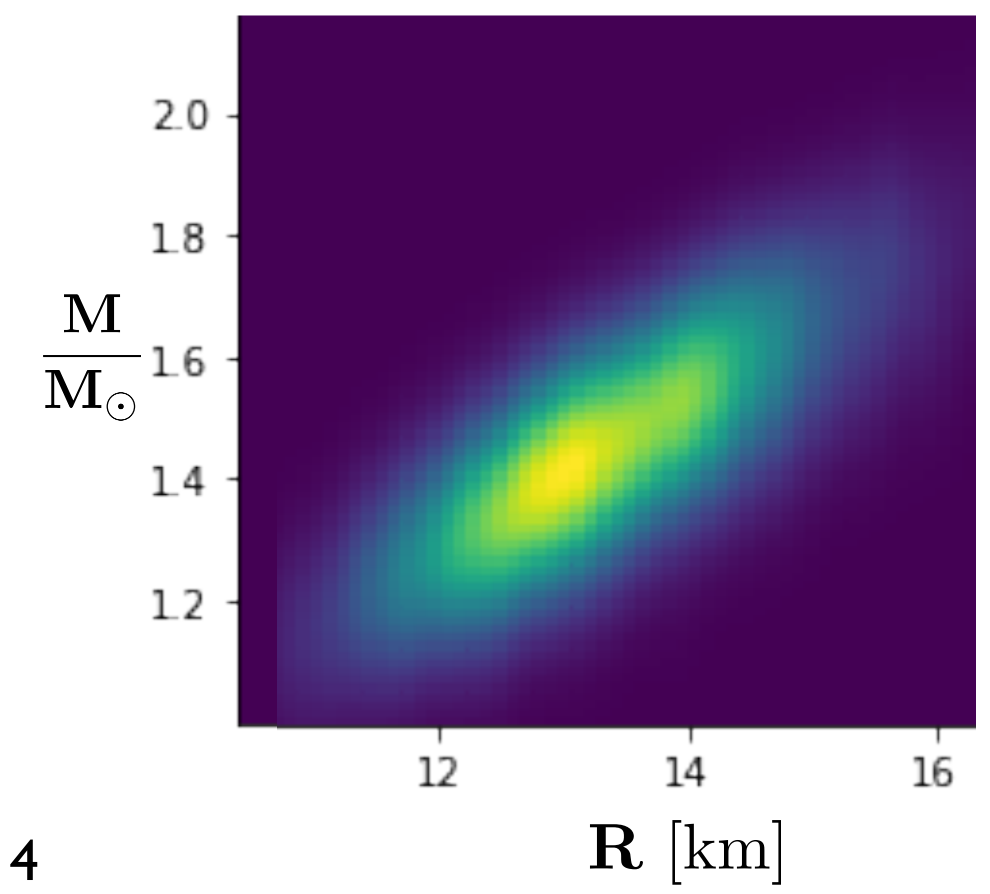
$$\boxed{\mathrm{M} = 1.34 \pm 0.16\,\mathrm{M}_\odot \qquad \mathrm{R} = 12.71^{+1.14}_{-1.19}\ \mathrm{km}}$$

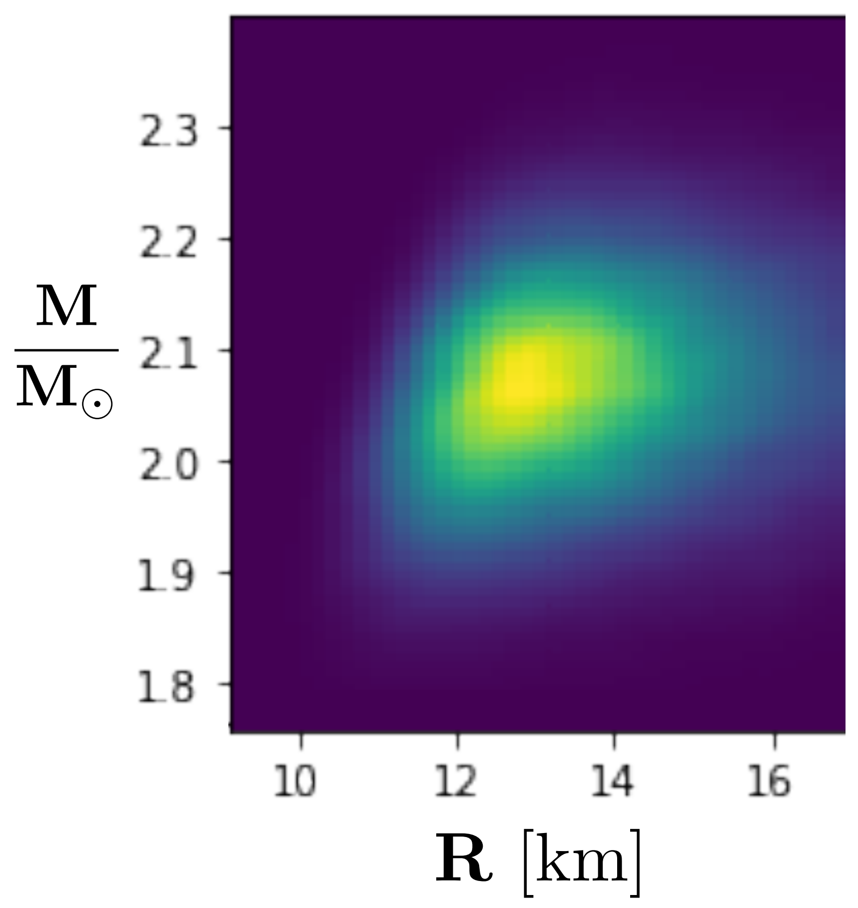
T.E. Riley et al. (NICER), Astroph. J. Lett. 887 (2019) L21

PSR J0740+6620

$$\boxed{\mathrm{M} = 2.07 \pm 0.07\,\mathrm{M}_\odot \qquad \mathrm{R} = 12.39^{+1.30}_{-0.98}\ \mathrm{km}}$$

T.E. Riley et al. (NICER + XMM Newton), Astroph. J. Lett. 918 (2021) L27





# NEUTRON STARS : DATA (contd.)

- **Very massive** and **fast rotating galactic neutron star**

PSR J0952-0607

$$\boxed{\mathrm{M} = 2.35 \pm 0.17\ \mathrm{M}_\odot}$$

R.W. Romano et al. : Astroph. J. Lett. 935 (2022) L17

→ **equivalent non-rotating mass after rotational correction :** $\mathrm{M} = 2.3 \pm 0.2\ \mathrm{M}_\odot$

(Keck Observatory)

- **Tidal deformabilities** from binary neutron star mergers (gravitational wave signals)

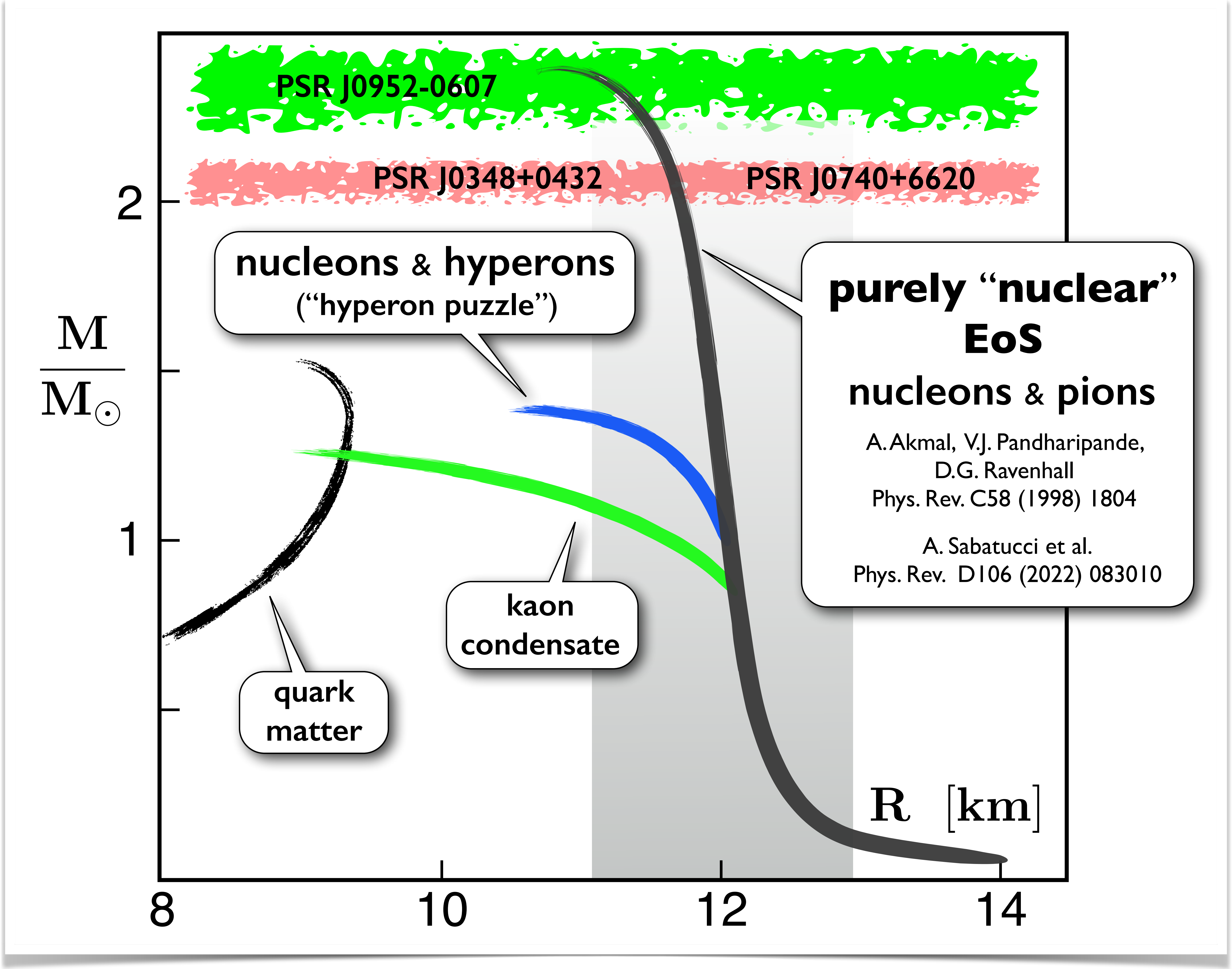
$$\tilde{\Lambda} = \frac{16}{13}\frac{(M_1 + 12\, M_2) M_1^4 \Lambda_1}{(M_1 + M_2)^5} + (1 \leftrightarrow 2)$$

GW170817 $\Lambda_{1.4} = 190^{+390}_{-120}$

B.P. Abbot et al. : Phys. Rev. Lett. 121 (2018) 161101

# CONSTRAINTS on EQUATION of STATE $P(\varepsilon)$

- from observations of massive neutron stars



**purely "nuclear" EoS**

nucleons & pions

A. Akmal, V.J. Pandharipande, D.G. Ravenhall
Phys. Rev. C58 (1998) 1804

A. Sabatucci et al.
Phys. Rev. D106 (2022) 083010

## **T**olman - **O**ppenheimer - **V**olkov Equations

$$\frac{dP(r)}{dr} = \frac{G\left[\varepsilon(r) + P(r)\right]\left[m(r) + 4\pi r^3 P(r)\right]}{r\left[r - 2\,G\,m(r)\right]}$$

$$\frac{dm(r)}{dr} = 4\pi r^2 \varepsilon(r)$$

$$M = m(R) = 4\pi \int_0^R dr\, r^2 \varepsilon(r)$$

- **Stiff equation-of-state $P(\varepsilon)$ required**
- **Simplest forms of exotic matter**
  (kaon condensate, quark matter, ...)
  **ruled out**

# SOUND VELOCITY and EQUATION of STATE

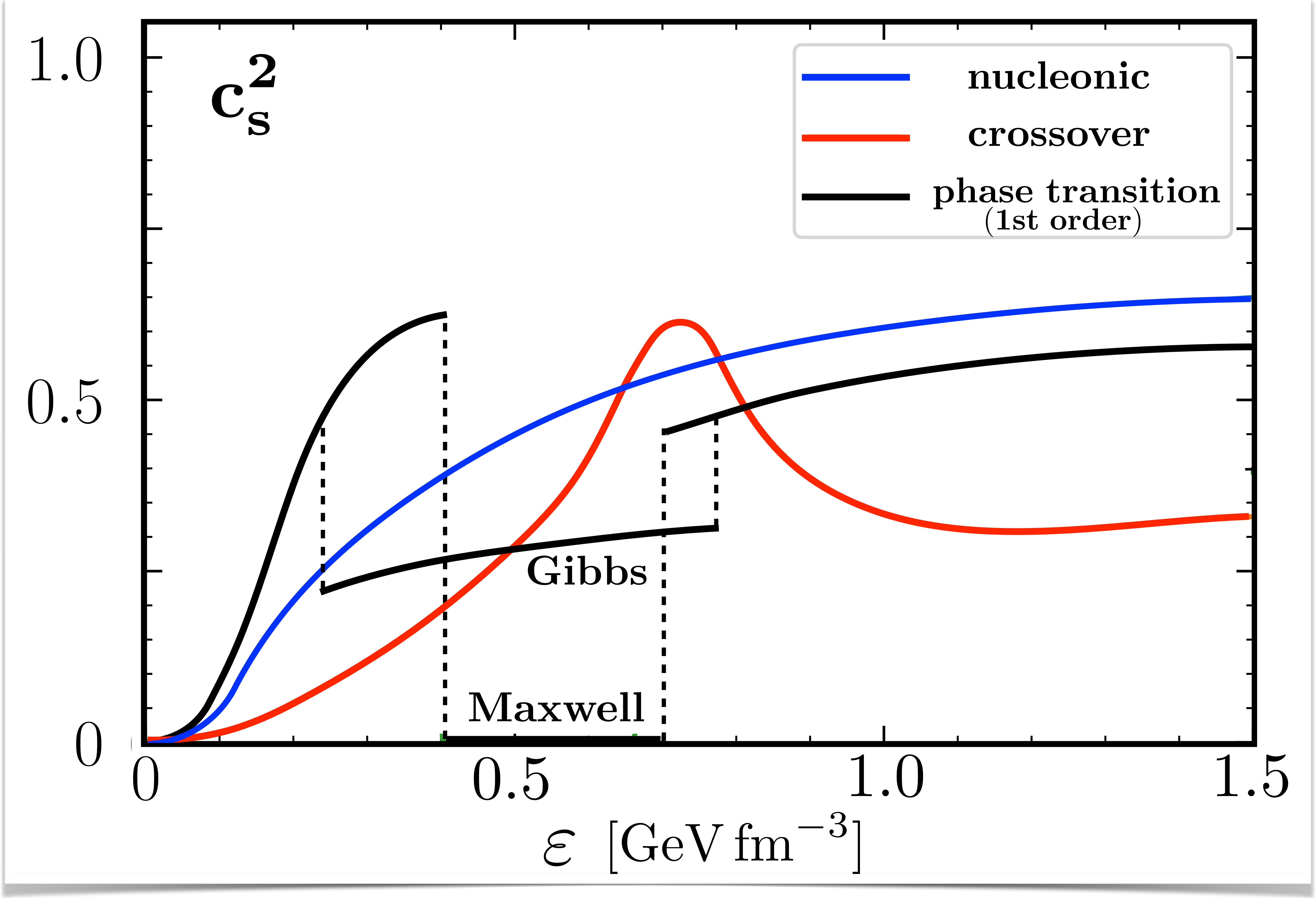
- Key quantity : **Speed of Sound**

$$\mathbf{c_s^2}(\varepsilon) = \frac{\partial \mathrm{P}(\varepsilon)}{\partial \varepsilon}$$

displays
characteristic signature
of
**phase transition**
or
**crossover**

- **E**quation **of S**tate :

