Correlations and Semi-Universal Relations Connecting Nuclear Matter and Neutron Stars

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Recent Collaborators:

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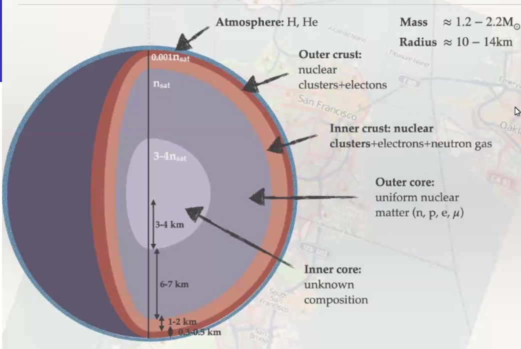
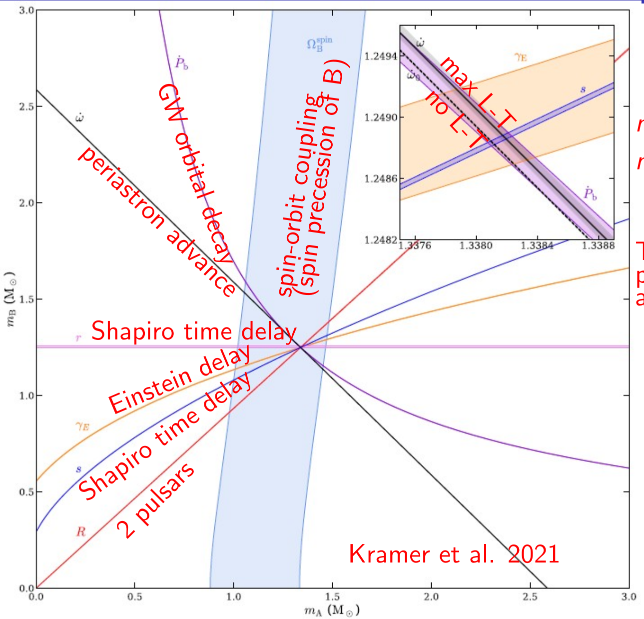
Neutron Stars: Basics



- Nearly all known NSs are pulsars (rapidly rotating and highly magnetized) that emit X-ray, optical or radio beams from their poles, like a lighthouse.
- The radii of most NSs are about 12 km.
- Most, if not all, NSs are formed in the gravitational collapse of massive stars at the ends of their lives; some of those collapses produce black holes instead. Some massive NSs may be formed in the aftermath of a binary merger of two lower-massed neutron stars.
- The minimum possible NS mass is $0.1 M_\odot$, but none are observed to be less massive than $1 M_\odot$.

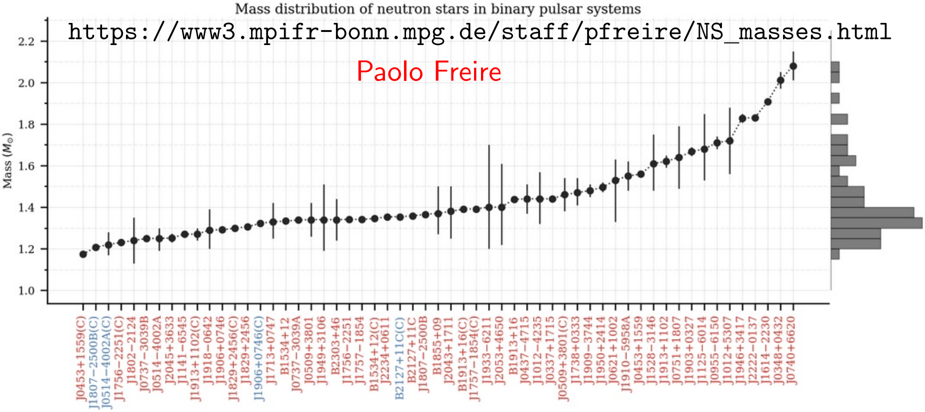
Pulsar Timing for PSR J0737-3039



$m_A = 1.338185^{+12}_{-14}\ M_\odot$
$m_B = 1.248868^{+13}_{-11}\ M_\odot$

These are the most precisely known masses of any astronomical objects.