$$P(\varepsilon) = \int_0^\varepsilon d\varepsilon'\, c_s^2(\varepsilon')$$



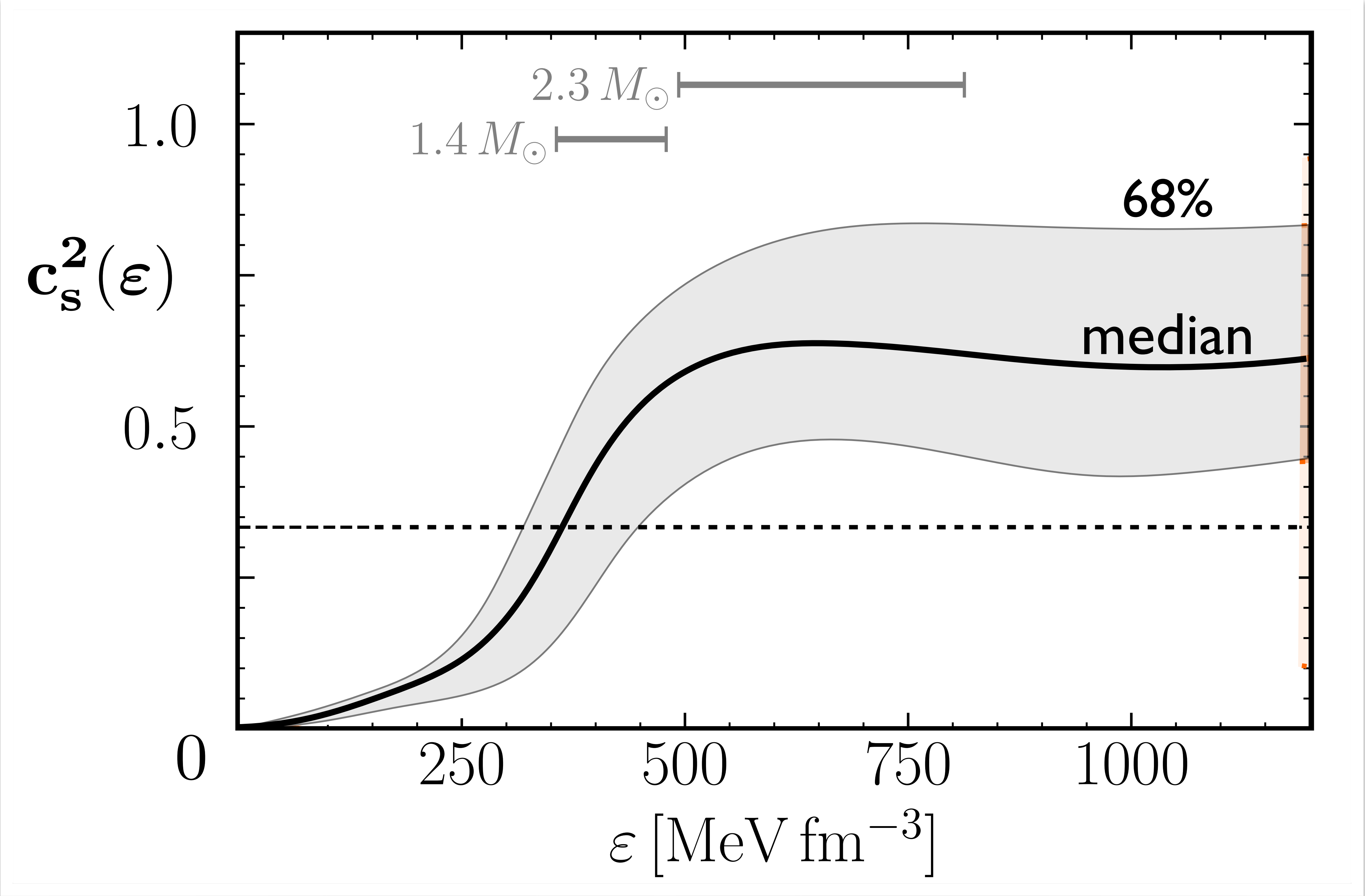
energy density

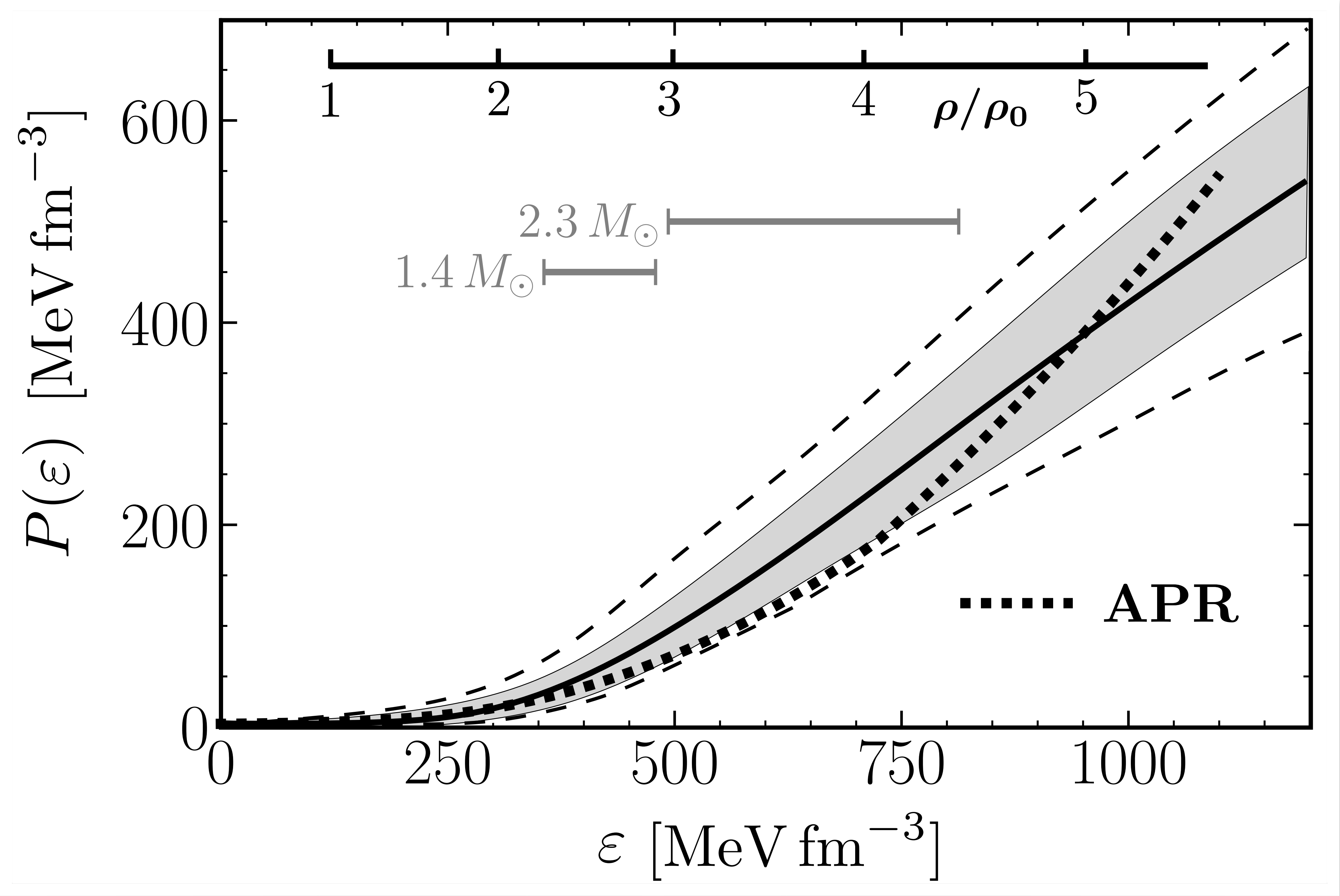
# NEUTRON STAR MATTER EQUATION of STATE

- Bayesian inference of **sound speed** and **EoS**

**PSR masses, NICER & GW data, low-density constraints** (ChEFT), **asymptotic constraints** (pQCD)

**L. Brandes**, W. W., N. Kaiser : Phys. Rev. D 107 (2023) 014011 ; Phys. Rev. D 108 (2023) 094014 - L. Brandes, W. W. : arXiv:2312.11937

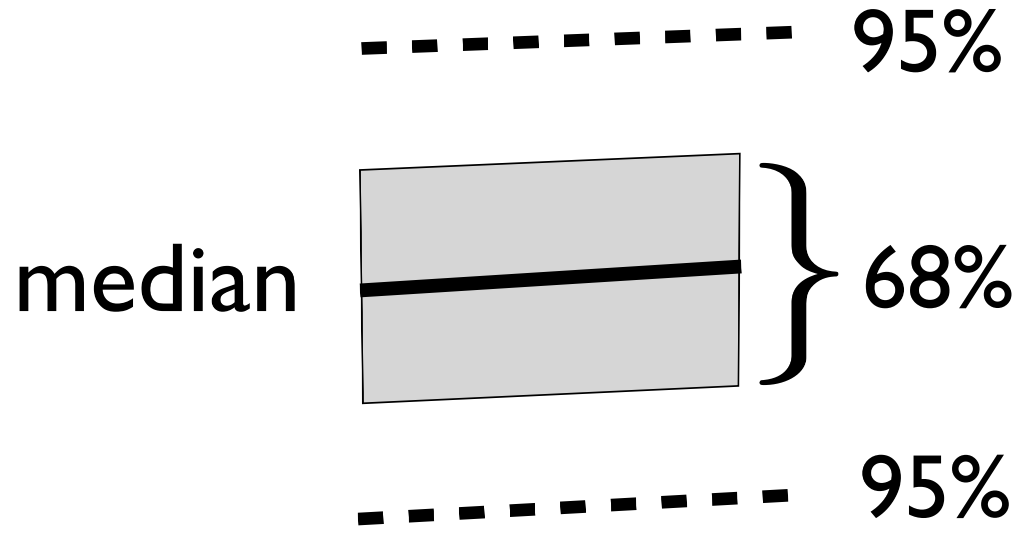




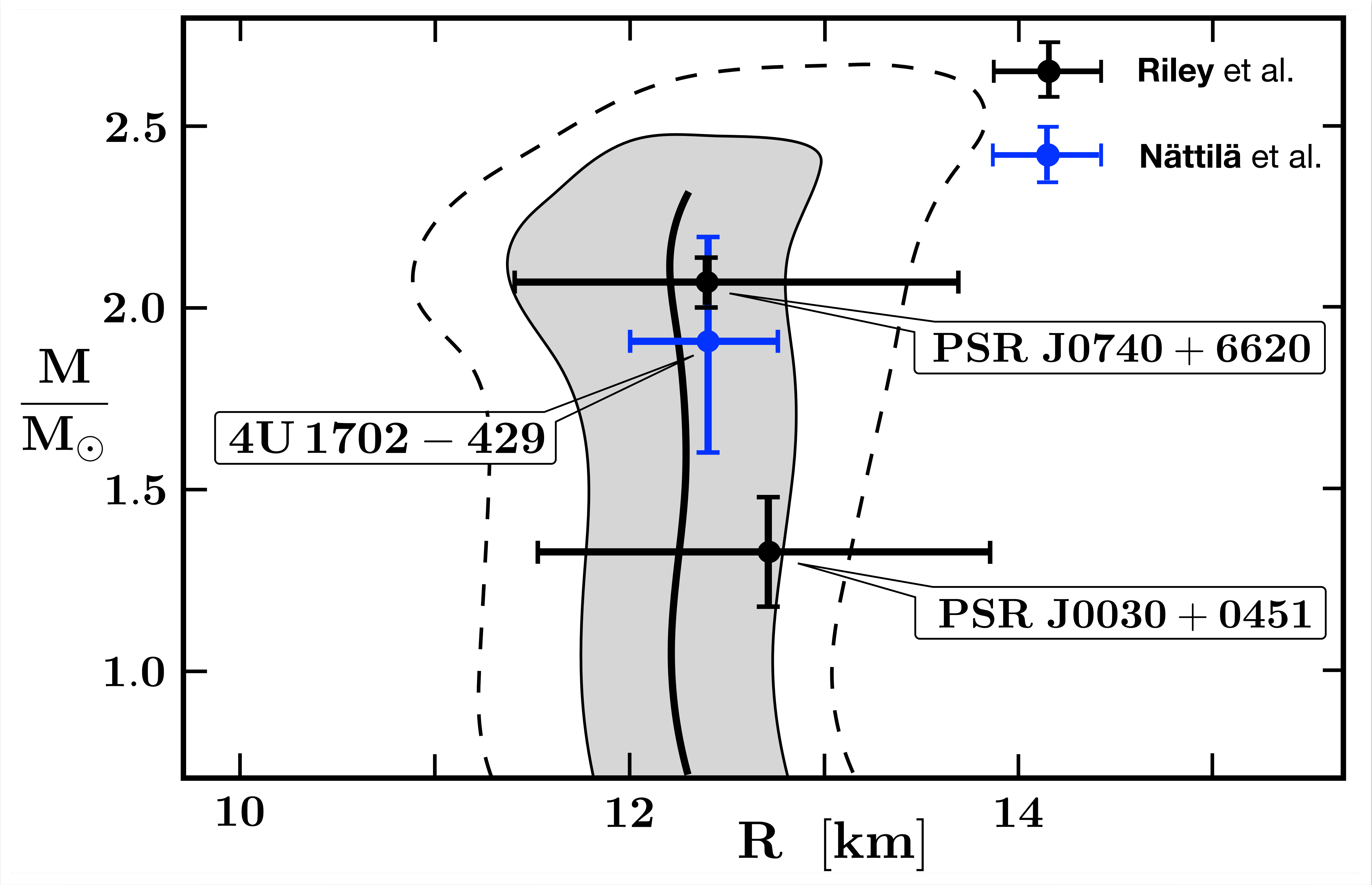
- Squared **speed of sound** exceeds conformal bound $c_s^2 = 1/3$ at densities $\rho > 3\rho_0$
- **Strongly repulsive correlations** at high baryon densities