Masses of Pulsars in Binaries from Pulsar Timing



https://www3.mpifr-bonn.mpg.de/staff/pfreire/NS_masses.html

Paolo Freire

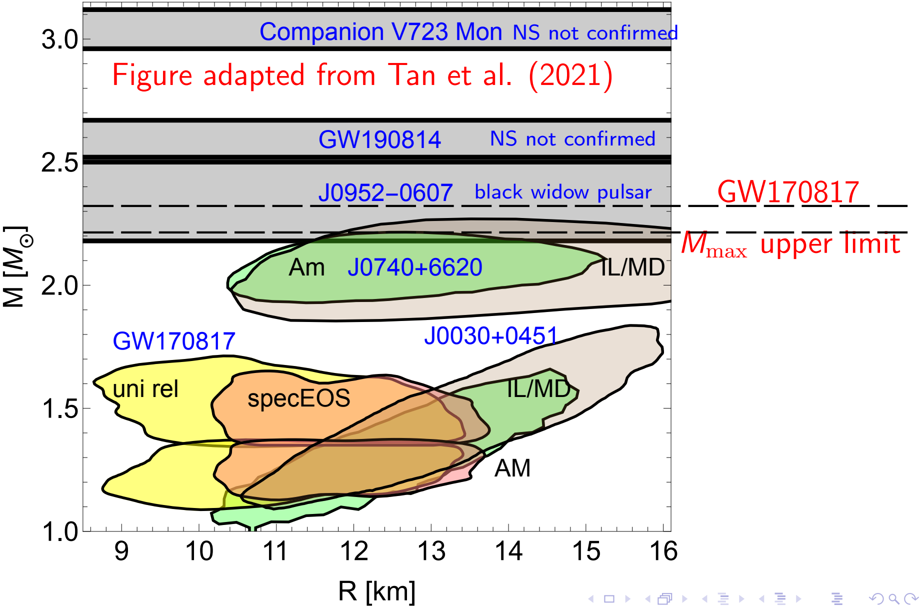
Largest: 2.08 ± 0.07 $M_\odot$
Smallest: 1.174 ± 0.004 $M_\odot$

Several other NS masses have been measured by other means, including some estimated to be more than $2M_\odot$ (e.g., black widow pulsars) and smaller than $1M_\odot$ (HESS J1731-347), but their mass uncertainties are generally large.

How Can a Neutron Star's Radius Be Measured?

- Flux $= \frac{\text{Luminosity}}{4\pi D^2} = \frac{4\pi R^2 \sigma_B T_s^4}{4\pi D^2} = \left(\frac{R}{D}\right)^2 \sigma_B T_s^4$
  X-ray observations of quiescent neutron stars in low-mass X-ray binaries measure the flux and surface temperature T_s. Distance D somewhat uncertain; GR effects introduce an M dependence.
- $F_{Edd} = \frac{GMc}{\kappa D^2}$ X-ray observations of bursting neutron stars in accreting systems measure the Eddington flux F_{Edd}. κ is the poorly-known opacity; GR effects introduce an R dependence.
- X-ray phase-resolved spectroscopy of millisecond pulsars with nonuniform surface emissions (hot spots). NICER: PSR J0030+0451, PSR J0437-4715 (closest and brightest millisecond pulsar) and PSR J0740+6620 (most massive pulsar).
- $R_{1.4} = (11.5 \pm 0.3) \frac{\mathcal{M}}{M_\odot} \left(\frac{\tilde{\Lambda}}{800}\right)^{1/6}$ km, $\qquad \mathcal{M} = \frac{(M_A M_B)^{3/5}}{(M_A + M_B)^{1/5}}$
  GW observations of neutron star mergers measure the chirp mass $\mathcal{M}$ and binary tidal deformability $\tilde{\Lambda}$ (GW170817).
- $I \propto M_A R_A^2$ Radio observations of extremely relativistic binary pulsars measure masses M_A, M_B and moment of inertia I_A from spin-orbit coupling [PSR J0737-3039 ($P_b = 0.102$d), PSR J1757-1854 (0.164 d), PSR J1946+2052 (0.078 d)].

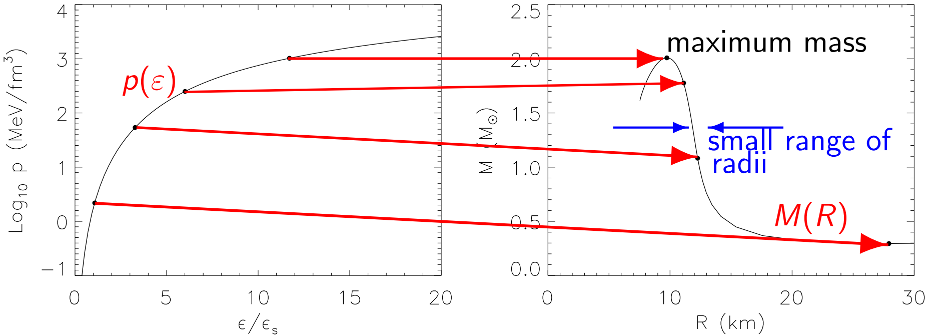
Summary of Astrophysical Observations



Neutron Star Structure

Tolman-Oppenheimer-Volkov equations

$$\frac{dp}{dr} = -\frac{G}{c^4}\frac{(mc^2+4\pi pr^3)(\varepsilon+p)}{r(r-2Gm/c^2)}$$
$$\frac{dm}{dr} = 4\pi\frac{\varepsilon}{c^2}r^2$$