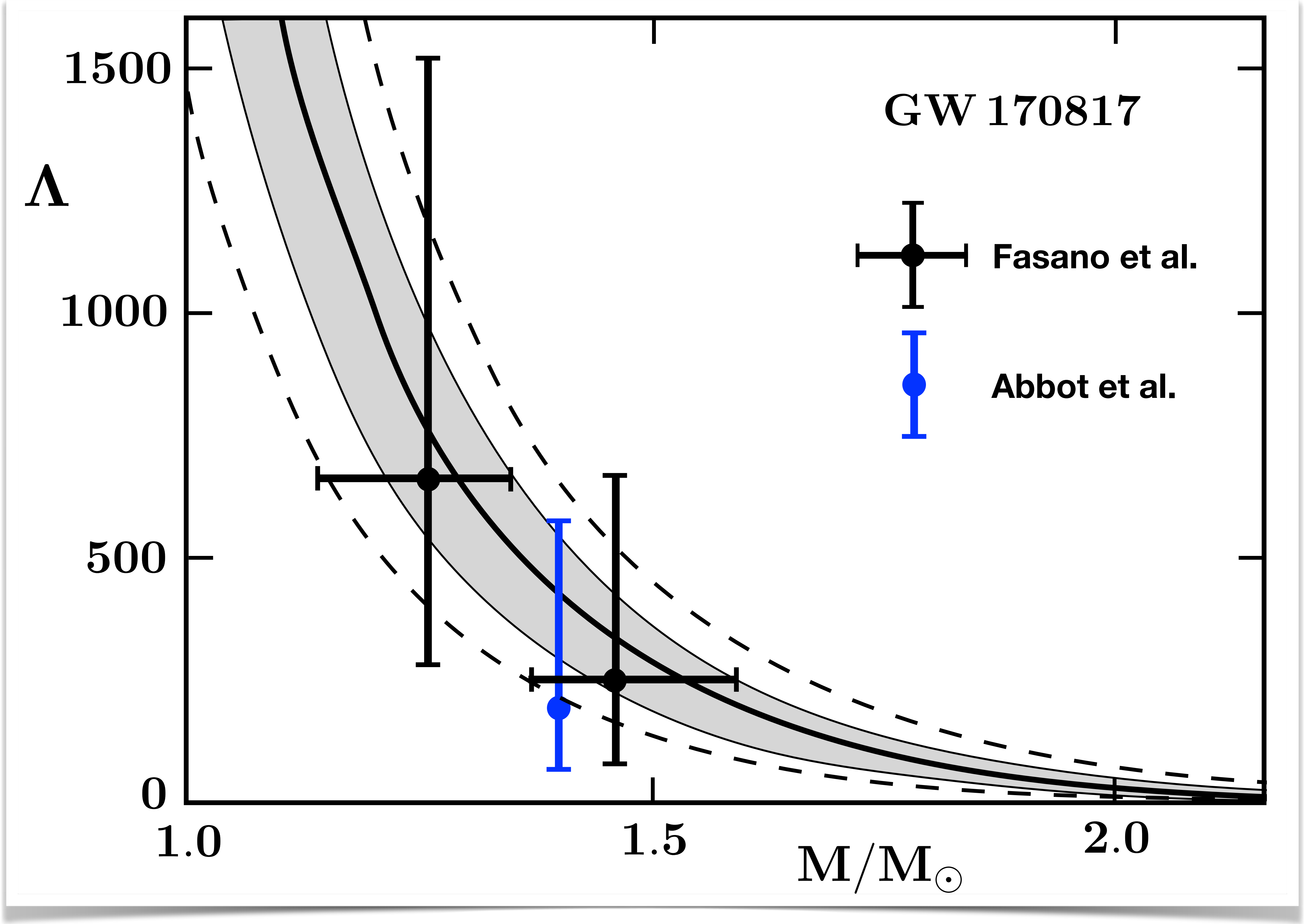
# NEUTRON STAR PROPERTIES

- Bayesian inference posterior credible bands



- Mass - Radius relation (TOV)
- Tidal deformability

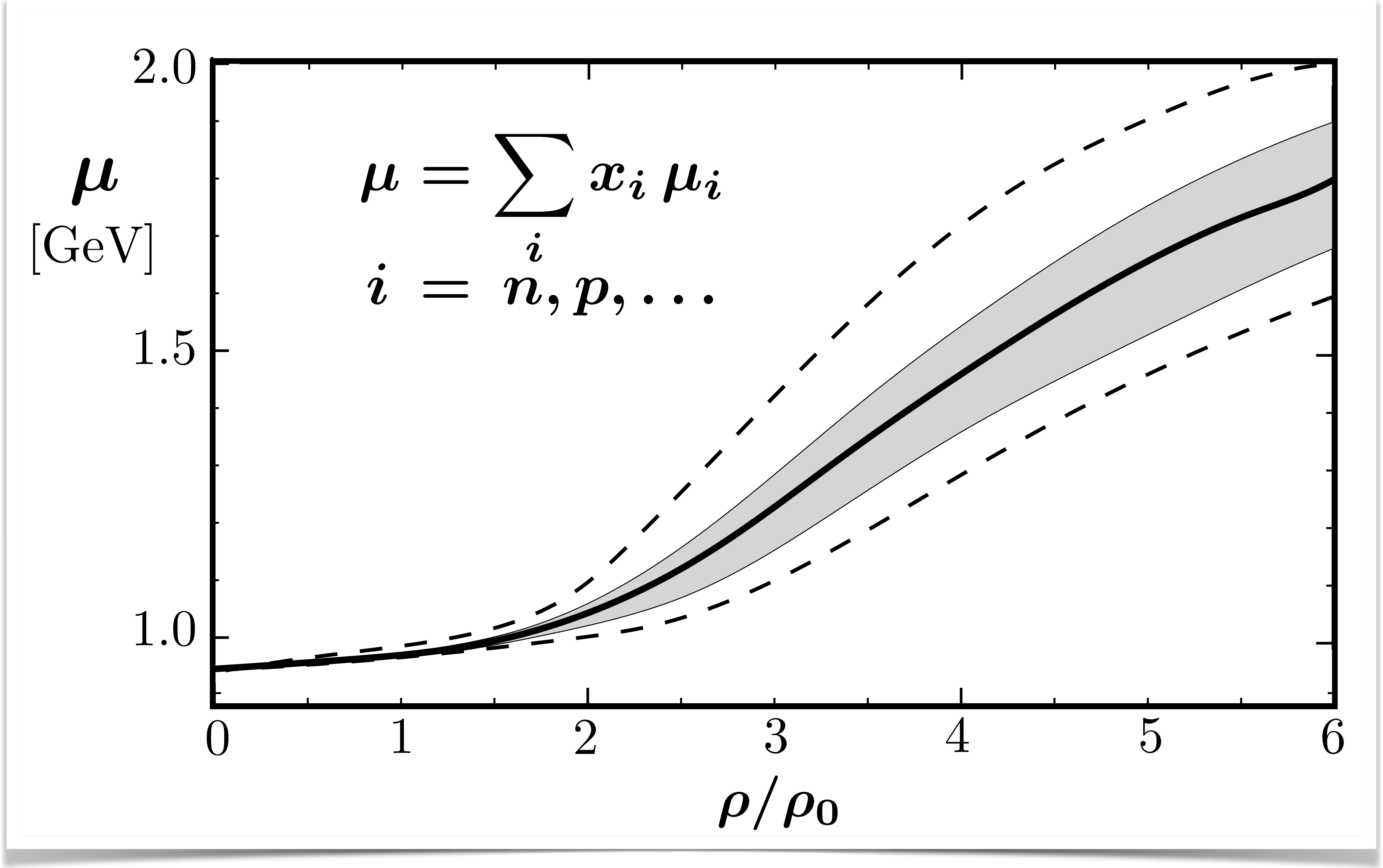




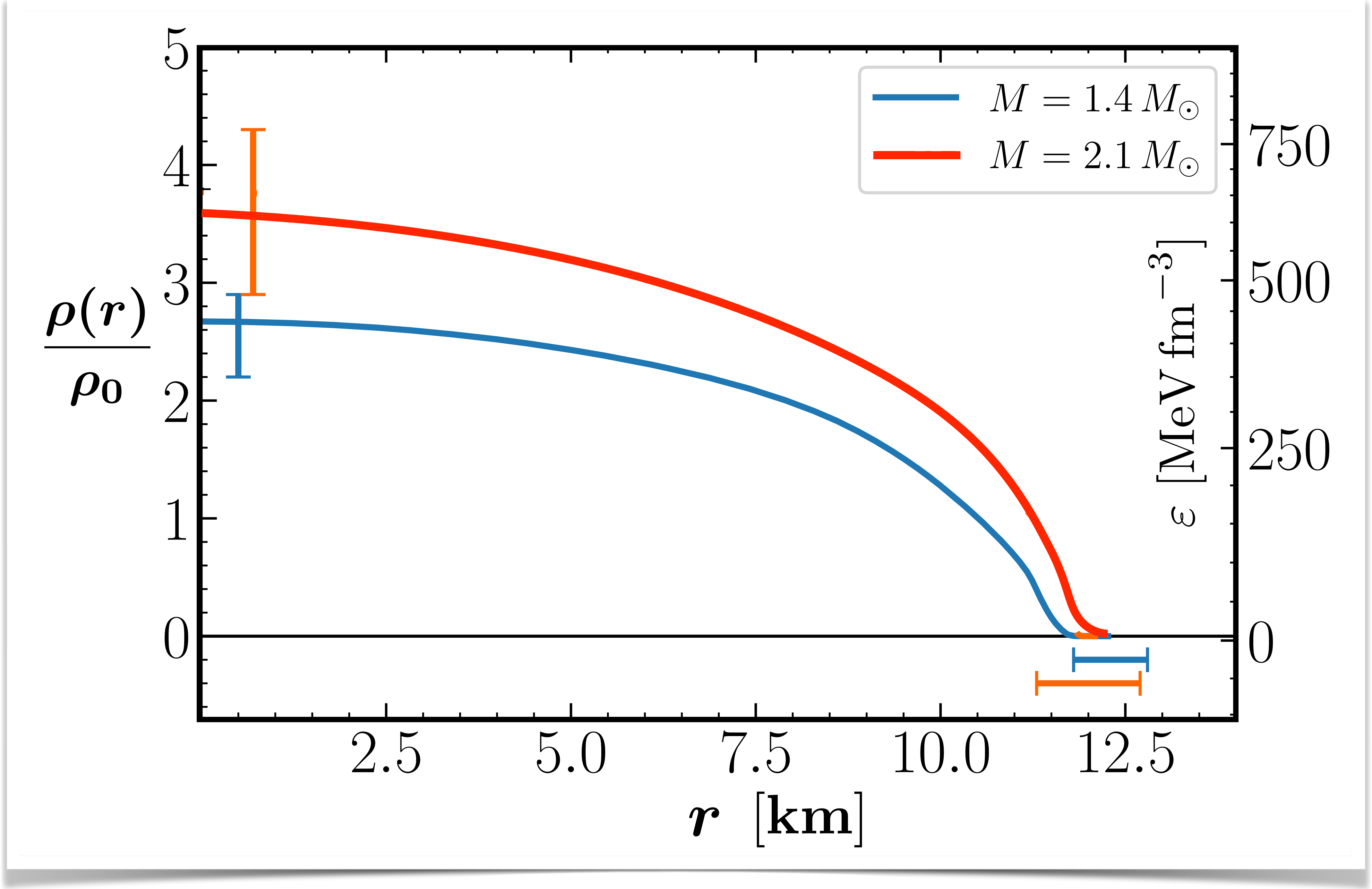
**L. Brandes**, W. W., N. Kaiser : Phys. Rev. D 107 (2023) 014011 ; Phys. Rev. D 108 (2023) 094014

# NEUTRON STAR PROPERTIES (contd.)

- **Baryon chemical potential**



- **Density profiles of neutron stars**



L. Brandes, W. W., N. Kaiser : Phys. Rev. D 107 (2023) 014011 ; Phys. Rev. D 108 (2023) 094014.

- **Stiff equation of state → central core densities in neutron stars are NOT extreme :**

$$\rho_c(1.4\,M_\odot) = 2.6^{+0.3}_{-0.4}\,\rho_0 \qquad \rho_c(2.1\,M_\odot) = 3.6 \pm 0.7\,\rho_0 \qquad \rho_c(2.3\,M_\odot) = 3.8 \pm 0.8\,\rho_0$$

$$(\rho_0 = 0.16\,\mathrm{fm}^{-3})$$

# Constraints on
# FIRST-ORDER PHASE TRANSITION in NEUTRON STAR MATTER

- Bayes factor analysis :

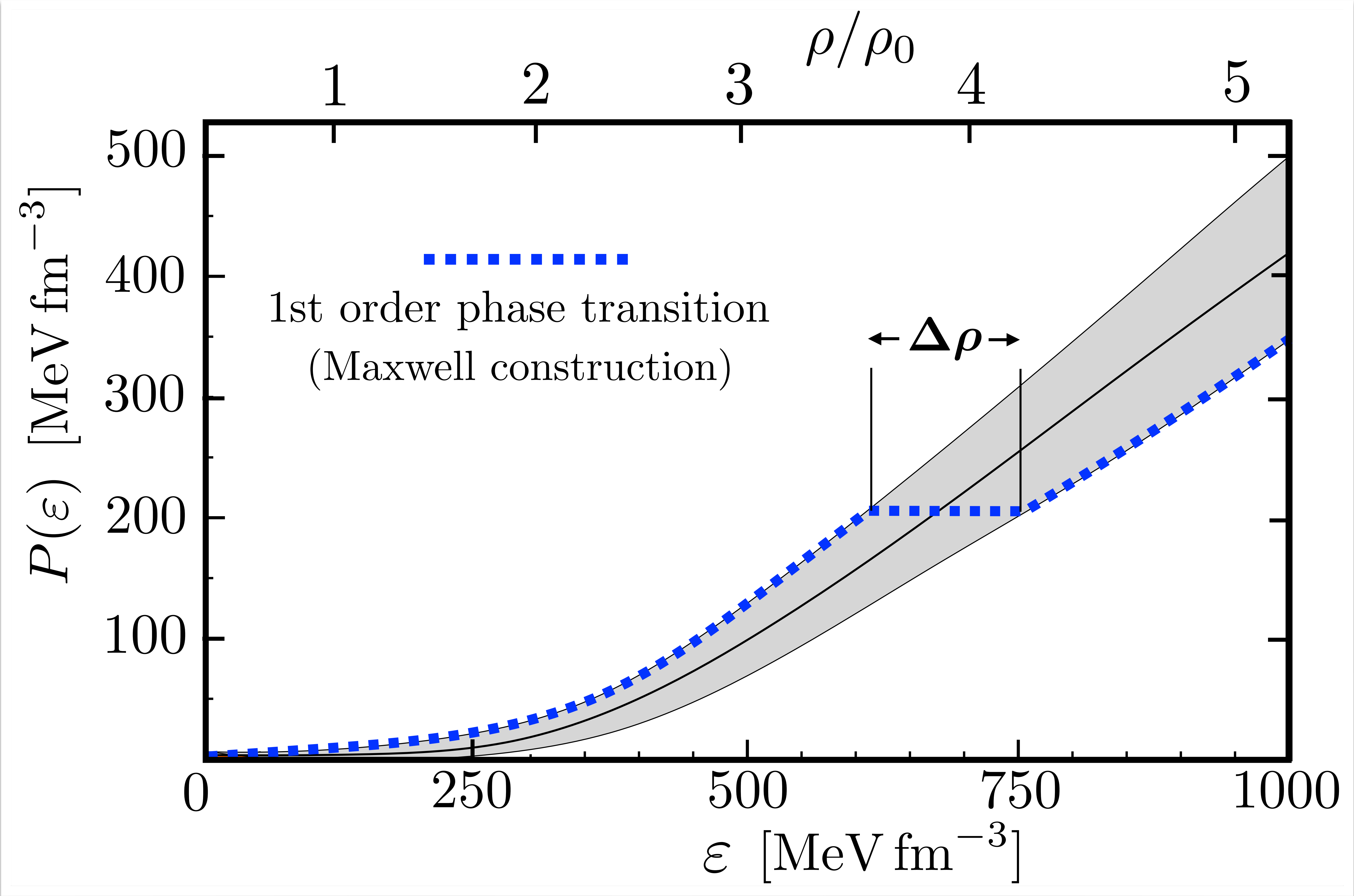
  → Extreme evidence for sound velocities $c_s > 0.5$ in cores of all neutron stars with

  $1.4 \leq M/M_\odot \leq 2.3$

- Evidence against **strong** 1st order transition :

  → Maximum possible extension of phase coexistence domain

  $\Delta\rho/\rho < 0.2$



L. Brandes, W. W., N. Kaiser : Phys. Rev. D 108 (2023) 094014 - L. Brandes, W. W. : arXiv:2312.11937

→ Compare with : Maxwell construction for nuclear liquid-gas phase transition $(\Delta\rho/\rho > 1)$

# INTERMEDIATE SUMMARY

- **Bayesian inference analysis**
  now including heavy $(\mathrm{M} = 2.35 \pm 0.17\,\mathrm{M}_\odot)$ galactic neutron star
  - → even **stiffer equation of state** required
  - → almost **constant neutron star radii** $(\mathrm{R} \simeq 12 \pm 1\ \mathrm{km})$ for all masses

- **Extreme evidence** for sound velocities $\mathrm{c_s} > 1/\sqrt{3}$ in neutron star cores
  - → **strongly repulsive correlations** at work

- **Evidence** against **strong 1st order phase transition** in neutron star cores
  - → **not** excluded: **baryonic** matter or **hadron-quark** continuous crossover

- **No extreme central core densities** even in the heaviest neutron stars:
  $\rho \lesssim 4.5\,\rho_0$ for $M \leq 2.3\,M_\odot$ → average baryon-baryon distance : $\mathrm{d} \gtrsim 1\,\mathrm{fm}$

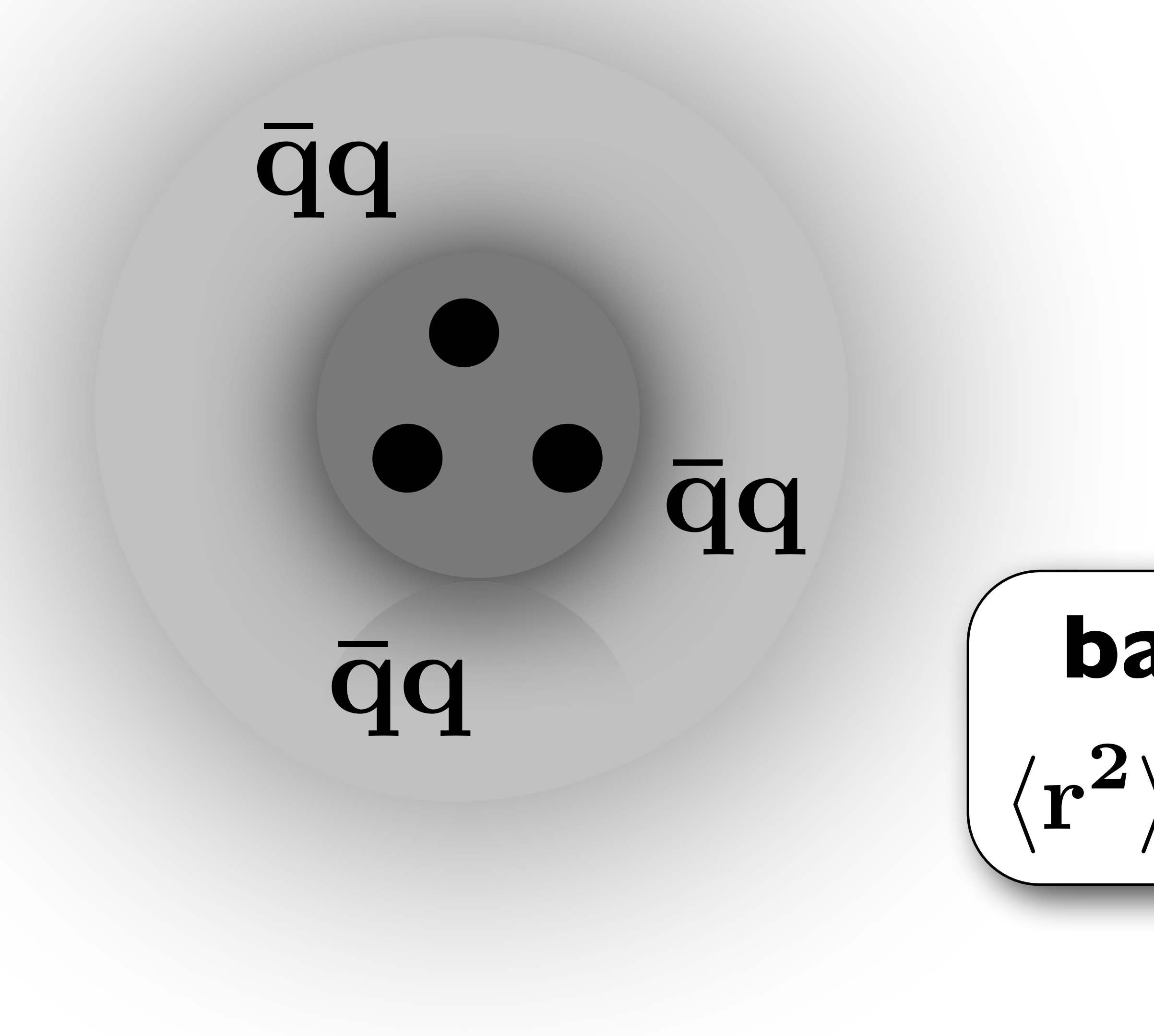
# Part Two

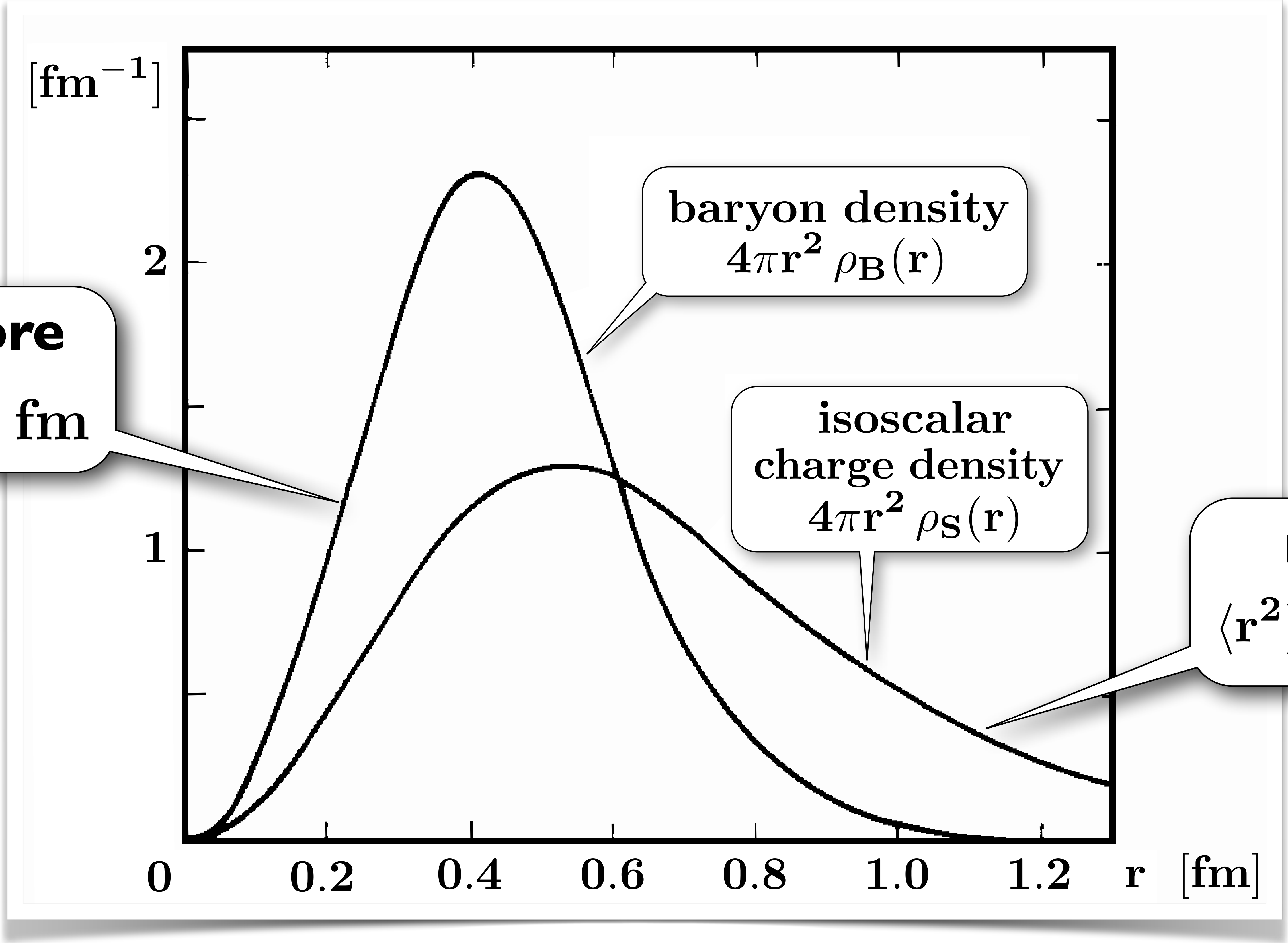
## Phenomenology, Models
## and
## Possible Dense Matter Scenarios

# Historical reminder: **SIZES** of the **NUCLEON**

Low-energy QCD: **spontaneously broken chiral symmetry + localisation (confinement)**

- **NUCLEON** : **compact valence quark core + mesonic** (multi $\bar{q}q$) **cloud**
  - Example: Chiral Soliton Model of the Nucleon





**baryonic core**

$\langle r^2 \rangle_B^{1/2} \simeq 0.5$ fm

**mesonic cloud**

$\langle r^2 \rangle_{E,\mathrm{isoscalar}}^{1/2} \simeq 0.8$ fm

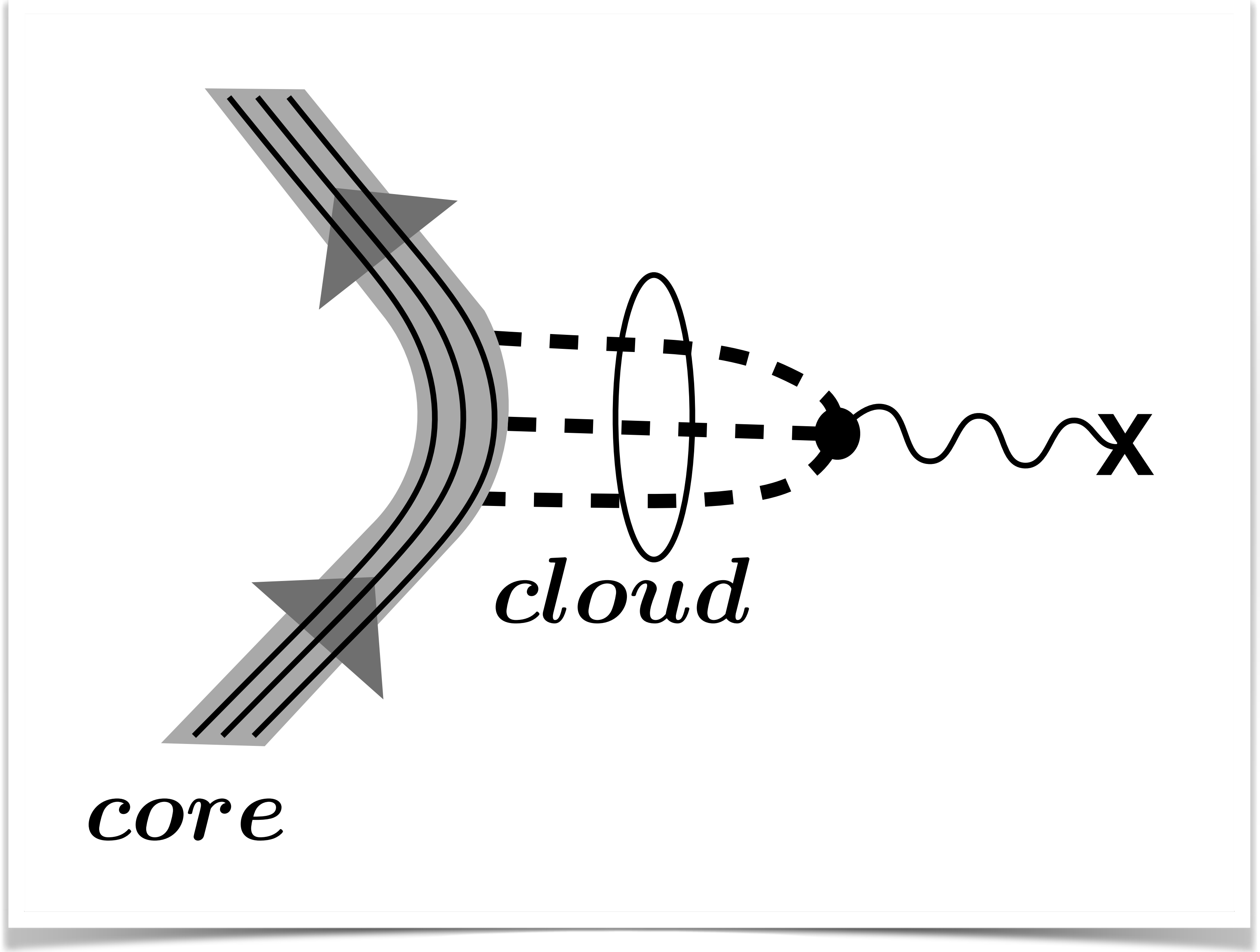
N. Kaiser,
U.-G. Meißner,
W.W.
Nucl. Phys. A466 (1987) 685

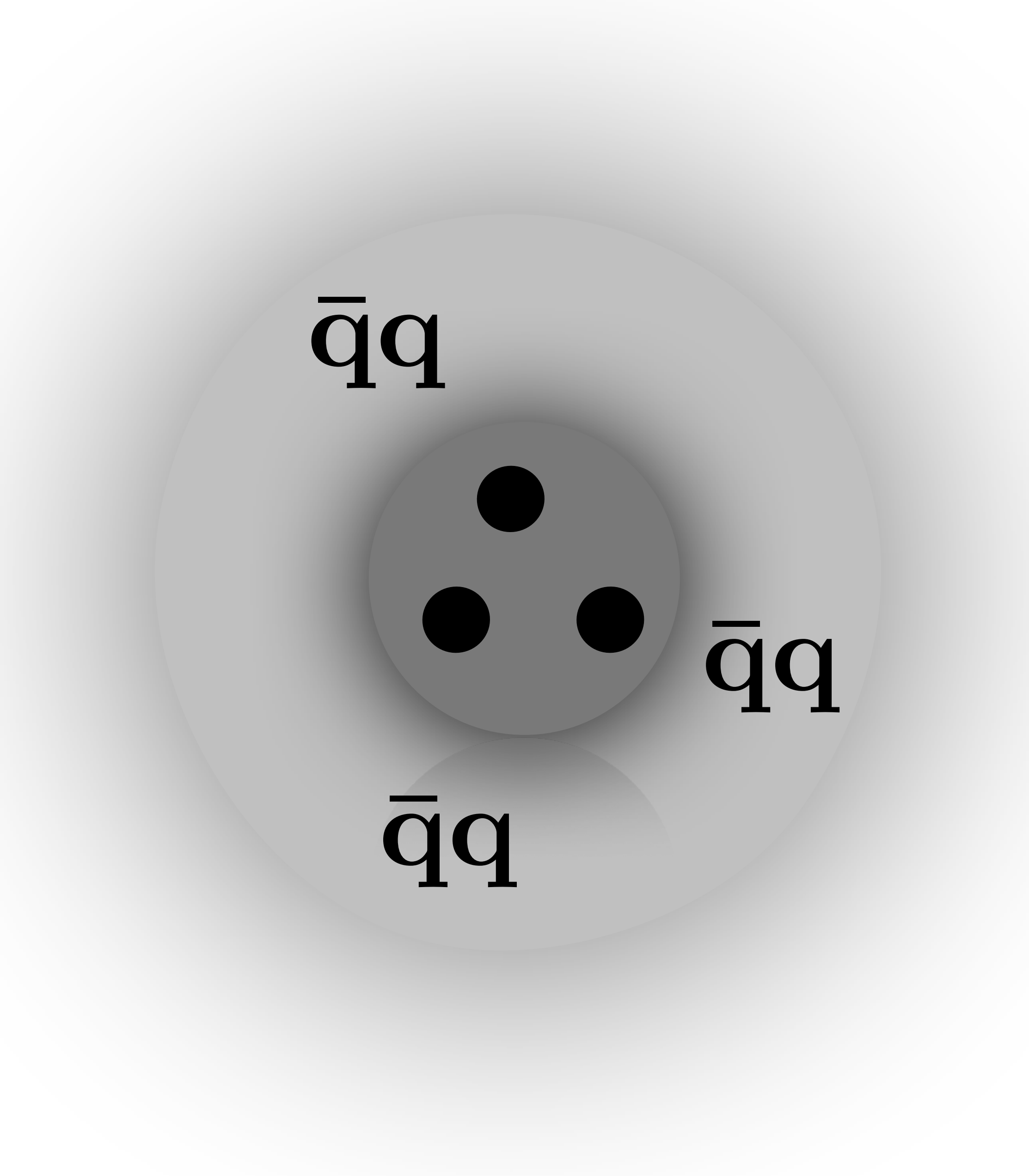
- **Separation of scales**

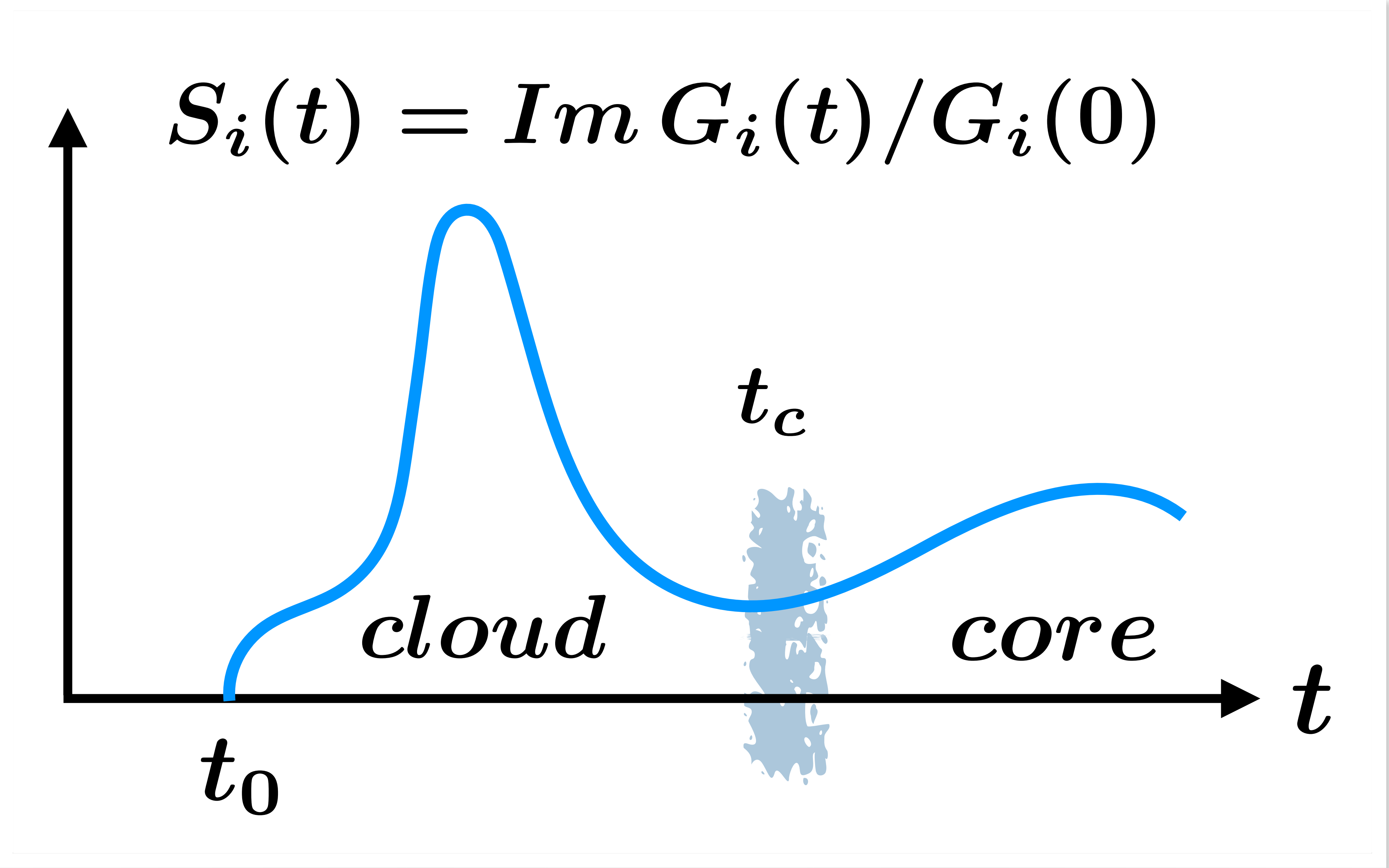
$$\left(\frac{R_{cloud}}{R_{core}}\right)^3 \gtrsim 5$$