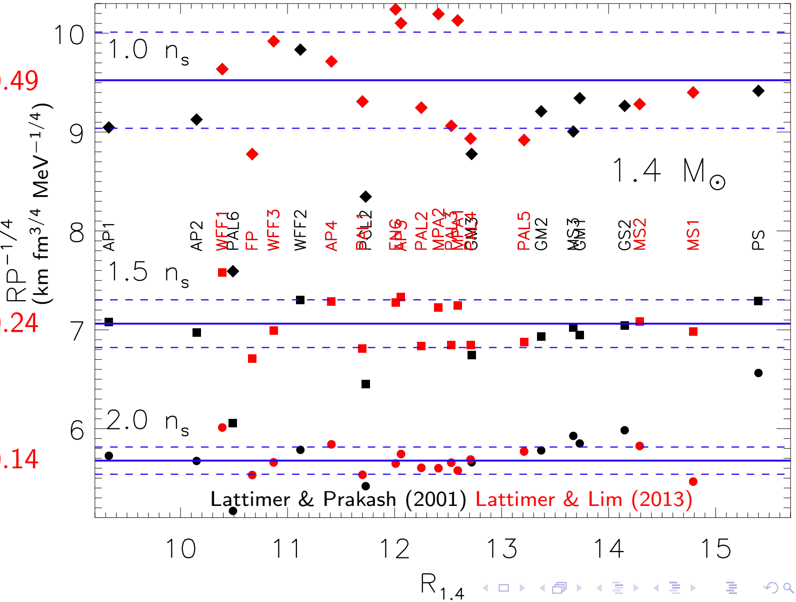
Equation of State ← Observations

The Radius – Pressure Correlation

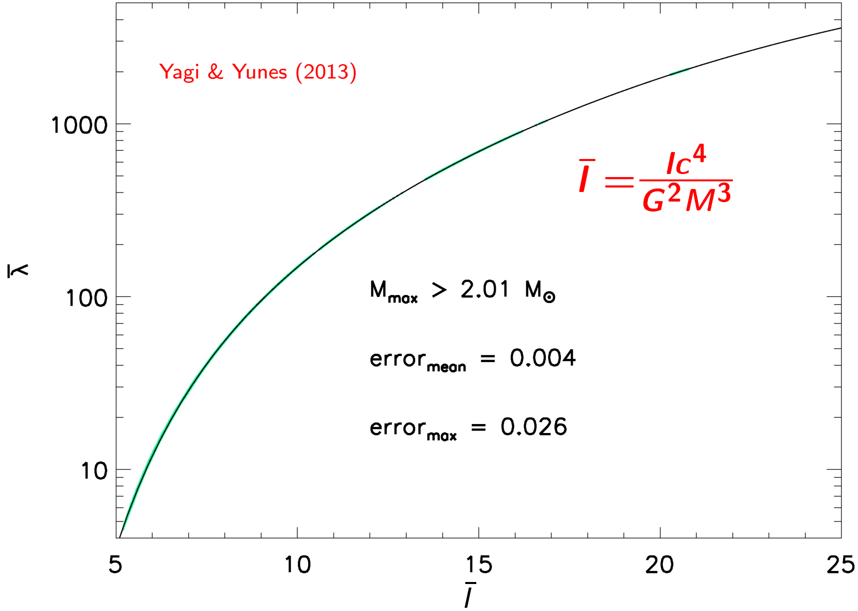
9.52 ± 0.49

7.06 ± 0.24

5.68 ± 0.14



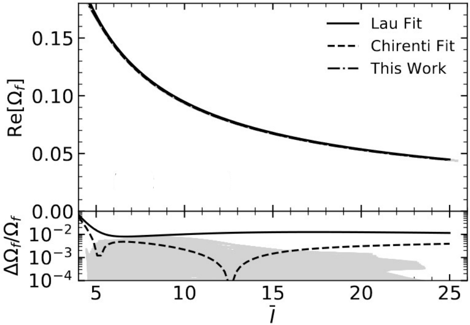
Tidal Deformatibility - Moment of Inertia