# FORM FACTORS of the NUCLEON

$$G_i(q^2) = G_i(0) + \frac{q^2}{\pi}\int_{t_0}^{\infty} dt \frac{Im\, G_i(t)}{t(t-q^2-i\epsilon)} \qquad \langle r_i^2\rangle = \frac{6}{G_i(0)}\frac{dG_i(q^2)}{dq^2}\Big|_{q^2=0} = \frac{6}{\pi}\int_{t_0}^{\infty}\frac{dt}{t^2}S_i(t)$$



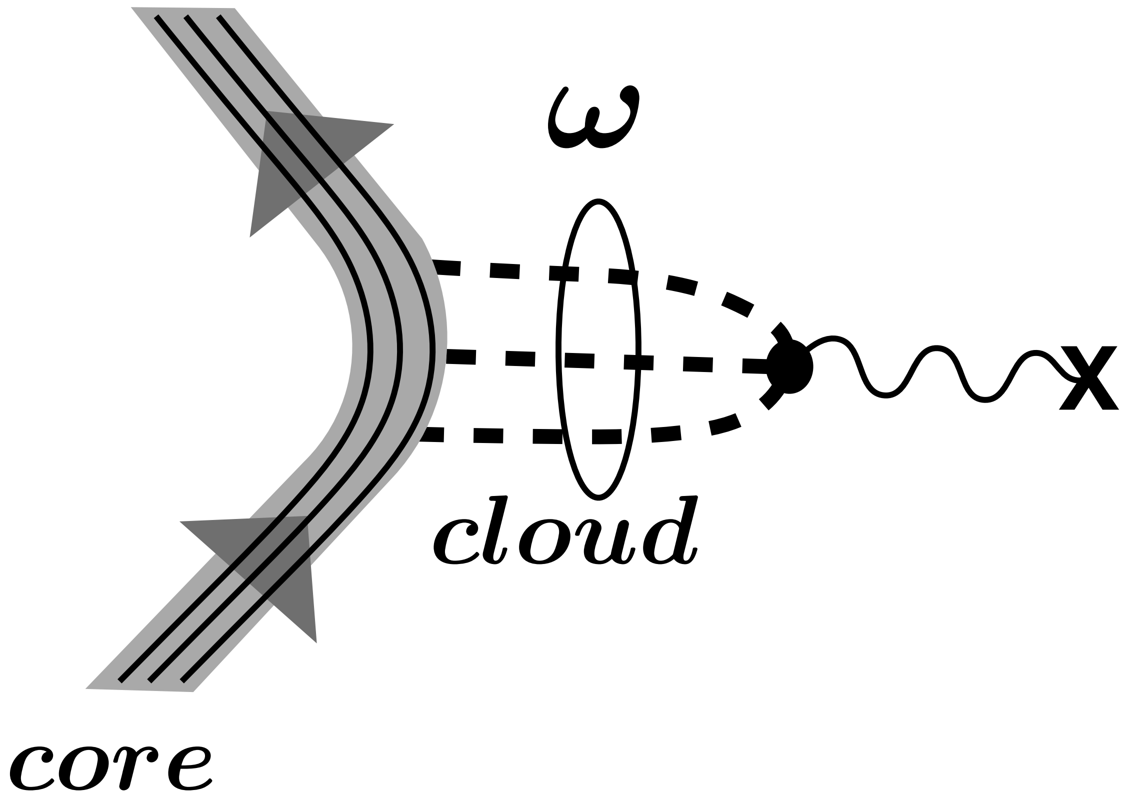




$$\langle r_i^2\rangle = \langle r_i^2\rangle_{cloud} + \langle r_i^2\rangle_{core} = \frac{6}{\pi}\left[\int_{t_0}^{t_c}\frac{dt}{t^2}S_i(t) + \int_{t_c}^{\infty}\frac{dt}{t^2}S_i(t)\right]$$

# Examples: **ISOSCALAR ELECTRIC** and **ISOVECTOR AXIAL FORM FACTORS** of the **NUCLEON**

- Isoscalar electric form factor $G_E^S(q^2) = \frac{1}{2}\left[G_E^p(q^2) + G_E^n(q^2)\right]$ $\langle r_{E,S}^2\rangle = \langle r_p^2\rangle + \langle r_n^2\rangle$



Empirical : $\langle r_p^2\rangle^{1/2} \simeq 0.84\,\mathrm{fm}$ $\langle r_n^2\rangle \simeq -0.11\,\mathrm{fm}^2$ $\langle r_{E,S}^2\rangle^{1/2} \simeq 0.77\,\mathrm{fm}$

Y.H. Lin,
H.-W. Hammer,
U.-G. Meißner
PRL 128 (2022) 052002

- Vector Dominance: "cloud" dominated by omega meson

$$\langle r_{E,S}^2\rangle \simeq \langle r_{core}^2\rangle + \frac{6}{m_\omega^2} \rightarrow \boxed{\langle r_{core}^2\rangle^{1/2} \equiv \langle r_B^2\rangle^{1/2} \simeq 0.5\,\mathrm{fm}}$$

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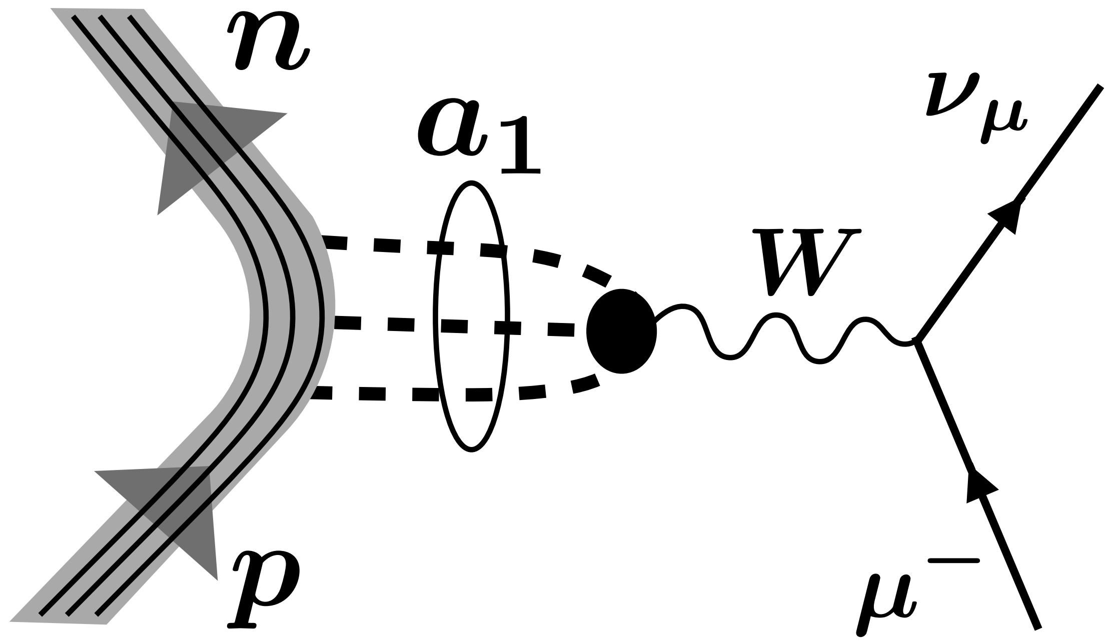
- Axial form factor $G_A(q^2) = g_A\left[1 + \frac{1}{6}\langle r_A^2\rangle q^2 + \ldots\right]$

R.J. Hill, P. Kammel, W.C. Marciano, A. Sirlin
Rep. Prog. Phys. 81 (2018) 096301



Empirical : $\langle r_A^2\rangle = (0.46 \pm 0.16)\,\mathrm{fm}^2$

$$\langle r_A^2\rangle \sim \langle r_A^2\rangle_{core} + \frac{6}{m_a^2} \rightarrow \boxed{\langle r_A^2\rangle_{core}^{1/2} \simeq (0.52 \pm 0.13)\,\mathrm{fm}}$$

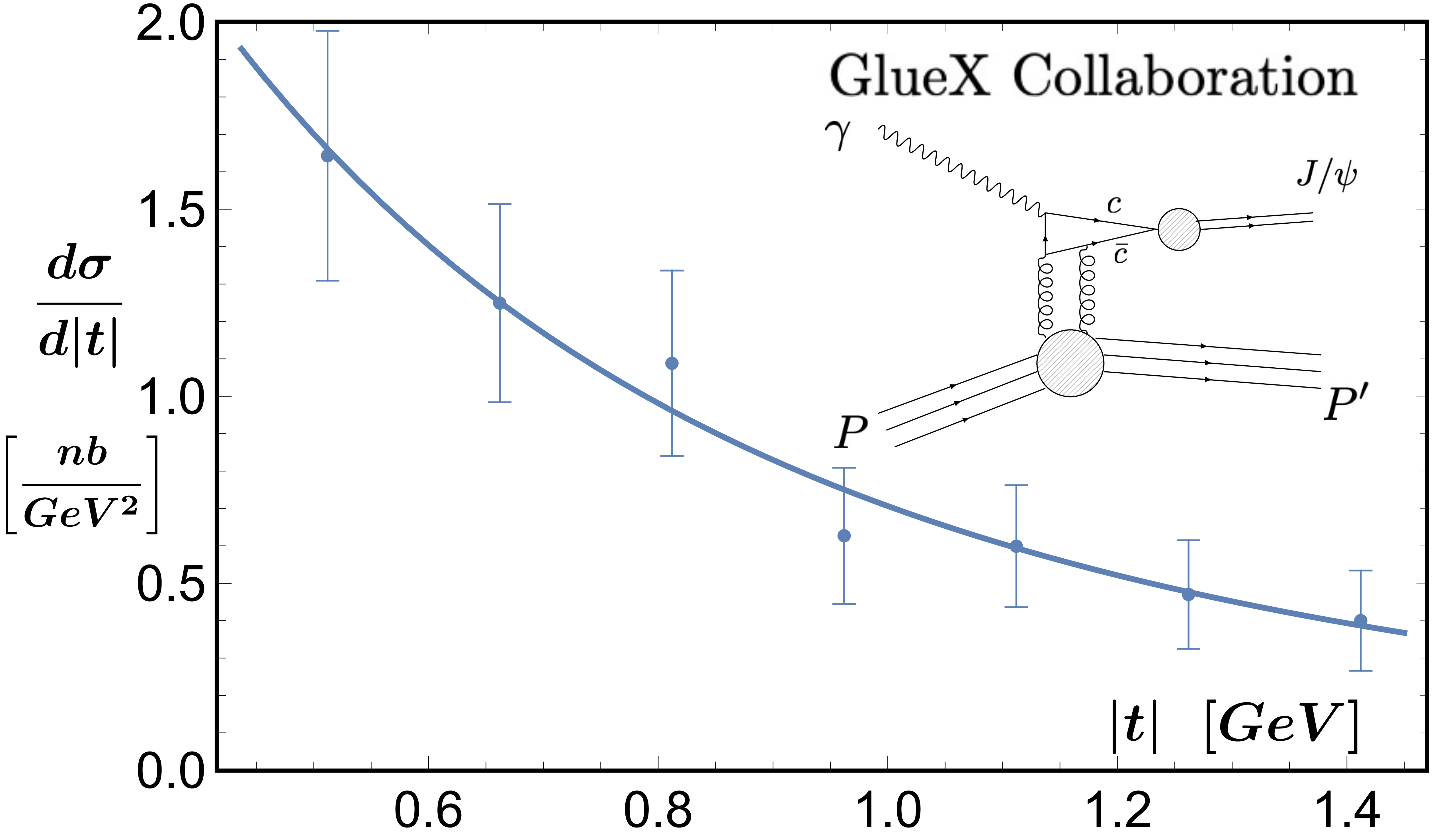
# MASS RADIUS of the NUCLEON

- Mass ("gravitational") form factor
- Trace of the QCD energy-momentum tensor

$$G_m(q^2) \sim \langle P'|T^\mu_\mu|P\rangle = \langle P'|\frac{\beta}{2g}G^{\mu\nu}_a G^a_{\mu\nu} + m_q(\bar{u}u + \bar{d}d) + m_s\bar{s}s|P\rangle$$

$$G_m(0) = M_N \simeq 0.94\,\mathrm{GeV}$$

$$\langle r_m^2\rangle = \frac{6}{M_N}\frac{dG_m(q^2)}{dq^2}\Big|_{q^2=0}$$



- Core (gluon) dominance plus small corrections from sigma terms

$$\langle r_m^2\rangle = \frac{1}{M_N}\left[M_0\langle r_{core}\rangle^2 + \sigma_N\langle r_\sigma^2\rangle + \sigma_s\langle r_s^2\rangle\right]$$

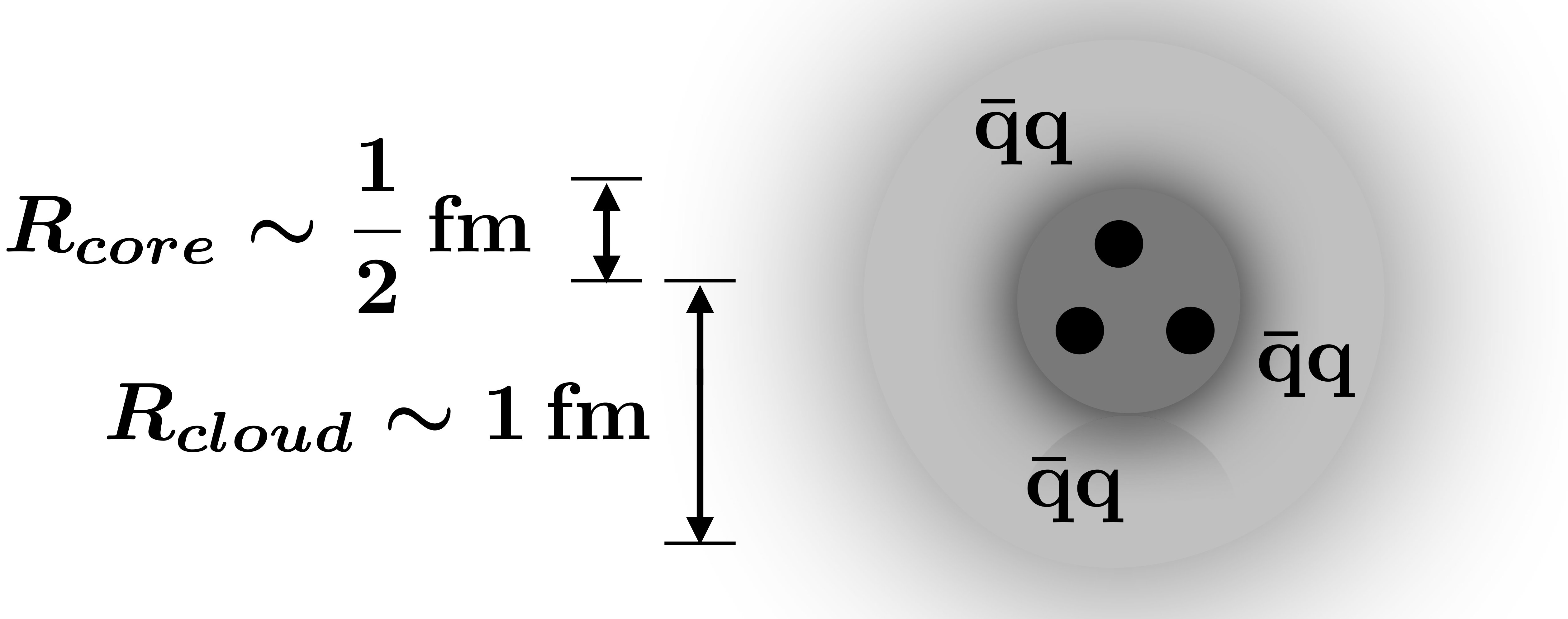
$$(M_0 \gtrsim 0.9\,M_N)$$

- **Mass radius** D. Kharzeev : Phys. Rev. D104 (2021) 054015

$$\boxed{\langle r_m^2\rangle^{1/2} = (0.55 \pm 0.03)\,\mathrm{fm}}$$

… equals "core" radius in the chiral limit

# TWO-SCALES Picture of the NUCLEON : implications for DENSE BARYONIC MATTER



$$R_{core} \sim \frac{1}{2}\ \mathrm{fm}$$

$$R_{cloud} \sim 1\ \mathrm{fm}$$

- **Separation of scales**

$$\left(\frac{R_{cloud}}{R_{core}}\right)^3 \gg 1$$

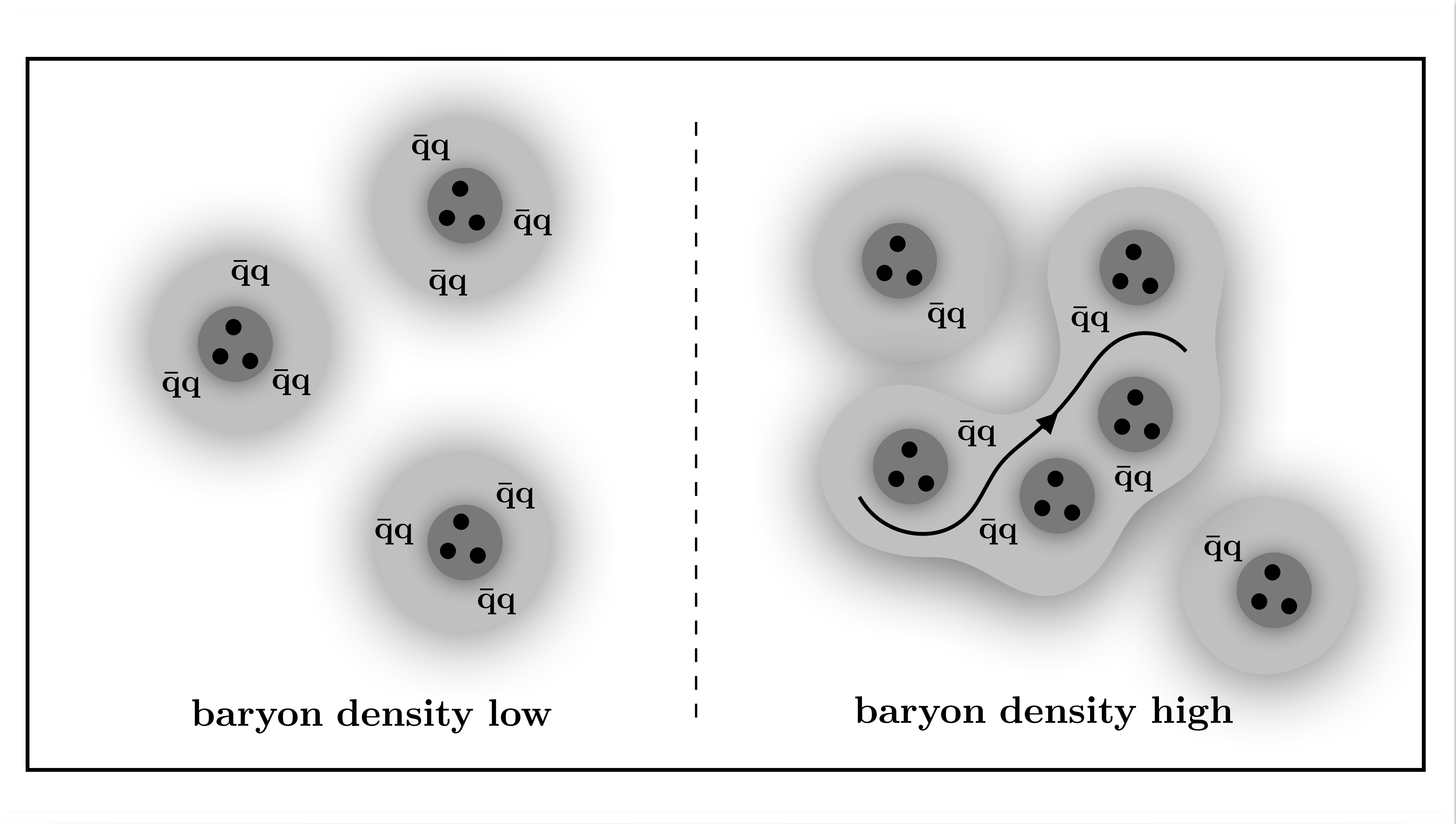
- **Soft mesonic (multi-pion) cloud**

  expected to **expand** with increasing baryon density along with decreasing **in-medium pion decay constant** $f_\pi^*(\rho)$

- **Hard baryonic core** governed by **gluon dynamics**

  expected to remain **stable** with increasing baryon density up until hard compact cores begin to touch and overlap

# TWO-SCALES Scenario for DENSE BARYONIC MATTER



- Baryon densities

  $\rho \sim \rho_0 = 0.16\,\mathrm{fm}^{-3}$

  tails of mesonic clouds overlap :
  two-body exchange forces
  between nucleons

- $\rho \gtrsim 2-3\,\rho_0$

  Soft $\bar{q}q$ clouds delocalize:
  **percolation** → many-body forces
  baryonic cores still separated, but subject to increasingly strong repulsive Pauli effects

- $\rho > 5\,\rho_0$ (beyond central densities of neutron stars)

  compact nucleon cores begin to touch and overlap at distances $d \lesssim 1\,\mathrm{fm}$
  (but still have to overcome repulsive NN hard core)

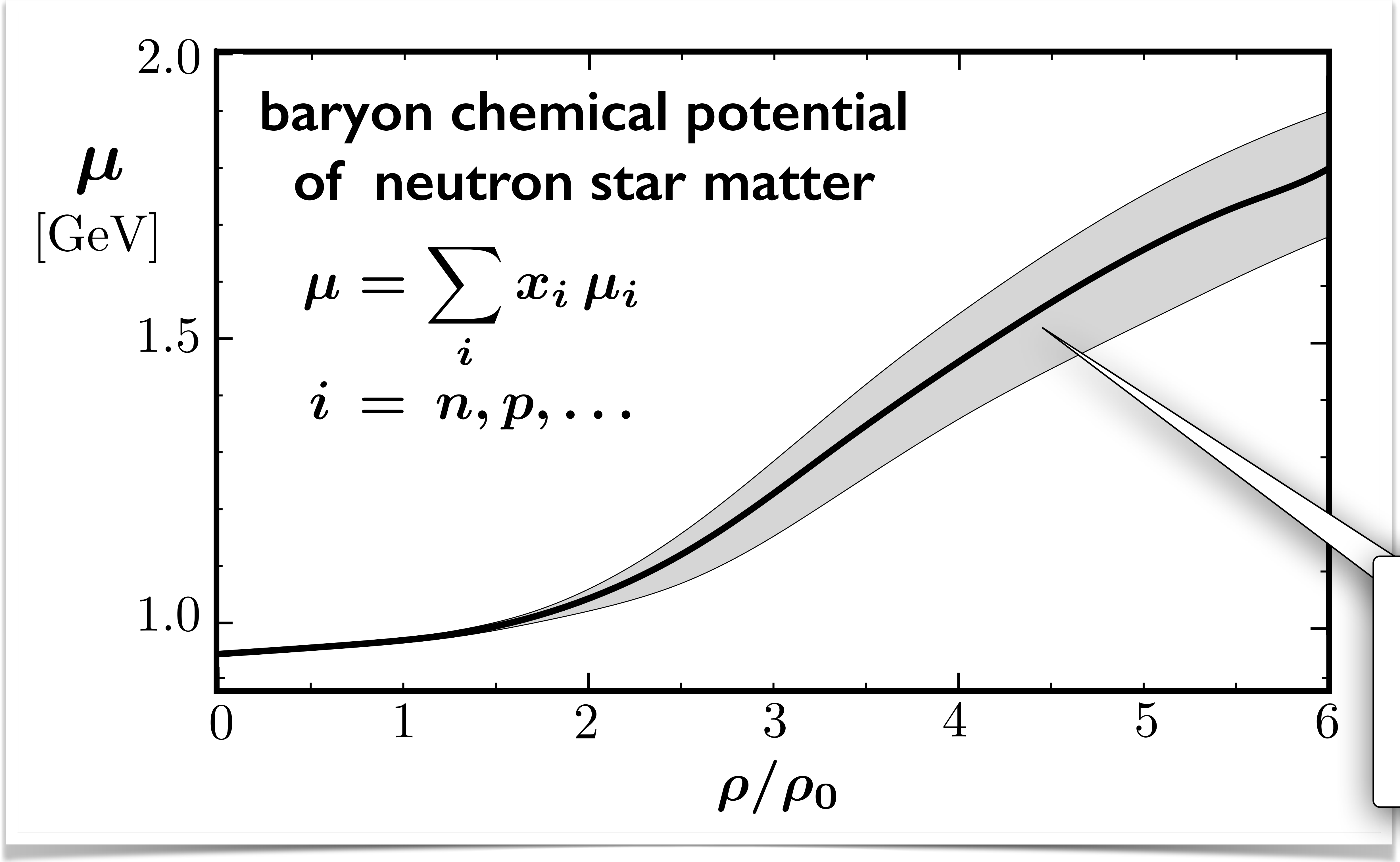
K. Fukushima, T. Kojo, W.W.
Phys. Rev. D 102 (2020) 096017

# DENSE BARYONIC MATTER in NEUTRON STARS as a RELAVISTIC FERMI LIQUID

B. Friman, W.W. : Rhys. Rev. C100 (2019) 065807

L. Brandes, W.W. : arXiv:2312.11937

- **Neutron Star Matter : Fermi liquid** / dominantly **neutrons** + ca. 5 % **protons**
- **Quasiparticles** :
  baryons "dressed" by their **strong interactions** and imbedded in **mesonic (multi-pion) field**



take median of $\mu(\rho)$ from Bayesian-inferred neutron star EoS

- **Landau effective mass**

$$m_L^*(\rho) = \sqrt{p_F^2 + m^2(\rho)}$$

- **Baryon chemical potential**

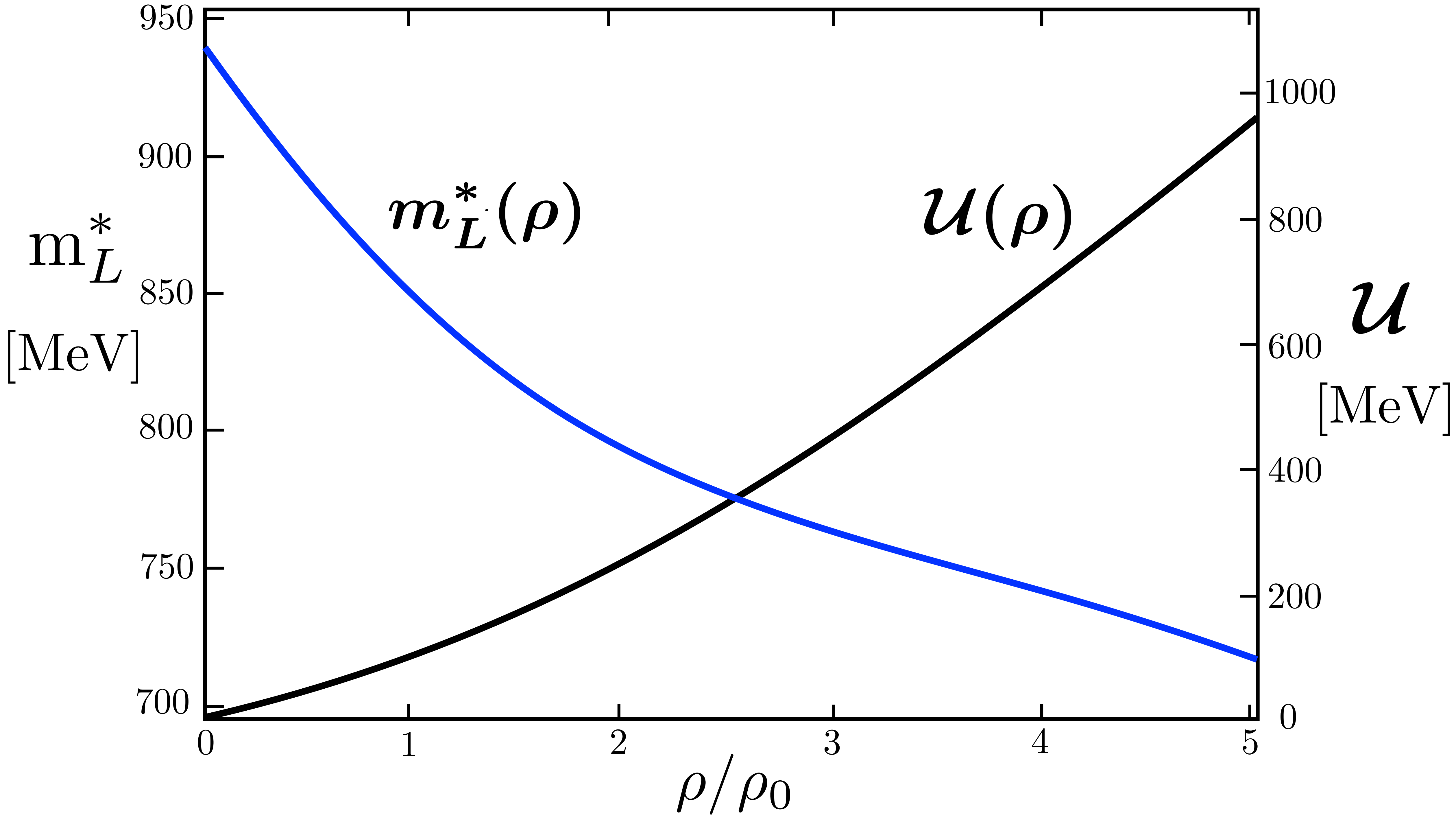
$$\mu(\rho) = m_L^*(\rho) + \mathcal{U}(\rho)$$

quasiparticle potential

# QUASIPARTICLE POTENTIAL and FERMI-LIQUID PARAMETERS

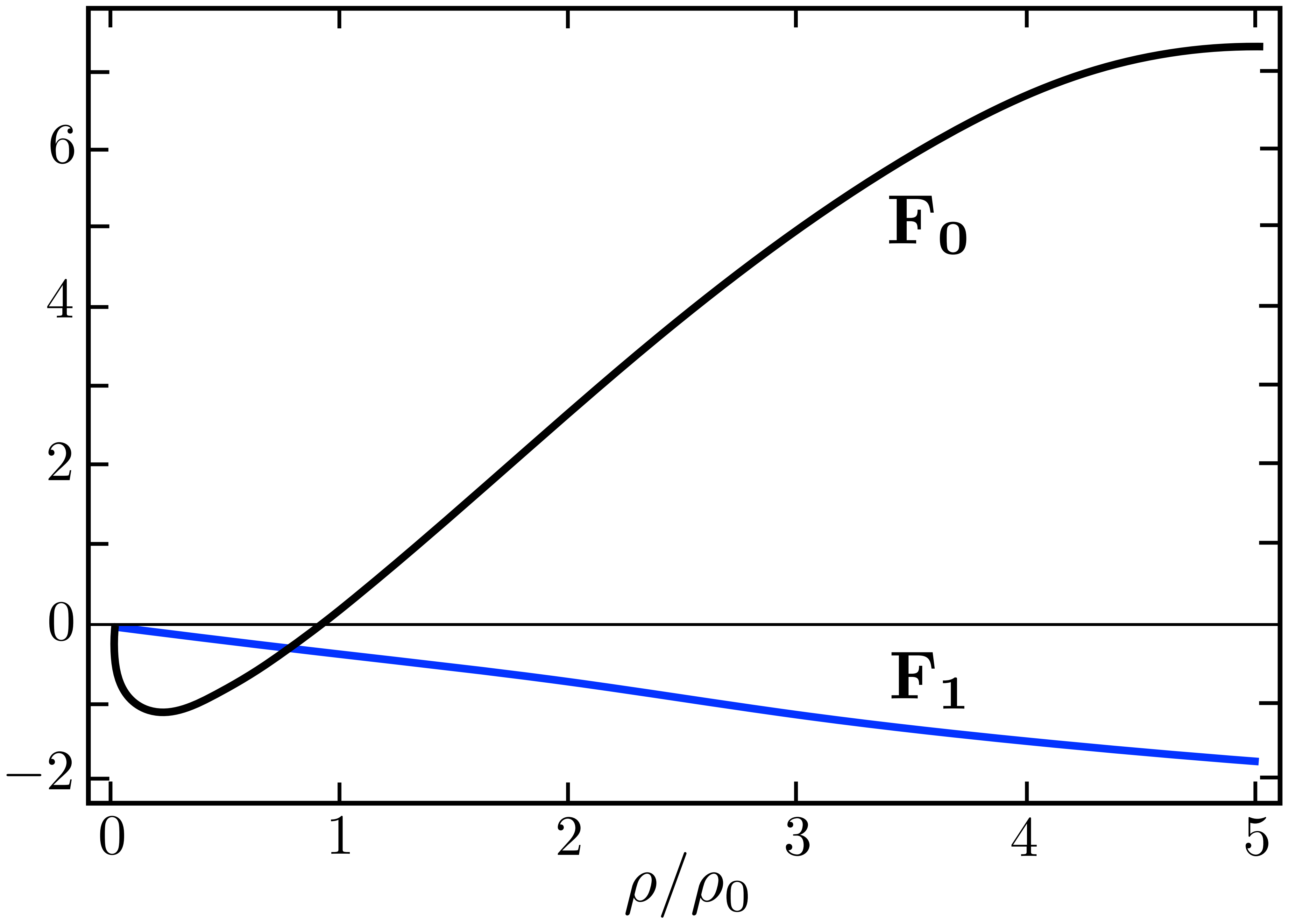
- $m_L^*(\rho)$ from **chiral nucleon-meson field theory** & Functional Renormalisation Group
- Quasiparticle effective potential

$$\mathcal{U}(\rho) = \sum_n u_n \left(\frac{\rho}{\rho_0}\right)^n$$



- Landau Fermi-Liquid parameters

$$F_0 = \frac{m_L^* p_F}{\pi^2}\frac{\partial\mu}{\partial\rho} - 1 \qquad F_1 = -\frac{3\,\mathcal{U}}{\mu}$$