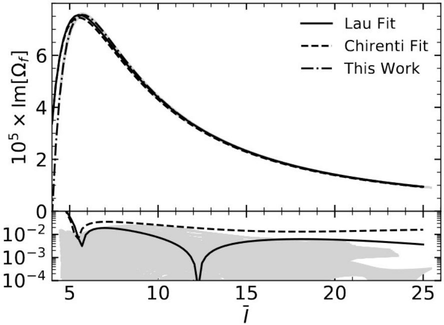
F-Mode Properties - Moment of Inertia

$$\Omega_f = \frac{GM\omega_f}{c^3}$$

Zhao & Lattimer 2022

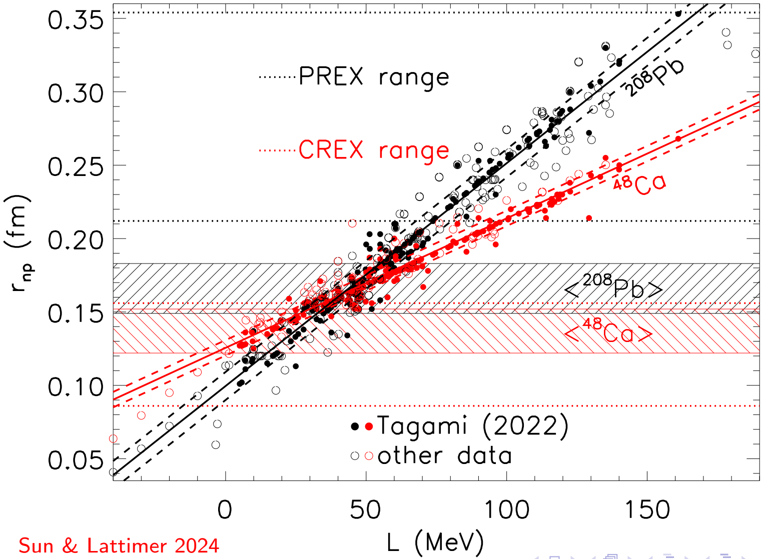


frequency



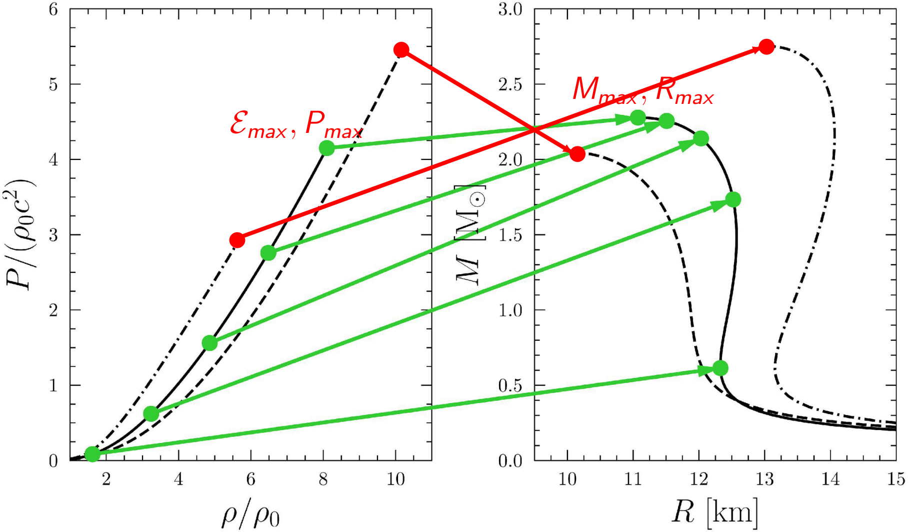
damping time

Neutron Skin Thickness - L

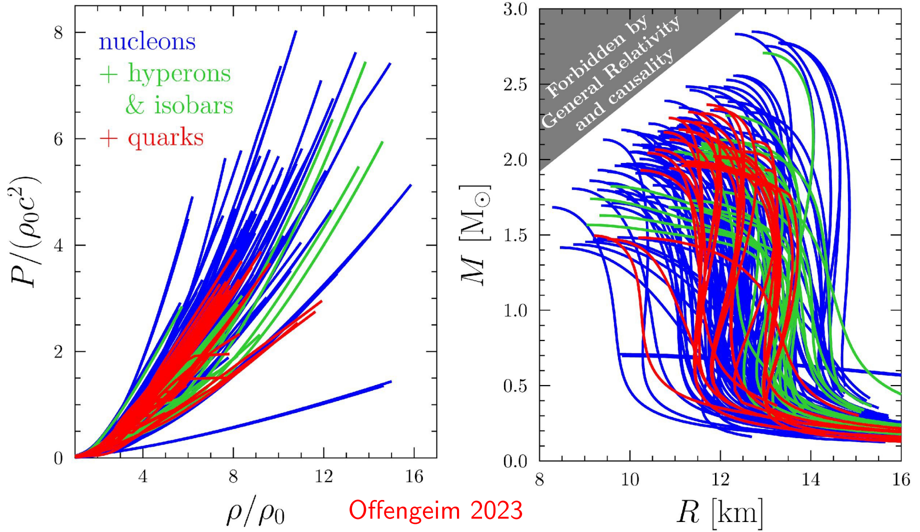


Sun & Lattimer 2024

Maximum Mass As a Unique Scaling Point



Varying the EOS



Offengeim 2023

$M_{\max}, R_{\max}, \mathcal{E}_{\max}, P_{\max}$ Correlations

• Ofengeim(2020) fitted $\mathcal{E}_{max}$ and P_{max} with the functions

$$\mathcal{E}_{max}, P_{max} \simeq \left[\frac{a_{\mathcal{E},P}}{R_{max}\cos\phi_{\mathcal{E},P} + (GM_{max}/c^2)\sin\phi_{\mathcal{E},P} + d_{\mathcal{E},P}}\right]^{s_{\mathcal{E},P}}$$

with accuracies of about 3% and 8%, respectively.

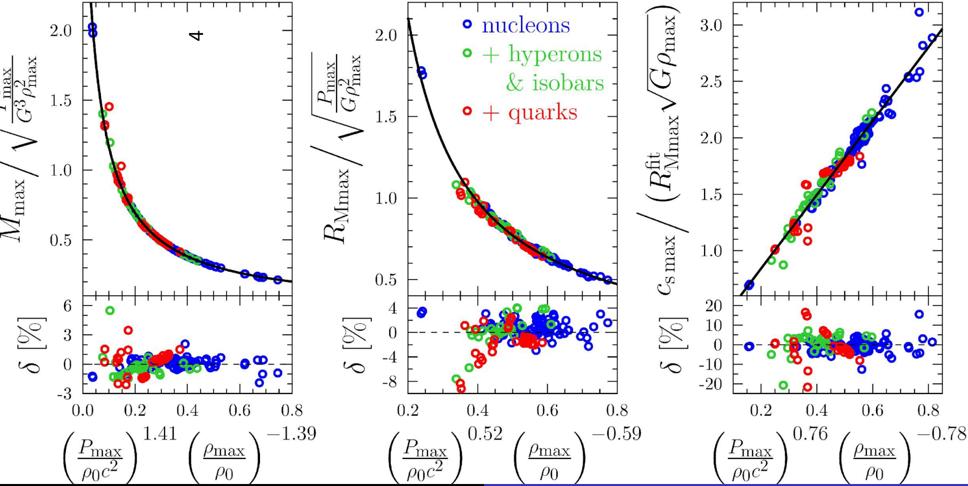
• Cai, Li and Zhang (2023) found a perturbative solution of the TOV equations in the parameter $x = P_c/\mathcal{E}_c$:

$$R \simeq \sqrt{\frac{3c^2}{2\pi G\mathcal{E}_c}}\left[\frac{x}{1+4x+3x^2}\right]^{1/2},$$
$$M \simeq \sqrt{\frac{54c^6}{\pi G^3\mathcal{E}_c}}\left[\frac{x}{1+4x+3x^2}\right]^{3/2}.$$

At M_{max}, accuracies are 7% and 8%; at $1.4M_\odot$, they are 2% and 6%.

- Ofengeim et al. (2023) suggested fits for M_{max}, R_{max}:

$$M_{max} \simeq \frac{a_M \mathcal{E}_{max}^{-1/2} x^{3/2}}{b_M + c_M x^{p_M} \mathcal{E}_{max}^{q_M}}, \quad R_{max} \simeq \frac{a_R \mathcal{E}_{max}^{-1/2} x^{1/2}}{b_R + c_R x^{p_R} \mathcal{E}_{max}^{q_R}}$$



$M_{\rm max}$, $R_{\rm max}$, $\mathcal{E}_{\rm max}$, $P_{\rm max}$ Correlation

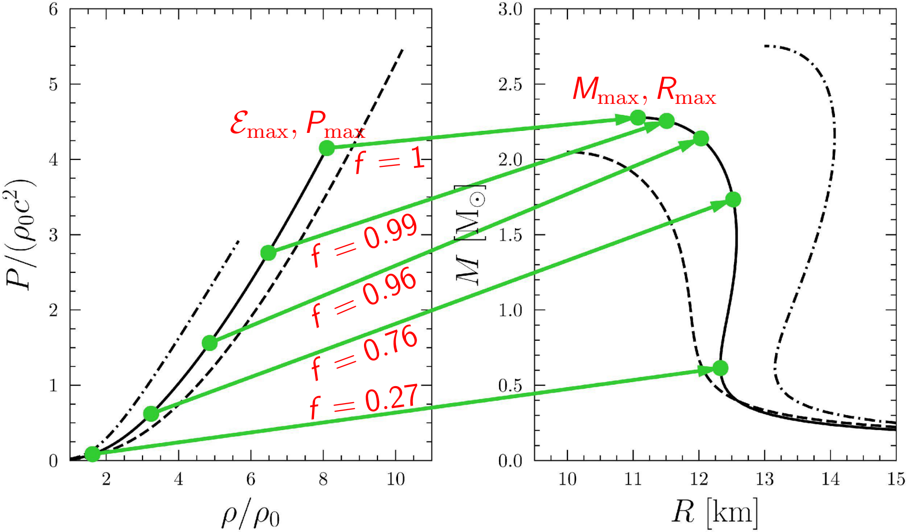
Ofengeim et al's finding suggest the power-law relations

$$\begin{aligned}
\mathcal{E}_{\rm c,max} &= (1.809 \pm 0.36)\left(\frac{R_{\rm max}}{\rm 10km}\right)^{-1.98}\left(\frac{M_{\rm max}}{M_\odot}\right)^{-0.171} {\rm GeV~fm}^{-3}, \\
P_{\rm c,max} &= (118.5 \pm 6.2)\left(\frac{R_{\rm max}}{\rm 10km}\right)^{-5.24}\left(\frac{M_{\rm max}}{M_\odot}\right)^{2.73} {\rm MeV~fm}^{-3},
\end{aligned}$$

We find, in addition, that additional points along the $M - R$ curve, at $M = fM_{\rm max}$, have similarly accurate correlations:

$$\begin{aligned}
\mathcal{E}_{c,f} &= a_{\mathcal{E},f}\left(\frac{R_{fM_{max}}}{\rm 10km}\right)^{b_{\mathcal{E},f}}\left(\frac{M_{max}}{M_\odot}\right)^{c_{\mathcal{E},f}}, \\
P_{c,f} &= a_{P,f}\left(\frac{R_{fM_{max}}}{\rm 10km}\right)^{b_{P,f}}\left(\frac{M_{max}}{M_\odot}\right)^{c_{P,f}},
\end{aligned}$$

Correlations at $M = fM_{\max}$



Correlations with $fM_{\rm max}$ and 2 R_f Values

$$\mathcal{E}_f = a_{\mathcal{E},f}\left(\frac{R_{f_1}}{10{\rm km}}\right)^{b_{\mathcal{E},f_1}}\left(\frac{R_{f_2}}{10{\rm km}}\right)^{c_{\mathcal{E},f_2}}\left(\frac{M_{max}}{M_\odot}\right)^{d_{\mathcal{E},f}},$$

$$P_f = a_{P,f}\left(\frac{R_{f_1}}{10{\rm km}}\right)^{b_{P,f_1}}\left(\frac{R_{f_2}}{10{\rm km}}\right)^{c_{P,f_2}}\left(\frac{M_{max}}{M_\odot}\right)^{d_{P,f}},$$