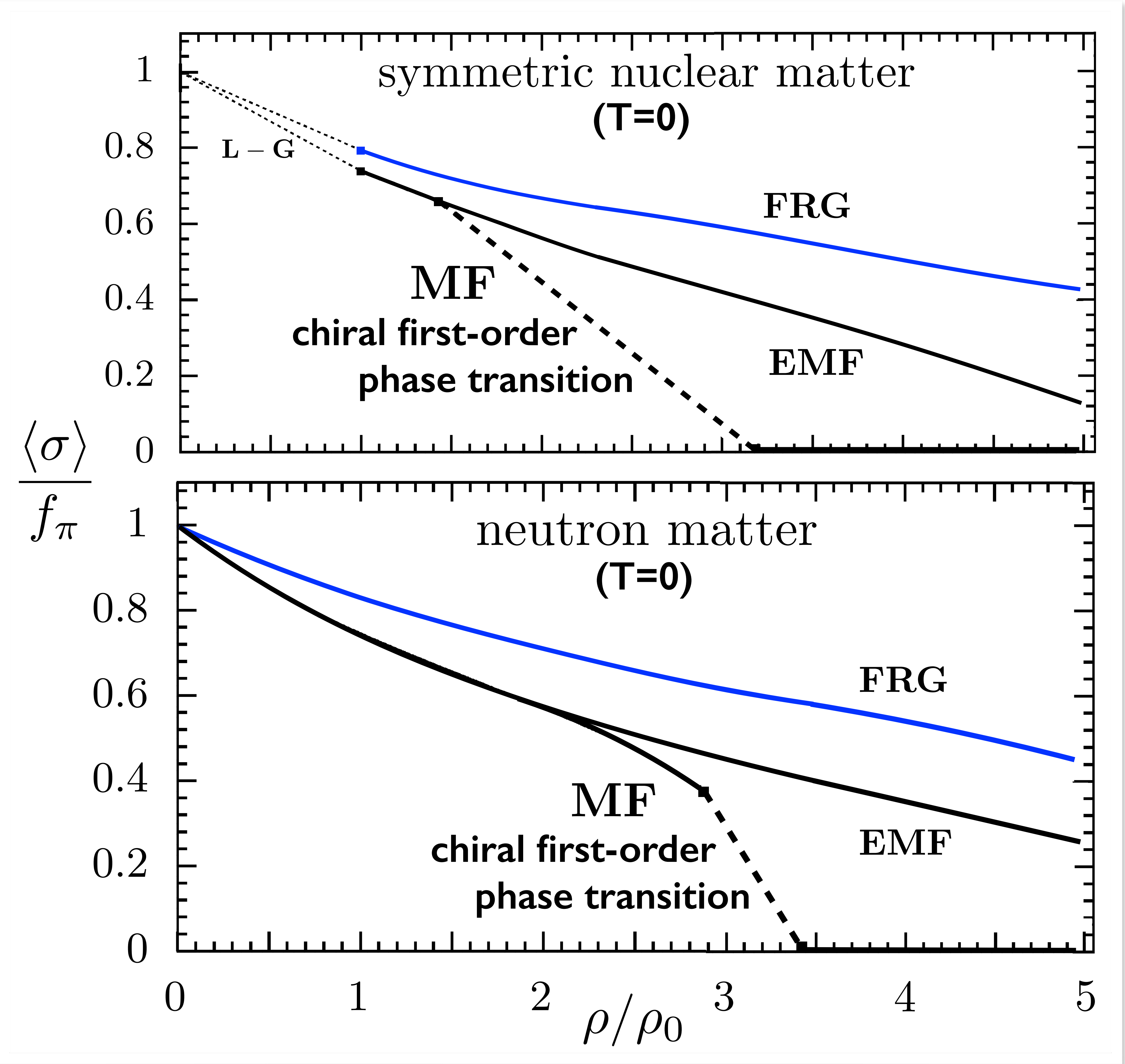
→ **Strongly repulsive correlations** including **many-body forces** with $n \geq 2$

# CHIRAL PHASE TRANSITION in DENSE BARYONIC MATTER ?

- **Studies in chiral nucleon-meson field theory**

M. Drews, W.W.: Prog. Part. Nucl. Phys. 93 (2017) 69 — L. Brandes, N. Kaiser, W.W.: Eur. Phys. J. A57 (2021) 243

- **Mean-field** approximation **(MF)**
  **chiral first-order phase transition**
  at baryon densities $\rho \sim 2 - 3\,\rho_0$
- **Vacuum fluctuations** (EMF)
  shift chiral transition to **high density**
  → **smooth crossover**
- **Functional Renormalisation Group** (FRG)
  with **non-perturbative loop corrections**
  involving **pions** & **nucleon-hole** excitations :
  → further enhancement of **stabilising effects**



**Chiral crossover transition at $\rho > 6\,\rho_0$**
**far beyond core densities in neutron stars**

# CONCLUSIONS

- **Constraints on phase transitions in neutron star matter**
  - **very stiff equation of state** implied by Bayesian inference results
  - **strong 1st order transition** unlikely in neutron star cores
  - **central baryon densities** in neutron stars : $\rho_c < 5\,\rho_0$
  - **chiral phase transition** shifted to **crossover** beyond $\rho > 6\,\rho_0$

- **Scenarios for cold dense matter in neutron stars**
  - **hadron-quark** continuity

    e.g. two-scales scenario: soft-surface delocalisation (percolation) followed by hard-core deconfinement at densities above $\rho_c$
  - neutron-dominated **baryonic** matter

    e.g. relativistic Fermi liquid featuring strongly repulsive **many-body forces** between **baryonic quasiparticles**

# Supplementary Materials

# INFERENCE of SOUND SPEED and RELATED PROPERTIES of NEUTRON STARS

- Introduce **general parametrization of sound velocity** by segment-wise representation :

$$c_s^2(\varepsilon,\theta) = \frac{(\varepsilon_{i+1}-\varepsilon)c_{s,i}^2 + (\varepsilon-\varepsilon_i)c_{s,i+1}^2}{\varepsilon_{i+1}-\varepsilon_i} \;, \quad \text{parameter set} \quad \theta = (c_{s,i}^2, \varepsilon_i) \;\; (i = 1,\ldots,N)$$

- Constrain parameters $\theta$ by **Bayesian inference** using **nuclear** and **astrophysical data** $\mathscr{D}$ :

$$\boxed{\Pr(\theta|\mathscr{D}) \propto \Pr(\mathscr{D}|\theta)\,\Pr(\theta)}$$

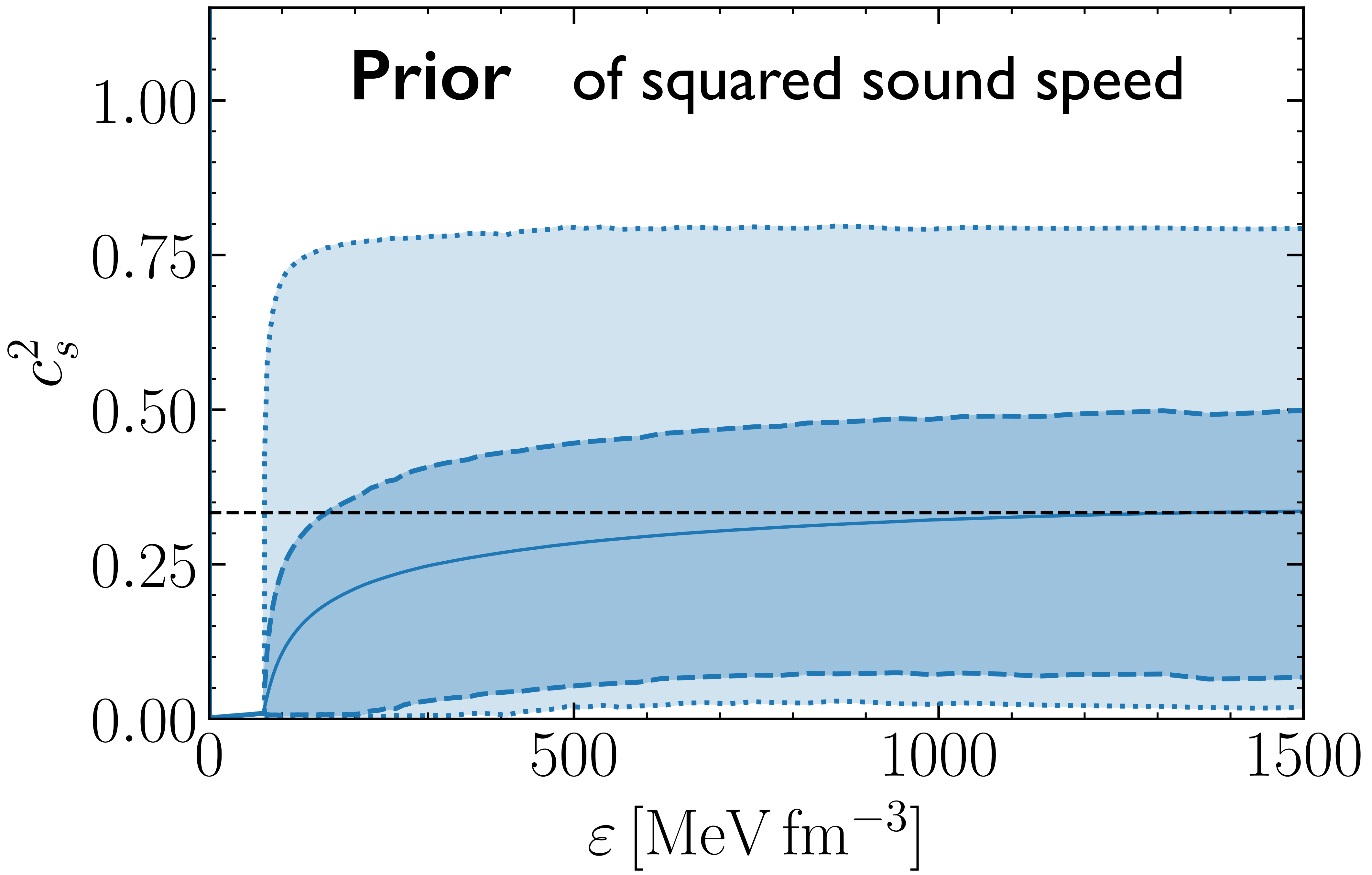
→ Choose **Prior** $\Pr(\theta)$

→ Compute **Posterior** $\Pr(\theta|\mathscr{D})$

from **Likelihood** $\Pr(\mathscr{D}|\theta)$

→ Quantify **Evidences** for hypotheses $H_0$ vs. $H_1$

in terms of **Bayes factors** $\mathscr{B}^{H_1}_{H_0} = \dfrac{\Pr(\mathscr{D}|H_1)}{\Pr(\mathscr{D}|H_0)}$