$f = M/M_{\rm max}$	$f_1 = M_1/M_{\rm max}$	$f_2 = M_2/M_{\rm max}$	$\Delta(\ln \mathcal{E}_f)$
1	1	1/2	0.0046
4/5	2/3	1/2	0.0036
2/3	2/3	3/5	0.0051
3/5	4/5	1/2	0.0025
1/2	2/3	1/2	0.0048
2/5	3/5	1/2	0.0047

$f = M/M_{\rm max}$	$f_1 = M_1/M_{\rm max}$	$f_2 = M_2/M_{\rm max}$	$\Delta(\ln P_f)$
1	1	1/2	0.019
4/5	4/5	1/2	0.0096
2/3	2/3	3/5	0.014
3/5	2/3	1/2	0.00069
1/2	3/5	1/2	0.020
2/5	3/5	1/2	0.032

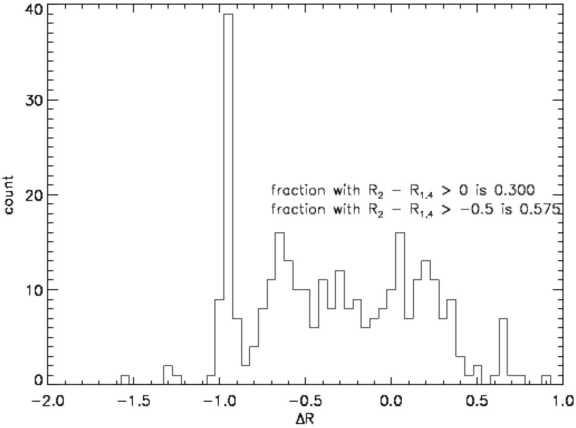
greatly reduced uncertaintites!

Importance of $\Delta R = R_{2.0} - R_{1.4}$

- J0030+0451: $R = 12.71^{+1.14}_{-1.19}$ km
- J0740+6620: $R = 12.39^{+1.30}_{-1.18}$ km

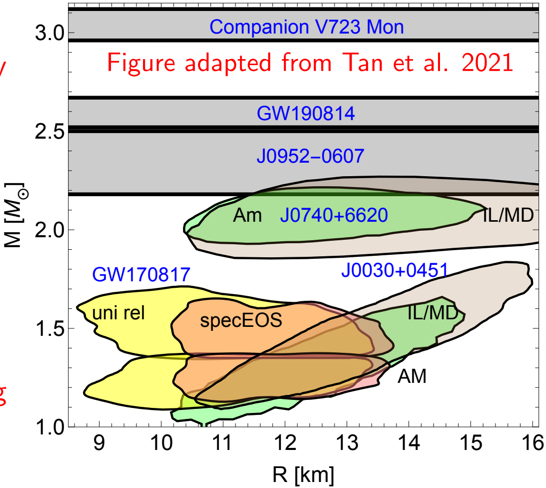
$\Delta R = -0.32^{+1.73}_{-1.68}$ km

313 Skyrme + RMF forces with $M_{max} \geq 2.0 M_\odot$



Applications

- Analytic method of directly inverting TOV equations. Accuracies can be made arbitrarily high (number of R_f values). Existing techniques use parameterized EOS models in probabilistic (Bayesian) approaches having unquantified systematic uncertainties stemming from the model choice and parameter ranges (prior distributions).



- Correlations of $c_{s,\max}$ with $M_{\max}$ and $R_{\max}$ also exist (Ofengeim et al. 2023 found $\sim 10\%$ uncertainty), but accuracies are improved using power law-relations with ≥ 2 R_f values. Accurate values for specific f values would be useful for interpolating within the $\mathcal{E}_f - P_f$ grid. They could also allow probing the composition of the neutron star interior (phase transitions, etc.).
- Another unique feature of the $M - R$ diagram is the value of $(d^2M/dR^2)_{max}$, which can be used in addition to M_{max} and R_{max} to improve accuracies.
- Correlations of $\tilde{\Lambda}, \bar{I}$ and BE$/M$ with M_{max} and R_{max} remain to be explored.