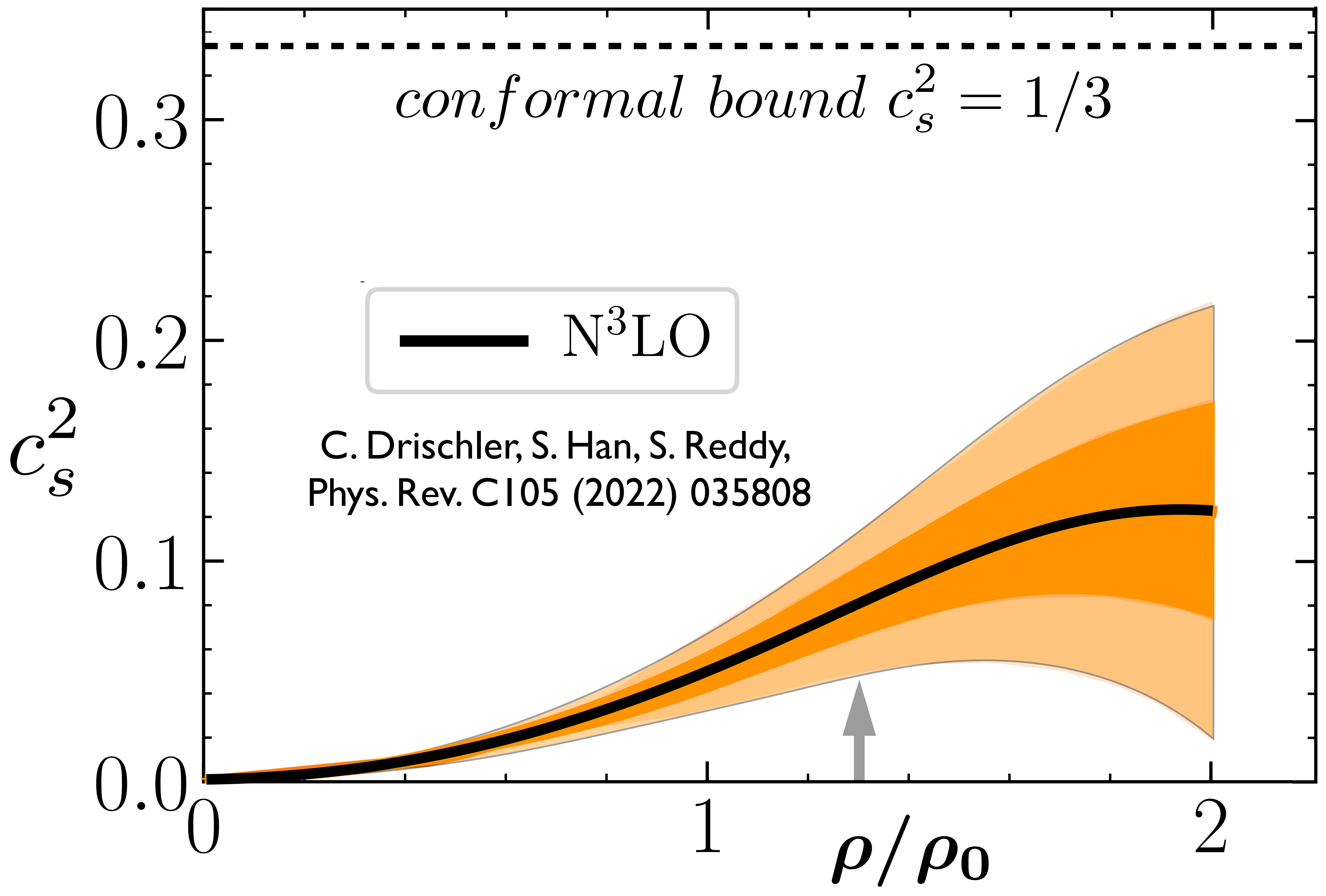


**Prior** of squared sound speed

# EQUATION of STATE and SOUND VELOCITY
## boundary conditions

- **Low densities :**

  **Chiral EFT @ $\rho \lesssim 2\,\rho_0$**

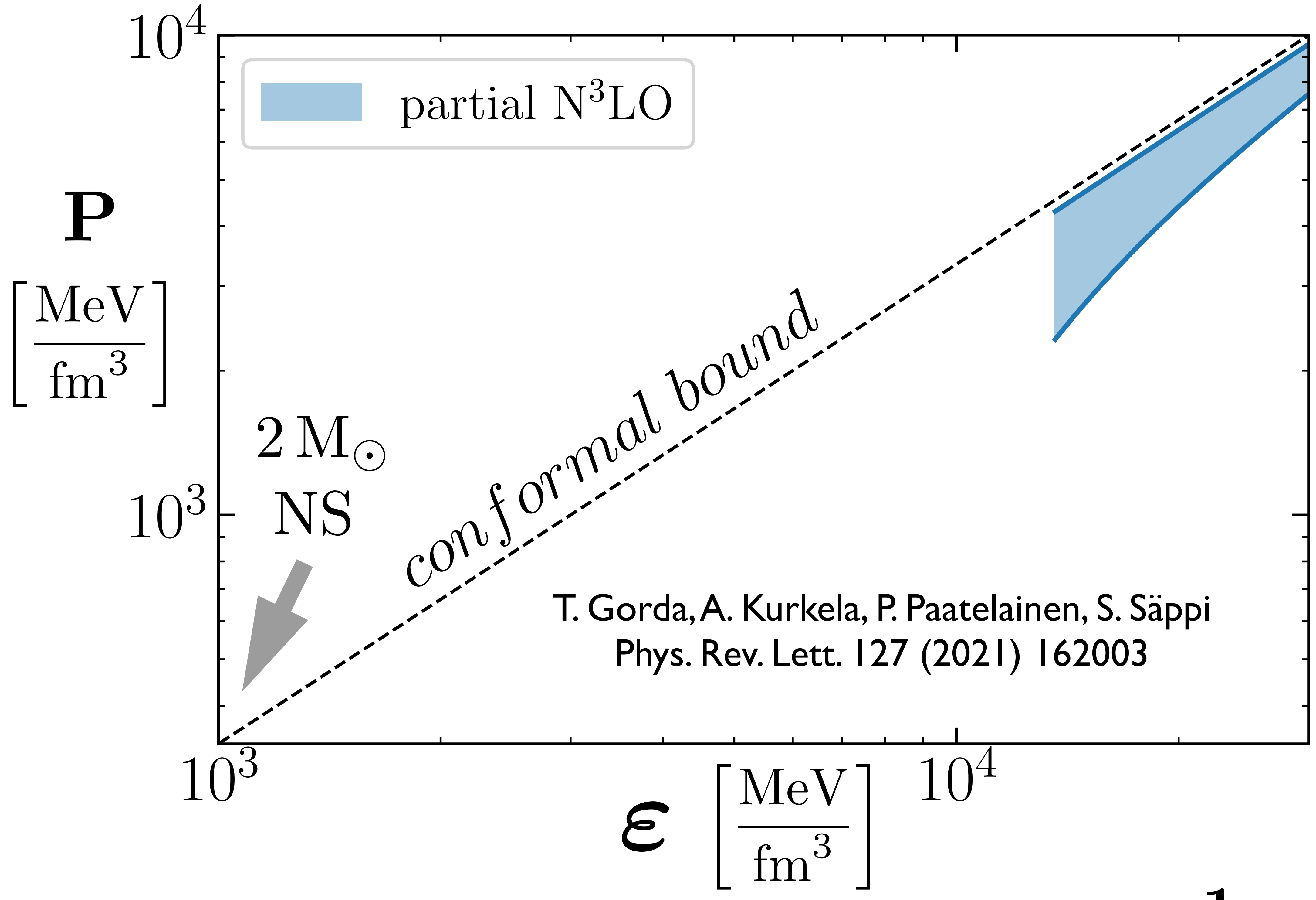


C. Drischler, S. Han, S. Reddy,
Phys. Rev. C105 (2022) 035808

- Employ ChEFT constraint at $\rho = 1.3\,\rho_0$ in Bayes inference as **Likelihood, not Prior**

- **Extremely high densities :**

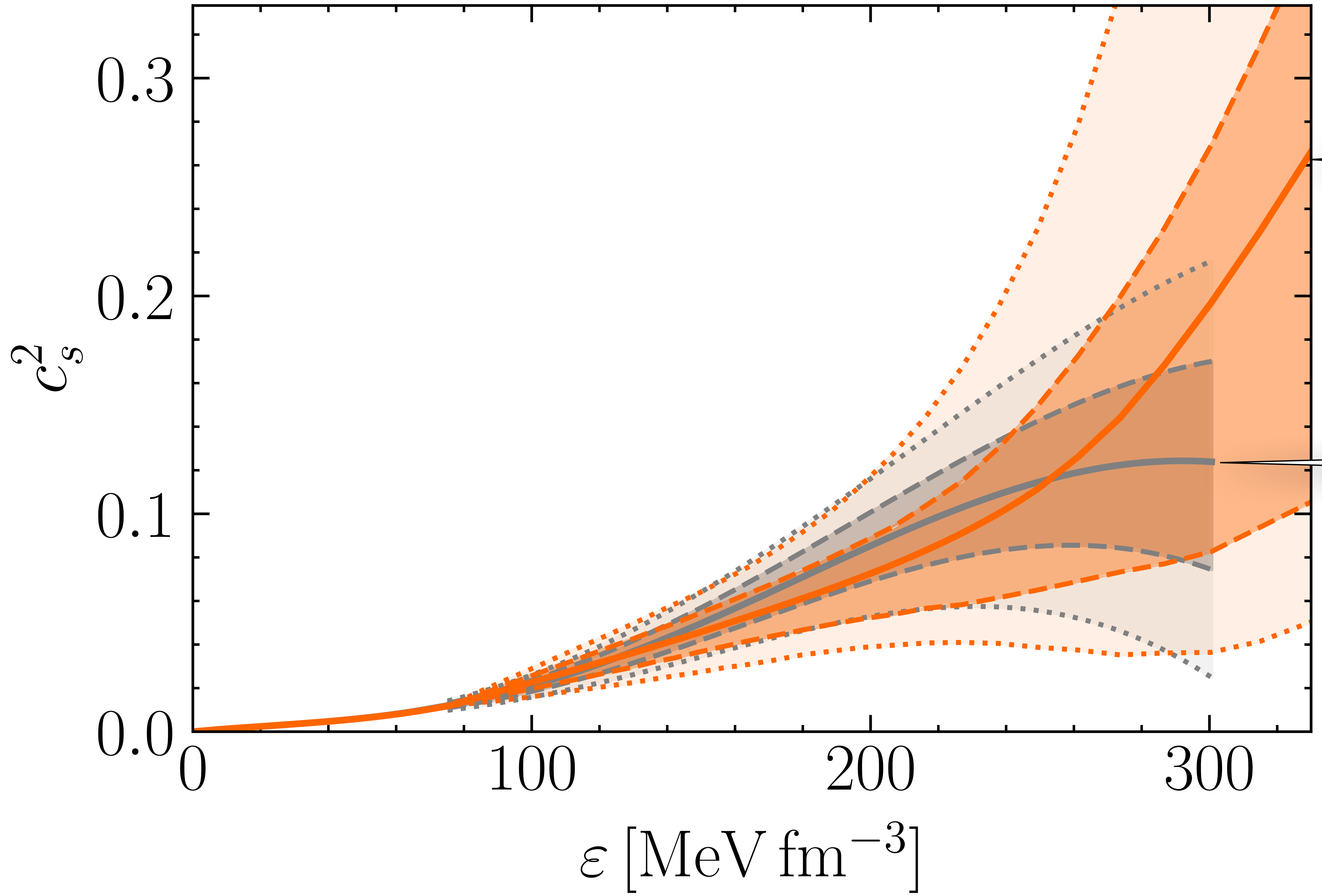
  **Perturbative QCD**



T. Gorda, A. Kurkela, P. Paatelainen, S. Säppi
Phys. Rev. Lett. 127 (2021) 162003

- **Conformal bound** $\mathbf{c_s^2 = \frac{1}{3}}$ reached asymptotically

# **EQUATION of STATE** and **SOUND VELOCITY**
## boundary conditions



**Bayesian - inferred squared sound speed**

L. Brandes, W.W., N. Kaiser
Phys. Rev. D 108 (2023) 094014

**Chiral EFT constraint**

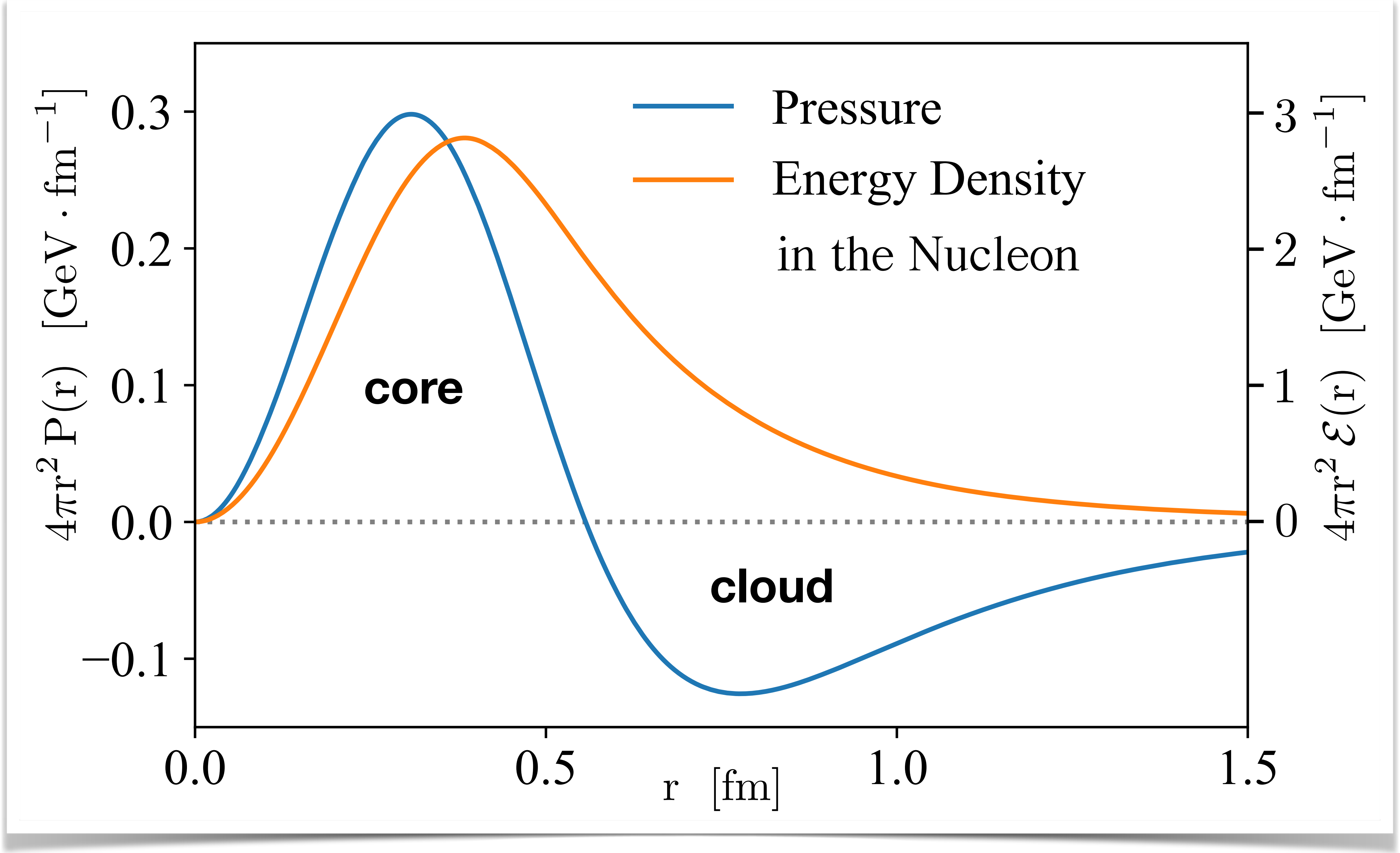
C. Drischler, S. Han, S. Reddy,
Phys. Rev. C105 (2022) 035808

- **Convergence issue in chiral EFT at densities** $\rho \sim 2\,\rho_0$

# COLD MATTER at EXTREME DENSITIES
## Hadron - Quark Continuity

K. Fukushima, T. Kojo, W.W. : Phys. Rev. D102 (2020) 096017

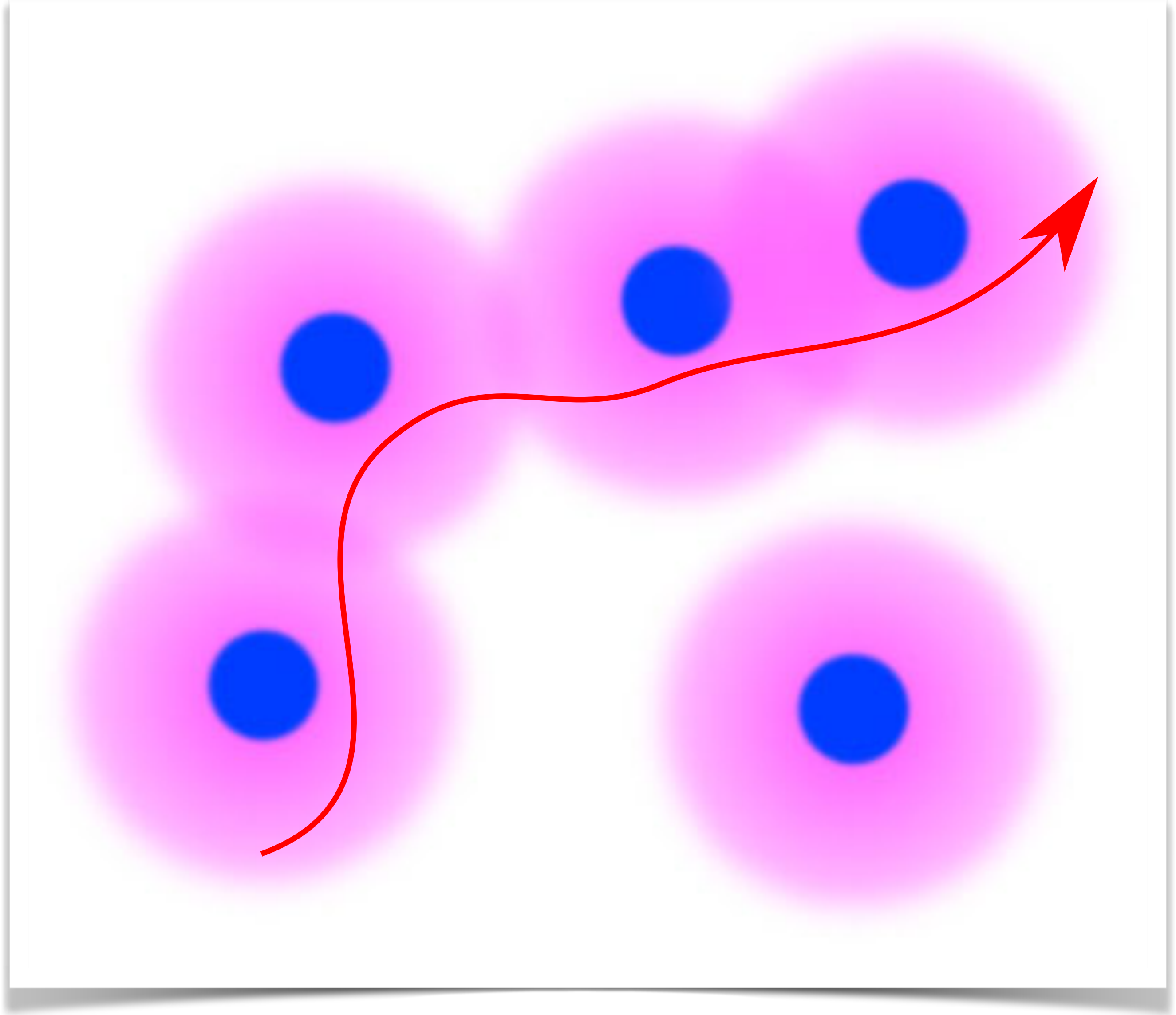
- Nucleonic scales : **HARD-CORE** deconfinement + **SOFT SURFACE** delocalisation



- Nucleon cores touch at baryon densities

$$\rho_B \sim 6\,\rho_0$$

- Percolation of mesonic clouds at lower densities inducing many-body correlations



F. Karsch, H. Satz
Phys. Rev.
D21 (1980) 1168

- Soft delocalisation and collective mobility of quark-antiquark pairs over larger distances

- No (first-order) phase transition expected at densities relevant to neutron stars